Re-appraising intraday trading patterns: what you didn't know you didn't know[*]

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Abstract

We document new intraday trading patterns indicative of the key roles of endogenous trading responses of investors to variations in imperfectly-competitive liquidity provision. When measured in trade times of fixed dollar values, price impacts and volatility *fall* sharply from open to close, and as trading activity rises. We also document reversions in trade-time returns in inactive markets, and priced, heavily-forecastable, order flow imbalances in active markets. Standard calendar-time aggregation approaches conceal these primitive trading patterns by matching up overly-balanced signed-trade observations with large price movements in active markets. Once one controls for over-aggregation, calendar-time patterns align with trade-time patterns.

"...we need statistics not only for explaining things but also in order to know precisely what there is to explain ..."— Schumpeter

1 Introduction

Researchers have traditionally aggregated trading outcomes over fixed calendar-time intervals to document regularities in intraday trading patterns.[1] Robust regularities obtain: trading activity and return volatility are positively related, and evolve according to $\cup$-shaped patterns over the trading day. In turn, theoretical researchers have taken these patterns as primitive stylized facts to be explained. Beginning with Admati and Pfleiderer (1988), which builds on Kyle (1985), most of these models revolve around how informed traders' incentives to conceal private information evolve when liquidity provision is competitive.

Our paper finds that calendar-time aggregation approaches mislead. We analyze intraday trading patterns by aggregating trading outcomes over variable "trade-time" intervals that correspond to realizations of a fixed dollar value. Thus, we aggregate over intervals whose lengths adjust to account for variation in trading activity.[2] Radically different patterns emerge in trade time. The empirical regularities that we document point to the role of imperfectly-competitive liquidity provision, and are at odds with predictions of models featuring asymmetric information in competitive liquid markets. We then establish that calendar-time aggregation approaches over-aggregate in active markets. Once we address the over-aggregation, we find that calendar-time patterns align with trade-time patterns.

Our trade-time analysis groups a stock's transactions into non-overlapping successive trade sequences with the same cumulative dollar volume. Thus, the inverse of the time duration of a trade-time sequence measures trading activity: shorter trade-time intervals in-

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[1]E.g., Wood et al. (1985); Jain and Joh (1986); Admati and Pfleiderer (1988); Heston et al. (2010).

[2]In practice, optimal trade execution strategies adjust the aggressiveness of trading in limit order markets according to real-time trading conditions, and our approach accommodates this. Adamic et al. (2017) show that execution strategies as reflected by trading network metrics are highly correlated with trading outcomes such as inter-trade time duration, liquidity, and volatility. While many studies examine trading outcomes over volume-indexed intervals (Clark 1973, Epps and Epps 1976, Harris 1987) or argue that relevant trading horizons reflect volume/business time (Easley et al. 2012, Kyle and Obizhaeva 2016), none has examined intraday patterns in trading outcomes using volume-indexed aggregation.

dicate greater trading activity. This measure of trading activity is ∪-shaped over the trading day, just like calendar-time trading volume. However, in contrast to the ∪-shaped volatility and price patterns found in calendar time, trade-time price impacts and return volatility *fall sharply* from open to close. Moreover, while volatility/price impacts and trading activity are positively related in calendar time, they are negatively related in trade time. After controlling for time-of-day, higher trading activity is associated with (a) *smaller* price impacts and return volatility, (b) larger transactions, and (c) trading strategies that rely more heavily on aggressive marketable orders. These trade-time patterns are economically relevant: The twin declines in price impacts and volatility from open to close and as trading activity rises from its lowest to its highest quartile, at a given time of day, are strikingly large, exceeding 20%.

These trade-time patterns suggest that imperfect and time-varying liquidity provision is a core feature of equity markets. They emerge naturally when investors (execution algorithms) actively respond to variations in extant liquidity to minimize overall price impacts of "parent orders."[3] To wit, when depth near best prices is sizable and persistent, traders aggressively remove liquidity via large marketable orders without inducing large price impacts. Thus, the resulting high trading activity is associated with small price impacts, larger transactions, and unbalanced order flow. In contrast, with little depth, large marketable orders would have large price impacts, so traders, instead, use passive trading strategies that employ non-marketable orders. This gives rise to the coexistence of low trading activity and large price impacts together with small transactions and balanced order flow.[4] Our findings extend the empirical literature on endogenously chosen trade sizes; Jones et al. (1994) find that the positive volatility-volume association is fully explained by variations in transaction size/frequency. They show that at a given trading volume level, volatility is greater when more trades occur per day, i.e., when transactions are smaller. This is consistent with a setting in which traders endogenously submit smaller marketable orders when markets are

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[3]Modern markets feature tiny spreads and minimal depth at best prices, inducing traders to adopt dynamic trading strategies that decompose large "parent orders" into small temporally-correlated "child orders" (O'Hara 2015). As such, trading costs associated with a parent order are cumulative in nature.

[4]These patterns are consistent with Latza et al. (2014) who find that fast aggressive trades feature low execution costs, while slow aggressive trades face higher execution costs.

not deep near good prices and price impacts are large, leading Jones et al. (1994) to argue as we do, that trading outcome patterns reflect variation in endogenous liquidity provision and consumption, rather than variation in the extent of information asymmetry.

We then provide further evidence of the impact of endogenous liquidity provision using the temporal correlations in trade-time returns at different trading activity levels. When trade times are longest—when price impacts and price movements are largest—trade-time returns exhibit reversion, indicating that liquidity providers can unwind positions over short horizons. This reversion, which vanishes in active markets, indicates that the associated high price impacts and volatility largely reflect compensation for liquidity provision, not information-driven price movements. This absence of Martingale pricing should not surprise—algorithmic trading strategies that seek to minimize trading costs switch from consuming liquidity to providing it in less active markets, and there is no reason to do so if liquidity provision is competitive. However, this consistency with algorithmic trading strategies indicates an inconsistency with the competitive foundations of classical models (e.g., the dealership models of Glosten and Milgrom 1985 or Kyle 1985, or limit order models like Glosten 2002).

Compelling evidence comes from the pricing of forecastable signed trade. Net signed trade is predictable, and its persistence rises sharply with activity: average auto-correlation coefficients on net signed trade rise from 0.15 to 0.5 as trading activity rises. This monotonic rise in the extent of auto-correlation is consistent with trade-time return dynamics whose reversals are stronger in less active markets. Were pricing competitive, forecastable signed trade would not be priced. Crucially, forecastable signed trade is strongly priced in active markets.

Our baseline trade-time aggregation uses trade sequences based on target dollar volumes of $80,000 + 0.025\%$ of a stock's market-cap. Our calendar-time analysis aggregates trading outcomes over 15-minute intervals—the length of a typical trade-time interval. Findings are robust to alternative effective aggregation horizons. Results are qualitatively unaffected if we (i) use a target dollar volume that is only proportional to market-cap, (ii) halve the target dollar volume, (iii) use a common target dollar volume for stocks with similar market-caps, (iv) account for variations associated with open and close auctions, (v) use a volume-based speci-

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fication, rather than market-cap, by constructing trade sequences with target dollar volumes equal to 4% of a stock's average daily dollar volume in the previous month, and (vi) exclude observations from windows around earnings announcements. So, too, intraday calendar-time patterns are not qualitatively affected by aggregating over 3- or 5-minute time intervals.

Our analysis shows that endogenous liquidity provision and consumption is a first-order feature of intraday trading.[5] Recent models deliver the negative association between trading activity and price impacts (Collin-Dufresne and Fos (2016, CDF), Hollifield et al. (2004, 2006)), but are either silent about time-of-day effects, or make assumptions that are inconsistent with declining price impacts from open to close.[6] Conversely, models of uninformed institutional traders who establish positions over the trading day (Choi, Larsen and Seppi (CLSa, 2018a, CLSb, 2018b), Boulatov, Bernhardt and Larianov (BBL, 2017)) match the time-of-day effects; but do not allow for endogenous liquidity consumption and provision. Our paper points to the need for models that generate both the activity and time-of-day patterns.

We complete our analysis by identifying and addressing why the two aggregation approaches generate different trading patterns. Figure 1 plots relative effective spread per 100 shares by *calendar-time* trading activity level at different times of the day.[7] This simple disaggregated measure of trading costs falls sharply by 25-40% from open to close and, fixing time of day, as trading volume rises. The same patterns hold in per dollar trading costs. These monotonic declines stand in marked contrast to the positive associations between volatility and trading activity found using traditional calendar-time aggregation, creating an "aggregation puzzle." In contrast, trade-time associations between volatility/price impacts and trading activity are strikingly similar to those in Figure 1.

Barardehi et al. (2019) use a stylized model to show how volume-indexed aggregation methods avoid over-aggregation biases by adjusting the aggregation horizon according to how

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[5]Collin-Dufresne and Fos (2015) show that insiders time trades to days with smallest price impacts.

[6]In CDF, a speculator has an incentive to delay trade in the hope that liquidity trade volatility will rise. To offset this, CDF need price impacts to rise, on average, over the trading day, but we find that they fall sharply.

[7]Effective spread equals the ratio of the difference between transaction price and the mid-point of NBBO at the previous millisecond divided by the midpoint. Average effective spread in a time interval is divided by the respective average transaction size (and rescaled by 100) to deliver relative effective spread per round lot, accounting for variation in trade size across different trading activity levels.

Figure 1: **Effective spread per round lot by calendar-time trading activity level.** The figure shows the relationships between effective spread per 100 shares against deciles of 15-minute dollar volumes by time-of-day window. Each trading day is parsed into 26 fifteen-minute intervals. Size-weighted mean effective spreads (the difference between transaction price and the mid point at the end of previous millisecond, divided by the midpoint) and mean trade sizes are calculated for each interval. Effective spread per 100 shares is the ratio of mean effective spread to mean trade size, re-scaled by 100. Each month, calendar-time intervals of each stock are sorted into deciles of 15-minute dollar volume by time-of-day window, with decile 1 reflecting the least active markets. Each month, medians of effective spreads per 100 shares are calculated for each stock by dollar volume decile and time-of-day. Cross-stock-month medians of these measures are plotted against trading activity (dollar volume) deciles by time-of-day. Sample contains all NYSE-listed stocks, 2009–2012.
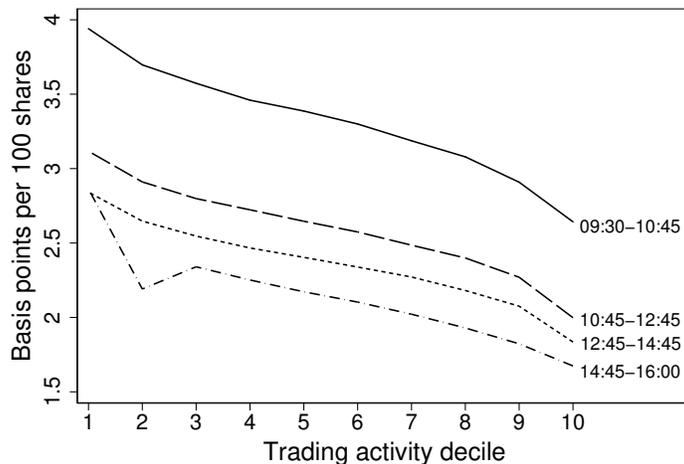
fast markets move. Here, we show that calendar-time aggregation approaches lump together **offsetting** buyer- and seller-initiated trades in active environments, biasing price impact measures up. We first provide evidence of this over-aggregation, i.e., of this lumping of offsetting trades, by documenting a negative association between calendar-time trading volume and signed trade imbalances that holds even for short (3- or 5-minute) aggregation horizons. This suggests that when trading volumes per unit time are high, overly "balanced" signed trade observations are matched up with large price changes biasing price impact measures up.

To confirm this, we calculate signed-trade imbalances for a fixed number of consecutive transactions in an interval, e.g., the first 60 trades, or last 60 trades in each trade- or calendar-time interval. Having the same number of trades in each interval equalizes the extent of aggregation across the two approaches and across different trading activity levels. Controlling for the extent of aggregation in this way ensures apples-to-apples comparisons across trading activity levels and aggregation approach (trade-time vs. calendar-time). Once we do this,

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calendar-time patterns *reverse*: rather than fall with trading activity, signed trade imbalances now rise sharply. More importantly, calendar-time patterns (e.g., in estimates of λ) now mirror their trade-time counterparts—falling over the trading day, and as activity rises.

These findings indicate that foundational stylized facts for many theoretical models of intraday trade have been distorted by varying degrees of over-aggregation over calendar-time intervals associated with different levels of trading activity. Our trade-time aggregation uncovers the economics of trading that calendar-time aggregation conceals.

2 Related literature

We next convey implications of our findings for existing models of intraday trade.[8]

Facts vs. Models. The seminal paper of Kyle (1985) considers an informed monopolist investor who has a single piece of long-lived private information about an asset's value. The speculator trades over the trading day in a competitive dealership market, trying to hide her trades behind noise trade. Kyle predicts that pricing follows a martingale, and that the price impact is constant over the day. Back and Pedersen (1998, BP) show that the prediction of a constant Kyle's λ extends when liquidity trade evolves in arbitrary deterministic ways: were λ lower at one point in the day, the speculator would trade more aggressively then—but then λ should be higher. Thus, liquidity and informed trade covary to deliver a constant λ.

Holden and Subrahmanyam (1992, HS) and Foster and Viswanathan (1996, FV) introduce multiple informed speculators who have correlated long-lived private information. Back, Cao and Willard (2000, BCW) nest these models in a continuous time framework. As long as correlation is not perfect, BCW show that the equilibrium features two phases: an initial racing phase, in which private information is positively correlated so that speculators race to trade at better prices before rivals; and then a waiting phase, in which private information is negatively correlated, and hence a speculator wants to slow down so that the trades of other

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[8]A sizable literature documents the positive relationship between trading activity and return volatility (e.g., Karpoff (1987)), which motivated work modeling stochastic volatility (e.g., Engle (1982)). Andersen and Bollerslev (1997) show how time-of-day patterns in return volatility undermine the ability of stochastic volatility models to capture intraday volatility dynamics.

speculators shift prices in a favorable direction. These models predict declining trading intensities and ∪-shaped price impacts. Contrary to predictions of a constant or ∪-shaped λ over the trading day, trade-time estimates of Kyle's λ fall sharply from open to close. Moreover, declining price impacts found as activity rises are contrary to BP's prediction of a constant λ.

Admati and Pfleiderer (1988, AP) introduce multiple speculators with short-lived private information, and discretionary liquidity traders who can time trades. AP predict low price impacts at the very beginning and ends of the trading day because concentrating liquidity trade draws more speculators who compete away information rents. However, they do not deliver the smooth ∪-shaped pattern in trading activity over the day, the smoothly declining price impact over the day, or the declining price impacts found as activity rises.

Recently, a new generation of models including CDF, CLSa, CLSb and BBL have emerged, motivated by modern trading strategies that reflect stochastically-varying liquidity provision or dynamic order splitting by traders who need to establish positions over a fixed time horizon. CDF modify BP to allow for stochastic volatility in liquidity trade, and show that the speculator tends to trade when liquidity trade is more volatile, resulting in smaller price impacts when activity is higher. Thus, CDF can reconcile the activity-price impact relationships that we find. However, in their model, the speculator has an incentive to delay trade in the hope that liquidity trade volatility will rise. To offset this, CDF need price impacts to rise, on average, over the trading day. In fact, we find that price impacts fall sharply.

Together, the (a) declining price impacts over the trading day, and (b) ∪-shaped trading activity pattern over time match the predictions of models that feature strategic investors who design trading strategies to minimize the expected trading costs of reaching target closing positions. CLSa introduce an inventory-rebalancing trader who receives a position shock that must be unwound by close to a variant of FV. The traders and market maker learn over time from information in net order flow. Trading volume can be smoothly ∪-shaped, high near open due to racing incentives, and high near close because the market maker does not price the predictable component of the rebalancing trader's order flow. Consistent with our findings, CLSa also predict that net order flow should be positively auto-correlated due to

market maker learning. However, their model cannot reconcile our findings that at a given moment in the trading day, price impacts and volatility fall sharply with trading activity, or that predictable net signed trade is heavily priced in active markets.

In BBL, an inventory-rebalancing trader faces parasites who begin and end the day with zero positions, but can trade to exploit the temporary and permanent price impacts of the rebalancing trader's orders. Trading strategies minimize transactions costs à la Almgren and Chriss (1999, 2001). Equilibria reproduce the time-of-day patterns found. Parasites exploit the price impacts of the rebalancing trader, trading in the same direction near open, before reversing trades to return to a zero position at close. The rebalancing trader's trading intensity falls near open due to racing incentives, but it rises near close to exploit the unwinding by parasites. Thus, early in the day, both trader types trade in the same direction, but near close, they trade in opposite directions. As a result, trading costs fall over the day, but volume is ∪-shaped. Also, like CLSa, the model can reconcile predictable signed net order flows, but not the declining volatility and price impacts as activity rises at a moment in the trading day.

We also find that signed trade is forecastable, highly persistent in active markets, and priced, indicating that liquidity provision is imperfect. Models with perfect liquidity provision also miss how real world trading strategies consume liquidity when extensive liquidity is available, but supply liquidity when it is absent. The limit order models of Goettler et al. (2005) and Hollifield et al. (2004, 2006) get at this, but their stylized models do not allow for dynamic order splitting. Trade execution service providers like ITG accommodate time-varying liquidity in ad hoc ways using a "proprietary parameterization of the temporary and permanent impact functions [of Almgren and Chriss (1999)] to compute transaction costs" (p.63), accounting for imperfect liquidity provision in a practical, but reduced form, way.

In sum, existing models that match some key features of intraday trading outcomes miss on other ones. Rich equilibrium models of imperfect liquidity provision do not yet exist. Our analysis suggests the promise of models that integrate inventory-rebalancing traders and stochastically observable liquidity trade.

Empirical work. Our work extends analyses of the role of trade-time in the price formation

process. Early studies, including Clark (1973), Epps and Epps (1976), and Harris (1987), investigate the statistical properties of stock returns measured over volume-indexed intervals using Mixture Distribution Models. Dufour and Engle (2000) found that shorter time durations between successive trades were associated with greater price impacts of individual trades and faster price adjustments (also see Hasbrouck (1999)). Engle and Russell (1998) and Engle (2000) develop Autoregressive Conditional Duration (ACD) models to estimate trade intensities. Post decimalization, dynamic order submission strategies and parasitic trade give rise to complex temporal dependence in individual transactions (O'Hara (2015) and Menkveld (2016)) that violates the i.i.d. error-term premise of ACD models. Trade-time aggregation can help: Barardehi (2015) shows that residuals obtained from applying ACD models to trade-time measures of trading activity satisfy the underlying assumptions well.

To "capture dependencies between intra-trade durations" Gouriéroux et al. (1999) measure trading activity as the time required to sell (buy) a predetermined value or volume. They study durations of trade sequences on the Paris Bourse. In contrast, we analyze how different trading outcomes vary over trade sequences of different trade times at different times of the day. Easley et al. (2012) highlight the need to aggregate to account for temporal correlation in trades due to dynamic order submission strategies. They bundle trades into volume groups with size equal to 1/50 of annual daily trade volume, so that a time duration *within* a day captures relative trading activity on *that* day. However, variations in daily trading volume mean that one cannot use their measure to compare *across* days.

3   Data, Summary Statistics, and Methodology

3.1   Data

Our sample runs from January 1, 2009 to December 31, 2012. Each year, our sample consists of U.S.-based NYSE-listed common shares that maintain a closing price of at least $1 over the entire year. Monthly observations on market-capitalization come from CRSP. We obtain trade prices, quantities and time stamps from the consolidated trade history from Daily

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TAQ. We consider all trades on U.S.-based venues during regular market hours, as well as trades flagged as market-on-close orders (which can be recorded after 4:00pm).[9] To estimate trading directions, we use quoted national best bids and offers: we construct National Best Bid and Offer prices at millisecond frequency from the consolidated quotes history as in Holden and Jacobsen (2014), matching each transaction with the prevailing mid-point at the end of the previous millisecond. Earnings announcement dates and analyst recommendation dates come from Compustat and I/B/E/S. We use NCUSIP from CRSP and CUSIP from TAQ, Compustat, and I/B/E/S to merge databases. We exclude stocks missing relevant identifying information in any database.

