Targeting target shareholders*

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Abstract

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Keywords: *Heterogeneous valuations; Mergers & Acquisitions; Optimal takeover offers*

JEL codes: *G34, D83*

1 Introduction

Heterogeneity—in beliefs, derived utility, tastes, etc.—is a pervasive characteristic of many economic settings. Our analysis focuses on financial markets, where such heterogeneity is manifest: different investors attach very different valuations to stocks, and some investors value their shares in a firm far above the market price.[1] We explicitly integrate such investor heterogeneity into a theory of takeovers, building an equilibrium model that accounts for heterogeneous investors on both sides of the acquisition. We investigate how the management of an acquiring firm should design its takeover bid—its size and cash/equity structure—in light of its own private valuations, deriving the consequences for takeover premia paid, target and acquiring firm returns, likelihood of successful takeovers, post-takeover offer share price dynamics, and social welfare changes created by a takeover. Our model reconciles a broad set of empirical regularities, shedding light on some features of corporate acquisitions that are otherwise difficult to reconcile with existing theories, and it generates several new testable implications.

In our model, a potential acquirer develops a synergy with a target firm and would thus gain from acquiring it. An acquisition offer consists of either an amount of cash in exchange for a target shareholder's ownership interest, or an equity stake in the joint (merged) firm. To succeed, a takeover offer must win approval from a majority of shareholders. If the majority agrees to sell their shares, the target is absorbed by the acquirer, becoming a single entity.

We capture the existing lack of consensus about a firm's value by assuming that different shareholders and management hold different private valuations of their firms. This is in the spirit of the literature on disagreement and differences of opinion between investors (see Harris and Raviv, 1993 or Morris, 1996), and the large asset pricing literature that inte-

[1]Anecdotes indicating this can be drawn from messages on various financial chat rooms:

"I would rather see PolyMedica Corporation (PLMD) continue to operate as a stand-alone company than be taken over by something BIG in the near future. A takeover premium of let's say 20% would certainly be nice, but it's game over for us as stockholders in PLMD.... I have more faith in management producing higher returns than that!" 15-Feb-06 03:41 am. Yahoo Message Board.

—PLMD closed at $43.43 on February 15, 2006. On August 28, 2007 Medco Health Solutions announced it would buy PLMD in an all-cash deal worth $1.5 billion. The purchase price valued PLMD at $53 per share, a 22% premium for that (presumably) disappointed shareholder.

grates heterogeneity of investors' beliefs with collateral or leverage constraints (e.g., Fostel and Geanakoplos, 2014, Simsek, 2013) to study asset price dynamics.[2] An implication of this work is that, in equilibrium, investors' marginal utilities (i.e., private valuations) for holding assets differ across both shareholders and non-shareholders.[3] In fact, institutional investors often sharply disagree over what a firm's future earnings and, hence, future share prices will be. One manifestation of this is the radically different one-year target share prices set for the same stock by analysts for different institutional investors.[4]

A firm's share price is determined by the private valuation of its marginal shareholder, who values the firm the least among all shareholders. However, a successful takeover offer must win approval from the median target shareholder, who attaches a higher valuation to the firm when shareholders disagree over their firm's value. This effectively endows target shareholders with bargaining power: because successful takeover offers must be at a premium over the extant share price, the marginal shareholder extracts significant rents. Consistent with this prediction, takeover premia are often high even when there is no evidence of other interested buyers who might engage in a bidding war (see Andrade et al., 2001, or Betton et al., 2008, for a survey; Fishman, 1988, provides an early theoretical treatment).

We go beyond this simple prediction to analyze the acquiring firm's choice of whether to use cash or equity in its takeover bid. Unlike cash offers, equity offers require an acquiring firm's manager to cede some of his private valuation for his firm, but allow target shareholders to retain greater stakes in the target and, thus, more of their private valuations. That is, equity offers mandate a transfer of private values from the acquiring firm's management and

[2]Similarly, Garleanu and Pedersen (2011) and Chabakouri (2013) introduce heterogeneity in risk aversion and collateral constraints. Osambela (2015), building on Gallmeyer and Hollifield (2008), shows how feedback effects of funding illiquidity, disagreement, and market liquidity can reconcile empirical features of liquidity and financial asset prices (see also Xiong and Yan, 2015). These papers share our starting point—investor heterogeneity—but focus on asset pricing implications rather than the market for corporate control.

[3]Miller (1977), Chen, Hong, and Stein (2002), and Bagwell (1991) also employ frameworks based on differences of opinion to study issues related to shareholder ownership and tendering decisions.

[4]Inspection of share price targets reveals that for larger firms (e.g., with market caps exceeding \$50 Billion), which are potential acquirers, the range of price targets is roughly 35-40% of their share prices; and for smaller firms with market caps between \$100 million and \$6 billion that are potential targets, the range of disagreement over price targets typically exceeds the outstanding share price. Papers that document upward-sloping supply curves for shares (i.e., heterogeneity in investor valuations) in takeover contexts include Bagwell (1992), Bradley, Desai and Kim (1987) and Moeller, Schlingemann, and Stulz (2007).

shareholders to target firm shareholders. The optimal means of payment therefore hinges on the private valuation of the acquirer's manager relative to the median target shareholder's—cash is optimal when the acquirer's manager has a relatively high private valuation, while equity is optimal when his private valuation is low relative to that of the median target shareholder. Our prediction on the means of payment therefore emphasizes the *contrast* in private valuations of management at the acquirer and the *median* shareholder at the target.[5]

We establish that the return to the combined firm in a cash acquisition is at least as high as that in an otherwise identical equity acquisition. We then show that an acquirer's stock price can fall following an optimal equity offer, but not after an optimal cash offer. This reflects that the interests of the acquiring firm's management and its shareholders are aligned for cash offers, but not necessarily for equity offers. That is, management and shareholders value cash similarly, so a cash offer that appeals to an acquirer's manager also appeals to its shareholders. In contrast, with heterogeneous valuations, an acquirer's manager values equity offers differently from its shareholders, and when the manager's private valuation is lower than those of shareholders, he may make an equity offer that acquirer shareholders do not like. Indeed, since equity offers are attractive to an acquirer's manager precisely when his private valuation is lower than the median shareholder's at the target firm, a decline in the acquirer's stock price upon a successful equity offer is a likely outcome.

We also show that the combined acquirer-target return in an equity acquisition can be negative even when synergies are positive. The intuition reflects that pre-merger, investors hold the firms they value most, but when firms merge, investors must hold both firms, diluting their claims to their preferred firms. Building on this intuition, we show that the return on the combined firm is a poor proxy for shareholder welfare that typically overstates welfare gains or understates welfare losses. This welfare bias reflects that combined firm returns do not capture the greater dilution losses of shareholders with higher private valuations.

[5]Gorbenko and Malenko (2014) also study the choice of the means of payment—cash or equity—in the context of a firm being auctioned to competing bidders. They suppose that an acquirer's synergy with the target is private information. This makes it costly for a high-synergy bidder to separate from a low-synergy bidder via equity offers since the cash-equivalent value is higher for a high-synergy bidder. As a result, bidders only use equity when cash constrained. In contrast, our model features no private information, and differences betwen cash and equity are driven by investors' heterogeneous valuations.

We then investigate the implications of the fact that a target's share price only reveals the private valuation of its *marginal* shareholder, leaving uncertainty about the *median* valuation. We show that if synergies are high then increased uncertainty about this median valuation—i.e., greater uncertainty about how high an offer must be to succeed—causes an acquirer to raise its offer in order to reduce the likelihood of a rejection that would lead to the synergies going unrealized. If, instead, synergies are small, increased uncertainty leads the acquirer to reduce its offer, since low offers are more likely to be accepted, and having an offer fail is less costly. Thus, whether greater uncertainty about target shareholder valuations raises or reduces offers hinges on the size of synergies.

A corollary of these findings is that offers sometimes fail even when synergies are large enough that both the median target shareholder and the acquirer could benefit from an acquisition. Likewise, takeover bids may be rejected by target shareholders even though they understand that rejection will reduce their share price relative to what would obtain if they approve the offer. We predict that a target's share price should always rise following a takeover offer that is possibly attractive to its median shareholder. The target's share price will rise further if a takeover succeeds, reflecting the beneficial resolution of takeover uncertainty, but fall if it fails. By contrast, an acquirer's share price will move in the same direction after a successful takeover as it moved after the announcement of the takeover offer. If, instead, a takeover offer fails, both share prices will return to their original levels.

Our paper makes two main contributions: First, we provide an intuitive way to think about the stylized facts of takeovers that is based on a feature—investor heterogeneity—that is recognized to be an important component of financial markets. This perspective allows us to rationalize a large set of stylized facts. Importantly, our model generates testable predictions that distinguish it from the literature on corporate takeovers motivated by moral hazard or asymmetries of information (see Betton, et al., 2008, for a survey).

For example, Andrade et al. (2001) find that market reactions to an acquirer's cash acquisitions are positive, but those to equity acquisitions are mostly negative.[6] In our setting, this

[6] So, too, Moeller, Schlingemann and Stulz (2005) find that around acquisition announcements, acquiring-firm shareholders lose 12 cents per dollar spent on acquisitions.

possible drop in market assessment of the acquirer reflects lower private valuations of management than shareholders and hence misaligned interests in equity offers—shareholders care more about the dilution of their claims to their preferred firm following an equity takeover. Acquirer returns can be negative when the synergies driving an acquisition are too small to overcome this misalignment (as long as the offer does not require acquiring firm shareholder approval, as is the case for most equity offers in the US). Theories based on moral hazard argue that the acquirer's share price may decline because the acquiring firm's manager may, for example, pursue value-reducing mergers in order to derive private benefits of control. To the best of our knowledge, such models do not have the differential implications between equity and cash offers that arise naturally in our model due to shareholder heterogeneity.

Asymmetric-information-based theories can also reconcile the decline in an acquirer's share price after an equity offer. In essence, an acquirer makes an equity offer when its CEO has private information that his firm is overvalued, and the equity offer reveals the assessed overvaluation. As a result, the acquirer's share price could fall due to the bad news revealed even though the equity offer is in the best interests of its shareholders. However, the subsequent predicted share price dynamics differ from ours: asymmetric-information-based theories predict that, as long as synergies are positive or target shareholder approval reflects a positive assessment by target shareholders, an acquirer's share price should rise when an offer is accepted and fall when a takeover fails. Findings in Savor and Lu (2009) are consistent with our model but not with asymmetric-information-based models: after an announcement that a takeover failed for exogenous reasons, the acquirer's share price *rises*, returning on average to the price when the takeover was first announced. Moreover, our model can reconcile the findings of Becht, Polo, and Rossi (2014) that an acquiring firm's share price rises after equity offers that require approval from its shareholders, but that it falls when approval is not required. In contrast, asymmetric-information-based arguments would not predict such a rise in the acquirer's share price when acquiring firm shareholder approval is required.

Rather than solely being "wealth destruction," our model suggests that the observed negative acquirer and combined acquirer-target returns just reflect what happens to the valuations of marginal shareholders. Relatedly, we can reconcile the more nuanced prediction of a "diversification discount" found by Berger and Ofek (1995), Lamont and Polk (2001), and Graham et al. (2002)—mergers between less-related firms are associated with lower returns. In our setting, the diversification discount arises naturally because shareholder valuations differ by more when the target and acquirer come from less-related industries, implying greater loss from dilution of shareholder valuations.

Our second key contribution is to provide novel insights into the market for corporate control. Our model provides a structure for endogenizing the choice of how a takeover is financed, and the consequences of this decision. It also provides a new perspective on the welfare consequences of acquisitions and derives new testable implications about how outcomes are affected when an acquirer is more uncertain about the median target shareholder's valuation. In addition, we provide fresh insights into why the combined acquirer-target return in a cash acquisition exceeds that in an equity acquisition. Our model suggests that a key driver of this empirical regularity is the interaction of two asymmetries—a size asymmetry (the acquirer is larger than the target in most takeovers) and an ownership asymmetry (the joint firm is held entirely by acquiring firm shareholders just after a cash acquisition, whereas it is held jointly by shareholders from both firms in an equity acquisition). We then derive the novel empirical implication that ceteris paribus, the difference in the combined acquirer and target returns between cash and equity acquisitions should rise with the size difference between the acquirer and the target. In sum, many predictions of our model are unique and stem largely from the simple premise that shareholders value their shares differently. This suggests that models incorporating such heterogeneity can help researchers better understand the dynamics of the takeover process.

Our paper is not the first to study the implications of heterogeneous shareholder valuations for acquisitions. Moeller, Schlingemann, and Stulz (2007) and Chatterjee, John, and Yan (2012) offer empirical analyses of stock market reactions to takeovers that are motivated by informal models featuring heterogeneous shareholder valuations. Focusing on bidding firms, Moeller et al. (2007) argue that greater diversity of opinion about a bidder's prospects leads to a steeper downward-sloping demand curves for its stock, resulting in lower announcement returns following its equity offer. Focusing on target firms, Chatterjee et al.

(2012) appeal to heterogeneity to justify a downward sloping demand curve for their shares, for which they find empirical support. Both of these papers reflect an increasing interest in using differences of opinion to explain takeover returns, and help explain some important underlying patterns. Our model goes beyond these analyses to provide a rigorous theoretical analysis. We incorporate investors on both sides of the takeover simultaneously, taking into account that with heterogeneous investors, not only do a firm's shareholders disagree on their firm's value, but shareholders also have higher valuations than non-holders, reflecting that investors hold stocks they deem "undervalued". We build an equilibrium model in which we derive equilibrium prices via market-clearing conditions, taking into account the valuations, wealth dynamics, and optimizing behavior of all parties to derive takeover outcomes and to distinguish between cash and equity offers. This formal modeling delivers a rich set of new implications that allow us to distinguish models based on heterogeneous valuations from competing theories.

We next present the model, and analyze optimal equity and cash offers. We then study which type of offer the acquiring firm finds optimal, and derive the consequences for market reactions. In Section 4 we present a number of novel (and testable) empirical predictions, and analyze how the extent of uncertainty about the median shareholder's valuation affects offers, probability of success, and share price movements following announcement and shareholder vote. Appendix A contains additional details of the analysis. Appendix B contains all proofs.

2 Base Model

Firms and Investors. The economy features a potential acquirer firm A and a potential takeover target T. We normalize each firm to have one share outstanding. Our base model focuses on two groups of risk-neutral investors who differ in their private valuations for the two firms. One group of investors consists of types $\epsilon_A \in [0, \infty)$ who place values $V_A + \epsilon_A$ on firm A and V_T on firm T; the other group of investors consists of types $\epsilon_T \in [0, \infty)$ who place values $V_T + \epsilon_T$ on firm T and V_A on firm A.[7] Thus, V_j represents a common component

[7]Our assumption that all acquiring-firm shareholders value the target at V_T and all target shareholders value the acquirer at V_A is designed to capture the fact that even within narrowly-defined industries (e.g.,

of valuation of firm j, and ϵ_i is an incremental private valuation that this investor type attaches, raising his total valuation of firm j to $V_j + \epsilon_i$.[8]

We assume that an investor can invest any amount in any firm(s) up to the amount of wealth that he holds (or equivalently, up to some maximum amount that the collateral-constrained investor can borrow). The limited access to capital means that the highest valuation investor does not hold the entire firm, giving rise to a downward sloping demand curve.

