CATalytic Insurance: the case of natural disasters.*

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Abstract

Insurance is usually costly, and particularly so for large catastrophic events. Moreover, developing countries have learned to expect assistance from the international community in the event of a natural disaster. Does this imply that the current scarcity of catastrophe insurance is the result of weak demand? Can insurance still play a useful role in the developing world? With the aid of a simple model, we show that insurance benefits help enhance financially constrained economies to access to private capital markets, even in the absence of risk aversion and in the presence of international safety nets. Thus, insurance, even at the current high costs, should be valuable for middle income economies because its catalytic effect on external finance.

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1 Introduction

Natural disasters have been in the rise in recent years (see Figure 1).[1] Hurricane Katrina has been the costliest ever in (absolute) economic terms (see Table 1.a), although its impact on the U.S. GDP (about 1 percent) was only "minor" relative to a number of hurricanes that have hit small island economies in the Caribbean or the Pacific (see Table 1.b): hurricane Hugo alone, in 1989, took more than 200 percent of Santa Lucia's GDP; hurricane Ivan more than 100 percent of Grenada's GDP, in 2004. Even if we abstract from small island economies, the costs of natural disasters are generally higher in developing countries than in the developed economies (see Table 1.c) that usually invest more in risk-management strategies.

Insert Figure 1 and Table 1

In addition to being more vulnerable to natural hazards, developing economies have also limited access to international capital market to finance reconstruction in the aftermath of a negative shock. How should then they cope with natural disasters? Catastrophe insurance, if available, is the natural option. However, developing countries have learned to rely on official lending (from multilateral banks or bilateral donors) in the aftermath of a negative shock. In light of the exceedingly high costs of catastrophe insurance (often a multiple of the fair price), one would expect that official lending crowds out all demand for insurance.

Is that the explanation for the scarcity of catastrophic insurance in the developing world? Does that mean that making insurance available (and cheaper) should not elicit any sizable demand from its prospective clients? This paper shows that this is not the case. Countries with access to official lending may in fact decide to purchase costly catastrophe insurance because of its "catalytic" role on external finance. More precisely, by guaranteeing resources that limit economic contraction in the aftermath of a shock, insurance makes default relatively more costly. This relaxes a country's borrowing constraint, increases its creditworthiness and enhances its access to capital markets. For large rare events such one the ones we consider here, such a benign effect outweighs cost considerations.

This is not surprising. Indeed, as we know from Elrich and Becker's (1972) seminal paper, market insurance is most effective when applied to a large but rare event. This is true for individuals as well as for sovereigns. Indeed, there are a few recent examples of countries that decided to insure themselves against natural catastrophes.[2] For instance, in May 2006, the Mexican government issued a 160 million US dollar parametric catastrophe bond to finance rescue and

[1] Global climate change, increasing population concentration and land erosion due to poor land use are often quoted among the major causes of the increase in the frequency and severity of natural disasters.

[2] See Hofman and Brukoff (2006) for a survey of the insurance opportunities available to developing counries against natural disasters.

rebuilding in the case a major earthquake hits some densely populated areas of the country.[3] In addition, a pool of Caribbean countries, with the help of the World Bank, is currently developing a Caribbean Catastrophe Risk Insurance Facility (CCRICF) to reduce the high costs of catastrophe insurance and guarantee access to the insurance market. However, even in the most "successful" cases, such as the one we just discussed, the cost of coverage is a multiple of the fair price (around three times for CCRICF).[4]

Why is insurance so costly? Several reasons are invoked in this regard, including supply-side constraints induced by either agency costs or adverse selection, problems of information opacity of tail events, and oligopolistic practices,[5] which could be compounded by lack of demand (due to Samaritan's dilemma considerations and other behavioral factors) and the resulting illiquidity of the market. As pointed out by Froot (2001), the securitization of catastrophic risk through the issuance of catastrophe bonds may in the future induce greater market discipline. However, at the moment, it has fall very short of reducing the costs of insurance towards fair levels.[6]

To understand why countries buy catastrophe insurance even if it is so expensive, in this paper, we develop a stylized model in which countries' access to the international capital market (as determined by their credit risk) depends on initial income levels, and on the incidence and magnitude of natural disasters. This set-up allows us to shed new light on why countries might decide to purchase catastrophe insurance even when premiums are exceedingly high, and why they would do so even in the presence of a multilateral catastrophe lending facility that may assist them–lending at the risk-free rate–in the aftermath of a negative shock.[7] Our analysis deliberately assumes high insurance premiums (that is, we assume that the premium to insure the stock of infrastructure against a natural disaster is higher than the expected return of rebuilding the same infrastructure) to acknowledge the fact that insurance is costly in practice, and risk neutrality to abstract from consumption smoothing motives to purchase expensive insurance. In this simplified setting, demand for insurance arises because of its "catalytic" role on external finance.

Not surprisingly, the countries that gain the most from the availability of catastrophe insurance are those medium-income countries that have limited ac-

[3] This was the the first tranch of a 450 milion US dollars insurage coverge plan. Payment are triggered if a earthquake of magnitude 7.5 or 8 hits some predefined zones of the country. See Nell and Richter (2004) for a discussion of the of parametric insurance, that is of insurance policies with payments linked to measurable events such as the magnitude af an eartquake, or the wind-speed of an hurricane.

[4] Note that a loading equal to two to three times the fair price is considerably lower industry averages that range from 5 to 6 times expected outlays.

[5] For a comprehensive discussion of the market for catastrophe risk, see Froot (2001).

[6] For a discussion of the securitization of catastrophe risk and the development of a catastrophe bond market, see Doherty (1997).

[7] To our knowledge, these issues have not been yet examined in the economic literature. By contrast, there is growing economic literature assessing the economic costs of natural disasters. See, inter alia, Mauro (2006), Ramcharan (2005), Raddatz (200X), and Calderon (200X), Toya and Skidmore (2006), and a well developed finance literature on catastrophe insurance, see, for instance...

cess to the international capital market and face financial constraints once they are hit by a shock (at the cost of important output losses) or even before the event, because of the credit risk associated with the possibility of the realization of the negative shock. Conversely, catastrophe insurance should not appeal to either poor countries without access to capital markets, or to rich countries that preserve (less costly) market access even in the aftermath of a disaster.

Even in those case in which it is beneficial for a country, insurance may not be an practical option due to its limited supply –particularly in the case of small sovereign borrowers, for which credit risk combines with event that are at once difficult to verify and hard to diversify.[8] Moreover, even if insurance is made available (as, for example, in the cited CCRIF), small low-income catastrophe-prone economies may be trapped in sort of Samaritan's dilemma by which insurance crowds out (belated but cheap) official loans, thereby detracting from the incentives to insure.

To explore whether this time inconsistency may preclude the demand for insurance, we study an extreme version of the implicit bailout: an *implicit or explicit* multilateral catastrophe lending facility that guarantees access to reconstruction funds at the risk-free interest rate in the event of a natural disaster. Unlike private lenders, multilateral lenders are assumed to enjoy a preferred creditor status that allows them to elude credit risk and the borrower constraint.[9] In such a set-up, we find that the introduction of the facility weakens but does not eliminate the demand for insurance, precisely because of the catalytic effect already mentioned. While insurance entails a positive transfer after the shock (hence, its catalytic effect), the repayment of the multilateral loan in the years following the shock tightens the borrowing constraint, crowds out private lending and reduce the country's investment opportunities in good times. This in turns implies that the more credit constrained a country is, the greater the drawback of the lending facility relative to catastrophe insurance, and the larger the demand for the latter.

The paper is organized as follows: Section 2 presents the model and characterizes the benchmark case. Section 3 introduces an discusses catastrophe insurance. Section 4 does the same for the multilateral catastrophe lending facility. Finally, Section 5 discusses the findings and concludes.

2 The benchmark

Consider an economy endowed with a two-factor Leontieff technology

$$Q = \rho \min\{\min\{1, L\}, K\} \tag{1}$$

to produce a single consumption good. The first factor, denoted by L, can be thought of as infrastructure, which we assume to be at its maximal level

[8] Despite the fact tat parametric insurance can help on the first count, its use in the insurance market has been disappointingly scarce.

[9] As documented by Jeanne and Zettelmeyer (2003) for the IMF, multilateral lending to middle income countries is virtually default risk-free. However, for our purposes it is sufficient to assume that the associated default costs are higher than for private claims.

$\overline{L} = 1$ at the beginning of the production cycle ($t = 0$). The second factor, K, represents installed productive units (or capital, for short), that we assume to be zero at $t = 0$, and needs to be externally financed (see below). ρ ($\rho > 1$) denotes a total factor productivity parameter. Finally, we also assume that the country is endowed with a storable stock of $\overline{x}$ units of the consumption good, $\overline{x} \in \Re^+$.

The timing of the model is as follows. At time $t = 0$, the country issues debt with private lenders (or bonds, for short) for an amount D_0 to finance capital investment $K = D_0$. The gross borrowing cost i is assumed to be equal to the risk free rate r_f (which we normalize to 1 without loss of generality) plus a risk premium δ, itself a function of the probability that the country defaults on the bond, so that $i = 1+\delta$. In the interim period[10] ($t = 1$), with a small probability π the country is hit by an exogenous shock (e.g., a natural disaster) that destroys a fraction $\beta > 0$ of its infrastructure. Faced with a negative shock, the country has the option to issue new bonds D_1 to finance infrastructure reconstruction, so that $L = 1 - \beta + D_1$. At the end of the production cycle ($t = 2$) output is realized and consumption takes place.

Denoting by the subscript b and g "bad" and "good" states of nature, according to whether the shock occurs or not, output X in period 2 can be written as:

$$X_g = \overline{x} + Q_g = \overline{x} + \rho \min\{1, D_0\}, \tag{2}$$

$$X_b = \overline{x} - Q_g = \overline{x} + \rho \min\{1 - \beta + D_1, D_0\}, \tag{3}$$

Crucially, the country's ability to raise new funds D_1 after suffering the adverse shock depends on its access to capital market, determined by its borrowing constraint. Whether this constraint is satisfied or not depends on a cost-benefit analysis of the default decision, which reflects the fact that the country will renege on its debt if the cost of servicing it exceeds the cost of default.

Specifically, following the "old" sovereign debt literature à la Sachs (19XX) , we assume that a default causes the country to lose a share $\gamma < 1$ of its current output X, a loss that is not fully appropriated by the lenders. For simplicity, and without great loss of generality, we also assume that no part of this lost income accrues to the lenders.

It follows that the country faces two distinct borrowing constraints, depending on whether default is ruled out altogether, or is expected only in the event of an adverse shock. In the first case, the constraint requires that default costs in bad states exceed the cost of servicing the debt, or:[11]

$$D_0 + D_1 < \gamma X_b = \gamma \left(\overline{x} + \rho \min\{1 - \beta + D_1, D_0\}\right). \tag{4}$$

In the second case, where (4) does not hold and lenders anticipate default in bad states and charge a risk-adjusted interest rate $i = \frac{1}{1-\pi}$, the borrowing

[10] For the sake of simplicity, and without great loss of generality, we assume that the interim period is close enough to the initial period so that the borrowing costs are the same in both periods.

[11] An alternative way of interpreting (4) is in terms of the collateral value of production, which increases with γ: the assumptions ensures that debt increases more rapidily than it contributes to build collateral, so that the constraint eventually binds.

constraint that ensures that default is avoided in good states can be written as:[12]

$$\frac{D_0}{1-\pi} < \gamma \left(\overline{x} + \rho \min\{1, D_0\} \right). \tag{5}$$

Finally, we assume that

$$\frac{1}{\gamma} > \rho > \frac{1}{1-\pi}, \tag{6}$$

where the first inequality implies that investment increases default costs by less than it increases debt so that a country without endowment has no access to finance. The second inequality ensures that investing in period 0 is always optimal.

Finally, we assume that consumers are risk neutral, and policy makers maximize expected income Y,

$$E^j(Y) = (1-\pi)Y_g^j + \pi Y_b^j \tag{7}$$

where superscript $j = \{d, nd\}$ denotes whether or not the country defaults if hit by an adverse shock, with

$$\begin{aligned} Y_g^{nd} &= X_g - D_0; \quad (8) \\ Y_b^{nd} &= X_b - D_0 - D_1; \\ Y_g^d &= X_b - iD_0; \quad (9) \\ Y_b^d &= (1-\gamma)X_b. \end{aligned}$$

Note that in this set-up, income (and welfare) are mostly determined by the borrowing constraints, and that the latter are, in turn, a function of the endowment $\overline{x}$, that is, they are more likely to bind in poor countries than in richer ones. In fact, given that default costs are proportional to total income, which, in turn, depends on endowments, the latter plays the role of an "implicit" collateral to the bond issuance: richer countries have more to lose if default is the avenue of choice.

We exploit this dimension in the characterization of the general solution of the benchmark case by distinguishing five intervals according to the value of the country's endowment $\overline{x}$. We refer the reader to the Appendix for the analytical details.

2.1 Case 1B: $\overline{x} \geq x_1^B \equiv \frac{1+\beta-\gamma\rho}{\gamma}$

In high-income countries ($\overline{x} \geq \overline{x}_1$)default costs are large enough to ensure that the borrowing constraint (4) is never binding. As a result, the country can borrow at the risk-free rate the optimal amount $D_0 = 1$ in period 0, and the optimal amount $D_1 = 1-\beta$ in period 1, if it is hit by a shock . Thus, production is always maximized, and so is expected income:

$$E(Y) = \overline{E}\,(Y) \equiv \overline{x} + \rho - 1 - \pi\beta, \tag{10}$$

[12] Note that in our set-up the borrowing constraint is more likely to be binding for poor than for rich countries. We exploit this dimension in the analysis that follows.

which amounts to the sum of endowment ($\overline{x}$) plus the (maximal) output (ρ), minus the cost of capital investment (1) and the expected cost of reconstruction ($\pi\beta$).