3.2 Stock-specific Trading Outcomes

We now describe the construction of trade-time intervals as well as related trade- and clanedar-time trading outcomes. We number transactions in stock j sequentially, using index n_j. For transaction n_j, $\tau_j(n_j)$, $Q_j(n_j)$, $P_j(n_j)$, $MP_j(n_j)$, and $D_j(n_j)$ respectively denote its: (i) time measured in seconds from the beginning of each year, (ii) size (in shares), (iii) price, (iv) corresponding midpoint of National Best Bid and Offer, and (v) trading direction, which is signed by comparing $P_j(n_j)$ to the mid-point of best quotes at the previous millisecond.[10] A **trade sequence** consists of consecutive transactions with an aggregate value of at least $V_{j,t}$ for stock j in month t. The first trade sequence begins with the first trade in a year. Each subsequent trade sequence begins with the first trade after the previous sequence.

As Figure 2 illustrates, we iteratively solve for the last trade of the kth trade sequence,

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[9]We exclude trade sequences with absolute returns that exceed 10%. These filters exclude less than 0.2% of observations, and are meant to rule out instances of extreme information arrival and data error. Qualitatively identical patterns obtain using winsorized samples at $\pm10\%$, $\pm20\%$, and $\pm30\%$ return.

[10]We identify buyer- and seller-initiated transactions using the Lee-Ready algorithm: $D_j(n_j) = 1$ for buyer-initiated transactions, $D_j(n_j) = -1$ for seller-initiated transactions, and $D_j(n_j) = 0$ for executions at the mid-point. Qualitative findings are robust to using trade classification routines proposed by Ellis et al. (2000) and Chakrabarty et al. (2006).

Figure 2: **Illustration of how trade time intervals of trade sequences with an aggregate value of at least $V_{j,t}$ are constructed.**

A: Dollar volume

B: Cumulative dollar volume
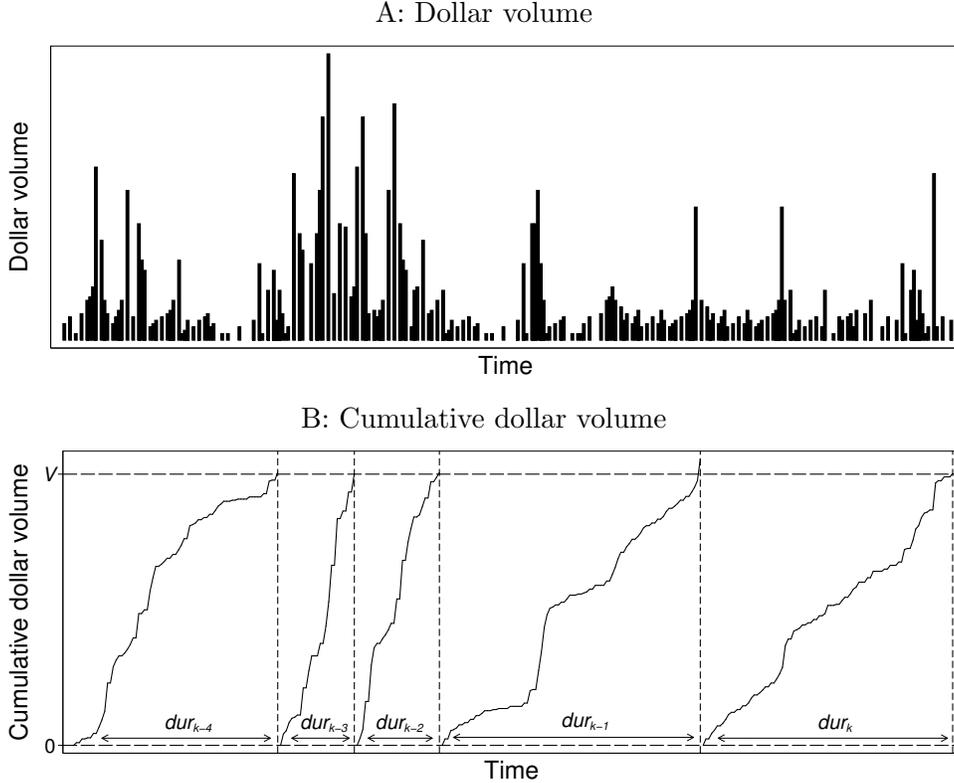
$k = \{1, 2, 3, \ldots\}$, as:

$$n_j^k = \underset{n^*}{\operatorname{argmin}} \left\{ \sum_{n=n_j^{k-1}+1}^{n^*} P_j^C(n) \times Q_j(n) \,\middle|\, \sum_{n=n_j^{k-1}+1}^{n^*} P_j^C(n) \times Q_j(n) \geq V_{j,t} \right\}, \qquad (1)$$

where $n_j^0 = 0$ and the value of aggregate trades is measured using the previous day's closing price, $P_j^C(n_j)$.[11] Using the previous day's closing price to calculate dollar volumes is crucial because it prevents contemporaneous price movements from affecting identification of trade sequences.[12] We construct trade sequences that span two trading days, but *exclude* them

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[11]The last quoted bid-ask midpoint is used when the closing price is not available.

[12]Using the previous day's closing price avoids introducing biases driven by current price movements. For example, with rapidly increasing prices, using today's prices can give rise to non-trivially growing dollar volumes, causing a downward bias in the time duration of the corresponding trade sequence. We cannot use contemporaneous price movements, else we risk introducing spurious relationships between trade-time price movements and time durations of trade-time intervals.

from the analysis; e.g., we drop a sequence that begins at 3:53pm but ends the next day at 9:31am. Calculating overnight trade sequences but excluding them: (1) delivers random starting points for initial trade sequences each day; (2) addresses overnight price adjustments or information arrival;[13] and (3) avoids mixing trading outcomes from near close with those just after open.

Excluding trade sequences that span overnight in this way tends to drop observations associated with the opening and closing auctions. To ensure that this does not affect inferences, we consider an alternative construction that also avoids trade sequences that span overnight. This approach constructs trade sequences by (1) starting at open of each trading day and stopping with the trade sequence that crosses 12:30pm; and (2) starting at close of a trading day, constructing trade sequences in reverse until they cross 12:30pm. We then discard trade sequences that overlap 12:30pm. This alternative construction includes all trades near open and close, but avoids spanning overnight. Appendix C establishes that results are virtually unchanged.

Our base formulation sets $V_{j,t} = 0.025\% M_{j,t-1} + \$80,000$. This generates a median duration (across stocks and years) of about 15 minutes, yielding about 26 trade-time observations on a typical trading day of a typical stock. In Appendix B, we show that a volume-based specification of V_{jt} equal to 4% of a stock's average daily dollar volume in the previous month (which also generates about 26 trade sequences per day), does not alter qualitative findings.[14]

Several considerations enter our specification: (1) $V_{j,t}$ is large enough to control for dynamic order splitting and division of orders against the book;[15] (2) $V_{j,t}$ is not so large that a single trade sequence spans very different activity levels; (3) the proportional-to-market-

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[13]For instance, we do not need to adjust $r_j(k)$ for stock splits or dividend distributions.

[14]An unreported robustness analysis verifies that our qualitative findings are not driven by the choice of $V_{j,t}$. For example, similar findings obtain using $V_{j,t} = 0.04\% M_{j,t-1}$ or $V_{j,t} = 0.0125\% M_{j,t-1} + \$40,000$.

[15]Trading strategies dynamically split large "parent orders" into small temporally-correlated "child orders" (O'Hara 2015). This can induce time-varying, autocorrelation in individual transaction level outcomes, that may be addressed by aggregation (Gouriéroux et al. 1999). To verify that we aggregate sufficiently, we estimate the stock-specific return autocorrelation over successive trade sequences. This autocorrelation is insignificantly different from zero—indeed, it is marginally *negative*—and it has no time-of-day patterns. See Appendix A.

cap component flattens out the distribution of trade time across market capitalizations (see Figure 3), so that trade times of small market-capitalization stocks are not too much longer, delivering enough observations for analysis; (4) the fixed-dollar amount in $V_{j,t}$ ensures a non-trivial target dollar-volume for stocks with small market capitalizations.

For each trade sequence k of stock j, we obtain its trade time,

$$dur_k^j = \tau_j(n_j^k) - \tau_j(n_j^{k-1} + 1), \tag{2}$$

as well as the corresponding volume-weighted average price (VWAP),

$$VWAP_k^j = \sum_{n=n_j^{k-1}+1}^{n_j^k} \frac{P_j(n) \times Q_j(n)}{\sum_{n=n_j^{k-1}+1}^{n_j^k} Q_j(n)}, \tag{3}$$

return,

$$r_k^j = \frac{P_j(n_j^k + 1)}{P_j(n_j^{k-1} + 1)} - 1, \tag{4}$$

midpoint return,

$$mr_k^j = \frac{MP_j(n_j^k + 1)}{MP_j(n_j^{k-1} + 1)} - 1, \tag{5}$$

volume-weighted return,

$$wr_k^j = \frac{VWAP_j(k)}{P_j(n_j^{k-1} + 1)} - 1, \tag{6}$$

proportions of buyer- and seller-initiated volume (where the $\mathbb{1}(.)$s are the associated indicator functions for buyer initiated or mid-point transactions),

$$BP_k^j = \frac{Q_j(n) \times \mathbb{1}(D_j(n) = 1)}{\sum_{n=n_j^{k-1}+1}^{n_j^k} Q_j(n)} + \frac{Q_j(n) \times \mathbb{1}(D_j(n) = 0)}{2\sum_{n=n_j^{k-1}+1}^{n_j^k} Q_j(n)} \quad \text{and} \quad SP_k^j = 1 - BP_k^j \tag{7}$$

trade imbalance,

$$IMB_k^j = \max\{BP_k^j, SP_k^j\}, \tag{8}$$

and average transaction size,

$$TS_k^j = \frac{\sum_{n=n_j^{k-1}+1}^{n_j^k} Q_j}{n_j^k - n_j^{k-1}}. \tag{9}$$

The median trade-time in our base formulation is 15 minutes. Accordingly, we construct calendar-time analogues of trading outcomes defined in equations (3)–(9) over 15-minute time intervals, using dollar volumes realized over these intervals to measure trading activity.

Figure 3: **Trade time by month and market-capitalization.** Average median trade time (in minutes) of a trade sequence with a cumulative aggregate value of at least $80,000 plus 0.025% of a firm's market capitalization. Median trade times are calculated stock-by-stock on a monthly basis. The average is then computed for stocks in a market-capitalization decile each month. An entire stock-month set of observations is excluded if the stock features fewer than 100 observations in that month. Size decile 1 contains the smallest firms; decile 10 contains the largest.
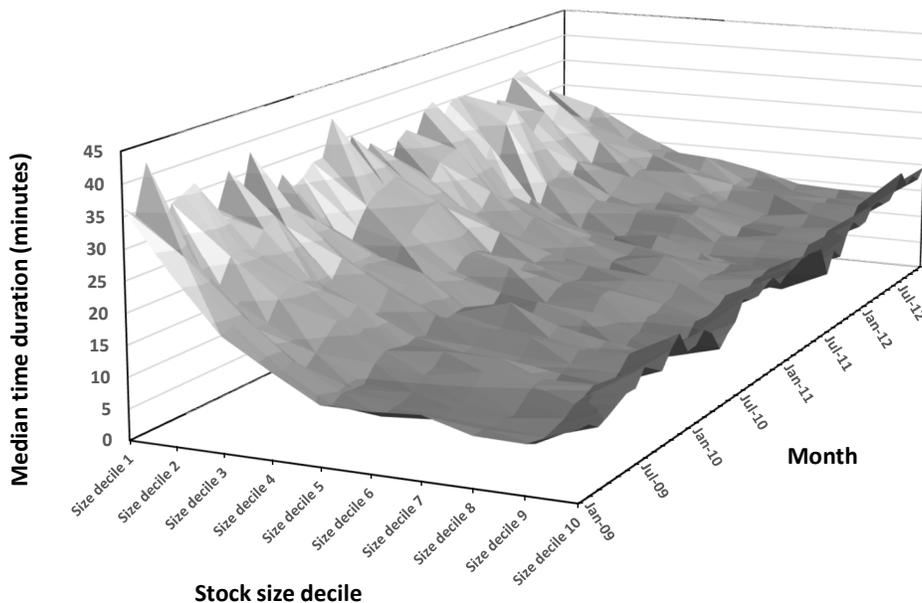
3.3 Trade-time Descriptive Statistics

Figure 3 shows how median trade times vary across stocks and time. We sort stocks into declises of end-of-previous month market-captilazation onservvations to control for size. We then calculate median trade times at the stock-month level, and find averages of these medians across stock size deciles every month. For a given size decile, there is minimal variation in the median trade time over the four-year sample period. The median trade time is a ∪-shaped function of stock size. The longer trade times of small stocks reflect their low trading activity; while the longer trade times of large stocks reflect their large targets $V_{j,t}$. To control for potential heterogeneity by stock size, we conduct our main analysis by three market-cap terciles.

Figure 4 presents the distribution of trade times (time durations of trade sequences) for three representative small, mid-sized, and large firms: Hanger, Inc. (HGR), Acuity Brands, Inc. (AYI, the median market capitalization stock), and International Business Machines

Corp. (IBM). The figure reveals how trade time varies at the stock level, conveying the time horizons over which we measure trading outcomes. Trade times can be as short as a few seconds or as long as an hour. Median time durations for these stocks are less than 25 minutes, and even for the small stock HGR, 80% of trade times are shorter than 40 minutes.

Figure 4: **Trade times for representative firms.** Cumulative distribution functions of trade times (in minutes) for International Business Machines Corp. (IBM), Acuity Brands, Inc. (AYI), and Hanger, Inc. (HGR) from January 1, 2011 to December 31, 2011. Trade times are computed based on $V_{j,t} = \$80,000 + 0.25\%$ market-cap.
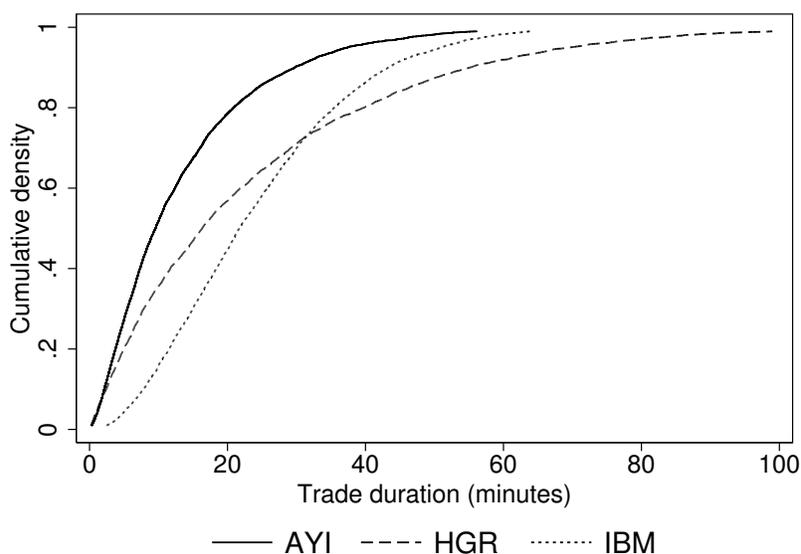
Episodes of non-trading are almost non-existent. Even conditional on a mid-sized or large stock being at its lowest quartile of trading activity, the median probability with which it trades in a given *minute* exceeds 0.99. For the smallest tercile of stocks, this probability is 0.92, and it is 0.99 at the other three quartiles of activity. Indeed, for AYI, at the 99th percentile of trade times ($\sim$50 minutes), $18k of trade occurs on average each minute: on average, trade is non-trivial even at the bottom *one* percent of trading activity.

3.4 Qualitative Time-of-day Patterns

We now compare time-of-day patterns in trading activity, volatility, and price impacts obtained using calendar- and trade-time aggregation.The calendar-time analysis parses a trading day into 13 time-of-day windows that each contain *two* 15-minute calendar-time intervals.[16] The trade-time analysis assigns a trade sequence to a time-of-day window in {09:30–10:00, 10:00–10:30, ... , 15:30–16:00} if the window includes the *mid-point* of the trade sequence. Thus, a trade sequence spanning 09:56:00 to 10:06:00 has a midpoint of 10:01:00, so it is assigned to the 10:00–10:30 time window.

We construct standardized measures of trading activity to facilitate comparisons across the two approaches. For each stock in each month we sort observations of 15-minute dollar volume and (inverse) trade-time durations into respective percentiles to compute order statistics of calendar- and trade-time trading activity. We then calculate each stock's average trading activity percentile by time-of-day window and month.[17] Finally, we calculate the cross-month-stock median of these averages in a given firm size category in each time-of-day window. Similarly, for return volatility, we calculate each stock's calendar- and trade-time return standard deviations by time-of-day and month, and then calculate the cross-month-stock median of these volatility measures in a given firm size category for each time-of-day window.

To estimate Kyle's λ, we follow Hasbrouck (2009). We estimate the sensitivity of calendar- or trade-time midpoint returns (equation (5)) to volume-neutral measures of net signed trade, controlling for the thirteen time-of-day windows and month fixed effects. Using the measure for proportion of buyer-initiated volume, BP, we construct $NOF = \text{sign}(BP - 0.5) \times \sqrt{|BP - 0.5|}$ for each interval. This estimate of net order flow normalizes for cross-sectional variations in trading volumes, controlling for the fact that trading volumes and target dollar values, V_{jt}, may vary across stocks (Wood et al. 1985). Thus, when buyer-initiated volume exceeds 50% of trading volume in a trade sequence, $BP > 0.5$; and $|BP-0.5|$ measures the extent to which estimated net signed trade deviates from being balanced. The

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[16]We drop those very rare intervals with zero trading volume.

[17]Stock-specific averages reflect about $2 \times 22 = 44$ monthly observations in a time-of-day window.

proportional specification of net signed trade maintains consistency with our trade-time analysis, where dollar volume targets are fixed across trade-time intervals of a given stock; the proportions of seller- versus buyer-initiated volumes capture variations in signed trade across trade-time intervals. Qualitatively-identical patterns obtain if we instead use "actual" net *signed dollar volume*, or a *linear* specification, $NOF = PB - 0.5$. To estimate Kyle's λ, we regress midpoint returns on NOFs for each stock and time-of-day window. We then calculate medians of Kyle's λ estimates by time-of-day window and firm size category.[18]

The top row of Figure 5 presents the familiar $\cup$-shaped time-of-day patterns in trading volume and volatility found in calendar time. Trading volume and return volatility are higher in early and late trading hours than in intermediate hours, with trading volume being highest near close, and volatility being highest near open. Estimates of Kyle's λ also follow a $\cup$-shaped pattern save that they fall in the last 30-minute window.

In striking contrast, the bottom row shows that in trade-time, return volatility and price impacts fall sharply over the day. The drops in trade-time volatility are massive both in absolute and relative terms, falling between open and close by 90 to 36bps for small stocks, 55 to 18bps for mid-sized stocks, and 41 to 14 bps for large stocks;[19] the drops in estimates of Kyle's λ are comparable. These declines represent first-order features of intraday trading. However, they are inconsistent with the constant λ predictions of classical models like Kyle, BP, the $\cup$-shaped predictions of FV or BCW, and the prediction of CDF that λ should rise. The declines *are* reconciled by models like CLSa or BBL that incorporate a trader who must reach a fixed targeted position by the end of a trading day.

These patterns reveal that the sources of price movements *change* over the course of a trading day. In earlier trading hours, price variations per unit traded drive the high volatility. In contrast, in later trading hours, volatility and price impact per unit traded falls, but the increased trading volume more than offsets this decline, driving up volatilities and price

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[18]Results are robust to using transaction-price returns (equation (4)) or VWAP-based returns (equation (3)) as dependent variables.