The marginal shareholder in firm j is the one with the lowest private valuation $\underline{\epsilon}_j$: all individuals with higher private valuations are fully invested, and all those with lower private valuations do not hold the firm. Firm j's trading price reflects that its marginal shareholder is indifferent between holding and selling,

$$P_j = V_j + \underline{\epsilon}_j, \quad j = A, T. \tag{1}$$

For each $j \in \{A, T\}$, we denote the cumulative wealth of type j investors with private valuations of at least ϵ_j by $\tilde{G}_j(\epsilon_j)$.[9] In Appendix A we show how $\underline{\epsilon}_j$ is pinned down by $\tilde{G}_j$ via market clearing, and we provide a sufficient condition on $\tilde{G}_j$ for this private valuation to be strictly positive (i.e., $\underline{\epsilon}_j > 0$), which we assume holds throughout our analysis.

Firm j's trading price does not reveal the exact form of $\tilde{G}_j(\cdot)$ save for what is revealed by market clearing. As a result, the acquiring firm is typically uncertain about the *median* target shareholder's private valuation, ϵ_T^*. We denote the distribution over ϵ_T^* conditional on the information in the market-clearing price P_T by $F_T(\cdot)$, with associated support $[\epsilon_T^l, \epsilon_T^h]$, where $\underline{\epsilon}_T < \epsilon_T^l \leq \epsilon_T^h$. Because a takeover's success requires median target shareholder approval, $F_T(\cdot)$ captures the relevant uncertainty faced by an acquiring firm about whether a particular offer will succeed.[10]

biotechnology), few investors have positive private values for any given pair of firms. We later relax this structure so that some investors have positive private valuations of both firms.

[8]We do not model the tradeoffs between risk and return that enter the market value of an asset through the valuation terms V_A and V_T for the acquirer and target that are common to all investors. This is consistent with the takeover literature that takes the current price as given rather than trying to derive it as the present value of future cash flows, embedding the risk and return tradeoff into the discount factor. The heterogeneity comes from the additional term ϵ_A or ϵ_T, and reflects heterogeneous opinions of investors about a firm's value.

[9]Letting $\tilde{G}_j(\cdot)$ converge to place all mass at $\epsilon_j = 0$ for $j \in \{A, T\}$ recovers the case with no disagreement.

[10]All results in Section 3 hold regardless of the specific form of $F_T(\cdot)$. In particular, they hold when F_T is degenerate (i.e., when the acquiring firm knows the median target shareholder's valuation).

Acquirer Management. Like its shareholders, the acquiring firm's management has a positive private valuation of firm A, attaching value $V_A + \epsilon_A^M$, where $\epsilon_A^M > 0$. We interpret $V_A + \epsilon_A^M$ as the manager's assessment of his firm's long-term value. The manager values the target at V_T. We assume that the manager maximizes the long-term profits of shareholders based on his assessment of the firm value, or alternatively, the manager has an equity stake in the firm and maximizes his own profit.

Timing. The sequence of events is as follows. At $t = 0$, a synergy $S > 0$ develops between firms A and T. The synergy S is public information and the valuation of the joint firm is additive for all investors. Thus, a type ϵ_A investor values the joint firm at $(V_A + \epsilon_A) + V_T + S$. At $t = 1$, the acquiring firm's management makes an offer. At $t = 2$, target shareholders decide whether to accept or reject the offer. The offer is accepted if and only if at least 50% of target shareholders vote in favor. We assume that following a favorable vote, there is a freeze-out of non-tendered shares, and the target is absorbed by the acquirer. This assumption mirrors general practice—freeze-outs occur in over 90% of US and UK takeovers (Gomes, 2001) in order to eliminate free riding.

Discussion. Our model structure is designed to capture two key dimensions of valuation heterogeneities. First, $\underline{\epsilon}_j$ represents the difference between how the marginal shareholder of firm j values firm j and how shareholders in other firms value firm j: it measures the extent to which shareholders of the two firms differ in their valuations of their respective firms. Such heterogeneity in valuations is a straightforward implication of models with differences of opinion (i.e., "agreeing to disagree"), where shareholders form different posterior valuations of financial assets even after information is aggregated via the trading process. Taking this heterogeneity as a primitive allows us to eschew complications arising from Bayesian updating while still incorporating the fundamental components of such frameworks.

Second, the difference $\epsilon_T^* - \underline{\epsilon}_T$ in the valuations of the median and marginal target shareholder is the key measure of dispersion in valuations among target shareholders, and it drives the offer premia. Empirical researchers often use the dispersion in analyst forecasts of one-year-ahead earnings to measure heterogeneity in investor beliefs (Moeller et al., 2007, Chatterjee et al., 2012). Researchers may also be able to use the information in one-year-ahead share price "targets" set by institutional analysts to obtain a proxy for $\epsilon_T^* - \underline{\epsilon}_T$. For example, as a proxy for ϵ_T^*, one could use the median of those price targets that exceed the outstanding share price (as these institutional investors are plausible shareholders).

Importantly for the ability of our model to deliver the takeover premia found in the data, the variation in share price targets and earnings forecasts is high relative to the share prices of potential takeover targets. For example, for moderate-sized biotech firms, the *range* of price targets often exceeds a firm's stock price. The implied large differences in private values mean that our model can reconcile the *magnitudes* of offer premia found in the data.

To ease analysis, we assume that the takeover opportunity is unexpected. What matters is that it is not fully anticipated. Market responses to takeover announcements make clear that this is the relevant scenario—share prices would not move were a takeover fully anticipated. If the market attaches a positive probability to a takeover, then the pre-merger share prices account for it, reducing the magnitudes of the predicted return effects, but not otherwise altering their qualitative properties. For work that considers the possibility that share prices may rise in anticipation of a takeover, see Edmans, Goldstein, and Jiang (2012).

3 Analysis

We collect all assumptions on parameter values below. Most results are established under a strict subset of the assumptions. In particular, if we say a result is established under **A3**, it means the result assumes that **A3** holds, but we do not require **A1** or **A2**. So, too, if we do not explicitly state any of **A1**, **A2**, or **A3**, then the result holds without their structure.

A1: The private valuation of the marginal shareholder in the acquiring firm $\underline{\epsilon}_A$ is nondecreasing in V_A.

A2: The *marginal* acquiring firm shareholder's private valuation exceeds the *marginal* target shareholder's: $\underline{\epsilon}_A \geq \underline{\epsilon}_T$.

A3: The *median* target shareholder's private valuation always exceeds the private valuation of the *marginal* acquiring firm shareholder: $\epsilon_T^l \geq \underline{\epsilon}_A$.

Assumption **A1** delivers an unambiguous interpretation of firms with larger market capitalization: they have larger private and common value components. This assumption is used for some comparative static results and facilitates interpretation.

To understand the economic rationale for Assumption **A2**, note that two possibly conflicting factors affect the relative magnitudes of $\underline{\epsilon}_A$ and $\underline{\epsilon}_T$. First, empirically, the acquiring firm is typically larger by a factor of three or more. This scale effect suggests that, consistent with **A1**, the marginal private valuation of the acquiring firm shareholder should be larger, i.e., $\underline{\epsilon}_A > \underline{\epsilon}_T$. Possibly offsetting this to some extent, the *percentage* of total assets that are intangible, a primary source of disagreement among investors and analysts, may be higher for target firms than acquiring firms. For example, in the pharmaceutical industry, targets are often small biotech firms developing drugs of unknown value. What Assumption **A2** says is that the scale effect dominates the greater percentage of intangibility in determining the magnitudes of private valuations. Indeed, a simple investigation of pharmaceutical firms indicates that while on a per dollar basis the extent of dispersion in analysts forecasts of earnings is higher for smaller firms that are likely targets, the absolute (total) dispersion in analyst forecasts is much greater for larger firms. Still, even though **A2** ($\underline{\epsilon}_A \geq \underline{\epsilon}_T$) holds in most takeover settings, it may not hold in all scenarios. We will provide intuition on how this condition is used to reconcile some of the stylized facts in takeovers, and we will also provide a qualitative discussion of what can happen when $\underline{\epsilon}_T$ is *sufficiently* larger than $\underline{\epsilon}_A$.

Assumption **A3** is not essential for our results—it is made for analytical ease and is useful in the context of equity acquisitions (see Lemma 1).

We start our analysis by examining equity and cash offers when the payment method (equity or cash) is exogenously determined. We then endogenize the method of payment.

Exogenous Equity Offers. In an equity offer, an acquirer offers I newly-created shares of the joint firm in exchange for all of the target shares. We denote the valuation of the target shareholder who is indifferent between accepting and rejecting this offer by $\epsilon(I)$. Without loss of generality, we focus on $\epsilon(I)$ that lie between the lower and upper bounds on the median target shareholder's private valuation,

$$\epsilon_T^l \leq \epsilon(I) \leq \epsilon_T^h, \tag{2}$$

because an offer where $\epsilon(I)$ exceeds ϵ_T^h always wins, and thus is dominated by offering I' such that $\epsilon(I') = \epsilon_T^h$; and offering I such that $\epsilon(I)$ is less than ϵ_T^l always loses, and is thus equivalent to offering I' such that $\epsilon(I') = \epsilon_T^l$.

If a takeover fails, the payoff of the shareholder with private valuation $\epsilon(I)$ is $V_T + \epsilon(I)$. If the takeover succeeds, then just after a successful equity offer, trade will occur if $\underline{\epsilon}_A \neq \underline{\epsilon}_T$. We denote the private valuation of a marginal shareholder in the joint firm by $\tilde{\underline{\epsilon}}_J$, where the tilde highlights that its realization will hinge on the realized distributions of shareholder wealth distributions, $\tilde{G}_A(\cdot)$ and $\tilde{G}_T(\cdot)$, via the market-clearing condition. The payoff of a shareholder with private valuation $\epsilon(I)$ following a successful takeover depends on the value of $\tilde{\underline{\epsilon}}_J$, which, via the indifference condition that equates the payoffs following successful and unsuccessful takeovers, yields a relationship between $\tilde{\underline{\epsilon}}_J$, $\epsilon(I)$ and I. Appendix A provides details on this relationship and on the market-clearing condition that determines the private valuation of the marginal shareholder of the joint firm, $\tilde{\underline{\epsilon}}_J$. The key features that emerge from that analysis that we exploit are: (i) $\tilde{\underline{\epsilon}}_J$ must lie between the private valuations of the acquirer and target's marginal shareholders, $\underline{\epsilon}_A$ and $\underline{\epsilon}_T$, i.e.,

$$\min\{\underline{\epsilon}_A, \underline{\epsilon}_T\} \leq \tilde{\underline{\epsilon}}_J \leq \max\{\underline{\epsilon}_A, \underline{\epsilon}_T\}, \tag{3}$$ [11]

and (ii)

$$I \leq \frac{V_T + \epsilon(I)}{V_A + S}. \tag{4}$$

To understand (4), note that just after an equity acquisition—but before equilibrium is reached—the per-share payoff of a target shareholder with private valuation $\epsilon(I)$ is $\frac{V_T+V_A+S+\epsilon(I)}{1+I}I$. If $\epsilon(I) < \tilde{\underline{\epsilon}}_J$, then this target shareholder will derive an additional benefit by selling his shares at the market price, which reflects $\tilde{\underline{\epsilon}}_J$; and since $\tilde{\underline{\epsilon}}_J$ exceeds $\epsilon(I)$, it is the source of an added benefit. Therefore, his post-takeover per-share payoff satisfies

$$\pi_E > \frac{V_T + V_A + S + \epsilon(I)}{1+I} I. \tag{5}$$

[11] The inequalities are strict if $\underline{\epsilon}_A \neq \underline{\epsilon}_T$ and the total measure of ϵ_j over the interval $(\min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}, \max\{\underline{\epsilon}_A, \underline{\epsilon}_T\})$ is strictly positive for $j \in \{A, T\}$.

If, instead, $\epsilon(I) \geq \tilde{\underline{\epsilon}}_J$, then a target shareholder with private valuation $\epsilon(I)$ will hold the joint firm and derive no additional benefit. Then, equation (5) holds as an equality. Combining these two scenarios yields

$$\pi_E \geq \frac{V_T + V_A + S + \epsilon(I)}{1+I} I. \tag{6}$$

Equation (6) and the indifference condition that π_E equals the target shareholder's pre-takeover per-share payoff of $V_T + \epsilon(I)$ give $\frac{V_T+V_A+S+\epsilon(I)}{1+I} I \leq V_T + \epsilon(I)$, which yields (4).

The following lemma provides a sufficient condition under which the indifferent target shareholder holds the joint firm (i.e., $\epsilon(I) \geq \tilde{\underline{\epsilon}}_J$):

Lemma 1 *Under **A3** ($\epsilon_T^l \geq \underline{\epsilon}_A$), for any equity offer that satisfies (2), the indifferent target shareholder will hold the joint firm. That is, (4) holds with equality.*

We next examine what happens after a successful offer, which is an offer that satisfies (2) and is accepted by a majority of target shareholders. After a successful equity offer, the joint firm's market value reflects the value attached by its marginal shareholder:

$$\widetilde{MV}_J \;=\; V_T + V_A + S + \tilde{\underline{\epsilon}}_J \;=\; P_T + P_A + S + \tilde{\underline{\epsilon}}_J - \underline{\epsilon}_A - \underline{\epsilon}_T. \tag{7}$$

Because $\tilde{\underline{\epsilon}}_J \leq \max\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$ (see (3)), an upper bound for the last four terms in (7) is:

$$\begin{aligned} S + \tilde{\underline{\epsilon}}_J - \underline{\epsilon}_A - \underline{\epsilon}_T &\leq S + \max\{\underline{\epsilon}_A, \underline{\epsilon}_T\} - \underline{\epsilon}_A - \underline{\epsilon}_T \\ &= S - \min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}. \end{aligned} \tag{8}$$

Thus, the (random) market value of the joint firm, $\widetilde{MV}_J$, is always less than the sum of the two firms' pre-merger valuations, $P_T + P_A$, whenever the synergy is small relative to the minimum of the marginal shareholders' valuations, $\min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$. This upper bound has implications for the combined return from holding equal positions in the acquirer and target:

$$\tilde{R}_E = \frac{\widetilde{MV}_J}{P_T + P_A} - 1 \;=\; \frac{P_T + P_A + S + \tilde{\underline{\epsilon}}_J - \underline{\epsilon}_A - \underline{\epsilon}_T}{P_T + P_A} - 1 \;=\; \frac{S + \tilde{\underline{\epsilon}}_J - \underline{\epsilon}_A - \underline{\epsilon}_T}{P_T + P_A}. \tag{9}$$

Result 1 *The combined acquirer–target return $\tilde{R}_E$ following an equity acquisition is negative if the synergy S is less than* $\min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$.