2.2 Case 2B: $x_1^B > \overline{x} \geq x_2^B \equiv \frac{1-\gamma\rho(1-\beta)}{\gamma}$

If $\overline{x} \leq x_1^B$, endowment no longer provides enough "collateral" value to overcome the borrowing constraint. As a result, the country cannot borrow $D_0 = 1$ in period 0 and $D_1 = \beta$ in the event of a shock. In this case, policy makers face a trade-off between maximizing period 0 investment, and "underinvesting" initially in order to "save" additional access in case the country is hit by an adverse shock in period 1.

However, it can be shown (see Appendix) that, if the shock is rare enough, the country always chooses to maximize period 0 investment at the expense of period 1 reconstruction funds in the event of an (unlikely) adverse shock.

As a result, $D_0 = 1$, $D_1 = \frac{\gamma\overline{x}-(1-\gamma\rho(1-\beta))}{1-\gamma\rho} < 1-\beta$, and

$$\begin{aligned} E(Y) &= \overline{x} + \rho - 1 - \pi\left[\rho(\beta - D_1) + D_1\right] \\ &= \overline{E}(Y) - \pi(\rho - 1)(\beta - D_1) \end{aligned} \tag{11}$$

where the cost of the shock now exceed the infrastructure damage β by an amount $\pi(\rho-1)(\beta-D_1)$ equal to the expected output loss associated with the failure to finance the reconstruction of infrastructure to its pre-shock level.

2.3 Case 3B: $x_2^B > \overline{x} \geq x_3^B \equiv \frac{\rho-\rho\gamma-1}{\gamma(\rho-1)(1-\pi)} - (1-\beta)\rho$

For $\overline{x} \leq x_2^B$, the borrowing constraint prevents the country from borrowing enough to fund the optimal amount of capital in period 0 while avoiding default in bad states. As before, for rare shocks, the country chooses to maximize borrowing in period 0 at the expenses of borrowing after a shock in period 1, and the solution in this case entails $D_0 \leq 1$, $D_1 = 0$ where the relevant borrowing constraint becomes

$$D_0 \leq D_0^{nd} \equiv \gamma(\overline{x} + \rho(1-\beta)) < 1, \tag{12}$$

where the superscript nd denotes non default in the bad state. Note, however, that the financially constrained country now has the option to increase its indebtedness up to the level that ensures repayment in good (but not in bad) states. Specifically, it can borrow up to (5), which now becomes:

$$D_0 \leq (1-\pi)\,\gamma\,(\overline{x} + \rho D_0) \tag{13}$$

from which

$$D_0 \leq D_0^d \equiv \min\left\{\frac{(1-\pi)}{1-(1-\pi)\gamma\rho}\gamma\overline{x}; 1\right\}. \tag{14}$$

where the superscript d denotes default in the bad state. It can be shown that, for $\overline{x} \in [x_3^B, x_2^B]$, total income is maximized by the lower (default-free) level of indebtedness, so that $D_0 = D_0^{nd} < 1$, and expected income is given by

$$\begin{aligned} E^{3B}(Y) &= \overline{x} + (1-\pi)(\rho-1) D_0^{nd} + \pi \left[\rho(1-\beta) - D_0^{nd}\right] \\ &= \overline{E}(Y) - \pi(\rho-1)\beta + (1-\pi)\rho\left(1-D_0^{nd}\right) - \left(1-D_0^{nd}\right). \quad (15) \end{aligned}$$

where the income cost of the shock now exceeds that for the unconstrained case by an amount that, in addition to the loss of output in bad states ($\pi(\rho-1)\beta$), reflects the output loss associated with suboptimal capital investment ($(1-\pi)\rho\left(1-D_0^{nd}\right)$) net of the savings in debt service $\left(1-D_0^{nd}\right)$.[13]

2.4 Case 4B: $\left(x_3^B > \overline{x} \geq x_4^B \equiv \frac{(\rho-1)(1-(1-\pi)\gamma\rho)}{(1-\pi)(\rho-1)\gamma-\pi}(1-\beta)\right)$

In this interval, the country chooses the higher level of indebtedness, borrowing and investing $K^{4B} = D_0^d$ in period 0, and defaulting whenever it is hit by a shock. In this case, income is given by

$$E^{4B}(Y) = \overline{x} + (1-\pi)\left(\rho - \frac{1}{1-\pi}\right) D_0^d + \pi\left[\rho(1-\beta) - \gamma\left(\overline{x} + \rho(1-\beta)\right)\right]. \quad (16)$$

Note that, comparing with the previous interval, the country increases its initial investment at the expense of a larger (*risk-adjusted*) interest rate $i = \frac{1}{1-\pi}$. Interestingly, income in bad states remain unchanged, since

$$Y_b^{3B} = \overline{x} + \rho(1-\beta) - D_0^d = \overline{x} + \rho(1-\beta) - \gamma\left(\overline{x} + \rho(1-\beta)\right) = Y_b^{4B}. \quad (17)$$

Thus, the default decision hinges entirely on the extra leverage that the country obtains by accepting a higher interest rate. More formally:

$$Y_g^{4B} = \overline{x} + \left(\rho - \frac{1}{1-\pi}\right) D_0^d > \overline{x} + (\rho-1) D_0^{nd} = Y_g^{3B} \iff \frac{D_0^d}{D_0^{nd}} > \frac{(\rho-1)}{\left(\rho - \frac{1}{1-\pi}\right)}. \quad (18)$$

2.5 Case 5B: $\overline{x} \geq x_4^B$

Finally, if $\overline{x} < x_4^B$, the country chooses, again, to restrain its borrowing in period 0 so as to avoid default if hit by a shock in period 1. We thus have $K^{5B} = D_0^{nd} = \gamma\left(\overline{x} + \rho(1-\beta)\right)$, and (because in this interval D_0^{nd} may be below $1-\beta$),

$$E^{5B}(Y) = \overline{x}(1-\pi)\rho D_0^{nd} + \pi(\rho \min\{(1-\beta), D_0^{nd}\} - D_0^{nd}. \quad (19)$$

[13] Note that the fact that these savings accrue in both states of nature indicates that capital investment is ex-post excessive in the event of a shock, as $D_0 > 1-\beta$.

2.6 Discussion

The above analysis is illustrated in **Figure 2.a**, where we plot D_0 and D_1 as a function of endowment $\overline{x}$, setting the rest of the intervening parameters to for reasonable (albeit arbitrary) values.[14] Intuitively, for rich countries ($\overline{x} > x_1^B$) creditworthiness is never a problem: endowments provide enough implicit collateral to ensure access to finance to exploit investment opportunities in good states, and to fully rebuild the infrastructure in bad states.

By contrast, all other countries face a trade-off between the amount they can invest in period 0 and what they can invest in period 1 if they are hit by a shock. For rare events, countries are better off investing more in period 0, even if this means losing access to finance in the (unlikely) event of a shock in period 1. In this context, relatively rich upper middle-income countries ($\overline{x} > x_2^B$) can still (partially) finance the rebuilding of infrastructure in period 1, this option is lost to when $\overline{x} < x_2^B$.

Moreover, because for $\overline{x} < x_2^B$ countries are forced to underinvest in period 0 in order to avoid default in period 1, they face the choice between borrowing more today and defaulting tomorrow in the event of a shock, and borrowing less today to avoid default costs tomorrow; a decision that ultimately depends on the extent to which investment can be expanded by accepting the higher risk-adjusted interest rate.

In the case of middle-income countries with good access to capital ($\overline{x} \in [x_3^B, x_2^B]$), the additional resources do not justify the higher rate, and default is avoided. These resources becomes relatively more valuable as endowments decline and the financing gap widens so that, for lower middle-income countries ($\overline{x} \in [x_4^B, \overline{x}_3^B]$), overborrowing is the preferred strategy. However, because of the higher interest rate charged to the overborrowing country, the financial constraint tightens faster in this case ($\frac{\partial D_0^d}{\partial \overline{x}} > \frac{\partial D_0^{nd}}{\partial \overline{x}}$), which, in turn, explains why low-income countries ($\overline{x} < x_4^B$) prefer to limit investment to avoid default.

3 Insurance

With the exception of case 1, in the benchmark case, income is suboptimal because of the presence of borrowing constraints that limit either initial investment or the post-shock rebuilding effort, or both, and may even induce substantial default costs. The natural arrangement to mitigate this problem is a standard insurance contract that offers to transfer reconstruction funds in bad states at the cost of a premium in good states. Specifically, in the context of the model, the country may purchase a contract that pays a pre-determined amount Z in the event of a shock, at the cost of a up-front premium paid in period 0.

Let's assume that in period 0 the country purchases insurance that pays Z in the interim period should the country be hit by a negative shock, at a premium $\varphi = \pi\nu$, which is paid in good times, where π is the net present value

[14] In particular, we assume that $\pi = 0.02, \gamma = 0.3, \rho = 1.25$, and $\beta = 0.4$.

of a unit expected insurance outlay, and ν is a margin that reflects, inter alia, intermediation costs and a risk premium (alternatively, the insurer's cost of capital, including potential increases if the event materializes). We assume that the insurance premium is paid up front and financed through debt issuance. In addition, we assume that

$$\nu \in \left[\rho, \rho \frac{1-\gamma}{1-\gamma\rho+\pi(\rho-1)}\right], \tag{20}$$

a non-empty interval for small enough π ($\pi < \gamma$), to explicitly model the fact that insurance is expensive[15] but not so expensive that the country would ever buy it. Furthermore, to simplify the analysis, we also assume that

$$\nu < \frac{1}{\beta}$$

which guarantees that no default occurs in the insurance case.[16]

Letting $D_0 = K_0$ for ease of comparison with the previous case, the expected income of a country that borrows D_0 to invest K_0 and φZ to purchase Z units of insurance is

$$E(Y) = \overline{x} + \rho\left((1-\pi)K_0 + \pi\rho \min\left\{K_0, (1-\beta) + Z + D_1\right\}\right) - (K_0 + \pi\nu Z + \pi D_1) \tag{21}$$

With this new instrument available, we revisit the country's choices as a function of the income levels, as in the previous case.

3.1 Case 1I: $\overline{x} \geq x_1^B$.

No Insurance is needed to attain the optimum (the borrowing constraint is not binding). Moreover, because $\nu > 1$, the effective cost of insurance exceeds that of international capital, and no insurance is purchased. The solution is then identical to benchmark case $1B$.

3.2 Case 2I: $x_1^B > \overline{x} \geq x_2^I \equiv \frac{1-\gamma\rho+\pi\beta\nu}{\gamma}$.

When $\overline{x} < x_1^B$, the borrowing constraint limits period 1 borrowing. As before, for rare events the country always chooses to maximize period 0 investment (which in this case attains the optimal $D_0 = 1$), so period 1 is the residual variable.

In this regard, insurance plays a complementary role: by ensuring the availability of resources in the aftermath of a shock, it increases output in bad states

[15] This stilyzed contract applies more directly to the case of a parameterized CAT bond with principal Z and coupon φ, which, in the case of a verifiable natural disaster, virtually eliminates the need for costly state verification. Standard CAT insurance, by contrast, are typically based on actual losses and cover pre-specified layers, defined by a deductible or "retention" (below which no losss is covered) and a "limit" above which no loss is covered.

[16] Such assumption does not alter the qualitative reuslts of our analysis (insurance always reduces the likelihood of default) but limit the number of cases that we have to analyze.

and, through this channel, relaxes the constraint. In this context, the borrowing constraint (4) becomes:

$$(D_0 + \pi\nu Z) + D_1 \leq \gamma(\overline{x} + \rho\left((1-\beta) + Z + D_1\right)) \tag{22}$$

so that

$$D_1 \leq D_1^{nd}(Z, D_0) \equiv \frac{\gamma\left(\overline{x} + \rho(1-\beta)\right) - D_0 + (\gamma\rho - \pi\nu)Z}{(1-\gamma\rho)}, \tag{23}$$

and

$$\frac{\partial D_1^{nd}}{\partial Z} = \frac{\gamma\rho - \pi\nu}{1-\gamma\rho} \geq 0, \tag{24}$$

This implies that insurance has a "catalytic" effect on private lending: by purchasing insurance the country enhances access to the international capital market in bad states. This effect explains why the country might be willing to purchase insurance even if it is expensive relative to capital markets. More precisely, the derivative of expected income on insurance is given by:

$$\frac{\partial E(Y)}{\partial Z} = \pi\left(-(\nu-\rho) + (\rho-1)\frac{\gamma\rho - \pi\nu}{1-\gamma\rho}\right) > 0 \tag{25}$$

where the expensive premium (the first RHS term) is counterbalanced by the positive catalytic effect (the second RHS term). On the other hand, given its costly nature, the country would purchase insurance only as a complement to market funds, so that

$$Z \equiv \beta - D_1^{nd}, \tag{26}$$

since increasing coverage beyond Z would simply substitute expensive insurance for less costly bonded debt. In this way, insurance fills in for private markets, both crowding in additional private funds and providing the resources that the market does not offer to allow the country to insure against the shock.