[19]In unreported results, we document similar calendar-time versus trade-time differences for alternative measures of price movements such as average absolute returns (see equations (3) and (6)).

Figure 5: **Time-of-day patterns of trading activity, return volatility, and price impacts measured in calendar-time versus trade-time.** **Top row:** Trading volume and return volatility are measured over 26 15-minute time intervals. Time-of-day windows {09:30–10:00, 10:00–10:30, ..., 15:30–16:00} are considered to study intraday seasonalities. For each stock, 15-minute dollar volume *percentiles* are calculated month-by-month. For each stock, average dollar-volume percentile and return standard deviation are calculated by time-of-day and month. Cross-month-stock medians of mean trading volume percentile and return volatility are calculated by time-of-day and stock size tercile. For each stock, Kyle's λ is estimated by regressing 15-minute returns on corresponding net signed trade (normalized by 15-minute volumes), controlling for month fixed effects, by time-of-day. Cross-stock medians are calculated month-by-month. Cross-month-stock medians are reported by time-of-day and stock size tercile. **Bottom row:** For each stock, trading activity (inverse of trade time) *percentile* and trade-time return standard deviation are calculated by time-of-day and month, stock-by-stock. Average trading activity percentile and trade-time return standard deviation are calculated by time-of-day and month. Cross-month-stock medians of mean trading activity percentile, return volatility, and mean VWAP-based absolute returns are calculated by time-of-day and stock size tercile. For each stock, Kyle's λ is estimated by regressing trade-time returns on corresponding net signed trade, controlling for month fixed effects, by time-of-day. Cross-stock medians are reported by time-of-day and stock size tercile.

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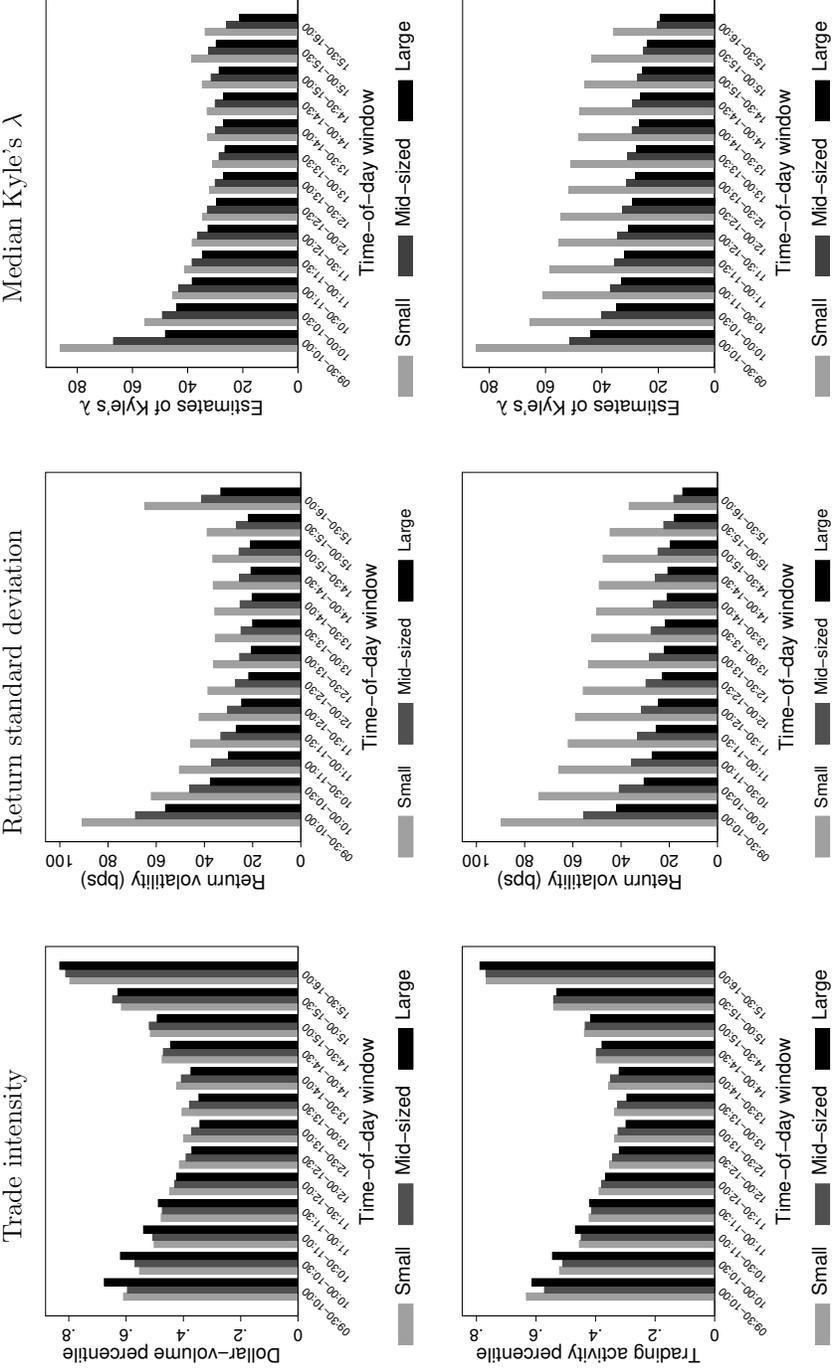
impacts measured in calendar-time. Robustness results in Appendix A reinforce these conclusions, for example showing that a variable autocorrelation structure in trade-time returns does not drive the different volatility patterns in trade- and calendar-time.

3.5   Estimation Methodology

The unique construction of our trade-time sample precludes the use of standard estimation approaches. In particular, panel analyses are infeasible because trading outcomes are measured over variable and stock-specific time intervals, so there is no way to align cross-sections at successive, equally-long time intervals. As is well-understood, estimating parameters by pooling across stocks would also be problematic: auto-correlated error terms would inflate t-statistics (Petersen 2009);[20] and influential outliers would skew t-statistics from a pooled sample. In our setting, further issues would arise due to variations in the number of observations (trade sequences) across stocks and variable target dollar volumes $V_{j,t}$.

As a result, we carry out estimations at the individual stock level, but base statistical inference on the magnitudes of cross-stock mean coefficients and the proportions of statistically-significant coefficients in the cross-section. Foster and Viswanathan (1994) employ a similar approach. We use one-sided tests to underscore the commonality of the sign of parameter estimates across stocks.[21] Estimating parameters at the stock level avoids issues with unobserved heterogeneity across stocks and the issues associated with pooling. Basing inference on the empirical distribution of parameter estimates is conservative because any cross-stock heterogeneity raises the dispersion in the distribution of estimates. To assess statistical significance we first use the traditional way of selecting type I error, setting $\alpha = 5\%$. We also employ the very conservative Bonferroni correction to adjust the type I error for multiple testing by dividing the selected significance level by the number of simultaneously-tested hypotheses (see e.g., Romano and Wolf 2005), reducing the "effective" type I error to about 0.01%.[22]

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[20]For example, cross-stock correlations in trading activity and short-term price movements may arise due to trading behaviors of fund managers who take/leave positions in a group of stocks on the same day.

[21]Commonality is important because cross-stock averages can mislead if estimated parameters differ in sign.

[22]Type I error becomes α/n, where $n \sim 450$ is the number of stocks in the sample. Adjusting individual tests' type I errors in this way when the tests are independent guarantees that the probability of observing

Our main analysis characterizes the associations between volatility/price impacts and trading activity accounting for the time-of-day patterns documented above. To ensure there are enough observations at each time-of-day, we widen time-of-day windows using windows {09:30-10:45, 10:45-12:45, 12:45-14:45, 14:45-16:00}, and sort calendar and trade-time intervals by level of trading activity at a time-of-day. We use shorter windows near open and close because they feature greater trading activity and hence contain more trade sequences: this delivers more equal numbers of observations across time-of-day windows. Time-of-day indicator functions $D_{i,k}^{j}$, with $i \in \{2, 3, 4\}$ index the later windows, and 09:30-10:45 serves as the base window. Thus, $D_{i,k}^{j} = 1$ (or $\delta_{i,l}^{j} = 1$) if trade sequence k (or 15-minute interval l) belongs to time-of-day window i, and $D_{i,k}^{j} = 0$ (or $\delta_{i,l}^{j} = 1$) otherwise.

To control for trading activity, we sort time intervals into *quartiles* of trading activity by time-of-day window. Indicator functions $Z_{s,k}^{jt}$, with $s \in \{1, 2, 3, 4\}$ index trading activity in trade-time. Thus, $Z_{s,k}^{jt} = 1$ if the activity level of trade sequence k is s, and $Z_{s,k}^{jt} = 0$ otherwise, where a higher s indexes a greater trading activity level. To index calendar-time trading activity, we use indicator functions $W_{s,l}^{jt}$, with $W_{s,l}^{jt} = 1$ if interval l's dollar volume falls in quartile s, and $W_{s,l}^{jt} = 0$ otherwise. Sorting time intervals by month and including month fixed effects ensures that any long-term trends do not affect findings. We include month-fixed effects using indicator variables MD^{jt}, where $MD^{jt} = 1$ if stock j's time intervals are from month t, and $MD^{jt} = 0$ otherwise. These month dummies control for standard features (e.g., evolving trading environments), and for the fact that $V_{j,t}$ is updated on a monthly basis.

For each trading outcome Y_k^{jt} and Y_l^{jt}, we estimate

$$Y_k^{jt} = \beta_{1,1}^{j} + \sum_{s=2}^{4} \beta_{1,s}^{j} Z_{s,k}^{jt} + \sum_{i=2}^{4} \sum_{s=1}^{4} \beta_{i,s}^{j} D_{i,k}^{j} Z_{s,k}^{jt} + \sum_{t=1}^{48} \gamma^{j} MD_k^{jt} + u_k^{jt} \tag{10}$$

and

$$Y_l^{jt} = \delta_{1,1}^{j} + \sum_{s=2}^{4} \delta_{1,s}^{j} W_{s,l}^{jt} + \sum_{i=2}^{4} \sum_{s=1}^{4} \delta_{i,s}^{j} D_{i,l}^{j} W_{s,l}^{jt} + \sum_{t=1}^{48} \gamma^{j} MD_l^{jt} + u_l^{jt} \tag{11}$$

for each stock j in our trade-time and calendar-time analyses, respectively.[23] The sixteen

at least one significant test when the null that a coefficient is zero holds for **every** stock is only α.

[23]In an unreported robustness check, we verify that our qualitative findings extend if we include controls

combinations of time-of-day windows and trading activity quartiles fully identify the relevant state space, i.e., separate inclusion of $D^j_{i,k}$ and $Z^{jt}_{s,k}$ would lead to perfect multi-collinearity. Statistical inference is based on the empirical distributions of $\hat{\beta}_{i,s}$ and $\hat{\delta}_{i,s}$ and their respective t-statistics. Our sample of 1354 stocks gives rise to rich empirical distributions of the estimates of each parameter that let us analyze how the size and significance of parameters vary with stock attributes (Foster and Viswanathan 1994).

In estimating equation (10), we sort trade-time intervals in a month t to define trading activity levels captured by $Z^{jt}_{s,k}$ in that month. One might wonder about look-ahead biases in how trading outcomes Y^{jt}_k are assigned to trading activity levels. A robustness analysis in Appendix D instead classifies trade-time intervals to trading activity levels by pseudo-matching them to the empirical distribution of trade times of $V_{j,t-1}$ in the *previous month*. Qualitatively identical patterns obtain: if anything, look-ahead biases attenuate the patterns.[24]

To estimate Kyle's λ we regress midpoint returns (in basis points) on net signed trade from trade-time intervals at a given time-of-day and trading activity level. For stock j, we estimate

$$
\begin{aligned}
mr^{jt}_k &= \lambda^j_{1,1}NOF^{jt}_k + \sum_{s=2}^{4}\bar{\lambda}^j_{1,s}Z^{jt}_{s,k}NOF^{jt}_k + \sum_{i=2}^{4}\sum_{s=1}^{4}\bar{\lambda}^j_{i,s}D^j_{i,k}Z^{jt}_{s,k}NOF^{jt}_k \qquad (12) \\
&\quad + \sum_{t=1}^{48}\gamma^j MD^{jt}_k + u^{jt}_k.
\end{aligned}
$$

$\lambda^j_{1,1}$ is the Kyle's λ estimate for the lowest quartile of trading activity near open. Estimates for other time-of-day window and trading activity quartile combinations are given by $\lambda^j_{i,s} = \lambda^j_{1,1} + \bar{\lambda}^j_{i,s}$.[25] Calendar-time estimates of Kyle's λ use the calendar-time analogues as regression inputs.

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for informational events such as earnings announcements or analyst recommendation dates in our estimating regression, or if we exclude observations from three-day widows around such event dates.

[24]Another source of look-ahead bias may arise due to endogenous responses of traders motivated by clustered volatility/activity. Traders (trading algorithms) may tend to trade more in the very near future after unusually high current activity. Such trading strategies would likely weaken the patterns that we document (i.e., the negative association between volatility and activity in trade-time), because by trading more in response to past high activity, traders increase price impacts and volatility. In an unreported robustness analysis, we verify this by defining current trade sequence's trading activity level based on the time duration of the preceding trade sequence. To make an even stronger case, we use the empirical distribution of time durations from the previous month to assign lagged trade sequences in the current month to activity levels. Qualitatively identical patterns obtain.

[25]Results are robust to including an intercept, and time-of-day and trading activity level fixed effects.

4 Trading Activity, Volatility, and Price Impacts

In this section we document the strikingly different associations between trading activity and volatility/price impacts that emerge when one aggregates outcome in trade-time rather than calendar-time. To analyze how trade-time return volatility varies over the trading day and with trading activity, we use $|r_k^j|$ and $|r_l^j|$ as trading outcomes in equations (10) and(11), respectively. Because average returns over the very short time horizons in our study are essentially zero, $E[|r^j|] \sim E[|r^j - \bar{r}^j|]$. Thus, $|r^j|$ closely approximates return volatility.[26]

Table 1 shows that calendar- and trade-time measures give rise to very different patterns. At each trading activity level, the trade-time return volatility of fixed-dollar positions falls sharply over the trading day: going from near open to near close, on average, the mean absolute trade-time return plunges by 30% for small stocks, 35% for mid-sized stocks, and 33% for large stocks (percentage changes are calculated with respect to the average intercept $\bar{\beta}_{11}$). After controlling for time-of-day, *higher* trading activity is associated with *smaller* trade-time return volatility: at each point in the trading day, return volatilities are smallest in most active markets, and largest in least active ones, falling by 10 to 18% on average. In contrast, in calendar-time, at a given activity level, volatility evolves according to a ∪-shaped pattern over time; while volatility and trading activity are highly *positively* related, not negatively related.[27] Concretely, after controlling for time-of-day, in calendar time, 15-minute return volatilities rise by an average of 33% as dollar volume rises from the bottom quartile to the top. Table 3 shows that similar results obtain for VWAP-based measures of volatility.

The negative association between trading activity and volatility in trade-time is remarkable. On average, the effects of time-of-day on return volatility are stronger than those of trading activity. But, to illustrate how strong both effects are, observe that, on average, for small stocks, the mean absolute trade-time return observed between 12:45 and 14:45 when trading is in the highest quartile of activity is almost 7bp lower than that near close when

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[26]Qualitatively identical results obtain using measures of spot volatility that sum squared trade-by-trade returns for each calendar- or trade-time interval (see Barardehi, Bernhardt, and Ruchti 2019).

[27]This positive association is muted near close, taking a ∪-shaped pattern, reflecting that the time-of-day evolution of calendar-time volatility changes with the level of trading activity.

Table 1: **Return volatility by time-of-day and trading activity level: Trade-time vs. Calendar-time.** Absolute contemporaneous returns ($|r_k^j|$ $|r_T^j|$) are modeled for each stock j using equations (10) and (11). Mean $\hat{\beta}_{i,s}^j$ ($\hat{\delta}_{i,s}^j$) and proportions of significantly **negative** (positive) and proportions of significantly **negative** (positive) estimates $\pi(\cdot)$, according to both traditional type I error α and that after Bonferroni correction, α/n, are reported by time-of-day, trading activity quartile, and stock size tercile. "High−Low" reports mean $\hat{\beta}_{i,4}^j - \hat{\beta}_{i,1}^j$ (mean $\hat{\delta}_{i,4}^j - \hat{\delta}_{i,1}^j$) and proportion of significantly **negative** (positive) differences by time-of-day and stock size tercile. "Late−Early" reports mean $\hat{\beta}_{4,s}^j - \hat{\beta}_{1,s}^j$ and mean $\hat{\delta}_{4,s}^j - \hat{\delta}_{1,s}^j$ and their respective proportions of significantly **negative** differences by activity level and stock size tercile. Stocks are ranked into terciles of average market-capitalization over 2009−2012, and n is the number of stocks in each tercile. Stock specific hypothesis tests use $\alpha = 5\%$.

Panel A: Trade-time volatility

Time-of-day		Small stocks					Mid-sized stocks					Large stocks				
		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30-10:45	Mean $\hat{\beta}$		−8.0	−12.9	−17.8	**−17.8**		−5.3	−9.1	−12.0	**−12.0**		−1.8	−3.4	−4.1	**−4.1**
	$\pi(\alpha)$		0.67	0.81	0.86	0.86		0.85	0.93	0.95	0.95		0.60	0.70	0.71	0.71
	$\pi(\alpha/n)$		0.34	0.55	0.70	0.70		0.62	0.81	0.87	0.87		0.34	0.55	0.60	0.60
10:45-12:45	Mean $\hat{\beta}$	−16.5	−23.4	−28.3	−35.4	**−19.0**	−13.2	−16.6	−19.5	−23.3	**−10.1**	−9.0	−10.2	−11.2	−12.9	**−3.8**
	$\pi(\alpha)$	0.91	0.98	0.99	1.00	0.93	0.99	1.00	1.00	1.00	0.97	0.99	0.99	0.98	0.97	0.80
	$\pi(\alpha/n)$	0.71	0.86	0.92	0.97	0.82	0.95	0.98	0.99	1.00	0.90	0.98	0.97	0.96	0.96	0.64
12:45-14:45	Mean $\hat{\beta}$	−24.8	−31.2	−34.8	−40.2	**−15.4**	−18.1	−21.1	−22.9	−25.8	**−7.8**	−12.0	−13.1	−13.7	−14.6	**−2.7**
	$\pi(\alpha)$	0.97	1.00	1.00	1.00	0.90	1.00	1.00	1.00	1.00	0.96	0.99	1.00	0.99	0.98	0.70
	$\pi(\alpha/n)$	0.85	0.93	0.97	0.98	0.75	0.98	0.99	1.00	1.00	0.84	0.99	0.99	0.98	0.96	0.51
14:45-16:00	Mean $\hat{\beta}$	−33.2	−38.4	−42.4	−46.7	**−13.5**	−23.3	−26.2	−28.5	−31.2	**−7.9**	−14.9	−16.4	−17.9	−20.2	**−5.3**
	$\pi(\alpha)$	0.99	1.00	1.00	1.00	0.91	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	0.94
	$\pi(\alpha/n)$	0.94	0.97	0.98	0.99	0.73	1.00	1.00	1.00	1.00	0.90	0.99	0.99	1.00	1.00	0.88
$\hat{\beta}_{4,s} - \hat{\beta}_{1,s}$		**−33.2**	**−30.4**	**−29.5**	**−28.9**		**−23.3**	**−20.9**	**−19.5**	**−19.2**		**−14.9**	**−14.6**	**−14.5**	**−16.1**	
	$\pi(\alpha)$	0.99	0.99	0.98	0.99		1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	
	$\pi(\alpha/n)$	0.94	0.92	0.93	0.92		1.00	1.00	0.98	0.98		0.99	1.00	0.99	0.99	

