An Evolutionary Theory of Monogamy*

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Abstract

This paper presents an evolutionary model to explain the widespread diffusion of lifelong monogamous families. The essential condition, unique to humans, is the overlap of children of different ages. Under this condition, together with the salience of paternal investment and fatherhood uncertainty, monogamy attains a greater survivorship than serial monogamy and polygyny. This result is robust to a number of extensions, including the presence of kin ties, resource inequality, group marriage, and the risk of adult mortality.

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1. Introduction

A large scientific literature shows that humans have formed and lived in a wide variety of sexual organizations, from monogamy to serial monogamy and polygyny. It is interesting that, among humans, long-term monogamous relationships are pervasive. In this paper we address the question of efficiency of family conventions formulating a game-theoretic evolutionary model in which individuals care only for the propagation of their own genes into future generations. Our main finding is that monogamy is the most efficient family configuration. The essential force behind this result, which is unique to humans, is the overlap of children of different ages.

To capture the effect of this force, we develop an overlapping generations model, in which paternal investment and uncertain paternity play also a key role. In the model, hunter-gatherers live for five periods, the first two as children, when they rely on parental investment, and the last three as adults. Opposite-sex adults mate and have children, all striving to maximize the expected number of their own offspring. Adult men allocate their resources into hunting, mate guarding, and casual (or extra-marital) sex. Adult women instead feed their progeny with the food they receive from men.\(^1\) The model fits not only primeval societies but also other environments in which women must rely on paternal provisioning to raise their offspring.

We characterize stationary symmetric pure strategy Nash equilibria for a range of family conventions. Life long monogamy is shown to be the most efficient family convention when compared to the other family types under analysis.\(^2\) To understand this result, consider for simplicity the comparison with serial monogamy. In a monogamous environment a man protects his paternity by guarding and giving food to the same woman during his entire adulthood. In the serially monogamous convention instead a man does not keep the same partner from one period to the next, and thus in a given period he has children to support from more than one woman. The efficiency difference between these two family types arises because mothers redistribute food among their offspring. If a woman receives food from two different men, there is a free-rider problem whereby each man realizes that his contribution to his children will be shared with other children who are most likely not his.\(^3\) Thus, every man will shift more of his resources toward extra-marital (casual) sex and consequently toward guarding, which are both socially wasteful activities. Life long monogamy is more efficient precisely because it is socially less wasteful and, by allowing individuals to channel more resources to child provisioning, it delivers a higher rate of

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\(^1\)All our results hold true even if we relax this full specialization assumption.

\(^2\)Greater efficiency here means that the monogamous family grows faster than the other family conventions.

\(^3\)Private underprovision of public goods is a standard result (Becker 1981/1991; Bergstrom, Blume, and Varian 1986). In Weiss and Willis’s (1985) model of divorce, the free-rider problem is between former spouses who cannot verify each others’ allocative decisions. In our model instead it is between unrelated men who cannot write down binding verifiable contracts.
population growth.

Similar considerations apply to the case of polygyny, where one man has multiple official partners to whom he provides food at the same time. Matched males have to defend their paternity claims from the harsh intrasexual competition of the unmatched men who spend their entire endowment on cheating, free riding on the food supplied by the matched males.

In one of the extensions of the model, we establish that, in equilibrium, altruistic ties (i.e., food transfers among siblings, which are efficiency enhancing) occur only in the context of monogamy and not in the context of the other family conventions. Monogamy is thus a family configuration in which an adult man values his siblings because they provide him with the assurance that some of his genes will survive into future generations. In another extension, we analyze the role played by the possibility of adult mortality. We find that, although it is possible that the risk of adult mortality weakens the efficiency advantage of lifelong monogamy, monogamy is less efficient only if we are willing to admit extremely high adult mortality rates.

The rest of the paper is organized as follows. Section 2 links our contribution to some of the most relevant studies in the literature. Section 3 develops our model of family formation. Focusing only on monogamy and serial monogamy, Section 4 establishes the existence of the two respective Nash equilibria, compares them, and analyzes their stability. Section 5 examines polygyny (including the case in which there is heterogeneity in male endowments) and extends the model to allow for kin ties among siblings. It also discusses group marriage as an alternative family structure, considers the presence of the risk of adult mortality, and provides an interpretation of conventions as social norms. Section 6 concludes. The proofs of the key results are in the Appendix. A thorough discussion of the main assumptions that underpin the model and the proofs of additional results are reported as supplementary material in the online appendix.