It is easy to verify that, in this region, we have that $K = D_0 = 1$, $D_1^{nd} = \frac{\gamma(\overline{x}+\rho)-(1+\pi\beta\nu)}{1-\pi\nu}$, $Z = \frac{1+\beta-\gamma(\overline{x}+\rho)}{(1-\pi\nu)}$, $L = 1 - \beta + D_1^{nd} + Z = 1$, and

$$E^{2I}(Y) = \overline{x} + \rho - 1 - \pi\left(D_1^{nd} + \nu Z\right) = \overline{x} + \rho - 1 - \pi\left(\beta + (\nu - 1)Z\right) \tag{27}$$

Comparing with (10), it can be see that, as expected, the only difference in income between this and the financially unconstrained case is the additional expense associated with the insurance premium $(\nu - 1)$.

3.3 Case 3I: $x_2^I > \overline{x} \geq x_3^I \equiv \frac{(1-\beta)(1-\gamma\rho)}{\gamma}$.

If $\overline{x} < \overline{x}_2^{IN}$, insurance will no longer have a catalytic effect, and the country cannot borrow in period 1.However, the country will still be able to buy insurance to increase access in period 0. Would it do it?

The borrowing constraint now becomes

$$(D_0 + \pi\nu Z) \leq \gamma(\overline{x} + \rho \min\{D_0, (1-\beta) + Z\}), \tag{28}$$

from which we have that:

$$D_0 \leq D_0^{nd} \equiv \gamma\left(\overline{x} + \rho(1-\beta)\right) + (\gamma\rho - \pi\nu)Z. \tag{29}$$

and, for small values of π, $\frac{\partial D_0^{nd}}{\partial Z} > 0$, and $\frac{\partial E(Y)}{\partial Z}$, so that, because insurance increases access to capital market in period 0, the country fully insures, that is, purchases insurance for an amount $Z = D_0^{nd} - (1-\beta)$ so as to bring infrastructure to $L = K$ in the event of a shock.

This, in turn, implies that in this region we have that: $K^{3I} = D_0^{nd} = \frac{\gamma\overline{x} + \pi\nu(1-\beta)}{1-(\gamma\rho-\pi\nu)}$, $Z = \frac{\gamma\overline{x} - (1-\gamma\rho)(1-\beta)}{1-(\gamma\rho-\pi\nu)}$, $L^{3I} = 1-\beta+Z$, and

$$E^{3I}(Y) = \overline{x} + \left((1-\pi)\rho - 1\right) D_0^{nd} + \pi\rho(1-\beta) - \pi\left(\nu - \rho\right) Z$$

It is important to not eat this stage that it is never in the best interest of

the country to overborrow at the risk of default in bad states. The intuition is straightforward. Consider a country that overborrows and defaults in bad states. The anticipation of default eliminates the catalytic effect of insurance (through its role in the now irrelevant (4)). In turn, without this effect, insurance is simply too expensive: no insurance would be purchased if default in bad states is inevitable.[17] But lack of insurance tightens the borrowing constraint of the overborrowing country to a point at which fewer funds are available than when borrowing is limited and default is prevented. Since no additional resources can be gained by overborrowing, the latter is never a solution.

3.3.1 Case 4I: LIC ($\overline{x} < x_3^I$)

For $x_3^I > \overline{x}$, $Z = 0$ (access to insurance no longer plays a role) and that $\overline{x}_4 = \frac{(1-\beta)(\frac{1}{(1-\pi)} - \gamma\rho)}{\gamma - \frac{\pi}{(\rho-1)}} > x_3^{IN}$ implies that we are back in benchmark, case 5 above.

$$E^{4I}(Y) = E^{5B}(Y) = \overline{x} + (1-\pi)\rho D_0^{nd} + \pi(\rho \min\{(1-\beta), D_0^{nd}\} - D_0^{nd}$$

3.4 The benefits of insurance: Discussion

The previous analysis is summarized in **Figure 2.b**, where we plot debt and insurance outlays under the insurance case for the same parameter set as in **Figure 2.a**. In addition, we assume that the overhead parameter $\nu = 1.3$. As noted, for high income the borrowing constraint does not bind, no insurance is purchased and the results coincide with the benchmark.

[17] Note that insurance does not work as a substitute for lending. Rather, it pays to insure only if the country reaps the crowding-in benefits, for which insurance funds need to work as collateral to prevent default in bad states. If default in bad states is anticipated, the collateral value of insurance vanishes.

For $\overline{x} < x_1^B$, by contrast, countries purchase insurance to increase their ability to rebuild infrastructure after the shock. Notice that in this case insurance plays two roles. On the one hand, by ensuring that reconstruction funds will be there, it increases output in bad states and, in turn, default costs, crowding in private lenders. As a result, in this region, insurance enlarges the amount of resources available in the aftermath of the shock relative to the benchmark, and complements privates funds. Relatively poorer countries ($\overline{x} < x_2^I$) purchase more insurance as they face growing financial constraints in international capital markets: insurance makes up for these constraints (at a considerable premium), bringing the country closer to the optimum, again improving upon the benchmark.

Finally, capital investment in low-income countries ($\overline{x} < x_3^I$) is not high enough to justify insurance and we are back in the benchmark case.[18]

4 The demand factor.

The previous analysis focused on the "supply side" of the problem to show that the availability of insurances allows the country to relax the borrowing constraint–albeit at (or despite) a considerable cost. However, because large events of a systemic nature (such as natural disasters) involve massive economic losses and affect a large number of people they typically trigger ex-post government intervention which agents correctly anticipate. This often creates Samaritan dilemmas problems that lead individuals to underinvest in catastrophe insurance. Similarly, at the international level, catastrophes in low-income countries elicit an almost immediate reaction by the international community in the form of (often concessional) loans for social expenditure and reconstruction. Why would a country bear the exorbitant insurance premiums if it is likely to have access to official resources at a small cost? Is this version of the Samaritan's dilemma what is behind the scarcity of catastrophe insurance in middle- and low–income countries? Would it be there any demand for insurance should the latter be made available to all?

We can adapt our model to look into this issues, by examining whether insurance is still purchased by the country in presence of a catastrophe lending facility that lends unlimited funds at the risk-free rate in the event a shock. This can be considered an extreme version of the implicit bailout story: For expositional purposes, is it easier to tackle this question in two steps, solving for the lending facility in the absence of insurance, and then putting the two arrangements together.

[18] Note that, in our framework, the availability of insurance not only increases investment for middle-income countries but also rules out defaults altogether because of it positive effect on output in bad states. We come back to this in the final section.

4.1 A catastrophe lending facility

Consider now the case in which a *multilateral lender* offers a one-period catastrophe lending facility from which the country can draw (only) in the event of a shock. It is easy to show that this facility cannot be offered by private markets because loan amounts will be restricted by the borrowing constraint in exactly the same way as D_1 was in the previous case. However, a multilateral lender could in principle exploit its preferred creditor status to provide access in period 1 beyond what the borrowing constraint allows. Indeed, preferred official creditors (the government at the national level, multilaterals and donors at the international level) are the ones that usually come to the rescue after large natural disasters.

In order to represent the preferred creditor status of the multilateral lender, we assume that defaulting on the multilateral is prohibitively costly so that multilateral loans are always repaid. Therefore, in this case, selective default on private creditors could be an equilibrium outcome.

Given the debt D_0 (with private lenders) and M (with official lenders), in period 2 the country faces two choices: repay or default on bonds (where "bonds" is short for "debt with the private sector").[19]

More formally, in period 2 the sovereign does not default on bonds if, and only if

$$D_0 \leq D_0^{nd}(M) \equiv \gamma \left[\overline{x} + \rho \left(1 - \beta + D_1 + M\right)\right] - D_1 - \gamma M, \tag{30}$$

whereas the unconstrained M is set to maximize period 2 output, i.e., $M = D_0 - (1 - \beta)$.[20]

Replacing M into (30), we obtain

$$D_0 \leq D_0^{nd}(M) \equiv \frac{\gamma \left(\overline{x} + 1 - \beta\right)}{1 - \gamma \left(\rho - 1\right)}. \tag{31}$$

Note that the fact that M is chosen ex post (i.e., the country cannot commit to not borrowing from the facility in period 1) simplifies the problem, which it now boils down to the choice of period 0 borrowing, D_0. Also note that, under the assumption that multilateral and private lending command the same interest rate, the actual composition of period 1 lending is immaterial for the current analysis. Then, without loss of generality, we can set $D_1 = 0$.

[19] We implicitly assume that the multilateral has no way of conditioning its lending on the continued service of the debt with private lenders, which in our setup simply reflects a sequencing issue: the fact that the contingent loan is disbursed in period 1, before the bond matures. However, we come back to this point below.

[20] Compare with the constraint under insurance:

$$D_0 \leq \gamma \left[\overline{x} + \rho \left(1 - \beta + Z + D_1\right)\right] - D_1 - \pi \nu Z,$$

note that multilateral lending relaxes the bond borrowing constraint because it increases the value at stake, although the crowding out is stronger than insurance because the burden of the loan is specific to bad states.

Expected income can be expressed as

$$\begin{aligned} E(Y) &= \overline{x} + (\rho - 1)\, D_0^{nd} - \pi \left[D_0 - (1-\beta) \right] \\ &= \overline{E}(Y) - (\rho - 1 - \pi)\left(1 - D_0^{nd}\right), \end{aligned} \tag{32}$$

where the difference with the unconstrained results reflects the lost output net of financial savings in both periods (both a function of the borrowing and investment gap, $1-D_0^{nd}$). On the other hand, the country can in principle borrow beyond the limit imposed by (32) and, after a shock, withdraw from the facility and default on the bond. In this case, expected income is characterized by

$$\begin{aligned} E(Y) &= (1-\pi)\left(\overline{x} + \rho D_0^d\right) + \pi (1-\gamma)\left[\overline{x} + (\rho-1)\, D_0^d + (1-\beta)\right] - D_0^d \\ &= (1-\pi\gamma)\left[\overline{x} + (\rho - 1)\, D_0^d\right] + \pi (1-\gamma)(1-\beta) \end{aligned} \tag{33}$$

As before, the equilibrium can be characterized by income brackets.

4.1.1 Case 1M: $\overline{x} \geq x_1^M \equiv \frac{1-\gamma(\rho-\beta)}{\gamma}$

From (31), we know that

$$D_0^{nd}(M) \geq 1 \Longleftrightarrow \overline{x} \geq x_1^M, \tag{34}$$

which tells us that the constraint is not binding: the country borrows and invests $K^{1B} = D_0^{nd} = 1$ in period 0, and $L^{1B} = \beta$, in period 1 in the event of a shock, and attains the maximum expected income $E(Y) = \overline{E}(Y)$.

The country can borrow the optimal amount in both periods, rendering the facility irrelevant. Recall that, in the absence of multilateral lending, the country could borrow unconstrained for $\overline{x} \geq x_1^B > \overline{x}_1^M$. The solution is then identical to the unconstrained benchmark case (case 1B), but in this case the interval in which the country attains the optimum is widened by the presence of the facility.

Note also that this case corresponds to cases $1I$ and $2I$, where borrowing is at the optimum in both periods.

4.1.2 Case 2M: $x_1^M > \overline{x} \geq x_2^M \equiv \frac{\rho-1-\pi}{\gamma(\rho-1)(1-\pi\gamma)} - (\rho - \beta)$

In this interval, the borrowing constraint binds: $D_0^{nd} < 1$ and, as a result, $M = D_0^{nd} - (1-\beta) < \beta$.

Relative to the insurance case, we observe that the multilateral loan introduces a weaker crowding in effect that yields a relatively lower period 0 borrowing, so that $K^{2M} = D_0^{nd} = \frac{\gamma[\overline{x}+1-\beta]}{1-\gamma(\rho-1)} < K^{2I}$, and $M < Z \Longrightarrow L^{2M} < L^{2I}$. Note, however, that this difference hinges entirely on the assumption that the loan is repaid in full in period 2 (while the insurance cost is distributed across states). We come back to this assumption in the next section.

4.1.3 Case 3M: $x_2^M > \overline{x} \geq x_3^M \equiv \frac{(1-\beta)(1-\pi\gamma)(\rho-1)(1-(1-\pi p)\gamma\rho)}{\pi^2(1+\gamma^2(\rho-1))+\gamma(\rho-1)-\pi(\gamma^2(\rho-1)+\rho)}$.

The constrained country chooses borrows $D_0^d = \min\left\{\frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}, 1\right\}$ in period 0 at a risk-adjusted rate $i = \frac{1}{1-\pi}$, and, if hit by the shock, borrows $M^d = D_0^d - (1-\beta)$ from the contingent credit line in period 1, and defaults in period 2. Then, we have $K^{3M} = D_0^d$, $L^{3M} = D_0^d - (1-\beta)$.

The intuition is similar to that in the benchmark case, except that now the overborrowing country still have access to financial resources in period 2. Indeed, overborrowing also increases output in bad states, since reconstruction funds are not restricted by the borrowing constraint and increase linearly with period 0 investment. For this reason, default has a smaller impact on income than in the benchmark.

4.1.4 Case 4M: $x_3^M > \overline{x} \geq x_3^I$)

$$M^{nd} = D_0^{nd} - (1-\beta) = \frac{\gamma\overline{x} - (1-\gamma\rho)(1-\beta)}{1-\gamma(\rho-1)} > 0 \Longleftrightarrow \overline{x} > \overline{x}_4^M. \qquad (35)$$

For low incomes, the country chooses to avoid default. Then, $K^{4M} = D_0^{nd}$, $L^{4M} = D_0^{nd} - (1-\beta)$.