Here, $\min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$ is a measure of the extent to which shareholders of the two firms differ in their valuations of their respective firms. Result 1 says that the combined return is negative if the synergy is less than this measure of the heterogeneity in valuations between the shareholders of the two firms. This result reflects that a merger forces investors to hold firms they may otherwise not hold, diluting their claims to their favorite firms.

Next note that because $\tilde{\underline{\epsilon}}_J \geq \min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$, a lower bound obtains for the combined return. Using

$$S + \tilde{\underline{\epsilon}}_J - \underline{\epsilon}_A - \underline{\epsilon}_T \geq S + \min\{\underline{\epsilon}_A, \underline{\epsilon}_T\} - \underline{\epsilon}_A - \underline{\epsilon}_T \tag{10}$$

$$= S - \max\{\underline{\epsilon}_A, \underline{\epsilon}_T\}, \tag{11}$$

we have a sufficient condition on synergies for the combined return to be positive:

Result 2 *The combined acquirer–target return $\tilde{R}_E$ following an equity acquisition is positive if the synergy S exceeds* $\max\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$.

The share price of the joint firm is:

$$\tilde{P}_J = \frac{\widetilde{MV}_J}{1+I} = \frac{V_T + V_A + S + \tilde{\underline{\epsilon}}_J}{1+I}. \tag{12}$$

Interpreting $\tilde{P}_J I$ as the cash equivalent of the equity offer, we have:

Proposition 1 *Under* ***A2*** *($\underline{\epsilon}_A \geq \underline{\epsilon}_T$), any successful equity offer has a cash equivalent that is at a premium over the target's market value: $\tilde{P}_J I > P_T$.*

A takeover dilutes a target shareholder's claim to the target—he now only has a claim of $\frac{I}{1+I}$ to his private valuation ϵ_T—for which he must be compensated. This dilution affects every target shareholder, but the resulting loss is more severe for the median target shareholder than the marginal shareholder. When the median target shareholder is indifferent between accepting and rejecting, the marginal target shareholder must be strictly better off. To understand the role of **A2**, suppose that $\underline{\epsilon}_A = \underline{\epsilon}_T$. Then, by (3), the marginal target shareholder is also the marginal shareholder in the joint firm. Hence $\tilde{P}_J I$, which is the value of the equity offer from the perspective of the marginal shareholder in the joint firm, is also the offer's value from the perspective of the marginal target shareholder. Since, as established above, the marginal target shareholder is strictly better off, it follows that $\tilde{P}_J I > P_T$. Now suppose that $\underline{\epsilon}_A > \underline{\epsilon}_T$. Then, by (3), the marginal shareholder in the joint firm has a higher private valuation than the marginal target shareholder, further raising $\tilde{P}_J I$ relative to P_T. If, instead, **A2** is violated, so that $\underline{\epsilon}_A < \underline{\epsilon}_T$, then (3) implies that the joint firm is priced by someone who values it less than the marginal target shareholder, i.e., $\underline{\epsilon}_A < \tilde{\underline{\epsilon}}_J < \underline{\epsilon}_T$. If $\underline{\epsilon}_T$ is sufficiently greater than $\underline{\epsilon}_A$ so that $\tilde{\underline{\epsilon}}_J$ is sufficiently less than $\underline{\epsilon}_T$, this may offset the more generous offer made to win median target shareholder's approval, reversing the result.

This discussion reveals that Proposition 1 holds even when **A2** does not hold, as long as $\underline{\epsilon}_A$ is not too much smaller than $\underline{\epsilon}_T$. Furthermore, even on the uncommon occasions where $\underline{\epsilon}_A$ is much smaller than $\underline{\epsilon}_T$, Proposition 1 may still hold when the underlying sources of disagreement move the components of heterogeneous valuations about the target in the same direction. That is, when the marginal target shareholder's valuation rises relative to non-shareholders, the median target shareholder's valuation is also likely to diverge from the marginal target shareholder's valuation. Thus, when $\underline{\epsilon}_T >> \underline{\epsilon}_A$, the median target shareholder is also likely to have a far higher private valuation than the marginal target shareholder, forcing the acquiring firm to make a more generous offer to win approval, raising $\tilde{P}_J I$.

Noting that $\tilde{P}_J I$ is also the target's stock price after a takeover, Proposition 1 implies:

Result 3 *Under **A2** ($\underline{\epsilon}_A \geq \underline{\epsilon}_T$), in a successful equity offer, the target's return $R_T = \frac{I\tilde{P}_J - P_T}{P_T}$ is always positive.*

In contrast, the *acquirer's* return after a successful equity takeover, $\frac{\tilde{P}_J - P_A}{P_A}$, is negative if synergies are sufficiently small. Under the simplifying condition **A3** (the *median* target shareholder's private valuation exceeds *marginal* acquiring firm shareholder's private valuation), we provide a sufficient condition for the acquirer's return to be negative:

Result 4 *Under **A3** ($\epsilon_T^l \geq \underline{\epsilon}_A$), in any successful equity offer, the acquirer's return is negative if $S < \underline{\epsilon}_A$.*

Intuitively, when the median target shareholder's private valuation is high, a large premium must be offered, which results in a negative return for the acquirer. More generally, the acquirer's return is always negative when the synergy is positive but sufficiently small. Thus, our model can reconcile the negative returns for acquirers that Moeller, Schlingemann, and Stulz (2005) find. Further, the combined return is negative when the synergy is small (Result 1), even though the target's return is always positive (Result 3). These results will hold when we endogenize the acquiring firm's optimal offer.

Exogenous Cash Offers. With a cash offer, the acquirer offers cash C to target shareholders in exchange for all of their shares. Immediately after a successful cash acquisition, the joint firm is held only by the acquiring firm's shareholders, while all previous target shareholders hold cash. Then, since type ϵ_A and ϵ_T investors value the joint firm at $V_A + V_T + S - C + \epsilon_A$ and $V_A + V_T + S - C + \epsilon_T$ respectively, any target shareholders with private values $\epsilon_T > \underline{\epsilon}_A$ will purchase claims to the joint firm from marginal acquiring firm shareholders. This transaction results in a new marginal holder of the joint firm, one with a higher private valuation. Therefore, the share price of the joint firm will satisfy

$$\tilde{P}_J \;>\; V_A + V_T + S - C + \underline{\epsilon}_A. \tag{13}$$

Rearranging (13) yields $\tilde{P}_J + C > V_A + V_T + S + \underline{\epsilon}_A$. Hence, provided that $\underline{\epsilon}_A \geq \underline{\epsilon}_T$, the combined return is

$$\begin{aligned}\tilde{R}_C &= \frac{\tilde{P}_J + C}{P_A + P_T} - 1 > \frac{V_A + V_T + S + \underline{\epsilon}_A}{P_A + P_T} - 1 = \frac{S - \underline{\epsilon}_T}{P_A + P_T} \\ &= \frac{S - \min\{\underline{\epsilon}_T, \underline{\epsilon}_A\}}{P_A + P_T} \geq \tilde{R}_E,\end{aligned}$$

where $\tilde{R}_E$ is the combined return in an equity offer and we have used equations (8) and (9). Summarizing, we have:

Result 5 *Under* ***A2*** *($\underline{\epsilon}_A \geq \underline{\epsilon}_T$), ceteris paribus, the combined acquirer and target return in a cash acquisition exceeds that in an equity acquisition.*

Result 5 shows that under **A2**, the combined acquirer and target return in a cash acquisition exceeds that in an equity acquisition. This result is a consequence of two effects, an "asymmetry effect" and a "cash effect." Just after a successful cash offer (but before the equilibrium is reached), the joint firm is held entirely by the original acquiring firm shareholders, whose private valuation is at least $\underline{\epsilon}_A$. In contrast, after a successful equity offer, the combined firm is held jointly by the original acquiring and target firm shareholders, and the valuation $\tilde{\epsilon}_J$ of the marginal holder of the joint firm is between $\underline{\epsilon}_A$ and $\underline{\epsilon}_T$ (see (3)). When $\underline{\epsilon}_A > \underline{\epsilon}_T$, the private valuation of the marginal holder of the joint firm just after a cash offer is higher than after an equity offer. The cash effect reflects that cash acquisitions provide the original target shareholders with new funds that allow them to purchase claims to the joint firm from original acquiring firm shareholders. This effect further drives up the price in a cash offer. The above two effects go in the same direction, making the combined return higher in cash than in equity acquisitions.[12] This result can explain the empirical finding that, on average, the combined returns in cash offers exceed those in equity offers (Andrade et al., 2001).

It is instructive to examine these two effects more closely. The "asymmetry" effect shows that the interaction of two asymmetries—a size asymmetry (the acquirer is larger than the target in most takeovers, the foundation of assumption **A2**) and an ownership asymmetry (the joint firm is held entirely by acquiring firm shareholders just after a cash acquisition, whereas it is held jointly by shareholders from both firms in an equity acquisition)—is important for explaining the difference in the combined return between cash and equity acquisitions; our model indicates that this asymmetry effect is a key driver of the empirically-observed differences in the acquirer's return between cash and equity offers.

This asymmetry effect also implies the following novel empirical prediction:

Implication 1: Under **A1** (monotonicity), ceteris paribus, the difference in the combined acquirer and target return between cash and equity acquisitions should rise with the difference in the sizes of the acquirer and the target.

The "cash" effect suggests that to the extent that a cash offer relaxes budget constraints of investors, some of the empirically-observed differences in an acquirer's return between cash

[12] In obtaining Result 5, we have implicitly assumed that the acquirer has the cash required to make the acquisition. If, instead, the acquirer pays out its cash to its shareholders—e.g., via a dividend—and then conducts an equity acquisition, the presence of higher-valuation investors in the market who now have available cash will drive up the joint firm's price, counteracting the negative effects in an equity acquisition.

and equity offers may reflect interactions between investors' increased supply of cash and heterogeneous valuations. This cash effect is likely second order in practice. In particular, it is dominated by the asymmetry effect whenever the acquirer is far larger than the target.

We now examine target shareholders' willingness to accept an offer. We let $\epsilon(C)$ denote the private valuation of the target shareholder who is indifferent between accepting and rejecting a cash offer of C. As with equity offers, one can focus on $\epsilon(C)$ lying between the lower and upper bounds of the median target shareholder's private valuation,

$$\epsilon_T^l \leq \epsilon(C) \leq \epsilon_T^h, \tag{14}$$

because a cash offer such that $\epsilon(C)$ exceeds ϵ_T^h always wins, and thus is dominated by offering C' such that $\epsilon(C') = \epsilon_T^h$; and offering C such that $\epsilon(C)$ is less than ϵ_T^l always loses, and is thus equivalent to making an offer such that $\epsilon(C') = \epsilon_T^l$.

Just after a successful cash offer—one that satisfies (14) and is accepted by a majority of target shareholders—former shareholders of the target for whom $V_A + V_T + S + \epsilon_T - C > \tilde{P}_J$ wish to buy shares in the joint firm, while original shareholders of the acquiring firm for whom $V_A + V_T + S + \epsilon_A - C < \tilde{P}_J$ want to sell. Market clearing determines the price of the joint firm, $\tilde{P}_J$. There are two possible scenarios:

(i) If $\frac{V_T+V_A+S-C+\epsilon(C)}{\tilde{P}_J} > 1$, then $\epsilon(C)$ exceeds the private valuation of the marginal shareholder of the joint firm. Thus, a target shareholder with private valuation $\epsilon(C)$ derives an added benefit by holding $\frac{C}{\tilde{P}_J}$ shares of the joint firm for each share he originally held in the target, receiving a per-share payoff of $(V_T + V_A + S - C^* + \epsilon(C))\,\frac{C}{\tilde{P}_J} > C$ from the takeover.

(ii) If $\frac{V_T+V_A+S-C+\epsilon(C)}{\tilde{P}_J} \leq 1$, then $\epsilon(C)$ is less than the private valuation of the marginal joint firm shareholder. A target shareholder with private valuation $\epsilon(C)$ will not hold the joint firm, hence gaining no additional benefit, so his post-takeover per-share payoff is just C.

As the offer C leaves the target shareholder with private valuation $\epsilon(C)$ indifferent between accepting and rejecting, the indifference conditions for these two scenarios yield:

$$\begin{aligned} &V_T + \epsilon(C) = \frac{V_T + V_A + S - C + \epsilon\,(C)}{\tilde{P}_J} C \quad \text{if } \frac{V_T + V_A + S - C + \epsilon\,(C)}{\tilde{P}_J} > 1 \quad (i) \\ &V_T + \epsilon(C) = C \qquad\qquad\qquad\qquad\qquad \text{otherwise.} \qquad\qquad\qquad\qquad (ii) \end{aligned} \tag{15}$$

Equation (15) (i) reveals that if the marginal joint firm shareholder has a lower private valuation than the indifferent target shareholder, then $C < V_T + \epsilon(C)$, i.e., the cash offer is less than the indifferent target shareholder's valuation. The indifferent target shareholder uses the cash received for his shares to purchase shares in the joint firm at its market price. As the marginal joint firm shareholder who determines this price has a lower private valuation, this purchase provides the indifferent target shareholder with an added private benefit, making him willing to tender at a lower price (as are all shareholders with lower valuations).

Even when the indifferent target shareholder holds the joint firm (Lemma 2 of Appendix A provides sufficient conditions for the indifferent target shareholder to hold/not hold the joint firm), so that $C < V_T + \epsilon(C)$, Proposition 2 below shows that the offer must still exceed the target firm's pre-acquisition price, $P_T = V_T + \underline{\epsilon}_T$, as long as the acquirer's market value is high enough. This is because when the joint firm is expensive, the indifferent target shareholder only purchases a small claim, so the added private benefit received is small. Thus, to make him indifferent, a premium relative to the pre-acquisition price must be offered. Indeed, as the acquirer's market value grows arbitrarily larger than the target's, the offer approaches $V_T + \epsilon(C)$:

Proposition 2 *Under* ***A2*** *($\underline{\epsilon}_A \geq \underline{\epsilon}_T$), in a successful cash offer, the offer represents a premium if the acquirer is sufficiently larger than the target. In particular, if either $P_A > 2P_T$ or $V_A + S > P_T$, then*

$$P_T < C \leq V_T + \epsilon(C). \tag{16}$$

Further, as the acquirer's market value grows arbitrarily larger than the target's value, the offer approaches the pre-acquisition value of the indifferent target shareholder, $V_T + \epsilon(C)$:

$$\lim_{\frac{V_T+\epsilon_T^h}{V_A} \to 0} C - (V_T + \epsilon(C)) = 0. \tag{17}$$

Corollary 1 *Under* ***A2****, the target's return is positive in a cash acquisition if either $P_A > 2P_T$ or $V_A + S > P_T$.*

Boone and Mulherin (2007) report that the median ratio of the target-to-bidder equity value is only 0.27. Andrade et al. (2001) and Moeller et al. (2004) document that the acquirer is especially likely to be much larger in cash acquisitions. Thus, Proposition 2 and Corollary 1 apply to most cash acquisitions.