Panel B: Calendar-time volatility

Time-of-day		Small stocks					Mid-sized stocks					Large stocks				
		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30-10:45	Mean $\hat{\delta}$		11.5	22.1	46.8	**46.8**		6.9	12.9	29.6	**29.6**		5.8	11.3	25.5	**25.5**
	$\pi(\alpha)$		0.00	0.00	0.00	1.00		0.00	0.00	0.00	1.00		0.00	0.00	0.00	1.00
	$\pi(\alpha/n)$		0.00	0.00	0.00	1.00		0.00	0.00	0.00	0.99		0.00	0.00	0.00	1.00
10:45-12:45	Mean $\hat{\delta}$	−16.6	−10.3	−4.2	9.9	**26.6**	−15.1	−11.0	−7.5	0.6	**15.7**	−12.0	−8.6	−5.7	1.2	**13.1**
	$\pi(\alpha)$	0.99	0.92	0.54	0.05	1.00	1.00	0.98	0.90	0.37	0.99	1.00	0.99	0.91	0.22	1.00
	$\pi(\alpha/n)$	0.89	0.65	0.28	0.01	0.99	0.97	0.91	0.75	0.13	0.98	0.97	0.93	0.72	0.05	0.98
12:45-14:45	Mean $\hat{\delta}$	−19.4	−14.6	−9.8	2.5	**21.9**	−17.8	−14.7	−11.9	−4.9	**12.8**	−14.1	−11.6	−9.2	−3.2	**10.9**
	$\pi(\alpha)$	1.00	0.98	0.85	0.28	1.00	1.00	1.00	0.97	0.76	0.99	1.00	0.99	0.98	0.73	0.99
	$\pi(\alpha/n)$	0.94	0.83	0.57	0.14	0.98	0.98	0.96	0.90	0.54	0.97	0.98	0.96	0.93	0.44	0.97
14:45-16:00	Mean $\hat{\delta}$	15.3	−7.9	−4.3	8.6	**−6.7**	3.8	−12.3	−8.8	−2.1	**−5.9**	3.7	−9.4	−5.9	−0.2	**−3.8**
	$\pi(\alpha)$	0.08	0.71	0.53	0.10	0.14	0.22	0.97	0.91	0.54	0.10	0.10	0.97	0.85	0.48	0.15
	$\pi(\alpha/n)$	0.03	0.52	0.27	0.03	0.08	0.09	0.90	0.73	0.34	0.04	0.04	0.90	0.66	0.26	0.08
$\hat{\delta}_{4,s} - \hat{\delta}_{1,s}$		**15.3**	**−19.4**	**−26.4**	**−38.2**		**3.8**	**−19.2**	**−21.7**	**−31.6**		**3.7**	**−15.2**	**−17.2**	**−25.7**	
	$\pi(\alpha)$	0.08	0.93	1.00	1.00		0.22	1.00	1.00	0.99		0.10	1.00	1.00	0.98	
	$\pi(\alpha/n)$	0.03	0.87	0.96	0.99		0.09	0.98	0.99	0.99		0.04	0.98	0.98	0.97	

trading is in the lowest quartile of activity. To appreciate the strong trade-time association between higher trading activity and reduced return volatility that holds at a point in the trading day, it is useful to reflect on the time-of-day patterns in trading activity and mean absolute returns. A naïve consideration of the time-of-day patterns in the bottom row of Figure 5 might lead to a conjecture that because (a) trading activity and mean absolute returns both tend to be high near open, e.g., due to price discovery and accumulated overnight trading decisions, then (b) greater activity (shorter trade times) near open should be associated with greater volatility. In fact, the *opposite* is true—near open, the mean absolute trade-time return is *smallest* when trading activity is highest, and *largest* when trading activity is lowest.

Similar contrasts between calendar- and trade-time patterns emerge using alternative volatility and price impact measures. Panel A in Table 2 shows that mean trade-time VWAP-based absolute returns fall sharply over the trading day and as trading activity rises.[28] Panel B reveals that at each point in the trading day, trade-time estimates of λ are smaller when trading activity is higher; and that these estimates fall over the trading day at each quartile of trading activity.[29] Table 3 presents the calendar-time analogues, revealing radical differences between calendar-time and trade-time patterns for VWAP-based absolute returns and estimates of Kyle's λ. In trade-time, higher trading activity is associated with smaller price impacts at any time-of-day window. The exact opposite obtains in calendar time, and

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[28]The VWAP-based absolute return for sequence k of stock j is given by the absolute volume-weighted return, $|wr_k^j|$. This measure incorporates prices from all transactions in a trade sequence. As a result, it is a less noisy measure of price movements than measures such as absolute contemporaneous returns that reflect the difference between price at the beginning and end of a trade sequence.

[29]Deuskar and Johnson (2011) note the endogeneity of prices to order flow, and offer alternative identification strategies to estimate Kyle's λ. Their concern is that both trading activity and order flow may be information-driven, and prices move in response to new information revealed via trading. This would bias estimates of Kyle's λ *upward* in active conditions. In fact, our estimates of λ based on trade-time observations are *smallest* when trading activity is highest. We conjecture that sampling over trade-time intervals mitigates the potential bias that Deuskar and Johnson highlight. That is, when trading volume contains information, using trade-time intervals distributes the bias more evenly across different market conditions because dollar-volume is fixed, i.e., volume-related unobserved information content is largely fixed. Indeed, our analysis highlights a mechanism that can bias estimates of Kyle's λ in the direction opposite that suggested by Deuskar and Johnson. We provide strong evidence that high trading activity is driven in part by endogenous consumption of unusually high liquidity, resulting in smaller estimates of Kyle's λ when markets are more active. Our findings highlight the importance of developing new measures of liquidity that control for this endogeneity. In fact, Barardehi et al. (2018) show that trade-time measures of liquidity outperform estimates of Kyle's λ in capturing cross-sections of institutional trading costs and in explaining the cross-sections of stock returns.

Table 2: **Price impacts of fixed-dollar positions by time-of-day and trading activity level.** *Panel A*: VWAP-based absolute returns ($|wr_k^j|$) are modeled for each stock j using equation (10). Mean $\hat{\beta}_{i,s}^j$ and proportion of significantly **negative** estimates $\pi(\cdot)$, according to both traditional type I error α and type I error after Bonferroni correction α/n are reported by time-of-day, trading activity quartile, and stock size tercile. *Panel B*: Kyle's λ is estimated for each stock j using equation (12). Mean $\hat{\lambda}_{i,s}^j$ and the proportion of significantly **positive** estimates $\pi(\cdot)$, according to traditional type I error α and that after Bonferroni correction, α/n, are reported by time-of-day, trading activity quartile, and stock size tercile. "High–Low" reports mean $\hat{\beta}_{i,4}^j - \hat{\beta}_{i,1}^j$ (or $\hat{\lambda}_{i,4}^j - \hat{\lambda}_{i,1}^j$) and proportion of significantly **negative** differences by time-of-day and stock size tercile. "Late–Early" reports mean $\hat{\beta}_{4,s}^j - \hat{\beta}_{1,s}^j$ (or $\hat{\lambda}_{4,s}^j - \hat{\lambda}_{1,s}^j$) and proportions of significantly **negative** differences by activity level and stock size tercile. Stocks are ranked into terciles of average market-capitalization over 2009–2012, and n is the number of stocks in each tercile. Stock specific hypothesis tests use $\alpha = 5\%$.

Panel A: VWAP-based absolute returns

Small stocks

Time-of-day		Trading activity level 1	2	3	4	High–Low
9:30–10:45	Mean $\hat{\beta}$	75.6	−7.1	−11.9	−16.7	**−16.7**
	$\pi(\alpha)$		0.77	0.88	0.91	0.91
	$\pi(\alpha/n)$		0.48	0.71	0.85	0.85
10:45–12:45	Mean $\hat{\beta}$	−11.9	−17.3	−21.4	−25.8	**−13.8**
	$\pi(\alpha)$	0.90	0.97	0.99	0.98	0.93
	$\pi(\alpha/n)$	0.73	0.87	0.94	0.95	0.83
12:45–14:45	Mean $\hat{\beta}$	−17.4	−22.2	−25.0	−28.3	**−10.8**
	$\pi(\alpha)$	0.96	0.99	1.00	1.00	0.88
	$\pi(\alpha/n)$	0.86	0.93	0.95	0.97	0.69
14:45–16:00	Mean $\hat{\beta}$	−23.0	−26.9	−29.5	−33.3	**−10.3**
	$\pi(\alpha)$	0.99	1.00	1.00	1.00	0.93
	$\pi(\alpha/n)$	0.95	0.97	0.97	0.98	0.76
Late-Early $\hat{\beta}_{4,s} - \hat{\beta}_{1,s}$		**−23.0**	**−19.8**	**−17.6**	**−16.6**	
	$\pi(\alpha)$	0.99	0.97	0.96	0.96	
	$\pi(\alpha/n)$	0.95	0.90	0.89	0.89	

Mid-sized stocks

Time-of-day		Trading activity level 1	2	3	4	High–Low
9:30–10:45	Mean $\hat{\beta}$	42.8	−4.8	−7.9	−11.1	**−11.1**
	$\pi(\alpha)$		0.91	0.97	0.98	0.98
	$\pi(\alpha/n)$		0.69	0.89	0.96	0.96
10:45–12:45	Mean $\hat{\beta}$	−8.8	−11.7	−13.9	−16.4	**−7.6**
	$\pi(\alpha)$	0.98	1.00	1.00	1.00	0.94
	$\pi(\alpha/n)$	0.94	0.97	0.99	0.99	0.88
12:45–14:45	Mean $\hat{\beta}$	−11.9	−14.4	−15.8	−17.7	**−5.8**
	$\pi(\alpha)$	0.98	1.00	1.00	1.00	0.96
	$\pi(\alpha/n)$	0.96	0.98	1.00	1.00	0.82
14:45–16:00	Mean $\hat{\beta}$	−15.5	−17.7	−19.2	−21.7	**−6.2**
	$\pi(\alpha)$	1.00	1.00	1.00	1.00	0.96
	$\pi(\alpha/n)$	0.99	1.00	1.00	1.00	0.91
Late-Early $\hat{\beta}_{4,s} - \hat{\beta}_{1,s}$		**−15.5**	**−12.9**	**−11.3**	**−10.6**	
	$\pi(\alpha)$	1.00	0.99	0.99	0.98	
	$\pi(\alpha/n)$	0.99	0.98	0.96	0.96	

Large stocks

Time-of-day		Trading activity level 1	2	3	4	High–Low
9:30–10:45	Mean $\hat{\beta}$	34.9	−2.7	−5.2	−6.4	**−6.4**
	$\pi(\alpha)$		0.68	0.81	0.83	0.83
	$\pi(\alpha/n)$		0.43	0.69	0.74	0.74
10:45–12:45	Mean $\hat{\beta}$	−4.1	−6.5	−8.1	−10.7	**−6.5**
	$\pi(\alpha)$	0.89	0.95	0.94	0.96	0.78
	$\pi(\alpha/n)$	0.87	0.89	0.91	0.93	0.65
12:45–14:45	Mean $\hat{\beta}$	−5.6	−7.9	−9.6	−11.3	**−5.7**
	$\pi(\alpha)$	0.91	0.95	0.97	0.98	0.70
	$\pi(\alpha/n)$	0.88	0.90	0.94	0.94	0.47
14:45–16:00	Mean $\hat{\beta}$	−8.9	−11.5	−13.3	−16.4	**−7.5**
	$\pi(\alpha)$	0.96	0.98	0.99	1.00	0.93
	$\pi(\alpha/n)$	0.93	0.96	0.98	0.99	0.86
Late-Early $\hat{\beta}_{4,s} - \hat{\beta}_{1,s}$		**−8.9**	**−8.8**	**−8.1**	**−10.0**	
	$\pi(\alpha)$	0.96	0.97	0.96	0.97	
	$\pi(\alpha/n)$	0.93	0.94	0.94	0.96	

Panel B: Kyle's λ estimates

Small stocks

Time-of-day		Trading activity level 1	2	3	4	High–Low
9:30–10:45	Mean $\hat{\lambda}$	99.7	84.4	73.6	60.7	**−39.1**
	$\pi(\alpha)$	1.00	0.99	0.99	0.98	0.79
	$\pi(\alpha/n)$	0.94	0.94	0.91	0.90	0.50
10:45–12:45	Mean $\hat{\lambda}$	88.5	74.8	63.2	46.1	**−42.4**
	$\pi(\alpha)$	1.00	1.00	1.00	0.98	0.89
	$\pi(\alpha/n)$	0.97	0.96	0.93	0.86	0.61
12:45–14:45	Mean $\hat{\lambda}$	76.8	63.9	55.7	40.9	**−35.9**
	$\pi(\alpha)$	1.00	0.99	0.99	0.98	0.85
	$\pi(\alpha/n)$	0.95	0.93	0.90	0.82	0.45
14:45–16:00	Mean $\hat{\lambda}$	62.4	52.7	45.3	34.8	**−27.6**
	$\pi(\alpha)$	1.00	0.99	0.99	0.98	0.82
	$\pi(\alpha/n)$	0.92	0.91	0.85	0.80	0.35
Late-Early $\hat{\lambda}_{4,s} - \hat{\lambda}_{1,s}$		**−37.3**	**−31.7**	**−28.3**	**−25.8**	
	$\pi(\alpha)$	0.75	0.71	0.71	0.70	
	$\pi(\alpha/n)$	0.47	0.43	0.40	0.38	

Mid-sized stocks

Time-of-day		Trading activity level 1	2	3	4	High–Low
9:30–10:45	Mean $\hat{\lambda}$	63.8	53.1	44.7	34.5	**−29.3**
	$\pi(\alpha)$	1.00	1.00	0.98	0.98	0.93
	$\pi(\alpha/n)$	0.97	0.97	0.94	0.94	0.79
10:45–12:45	Mean $\hat{\lambda}$	51.6	43.6	36.3	25.0	**−26.6**
	$\pi(\alpha)$	1.00	1.00	1.00	0.98	0.95
	$\pi(\alpha/n)$	0.97	0.97	0.96	0.92	0.80
12:45–14:45	Mean $\hat{\lambda}$	43.4	36.6	31.8	22.3	**−21.0**
	$\pi(\alpha)$	1.00	0.99	0.99	0.98	0.93
	$\pi(\alpha/n)$	0.97	0.95	0.95	0.91	0.69
14:45–16:00	Mean $\hat{\lambda}$	34.5	28.8	24.4	17.9	**−16.5**
	$\pi(\alpha)$	0.99	1.00	0.99	0.99	0.92
	$\pi(\alpha/n)$	0.96	0.95	0.95	0.92	0.64
Late-Early $\hat{\lambda}_{4,s} - \hat{\lambda}_{1,s}$		**−29.3**	**−24.3**	**−20.3**	**−16.6**	
	$\pi(\alpha)$	0.89	0.88	0.87	0.83	
	$\pi(\alpha/n)$	0.76	0.71	0.65	0.62	

Large stocks

Time-of-day		Trading activity level 1	2	3	4	High–Low
9:30–10:45	Mean $\hat{\lambda}$	50.2	44.7	39.6	32.0	**−18.2**
	$\pi(\alpha)$	1.00	0.99	0.99	0.98	0.87
	$\pi(\alpha/n)$	0.98	0.98	0.98	0.97	0.63
10:45–12:45	Mean $\hat{\lambda}$	40.7	36.5	32.5	24.1	**−16.6**
	$\pi(\alpha)$	1.00	1.00	0.99	0.99	0.90
	$\pi(\alpha/n)$	0.98	0.97	0.97	0.96	0.67
12:45–14:45	Mean $\hat{\lambda}$	35.9	32.4	29.4	22.5	**−13.4**
	$\pi(\alpha)$	1.00	0.99	1.00	0.98	0.85
	$\pi(\alpha/n)$	0.98	0.98	0.97	0.95	0.54
14:45–16:00	Mean $\hat{\lambda}$	30.7	26.9	23.5	17.6	**−13.1**
	$\pi(\alpha)$	1.00	1.00	0.99	1.00	0.90
	$\pi(\alpha/n)$	0.98	0.98	0.97	0.95	0.59
Late-Early $\hat{\lambda}_{4,s} - \hat{\lambda}_{1,s}$		**−19.4**	**−17.8**	**−16.1**	**−14.4**	
	$\pi(\alpha)$	0.89	0.89	0.87	0.86	
	$\pi(\alpha/n)$	0.66	0.68	0.66	0.68	

estimates of Kyle's λ rise sharply with trading volume, increasing by 100-400%. So, too, trade-time estimates of Kyle's λ fall by about 40% over the trading day, but calendar-time estimates tend to evolve according to a $\cup$-shaped pattern. Similar calendar-time patterns obtain for VWAP-based absolute returns measured over 15-minute intervals.

These findings highlight the fundamentally different pictures that obtain using trade-time measures rather than calendar-time measures. The positive calendar-time association between trading activity and estimates of Kyle's λ might suggest a setting where increases in trading activity are driven by the increased presence of privately-informed speculators. This interpretation has been the empirical foundation for theoretical analyses in which price movements reflect activities of informed investors. Our trade-time analysis casts doubt on this interpretation, as it delivers the opposite relationship between trading activity and price impacts. We next provide evidence that the link between price movements and trading activity primarily reflects non-competitive and time-varying liquidity provision.

5 Trade aggressiveness and Imperfect Liquidity

We now link the associations between trading activity level and volatility/price impacts to basic measures of trade execution strategies and outcomes, analyzing how signed trade imbalances and transaction sizes vary with trading activity. Our findings are consistent with endogenous choices of trade aggressiveness in response to extant liquidity.

Table 4 shows that in trade-time, both signed trade imbalances and average trade size rise sharply with trading activity. At any given point in the trading day, as trading rises from the bottom quartile of activity to the top quartile, average percent trade imbalances rise by 7-9% depending on market cap, and average transaction size grows by 300-400 shares.[30] That is, large, aggressive execution strategies dominate more active trading markets, which we just showed also feature *smaller* volatility/price impacts. This finding is consistent with Latza

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[30]In unreported robustness analyses, we verify that the positive association between average transaction size and trading activity is not driven by extremely large transactions that skew mean transaction sizes upwards. For example, prior to constructing trade sequences of V_{jt} for stock j in month t, we exclude transactions whose dollar values exceed $0.2V_{jt}$. Qualitatively identical results obtain.

Table 3: **Calendar-time 15-minute price impacts by time-of-day and trading activity level.** *Panel A*: VWAP-based absolute returns ($|wr_{j,l}|$) are modeled for each stock j using equation (11). Mean $\hat{\delta}^j_{i,s}$ and proportion of significantly **positive** estimates $\pi(\cdot)$, according to both traditional type I error α and type I error after Bonferroni correction α/n are reported by time-of-day, trading activity quartile, and stock size tercile. *Panel B*: Kyle's λ is estimated for each stock j using the **calendar-time** version of equation (12). Mean $\hat{\lambda}^j_{i,s}$ and the proportion of significantly **positive** estimates $\pi(\cdot)$, according to traditional type I error α and that after Bonferroni correction α/n, are reported by time-of-day, trading activity quartile, and stock size tercile. "High−Low" reports mean $\hat{\delta}^j_{i,4} - \hat{\delta}^j_{i,1}$ (or $\hat{\lambda}^j_{i,4} - \hat{\lambda}^j_{i,1}$) and proportion of significantly **positive** differences by time-of-day and stock size tercile. "Late−Early" reports mean $\hat{\delta}^j_{4,s} - \hat{\delta}^j_{1,s}$ (or $\hat{\lambda}^j_{4,s} - \hat{\lambda}^j_{1,s}$) and proportions of significantly **negative** differences by activity level and stock size tercile. Stocks are ranked into terciles of average market-capitalization over 2009–2012, and n is the number of stocks in each tercile. Stock specific hypothesis tests use $\alpha = 5\%$.