4.1.5 Case 5M: LIC ($\overline{x} < x_3^I$)

For low incomes, the country chooses to avoid default and, given that $M^{nd} = D_0^{nd} - (1-\beta) = 0$, the facility no longer plays a role. We are back to benchmark case $5B$: $K^{4M} = K^{5B}$, $L^{4M} = L^{5B}$.

4.2 Multilateral lending: Discussion

A visual comparison with the benchmark see **Figure 2.c** reveals that the presence of the contingent credit line improves the country's welfare along two dimensions. First, it narrows the interval in which the country chooses to default. This is so because the contingent credit line increase the value at stake in case of a default. Given that default costs in this setup are proportional to output, the benign output effect of the contingent line increases the cost of defaults and reducing their incidence –even though the defaulting country still has access to the multilateral loan.

In turn, comparing with the insurance case, borrowing is never higher under the lending facility. Again, the intuition is relatively straightforward: whereas the insurance premium entails a transfer from good to bad states (and, in particular, is arbitrarily small for rare events), the catastrophe loan transfers the cost of the shock intertemporally within bad states (that is, states marked by the occurrence of the shock), creates a sharp asymmetry between good and bad states, tightening the borrowing constraint associated with the latter. Hence, the lower borrowing amounts (due to the crowding out of period 0 bond borrowing by period 1 multilateral lending) and the positive probability of default.

Regarding this point, note that we assumed for simplicity that the lending facility extended one-period loans. While this realistically reflect the short-run nature of most emergency and concessional lending, it bears the question of whether a longer loan can substitute insurance in those cases in which, because of market imperfections or political economy reasons, supply or demand for insurance is likely to be insufficient. More specifically, can a 1 in 30 years event be covered indistinctly by insurance and by a 30-year contingent loan?

According to the previous analysis, it cannot. A country that optimally borrows from the facility after it is hit by a shock inherits the full stock of debt, irrespective of the lengthening of the loan. In other words, since default in this case is not the result of a liquidity crisis but rather the consequence of a cost-benefit analysis, it is the stock of debt rather than its flow cost that determines the decision.

4.3 Catastrophe lending and insurance

Consider now the case in which the country has access to both insurance and the lending facility. Would the country still purchase insurance in this case, or would it rely entirely on catastrophe lending? In other words, does the facility make the supply of insurance redundant for the country?

To answer the question, first note that if $x \geq \overline{x}_1^M$, lending is clearly superior to insurance, because it allows the country to circumvent the borrowing constraint at a lower cost. On the other hand, it is easy to verify that, for $x < \overline{x}_1^M$, insurance is always demanded.

In particular, for $x \in [\overline{x}_1^M, \overline{x}_2^I]$, the country's problem consists in investing $L_0 = 1$, and $L_1 = \beta$, at the lowest cost, which in turn implies minimizing the amount of (costly) insurance compatible with that objective. In this context, the borrowing constraint (4) becomes:

$$(D_0 + \pi\nu Z) \leq \gamma(\overline{x} + \rho\left((1-\beta) + Z + M\right)) - \gamma M, \tag{36}$$

and substituting

$$M = D_0 - (1-\beta) - Z \tag{37}$$

(91) can be rewritten as:

$$D_0 \leq \widetilde{D}_0^{MIN} \equiv \frac{(\overline{x} + 1 - \beta + Z)\gamma - \pi\nu Z}{1 - \gamma(\rho - 1)}. \tag{38}$$

It is easy to verify (see Appendix) that within this interval, both insurance and multilateral lending coexist, where the former crowds in the latter much in the same way it does in the absence of a lending facility.

On the other hand, for $\overline{x} \leq \overline{x}_2^{IN}$, low period 0 investment levels make the lending facility redundant, and insurance becomes the only source of funding in bad states. Thus, we are back to the insurance case discussed previously.

Overall, the demand for insurance is weakened by the presence of the facility only for relatively high endowments for which the facility is enough to lift the borrowing constraint. However, because multilateral lending crowds out access to capital markets in period 0, insurance still plays a helpful role reducing the burden of period 1 borrowing, thereby relaxing the borrowing constraint. In other words, while the Samaritan's dilemma considerations eliminate the need for insurance as a source of funds in bad states, it does not eliminate its catalytic role in good states.

5 What's best? What's feasible? (very preliminary)

So far, we concentrated on the mechanics of the model and its implications in terms of access to finance in both states. Naturally, there is more to this exercise than simply comparing access. Indeed, an evaluation of the different alternatives would have to ponder their consequences in terms of expected income. We summarize our welfare analysis in **Figure 3**. The top panel of the figure plots net income from production in good and bad states (that is, output net of borrowing costs and endowments, or $Y_{g,b} - \overline{x}$), for each of the three main scenarios under study: the benchmark, up-front insurance and ex-post catastrophe lending. The second panel does the same for expected income. As can be seen, both insurance and catastrophe lending are (weakly) superior to the benchmark: income under each alternative (and in both states) is always greater or equal than in their absence. But their relative benefits differ according to the country's endowment.

If access to finance is not critical (richer countries), the insurance option yields a lower expected income than the less expensive multilateral lending facility. However, in the case of low-middle income countries the multilateral facility may, at the same time, crowd out private lending and be ineffective in avoiding costly default. Since for these countries access to finance is critical it is not surprising that higher levels of expected income are associated with the insurance option. What is somehow more surprising is that for a large set of endowment values a country may enjoys higher income in both states of nature if it relies on insurance rather than on the multilateral facility.

Such welfare trade-off are clearly illustrated in **Figure 4**, where we cast a closer look at the situation in which insurance and the catastrophe lending coexist. As can be seen, the demand for insurance kicks at the endowment level for which the borrowing constraint starts limiting investment in period 0. Thus, by crowding in private lending in period 0, insurance enables a financially constrained country to reach the optimal level of investment, albeit at a premium that detracts from the optimal expected income.

A potentially undesirable characteristic of the lending facility examined above is the fact that it involves multilateral lending to a country at a time when the country is expected to default on its private creditors. Unlike implicit arrangements, an explicit facility could still condition access to the facility ex ante, so as to make sure that the borrower has the incentives to avoid default.[21] This is not far from standard multilateral practice: multilateral loans are often granted provided that the recipient country meets certain debt sustainability criteria.[22] In this way, the official lender ensures that the country does not take the new money the minute before it defaults on third parties. Intuitively, to the extent that overborrowing excludes the country from the facility, this new condition should detract from the incentives to default, and reduce its incidence.

The solution for a *contingent* catastrophe lending facility does not differ much from the one presented in the previous section (see Appendix). Interestingly, a comparison between the contingent and the uncontingent facility reveals the latter to be better, at least in terms of expected income (**Figure 5**). The reasons is that, for those endowment levels for which the two differ, the contingent facility saves the default costs at the expense of leaving the country underfinanced after an adverse shock. However, because the shock is exogenous, the situation involves no moral hazard and no value is created by reducing the incidence of default. On the contrary, the punishment (exclusion from the facility) translates in a lower overall welfare.

The previous discussion, however, needs to be qualified in two ways. First, as previously noted two options differ in one crucial aspect: the loan has to be paid after a bad shock while the cost of insurance is transferred to good states. In other words, income (and consumption) volatility is bound to be lower with insurance than in any other scenario. Trivially, if income smoothing–from which we deliberately abstracted so far–were a policy objective, insurance would become relatively more appealing, a results that would only add to the case for insurance that our findings support.[23]

The second qualification, however, works in the opposite direction. While insurance (particularly when reasonably priced) would seem the logical option for disaster-prone middle-income countries, the fact that it entails a payment up front in exchange for an infrequent positive transfer makes this type of arrangements a political hard sell. Thus, even if a multilateral agency were willing to offer this facility at a fair premium, it may face a disappointingly weak demand from potential clients.[24] Thus, from a policy perspective, a multilateral catastrophe lending facility may be the only feasible alternative even for those

[21] To enhance incentives without distorting its automatic nature, the facility could involve temporary subscriptions on a rolling basis, to ensure that the country is not cut off overnight but still faces frequent exams.

[22] However, in the aftermath of a natural catastrophe it is not unlikely that other creditors (especially bilateral) agree to write-off part of their credit to allow multilateal lending.

[23] Needless to say, the inclusion of risk or loss aversion would modify the solutions and the charts in each case, increasing period 1 borrowing at the expense of period 0 borrowing, and raising insurance benefits and the amount of insurance purchased. However, the qualitative nature of the results –and of the differences between alternative cases– should remained unchanged.

[24] Note that this problem is different from the signaling problem associated with liquidity facilities such as the IMF's failed Contingent Credit Line, where the requesting country may have private information about its financial conditions or its planned policies that may be revealed by applying to the facility. In the case of catastrophe insurance, the shock is exogenous and any relevant information is common knowledge, so there should be no signaling problem.

cases in which insurance is the possible choice.

6 Conclusions (to be added)

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7 APPENDIX (Under construction)

7.1 The benchmark

7.1.1 Case 1B: High-income countries (HIC) $\left(\overline{x} \geq x_1^B \equiv \frac{1+\beta-\gamma\rho}{\gamma}\right)$

Setting $D_0 = 1$, and $D_1 = \beta$, it is immediate to verify that (4) becomes

$$1 + \beta \leq \gamma(\overline{x} + \rho), \tag{39}$$

and is satisfied iff $\Longleftrightarrow \overline{x} \geq x_1^B$.

In turn, $K = D_0 = 1$, $L = D_1 = \beta$ yield

$$E^{1B}(Y) = \overline{x} + \rho - (1 + \pi\beta). \tag{40}$$

7.1.2 Case 2B: Upper-middle-income countries (UMIC) $\left(x_1^B > \overline{x} \geq x_2^B \equiv \frac{1-\gamma\rho(1-\beta)}{\gamma}\right)$

If $\overline{x} \leq x_1^B$, the country cannot borrow $D_0 = 1$ in period 0 and still have access to $D_1 = \beta$ in bad states. Therefore, it faces a trade-off between maximizing period 0 investment, or "underinvesting" initially in order to "save" additional access in bad times.

Note that in equilibrium $D_1 \leq D_0 - (1-\beta)$,[25] so that the relevant borrowing constraint can be written as:

$$D_0 + D_1 \leq \gamma\left(\overline{x} + \rho\left[(1-\beta) + D_1\right]\right), \tag{41}$$

or, expressing the maximum amount that the country can borrow in period 1 (without defaulting if hit by a bad shock) as a function of period 0 borrowing:

$$D_1 \leq D_1^{nd} \equiv \frac{\gamma\overline{x} + \gamma\rho(1-\beta) - D_0}{1-\gamma\rho}. \tag{42}$$

In turn, we can express the trade-off between increasing consumption in good and bad states in terms of the country's problem at time 0:

$$\max_{D_0} E((Y(D_0, D_1^{nd}) = \overline{x} + \rho\left((1-\pi)D_0 + \pi(1-\beta)\right) - D_0 - \pi(\rho-1)\widehat{D}_1 \tag{43}$$

$$\text{subject to } 0 \leq D_1^{nd} \leq D_0 - (1-\beta) \tag{44}$$

It is then easy to verify that, for π low enough,

$$\frac{\partial E(Y)}{\partial D_0} = \rho(1-\pi) - 1 - \pi(\rho-1)\frac{1}{1-\gamma\rho} > 0, \tag{45}$$

[25] Note that if this were not the case, then we would have that

$$D_1 > D_0 - (1-\beta) \Longrightarrow L_1 = (1-\beta) + D_1 > K_0 = D_0$$

which would imply that marginal returns to investment in L in period 1 are zero.

which indicates that the country maximizes period 1 investment ($D_0 = 1$) at the expense of lower investment in the event of an adverse shock.

Finally, we need to verify that the country can invest a positive amount in the second period while avoiding default. This condition can be written as:

$$D_1^{nd}\big|_{D_0=1} = \frac{\gamma\overline{x} - (1-\gamma\rho(1-\beta))}{1-\gamma\rho} \geq 0 \Longleftrightarrow \overline{x} \geq x_2^B \equiv \frac{1-\gamma\rho(1-\beta)}{\gamma}. \quad (46)$$

In sum, for $\overline{x} \in [\overline{x}_1, \overline{x}_2]$, we have that $K^{2B} = D_0 = 1$, and, $L^{2B} = D_1 = \frac{\gamma\overline{x}-(1-\gamma\rho(1-\beta))}{1-\gamma\rho}$, and

$$E^{2B}(Y) = \overline{x} + (1-\pi)\rho + \pi\rho(1-\beta+D_1) - (1+\pi D_1). \quad (47)$$

7.1.3 Case 3B: Middle-income countries (MIC) $\left(x_2^B > \overline{x} \geq x_3^B \equiv \frac{\rho-\rho\gamma-1}{\gamma(\rho-1)(1-\pi)} - (1-\beta)\rho\right)$

It follows from (46) that, in this interval, $D_1 = 0$. Moreover, the country cannot borrow the optimal amount of capital in period 0 without risking default if hit by a shock.