Assumption **A2** ($\underline{\epsilon}_A \geq \underline{\epsilon}_T$) plays a key role in Proposition 2 for the same reason it is important to ensure there is a premium with equity. Were all target shareholders to have the same private valuation ($\underline{\epsilon}_T$), and $\underline{\epsilon}_A = \underline{\epsilon}_T$, there would be no premium, i.e., $P_T = C$. Since, in fact, most shareholders have private valuations greater than $\underline{\epsilon}_T$, the offer has to be raised to appeal to them, generating a positive premium. However, if $\underline{\epsilon}_A << \underline{\epsilon}_T$, then the cash offer can be less generous because the indifferent investor with private valuation $\epsilon(C)$ can use the cash to purchase shares at a low price from acquiring firm shareholders who place a far lower value on the joint firm.

3.1 Optimal Payment Method

We now let the acquirer choose the type of offer—cash, equity, no offer—to make.[13] We first examine an acquiring firm manager's willingness to make an equity offer. Prior to a takeover, his per-share payoff is $\pi_{AM} = V_A + \epsilon_A^M$; if an offer I is accepted, his post-merger per-share payoff is $\frac{V_T+V_A+S+\epsilon_A^M}{1+I}$. Thus, the manager's expected per-share payoff is

$$\pi_{AM}^E\left(I\right) = \Pr(\epsilon(I) \geq \epsilon_T^*)\frac{V_T + V_A + S + \epsilon_A^M}{1+I} + \Pr(\epsilon(I) < \epsilon_T^*)\left(V_A + \epsilon_M^A\right)], \tag{18}$$

where $\Pr(\epsilon(I) \geq \epsilon_T^*)$ is the probability that offer I is accepted since $\epsilon(I)$ is the private valuation of the indifferent target shareholder given offer I, and ϵ_T^* is the realization of the median target shareholder's valuation.

The optimal offer I^* maximizes π_{AM}^E, trading off between the probability of winning and the size of the payoff when a takeover succeeds. That is,

$$I^* = \arg\max_I \pi_{AM}^E\left(I\right).$$

If, instead, the acquirer makes a cash offer C, then its manager's expected per-share

[13] In practice, an acquirer may not always be able to choose between cash or equity offers; for example, financial constraints may mandate equity offers. Our main results extend to those situations.

payoff is

$$\pi_{AM}^{C}(C) = \Pr(\epsilon(C) \geq \epsilon_T^*) \left(V_T + V_A + S + \epsilon_A^M - C\right) + (1 - \Pr(\epsilon(C) \geq \epsilon_T^*)) \left(V_A + \epsilon_M^A\right)], \quad (19)$$

where $\epsilon(C)$ is the private valuation of the indifferent target shareholder given offer C. As with the optimal equity offer, the optimal cash offer C^* maximizes π_{AM}^C, trading off between the probability of winning and the size of the payoff when a takeover succeeds:

$$C^* = \arg\max_{C} \pi_{AM}^{C}(C).$$

The choice between cash and equity boils down to whether $\pi_{AM}^{C}(C^*) > \pi_{AM}^{E}(I^*)$, in which case a cash offer is made, or $\pi_{AM}^{C}(C^*) < \pi_{AM}^{E}(I^*)$, in which case an equity offer is optimal. Finally, if $\pi_{AM}^{C}(C^*) = \pi_{AM}^{E}(I^*) = \pi_{AM}$, where π_{AM} is the manager's per-share payoff prior to the takeover, then the acquirer optimally makes no offer.

Cash and equity have competing merits. Equity offers require an acquiring firm's manager to cede some of his private valuation for his firm. This works in favor of using cash, and further favors cash as the manager's valuation of his firm, ϵ_A^M, increases. Conversely, equity offers allow target shareholders to retain stakes in the target and thus some of their private valuations. This works in favor of using equity since the retention of the private valuations by target shareholders allows the acquirer to make a relatively lower offer, an effect that rises with the median target shareholder's valuation, ϵ_T^*. One additional secondary effect may also arise and affect the offer choice. If the indifferent target shareholder (1) does not hold the joint firm in an equity offer, or (2) holds the joint firm in a cash offer, then he could derive an additional benefit by (1) selling his shares in an equity offer, or (2) using the available cash to purchase claims to the joint firm in a cash offer. This would allow the acquirer to reduce its offer, making that offer type more attractive.

The resulting choice of means of payment depends on how the private valuation of the acquirer's management compares to that of the median target shareholder, and how the private valuation of the marginal holder of the joint firm compares to that of the indifferent target shareholder (which affects the occurrence of the additional benefit):

Proposition 3 *(i) If (a) the acquirer manager's private valuation always exceeds the median target shareholder's (i.e., if $\epsilon_A^M \geq \epsilon_T^h$), and (b) following the optimal equity offer, the indifferent target shareholder holds the joint firm (for which Lemma 1 provides a sufficient condition), then the acquirer's manager prefers to make a cash offer, i.e., $\pi_{AM}^C(C^*) \geq \pi_{AM}^E(I^*)$.*

(ii) If, instead, (a) the median target shareholder's private valuation always exceeds the acquirer manager's, i.e., if $\epsilon_A^M \leq \epsilon_T^l$, and (b) following the optimal cash offer, the median target shareholder does not hold the joint firm (a sufficient condition for which is $\epsilon_T^h \leq \underline{\epsilon}_A$), then the acquirer's manager prefers to make an equity offer, i.e., $\pi_{AM}^E(I^*) \geq \pi_{AM}^C(C^*)$.

The proposition provides sufficient conditions for equity or cash offers to be made. Quite generally, however, cash is always optimal if the acquiring manager's private valuation is sufficiently high relative to the median target shareholder's; and equity is always optimal when the reverse is true. That is, the key determinant for the means of payment is how the acquiring manager's private valuation compares to that of the median target shareholder, with cash offers becoming more attractive as ϵ_A^M increases. To gain intuition, consider a simple case in which the median target shareholder's private valuation ϵ_T^* is known with certainty and the median target shareholder derives no additional benefits (from purchasing or selling the joint firm) in both optimal cash and optimal equity offers (e.g., if $\underline{\epsilon}_A = \underline{\epsilon}_T$ and ϵ_T^* is sufficiently close to $\underline{\epsilon}_T$). Then, regardless of whether equity or cash is used, the acquiring firm's optimal offer leaves the median target shareholder indifferent between accepting and rejecting the offer. Thus, the acquiring firm's management prefers cash to equity if and only if the sum of its per-share payoff plus the per-share payoff of the median target shareholder is higher with cash. Equity and cash offers differ in their impacts on the loss of private valuations in a merger. With equity, the acquirer holds a fraction $\frac{1}{1+I}$ of the joint firm and the target holds the remaining fraction $\frac{I}{1+I}$. Hence, the total loss of private valuation with an equity offer is $\frac{I}{1+I}\epsilon_A^M + \frac{1}{1+I}\epsilon_T^*$. In contrast, the loss with a cash offer is ϵ_T^*. Thus, the loss with the equity offer is greater if and only if

$$\frac{I}{1+I}\epsilon_A^M + \frac{1}{1+I}\epsilon_T^* \geq \epsilon_T^* \;\Leftrightarrow\; \epsilon_A^M \geq \epsilon_T^*,$$

which is exactly the condition from the proposition.

Existing theories (e.g., Chatterjee, John, and Yan, 2012) predict that a manager wants to use equity when the market overvalues his firm's equity. Proposition 3 is consistent with such theories in that it shows that equity is preferred when an acquirer's private valuation is low relative to its marginal shareholder's private valuation $\underline{\epsilon}_A$ (i.e., its equity is overvalued). However, our analysis provides additional insights, showing that the choice between cash and equity should also reflect the private valuations of target shareholders: equity is preferred to cash when the acquiring firm's manager has a low private valuation *relative* to the *median* target shareholder. Thus, the target's market value, as determined by its marginal shareholder, does not directly enter this calculation.

Proposition 3 establishes that the acquirer is more likely to use equity if its manager's private valuation for his firm is lower. Empirically, one can interpret the acquiring firm's manager as its CEO. Provided that a manager's shareholding in his firm rises with his private valuation, we can use the manager's stake to proxy for the valuation, yielding the following testable prediction:

Implication 2: The smaller is an acquirer manager's holding of his company, the more likely the acquirer is to offer equity.

3.2 Stock Price Effects of Optimal Offers

Having derived how an acquiring firm's manager designs optimal offers, we now characterize the stock price consequences. We first observe that, given sufficient conditions that describe most takeovers, target shareholders receive positive returns after both equilibrium cash and equity offers.

Proposition 4 *(i) Under* ***A2*** *($\underline{\epsilon}_A \geq \underline{\epsilon}_T$), a target firm's share price always rises following an equilibrium equity offer;*
(ii) Under ***A2*** *and $P_A \geq 2P_T$ (or $V_A + S > P_T$), a target's share price always rises following an equilibrium cash offer.*

This result is a direct corollary of Propositions 1 and 2, which provide sufficient conditions for target shareholders to receive positive returns from optimal equity and cash offers. We now contrast this result with what acquiring firm shareholders may experience.

Proposition 5 *(i) If the acquirer manager's private valuation is less than the marginal shareholder's* ($\epsilon_A^M < \underline{\epsilon}_A$)*, the synergy is less than* $\min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$*, and* ***A2*** ($\underline{\epsilon}_A \geq \underline{\epsilon}_T$) *holds, then the acquiring firm's share price falls following an equilibrium equity offer, i.e.,* $\tilde{P}_J < P_A$*.*
(ii) An acquiring firm's share price always rises after an equilibrium cash offer.

Thus, an equity offer that is optimally chosen by the acquirer manager may succeed, and yet cause the acquirer's stock price to fall. Intuitively, if the acquirer management has a lower private valuation than its shareholders, it means that the shareholders care more about the dilution associated with an equity offer. To succeed an equity offer must be generous enough to win approval from the median target shareholder. Such an offer may fail to leave enough to compensate the marginal acquiring firm shareholder for the dilution to his private valuation when the synergy is too small—but when $\epsilon_A^M < \underline{\epsilon}_A$, the acquiring firm's manager may be willing to make the equity offer. Indeed, equity offers are especially attractive precisely when ϵ_A^M is small.

In contrast, any cash offer that is individually rational for an acquiring firm's manager is also preferred by its marginal shareholder, implying that the acquiring firm's share price rises. Intuitively, all parties value cash in the same way. Hence, a cash offer that appeals to the acquiring firm's management also appeals to its shareholders. This result is consistent with Andrade et al.'s (2001) empirical finding that acquiring firms' share prices tend to drop following stock acquisitions, but not cash acquisitions.

We next establish that not only may the acquirer's share price fall following an equilibrium equity offer, but it can fall by so much that the combined return is negative:

Proposition 6 *(i) If the acquirer manager's private valuation is less than the marginal shareholder's* ($\epsilon_A^M < \underline{\epsilon}_A$) *and the synergy is less than* $\min\{\underline{\epsilon}_A, \underline{\epsilon}_T\}$*, then the combined acquirer and target return is negative following an equilibrium equity offer, i.e.,* $R_E < 0$*.*

*(ii) Under **A2** and $P_A \geq 2P_T$ (or $V_A + S > P_T$) the combined return to the target and acquiring firm is always positive following an equilibrium cash offer.*

The result that the combined return is always positive following an equilibrium cash offer follows directly from the results that the acquirer and target each experience positive returns. The result that the combined return can be negative following an equilibrium equity offer implies the previous result that the acquirer's return can be negative.

Proposition 6 reveals that a negative combined return following an equilibrium equity offer does not mean that synergies are negative. Rather, combined returns can be negative even when synergies are positive because pre-merger, shareholders hold the firms they value most, but post-merger, they must hold both firms, diluting their claims to their preferred firms. In Section 3.3, we investigate aggregate shareholder welfare and its relationship with the size of the synergy and the combined return.

The size of the lost value to an acquiring shareholder increases in his private valuation and the extent of the dilution of his claim to that private valuation. If the private valuation of the acquiring firm's management is less than that of its marginal shareholder's, the marginal shareholder may suffer a loss when its management's payoff is positive, but sufficiently small. Further, the attraction of equity offers relative to cash offers rises when ϵ_A^M is smaller—precisely because the acquiring firm's management does not mind diluting its private valuation by as much. Here, $\epsilon_A^M < \underline{\epsilon}_A$ captures shareholders who attach higher valuations to the firm's assets than management.[14] More generally, more extensive investor heterogeneity, as captured by a larger value of $\underline{\epsilon}_A$, can cause the acquiring firm's share price to fall.

We conclude by comparing combined acquirer and target returns for equilibrium acquisitions. Recall that when the payment method was exogenous and $\underline{\epsilon}_A \geq \underline{\epsilon}_T$, the combined return in a cash acquisition exceeded that in an equity acquisition (Result 5). This result extends when the acquiring firm's manager selects his preferred payment method:

[14] Agency considerations (e.g., a manager's empire building motives) could also lead to a decrease in the acquirer's return, just as when the manager's private valuation differs from target shareholders. However, a difference exists: a manager's private benefit of control does not vary with the payment method, so agency considerations do not have the same differential implications for the choice between cash and equity that differences between a manager's private valuation and that of the median target shareholder do.

Proposition 7 *Consider two equilibrium takeovers in settings that are identical save for the private valuation of the acquiring firm's management* ϵ_A^M*, so that one acquisition is with equity and the other is with cash. Under* ***A2*** *(*$\underline{\epsilon}_A \geq \underline{\epsilon}_T$*), the combined return in the cash acquisition exceeds that in the equity acquisition.*

Summarizing these results reveals that, consistent with empirical findings, cash acquisitions are associated with positive and higher returns than equity acquisitions, the target experiences positive returns, but equity acquisitions can be associated with negative combined acquirer-target returns, even when equity acquisitions are optimal.

Share Price Dynamics Over the Takeover Process. Following a cash offer, the target's share price will rise to reflect that the offer is at a premium over the target's stand-alone price, and that with positive probability the median shareholder's private valuation will be low enough that the takeover succeeds. If the offer is accepted, the target's share price will rise further to reflect the beneficial resolution of uncertainty from the perspective of its marginal shareholder. However, if the median shareholder's private valuation is higher, the offer will be rejected, and the target's share price will fall to its pre-takeover value, $V_T + \underline{\epsilon}_T$.

Moreover, since a cash offer that appeals to the acquiring firm's management also appeals to its shareholders, following a cash bid the acquiring firm's share price will rise to reflect the positive probability that the bid will succeed. The share price would rise further upon acceptance, reflecting the beneficial resolution of takeover uncertainty from the perspective of the acquirer; but fall to its level prior to the emergence of synergies whenever its offer is rejected. Hence, we have the following testable predictions:

Corollary 2 *Suppose that* ***A2*** *(*$\underline{\epsilon}_A \geq \underline{\epsilon}_T$*) and* $P_A \geq 2P_T$ *(or* $V_A + S > P_T$*) hold. Then, the share prices of both the target and acquiring firm rise when synergies emerge and a cash bid is made, and rise further if the bid succeeds. Both firms' share prices fall if the bid fails.*

If, instead, an acquirer makes an equity bid rather than cash, the *target*'s share price exhibits similar dynamics. However, the *acquirer*'s share price dynamics are unchanged *only* if synergies are high enough that a successful takeover results in positive acquirer returns.