Panel A: VWAP-based absolute returns

Time-of-day		Small stocks					Mid-sized stocks					Large stocks				
		Trading activity level					Trading activity level					Trading activity level				
		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30–10:45	Mean $\hat{\delta}$		10.3	18.6	37.4	**37.4**		5.1	9.2	20.5	**20.5**		2.8	5.9	14.6	**14.6**
	$\pi(\alpha)$		0.00	0.00	0.00	1.00		0.00	0.00	0.00	0.98		0.04	0.04	0.04	0.94
	$\pi(\alpha/n)$		0.00	0.00	0.00	1.00		0.00	0.00	0.00	0.97		0.01	0.02	0.03	0.92
10:45–12:45	Mean $\hat{\delta}$	−9.1	−2.7	2.4	13.2	**22.3**	−9.3	−6.3	−4.1	1.0	**10.3**	−6.0	−5.2	−4.5	−1.3	**4.7**
	$\pi(\alpha)$	0.94	0.59	0.31	0.02	1.00	0.98	0.93	0.83	0.26	0.98	0.91	0.92	0.89	0.30	0.91
	$\pi(\alpha/n)$	0.77	0.42	0.14	0.00	1.00	0.93	0.84	0.68	0.07	0.96	0.87	0.84	0.74	0.10	0.89
12:45–14:45	Mean $\hat{\delta}$	−10.4	−4.9	−0.6	8.7	**19.1**	−11.0	−8.6	−6.7	−2.4	**8.6**	−7.2	−6.9	−6.2	−3.6	**3.6**
	$\pi(\alpha)$	0.95	0.66	0.51	0.12	1.00	0.99	0.96	0.92	0.64	0.98	0.92	0.94	0.93	0.72	0.92
	$\pi(\alpha/n)$	0.79	0.55	0.35	0.04	1.00	0.94	0.89	0.83	0.44	0.96	0.88	0.87	0.86	0.46	0.87
14:45–16:00	Mean $\hat{\delta}$	−6.8	−0.9	3.8	15.0	**21.8**	−10.5	−6.9	−4.6	0.2	**10.7**	−5.7	−6.5	−5.4	−3.0	**2.7**
	$\pi(\alpha)$	0.80	0.53	0.26	0.03	0.98	0.96	0.93	0.80	0.35	0.96	0.87	0.93	0.87	0.44	0.88
	$\pi(\alpha/n)$	0.69	0.35	0.11	0.01	0.97	0.93	0.81	0.60	0.16	0.95	0.86	0.85	0.68	0.21	0.86
Late−Early	$\hat{\delta}_{4,s} - \hat{\delta}_{1,s}$	**−6.8**	**−11.2**	**−14.8**	**−22.4**		**−10.5**	**−12.0**	**−13.7**	**−20.4**		**−5.7**	**−9.3**	**−11.4**	**−17.6**	
	$\pi(\alpha)$	0.80	0.93	0.98	0.98		0.96	0.99	0.99	1.00		0.87	0.96	0.98	0.98	
	$\pi(\alpha/n)$	0.69	0.84	0.90	0.96		0.93	0.97	0.97	0.99		0.86	0.90	0.92	0.94	

Panel B: Kyle's λ estimates

Time-of-day		Small stocks					Mid-sized stocks					Large stocks				
		Trading activity level					Trading activity level					Trading activity level				
		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30–10:45	Mean $\hat{\lambda}$	43.3	65.0	81.2	105.9	**62.6**	50.1	62.2	68.9	74.0	**23.9**	44.7	52.2	55.5	57.7	**12.9**
	$\pi(\alpha)$	1.00	1.00	1.00	1.00	0.96	1.00	0.99	1.00	0.99	0.68	1.00	1.00	0.99	0.98	0.43
	$\pi(\alpha/n)$	0.93	0.99	1.00	0.99	0.82	0.97	0.99	0.99	0.98	0.42	0.97	0.97	0.97	0.95	0.22
10:45–12:45	Mean $\hat{\lambda}$	24.3	37.4	47.4	63.7	**39.4**	30.2	39.1	44.9	47.4	**17.2**	29.2	35.9	39.5	41.4	**12.2**
	$\pi(\alpha)$	0.97	0.99	1.00	1.00	0.96	1.00	1.00	1.00	1.00	0.73	0.99	1.00	1.00	0.99	0.55
	$\pi(\alpha/n)$	0.82	0.95	0.97	0.99	0.78	0.96	0.97	0.99	0.97	0.38	0.95	0.98	0.98	0.97	0.24
12:45–14:45	Mean $\hat{\lambda}$	20.9	31.1	39.2	53.7	**32.8**	25.5	32.0	36.0	40.2	**14.8**	24.9	29.7	32.8	35.4	**10.5**
	$\pi(\alpha)$	0.97	0.99	1.00	1.00	0.95	1.00	1.00	0.99	1.00	0.68	1.00	1.00	1.00	0.99	0.54
	$\pi(\alpha/n)$	0.76	0.89	0.95	0.99	0.70	0.93	0.96	0.96	0.97	0.31	0.95	0.95	0.96	0.96	0.20
14:45–16:00	Mean $\hat{\lambda}$	13.4	36.0	48.1	72.2	**58.8**	14.9	35.6	41.6	52.8	**37.9**	11.5	33.2	38.4	47.8	**36.3**
	$\pi(\alpha)$	0.69	0.98	1.00	1.00	0.99	0.80	0.99	0.99	1.00	0.95	0.77	0.98	0.99	0.99	0.95
	$\pi(\alpha/n)$	0.38	0.86	0.96	0.99	0.91	0.49	0.94	0.95	0.97	0.82	0.47	0.92	0.94	0.95	0.84
Late−Early	$\hat{\lambda}_{4,s} - \hat{\lambda}_{1,s}$	**−29.9**	**−29.0**	**−33.1**	**−33.7**		**−35.2**	**−26.6**	**−27.3**	**−21.2**		**−33.2**	**−19.0**	**−17.1**	**−9.9**	
	$\pi(\alpha)$	0.90	0.93	0.92	0.79		0.96	0.93	0.89	0.63		0.97	0.74	0.66	0.40	
	$\pi(\alpha/n)$	0.60	0.50	0.53	0.45		0.85	0.51	0.51	0.33		0.84	0.28	0.27	0.21	

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et al. (2014), which finds that faster aggressive transactions incur smaller execution costs.

These findings collectively represent comprehensive evidence that endogenous choices by traders of whether to consume or supply liquidity drives outcomes. That is, when the order book is deep near good prices on one side of the market, traders seeking to establish positions on the other side have incentives to submit aggressive large marketable orders that consume this liquidity. This leads to the coexistence of small price impacts, imbalanced signed trades, and large transaction sizes. In contrast, when there is little depth, traders on the other side have incentives to shift toward passive orders, i.e., to provide liquidity themselves in an effort to obtain better prices. As such, little extant liquidity leads to the coexistence of large price impacts, more balanced trade, and small transaction sizes.

Table 5 provides the calendar-time analogues. Transaction sizes are positively related with trading activity, just as they are in trade-time. However, relationships between signed trade imbalance and trading activity in calendar time radically differ from their trade-time counterparts. In calendar time, signed trade imbalances *fall* as 15-minute trading volumes rise from the bottom quartile to the top. This negative association is inconsistent with variations in informed trading driving variations in trading activity: if informed trading drove high trading volumes, signed trade imbalances would be higher in high volume times—informed traders with more private information trade more. We next show that this decline in signed trade imbalances in more active markets is a symptom of over-aggregation in calendar-time that lumps together sequentially-offsetting buyer- and seller-dominated order flow in a single calendar-time interval, consistent with the stylized model of Barardehi et al. (2019).

6 Over-aggregation and Mis-measured Price Impacts

We now show that the opposing signed-trade imbalance/activity relationships found using trade- and calendar-time measures reflect that calendar-time measures over-aggregate in active markets. This over-aggregation results in mis-measured, overly-balanced net signed trades being matched up with large movements in prices in active markets, creating a false

Table 4: **Trade imbalances and mean transaction sizes of fixed-dollar positions by time-of-day and trading activity level.** Trade imbalances (IMB_k^j) and mean transaction sizes (TS_k^j) are estimated stock-by-stock using equation (10). Mean $\hat\beta_{i,s}$ and the proportion of significantly **positive** estimates $\pi(\cdot)$, according to both traditional type I error α and Type I error after Bonferroni correction α/n, are reported by time-of-day, trading activity quartile, and stock size tercile. "High–Low" reports mean $\hat\beta_{i,4}^j - \hat\beta_{i,1}^j$ and proportions of significantly **positive** differences by time-of-day and stock size tercile. "Late–Early" reports mean $\hat\beta_{4,s}^j - \hat\beta_{1,s}^j$ and proportion of significantly **negative** differences by activity level and stock size tercile. Stocks are ranked into terciles of average market-capitalization over 2009–2012, and n is the number of stocks in each tercile. Stock specific hypothesis tests use $\alpha = 5\%$.

Panel A: Trade imbalance (%)

Time-of-day	stat	Small stocks 1	2	3	4	High–Low	Mid-sized stocks 1	2	3	4	High–Low	Large stocks 1	2	3	4	High–Low
9:30–10:45	Mean $\hat\beta$		2.1%	4.1%	8.9%	**8.9%**		1.7%	3.5%	7.9%	**7.9%**		1.2%	2.5%	6.0%	**6.0%**
	$\pi(\alpha)$		0.80	0.98	1.00	1.00		0.94	0.99	1.00	1.00		0.92	1.00	1.00	1.00
	$\pi(\alpha/n)$		0.36	0.77	0.98	0.98		0.58	0.90	1.00	1.00		0.59	0.95	1.00	1.00
10:45–12:45	Mean $\hat\beta$	-0.5%	1.3%	3.2%	7.8%	**8.3%**	-0.1%	1.3%	2.9%	7.2%	**7.3%**	0.0%	1.1%	2.2%	5.7%	**5.6%**
	$\pi(\alpha)$	0.06	0.56	0.91	1.00	1.00	0.14	0.80	0.97	1.00	1.00	0.18	0.86	0.99	1.00	1.00
	$\pi(\alpha/n)$	0.00	0.19	0.64	0.98	0.99	0.04	0.38	0.86	0.99	1.00	0.06	0.45	0.92	1.00	1.00
12:45–14:45	Mean $\hat\beta$	-0.6%	1.0%	2.9%	7.6%	**8.2%**	-0.2%	1.1%	2.6%	6.9%	**7.1%**	0.0%	1.0%	2.1%	5.4%	**5.4%**
	$\pi(\alpha)$	0.06	0.45	0.87	1.00	1.00	0.19	0.65	0.95	1.00	1.00	0.22	0.76	0.98	1.00	1.00
	$\pi(\alpha/n)$	0.01	0.16	0.58	0.96	0.99	0.08	0.31	0.81	0.99	1.00	0.10	0.39	0.89	1.00	1.00
14:45–16:00	Mean $\hat\beta$	-0.3%	1.4%	3.2%	7.9%	**8.2%**	0.0%	1.3%	2.8%	7.1%	**7.1%**	0.2%	1.2%	2.3%	5.4%	**5.2%**
	$\pi(\alpha)$	0.15	0.55	0.89	1.00	1.00	0.24	0.59	0.93	1.00	1.00	0.35	0.70	0.96	1.00	1.00
	$\pi(\alpha/n)$	0.07	0.22	0.60	0.97	1.00	0.18	0.36	0.76	0.99	1.00	0.19	0.47	0.81	1.00	1.00
$\hat\beta_{4,s}-\hat\beta_{1,s}$ Late–Early	Mean	-0.3%	-0.7%	-0.9%	-0.9%		0.0%	-0.5%	-0.7%	-0.8%		0.2%	0.0%	-0.2%	-0.6%	
	$\pi(\alpha)$	0.15	0.12	0.11	0.11		0.24	0.20	0.19	0.19		0.35	0.29	0.25	0.20	
	$\pi(\alpha/n)$	0.07	0.06	0.05	0.03		0.18	0.15	0.14	0.11		0.19	0.18	0.18	0.17	

Panel B: Average transaction size (# of shares)

Time-of-day	stat	Small stocks 1	2	3	4	High–Low	Mid-sized stocks 1	2	3	4	High–Low	Large stocks 1	2	3	4	High–Low
9:30–10:45	Mean $\hat\beta$		45.6	100.2	402.1	**402.1**		31.3	75.6	337.9	**337.9**		17.8	42.9	265.5	**265.5**
	$\pi(\alpha)$		0.09	0.47	0.98	0.98		0.10	0.53	0.99	0.99		0.04	0.33	0.97	0.97
	$\pi(\alpha/n)$		0.00	0.08	0.83	0.83		0.00	0.09	0.93	0.93		0.00	0.04	0.87	0.87
10:45–12:45	Mean $\hat\beta$	-15.2	19.7	71.7	319.6	**334.8**	-3.1	20.0	52.1	267.7	**270.8**	0.1	15.4	36.1	218.7	**218.5**
	$\pi(\alpha)$	0.00	0.03	0.20	0.95	0.97	0.00	0.03	0.33	0.98	0.98	0.00	0.02	0.18	0.94	0.94
	$\pi(\alpha/n)$	0.00	0.00	0.02	0.73	0.82	0.00	0.00	0.03	0.85	0.90	0.00	0.00	0.01	0.79	0.81
12:45–14:45	Mean $\hat\beta$	-18.5	8.6	50.9	299.9	**318.4**	-1.9	17.1	46.7	250.2	**252.1**	2.0	15.7	35.4	233.5	**231.5**
	$\pi(\alpha)$	0.00	0.01	0.15	0.95	0.97	0.00	0.02	0.24	0.97	0.98	0.01	0.02	0.16	0.94	0.94
	$\pi(\alpha/n)$	0.00	0.00	0.01	0.69	0.77	0.00	0.00	0.01	0.84	0.87	0.00	0.00	0.01	0.80	0.78
14:45–16:00	Mean $\hat\beta$	-17.5	8.3	50.7	241.5	**259.0**	0.2	17.3	42.3	197.3	**197.1**	4.5	20.3	41.5	181.4	**176.8**
	$\pi(\alpha)$	0.00	0.01	0.12	0.93	0.96	0.00	0.04	0.27	0.94	0.96	0.01	0.06	0.30	0.93	0.94
	$\pi(\alpha/n)$	0.00	0.00	0.01	0.61	0.73	0.00	0.00	0.03	0.76	0.83	0.00	0.01	0.03	0.76	0.78
$\hat\beta_{4,s}-\hat\beta_{1,s}$ Late–Early	Mean	-17.5	-37.3	-49.6	-160.6		0.2	-14.0	-33.3	-140.6		4.5	2.5	-1.4	-84.1	
	$\pi(\alpha)$	0.03	0.11	0.25	0.69		0.01	0.06	0.20	0.79		0.02	0.03	0.08	0.64	
	$\pi(\alpha/n)$	0.00	0.01	0.03	0.40		0.00	0.00	0.03	0.51		0.00	0.00	0.02	0.34	

Table 5: **Calendar-time 15-minute trade imbalances and mean transaction sizes by time-of-day and trading activity level.** Trade imbalances ($IMB_{j,l}$) and mean transaction sizes ($TS_{j,l}$) are estimated stock-by-stock using equation (11). Mean $\hat{\delta}_{i,s}$ and the proportion of significantly **positive** estimates $\pi(\cdot)$, according to both traditional type I error α and Type I error after Bonferroni correction α/n, are reported by time-of-day, trading activity quartile, and stock size tercile. "High−Low" reports mean $\hat{\delta}_{i,4} - \hat{\delta}_{i,1}$ and proportions of significantly **negative** and significantly **positive** differences by time-of-day and stock size tercile for trade imbalances and trasnaction size, respectively. "Late−Early" reports mean $\hat{\delta}_{4,s} - \hat{\delta}_{1,s}$ and proportion of significantly **positive** and proportion of significantly **negative** differences by activity level and stock size tercile. Stocks are ranked into terciles of average market-capitalization over 2009–2012, and n is the number of stocks in each tercile. Stock specific hypothesis tests use $\alpha = 5\%$.

Panel A: Trade imbalance (%)

Time-of-day		Small stocks					Mid-sized stocks					Large stocks				
		Trading activity level					Trading activity level					Trading activity level				
		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30–10:45	Mean $\hat{\delta}$		−4.7%	−6.2%	−6.1%	−6.1%		−1.4%	−1.7%	−0.4%	−0.4%		−0.2%	0.0%	1.5%	1.5%
	$\pi(\alpha)$		0.89	0.91	0.79	0.79		0.56	0.60	0.29	0.29		0.15	0.14	0.05	0.05
	$\pi(\alpha/n)$		0.68	0.75	0.66	0.66		0.24	0.31	0.16	0.16		0.03	0.05	0.03	0.03
10:45–12:45	Mean $\hat{\delta}$	2.2%	−3.3%	−5.0%	−5.8%	−8.0%	1.1%	−0.8%	−1.4%	−0.5%	−1.6%	0.9%	0.2%	0.2%	1.2%	0.3%
	$\pi(\alpha)$	0.01	0.72	0.79	0.79	0.95	0.02	0.45	0.51	0.34	0.58	0.01	0.13	0.14	0.04	0.19
	$\pi(\alpha/n)$	0.00	0.58	0.69	0.68	0.88	0.00	0.21	0.32	0.21	0.41	0.00	0.03	0.04	0.02	0.10
12:45–14:45	Mean $\hat{\delta}$	2.0%	−3.5%	−5.2%	−5.7%	−7.7%	1.2%	−0.9%	−1.4%	−0.5%	−1.8%	1.3%	0.4%	0.3%	1.3%	0.0%
	$\pi(\alpha)$	0.02	0.74	0.80	0.76	0.96	0.03	0.51	0.55	0.35	0.65	0.02	0.16	0.15	0.04	0.24
	$\pi(\alpha/n)$	0.00	0.59	0.69	0.66	0.87	0.00	0.28	0.37	0.23	0.48	0.00	0.05	0.06	0.02	0.15
14:45–16:00	Mean $\hat{\delta}$	3.1%	−6.1%	−8.3%	−9.5%	−12.6%	6.2%	−3.0%	−3.5%	−3.3%	−9.5%	7.9%	−0.9%	−1.1%	−1.1%	−9.0%
	$\pi(\alpha)$	0.20	0.90	0.97	0.98	1.00	0.00	0.82	0.88	0.86	1.00	0.01	0.57	0.59	0.59	1.00
	$\pi(\alpha/n)$	0.12	0.81	0.88	0.92	0.99	0.00	0.69	0.75	0.71	0.99	0.00	0.36	0.37	0.36	0.99
Late−Early	$\hat{\delta}_{4,s} - \hat{\delta}_{1,s}$	3.1%	−1.5%	−2.2%	−3.4%		6.2%	−1.6%	−1.8%	−2.9%		7.9%	−0.7%	−1.2%	−2.6%	
	$\pi(\alpha)$	0.20	0.63	0.77	0.87		0.00	0.74	0.80	0.92		0.01	0.62	0.75	0.92	
	$\pi(\alpha/n)$	0.12	0.27	0.47	0.68		0.00	0.52	0.59	0.80		0.00	0.36	0.50	0.83	

Panel B: Average transaction size (# of shares)

Time-of-day		Small stocks					Mid-sized stocks					Large stocks				
		Trading activity level					Trading activity level					Trading activity level				
		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30–10:45	Mean $\hat{\delta}$		33.8	67.3	245.0	245.0		12.6	26.3	106.4	106.4		11.9	24.0	73.1	73.1
	$\pi(\alpha)$		0.61	0.89	1.00	1.00		0.47	0.86	0.99	0.99		0.26	0.63	0.93	0.93
	$\pi(\alpha/n)$		0.13	0.49	0.97	0.97		0.01	0.38	0.96	0.96		0.01	0.17	0.75	0.75
10:45–12:45	Mean $\hat{\delta}$	−15.1	6.1	29.3	127.9	143.0	−10.6	−0.5	10.7	70.7	81.3	−5.8	4.2	15.2	55.0	60.8
	$\pi(\alpha)$	0.00	0.05	0.47	0.99	1.00	0.00	0.01	0.33	0.99	0.99	0.00	0.03	0.28	0.89	0.93
	$\pi(\alpha/n)$	0.00	0.01	0.10	0.88	0.94	0.00	0.00	0.02	0.92	0.96	0.00	0.00	0.03	0.65	0.74
12:45–14:45	Mean $\hat{\delta}$	−14.0	3.8	25.3	119.0	133.0	−4.2	1.0	12.0	72.7	77.0	12.3	11.1	21.4	78.3	66.1
	$\pi(\alpha)$	0.00	0.03	0.42	0.99	1.00	0.04	0.04	0.42	0.99	0.99	0.26	0.15	0.50	0.98	0.88
	$\pi(\alpha/n)$	0.00	0.00	0.06	0.87	0.91	0.00	0.00	0.05	0.93	0.96	0.02	0.01	0.10	0.81	0.73
14:45–16:00	Mean $\hat{\delta}$	6.8	11.1	26.3	88.5	81.7	21.7	3.3	13.9	53.6	31.9	54.9	9.6	22.3	54.3	−0.6
	$\pi(\alpha)$	0.23	0.11	0.43	0.93	0.89	0.58	0.08	0.46	0.97	0.79	0.80	0.14	0.49	0.89	0.40
	$\pi(\alpha/n)$	0.05	0.00	0.05	0.73	0.64	0.27	0.00	0.07	0.82	0.49	0.48	0.01	0.11	0.60	0.19
Late−Early	$\hat{\delta}_{4,s} - \hat{\delta}_{1,s}$	6.8	−22.6	−41.0	−156.5		21.7	−9.3	−12.4	−52.8		54.9	−2.4	−1.6	−18.8	
	$\pi(\alpha)$	0.04	0.39	0.69	0.96		0.01	0.41	0.46	0.85		0.01	0.18	0.20	0.52	
	$\pi(\alpha/n)$	0.01	0.10	0.25	0.87		0.00	0.07	0.10	0.68		0.00	0.02	0.03	0.31	

image of larger price impacts. We then show that this underlies the different relationships between trading activity and price impacts found using trade- vs. calendar-time aggregation.