From (12) and (14),

$$D_0^{nd} = \gamma(\overline{x} + \rho(1-\beta)) < 1. \quad (48)$$

and

$$D_0^d \equiv \min\left\{\frac{(1-\pi)}{1-(1-\pi)\gamma\rho}\gamma\overline{x}; 1\right\}, \quad (49)$$

In turn,

$$D_0^d = 1 \Longleftrightarrow x > \widetilde{x} \equiv \frac{1-\gamma\rho(1-\pi)}{\gamma(1-\pi)} \quad (50)$$

and

$$\widetilde{x} \leq \overline{x}_2 \Longleftrightarrow \pi < \frac{\gamma\rho\beta}{1+\gamma\rho\beta}, \quad (51)$$

so that for a sufficiently small π, there exists a non-empty interval $[\widetilde{x}, \overline{x}_2]$ such that $D_0^d = 1$. We also have that

$$D_0^{nd} > (1-\beta) \Longleftrightarrow x > \frac{(1-\beta)(1-\gamma\rho)}{\gamma} \equiv \widetilde{x}_a$$

and

$$\widetilde{x}_a < \widetilde{x} \Longleftrightarrow \pi < 1 - \frac{1}{1-\beta(1-\gamma\rho)}$$

so that for a sufficiently small π (notice that $\rho < 1/\gamma \Longleftrightarrow 1 - \frac{1}{1-\beta(1-\gamma\rho)} > 0$), in the interval $[\widetilde{x}, \overline{x}_2]$, $\min\{D_0^d, D_0^{nd}\} > 1-\beta$.

Finally, substituting the values for D_0^d, and D_0^{nd} in the income expressions in the non default and default case, we get

$$\begin{aligned} EY((D_0^{nd})) &= (1-\pi)(\overline{x}+\rho D_0^{nd})+\pi\left[\overline{x}+\rho(1-\beta)\right]-D_0^{nd}, && (52)\\ E(Y(D_0^d)) &= (1-\pi)(\overline{x}+\rho D_0^d)+\pi\left(1-\gamma\right)\left[\overline{x}+\rho(1-\beta)\right]-D_0^d. && (53)\end{aligned}$$

from which we can define

$$\begin{aligned} \Delta\left(D_0^d, D_0^{nd}\right) &\equiv E\left(Y\left(D_0^d\right)\right)-E\left(Y(D_0^{nd})\right) && (54)\\ &= 1-(1-\pi)\overline{x}\gamma(\rho-1)-(1-\pi)(1-(1-\beta)\gamma(\rho-1))\rho && (55)\end{aligned}$$

which is linear in $\overline{x}$.

Trivially, for $x = x_2^B$, we have that $D_0^d = D_0^{nd} = 1$, and $\Delta\left(D_0^d, D_0^{nd}\right) = \pi > 0$. On the other hand, for π small enough

$$\begin{aligned} \lim_{x\to\widetilde{x}}\Delta\left(D_0^d, D_0^{nd}\right) &= \lim_{x\to\widetilde{x}}\Delta\left(1, D_0^{nd}\right) && (56)\\ &= \rho(\pi-(1-\pi)\beta\gamma(\rho-1)) < 0. \end{aligned}$$

Then, it follows that there is a unique value of $\overline{x}_3 \in [\widetilde{x}, x_2^B]$ such that $x_3^B \leq \overline{x} \leq x_2^B \iff E(Y(D_0^d)) - E(Y(D_0^{nd})) < 0$. It is easy to verify that $x_3^B = \frac{1}{\gamma}\left(\frac{\rho-\rho\gamma-1}{(\rho-1)(1-\pi)}\right) - (1-\beta)\rho$, so that in the interval $[\overline{x}_3, \overline{x}_2]$ the country chooses to borrow $K^{3B} = D_0^{nd} = \gamma\left(\overline{x}+(1-\beta)\rho\right)$ and default is avoided. Income is given by

$$E^{3B}(Y) = (1-\pi)\left(\overline{x}+\rho D_0^{nd}\right)+\pi(\overline{x}+\rho(1-\beta)\}-D_0^{nd}. \tag{57}$$

7.1.4 Case 4B: Lower Middle–Income countries (LMIC): procyclical policies, default $\left(x_3^B > \overline{x} \geq x_4^B \equiv \frac{(\rho-1)(1-(1-\pi)\gamma\rho)}{(1-\pi)(\rho-1)\gamma-\pi}(1-\beta)\right)$

From the previous proof, it follows that for $\widetilde{x} \leq \overline{x} \leq x_3^B$, $E(Y(D_0^d)) - E(Y(D_0^{nd})) > 0$, and $K_0 = D_0^d = 1$.

Consider now the interval $[\widetilde{x}; \overline{x}_4]$ for which:

$$D_0^d = \frac{(1-\pi)}{1-(1-\pi)\gamma\rho}\gamma\overline{x} < 1,$$

First of all let's verify that also in this interval $\min\{D_0^d, D_0^{nd}\} > 1-\beta$. Indeed, we can define:

$$\begin{aligned} x_a^d &= x : D_0^d = (1-\beta) = (1-\beta)\left(\frac{1}{(1-\pi)\gamma}-\rho\right), \text{and } D_0^d \geq (1-\beta) \text{ if } x > x_a^d;\\ x_a^{nd} &= x : D_0^{nd} = (1-\beta) = \frac{(1-\beta)(1-\gamma\rho)}{\gamma}; \text{and } D_0^{nd} \geq (1-\beta) \text{ if } x > x_a^{nd}; \end{aligned}$$

from which we have that

$$x_a^d - x_a^{nd} = \frac{\pi(1-\beta)}{(1-\pi)\gamma} > 0$$

and, in turn, that in the interval $[x_4^B, x_3^B]$, $x_4^B > x_a^{nd} \Longrightarrow \min\{D_0^d, D_0^{nd}\} > 1-\beta$. Indeed,

$$x_4^B - x_a^{nd} = \frac{\pi(1-\beta)(1-(1-\pi)\gamma\rho}{(1-\pi)\gamma((1-\pi)\gamma\rho-(1-\pi)\gamma-\gamma)} > 0 \Longleftrightarrow \pi < 1 - \frac{1}{\gamma\rho}$$

which is always verified for π small enough. We can now substitute the values value into D_0^d, and D_0^{nd} in (52) and (53) to get

$$\begin{aligned} \Delta\left(D_0^d, D_0^{nd}\right) &= \frac{(1-\pi)\,\gamma\rho\left((1-\beta-\gamma\left(x+\rho(1-\beta)\right)(\rho-1)\right)+\pi\left((1-\beta)\,\gamma\rho-1)\rho+x\left(1-\gamma(1-\rho)\right)\right)}{1-(1-p)\,\gamma\rho} \geq \\ &\Longleftrightarrow \quad \overline{x} \geq x_4^B \equiv \frac{(\rho-1)(1-\beta)(1-(1-\pi)\gamma\rho)}{(1-\pi)(\rho-1)\gamma-\pi}. \end{aligned}$$

In sum, for $\overline{x} \in [x_4^B, x_3^B]$, $\Delta\left(D_0^d, D_0^{nd}\right) > 0$, and the country chooses to borrow $K_0^{4B} = D_0^d = \min\{\frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}; 1\}$ in period 0, and defaults if hit by a shock in period 1. In this case, income is given by

$$E^{4B}(Y) = (1-\pi)(\overline{x}+\rho D_0^d) + \pi\left(1-\gamma\right)\left(\overline{x}+\rho(1-\beta)\right) - D_0^d. \tag{58}$$

7.1.5 Case 5B: Low Income countries (LIC) $\left(x_4^B > \overline{x} \geq 0\right)$

Finally, it is easy to check that for $x < x_4^B$, $K_0^{5B} = D_0^{nd} = \gamma(\overline{x}+\rho(1-\beta))$ and the country does not default.

In this case, expected income is given by, in case 3B, by

$$E^{5B}(Y) = (1-\pi)\left(\overline{x}+\rho D_0^{nd}\right) + \pi(\overline{x}+\rho\min\{(1-\beta), D_0^{nd}\} - D_0^{nd}. \tag{59}$$

7.2 Insurance

7.2.1 Case 2I $\left(\overline{x}_1 > \overline{x} \geq \overline{x}_2^{IN} \equiv \frac{1-\gamma\rho+\pi\beta\nu}{\gamma}\right)$

As in case $2B$, the borrowing constraint determines period 1 borrowing. However, unlike in the benchmark, insurance plays a complementary role by increasing the collateral and relaxing the constraint. Specifically, we can write the country's problem at time 0 in terms of D_0, D_1 and Z as:

$$\max_{D_0,Z,D_1} E\left[Y(D_0,Z,D_1\right] = \overline{x}+((1-\pi)\rho-1)\,D_0+\pi\rho(1-\beta)+\pi(\rho-1)D_1-\pi\left(\nu-\rho\right)Z \tag{60}$$

subject to the borrowing constraint

$$(D_0+\pi\nu Z)+D_1 \leq \gamma\left[\overline{x}+\rho\left(1-\beta\right)+Z+D_1)\right] \tag{61}$$

from which

$$D_1 \leq D_1^{nd}(Z, D_0) \equiv \frac{\gamma(\overline{x} + \rho(1-\beta)) - D_0 + (\gamma\rho - \pi\nu) Z}{(1-\gamma\rho)}, \tag{62}$$

with $\frac{\partial D_1^{nd}}{\partial Z} = \frac{\gamma\rho - \pi\nu}{1-\gamma\rho} \geq 0$.

Substituting (62) into (60), we have that, for any given Z, and for small π,

$$\pi < \frac{(\rho-1)(1-\gamma\rho)}{\rho(1-\gamma\rho) + (\rho-1)}, \tag{63}$$

$$\frac{\partial E(Y)}{\partial D_0} = (1-\pi)\rho - 1 - \frac{\pi(\rho-1)}{(1-\gamma\rho)} > 0.$$

which implies that the country maximizes period 0 investment and tells us, in particular, that $D_1^{nd} > 0 \Longrightarrow D_0 = 1$.

Consider the case in which endowment $\overline{x}$ is large enough to allow the country to borrow $D_0 = 1$. Differentiating (60) with respect to Z we obtain:

$$\frac{\partial E(Y)}{\partial Z} = \pi\left(-(\nu-\rho) + (\rho-1)\frac{\gamma\rho - \pi\nu}{1-\gamma\rho}\right) > 0 \tag{64}$$

where the positive sign comes from (20), so that the country purchases insurance subject to

$$Z \leq Z^{nd} \equiv \beta - D_1^{nd}. \tag{65}$$

Finally, substituting (65) into (62) it is easy to verify that:

$$\begin{aligned} D_1^{nd}\left(\widehat{Z}, 1\right) &= \frac{\gamma(\overline{x}+\rho) - (1+\pi\nu\beta)}{1-\pi\nu} \geq 0 \\ &\Longleftrightarrow \quad \overline{x} \geq \frac{1-\gamma\rho+\pi\nu\beta}{\gamma} \equiv \overline{x}_2^{IN} \end{aligned} \tag{66}$$

and that

$$Z^{nd} = \frac{1+\beta-\gamma(\overline{x}+\rho)}{(1-\pi\nu)} \geq 0 \Longleftrightarrow \overline{x} \leq \frac{1+\beta-\gamma\rho}{\gamma} = \overline{x}_1 \tag{67}$$

so that in the interval $\left[\overline{x}_2^{IN}; \overline{x}_1\right]$, $D_0 = 1$, $D_1^{nd} > 0$ and $Z^{nd} > 0$.

7.2.2 Case 3I $\left(\overline{x}_2^{IN} > \overline{x} \geq \overline{x}_3^{IN} \equiv \frac{(1-\beta)(1-\gamma\rho)}{\gamma}\right)$

If $\overline{x} < \overline{x}_2^{IN}$, the country fully exhausts its access to capital in period 0, so the "catalytic" effect of insurance is reflected directly in the borrowing constraint determining access in period 0. Specifically, insurance increases income in period 1 (the value at stake in bad states), at a cost proportional to the likelihood of the insured event, relaxing the borrowing constraint that now becomes

$$(D_0 + \pi\nu Z) \leq \gamma(\overline{x} + \rho\min\{D_0, (1-\beta) + Z\}), \tag{68}$$

or

$$D_0 \leq D_0^{nd} \equiv \gamma\left[\overline{x}+\rho\left(1-\beta\right)\right]+\left(\gamma\rho-\pi\nu\right)Z. \tag{69}$$

Substituting D_0^{nd} in the objective function

$$E\left(Y\left(D_0^{nd},Z\right)\right)=\overline{x}+\left(\left(1-\pi\right)\rho-1\right)D_0^{nd}+\pi\rho(1-\beta)-\pi\left(\nu-\rho\right)Z$$

we have:

$$\frac{\partial E(Y)}{\partial Z}\geq 0 \Longleftrightarrow \gamma\rho-\pi\nu>\frac{\gamma-\pi}{1-\pi}. \tag{70}$$

which is trivially verified for a small enough π.

This in turn implies that the country always purchase insurance subject to

$$Z\leq Z^{nd}=D_0-(1-\beta),$$

where Z^{nd} denotes the point at which insurance funds allow the country to rebuild infrastructure to the level $L=K$.