2. Related Literature

Since Becker’s pioneering works (1973, 1974, and 1981/1991), the family has become a prominent area of investigation among economists. Becker (1973, pp. 818–820) emphasizes the importance of own children as the explanation of why men and women live together in an environment in which there is complementarity of male and female time inputs in production. Because own children are important, the notion of uncertain paternity is implicitly called upon in order to justify why unions of several men to one or several women are uncommon. In Becker’s analysis, however, men are not allowed to respond to fatherhood uncertainty (for example with the introduction of implicit contracts or with guarding), and all non-monogamous unions are essentially assumed to be less efficient than monogamous partnerships.

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Somewhat closer to our approach is a set of studies that emphasize the importance of fatherhood uncertainty. These include Edlund and Korn (2002), Edlund (2006), Saint-Paul (2008), and Bethmann and Kvasnicka (2011). But, unlike ours, these papers rule out casual sex and mate guarding. Rather, they underline the explicit or implicit transfers that take place between a man and a woman engaged in sexual reproduction (as in the case of legal marriages, which are seen as a contractual form of establishing paternity presumption and custodial rights to the man), and examine their consequences in terms of, for example, the matching patterns in marriage markets and the dynamics of human capital accumulation, and in response to environmental changes that might have altered the demand for marriage (e.g., the introduction of oral contraceptives). By looking at the question of why humans started to form lifelong families, we exclude the possibility of binding commitments and enforceable contracts and, rather, concentrate on mate guarding as men’s strategic adaptation to casual sex and uncertain paternity.

Another strand of economic research explicitly incorporates biological considerations into individual or household behavior (Bergstrom 1995, 2007; Robson 2001; Cox 2007) as well as economic development and growth (Galor and Michalopoulos 2012; Ashraf and Galor 2013). Siow (1998) investigates how differential fecundity interacts with market structure to affect gender roles in monogamous societies. Alger and Weibull (2010) examine the strategic interactions between mutually altruistic siblings to assess the extent to which family ties may vary in relation to factors such as output variability and marginal returns to effort.

Becker (1973 and 1974) shows that polygyny can be explained by male wealth inequality and gender differences in the constraints on reproduction. Gould, Moav, and Simhon (2008) provide an alternative explanation based on the increasing relevance of female human capital. They argue that educated men increasingly value educated women for their ability to raise educated children, and this drives up the value of educated women to the point where educated men prefer one educated partner to multiple unskilled wives. In the absence of male and female heterogeneity, instead, we show the evolutionary advantage of monogamy relying on the interactions among the overlap of children, paternal investment, and uncertain paternity.

Diamond and Locay (1989) stress the role played by uncertain paternity in explaining kin ties. They note that men invest in their sisters’ children even at high paternity probabilities and that, in many societies, they invest in the children of both their official partner and their sisters. Like in our case, Diamond and Locay’s explanation is that a male values his sister’s children in part because they provide him with the assurance that some of his genes will survive into future generations. Our analysis emphasizes an even more fundamental link, that is, the link between older and younger siblings rather than that between an adult man and his sororal nephews and nieces.

5Cox (2007) provides an interesting survey of recent contributions.
We conclude this review by highlighting the salience of our three fundamental conditions — overlap of children, paternal investment, and uncertain paternity. Many have recognized the relationship between overlapping cohorts of children and the necessity of protracted parental care (e.g., Chapais 2008), but failed to emphasize parenting as an activity that requires parents to support multiple children of different ages at the same time. A prolonged overlap of needy children pushes up the demand for food. This underlies the salience of parental investment (Trivers 1972), something humans share with non-familial eusocial insects and the great apes. But chimpanzee mothers can provide for themselves and their infants, while primeval human mothers could not (Kaplan et al. 2000; Kaplan and Robson 2002). Here is our second key ingredient: women look to men in order to make up their energy (or resource) deficit.

Fatherhood uncertainty is also not unique to humans, as it is shared, for example, by all males of mammal species in which female ovulation is concealed and fertilization is internal. But most of these other species do not form long lasting families (e.g., Emlen 1995). In the case of humans, instead, fatherhood uncertainty must be linked to the other two previous ingredients. In particular, a man will not have an incentive to invest unless he has a relatively high confidence in his paternity. When fatherhood is uncertain, a man must resort to guarding his opposite-sex partner(s) if he wishes to reduce the chances that other males steal his paternity and wants to enhance confidence in his biological association with the children he feeds (Hawkes, Rogers, and Charnov 1995). Our theory therefore is based on the combination of these three components to explain the emergence of monogamy.

3. The Model

A. Setup

Consider an overlapping generations model in which identical men and identical women live for five periods. The first two periods define childhood, during which individuals are infertile, unproductive, and dependent. Individuals are infertile also in the last period when they are old. The remaining two periods define adulthood and are the centre piece of our analysis. In each of these periods (labeled 1 and 2), opposite-sex individuals from the same cohort are matched to have children. Agents are programmed to maximize the propagation of own genes into future generations.