To control for over-aggregation, we *fix* the number of transactions to be the same in all sequences, using only the *first 60* consecutive transactions in a sequence, dropping intervals with fewer than 60 transactions. After dropping these observations, we only include stocks with at least 100 observations at each time-of-day and activity combination.[31] We calculate signed trade imbalances for this fixed number of transactions at each time interval, and then match time intervals with trading activity (trading volume) deciles that would obtain in the original sample, prior to dropping any intervals.[32] As such, we condition on trading activity environments used to obtain the original calendar- and trade-time patterns. Fixing the number of transactions fixes the "extent" of aggregation to establish comparable metrics in calendar- and trade-time. Most importantly, it controls for variation due to variations in the number of trades across trading activity levels, and hence the extent of over-aggregation.

Figure 6 shows the dramatic consequences. Once we control for over-aggregation, calendar-time patterns *reverse*. Now, rather than fall with trading activity, signed trade imbalances using calendar-time measures rise sharply with activity. Indeed, the rise is roughly parallel to that for trade-time measures.[33] Importantly, these very different patterns link variations in signed trade imbalances to the levels of calendar- or trade-time trading activity that obtain based on the full sample. Thus, this analysis reveals the primitive positive relationship between signed trade imbalances and trading activity that is concealed by the over-aggregation in calendar-time.[34] Trade-time aggregation gets the signed-trade imbalance–activity rela-

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[31]1082 stocks in our trade-time analysis and 857 stocks in our calendar-time analysis survive this filter. For these firms, 90-92% of the time intervals contain at least 60 transactions. Findings are qualitatively unchanged if we consider the first 40 or 80 transactions in a sequence.

[32]Defining trading activity deciles based on the modified sample leads to qualitatively identical findings.

[33]Fixing the number of transactions results in only a moderately less steep relationship for trade-time measures indicating that relative over-aggregation by trade-time measures in less active markets is modest.

[34] In an unreported analysis, we find additional evidence of the impact of over-aggregation using calendar-time measures in active markets by dividing each 15-minute interval into three 5-minute intervals (or five 3-minute intervals). Doing so shows that signed trade imbalances calculated over shorter calendar-time intervals at a given level of activity are much higher, but they display negative associations with calendar-time dollar volumes. In contrast, using trade-time measures, when one cuts the target dollar trading volume in half, it leads to a marginally *steeper* signed trade imbalance-activity relationship,

Figure 6: **Over-aggregation by calendar-time measures in high-volume times.** The figure shows the relationships between mean signed trade imbalances and trading activity in calendar-time and trade-time based on the **first 60 transactions** in each time interval. **Calendar-time:** Each month, calendar-time intervals of each stock are sorted into deciles of 15-minute dollar volume, with decile 1 reflecting the least active markets. Each month, trade imbalances of each stock are calculated by trading activity decile. Medians of averages are calculated across stocks and months by trading activity decile. **Trade-time:** Each month, trade-time intervals of each stock are sorted into deciles of *inverted trade times*, with decile 1 reflecting the least active markets. Each month, trade-time trade imbalances of each stock are calculated by trading activity decile. Medians of averages are calculated across stocks and months by trading activity decile.

All transactions | First 60 transactions
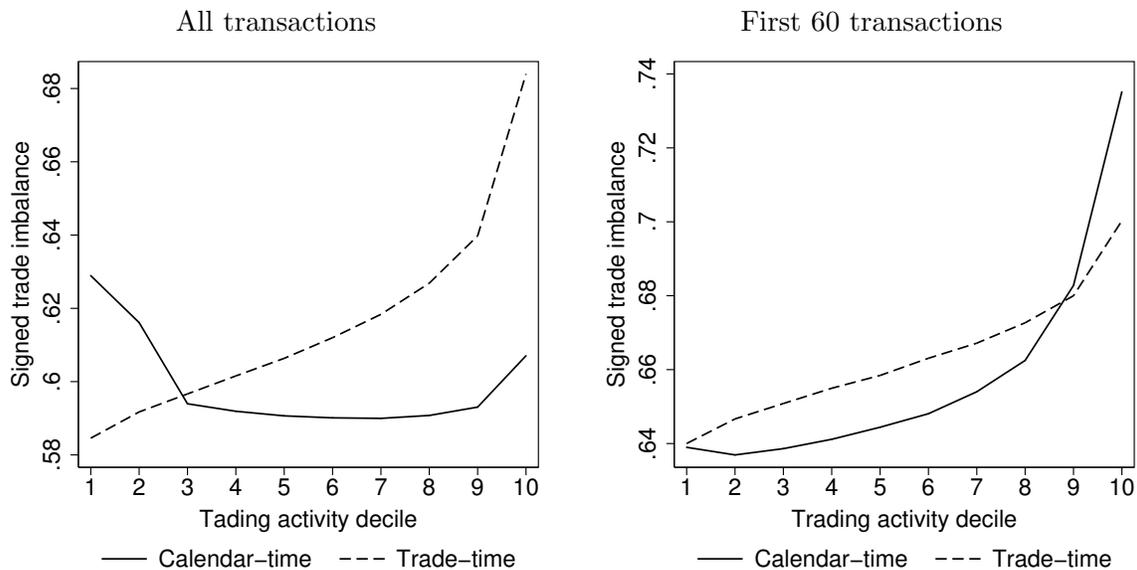
tionship 'right' because it adjusts by varying the time-aggregation horizon to the activity level—horizons shrink when activity rises.

Table 6 shows that the over-aggregation of net signed trades by calendar-time measures drives the larger estimates of Kyle's λ when trading volumes are higher: the mis-aggregation means that large price movements are associated with more balanced net signed trades, mechanically inflating estimates, and driving the positive association between calendar-time estimates of Kyle's λ and trading volume. Once we correct for this over-aggregation by fixing the number of observations in a sequence, the pattern reverses.

In sum, trade-time aggregation better measures signed-trade imbalances by adjusting

qualitatively confirming the patterns that we document.

Table 6: **Over-aggregation by calendar-time estimates of Kyle's λ.** *Panel A*: Kyle's λ is estimated for each stock j using the **calendar-time** version of equation (12). Estimates are carried out using the data from the first 60 transactions of each time intervals, and are compared to those obtained using all transactions in each calendar-time interval. Mean $\hat{\lambda}^j_{i,s}$ and the proportion of significantly **positive** estimates $\pi(\cdot)$, according to traditional type I error α and that after Bonferroni correction, α/n, are reported by time-of-day and trading activity quartile. "High−Low" reports mean $\hat{\lambda}^j_{i,4} - \hat{\lambda}^j_{i,1}$ and the proportion of significantly **positive** (**negative**) differences by time-of-day when estimates **rise** (**fall**) with trading activity. "Late−Early" reports mean $\hat{\lambda}^j_{i,4} - \hat{\lambda}^j_{i,1}$ and proportions of significantly **negative** differences by activity level and stock size tercile. n is the number of stocks with 100+ time intervals in each time-of-day/activity level combination that contain at least 60 transactions. Stock-specific hypothesis tests use $\alpha = 5\%$. *Panel B*: is the analogue for *Panel A* based on trade-time aggregation.

		Panel A: Calendar-time analysis									
		First 60 transactions					All transactions				
		Trading activity level					Trading activity level				
Time-of-day		1	2	3	4	High−Low	1	2	3	4	High−Low
	Mean $\hat{\lambda}$	31.8	29.8	28.3	25.7	**−6.1**	46.4	59.8	68.8	79.3	**32.9**
9:30–10:45	$\pi(\alpha)$	0.99	0.99	0.99	0.98	0.58	1.00	1.00	1.00	0.99	0.67
	$\pi(\alpha/n)$	0.96	0.97	0.97	0.96	0.25	0.96	0.98	0.99	0.98	0.49
	Mean $\hat{\lambda}$	23.5	21.8	20.0	17.1	**−6.4**	27.9	37.5	43.8	50.9	**23.0**
10:45–12:45	$\pi(\alpha)$	0.99	0.99	0.99	0.99	0.66	0.99	1.00	1.00	1.00	0.74
	$\pi(\alpha/n)$	0.93	0.97	0.97	0.96	0.17	0.91	0.97	0.98	0.98	0.46
	Mean $\hat{\lambda}$	20.5	18.5	17.1	14.7	**−5.8**	23.8	30.9	36.1	43.2	**19.4**
12:45–14:45	$\pi(\alpha)$	0.99	0.99	0.99	0.99	0.61	0.99	1.00	1.00	1.00	0.72
	$\pi(\alpha/n)$	0.90	0.96	0.96	0.95	0.08	0.89	0.93	0.96	0.97	0.40
	Mean $\hat{\lambda}$	17.4	15.2	13.7	14.5	**−2.8**	13.1	35.1	42.9	58.2	**45.1**
14:45–16:00	$\pi(\alpha)$	0.98	0.98	0.98	0.98	0.27	0.76	0.98	0.99	1.00	0.97
	$\pi(\alpha/n)$	0.87	0.92	0.93	0.92	0.01	0.44	0.91	0.96	0.98	0.88
	$\hat{\lambda}_{4,s} - \hat{\lambda}_{1,s}$	**−14.4**	**−14.6**	**−14.5**	**−11.2**		**−33.3**	**−24.7**	**−25.9**	**−21.1**	
Late−Early	$\pi(\alpha)$	0.88	0.90	0.90	0.73		0.95	0.86	0.82	0.60	
	$\pi(\alpha/n)$	0.53	0.66	0.66	0.47		0.77	0.42	0.43	0.32	
		Panel B: Trade-time analysis									
		First 60 transactions					All transactions				
		Trading activity level					Trading activity level				
Time-of-day		1	2	3	4	High−Low	1	2	3	4	High−Low
	Mean $\hat{\lambda}$	41.0	36.6	34.0	34.4	**−6.6**	68.5	58.2	50.4	40.2	**−28.2**
9:30–10:45	$\pi(\alpha)$	1.00	0.99	0.99	0.99	0.64	1.00	1.00	0.99	0.99	0.90
	$\pi(\alpha/n)$	0.98	0.97	0.97	0.96	0.38	0.98	0.98	0.97	0.96	0.69
	Mean $\hat{\lambda}$	31.6	27.7	25.1	22.9	**−8.7**	57.1	48.9	41.7	30.1	**−27.1**
10:45–12:45	$\pi(\alpha)$	1.00	0.99	0.99	0.99	0.74	1.00	1.00	0.99	0.99	0.93
	$\pi(\alpha/n)$	0.98	0.97	0.97	0.94	0.35	0.98	0.98	0.97	0.95	0.74
	Mean $\hat{\lambda}$	26.4	23.2	21.6	20.1	**−6.3**	49.0	41.8	36.8	27.1	**−21.9**
12:45–14:45	$\pi(\alpha)$	0.99	0.99	0.99	0.99	0.64	1.00	0.99	0.99	0.99	0.90
	$\pi(\alpha/n)$	0.96	0.96	0.95	0.93	0.17	0.98	0.97	0.97	0.93	0.61
	Mean $\hat{\lambda}$	21.1	18.4	16.6	15.6	**−5.4**	40.0	33.6	29.0	21.8	**−18.2**
14:45–16:00	$\pi(\alpha)$	0.99	0.99	0.99	0.98	0.71	1.00	1.00	0.99	0.99	0.92
	$\pi(\alpha/n)$	0.95	0.94	0.93	0.91	0.30	0.98	0.97	0.96	0.94	0.58
	$\hat{\lambda}_{4,s} - \hat{\lambda}_{1,s}$	**−19.9**	**−18.2**	**−17.4**	**−18.8**		**−28.5**	**−24.5**	**−21.5**	**−18.5**	
Late−Early	$\pi(\alpha)$	0.93	0.93	0.92	0.92		0.88	0.87	0.86	0.84	
	$\pi(\alpha/n)$	0.76	0.74	0.73	0.76		0.69	0.66	0.63	0.61	

calendar-time horizons to the level of real-time trading activity. The trade-time analysis reveals that price impacts fall over the trading day, and as activity rises. The higher price

impacts in less active markets are indicative of lower liquidity. We next investigate trade-time dynamics, documenting compelling evidence of imperfectly-competitive liquidity provision.

7   Non-competitive Liquidity Provision

We now highlight predictable dynamics of pricing and net signed trade, analyzing the correlation structure of trade-time returns and net signed trades over successive trade sequences. Not only do prices not follow martingale processes, but the extent and nature of temporal correlation varies significantly with trading activity, in ways consistent with greater liquidity provision in active markets. We then show that net signed trades are highly autocorrelated, especially in active markets, and that expected net signed trade is strongly priced in active markets, contrary to the competitive foundations of classical models.

To analyze return dynamics over pairs of successive trade sequences that belong to the same trading day, we estimate[35]

$$
\begin{aligned}
Y_k^{jt} &= \beta_{1,1}^j + \sum_{s=2}^{4} \beta_{1,s}^j Z_{s,k}^{jt} + \sum_{i=2}^{4}\sum_{s=1}^{4} \beta_{i,s}^j D_{i,k}^j Z_{s,k}^{jt} + \phi_{1,1}^j Y_{k-1}^{jt} \\
&+ \sum_{s=2}^{4} \bar{\phi}_{1,s}^j Z_{s,k}^{jt} Y_{k-1}^{jt} + \sum_{i=2}^{4}\sum_{s=1}^{4} \bar{\phi}_{i,s}^j D_{i,k}^j Z_{s,k}^{jt} Y_{k-1}^{jt} + \sum_{t=1}^{48} \gamma^j M D_k^{jt} + u_k^{jt}
\end{aligned}
\tag{13}
$$

given $Y_k^{jt} = r_k^{jt}$ to analyze return dynamics by time-of-day and trading activity. Here, $\phi_{1,1}^{j*}$ denotes the auto-correlation near open at the lowest trading activity quartile, i.e., $i = 1$ and $s = 1$; and $\phi_{i,s}^j = \phi_{1,1}^j + \bar{\phi}_{i,s}^j$ for other time-of-day and activity quartile combinations.[36] At a trading activity level and time-of-day window, the proportion of significantly negative $\phi_{i,s}^j$ estimates measure the extent of short-term price reversals. The averages and proportions of significantly *positive* differences $\hat{\phi}_{i,4}^j - \hat{\phi}_{i,1}^j$ reveal how auto-correlations rise with trading activity. Panel A in Table 7 shows that returns revert when activity is low. The nature of

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[35]Qualitatively identical patterns obtain if we control for bid-ask bounce biases by using returns based on average best quoted prices at the beginning and end of trade sequences rather than actual transaction prices.

[36]Because average return is virtually zero, the intercept terms, i.e., the $\beta_{i,s}^j$s, are all essentially zero. Thus, return dynamics may be discerned from the $\phi_{i,s}^j$s.

return dynamics changes as trading activity rises: the first order serial correlation coefficient goes from distinctly negative in least active markets to marginally positive in most active markets, and roughly 2/3 of the changes are significantly positive. Moreover, these dynamics are not a small firm effect—magnitudes of autocorrelation coefficients vary little with stock size. The high price impacts and the reversion of trade-time returns in less active times are symptoms of limited liquidity: the high price impacts reveal that liquidity providers demand higher premia, and the reversion in returns suggests that liquidity providers can unwind positions just taken on over short horizons.

Most models of dynamic intraday trade featuring informed agents are closed by a zero expected profit condition for liquidity providers that delivers martingale pricing—contrary to the reversion found in less active markets. We next analyze temporal correlation in net signed trades, estimating equation (13) with $Y_k^{jt} = NF_k^{jt} \equiv BP_k^{jt} - 0.5$ measuring net percent signed trade. We then establish how the predictable and unpredictable components of net signed trade are priced. Panel B in Table 7 shows that higher trading activity is associated with far larger auto-correlations in net signed trade. Mean autocorrelations in net signed trade are only about 0.15 in less active markets, averaged across stock size and time of day, but they exceed 0.5 in active markets. Thus, net signed trade is highly predictable, and the extent of persistence in signed trade rises sharply with trading activity. In CLSa, the greater predictability in more active markets, especially near close, indicates *relatively less* informed trading, consistent with the smaller estimated price impacts in those markets.

To showcase the role of imperfectly-competitive liquidity provision, we document radically different pricing of expected versus unexpected signed trade, estimating Kyle's λ by time-of-day and trading activity using expected and unexpected signed trade. We use predicted and residual values from equation (13), with $Y = NF$, to construct NOF that enters equation (12). Thus, we separately estimate Kyle's λ using expected/predicted signed trade for a trade sequence, and then do so using unexpected/residual signed trade. Models like CLSa predict that only the unexpected (from the econometrician's perspective) component of order flow should be priced; and standard models assume that all order flow is unexpected.

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Table 7: **Trade-time return and net signed trade dynamics versus trading activity level by time-of-day and stock size.** First order auto-correlation in returns (Panel A) and signed percent signed trade (Panel B) are examined against trading activity levels via estimating equation (13) for each stock. Mean $\hat{\phi}_{i,s}$ and the proportion of significantly **negative** estimates $\pi(\cdot)$, according to both traditional type I error α and Type I error after Bonferroni correction α/n, are reported by time-of-day and activity quartile. "High−Low" reports mean $\hat{\phi}_{i,4}^{j} - \hat{\phi}_{i,1}^{j}$ and proportions of significantly **positive** differences by time-of-day and stock size tercile. Stocks are ranked into terciles of average market-capitalization over 2009–2012, and n is the number of stocks in each tercile. Stock specific hypothesis tests use $\alpha = 5\%$.