Finally, substituting Z^{nd} in (**??**) we have that

$$Z^{nd}\geq 0 \Longleftrightarrow D_0=\frac{\overline{x}\gamma+(1-\beta)\pi\nu}{1-\gamma\rho+\pi\nu}\geq(1-\beta) \Longleftrightarrow \overline{x}\geq\overline{x}_3^{IN}\equiv\frac{(1-\beta)(1-\gamma\rho)}{\gamma}$$

In this interval the country has the option to borrow D_0^d, such that $1\geq D_0^d>D_0^{nd}$, at a risk-adjusted rate, and default in bad states. If so, the borrowing constraint (5) becomes:

$$\frac{1}{1-\pi}\left(D_0+\pi\nu Z\right)\leq\gamma(\overline{x}+\rho D_0) \tag{71}$$

from which

$$D_0\leq D_0^d\equiv\min\left\{\frac{\left(1-\pi\right)\gamma\overline{x}-\pi\nu Z}{1-\left(1-\pi\right)\gamma\rho};1\right\}, \tag{72}$$

and expected income modifies to

$$E\left(Y(D_0^d,Z)\right)=(1-\gamma\pi)\overline{x}+\left(1-\pi\right)\left[\rho-\frac{1}{\left(1-\pi\right)}\right]D^d+\pi\left[\rho(1-\gamma)(1-\beta)-\left[\nu-\rho(1-\gamma)\right]Z\right)\right].$$

The first thing to stress is that no insurance is purchased if default is anticipated. To see that, note that, for $D_0^d=1$,

$$\frac{\partial E((Y(1,Z))}{\partial Z}=\pi\left[\nu-\rho(1-\gamma)\right]<0 \Longleftrightarrow \nu>\rho(1-\gamma), \tag{73}$$

whereas for $D_0^d<1$,

$$\frac{\partial E((Y(D_0^d,Z))}{\partial Z}=\pi\left[\rho(1-\gamma)-\nu\right]-\pi\nu\frac{\left[\rho\left(1-\pi\right)-1\right]}{1-\left(1-\pi\right)\gamma\rho}<0 \Longleftrightarrow \nu>\frac{1}{\left(1-\pi\right)}-\gamma\rho. \tag{74}$$

From (20) and (6), we know that $\upsilon>\rho>\frac{1}{(1-\pi)}$, so both conditions hold.

Then, substituting $Z = 0$ into (72),

$$D_0^d = 1 \Longleftrightarrow \overline{x} \geq \widetilde{x} \equiv \frac{1 - (1-\pi)\gamma\rho}{(1-\pi)\gamma} \tag{75}$$

Also, for the default option to be the equilibrium we need that

$$D_0^d = \min\left\{\frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}; 1\right\} > D_0^{nd} = \frac{\gamma\overline{x} + (1-\beta)\pi\nu}{1-\gamma\rho+\pi\nu} > \frac{\gamma\overline{x}}{1-\gamma\rho+\pi\nu},$$

which in turn implies that

$$\left.\frac{\partial D_0^d}{\partial \overline{x}}\right|_{\overline{x}<\widetilde{x}} = \frac{(1-\pi)\gamma}{1-(1-\pi)\gamma\rho} > \frac{\partial D_0^{nd}}{\partial \overline{x}} = \frac{\gamma\overline{x}}{1-\gamma\rho+\pi\nu} > \left.\frac{\partial D_0^d}{\partial \overline{x}}\right|_{\overline{x}=\widetilde{x}} = 0.$$

Therefore, to show that the country never chooses the default option, it suffices to show that it is so for $\overline{x} = \widetilde{x}$, the point at which the additional borrowing in period 0 relative to the non-default case is maximized.

Finally,

$$D_0^d(\overline{x} = \widetilde{x}) = 1 > D_0^{nd} = \frac{\gamma\widetilde{x} + (1-\beta)\pi\nu}{1-\gamma\rho+\pi\nu}.$$

which would imply

$$\nu > \frac{1}{(1-\pi)\beta},$$

which is vever verified for small π, since $\nu < \frac{1}{\beta}$

7.2.3 Case 4I $\left(\overline{x}_3^{IN} > \overline{x} \geq 0\right)$

If $\overline{x} < \overline{x}_3$, the country does not purchase insurance and we are back in the benchmark case. To ensure that no default occurs, it is sufficient to show that

$$\overline{x}_3^{IN} - \overline{x}_4 = \frac{\pi(1-\beta)(1-\gamma)}{\gamma\left(\gamma\rho(1-\pi) + \pi\gamma - \pi - \gamma\right)} > 0$$

which is always the case for sufficiently small values of π.

7.3 Catastrophe lending facility

As before, for high initial incomes ($\overline{x} \geq \overline{x}_1^M$), the borrowing constraint (30) does not bind and the country invests the optimum in both states. In turn, for $\overline{x}_1^M > \overline{x}$,

$$D_0^{nd}(M) = \frac{\gamma(\overline{x} + 1 - \beta)}{1-\gamma(\rho-1)} \leq 1, \tag{76}$$

and

$$M = D_0^{nd} - (1-\beta) = \frac{\gamma\overline{x} - (1-\gamma\rho)(1-\beta)}{1-\gamma(\rho-1)} > 0 \Longleftrightarrow \overline{x} > \overline{x}_4^M \equiv \frac{(1-\gamma\rho)(1-\beta)}{\gamma} = \overline{x}_3^{IN}, \tag{77}$$

so that, using (30), we know that $\pi < \frac{\gamma}{\nu} \Longrightarrow D_0^{nd}(M) = \frac{\gamma[\overline{x}+1-\beta]}{1-\gamma(\rho-1)} < D_0^{nd} = \frac{\gamma\overline{x}+\pi\nu(1-\beta)}{1-(\gamma\rho-\pi\nu)} \Longrightarrow M = \frac{\gamma\overline{x}-(1-\gamma\rho)(1-\beta)}{1-\gamma(\rho-1)} < Z = \frac{\gamma\overline{x}-(1-\gamma\rho)(1-\beta)}{1-(\gamma\rho-\pi\nu)}$.

Alternatively, a financially constrained country may choose to increase borrowing from private lenders (at a risk-adjusted rate $i = \frac{1}{1-\pi}$) at the expense of defaulting on bonds and if hit by a shock. However, unlike in the benchmark, now the country would still have access to multilateral lending in period 2. In this scenario, period 0 borrowing D_0^d should be such that the country repays in good states:

$$D_0^d \leq (1-\pi)\,\gamma(\overline{x}+\rho D_0^d) \tag{78}$$

from which

$$D_0^d = \frac{(1-\pi)\,\gamma\overline{x}}{1-(1-\pi)\,\gamma\rho} \leq 1 \Longleftrightarrow \overline{x} \leq \widetilde{x}_2^M \equiv \frac{1-(1-\pi)\,\gamma\rho}{(1-\pi)\,\gamma}, \tag{79}$$

with $\widetilde{x}_2^M \leq \overline{x}_1^M$ for small π, and

$$\begin{aligned} M^d &= D_0^d - (1-\beta) = \frac{(1-\pi)\,\gamma\overline{x}}{1-(1-\pi)\,\gamma\rho} - (1-\beta) \geq 0 \\ \Longleftrightarrow &\quad \overline{x} \geq \widetilde{x}_3^M \equiv (1-\beta)\left[\frac{\frac{1}{(1-\pi)}-\gamma\rho}{\gamma}\right] > \overline{x}_4^M \end{aligned}$$

From (32) and (33), we know that the condition for a default on bonds is linear in $\overline{x}$:

$$\begin{aligned} E(Y(D_0^d, M^d) &= (1-\pi)\left(\overline{x}+\rho D_0^d\right) + \pi\,(1-\gamma)\left[\overline{x}+\rho(1-\beta+M^d)-M^d\right] - D_0^d \\ &> (1-\pi)\left(\overline{x}+\rho D_0^{nd}\right) + \pi\left[\overline{x}+\rho(1-\beta+M^{nd})-M^{nd}\right] - D_0^{nd} = E(Y(D^{nd}, M^{nd}) \end{aligned} \tag{80}$$

and can be written as[26]

$$\Delta\left(D_0^d, D_0^{nd}, M^d, M^{nd}\right) = D_0^d - D_0^{nd} - \frac{\pi}{(1-\pi)\rho-1}\left\{\gamma\left[\overline{x}+\rho(1-\beta)\right] - (\rho-1)\left[(1-\gamma)\,M^d - M^{nd}\right]\right\}. \tag{81}$$

As before, we can distinguish three intervals:

- $\left[\widetilde{x}_2^M, \overline{x}_1^M\right]$ where $D_0^d = 1 > D_0^{nd} = \frac{\gamma[\overline{x}+1-\beta]}{1-\gamma(\rho-1)}$,
- $\left[\widetilde{x}_3^M, \widetilde{x}_2^M\right]$ where $D_0^d = \frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}$; $D_0^{nd} = \frac{\gamma[\overline{x}+1-\beta]}{1-\gamma(\rho-1)} < 1$, $M^{nd} > 0$; $M^d > 0$,

[26] Comparing (??) with the "default" condition under the benchmark:

$$\widetilde{D}_0 - \widehat{D}_0 > \pi\gamma\frac{\overline{x}+\rho(1-\beta)}{(1-\pi)\rho-1},$$

note the country now has access in period 2 ($M > 0$) in both cases, but borrowing is higher in the default case: due to the complementarities in the two types of capital, by borrowing more today, we increase optimal borrowing tomorrow. On the other hand, default costs detract from this expected income. The net effect, proportional to $(1-\gamma)\,\widetilde{M} - \widehat{M} = \left[\left(\widetilde{D}_0 - \widehat{D}_0\right) - \gamma\left(\widetilde{D}_0 - (1-\beta)\right)\right]$, is a priori ambiguous.

- $\left[\overline{x}_4^M, \widetilde{x}_3^M\right]$ where $D_0^d = \frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho} < 1-\beta < D_0^{nd} = \frac{\gamma[\overline{x}+1-\beta]}{1-\gamma(\rho-1)}$, and $M^{nd} > M^d = 0$, and no default occurs

Given the linearity of (81), to characterize the equilibrium it suffices to check the thresholds for the first two intervals.

Trivially, for $\overline{x} = \overline{x}_1^M$, (81) does not hold and no default occurs, since $D_0^d = D_0^{nd} = 1$. In turn, for $\overline{x} = \widetilde{x}_2^M$, $D_0^d = 1 > D_0^{nd}$ implies that (81) always holds for small enough π. Finally, for $\overline{x} = \widetilde{x}_3^M = (1-\beta)\left[\frac{\frac{1}{(1-\pi)} - \gamma\rho}{\gamma}\right]$,

$$D_0^d\left(\widetilde{x}_3^M\right) = 1-\beta < \left[\frac{1}{(1-\pi)} + \rho\left(1-\gamma\right)\right]\left(1-\beta\right) = D_0^{nd}\left(\widetilde{x}_3^M\right) \tag{82}$$

and, again, the country does not default.

It follows that there is an interval $\left[\overline{x}_3^M, \overline{x}_2^M\right]$, such that $\overline{x}_1^M > \overline{x}_2^M > \widetilde{x}_2^M > \overline{x}_3^M > \widetilde{x}_3^M > \overline{x}_4^M$, within which the country borrows $D_0^d = \min\left\{\frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}, 1\right\}$ in period 0, and, if hit by the shock, borrows $M^d = D_0^d - (1-\beta)$ from the contingent loan credit and defaults.

The thresholds for the default interval are obtained directly from (81) as:

$$\overline{x}_2^M \equiv \overline{x} : \Delta\left(1, D_0^{nd}, \beta, M^{nd}\right) = 0 = \frac{(\rho-1)-\pi}{\gamma\left(1-\pi\gamma\right)\left(\rho-1\right)} - \left(\rho-\beta\right) \tag{83}$$

and

$$\overline{x}_3^M \equiv \overline{x} : \Delta\left(D_0^d, D_0^{nd}, M^d, M^{nd}\right) = 0 = \frac{\left(1-\beta\right)\left(1-\pi\gamma\right)\left(\rho-1\right)\left(1-\left(1-\pi p\right)\gamma\rho\right)}{\pi^2\left(1+\gamma^2\left(\rho-1\right)\right) + \gamma\left(\rho-1\right) - \pi\left(\gamma^2\left(\rho-1\right)+\rho\right)}. \tag{84}$$

7.4 A contingent catastrophe lending facility

The model can be readily modified to represent this case: we simply need to note that no multilateral assistance is forthcoming in the event of default ($M^d = 0$), which tilts the balance against the default decision: default, while still possible, is associated with narrower interval. The appendix provides a derivation of the new intervals.[27] Figure...illustrates this new case.

Under this new assumption, $M^d = 0$, and the borrowing constraint (30) becomes

$$\Delta\left(D_0^d, D_0^{nd}, M^d, M^{nd}\right) = D_0^d - D_0^{nd} - \frac{\pi}{(1-\pi)\rho - 1}\left\{\gamma\left[\overline{x} + \rho(1-\beta)\right] + \left(\rho-1\right)M^{nd}\right\}. \tag{85}$$

[27] Naturally, the multilateral lender could in principle condition the disbursements on the country's being current on its bonded debt. It is easy to verify that, in this case, default conditions tighten even further, as now the multilateral loan "rewards" the well-behaved (see appendix). However, such conditionality would introduce an additional source of uncertainty, in terms of access as well as the timing of the assistance, rendering the arrangement closer to standard multilateral loans and farther away from the concept of a standing facility.

It is immediate to verify that, for $\overline{x} = \widetilde{x}_3^M$, $D_0^d = 1$,

$$D_0^{nd}\Big|_{\overline{x}=\widetilde{x}_2^M} = \frac{(1-\gamma\rho+\gamma(1-\beta))}{1-\gamma(\rho-1)} < 1$$

and

$$\Delta\left(1, D_0^{nd}, M^{nd}\right) = 1 - D_0^{nd} - \frac{\pi}{(1-\pi)\rho-1}\left\{\gamma\left[\overline{x}+\rho(1-\beta)\right]+(\rho-1)M^{nd}\right\} > 0 \tag{86}$$

for small π. On the other hand, for $\overline{x} = \widetilde{x}_3^M$, $M^d = 0$ and we are back in the previous case, where from (82) we know that the country chooses to borrow less and avoid default.