Otherwise, the acquiring firm's share price will fall after an equity takeover bid, *and fall further if the takeover succeeds*. This prediction is the opposite of that implied by takeover theories based on asymmetric information: when an acquiring firm has private information about its value, equity offers would suggest that its stock is overvalued, so that its share price could fall following an equity offer due to the bad news revealed. However, subsequently, the acquirer's share price should rise with approval as long as synergies are positive, or if approval reflects positive private target shareholder information; and it should fall when takeovers fail due to any negative information revealed by the rejection about the acquirer and the loss of synergies. In contrast, in our setting, if an acquiring firm's stock price falls following an equity offer and there is uncertainty over whether the offer would be accepted, then it should fall further following acceptance, but rise following rejection.

It is difficult to test these predictions directly due to the endogeneity and selection issues associated with accepted and rejected offers (for example, outside of our model, the takeover negotiation process may feature the possibility of a subsequent offer if an initial offer is rejected). An insight from Savor and Lu (2009) is that one can get clean identification by focusing on takeovers that fail for exogenous reasons, an approach that Masulis et al. (2012) also employ. Then our model predicts that the acquirer's share price should rise back to its original level when the failure of a takeover is announced (as the transaction is unwound). Consistent with our model, in the three day window around the announcement of a takeover's failure, Savor and Lu (2009) find abnormal acquirer returns of 3 percent, which just offset the negative abnormal acquirer's returns of 3 percent when a takeover with equity was first announced. These twin results provide strong confirmation of our theory.

3.3 Welfare Effects

In this section we examine the welfare consequences of mergers. With homogeneous valuations, combined returns are a good measure of welfare gains or losses. However, the literature also uses combined returns as a proxy for changes in social welfare in settings where investors have heterogeneous valuations. Here, we take a closer look at the welfare consequences of mergers. We show that with heterogeneous valuations the combined return ceases to be a good measure of welfare change, and that it typically overestimates welfare gains. For simplicity, we focus on equity offers (cash offers have the same qualitative features).

To proceed, it is useful to calculate the fraction of (pre-merger) firm j held by type-j investors with private valuations between $\underline{\epsilon}_j$ and ϵ_j, for each $j \in \{A, T\}$. From equation (26) in the appendix, this fraction is given by $L_j(\epsilon_j) = 1 - \frac{\tilde{G}_j(\epsilon_j)}{V_A + \epsilon_j}$ for $\epsilon_j \in [\underline{\epsilon}_j, \infty)$, where $\tilde{G}_j(\epsilon_j)$ is the cumulative wealth of type j investors with private valuations of at least ϵ_j, where $L_j(\underline{\epsilon}_j) = 0$ and $L_j(\infty) = 1$.

Pre-merger, the total utility of all shareholders of the acquiring and target firms is

$$\int_{\underline{\epsilon}_A}^{\infty} (\epsilon_A + V_A)\, dL_A(\epsilon_A) + \int_{\underline{\epsilon}_T}^{\infty} (\epsilon_T + V_T)\, dL_T(\epsilon_T) = V_A + V_T + \int_{\underline{\epsilon}_A}^{\infty} \epsilon_A dL_A(\epsilon_A) + \int_{\underline{\epsilon}_T}^{\infty} \epsilon_T dL_T(\epsilon_T).$$

Just after the merger (before any subsequent trading), the total utility of these investors is

$$\begin{aligned}
&\frac{1}{I+1}\int_{\underline{\epsilon}_A}^{\infty} (S + \epsilon_A + V_A + V_T)\, dL_A(\epsilon_A) + \frac{I}{I+1}\int_{\underline{\epsilon}_T}^{\infty} (S + \epsilon_T + V_A + V_T)\, dL_T(\epsilon_T) \\
&= S + V_A + V_T + \frac{1}{I+1}\int_{\underline{\epsilon}_A}^{\infty} \epsilon_A dL_A(\epsilon_A) + \frac{I}{I+1}\int_{\underline{\epsilon}_T}^{\infty} \epsilon_T dL_T(\epsilon_T).
\end{aligned}$$

The difference between these two measures represents the increase or decrease in social welfare immediately after the merger. We write this change in social welfare as $S - \Delta\pi_1$, where

$$\Delta\pi_1 = \frac{I}{I+1}\int_{\underline{\epsilon}_A}^{\infty} \epsilon_A dL_A(\epsilon_A) + \frac{1}{I+1}\int_{\underline{\epsilon}_T}^{\infty} \epsilon_T dL_T(\epsilon_T). \tag{20}$$

Recall that trade will occur after a successful equity offer if $\underline{\epsilon}_A \neq \underline{\epsilon}_T$. In equilibrium, the private valuation of a marginal shareholder in the joint firm $(\tilde{\underline{\epsilon}}_J)$ satisfies (3). The gains from this trading increase social welfare. Denoting this increase by $\Delta\pi_2$, we have

$$0 \leq \Delta\pi_2 \leq \begin{cases} \frac{I(\underline{\epsilon}_A - \underline{\epsilon}_T)}{I+1} & \text{if} \quad \underline{\epsilon}_A \geq \underline{\epsilon}_T \\ \frac{(\underline{\epsilon}_T - \underline{\epsilon}_A)}{I+1} & \text{if} \quad \underline{\epsilon}_A < \underline{\epsilon}_T \end{cases}. \tag{21}$$

To understand (21), suppose that $\underline{\epsilon}_A \geq \underline{\epsilon}_T$. The fraction of the joint firm held by the original target shareholders with private valuations between $\underline{\epsilon}_T$ and $\underline{\epsilon}_A$ cannot exceed $\frac{I}{I+1}$. These target shareholders trade with type-ϵ_A investors with private valuations below $\underline{\epsilon}_A$ who had not previously held the acquiring firm and hence have cash available for trade. It follows that the gain in social welfare for each share traded cannot exceed $\underline{\epsilon}_A - \underline{\epsilon}_T$.

The total change in welfare due to the merger is then

$$\Delta W = S - \Delta\pi_1 + \Delta\pi_2. \tag{22}$$

We next characterize the relation between S and ΔW in terms of model primitives. This will prove useful for understanding the relationship between combined firm returns and welfare.

Result 6 $S - \Delta\pi^* - \max\{\underline{\epsilon}_T, \underline{\epsilon}_A\} \leq \Delta W \leq S - \Delta\pi^* - \min\{\underline{\epsilon}_T, \underline{\epsilon}_A\}$, *where* ΔW *is the total change in social welfare and*

$$\Delta\pi^* = \frac{I}{I+1}\int_{\underline{\epsilon}_A}^{\infty}(\epsilon_A - \underline{\epsilon}_A)\,dL_A(\epsilon_A) + \frac{1}{I+1}\int_{\underline{\epsilon}_T}^{\infty}(\epsilon_T - \underline{\epsilon}_T)\,dL_T(\epsilon_T). \tag{23}$$

Any increase in total shareholder welfare is always less than the size of the synergy. This reflects that, with heterogeneous valuations, a merger dilutes an investor's holding of his preferred firm, and the resulting loss of social welfare is too large to be compensated by the welfare increase due to subsequent trading.

We next compare this change in welfare with a total firm return-based proxy, defined as

$$\Delta W_{return} = \widetilde{MV}_J - P_T - P_A = S + \tilde{\underline{\epsilon}}_J - \underline{\epsilon}_A - \underline{\epsilon}_T, \tag{24}$$

where $\widetilde{MV}_J$ is the market value of the joint firm defined in (7). We show that with heterogeneous valuations, this proxy incorrectly measures welfare, typically over-stating the welfare gain (or underestimates the welfare loss), ΔW. We first derive a bound from (8) and (11):

$$S - \max\{\underline{\epsilon}_T, \underline{\epsilon}_A\} \leq \Delta W_{return} \leq S - \min\{\underline{\epsilon}_T, \underline{\epsilon}_A\}. \tag{25}$$

Combining these bounds with Result 6 yields bounds on $\Delta W_{return} - \Delta W$:

Result 7 $\Delta\pi^* - |\underline{\epsilon}_A - \underline{\epsilon}_T| \leq \Delta W_{return} - \Delta W \leq \Delta\pi^* + |\underline{\epsilon}_A - \underline{\epsilon}_T|$, *where* $\Delta\pi^*$ *is defined in (23).*

When $|\underline{\epsilon}_A - \underline{\epsilon}_T|$ is small (see the discussion of Assumption **A1** in Section 3), Result 7 shows that ΔW_{return} over-estimates the welfare gains (understates the losses) from the merger of ΔW by about $\Delta\pi^*$. Economically, $\int_{\underline{\epsilon}_j}^{\infty}\left(\epsilon_j - \underline{\epsilon}_j\right)dL_j(\epsilon_j)$ in $\Delta\pi^*$ is the difference between the average private valuation of firm j's shareholder and the private valuation of its marginal shareholder. Empirically, this term could be estimated using the one-year-ahead share price "targets" set by institutional analysts: one could take the average of those price targets that exceed the outstanding share price and subtract the outstanding share price (and multiply by the number of shares outstanding).

Intuitively, the return-based proxy overestimates welfare gains because returns reflect changes in the utilities of the marginal shareholders, whereas the average shareholders have higher private valuations and hence incur larger losses due to a merger's dilution of their private valuations. As a result, total surplus may fall even if stock returns are positive; and greater positive skewness in the distribution of private values in target firm shareholders magnify these welfare losses.

Not only do heterogeneous valuations result in welfare losses that exceed those implied by a combined return proxy, they also introduce ex-post inefficiencies: due to the standard freeze-out condition for non-tendering shareholders, target shareholders with high valuations suffer larger losses since these minority shareholders must take an offer that appeals to the majority shareholders who suffer less from the dilution in their private valuations. Such losses are larger when the distribution of shareholder valuations is more positively skewed.

4 Comparative Statics

In this section, we derive testable implications related to an acquirer's offer, its probability of success, and share prices. To ease presentation and analysis, we add structure, assuming that $\underline{\epsilon}_A = \underline{\epsilon}_T \equiv \underline{\epsilon}$, and that the median target shareholder's private valuation, ϵ_T^*, is uniformly distributed on $[\hat{\epsilon} - \alpha, \hat{\epsilon} + \alpha]$, where $\hat{\epsilon} - \alpha > \underline{\epsilon}$. The expected private valuation, $\hat{\epsilon}$, of the median target shareholder measures the extent of heterogeneity in valuations for the target firm between shareholders of the acquiring firm and those of the target firm; and α measures the extent to which an acquiring firm is uncertain about the median target shareholder's private valuation. We focus on cash offers; equity offers have qualitatively similar features. To avoid the complications in cash offers when the median target shareholder derives private benefits from holding the joint company, we assume that $\frac{V_T+\hat{\epsilon}+\alpha}{V_A} << 1$, in which case we can approximate this additional private benefit as zero (see (17)).

Proposition 8 characterizes how the optimal offer and probability of success vary with the primitive parameters:

Proposition 8 (i) *As the mean private valuation $\hat{\epsilon}$ of the median target shareholders rises, the optimal offer C^* increases, but the probability of acceptance falls.*

(ii) *As the value of the synergy S rises, both C^* and the probability of acceptance rise.*

(iii) *As the uncertainty α about the median target shareholder's private valuation rises: (a) if $S < \hat{\epsilon}$, then C^* falls, but the probability of acceptance rises; (b) if $S > \hat{\epsilon}$, C^* first rises and then falls, but the probability of acceptance always falls.*

The result in part (i) that C^* rises with the degree of heterogeneity in private values of target firm shareholders reflects the intuition that a successful offer must win approval from at least 50% of shareholders, who have higher valuations than the marginal shareholder who determines the price. The result in part (ii) that C^* rises with the synergy S is also intuitive, reflecting that the opportunity cost of rejection rises in S. The reason why increased uncertainty can cause an acquirer to reduce its offer, as illustrated in part (iii), is that greater uncertainty raises the likelihood that low offers are accepted; and when synergies are small, the opportunity cost of having an offer rejected is small, making lower offers optimal. If, instead, synergies are large, there is a range where the offer initially rises in α because the acquirer does not want to risk a failed offer. However, as the extent of uncertainty grows, the only way to ensure success is to keep raising the offer, which eventually becomes too costly. Beyond this point, the marginal increase in the probability that a higher offer succeeds is too small to justify increasing the offer further, and the optimal offer C^* begins to fall with α.

The optimal offer rises less than one-for-one with the median target shareholder's expected private valuation $\hat{\epsilon}$, reflecting that the acquirer trades off between the amount paid for a successful offer and the probability of success. Greater synergies induce the acquiring firm to raise its offer, raising the probability it is accepted. To understand why the probability of success rises with the extent of uncertainty α when synergies are small, observe that when synergies are low, the realized valuation of the median target shareholder must be low for the offer to be accepted, and a higher α raises this probability. In contrast, when synergies are high, the probability of success falls with α.[15] This reflects that with increased dispersion in the median target shareholder's possible valuation, it becomes more and more costly for the acquiring firm to maintain a high acceptance probability, leading to a reduced probability of success.

Parts (i) and (iii) of Proposition 8 can be viewed as providing foundations for Hypothesis 1 in Chatterjee et al. (2012), and extending it to consider the consequences of uncertainty about the median target shareholder's valuation. There are two key components to increased target shareholder heterogeneity: (1) the difference between the expected private valuation of the median and marginal target shareholders, $\mathrm{E}[\epsilon_T^*] - \underline{\epsilon}_T$, rises; (2) the extent α of uncertainty about the median target shareholder's valuation also rises. Chatterjee et al.'s (2012) intuition derives from the first source. The premium in the optimal offer rises with $E\left[\epsilon_T^*\right] - \underline{\epsilon}_T$ to raise the likelihood that the median target shareholder accepts it, consistent with their Hypothesis 1. However, part (iii) of Proposition 8 shows that greater uncertainty about the median shareholder's valuation can lower the offer premium when synergies are low. This discussion suggests that there may be value added from estimation approaches that include measures of synergies.

Our core model captures the observation that individual investors focus on a few stocks and are unlikely to have positive private valuations in any given two stocks, by assuming that an individual investor only has a positive private valuation of the target or the acquiring firm. In practice, some investors may have private valuations of both firms. We now provide a comparative static result that illustrates the qualitative consequences. We consider three groups of investors: a fraction $\frac{1-\rho}{2}$ place values $V + \epsilon_A$ on firm A and V on firm T; a fraction $\frac{1-\rho}{2}$ place values $V + \epsilon_T$ on firm T and V on firm A; and a fraction ρ place the same value $V + \epsilon_{AT}$ on both firms T and A. The fraction ρ of investors with positive private valuations

[15]When synergies are so high that the acquiring firm finds it optimal to make an offer that always succeeds, the decrease is only weak—the probability of success stays constant at 100%.

of both firms captures the closeness of the two firms. For example, ρ may be higher when the industries in which the firms operate are more similar.