Panel A: Return auto-correlation

Time-of-day		Small stocks					Mid-sized stocks					Large stocks				
		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30–10:45	Mean $\hat{\phi}$	−0.10	−0.04	0.00	0.07	**0.16**	−0.10	−0.04	0.00	0.06	**0.15**	−0.12	−0.07	−0.03	0.03	**0.15**
	$\pi(\alpha)$	0.59	0.32	0.16	0.07	0.68	0.77	0.36	0.15	0.10	0.82	0.89	0.53	0.26	0.17	0.84
	$\pi(\alpha/n)$	0.29	0.09	0.03	0.04	0.42	0.48	0.14	0.06	0.07	0.63	0.67	0.26	0.17	0.15	0.70
10:45–12:45	Mean $\hat{\phi}$	−0.09	−0.03	0.01	0.05	**0.14**	−0.09	−0.03	0.01	0.06	**0.14**	−0.13	−0.08	−0.03	0.03	**0.17**
	$\pi(\alpha)$	0.69	0.27	0.13	0.07	0.71	0.78	0.30	0.10	0.07	0.84	0.90	0.57	0.20	0.17	0.84
	$\pi(\alpha/n)$	0.27	0.07	0.04	0.03	0.36	0.40	0.08	0.06	0.05	0.56	0.63	0.24	0.15	0.15	0.69
12:45–14:45	Mean $\hat{\phi}$	−0.09	−0.03	0.01	0.05	**0.14**	−0.09	−0.05	0.00	0.04	**0.13**	−0.14	−0.09	−0.05	0.02	**0.16**
	$\pi(\alpha)$	0.66	0.25	0.10	0.05	0.68	0.73	0.38	0.13	0.07	0.77	0.87	0.61	0.24	0.15	0.83
	$\pi(\alpha/n)$	0.23	0.05	0.03	0.03	0.28	0.32	0.09	0.06	0.05	0.39	0.61	0.22	0.15	0.15	0.60
14:45–16:00	Mean $\hat{\phi}$	−0.07	−0.03	0.01	0.07	**0.13**	−0.08	−0.05	−0.02	0.03	**0.11**	−0.12	−0.08	−0.06	−0.01	**0.11**
	$\pi(\alpha)$	0.53	0.27	0.10	0.04	0.64	0.69	0.42	0.17	0.10	0.65	0.82	0.51	0.30	0.21	0.69
	$\pi(\alpha/n)$	0.15	0.06	0.03	0.03	0.19	0.26	0.12	0.07	0.06	0.19	0.46	0.23	0.18	0.18	0.33

Panel B: Net signed trade auto-correlation

Time-of-day		Small stocks					Mid-sized stocks					Large stocks				
		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30–10:45	Mean $\hat{\phi}$	0.14	0.23	0.36	0.55	**0.41**	0.14	0.21	0.32	0.52	**0.38**	0.14	0.20	0.28	0.45	**0.31**
	$\pi(\alpha)$	0.64	0.90	0.97	1.00	0.95	0.88	0.94	0.99	1.00	0.98	0.95	0.98	1.00	1.00	0.96
	$\pi(\alpha/n)$	0.25	0.64	0.86	0.98	0.75	0.52	0.83	0.94	1.00	0.89	0.67	0.88	0.96	0.99	0.87
10:45–12:45	Mean $\hat{\phi}$	0.14	0.21	0.31	0.52	**0.38**	0.15	0.20	0.30	0.48	**0.33**	0.14	0.18	0.26	0.43	**0.29**
	$\pi(\alpha)$	0.82	0.92	0.99	1.00	0.98	0.94	0.97	0.99	1.00	0.98	0.98	0.99	0.99	1.00	0.97
	$\pi(\alpha/n)$	0.39	0.69	0.90	0.99	0.86	0.77	0.90	0.96	0.99	0.93	0.81	0.93	0.96	0.99	0.91
12:45–14:45	Mean $\hat{\phi}$	0.16	0.19	0.23	0.32	**0.17**	0.16	0.20	0.22	0.28	**0.12**	0.15	0.17	0.18	0.23	**0.08**
	$\pi(\alpha)$	0.87	0.93	0.97	0.99	0.75	0.96	0.97	0.99	0.99	0.77	0.97	0.99	0.98	0.99	0.63
	$\pi(\alpha/n)$	0.45	0.67	0.82	0.93	0.32	0.80	0.87	0.92	0.96	0.35	0.82	0.91	0.92	0.97	0.22
14:45–16:00	Mean $\hat{\phi}$	0.18	0.22	0.27	0.66	**0.48**	0.19	0.21	0.24	0.66	**0.48**	0.16	0.18	0.20	0.64	**0.48**
	$\pi(\alpha)$	0.92	0.96	0.98	1.00	0.99	0.98	0.99	0.99	1.00	1.00	0.98	0.99	0.99	1.00	1.00
	$\pi(\alpha/n)$	0.60	0.78	0.89	1.00	0.95	0.89	0.92	0.95	1.00	0.98	0.89	0.91	0.96	1.00	0.99

Table 8: **Price impacts of expected vs. unexpected signed trades by time-of-day and trading activity level.** Kyle's λ is estimated for each stock j using equation (12). In Panel A, predicted values from equation (13), given $Y = NF$, are used to construct NOF. In Panel B, residuals from equation (13), given $Y = NF$, are used to construct NOF. Mean $\hat{\lambda}^j_{i,s}$ and the proportion of significantly **positive** estimates $\pi(\cdot)$, according to traditional type I error α and that after Bonferroni correction, α/n, are reported by time-of-day, trading activity quartile, and stock size tercile. "High−Low" reports mean $\hat{\beta}^j_{i,4} - \hat{\beta}^j_{i,1}$ (or $\hat{\lambda}^j_{i,4} - \hat{\lambda}^j_{i,1}$) and proportions of significantly **positive** (Panel A) or significantly **negative** (Panel B) differences by time-of-day and stock size tercile. "Late−Early" reports mean $\hat{\beta}^j_{4,s} - \hat{\beta}^j_{1,s}$ (or $\hat{\lambda}^j_{i,4} - \hat{\lambda}^j_{i,1}$) and proportions of significantly **negative** differences by activity level and stock size tercile. Stocks are ranked into terciles of average market-capitalization over 2009−2012, and n is the number of stocks in each tercile. Stock specific hypothesis tests use $\alpha = 5\%$.

*Panel A: Kyle's λ estimates using **expected** net signed trade*

Small stocks

Time-of-day		1	2	3	4	High−Low
9:30−10:45	Mean $\hat{\lambda}$	−7.5	7.2	15.4	28.9	**36.4**
	$\pi(\alpha)$	0.07	0.28	0.50	0.71	0.61
	$\pi(\alpha/n)$	0.00	0.09	0.27	0.53	0.35
10:45−12:45	Mean $\hat{\lambda}$	−7.5	2.3	10.5	21.0	**28.5**
	$\pi(\alpha)$	0.04	0.16	0.52	0.74	0.65
	$\pi(\alpha/n)$	0.00	0.02	0.23	0.49	0.35
12:45−14:45	Mean $\hat{\lambda}$	−6.1	1.9	6.5	14.1	**20.2**
	$\pi(\alpha)$	0.02	0.07	0.23	0.52	0.54
	$\pi(\alpha/n)$	0.00	0.00	0.00	0.10	0.11
14:45−16:00	Mean $\hat{\lambda}$	−3.3	1.1	6.2	19.9	**23.1**
	$\pi(\alpha)$	0.02	0.07	0.18	0.86	0.71
	$\pi(\alpha/n)$	0.00	0.00	0.01	0.57	0.22
Late−Early	$\hat{\lambda}_{4,s}-\hat{\lambda}_{1,s}$	**4.3**	**−6.1**	**−9.2**	**−9.0**	
	$\pi(\alpha)$	0.08	0.23	0.33	0.43	
	$\pi(\alpha/n)$	0.00	0.06	0.15	0.23	

Mid-sized stocks

Time-of-day		1	2	3	4	High−Low
9:30−10:45	Mean $\hat{\lambda}$	−5.9	0.3	8.6	11.2	**17.1**
	$\pi(\alpha)$	0.03	0.24	0.55	0.77	0.75
	$\pi(\alpha/n)$	0.00	0.07	0.33	0.56	0.43
10:45−12:45	Mean $\hat{\lambda}$	−4.9	1.4	6.3	11.1	**16.0**
	$\pi(\alpha)$	0.02	0.17	0.50	0.79	0.78
	$\pi(\alpha/n)$	0.00	0.04	0.23	0.56	0.49
12:45−14:45	Mean $\hat{\lambda}$	−4.6	−0.7	2.0	5.9	**10.5**
	$\pi(\alpha)$	0.02	0.04	0.15	0.54	0.63
	$\pi(\alpha/n)$	0.00	0.00	0.02	0.09	0.17
14:45−16:00	Mean $\hat{\lambda}$	−3.3	−0.9	1.2	6.5	**9.8**
	$\pi(\alpha)$	0.02	0.04	0.11	0.85	0.80
	$\pi(\alpha/n)$	0.00	0.01	0.01	0.61	0.32
Late−Early	$\hat{\lambda}_{4,s}-\hat{\lambda}_{1,s}$	**2.6**	**−1.2**	**−7.4**	**−4.7**	
	$\pi(\alpha)$	0.05	0.20	0.45	0.49	
	$\pi(\alpha/n)$	0.01	0.05	0.20	0.28	

Large stocks

Time-of-day		1	2	3	4	High−Low
9:30−10:45	Mean $\hat{\lambda}$	−7.1	−1.2	5.3	12.6	**19.8**
	$\pi(\alpha)$	0.03	0.16	0.50	0.80	0.83
	$\pi(\alpha/n)$	0.00	0.06	0.28	0.56	0.55
10:45−12:45	Mean $\hat{\lambda}$	−5.9	−1.8	5.0	11.1	**17.0**
	$\pi(\alpha)$	0.03	0.10	0.41	0.78	0.83
	$\pi(\alpha/n)$	0.01	0.04	0.20	0.52	0.58
12:45−14:45	Mean $\hat{\lambda}$	−6.5	−1.7	0.8	6.2	**12.7**
	$\pi(\alpha)$	0.02	0.04	0.07	0.49	0.73
	$\pi(\alpha/n)$	0.01	0.00	0.00	0.08	0.21
14:45−16:00	Mean $\hat{\lambda}$	−4.2	−1.1	0.5	8.8	**13.0**
	$\pi(\alpha)$	0.02	0.03	0.06	0.83	0.82
	$\pi(\alpha/n)$	0.00	0.00	0.01	0.58	0.41
Late−Early	$\hat{\lambda}_{4,s}-\hat{\lambda}_{1,s}$	**3.0**	**0.1**	**−4.8**	**−3.8**	
	$\pi(\alpha)$	0.04	0.17	0.43	0.50	
	$\pi(\alpha/n)$	0.00	0.05	0.19	0.30	

*Panel B: Kyle's λ estimates using **unexpected** signed trade*

Small stocks

Time-of-day		1	2	3	4	High−Low
9:30−10:45	Mean $\hat{\lambda}$	45.6	41.7	37.6	33.6	**−12.0**
	$\pi(\alpha)$	0.90	0.91	0.90	0.91	0.49
	$\pi(\alpha/n)$	0.69	0.76	0.74	0.78	0.27
10:45−12:45	Mean $\hat{\lambda}$	45.9	40.0	34.2	28.5	**−17.4**
	$\pi(\alpha)$	0.96	0.98	0.96	0.95	0.70
	$\pi(\alpha/n)$	0.86	0.88	0.86	0.84	0.41
12:45−14:45	Mean $\hat{\lambda}$	43.2	35.8	31.5	25.7	**−17.6**
	$\pi(\alpha)$	0.98	0.98	0.98	0.96	0.73
	$\pi(\alpha/n)$	0.90	0.88	0.86	0.80	0.36
14:45−16:00	Mean $\hat{\lambda}$	35.4	29.7	25.0	24.2	**−11.3**
	$\pi(\alpha)$	0.98	0.97	0.98	0.98	0.64
	$\pi(\alpha/n)$	0.88	0.86	0.80	0.87	0.28
Late−Early	$\hat{\lambda}_{4,s}-\hat{\lambda}_{1,s}$	**−10.2**	**−12.0**	**−12.6**	**−9.4**	
	$\pi(\alpha)$	0.45	0.54	0.58	0.53	
	$\pi(\alpha/n)$	0.25	0.26	0.30	0.31	

Mid-sized stocks

Time-of-day		1	2	3	4	High−Low
9:30−10:45	Mean $\hat{\lambda}$	31.0	26.7	23.9	19.2	**−11.9**
	$\pi(\alpha)$	0.97	0.98	0.97	0.96	0.77
	$\pi(\alpha/n)$	0.90	0.90	0.90	0.92	0.55
10:45−12:45	Mean $\hat{\lambda}$	28.3	24.6	20.8	16.3	**−12.0**
	$\pi(\alpha)$	0.99	1.00	0.99	0.97	0.86
	$\pi(\alpha/n)$	0.95	0.95	0.95	0.92	0.68
12:45−14:45	Mean $\hat{\lambda}$	24.8	20.9	18.1	14.5	**−10.3**
	$\pi(\alpha)$	0.99	0.98	0.98	0.98	0.88
	$\pi(\alpha/n)$	0.95	0.94	0.93	0.91	0.55
14:45−16:00	Mean $\hat{\lambda}$	19.1	15.7	13.3	11.3	**−7.9**
	$\pi(\alpha)$	0.99	0.99	0.98	0.98	0.83
	$\pi(\alpha/n)$	0.94	0.93	0.92	0.92	0.50
Late−Early	$\hat{\lambda}_{4,s}-\hat{\lambda}_{1,s}$	**−11.9**	**−11.0**	**−10.6**	**−7.9**	
	$\pi(\alpha)$	0.76	0.76	0.76	0.74	
	$\pi(\alpha/n)$	0.54	0.53	0.54	0.52	

Large stocks

Time-of-day		1	2	3	4	High−Low
9:30−10:45	Mean $\hat{\lambda}$	28.3	25.2	23.3	21.1	**−7.3**
	$\pi(\alpha)$	0.98	0.98	0.98	0.97	0.71
	$\pi(\alpha/n)$	0.93	0.93	0.94	0.94	0.46
10:45−12:45	Mean $\hat{\lambda}$	25.1	23.2	21.0	17.4	**−7.7**
	$\pi(\alpha)$	0.99	0.99	0.98	0.98	0.74
	$\pi(\alpha/n)$	0.95	0.95	0.93	0.92	0.51
12:45−14:45	Mean $\hat{\lambda}$	22.6	20.4	18.5	16.0	**−6.5**
	$\pi(\alpha)$	0.99	0.98	0.99	0.98	0.72
	$\pi(\alpha/n)$	0.94	0.94	0.93	0.93	0.31
14:45−16:00	Mean $\hat{\lambda}$	18.5	16.4	14.7	12.1	**−6.4**
	$\pi(\alpha)$	0.99	0.99	0.99	0.97	0.76
	$\pi(\alpha/n)$	0.93	0.93	0.92	0.92	0.44
Late−Early	$\hat{\lambda}_{4,s}-\hat{\lambda}_{1,s}$	**−9.9**	**−8.9**	**−8.5**	**−9.0**	
	$\pi(\alpha)$	0.78	0.76	0.75	0.79	
	$\pi(\alpha/n)$	0.51	0.51	0.56	0.59	

Table 8 shows that the pricing of unexpected signed trade falls as activity rises, but the *opposite* happens for expected signed trade. Panel A shows that while expected signed trade is not priced when trading activity is low, it is significantly priced when trading activity is high, contrary to CLSa and classical models. The combined facts that in active markets (1) signed trade is quite predictable, and (2) this expected signed trade is priced is further indicative of imperfect liquidity provision. That is, they suggest that when trading activity is high, the market is not sufficiently liquid to absorb the high expected signed trade. Panel B shows that estimates of Kyle's λ based on unexpected signed trade fall with trading activity, and are significantly priced at all activity levels. These pricing patterns are so strong that they more than offset the opposing patterns in expected signed trade, driving the negative relationship between price impacts of *total* signed trade and trading activity that we first documented.

8 Conclusion

Researchers have long known key calendar-time properties of intraday trading outcomes, in particular that trading volume, price impacts and return volatility are positively related and evolve according to a $\cup$-shaped pattern over the trading day. Our paper first shows that very different patterns arise when one instead aggregates trading outcomes over trade-time intervals that group transactions into successive non-overlapping trade sequences of a *fixed* dollar-value. In trade time, return volatility and price impact measures such as Kyle's λ all *fall* sharply by 40-60% over the trading day. Controlling for time-of-day reveals that shorter trade times that correspond to episodes of higher calendar-time trading volume are associated with *far smaller* trade-time volatilities and price impacts.

The smaller trade-time volatility/price impacts found after controlling for time of day suggest the endogenous responses of traders to extant liquidity. Shorter trade times are associated with larger marketable orders and greater signed trade imbalances that enjoy smaller price impacts. As such, we relate trade-time price impacts to trading costs. Our dynamic analysis provides more evidence of imperfect liquidity provision: in less active times, trade-

time returns over successive trade sequences revert; while in more active times, expected signed net order flow is priced, indicating price pressure from forecastable components.

The root cause of the contrasting intraday patterns found using calendar- and trade-time approaches lies in the over-aggregation by calendar-time measures in active markets. Once we control for over-aggregation of off-setting signed trades in active markets, calendar-time patterns mirror their trade-time counterparts, confirming the relationships uncovered by trade-time aggregation. The striking time-of-day and activity patterns highlight a key role for imperfectly-competitive and time-varying liquidity provision. Collectively, our findings suggest the merits of melding stochastically-varying liquidity provision into theoretical models with strategic inventory-rebalancing traders.

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Appendix

A Return auto-correlations and time-of-day patterns

We verify that the differences in time-of-day patterns in calendar time and trade time are not driven by the correlation structure of returns. One may posit that the correlation structure of returns varies by time-of-day, and that this may mechanically generate the higher calendar-time volatility near close despite the reduced trade-time volatility. To see how this could happen, suppose that return auto-correlations tend to rise over the day, becoming *positive* near close. Then, near close, small incremental price movements over successive *short* trade-time intervals aggregate the momentum to large price movements over fixed *long* (15-minute) calendar-time intervals. This leads us to first estimate calendar-time return auto-correlations on a stock-by-stock basis, controlling for time-of-day. For each stock j, we estimate

$$r_k^j = c_0^{1j} + \sum_{i=2}^{13} c_0^{ij} D_i + \rho_1^j r_{k-1}^j + \sum_{i=2}^{13} \bar{\rho}_{ij} D_i \times r_{k-1}^j + e_k^j \tag{A.1}$$

using all observations save those in three-day windows around earnings announcements or analyst recommendation dates.[37] Here, r_k^j and r_{k-1}^j are returns over two successive trade sequences that belong to the same trading day and D_i's are dummy variables that identify 15-minute time-of-day windows. The AR(1) coefficient for the first time-of-date window is ρ_1^j; AR(1) parameters for the other windows are given by $\rho_i^j = \rho_1^j + \bar{\rho}_i^j$, with $i \in \{2, \ldots, 13\}$.

Figure A.1 shows the inter-tercile ranges of stock-specific estimated ρ_i's for different time-of-day windows. Medians of ρ_i estimates are close to zero at all time-of-day windows, indicating that our approach of aggregating transactions into trade-time intervals successfully controls for temporally-dependent order flow at the transaction-by-transaction frequency.

About 30% of these individual stock return auto-correlation estimates take on positive values. To reinforce that return auto-correlations do not drive our results, we decompose each time-of-day sample into stocks whose AR(1) parameter estimates falls in the first tercile and

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[37]Qualitatively identical results obtain when we include all observations and control for earnings announcements or analyst recommendation dates.