Thus, following the steps of the previous proof, it can be shown that there is an interval $\left[\overline{x}_2^{M'}, \overline{x}_3^{M'}\right]$ such that $\overline{x}_1^M > \overline{x}_2^{M'} > \widetilde{x}_2^M > \overline{x}_3^{M'} > \widetilde{x}_3^M > \overline{x}_4^M$, within which the country borrows $D_0^d = \min\left\{\frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho}, 1\right\}$ in period 0, and, if hit by the shock, defaults.

The thresholds of this interval are defined by the zeros of

$$\Delta\left(1, D_0^{nd}, M^{nd}\right) = 1 - \frac{\gamma(\overline{x}+1-\beta)}{1-\gamma(\rho-1)}\left[1+\frac{\pi}{(1-\pi)\rho-1}(\rho-1)\right] \tag{87}$$

$$-\frac{\pi}{(1-\pi)\rho-1}\left\{\gamma\left[\overline{x}+\rho(1-\beta)\right]-(\rho-1)(1-\beta)\right\} \tag{88}$$

and

$$\Delta\left(D_0^d, D_0^{nd}, M^{nd}\right) = \frac{(1-\pi)\gamma\overline{x}}{1-(1-\pi)\gamma\rho} - \frac{\gamma(\overline{x}+1-\beta)}{1-\gamma(\rho-1)}\left[1+\frac{\pi}{(1-\pi)\rho-1}(\rho-1)\right] \tag{89}$$

$$-\frac{\pi}{(1-\pi)\rho-1}\left\{\gamma\left[\overline{x}+\rho(1-\beta)\right]-(\rho-1)(1-\beta)\right\}, \tag{90}$$

from which $\overline{x}_2^{M'} = \frac{(\rho-1)(1+\beta\gamma-\gamma\rho)-\pi(1+(\rho-1)(\beta-(\beta(1-\gamma)+\gamma)\gamma\rho))}{\gamma(\rho-1)(1-\pi\gamma)}$, and $x_3^{M'} = \frac{(\rho-1)(1-\beta)(1-(1-\pi)\gamma\rho)(\gamma+\pi(1-\gamma)\gamma\rho-\pi)}{\gamma(\pi+\gamma(\rho-1)-\pi(2-\gamma)(1+\gamma)\rho+\pi^2(1-(1-\gamma)\gamma(\rho-1))\rho+\pi(1-\gamma)\gamma\rho^2)}$.

7.5 Catastrophe lending and insurance

For $x \in [\overline{x}_1^M, \overline{x}_2^{IN}]$, the country minimizing the amount of (costly) insurance such that it still attains $L_0 = 1$, and $L_1 = \beta$. The borrowing constraint (4) then becomes:

$$(D_0 + \pi\nu Z) \leq \gamma(\overline{x} + \rho((1-\beta) + Z + M)) - \gamma M, \tag{91}$$

which, substituting,

$$M = D_0 - (1-\beta) - Z$$

yields

$$D_0 \leq \frac{(\overline{x}+1-\beta+Z)\gamma - \pi\nu Z}{1-\gamma(\rho-1)} \equiv \widetilde{D}_0^{MIN} \tag{92}$$

Finally, we have that

$$\widetilde{D}_0^{MIN} \geq 1 \Longleftrightarrow Z \geq \widetilde{Z}^{MIN} \equiv \frac{1-\gamma(\overline{x}-\beta-\rho)}{\gamma-\pi\nu},$$

and, in addition, for M to be non negative, we need

$$Z \leq \beta.$$

The two conditions are simultaneously verified for

$$\overline{x} \geq \frac{1+\pi\beta\nu-\gamma\rho}{\gamma} = \overline{x}_2^{IN}$$

Consider now the case $\overline{x} \leq \overline{x}_2^{IN}$. We can now substitute D_0 from (91) in the expression for expected income, so that

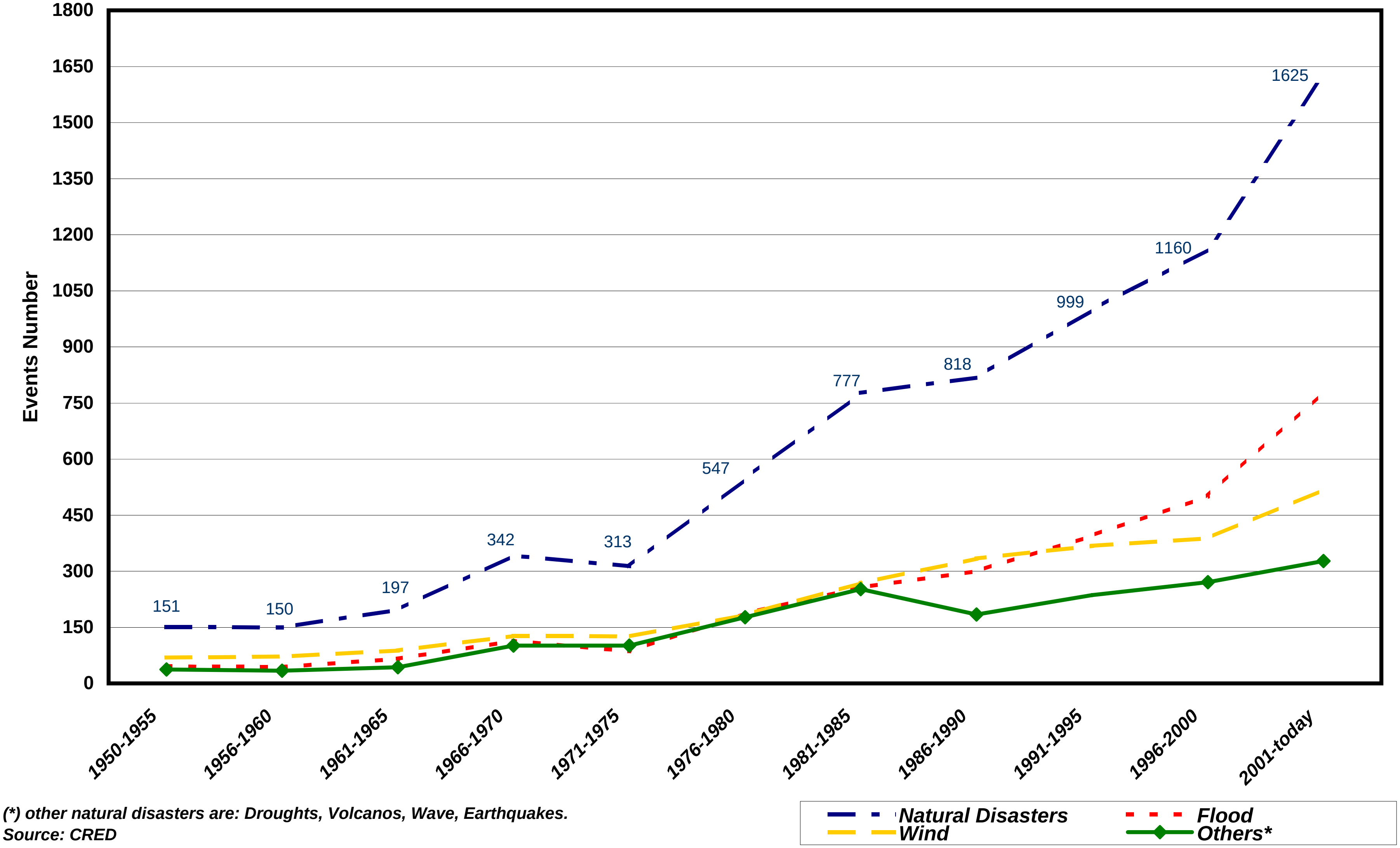
$$E(Y) = (1-\pi)\left(\overline{x}+\rho D_0^{bl}\right) + \pi(\overline{x}+\rho(1-\beta+M+Z)-M) - D_0^{bl} - \pi\nu Z.$$

and, differentiating with respect to Z, we get

$$\frac{\partial E(Y)}{\partial Z} = \frac{(\pi+\gamma-\pi\,(1-\pi-\gamma)\,\nu)\,\rho-\gamma(1+\pi\nu+\pi\rho^2)}{1-\gamma\,(\rho-1)},$$

which is always positive for small enough π. Thus for $\overline{x} \leq \overline{x}_2^{IN}$ we are back to the insurance case discussed previously.

Figure 1
Natural Disasters in the last fifty years



(*) other natural disasters are: Droughts, Volcanos, Wave, Earthquakes.
Source: CRED

TABLE 1

Table 1.a: Major Disasters in the last forty years - per damage (absolute values)

Year	Natural Disaster	Country	Region	Damage (mil. USD)	Damage/GDP*
2005	Hurricane (Katrina)	United States	North America	125,000,000.00	1.1%
1995	Earthquake	Japan	East Asia	100,000,000.00	3.2%
1998	Flood	China	East Asia	30,000,000.00	0.7%
2004	Earthquake	Japan	East Asia	28,000,000.00	0.8%
1992	Hurricane (Andrew)	United States	North America	26,500,000.00	0.4%

(*) GDP constant 2000, USD

Sources: CRED, World Bank

Table 1.b: Major Disasters in the last forty years - per damage/GDP - Small Islands Economies

Year	Natural Disaster	Country	Region	Damage (mil. USD)	Damage/GDP*
1988	Hurricane (Gilbert)	St Lucia	Caribbean	1,000,000.00	204%
2004	Hurricane (Ivan)	Grenada	Caribbean	889,000.00	114%
1991	Cyclone (Val & Wasa)	Samoa	Oceania	278,000.00	61%
1995	Hurricane (Luis)	St Kitts and Nevis	Caribbean	197,000.00	52%
1995	Hurricane (Luis)	Antigua and Barbuda	Caribbean	300,000.00	49%

(*) GDP constant 2000, USD

Sources: CRED, World Bank

Table 1.c: Major Disasters in the last forty years - per damage/GDP - Developing Countries with population > 5 milions

Year	Natural Disaster	Country	Region	Damage (mil. USD)	Damage/GDP*
1998	Hurricane (Mitch)	Honduras	Central America	3,793,600.00	24%
1981	Drought	Zimbabwe	East Africa	2,500,000.00	12%
2001	Earthquake	El Salvador	Central America	1,500,000.00	5%
1980	Earthquake	Algeria	North Africa	5,200,000.00	5%
1987	Earthquake	Ecuador	South America	1,500,000.00	5%