When the distributions of ϵ_A, ϵ_T and ϵ_{AT} are uniform and each investor has the same wealth, our companion working paper establishes that (1) for all $\rho < 1$, the price of the merged firm is at a discount from the sum of the two firms if the synergy is small enough (but still positive); and (2) the magnitude of this discount rises as ρ decreases. Thus, our model suggests why mergers between less-related firms tend to be associated with lower returns (see e.g., Berger and Ofek, 1995, Lamont and Polk, 2001, or Graham et al., 2002)—the fraction of investors who views the two firms similarly and thus assign a positive private value to both firms is smaller. Our core model delivers the intuition that this "diversification discount" reflects the differences in valuations between target and acquiring firm shareholders of each other's firm, as a merger dilutes a shareholder's holdings of his preferred firm. That is, a greater diversification discount need not reflect lower or more negative synergies, but rather that shareholders in the two firms differ more substantively in their valuations of each other's firm. When some investors have positive private valuations of both firms, this intuition extends in that the diversification discount reflects a measure of average differences in valuations between target and acquiring firm shareholders of each other's firm. Moreover, presuming that the magnitude of such differences is larger when the two firms are from less-related industries, our model predicts that the discount should be larger, consistent with the data.

5 Further Discussion

Approval of Acquirer Shareholders. Our model assumes that voting by shareholders of the acquiring firm is not needed for the takeover to be effected. This is consistent with practice in the U.S., where shareholder voting on acquisitions is optional for cash deals, and is only required for equity financed deals when new share issuance exceeds 20% of an acquirer's outstanding equity. Moreover, even for these cases, Becht, Polo, and Rossi (2014) observe that an acquiring firm's management can circumvent this voting requirement by funding the deal using less equity and more debt or cash.

When acquiring firm shareholders must approve an offer, the offer must make the *median* acquirer shareholder better off. This has implications for the acquirer's share price after an equity acquisition. An equity acquisition dilutes acquirer shareholders' claims to the acquiring firm, generating losses of private valuation. Because the median acquirer shareholder's valuation exceeds the marginal shareholder's valuation, if the median shareholder gains from an acquisition, so does the original marginal shareholder in the acquiring firm. For all practical purposes, this precludes the possibility that an acquirer's share price may fall after equity acquisitions that require acquirer shareholder approval, in contrast to equity acquisitions that do not (to which Proposition 5 (i) applies).[16] Becht, Polo, and Rossi (2014) provide strong empirical support for this contrast. In the UK, shareholder voting is mandatory for deals that exceed a relatively low valuation threshold, but is discretionary below that threshold. Becht, Polo, and Rossi (2014) exploit this discontinuity to establish that when approval is required, the acquiring firm's share price rises by 8% on average, but its share price falls when approval is not required.[17] Importantly, private-information based explanations predict the same decline in the acquiring firm's share price following an equity offer (which indicates that the acquiring firm's management believes its shares are overpriced), *regardless* of whether acquirer shareholder approval is required. Hence, this evidence provides key support for our model.

External Financing. In establishing Result 5 and Proposition 7, we implicitly assume that the acquiring firm has enough cash to make a cash acquisition. Here, we discuss what happens when the acquiring firm lacks cash, so that its cash offer is financed by issuing equity.

First consider a benchmark setting of auctions, where the main friction is information asymmetry between bidders and the seller (see Skrzypacz, 2013). In such settings, bidding with cash and financing entirely by equity issuance has the same revenue consequence as

[16]Matvos and Ostrovsky (2008) show that when institutional investors hold both sides of a transaction, owning shares in both the acquiring firm and the target firm, they are more likely to vote in favor of the acquisition when shareholder meetings at the acquiring firm call for such a vote. While our model does not include shareholders who hold positions in both firms (see Section 4 for a discussion of this direction), including such shareholders would be an interesting extension, and when such investors have similar private valuations of the two firms, we can reconcile their findings.

[17]Relatedly, Hsieh and Wang (2008) report that an acquirer's share price is more likely to fall when acquisitions are structured to bypass shareholder scrutiny by funding a deal with less equity and more debt or cash.

bidding with equity; and if bidders can finance a cash acquisition by different means, a pecking order obtains: bidders prefer to finance via internal capital to equity (Liu 2012).[18]

In contrast, in our setting, the key friction stems from heterogeneous valuations. As in auction settings, paying with cash and financing an acquisition entirely by equity issuance leads to the same outcome as paying with equity.[19] However, the implications for the pecking order are different, reflecting the difference in underlying frictions. Proposition 3 reveals that an acquirer's preferred means of financing hinges on the private valuation of its manager vs. that of the median target shareholder—internal cash is optimal when the manager has a relatively higher private valuation, and equity financing is optimal when the reverse holds.

6 Conclusion

In this paper, we integrate heterogeneity and uncertainty in investor valuations of firms into a model of takeovers, and study the choice between cash and equity offers that are geared toward maximizing the payoff of the acquiring firm's manager. In equilibrium, share prices are determined via market-clearing conditions and reflect the valuations and optimizing behavior of all parties. These elements allow us to reconcile an array of empirical regularities and provide new testable predictions. For instance, our model implies that combined target-acquirer returns are higher after cash acquisitions than after equity acquisitions when the method of payment is chosen optimally; that the difference in the combined acquirer and target return between cash and equity acquisitions should rise with the difference in the sizes of the acquirer and the target; and that CEOs of acquiring firms with greater shareholdings should be more likely to use cash. We also derive implications for the patterns of share price dynamics following successful and unsuccessful takeover offers which can be distinguished

[18]If a bidder has to obtain post-auction financing, he has incentives to signal high valuations by over-bidding during the auction, which influences post-auction financing terms. Bidders dislike equity financing because signaling is costly. Moreover, if a bidder could finance by debt, then because debt financing induces smaller signaling incentives than equity financing (the value of debt is less information-sensitive than the value of equity), a bidder's preferred means of financing is first internal capital, then debt, and finally equity.

[19]This equivalence presumes the market anticipates the subsequent equity issuance at the time of the cash offer. We also assume the financing is entirely by equity issuance; if, instead, the financing is a mix of equity and cash, then our qualitative results concerning the differences between cash and equity acquisitions extend.

from alternative theories, such as those based on asymmetric information.

An issue not analyzed here, but of empirical relevance, is the role of management at the target firm. If target management has private information about target assets and can make recommendations to shareholders about whether to accept or reject offers, then its own valuation for its firm can play an important role in these recommendations. Endogenizing whether a merger offer is made based on the expectation of managerial support or resistance to an offer would have implications for the probability of acceptance by shareholders conditional on target management's endorsement (i.e., a "merger") or resistance (i.e., a "hostile" tender offer). Because recommendations reflect both target management's private valuation and its private information, target shareholders may not be able to fully infer its management's private information from the recommendation given. This, in turn, has implications for how the recommendation influences shareholders' voting decisions and, ultimately, for how the acquiring firm structures its offer.

Another interesting aspect to consider is whether and when firms want to put themselves "in play" and try to attract potential buyers. In line with a large literature on takeovers, we take the possibility of an acquisition as given, arising due to synergies that materialize. However, the study of when a firm wants to initiate a sale by approaching potential buyers may be important to the extent that it speaks to the timing of acquisitions. Gorbenko and Malenko (2014) study this in the context of a real option model where a seller has to decide whether to approach possible buyers, or wait to be approached. We leave analysis of this issue to the future.

One additional interesting aspect to consider would be how a firm decides to be an acquirer for a specific target firm. In principle, one could imagine that some firms view themselves as potential buyers and search for reasonable targets. Such competition could determine the value of the synergy relative to other variables (to the extent that potential acquirers may have heterogeneous synergies for a given target), as well as which firms or managers see the deal as attractive from the perspective of giving away the least value in the acquisition process, particularly in the case of equity acquisitions. Incorporating such an analysis raises considerations that go well beyond the scope of this paper, and is left for future research.

7 Appendix A

7.1 Model Foundations

Relation between wealth function and mass measure: For each $j \in \{A, T\}$, we denote the measure of type j investors by $Y_j(\cdot) : [0, \infty) \to [0, M_j]$; that is, $Y_j(\epsilon_j)$ denotes the mass of those type j investors whose private valuations do not exceed ϵ_j, and M_j denotes the total mass of all type j investors. The wealth function $\tilde{G}_j(\cdot)$ is related to type j investors' measure by $\tilde{G}_j(\epsilon_j) = \int_{\epsilon_j}^{\infty} W_j(z)\, dY(z)$, where $W_j(\cdot)$ denotes the wealth density of type ϵ_j investors.

Market-clearing: Market clearing pins down the private valuation $\underline{\epsilon}_j$ of the marginal shareholder of firm j:

$$V_j + \underline{\epsilon}_j = \tilde{G}_j\left(\underline{\epsilon}_j\right). \tag{26}$$

Equation (26) reflects optimization by investors: they invest in neither firm if each firm is deemed to be overvalued, or they invest all their wealth in the firm deemed to be most undervalued. Thus, a type ϵ_j investor invests all his wealth in firm j if $\epsilon_j > \underline{\epsilon}_j$, and invests in neither firm if $\epsilon_j < \underline{\epsilon}_j$. From (26), if $\tilde{G}_j(0) > V_j$, then $\underline{\epsilon}_j > 0$, and (1) and (26) give

$$P_j = \tilde{G}_j\left(\underline{\epsilon}_j\right), \quad j = A, T. \tag{27}$$

Median target shareholder valuation: The median target shareholder's valuation ϵ_T^* solves $\tilde{G}_T(\epsilon_T^*) = \frac{1}{2}\tilde{G}_j\left(\underline{\epsilon}_j\right)$, which, by (26), is equivalent to

$$\tilde{G}_T(\epsilon_T^*) = \frac{1}{2}\left(V_T + \underline{\epsilon}_T\right). \tag{28}$$

From equation (27), the trading price of firm j does not reveal the exact form of $\tilde{G}_j(\cdot)$. Thus, even conditional on observing the trading price, uncertainties may exist concerning the form of $\tilde{G}_j(\cdot)$, and hence on ϵ_T^*, which is the solution for (28).

7.2 Exogenous Equity Offers

As Section 3 details, just after a successful equity offer, trade occurs if $\underline{\epsilon}_A \neq \underline{\epsilon}_T$. There are two scenarios:

(i) If $\epsilon(I) \geq \underline{\tilde{\epsilon}}_J$, then the median target shareholder will hold the joint firm, so his post-takeover per-share payoff is $\pi_E = \frac{V_T+V_A+S+\epsilon(I)}{1+I}I$.

(ii) If $\epsilon(I) < \underline{\tilde{\epsilon}}_J$, then the median target shareholder will not hold the joint firm. Instead, he will sell his shares at the market price, which is determined by the marginal holder of the joint firm, so his (random) post-takeover per-share payoff is $\pi_E = \frac{V_T+V_A+S+\underline{\tilde{\epsilon}}_J}{1+I}I$.

Summing over the two possibilities, we have

$$\pi_E = \frac{V_T + V_A + S + \max\left(\underline{\tilde{\epsilon}}_J, \epsilon(I)\right)}{1+I} I.$$

Then the indifference condition implies that $\epsilon(I)$ solves

$$\frac{V_T + V_A + S + \max\left(\underline{\tilde{\epsilon}}_J, \epsilon(I)\right)}{1+I} I = V_T + \epsilon(I). \tag{29}$$

The value of $\underline{\tilde{\epsilon}}_J$ is pinned down by the market-clearing condition:

$$\begin{aligned} &\frac{I}{1+I}\left(1 - \frac{\tilde{G}_T\left(\underline{\tilde{\epsilon}}_J\right)}{\tilde{G}_T\left(\underline{\epsilon}_T\right)}\right)\left(V_T + V_A + S + \underline{\tilde{\epsilon}}_J\right) = G_A\left(\underline{\tilde{\epsilon}}_J\right) - \tilde{G}_A\left(\underline{\epsilon}_A\right) \quad \text{if } \underline{\epsilon}_A > \underline{\epsilon}_T \quad (i) \\ &\frac{1}{1+I}\left(1 - \frac{\tilde{G}_A\left(\underline{\tilde{\epsilon}}_J\right)}{\tilde{G}_A\left(\underline{\epsilon}_A\right)}\right)\left(V_T + V_A + S + \underline{\tilde{\epsilon}}_J\right) = \tilde{G}_T\left(\underline{\tilde{\epsilon}}_J\right) - \tilde{G}_T\left(\underline{\epsilon}_T\right) \quad \text{if } \underline{\epsilon}_T > \underline{\epsilon}_A \ (ii). \end{aligned} \tag{30}$$

To understand part (i) of (30), note that $\frac{I}{1+I}$ on the left-hand-side is the fraction of the joint firm held by the original target shareholders, $\left(1 - \frac{\tilde{G}_T(\underline{\tilde{\epsilon}}_J)}{\tilde{G}_T(\underline{\epsilon}_T)}\right)$ is the fraction of the original target shareholders who want to sell their holdings of the joint firm (these shareholders have private valuations below $\underline{\tilde{\epsilon}}_J$), and $(V_T + V_A + S + \underline{\tilde{\epsilon}}_J)$ is the joint firm's market value. Thus, the left-hand-side is the total dollar amount that those original target shareholders (who wish to sell) can sell for, which must equal the right-hand-side, which is the total wealth of those type ϵ_A investors who will buy the joint firm (these investors have private valuations exceeding $\underline{\tilde{\epsilon}}_J$). Part ($ii$) of (30) follows from a similar structure: $\frac{1}{1+I}$ is the fraction of the joint firm held by the original acquirer shareholders, $\left(1 - \frac{\tilde{G}_A(\underline{\tilde{\epsilon}}_J)}{\tilde{G}_A(\underline{\epsilon}_A)}\right)$ is the fraction of the original acquirer shareholders who want to sell their holdings of the joint firm, and $(V_T + V_A + S + \underline{\tilde{\epsilon}}_J)$ is the joint firm's market value. Thus, the left-hand-side is the total dollar amount that those original acquirer shareholders (who wish to sell) can sell for, which must equal the right-hand-side, which is the total wealth of those type ϵ_T investors who will buy the joint firm. The system of equations, (29) and (30), jointly determine the values of I and $\underline{\tilde{\epsilon}}_J$.