In each period 1 and 2, a man splits his resource (time or energy) endowment into three activities: (a) hunting, which provides food necessary for child survival; (b) mate guarding, which increases the probability that the children he supports carry his genes; and (c) cheating or casual (or extra-marital) sex, which increases the chance that his genes

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are represented in the next generation’s gene pool. Women are carers and have monopoly power to allocate the food they receive from their partners among their children.\footnote{This full gender specialization in home and market tasks echoes Becker (1981/1991). Interestingly, for hunter-gatherer societies, Robson and Kaplan (2006) provide convincing evidence according to which, after taking own consumption into account, women supply 3 percent of the calories to offspring while men provide the remaining 97 percent. See also Kaplan et al. (2000). Notice, however, that assuming that mothers are not food providers is done for simplicity only. It is easy to show that relaxing this assumption does not alter any of our main results.} The food a woman receives from one man is a perfect substitute of the food she receives from another man. In the last period of life (labeled 3), men are only food providers because they (as well as all the women in their cohort) are no longer fertile and thus do not engage in guarding and cheating.\footnote{All adults (males and females alike) are self-sufficient, that is, they have a given endowment of material resources which is enough for their survival. Without loss of generality, we normalize such an endowment to zero.}

There are two distinct types of sexual activity in the model. One is within the marital environment and entails a food-for-sex type of exchange between partners. The other type of sex (what we interchangeably call “casual” or “extra-marital” sex) occurs outside the marital bond and does not involve any food transfer from men to women. The asymmetry with which the two types of sex are characterized is driven by the differential reproductive success of men and women (Bateman 1948): a male’s reproductive success is not limited by his ability to produce sex cells but by his ability to fertilize eggs with these cells, while a female’s reproductive success is not limited by her ability to have her eggs fertilized but by her ability to produce eggs.\footnote{Much evolutionary research on human mating has emphasized the larger and more direct benefits gained by men seeking brief sexual encounters outside marriage (Baker and Bellis 1995). Several studies focus on the costs and benefits of casual sex faced by women (e.g., Greiling and Buss 2000).}

Our goal is to characterize sexual configurations (or conventions). A convention specifies the matching protocol between men and women, i.e., it defines the women with whom a man can have a food-for-sex exchange. Within a convention, individual actions are programmed to allocate resources optimally given the actions of other individuals. This leads to equilibrium within a convention. An efficient convention is one with the highest (population) growth rate. Genes therefore manifest themselves in the processes that underpin both individual survivorship and sexual organization of the family.\footnote{In all family configurations, individuals are not assumed to be part of a monomorphic population, even though genes do not have any expression other than instructing individuals to follow a given convention. One exception is in subsection 5.A, where genetic differences are allowed to be revealed in terms of inequality in male resource endowments.}

To highlight the key point of the paper, we formally consider two mutually exclusive conventions: (a) monogamy, denoted by $C_M$, where a female is matched with the same male in periods 1 and 2; and (b) serial monogamy, denoted by $C_S$, where each female is matched with a male in period 1 and another male in period 2.

It is useful to see how our three basic components are embedded in this setup. First, the overlapping generations structure with two childhood periods gives us the simplest way to
capture the overlap of needy children. Second, full gender specialization is an extreme (but harmless) assumption to emphasize the salience of paternal investment. Third, guarding is the direct result of fatherhood uncertainty.

B. Technologies and Actions

Let $g_t$ denote the amount of resources a man devotes to guarding his mate(s) in period $t$, $t = 1, 2, k_t$ be the amount he spends in casual sex, and $\hat{k}_t$ the average amount of resources spent on cheating by all other men in period $t$. The probability that guarding is successful is given by $\Pi(g_t, \hat{k}_t)$ and the probability that an adult man’s guarding is not successful is $1 - \Pi(g_t, \hat{k}_t)$.