Figure A.1: **Inter-tercile ranges of AR(1) estimates for trade-time returns by time-of-day.** Equation (A.1) is fit stock-by-stock to obtain 13 trade-time return AR(1) coefficient estimates, $\hat{\rho}_i$, per stock. $\hat{\rho}_i$'s are sorted into three equally-sized groups by time-of-day window. For each time-of-day window i, a bar reflects the difference between the 66th and the 33rd percentile of ρ estimates.
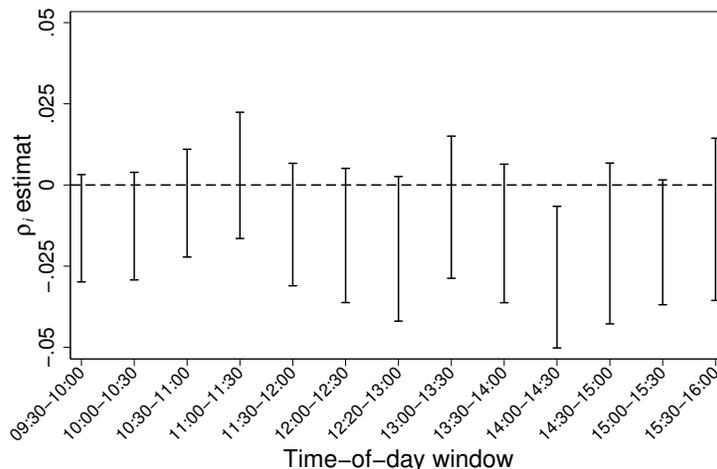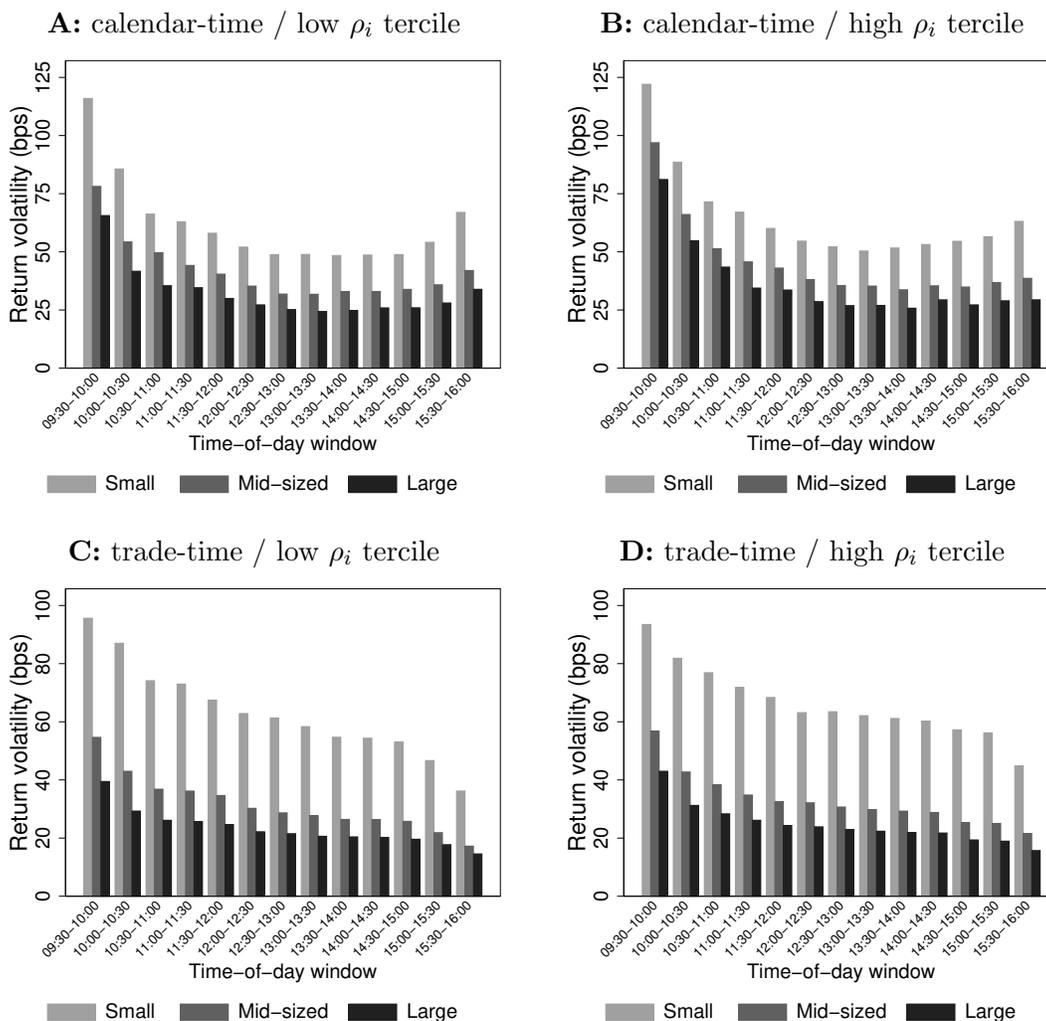

those whose AR(1) parameters fall in the third tercile. We then compare the calendar-time and trade-time time-of-day volatility patterns across the two sub-samples.

Figure A.2 shows that the return auto-correlation does not affect the calendar-time and trade-time patterns of volatility over the trading day (similar patterns obtain for VWAP-based volatility and Kyle's λ). As one would expect, the positive auto-correlation sample features slightly higher return volatility. However, regardless of the return auto-correlation, volatility measured in calendar-time evolves according to a ∪-shape over the trading day, while trade-time return volatility falls over the trading day, especially near open and close.

B   Robustness to employing volume-based trade sequence targets

We show robustness of our findings to an alternative choice of V_{jt}. With our base specification of V_{jt}, a typical stock has about 26 trade sequences per trading day. To mimic this, on average, with a volume-based V_{jt} target, we use 4% of the previous month's average daily dollar volume of a stock as an alternative cutoff. After constructing trade sequences based on this target value, the analysis proceeds exactly as in our main analysis. Table B.1 shows

Figure A.2: **Time-of-day patterns calendar-time versus trade-time return volatility *by tercile of trade-time return auto-correlation*** .Calendar-time and trade-time return standard deviations are calculated by time-of-day, month, *and ρ_i estimates tercile*, stock-by-stock. Cross-stock-month medians of return volatility are calculated by time-of-day and stock size tercile for the two samples of high and low trade-time return auto-correlation.

A: calendar-time / low ρ_i tercile

B: calendar-time / high ρ_i tercile

C: trade-time / low ρ_i tercile

D: trade-time / high ρ_i tercile

that the qualitative association between trade-time return volatility and trading activity iss unchanged by this alternative V_{jt} specification. Similar results obtain for all other outcomes.

C Robustness to inclusion of open and close transactions

To ensure that excluding transactions at open and close does not materially affect inferences, we consider an alternative trade sequence construction. This approach to avoiding trade se-

Table B.1: **Return volatility of fixed-dollar positions by time-of-day and trading activity level.** Trade sequences are constructed based on V_{jt} being equal to 4% of stock j's average daily dollar volume in month $t-1$. Absolute contemporaneous returns ($|r_k^j|$) are modeled for each stock j using equation (10). Mean $\hat{\beta}_{i,s}^j$ and proportion of significantly **negative** estimates $\pi(\cdot)$, according to both traditional type I error α and that after Bonferroni correction, α/n, are reported by time-of-day, trading activity quartile, and stock size tercile. "High−Low" reports mean $\hat{\beta}_{i,4}^j - \hat{\beta}_{i,1}^j$ and proportion of significant **negative** differences by time-of-day and stock size tercile. "Late−Early" reports mean $\hat{\beta}_{4,s}^j - \hat{\beta}_{1,s}^j$ and proportion of significantly **negative** differences by activity level and stock size tercile. Stocks are ranked into terciles of average market-capitalization over 2009−2012, and n is the number of stocks in each tercile. Stock specific hypothesis tests use $\alpha = 5\%$.

Time-of-day		Small stocks					Mid-sized stocks					Large stocks				
		Trading activity level					Trading activity level					Trading activity level				
		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30–10:45	Mean $\hat{\beta}$		−8.6	−14.9	−22.0	**−22.0**		−5.6	−9.4	−12.1	**−12.1**	0.00	−1.4	−2.8	−2.9	**−2.9**
	$\pi(\alpha)$		0.78	0.92	0.96	0.96		0.81	0.91	0.91	0.91	0.00	0.41	0.58	0.56	0.56
	$\pi(\alpha/n)$		0.46	0.74	0.89	0.89		0.48	0.76	0.83	0.83	0.00	0.16	0.36	0.41	0.41
10:45–12:45	Mean $\hat{\beta}$	−17.5	−25.8	−30.3	−31.7	**−14.2**	−13.6	−18.4	−21.4	−23.2	**−9.6**	−9.4	−11.2	−12.2	−13.5	**−4.1**
	$\pi(\alpha)$	0.97	1.00	1.00	1.00	0.95	0.99	1.00	1.00	1.00	0.96	0.98	0.99	0.98	0.98	0.75
	$\pi(\alpha/n)$	0.87	0.96	0.98	0.99	0.82	0.95	0.99	0.99	0.99	0.85	0.94	0.94	0.92	0.95	0.52
12:45–14:45	Mean $\hat{\beta}$	−26.1	−31.9	−35.1	−37.9	**−11.8**	−20.7	−23.9	−25.6	−27.5	**−6.8**	−14.0	−15.1	−15.6	−15.8	**−1.8**
	$\pi(\alpha)$	0.99	1.00	1.00	1.00	0.88	1.00	1.00	1.00	1.00	0.88	1.00	1.00	0.99	0.98	0.51
	$\pi(\alpha)$	0.95	0.99	0.99	0.99	0.71	0.98	1.00	1.00	1.00	0.68	0.98	0.98	0.97	0.94	0.27
14:45–16:00	Mean $\hat{\beta}$	−31.6	−37.9	−41.8	−45.8	**−14.2**	−26.3	−29.6	−32.1	−35.2	**−8.9**	−17.5	−19.1	−20.8	−23.3	**−5.9**
	$\pi(\alpha)$	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	0.94
	$\pi(\alpha/n)$	0.97	0.99	1.00	1.00	0.84	1.00	1.00	1.00	1.00	0.89	0.99	0.99	0.99	0.99	0.82
Late−Early	$\hat{\beta}_{4,s} - \hat{\beta}_{1,s}$	−31.6	−29.2	−26.9	−23.8		−26.3	−24.1	−22.8	−23.1		−17.5	−17.7	−17.9	−20.4	
	$\pi(\alpha)$	1.00	0.99	0.99	0.98		1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	
	$\pi(\alpha/n)$	0.97	0.97	0.95	0.93		1.00	1.00	0.99	0.98		0.99	0.99	0.99	0.99	

quences that span overnight constructs trade sequences as follows: each trading day, (1) it starts at open and stops with the trade sequence that crosses 12:30pm; (2) it starts at close, constructing trade sequences in reverse until they cross 12:30pm; and (3) it discards trade sequences that contain transactions that overlap 12:30pm. The resulting trading sequences include all trades near open and close, but not those that span overnight. We then use these trade sequences to estimate equation (10). Table C.1 shows that including transactions at the very beginning and end of each day does not qualitatively alter the relationship between trading activity and trade-time volatility. Similar results obtain for all other outcomes.

D   Robustness to controlling for look-ahead biases

We show that the associations between trading activity and trading outcomes, measured in trade time, are robust to assigning current month's trade sequences to trading activity levels based on the empirical distribution of trade times of $V_{j,t-1}$ from the previous month.

To control for trading activity, we first sort trade sequences **each month** into quartiles of trading activity (the inverse of a trade time, dur_k^j) by time-of-day window. We then use quartile statistics of trade times from the previous month to construct indicator functions $\tilde{Z}_{\tilde{s},k}^{jt}$, with $\tilde{s} \in \{1, 2, 3, 4\}$ indexing quartiles of trading activity in month $t-1$, where a higher s indexes a greater trading activity level. Thus, $\tilde{Z}_{\tilde{s},k}^{jt} = 1$ if trade sequence k has a time duration that corresponds to $\tilde{s}$, and $\tilde{Z}_{\tilde{s},k}^{jt} = 0$ otherwise.

For a given trading outcome Y_k^{jt}, we estimate

$$Y_k^{jt} = \beta_{1,1}^j + \sum_{\tilde{s}=2}^{4} \beta_{1,\tilde{s}}^j \tilde{Z}_{\tilde{s},k}^{jt} + \sum_{i=2}^{4} \sum_{\tilde{s}=1}^{4} \beta_{i,\tilde{s}}^j D_{i,k}^j \tilde{Z}_{\tilde{s},k}^{jt} + \sum_{t=1}^{48} \gamma_3^j MD_k^{jt} + u_k^{jt}. \tag{D.1}$$

Table D.1 shows that the association between trading activity and trade-time volatility is robust to controlling for look-ahead biases. Similar results obtain for other trading outcomes.

Table C.1: **Return volatility of fixed-dollar positions by time-of-day and trading activity level.** The table reports analogues of the results presented in Table 1 when trade sequence are constructed so that all transactions near open and close of each trading day are included. Absolute contemporaneous returns ($|r_k^j|$) are modeled for each stock j using equation (10). Mean $\hat{\beta}_{i,s}^j$ and proportion of significantly **negative** estimates $\pi(\cdot)$, according to both traditional type I error α and that after Bonferroni correction, α/n, are reported by time-of-day, trading activity quartile, and stock size tercile. "High−Low" reports mean $\hat{\beta}_{i,4}^j - \hat{\beta}_{i,1}^j$ and proportion of significantly **negative** differences by time-of-day and stock size tercile. "Late−Early" reports mean $\hat{\beta}_{4,s}^j - \hat{\beta}_{1,s}^j$ and proportion of significantly **negative** differences by activity level and stock size tercile. Stocks are ranked into terciles of average market-capitalization over 2009–2012, and n is the number of stocks in each tercile. Stock specific hypothesis tests use $\alpha = 5\%$.

		Small stocks					Mid-sized stocks					Large stocks				
		Trading activity level					Trading activity level					Trading activity level				
Time-of-day		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30–10:45	Mean $\hat\beta$		−6.8	−12.2	−19.0	**−19.0**		−4.9	−8.5	−11.7	**−11.7**		−1.3	−2.6	−3.2	**−3.2**
	$\pi(\alpha)$		0.61	0.79	0.88	0.88		0.79	0.88	0.94	0.94		0.47	0.59	0.62	0.62
	$\pi(\alpha/n)$		0.27	0.51	0.72	0.72		0.57	0.76	0.85	0.85		0.27	0.43	0.50	0.50
10:45–12:45	Mean $\hat\beta$	−20.0	−29.1	−34.7	−34.2	**−14.2**	−14.6	−19.1	−22.0	−23.9	**−9.4**	−9.6	−11.2	−12.3	−13.8	**−4.2**
	$\pi(\alpha)$	0.92	0.98	0.99	1.00	0.86	1.00	1.00	1.00	1.00	0.97	1.00	0.99	0.98	0.98	0.86
	$\pi(\alpha/n)$	0.72	0.88	0.92	0.94	0.65	0.94	0.98	1.00	1.00	0.89	0.98	0.97	0.97	0.97	0.69
12:45–14:45	Mean $\hat\beta$	−33.4	−39.8	−45.4	−48.2	**−14.9**	−21.2	−24.3	−26.9	−30.9	**−9.8**	−13.2	−14.3	−15.1	−17.2	**−4.0**
	$\pi(\alpha)$	0.99	1.00	1.00	0.99	0.90	1.00	1.00	1.00	0.99	0.96	1.00	1.00	0.98	0.96	0.90
	$\pi(\alpha)$	0.95	0.97	0.99	0.99	0.71	1.00	1.00	1.00	0.99	0.91	0.99	0.99	0.97	0.95	0.84
14:45–16:00	Mean $\hat\beta$	−37.9	−43.5	−46.7	−51.4	**−13.4**	−25.5	−28.4	−30.6	−33.3	**−7.8**	−15.9	−17.4	−18.7	−20.8	**−4.9**
	$\pi(\alpha)$	1.00	1.00	1.00	1.00	0.85	1.00	1.00	1.00	1.00	0.96	1.00	1.00	1.00	1.00	0.92
	$\pi(\alpha/n)$	0.92	0.96	0.96	0.98	0.61	1.00	1.00	1.00	1.00	0.86	0.99	0.99	0.99	0.99	0.82
Late−Early	$\hat\beta_{4,s} - \hat\beta_{1,s}$	**−37.9**	**−6.7**	**−34.6**	**−32.4**		**−25.5**	**−23.5**	**−22.1**	**−21.6**		**−15.9**	**−16.1**	**−16.1**	**−17.7**	
	$\pi(\alpha)$	1.00	0.99	0.98	0.99		1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00	
	$\pi(\alpha/n)$	0.92	0.95	0.91	0.90		1.00	1.00	0.99	0.99		0.99	1.00	0.99	0.99	

Table D.1: **Return volatility of fixed-dollar positions by time-of-day and trading activity level.** Absolute contemporaneous returns ($|r_k^j|$) are modeled for each stock j using equation (D.1). Mean $\hat{\beta}_{i,\tilde{s}}^j$ and proportion of significantly **negative** estimates $\pi(\cdot)$, according to both traditional type I error α and that after Bonferroni correction, α/n, are reported by time-of-day, trading activity quartile, and stock size tercile. "High−Low" reports mean $\hat{\beta}_{i,4}^j - \hat{\beta}_{i,1}^j$ and proportion of significantly **negative** differences by time-of-day and stock size tercile. "Late−Early" reports mean $\hat{\beta}_{4,\tilde{s}}^j - \hat{\beta}_{1,\tilde{s}}^j$ and proportion of significantly **negative** differences by activity level and stock size tercile. Stocks are ranked into terciles of average market-capitalization over 2009–2012, and n is the number of stocks in each tercile. Stock specific hypothesis tests use $\alpha = 5\%$.

| | | Small stocks | | | | | Mid-sized stocks | | | | | Large stocks | | | | |
| | | Trading activity level | | | | | Trading activity level | | | | | Trading activity level | | | | |
Time-of-day		1	2	3	4	High−Low	1	2	3	4	High−Low	1	2	3	4	High−Low
9:30–10:45	Mean $\hat{\beta}$	−14.1	−9.9	−13.8	−21.1	**−21.1**		−6.4	−10.0	−13.8	**−13.8**		−2.8	−4.1	−5.5	**−5.5**
	$\pi(\alpha)$	0.82	0.69	0.75	0.89	0.89		0.83	0.91	0.97	0.97		0.67	0.74	0.80	0.80
	$\pi(\alpha/n)$		0.51	0.59	0.79	0.79		0.70	0.82	0.91	0.91		0.50	0.62	0.70	0.70
10:45–12:45	Mean $\hat{\beta}$	−22.7	−27.4	−35.6	**−21.5**	−11.8	−16.0	−18.9	−23.2	**−11.3**	**−8.6**	−9.9	−11.1	−12.9	**−4.3**	
	$\pi(\alpha)$	0.92	0.94	0.98	0.94	0.96	0.97	0.99	1.00	0.98	0.98	0.98	0.98	0.97	0.81	
	$\pi(\alpha/n)$	0.66	0.81	0.86	0.94	0.86	0.92	0.95	0.97	0.99	0.94	0.96	0.95	0.96	0.95	0.69
12:45–14:45	Mean $\hat{\beta}$	−22.5	−29.2	−34.3	−38.8	**−16.3**	−16.7	−20.0	−22.2	−25.0	**−8.3**	−11.5	−12.7	−13.5	−14.6	**−3.1**
	$\pi(\alpha)$	0.90	0.96	0.97	0.99	0.92	0.98	1.00	1.00	1.00	0.97	0.99	0.99	0.98	0.98	0.78
	$\pi(\alpha/n)$	0.81	0.87	0.92	0.95	0.76	0.96	0.96	0.98	1.00	0.89	0.97	0.97	0.96	0.96	0.60
14:45–16:00	Mean $\hat{\beta}$	−31.3	−37.3	−41.7	−47.0	**−15.7**	−22.1	−25.4	−27.7	−31.0	**−9.0**	−14.5	−16.2	−17.7	−20.3	**−5.8**
	$\pi(\alpha)$	0.97	0.98	0.99	1.00	0.94	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	0.95
	$\pi(\alpha/n)$	0.89	0.94	0.97	0.98	0.81	0.98	0.99	0.99	1.00	0.92	0.98	0.98	0.99	0.99	0.89
Late−Early	$\hat{\beta}_{4,\tilde{s}} - \hat{\beta}_{1,\tilde{s}}$	**−31.3**	**−27.4**	**−27.9**	**−25.9**		**−22.1**	**−19.0**	**−17.7**	**−17.2**		**−14.5**	**−13.4**	**−13.6**	**−14.8**	
	$\pi(\alpha)$	0.97	0.95	0.96	0.97		1.00	1.00	0.99	0.99		1.00	0.99	0.99	0.99	
	$\pi(\alpha/n)$	0.89	0.88	0.88	0.91		0.98	0.98	0.97	0.98		0.98	0.98	0.98	0.98	