7.3 Sufficient Conditions in Exogenous Cash Offers

Denote the upper bound of the support of private valuations for shareholders of the acquiring firm by $\bar{\epsilon}_A$. Lemma 2 identifies sufficient conditions for the median target shareholder to hold or not hold the joint firm:

Lemma 2 *Define* $\tilde{F}_A(\epsilon_A) \equiv 1 - \frac{\tilde{G}_A(\epsilon)}{V_A+\epsilon_A}$ *for all* $\epsilon_A \in [\underline{\epsilon}_A, \bar{\epsilon}_A]$*. For any cash offer that satisfies (14), if* $(V_A + S)\tilde{F}_A\left(\min\left\{\epsilon_T^l, \bar{\epsilon}_A\right\}\right) > V_T + \epsilon_T^h$*, the original median target shareholder holds the joint firm, and* $C < V_T + \epsilon(C)$*. If, instead,* $\underline{\epsilon}_A > \epsilon_T^h$*, the original median target shareholder does not hold the joint firm, and* $C = V_T + \epsilon(C)$*.*

Proof of Lemma 2: To prove the first part of the lemma, suppose the conclusion is false, i.e., that $\tilde{P}_J \geq V_T + V_A + S - C + \epsilon(C)$, so that $C = V_T + \epsilon(C)$. After a successful cash offer, original shareholders of the acquiring firm for whom $V_A + V_T + S + \epsilon_A - C < \tilde{P}_J$ want to sell their shares. The value of their shares is $\tilde{P}_J\tilde{F}_A(\min\{(P_J - V_A - V_T - S + C), \bar{\epsilon}_A\})$. Substituting for $\tilde{P}_J$ and C, the value of their shares is at least

$$\begin{aligned}(V_T + V_A + S - C^* + \epsilon(C))\,\tilde{F}_A(\min\{\epsilon(C), \bar{\epsilon}_A\}) &= (V_A + S)\tilde{F}_A(\min\{\epsilon(C), \bar{\epsilon}_A\}) \\ &\geq (V_A + S)\tilde{F}_A\left(\min\left\{\epsilon_T^l, \bar{\epsilon}_A\right\}\right).\end{aligned}$$

On the demand side, shareholders of the original target for whom $V_A + V_T + S + \epsilon_T - C > \tilde{P}_J$ wish to buy shares in the joint firm, and they have cash not exceeding $C = V_T + \epsilon(C) \leq V_T + \epsilon_T^h$ to invest. Thus, equating total demand with the value of the shares supplied yields $\left(V_T + \epsilon_T^h\right) \geq (V_A + S)\tilde{F}_A\left(\min\left\{\epsilon_T^l, \bar{\epsilon}_A\right\}\right)$, contradicting the lemma's premise, thus establishing the first part of the lemma. The proof of the second part of the lemma is in the text. $\square$

To understand the intuition for Lemma 2, note that $\tilde{F}_A(\epsilon_A)$ is the number of shares of the acquiring firm held by investors with private valuation below ϵ_A. The first part of the lemma essentially says that if the value of the synergies plus the market value of the portion of the acquiring firm held by shareholders whose private valuations are less than the median target shareholder's is large relative to the target's market value, then $\tilde{P}_J$ becomes high relative to the cash that target shareholders receive. As a result, target shareholders do not purchase enough of the joint firm to drive its price up past the value to the original median target shareholder. The second part of the lemma follows from (13): in the less plausible scenario where the private valuation of the *marginal* holder of the acquiring firm always exceeds the *median* target shareholder's value, the median target shareholder will not hold the joint firm.

8 Appendix B: Proofs

Proof of Lemma 1: By **A3** and the fact that $\epsilon_T^l \geq \underline{\epsilon}_T$, (2) yields $\epsilon(I) \geq \max\{\underline{\epsilon}_T, \underline{\epsilon}_A\}$. Then, by (3), we have $\epsilon(I) \geq \tilde{\underline{\epsilon}}_J$. The lemma follows. □

Proof of Proposition 1: The indifference condition (29) yields

$$\frac{I}{1+I} = \frac{V_T + \epsilon(I)}{V_T + V_A + S + \max\left(\tilde{\underline{\epsilon}}_J, \epsilon(I)\right)}, \tag{31}$$

which gives

$$\begin{aligned}
\tilde{P}_J I &= \frac{V_T + V_A + S + \tilde{\underline{\epsilon}}_J}{1+I} I \\
&= \frac{V_T + V_A + S + \tilde{\underline{\epsilon}}_J}{V_T + V_A + S + \max\left(\tilde{\underline{\epsilon}}_J, \epsilon(I)\right)} \left(V_T + \epsilon(I)\right).
\end{aligned}$$

If $\tilde{\underline{\epsilon}}_J \geq \epsilon(I)$, then $\tilde{P}_J I = V_T + \epsilon(I) > V_T + \underline{\epsilon}_T = P_T$, establishing the proposition. Now suppose that $\tilde{\underline{\epsilon}}_J < \epsilon(I)$. Since $\underline{\epsilon}_T \geq \underline{\epsilon}_A$, then because $\tilde{\underline{\epsilon}}_J$ is between $\underline{\epsilon}_T$ and $\underline{\epsilon}_A$, we have $\tilde{\underline{\epsilon}}_J \geq \underline{\epsilon}_T$. Thus,

$$\begin{aligned}
\tilde{P}_J I - P_T &= \frac{V_T + V_A + S + \tilde{\underline{\epsilon}}_J}{V_T + V_A + S + \epsilon(I)} \left(V_T + \epsilon\left(I\right)\right) - V_T - \underline{\epsilon}_T \\
&\geq \frac{V_T + V_A + S + \underline{\epsilon}_T}{V_T + V_A + S + \epsilon(I)} \left(V_T + \epsilon(I)\right) - V_T - \underline{\epsilon}_T \\
&= V_T + \epsilon(I) - \frac{\left(\epsilon(I) - \underline{\epsilon}_T\right)\left(V_T + \epsilon(I)\right)}{V_T + V_A + S + \epsilon(I)} - V_T - \underline{\epsilon}_T \\
&= \frac{\left(\epsilon(I) - \underline{\epsilon}_T\right)\left(V_A + S\right)}{V_T + V_A + S + \epsilon(I)} \\
&> 0,
\end{aligned} \tag{32}$$

establishing the proposition.□

Proof of Result 4: By Lemma 1, (4) holds with equality. Substitute it into (12) yields for the joint firm's share price

$$\tilde{P}_J = \frac{V_T + V_A + S + \tilde{\underline{\epsilon}}_J}{V_T + V_A + S + \epsilon(I)} \left(V_A + S\right). \tag{33}$$

Note from equation (33) that $\tilde{P}_J < V_A + S$ (because $\epsilon(I) > \tilde{\underline{\epsilon}}_J$). Therefore, from equation (1), if $S < \underline{\epsilon}_A$, then $\tilde{R}_A = \frac{\tilde{P}_J - P_A}{P_A} < 0$. □

Proof of Proposition 2: To prove the first statement, suppose instead that $C \leq P_T = V_T + \underline{\epsilon}_T$. Because $\underline{\epsilon}_T < \epsilon(C)$, we have $C < V_T + \epsilon(C)$. Then the target shareholder with private valuation $\epsilon(C)$ must hold the joint firm, i.e., $V_T + V_A + S - C + \epsilon(C) > \tilde{P}_J$ and equation (15) (i) holds.

We now assume the condition $P_A > 2P_T$. Upon rearranging terms, equation (15) (i) gives

$$V_T = \frac{V_T + V_A + S - C + \epsilon(C)}{\tilde{P}_J} C - \epsilon(C). \tag{34}$$

Claim 1: The right-hand side of (34), when treating C and $\epsilon(C)$ as two independent variables, strictly decreases in $\epsilon(C)$.

Claim 1 follows since the derivative of this right-hand side with respect to $\epsilon(C)$ is

$$\begin{aligned} \frac{C}{\tilde{P}_J} - 1 \ = \ \frac{2C - V_T - V_A - S - \epsilon_J}{\tilde{P}_J} &< \frac{2C - V_T - V_A - \epsilon_A}{\tilde{P}_J} \\ &< \frac{2P_T - V_T - P_A}{\tilde{P}_J} < \frac{2P_T - P_A}{\tilde{P}_J} < 0. \end{aligned}$$

Because $\epsilon(C) > \underline{\epsilon}_T$, (34) together with the claim yield

$$V_T < \frac{V_T + V_A + S - C + \underline{\epsilon}_T}{\tilde{P}_J} C - \underline{\epsilon}_T < C - \underline{\epsilon}_T,$$

or $C > V_T + \underline{\epsilon}_T = P_T$, a contradiction to the premise that $C \leq P_T$.

We next assume the condition $V_A > P_T - S$ rather than $P_A > 2P_T$. It is straightforward to show the following:

Claim 2: The expression $(V_T + V_A + S - C + \epsilon(C))\, C$, when treating C and $\epsilon(C)$ as two independent variables, increases in C for all $C \in [0, P_T]$.

Therefore, (15) (i) yields

$$
\begin{aligned}
V_T + \epsilon(C) \; = \; \frac{V_T + V_A + S - C + \epsilon(C)}{\tilde{P}_J} C &\leq \frac{V_T + V_A + S - P_T + \epsilon(C)}{\tilde{P}_J} P_T \\
&= \frac{V_A + S + \epsilon(C) - \underline{\epsilon}_T}{\tilde{P}_J} (V_T + \underline{\epsilon}_T) \\
&\leq \frac{V_A + S + \epsilon(C) - \underline{\epsilon}_T}{V_T + V_A + S + \underline{\epsilon}_T - C} (V_T + \underline{\epsilon}_T),
\end{aligned}
$$

where the first inequality follows from Claim 2 and the premise $C \leq P_T$, and the second inequality follows from $\tilde{P}_J \geq V_T + V_A + S + \min(\underline{\epsilon}_T, \underline{\epsilon}_A) - C$ and $\underline{\epsilon}_T \leq \underline{\epsilon}_A$. From this, we have

$$
\begin{aligned}
C &\geq V_T + V_A + S + \underline{\epsilon}_T - \frac{V_T + \underline{\epsilon}_T}{V_T + \epsilon(C)} (V_A + S + \epsilon(C) - \underline{\epsilon}_T) \\
&= P_T + (\epsilon(C) - \underline{\epsilon}_T) \frac{V_A + S - P_T}{V_T + \epsilon(C)} \; > \; P_T,
\end{aligned}
$$

contradicting the premise that $C \leq P_T$. This completes the proof of the first statement in the proposition.

To prove the second statement, examine equation (15) (i):

$$
\begin{aligned}
C \; = \; (V_T + \epsilon(C)) \frac{\tilde{P}_J}{V_T + V_A + S - C + \epsilon(C)} &\geq (V_T + \epsilon(C)) \frac{V_T + V_A + S - C + \underline{\epsilon}_T}{V_T + V_A + S - C + \epsilon(C)} \\
&= (V_T + \epsilon(C)) - \frac{(\epsilon(C) - \underline{\epsilon}_T)(V_T + \epsilon(C))}{V_T + V_A + S - C + \epsilon(C)} \\
&\geq (V_T + \epsilon(C)) - \frac{V_T + \epsilon(C)}{V_A + S} (\epsilon(C) - \underline{\epsilon}_T).
\end{aligned}
$$

Rearranging, we have

$$
C - (V_T + \epsilon(C)) \geq -\frac{V_T + \epsilon(C)}{V_A + S} (\epsilon(C) - \underline{\epsilon}_T) \geq -\frac{V_T + \epsilon_T^h}{V_A} \left(\epsilon_T^h - \underline{\epsilon}_T\right).
$$

Taking limits on both sides yields

$$
\lim_{\frac{V_T + \epsilon_T^h}{V_A} \to 0} C - (V_T + \epsilon(C)) \geq 0.
$$

However, because $C \leq (V_T + \epsilon(C))$, we also have $\lim_{\frac{V_T + \epsilon_T^h}{V_A} \to 0} C - (V_T + \epsilon(C)) \leq 0$. Thus, the relationship must hold as an equality. $\square$

Proof of Proposition 3: To prove the first part, note (4) holds with equality. Substituting for I into (18) and omitting the I index yields the acquiring manager's expected per-share payoff in cash acquisition:

$$\pi_{AM}^{E}(\epsilon) = F_T(\epsilon)(V_A + S)\left(\frac{V_T + V_A + S + \epsilon_A^M}{V_T + V_A + S + \epsilon}\right) + (1 - F_T(\epsilon))\left(V_A + \epsilon_M^A\right). \tag{35}$$

From equation (19), the acquirer manager's per-share expected profit in cash acquisition is

$$\pi_{AM}^{C}(C) - \pi_{AM} = \Pr(\epsilon_C(C) \geq \epsilon_T^*)(V_T + S - C). \tag{36}$$

This, combined with $C \leq V_T + \epsilon_C(C)$, yields a lower bound on the manager's profit:

$$\pi_{AM}^{C}(C) - \pi_{AM} \geq \Pr(C - V_T \geq \epsilon_T^*)(V_T + S - C) \tag{37}$$

$$= F_T(C - V_T)(V_T + S - C). \tag{38}$$

For any $\epsilon \in \left[\epsilon_T^l, \epsilon_T^h\right]$, (35) and (38) yield

$$\begin{aligned}\pi_{AM}^{C}(C = V_T + \epsilon) - \pi_{AM}^{E}(\epsilon) &\geq F_T(\epsilon)\left[\epsilon_A^M - \epsilon - \frac{\epsilon_A^M - \epsilon}{V_T + V_A + S + \epsilon}(V_A + S)\right] \\ &= F_T(\epsilon)\left(\epsilon_A^M - \epsilon\right)\frac{V_T + \epsilon}{V_T + V_A + S + \epsilon}.\end{aligned} \tag{39}$$

When $\epsilon_A^M \geq \epsilon_T^h$, this expression is nonnegative for all $\epsilon \in \left[\epsilon_T^l, \epsilon_T^h\right]$, and in particular, for $\epsilon = \epsilon(I^*)$. Thus, $\pi_{AM}^{C}(C = V_T + \epsilon) \geq \pi_{AM}^{E}(I^*)$. As $\pi_{AM}^{C}(C^*) \geq \pi_{AM}^{C}(C = V_T + \epsilon)$, we have $\pi_{AM}^{C}(C^*) \geq \pi_{AM}^{E}(I^*)$. This proves the first part. To prove the second part, note under condition (b) of this part, (39) holds with equality. Further, under condition (a) of this part, the right-hand side of (39) is non-positive. In addition, the inequality in (4) implies (35) holds with an inequality. Following the logic in the proof of the first part establishes the result. $\square$

Proof of Proposition 4: Follows from Propositions 1 and 2. $\square$

Proof of Proposition 5: Part (i) follows from Proposition 4 (i) and Proposition 6 (i). To show part (ii), note the marginal holder of the joint firm in a cash offer has a private valuation of at least $\underline{\epsilon}_A$, the price of the joint firm satisfies

$$\tilde{P}_J \geq V_A + V_T + S - C + \underline{\epsilon}_A = P_A + V_T + S - C.$$

By (36), we have

$$V_T + S - C > 0.$$

Combining these inequalities yields $\tilde{P}_J > P_A$. $\square$

Proof of Proposition 6: We first prove part (i). We start by identifying situations in which the conditions in part (i) are satisfied and an equilibrium equity offer is made.