(*) GDP constant 2000, USD

Sources: CRED, World Bank

Figure 2

Figure 2a: Benchmark Case

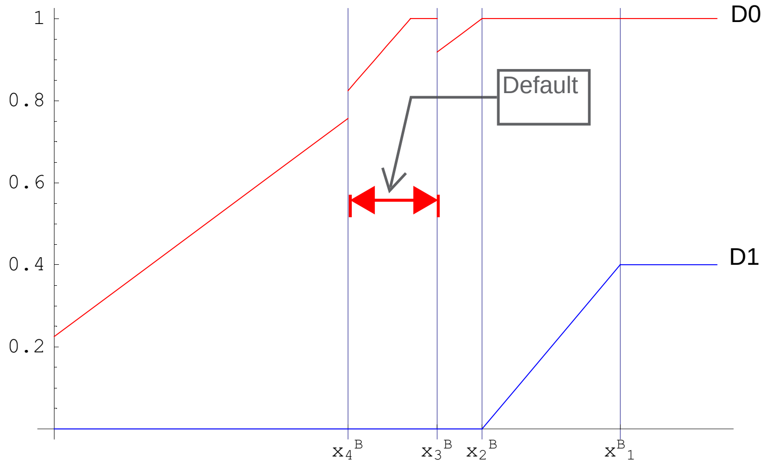


Figure 2b: Insurance Case

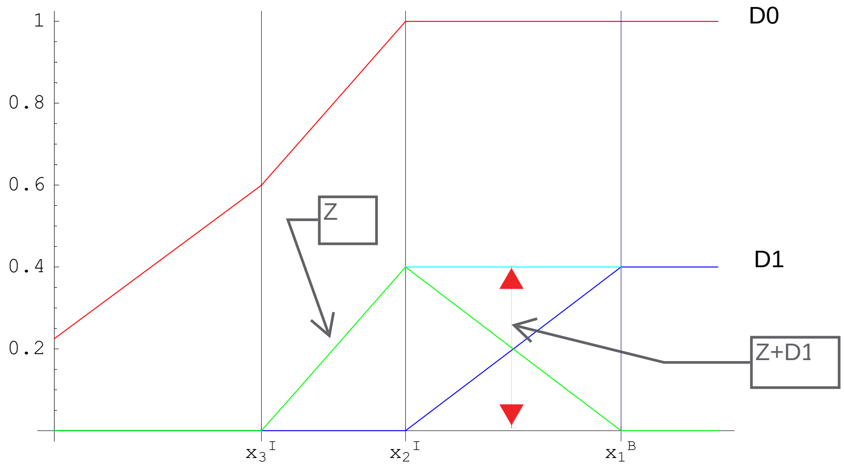


Figure 2c: Multilateral Lending

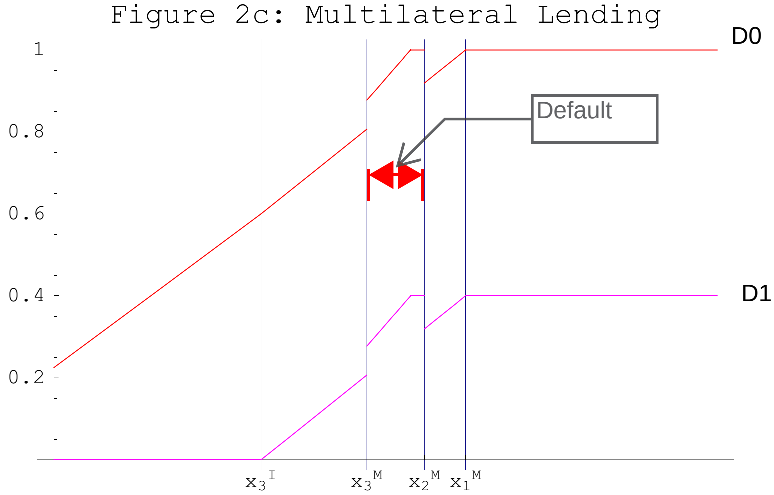


Figure 3

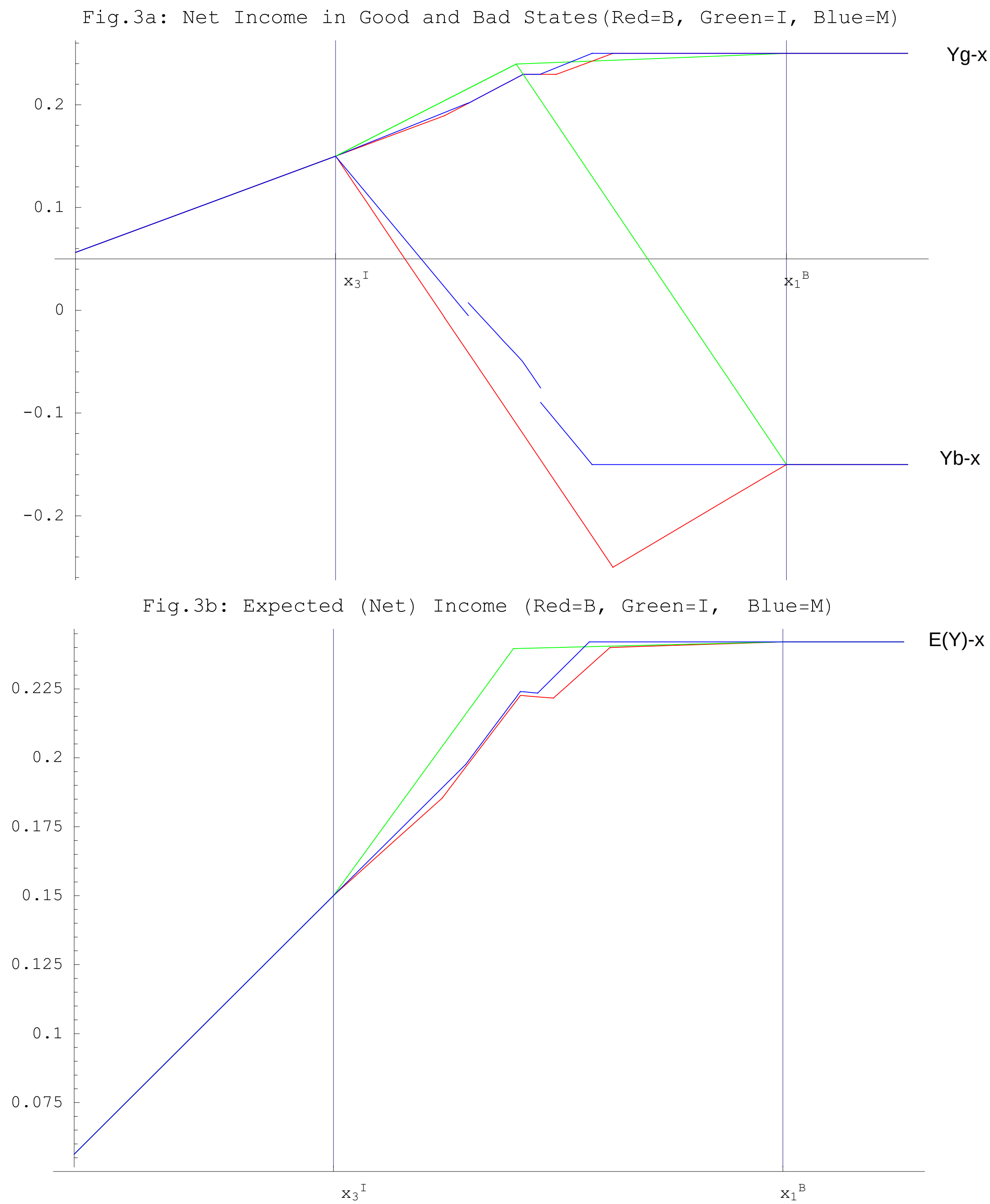


Fig.3a: Net Income in Good and Bad States(Red=B, Green=I, Blue=M)
Yg-x
Yb-x
0.2
0.1
0
-0.1
-0.2
x3I
x1B
Fig.3b: Expected (Net) Income (Red=B, Green=I, Blue=M)
E(Y)-x
0.225
0.2
0.175
0.15
0.125
0.1
0.075
x3I
x1B

Figure 4

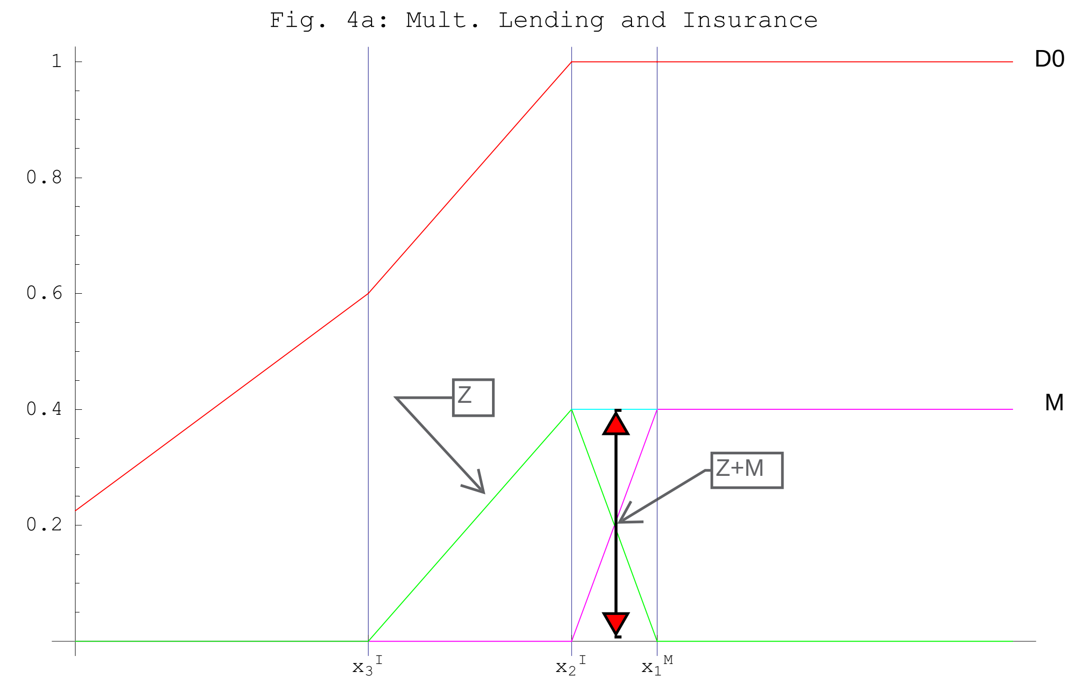


Fig. 4a: Mult. Lending and Insurance
1
0.8
0.6
0.4
0.2
D0
M
Z
Z+M
x_3^I
x_2^I
x_1^M

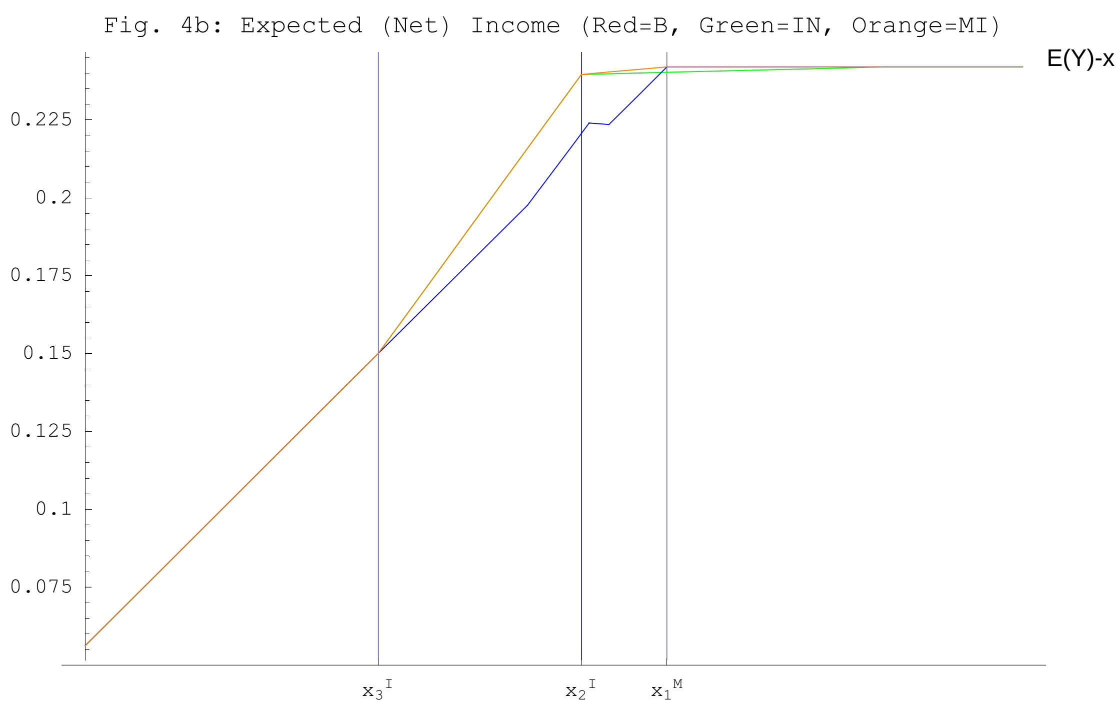


Fig. 4b: Expected (Net) Income (Red=B, Green=IN, Orange=MI)
0.225
0.2
0.175
0.15
0.125
0.1
0.075
E(Y)-x
x_3^I
x_2^I
x_1^M

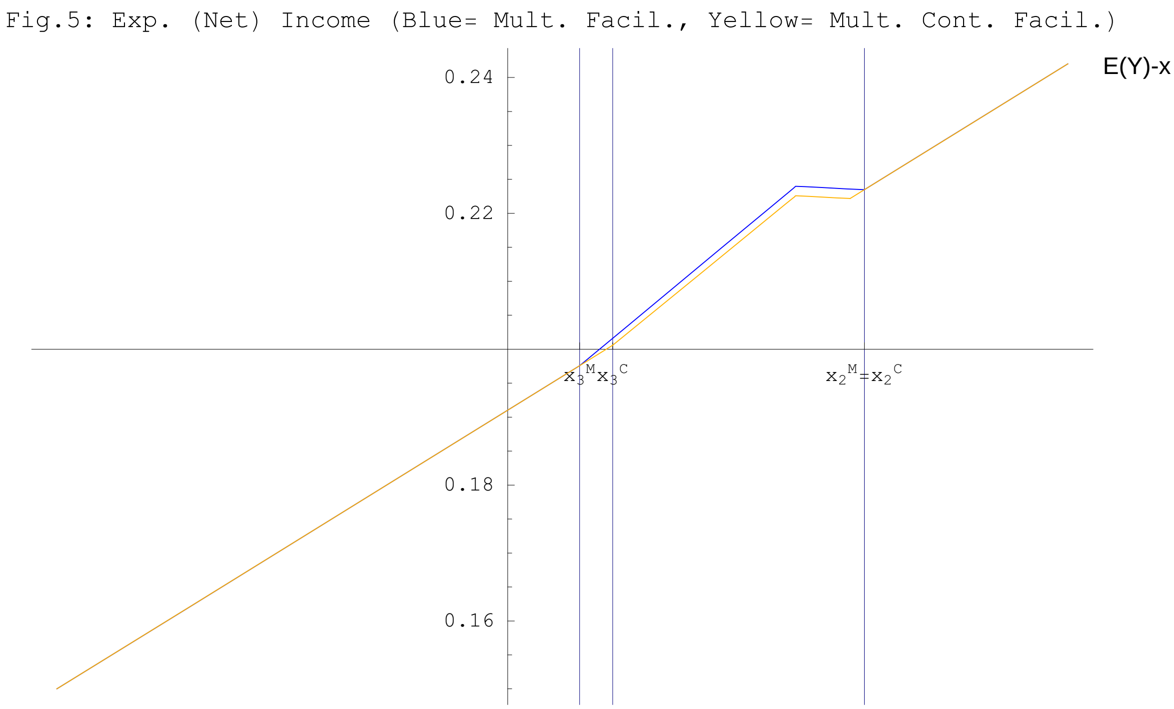


Fig.5: Exp. (Net) Income (Blue= Mult. Facil., Yellow= Mult. Cont. Facil.)
E(Y)-x
0.24
0.22
0.18
0.16
x3M x3C
x2M=x2C