Claim 1: The acquirer manager is strictly better offer in an optimal equity offer if

$$S > \frac{V_T + S + \epsilon_T^h}{V_T + V_A + S + \epsilon_A^M}\epsilon_A^M + \frac{V_A}{V_T + V_A + S + \epsilon_A^M}\epsilon_T^h. \tag{40}$$

To prove the claim, note that for all $\epsilon \in \left[\epsilon_T^l, \epsilon_T^h\right]$, by (4), we have

$$\begin{aligned}
\pi_{AM}^E\left(I\left(\epsilon\right)\right) - \pi_{AM} &\geq F_T\left(\epsilon\right)\left[S + \frac{\epsilon_A^M - \epsilon}{V_T + V_A + S + \epsilon}\left(V_A + S\right) - \epsilon_A^M\right] \\
&= F_T\left(\epsilon\right)\left[\frac{V_T + V_A + S + \epsilon_A^M}{V_T + V_A + S + \epsilon}S + \frac{\left(\epsilon_A^M - \epsilon\right)V_A}{V_T + V_A + S + \epsilon} - \epsilon_A^M\right] \\
&= F_T\left(\epsilon\right)\left[\frac{V_T + V_A + S + \epsilon_A^M}{V_T + V_A + S + \epsilon}S - \frac{V_T + S + \epsilon}{V_T + V_A + S + \epsilon}\epsilon_A^M - \frac{V_A}{V_T + V_A + S + \epsilon}\epsilon\right] \\
&= F_T\left(\epsilon\right)\frac{V_T + V_A + S + \epsilon_A^M}{V_T + V_A + S + \epsilon}\left(S - \frac{V_T + S + \epsilon}{V_T + V_A + S + \epsilon_A^M}\epsilon_A^M - \frac{V_A}{V_T + V_A + S + \epsilon_A^M}\epsilon\right).
\end{aligned} \tag{41}$$

Note that if (40) holds, then, for $\epsilon^* \equiv \frac{1}{2}\epsilon_T^l + \frac{1}{2}\epsilon_T^h$, we have, using (41), that $\pi_{AM}^E\left(I\left(\epsilon^*\right)\right) - \pi_{AM} > 0$. As $\pi_{AM}^E\left(I^*\right) \geq \pi_{AM}^E\left(I\left(\epsilon^*\right)\right)$, it follows that $\pi_{AM}^E\left(I^*\right) > \pi_{AM}$.

Next, consider a case in which $\underline{\epsilon}_A = \underline{\epsilon}_T \equiv \underline{\epsilon}$. Consider the limiting case in which ϵ_T^h is arbitrarily close to $\underline{\epsilon}$. Then the RHS of (40) approaches

$$\begin{aligned}
\frac{V_T + S + \underline{\epsilon}}{V_T + V_A + S + \epsilon_A^M}\epsilon_A^M + \frac{V_A}{V_T + V_A + S + \epsilon_A^M}\underline{\epsilon} &= \frac{\left(V_T + S\right)\epsilon_A^M + \underline{\epsilon}\epsilon_A^M + \underline{\epsilon}V_A}{V_T + V_A + S + \epsilon_A^M} \\
&< \frac{\left(V_T + S\right)\underline{\epsilon} + \epsilon_A^M\underline{\epsilon} + V_A\underline{\epsilon}}{V_T + V_A + S + \epsilon_A^M} = \underline{\epsilon}.
\end{aligned}$$

It then follows that there exists $S > 0$ such that the RHS of (40) $< S < \underline{\epsilon}$. In light of Claim 1, an equity offer can be made that maximizes and strictly increases the payoff of the acquirer's management. Furthermore, in light of Proposition 3, the equity offer is preferred to a cash offer. This establishes the existence result. By (3) and (9), part (i) of the proposition follows. Part (ii) follows from Proposition 4 (ii) and Proposition 5 (ii). $\square$

Proof of Proposition 7: Follows from the same arguments as for Result 5. $\square$

Proof of Result 6: Assume that $\underline{\epsilon}_A \geq \underline{\epsilon}_T$ (the proof follows similarly if $\underline{\epsilon}_A < \underline{\epsilon}_T$). Then (20) and (21) yield

$$\begin{aligned}\Delta\pi_1 - \Delta\pi_2 &\geq \frac{I}{I+1}\int_{\underline{\epsilon}_A}^{\infty} \epsilon_A dL_A(\epsilon_A) + \frac{1}{I+1}\int_{\underline{\epsilon}_T}^{\infty} \epsilon_T dL_T(\epsilon_T) - \frac{I(\underline{\epsilon}_A - \underline{\epsilon}_T)}{I+1} \\ &= \frac{I}{I+1}\int_{\underline{\epsilon}_A}^{\infty} (\epsilon_A - \underline{\epsilon}_A)\, dL_A(\epsilon_A) + \frac{1}{I+1}\int_{\underline{\epsilon}_T}^{\infty} (\epsilon_T - \underline{\epsilon}_T)\, dL_T(\epsilon_T) + \underline{\epsilon}_T.\end{aligned}$$

Next, $\Delta\pi_2 \geq 0$ yields $\Delta\pi_1 - \Delta\pi_2$

$$\begin{aligned}&\leq \frac{I}{I+1}\int_{\underline{\epsilon}_A}^{\infty} (\epsilon_A - \underline{\epsilon}_A)\, dL_A(\epsilon_A) + \frac{1}{I+1}\int_{\underline{\epsilon}_T}^{\infty} (\epsilon_T - \underline{\epsilon}_T)\, dL_T(\epsilon_T) + \frac{I\underline{\epsilon}_A}{I+1} + \frac{\underline{\epsilon}_T}{I+1} \\ &\leq \frac{I}{I+1}\int_{\underline{\epsilon}_A}^{\infty} (\epsilon_A - \underline{\epsilon}_A)\, dL_A(\epsilon_A) + \frac{1}{I+1}\int_{\underline{\epsilon}_T}^{\infty} (\epsilon_T - \underline{\epsilon}_T)\, dL_T(\epsilon_T) + \max\{\underline{\epsilon}_A, \underline{\epsilon}_T\}.\end{aligned}$$

Plugging the above into (22) establishes the result. $\square$

Proof of Result 7: Follows directly from Result 6 and (25). $\square$

Proof of Proposition 8: A target shareholder with private valuation ϵ accepts cash offer C if and only if $C \geq V_T + \epsilon$. The probability offer C is accepted is

$$\Pr(C) = \begin{cases} 1 & \text{if } \frac{C - V_T - (\hat{\epsilon} - \alpha)}{2\alpha} > 1 \\ \frac{C - V_T - (\hat{\epsilon} - \alpha)}{2\alpha} & \text{if } 0 < \frac{C - V_T - (\hat{\epsilon} - \alpha)}{2\alpha} \leq 1 \\ 0 & \text{if } \frac{C - V_T - (\hat{\epsilon} - \alpha)}{2\alpha} \leq 0. \end{cases}$$

We focus on offers where $C \in [V_T + \hat{\epsilon} - \alpha, V_T + \hat{\epsilon} + \alpha]$. As a function of C, the expected payoff of the acquiring firm's management is:

$$\begin{aligned}\Pi_A &= \Pr(C)(V_T + S - C) + \left(V_A + \epsilon_A^M\right) \\ &= \frac{C - V_T - \hat{\epsilon} + \alpha}{2\alpha}(V_T + S - C) + V_A + \epsilon_A^M.\end{aligned}$$

Differentiating with respect to C yields the first-order condition:

$$\frac{d\Pi_A}{dC} = 0 = \frac{S + \hat{\epsilon} + 2V_T - \alpha - 2C}{2\alpha}. \tag{42}$$

Since the second-order conditions are satisfied, (42) defines a global maximum. In addition, if the optimal offer C exceeds $V_T + \hat{\epsilon} - \alpha$, the offer must be individually rational because the acquiring firm could always offer $C = V_T + \hat{\epsilon} - \alpha$ and have its offer be rejected. Allowing for a boundary solution, the general solution for the optimal offer C^* is

$$C^* = \begin{cases} \text{no offer} & \text{if} \quad S < \hat{\epsilon} - \alpha \\ V_T + \frac{S+\hat{\epsilon}-\alpha}{2} & \text{if} \quad \hat{\epsilon} - \alpha < S < \hat{\epsilon} + 3\alpha \\ V_T + \hat{\epsilon} + \alpha & \text{if} \quad S \geq \hat{\epsilon} + 3\alpha. \end{cases} \tag{43}$$

We then solve for how the synergies and degree of uncertainty faced by the acquiring firm affect the equilibrium likelihood of a successful takeover. Substituting for C^* yields

$$\Pr(C^*) = \begin{cases} 0 & \text{if} \quad S \leq \hat{\epsilon} - \alpha & (i) \\ \frac{S-\hat{\epsilon}+\alpha}{4\alpha} & \text{if} \quad \hat{\epsilon} - \alpha < S < \hat{\epsilon} + 3\alpha & (ii) \\ 1 & \text{if} \quad S \geq \hat{\epsilon} + 3\alpha. & (iii) \end{cases} \tag{44}$$

The results now follow directly from equations (43) and (44). Note that the condition in the third bullet of the proposition is $S < \hat{\epsilon}$, which differs from the condition $S < \hat{\epsilon} + 3\alpha$, as in the second line of equation (43) because in the proposition we consider what happens when α increases from zero. $\square$

References

[1] Andrade Gregor, Mark Mitchell and Erik Stafford, 2001, "New Evidence and Perspectives on Mergers," *Journal of Economic Perspectives*, 15, 103-120.

[2] Bagwell, Laurie Simon, 1991, "Share Repurchase and Takeover Deterrence," *Rand Journal of Economics,* 22, 72-88.

[3] Bagwell, Laurie Simon, 1992, "Dutch Auction Repurchases: An Analysis of Shareholder Heterogeneity," *Journal of Finance*, Vol. 47, No. 1, pp. 71-105.

[4] Becht, Marco, Andrea Polo, and Stefano Rossi, 2014, "Does Mandatory Shareholder Voting Prevent Bad Acquisitions?" Working Paper.

[5] Berger, Phillip G. and Eli Ofek 1995, "Diversification's effect on firm value," *Journal of Financial Economics*, 37, 39-65.

[6] Betton, Sandra, Espen Eckbo, and Karin Thorburn, 2008, "Corporate Takeovers," in B. E. Eckbo (ed.), *Handbook of Corporate Finance: Empirical Corporate Finance, Volume 2,* North-Holland/Elsevier, Handbooks in Finance Series, Chapter 15, 291-430.

[7] Boone, Audra, and Harold Mulherin, 2007, "How Are Firms Sold?" *Journal of Finance*, 62, 847-75.

[8] Chabakauri, Georgy, 2013, "Dynamic equilibrium with two stocks, heterogeneous investors, and portfolio constraints," *Review of Financial Studies*, Vol. 26, No. 2, pp. 3104-41.

[9] Chatterjee, Sris, Kose John and An Yan, 2012, "Takeovers and Divergence of Opinion", *Review of Financial Studies*, 25, 227-277.

[10] Chen, Joseph, Harrison Hong and Jeremy C. Stein, 2002, "Breadth of Ownership and Stock Returns," *Journal of Financial Economics,* 66, 171-205.

[11] Edmans, Alex, Itay Goldstein, and Wei Jiang, 2012, "The Real Effects of Financial Markets: The Impact of Prices on Takeovers," *Journal of Finance*, Vol. 67, No. 3, pp. 933-971.

[12] Fishman, Michael, 1988, "A Theory of Preemptive Takeover Bidding," *RAND Journal of Economics,* 19(1): 88-101.

[13] Gallmeyer, Michael, and Burton Hollifield, 2008, "An examination of heterogeneous beliefs with a short-sale constraint in a dynamic economy," *Review of Finance*, Vol. 12, pp. 323-64.

[14] Garleanu, Nicolae, and Lasse Pedersen, 2011, "Margin-based asset pricing and deviations from the Law of One Price," *Review of Financial Studies*, Vol. 26, No. 6, pp. 1980-2022.

[15] Geanakoplos, John, and Ana Fostel, 2014, "Endogenous collateral constraints and the leverage cycle," *Annual Review of Economics*, Vol. 6, pp. 771-99.

[16] Gomes, Armando, 2001, "Takeovers, Freezeouts, and Risk Arbitrage," *Working Paper,* University of Pennsylvania.

[17] Gorbenko, Alexander, and Andrey Malenko, 2014, "Means of Payment and Timing of Mergers and Acquisitions in a Dynamic Economy," Working paper.

[18] Gorbenko, Alexander, and Andrey Malenko, 2014, "Auctions with endogenous initiation," Working paper.

[19] Graham, John R., Michael L. Lemmon and Jack G. Wolf, 2002, "Does Corporate Diversification Destroy Value," *Journal of Finance, 57, 695-720.*

[20] Harris, Milton and Artur Raviv, 1993, "Differences of Opinion Make a Horse Race," *Review of Financial Studies*, 6, 473-506.

[21] Jensen, Michael, 1986, "Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers," *American Economic Review Papers and Proceedings*, Vol. 76, No. 2, pp. 323-329.

[22] Lamont, Owen and Christopher Polk, 2001, "The Diversification Discount: Cash Flows versus Returns," *Journal of Finance*, 56, 1693-1721.

[23] Liu, Tingjun, 2012, "Takeover Bidding with Signaling Incentives," *Review of Financial Studies*, Volume 2, 522-556.

[24] Masulis, Ronald, Peter Swan and Brett Tobiansky, 2012, "Do Wealth Creating Mergers and Acquisitions Really Hurt Acquirer Shareholders?", AFA 2012 Chicago Meetings Paper.

[25] Matvos, Gregor, and Michael Ostrovsky, 2008, "Cross-ownership, returns, and voting in mergers," *Journal of Financial Economics*, Vol. 89, pp. 391-403.

[26] Miller, Edward, 1977, "Risk, Uncertainty, and Divergence of Opinion," *Journal of Finance*, 32, 1151-1168.

[27] Moeller, Sara, Frederik Schlingemann, and Rene Stulz, 2004, "Firm size and the gains from acquisitions," *Journal of Financial Economics*, 73, 201–228.

[28] Moeller, Sara, Frederik Schlingemann, and Rene Stulz, 2005, "Wealth Destruction on a Massive Scale? A Study of Acquiring-Firm Returns in the Recent Merger Wave," *Journal of Finance*, 757-782.

[29] Moeller, Sara, Frederik Schlingemann, and Rene Stulz, 2007, "How do Diversity of Opinion and Information Asymmetry Affect Acquirer Returns?" *Review of Financial Studies*, Vol. 20, No. 6, pp. 2047-2078.

[30] Morris, Stephen, 1996, "Speculative Investor Behavior and Learning," *Quarterly Journal of Economics*, 111, 1111-1133.

[31] Osambela, Emilio, 2015, "Differences of opinion, endogenous liquidity, and asset prices," forthcoming, *Review of Financial Studies*.

[32] Rhodes-Kropf, Matthew and S. Viswanathan, 2004, "Market Valuation and Merger Waves", *Journal of Finance*, 59, 2685-2718.

[33] Savor, Pavel and Qi Lu, 2009, " Do Stock Mergers Create Value for Acquirers?" *Journal of Finance*, 64, 1061–1097.

[34] Shleifer, Andrei and Robert Vishny, 2003, "Stock Market Driven Acquisitions," *Journal of Financial Economics*, 70, 295-311.

[35] Simsek, Alp, 2013, "Belief Disagreements and collateral constraints," *Econometrica*, Vol. 81, No. 1, pp. 1-53.

[36] Skrzypacz, A., 2013, "Auctions with Contingent Payments - an Overview," *International Journal of Industrial Organization* 31, 666-675.

[37] Xiong, Wei, and Hongjun Yan, 2010, "Heterogeneous expectations and bond markets," *Review of Financial Studies*, Vol. 23, No. 4, pp. 1433-66.