**Assumption 1.** The function $\Pi$ exhibits the following properties: (i) $\Pi(g, k) > 0$, $\Pi_k(g, k) < 0$, for all $k > 0$ and $g > 0$; (ii) $\Pi_{gg}(g, k) < 0$, $\Pi_{kk}(g, k) > 0$, for all $k > 0$ and $g > 0$; (iii) $\Pi(g, 0) = 1$ for any $g > 0$, $\Pi(0, k) = 0$, for all $k \geq 0$; (iv) $\lim_{g \to 0} \Pi(g, k) = +\infty$, for all $k > 0$; and $\lim_{k \to 0} \Pi_k(g, k) = -\infty$, for all $g > 0$; (v) there exists a twice differentiable mapping $P : \mathbb{R}_+ \to \mathbb{R}_+$ such that:

$$\Pi(g, k) = P \circ \gamma(g, k), \text{ where } \gamma(g, k) = g/(g + k) \text{ for all } g, k > 0. \quad (1)$$

Assumption 1(i)–(iv) states standard regularity conditions according to which $\Pi$ is increasing and concave in $g$ and decreasing and convex in $k$. With Assumption 1(v), which is introduced just for simplicity, $\Pi$ is parameterized in such a way that the probability of successful guarding is a function of the proportion of resources devoted to guarding relative to the total resources spent on guarding and cheating.\(^{11}\) The function $\gamma$ in (1) belongs to the class of so-called Tullock contest success functions, which have been extensively used in the economics of contests and conflict (Nti 1997; Garfinkel and Skaperdas 2007; Konrad 2009) and in evolutionary biology in similar contexts to ours (Hawkes, Rogers, and Charnov 1995; Gavrilets 2012).\(^{12}\) Assumption 1 guarantees that, for any given level of guarding, the likelihood of paternity is inversely related to the average level of casual sex of all other men.

Child survival is stochastic and depends only on the amounts of food children receive during the first two periods of life. These amounts are denoted by $u$ and $v$ respectively. The expected number of children of a given cohort who survive into adulthood is thus given by $F(u, v)$. The function $F$ is assumed to satisfy the following conditions.\(^{13}\)

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\(^{11}\)It is easy to verify there exists an open set of functions satisfying conditions (i)-(v) of Assumption 1. An example of a class of such functions is given in the Appendix.

\(^{12}\)A problem with $\gamma$ is that it is not defined at $(0, 0)$ and the function cannot be extended continuously to that point. A customary way to circumvent this discontinuity is to define the function $\gamma_a$, with the parameter $a > 0$ so that $\gamma_a(g, k) = \frac{g}{g + k + a}$, and to consider the limit as $a \to 0$.

\(^{13}\)For analytical convenience only, we assume that the actual death of children born in a given period can occur only after the two periods of childhood at the beginning of the adulthood stage. Thus, $F$ can be interpreted as the number of children who are expected to reach adulthood as fertile and productive individuals. All children reach the adulthood stage but, without adequate parental investment, they will
**Assumption 2.** The function $F$ has the following properties: (i) $F$ is increasing and strictly concave; (ii) $F(0,v) = F(u,0) = 0$; (iii) $\lim_{u \to 0} F_u(u,v) = +\infty$ for all $v > 0$ and $\lim_{v \to 0} F_v(u,v) = +\infty$ for all $u > 0$; (iv) $F_{uv} > 0$ for all $u > 0$ and $v > 0$; (v) $F(u,v) = F(v,u)$.

Through Assumption 2(i)–(iii), $F$ shares a number of properties with a standard neo-classical production function. Assumption 2(iv) ensures that $F$ is supermodular, which in our context is rather natural and amounts to the dynamic complementarity property discussed by Cunha and Heckman (2007). Finally, Assumption 2(v), which is invoked for analytical convenience, imposes a symmetry condition on the timing effect of parental investment on survivorship: that is, the probability that a child survives with transfer $u$ in the first period of childhood and $v$ in the second is the same as the survival probability obtained when the time order of the transfers is reversed to $v$ first and $u$ second. This, in the context of the skill formation model proposed by Cunha and Heckman (2007), implies that early and late childhood investments are equally critical and equally sensitive.

An adult man is productive only in periods 1 and 2 and can store food from period to period at no cost. Let $x_1$ and $x_2$ be the time or energy equivalent amounts of food a man transfers to the mother of his first-period children in periods 1 and 2, respectively. Similarly, he gives $y_2$ and $y_3$ to the mother of his second-period children in periods 2 and 3. A male then will face the following lifetime resource constraint

$$\omega = \sum_{t=1}^{2} (g_t + k_t + x_t + y_{t+1}),$$

which is defined over the entire adulthood period. Each man is endowed with $\omega$ units of resources (time or energy), that can be allocated either to seek paternity, or to engage in casual sex, or to hunt and provide food to his mate. The exact timing of such decisions is irrelevant, except that guarding and cheating will not be carried out in the last period because all adults of the same generation (men and women alike) are no longer fertile.

A mother then allocates the food she receives in a given period among her offspring. This allocation is straightforward in period 1, when she gives all the food she receives, $x_1$, to her first-period children. Similarly in period 3 she also cannot affect the amount of food supplied by the male to the second-born for her children’s second stage of childhood, $y_3$. In period 2 instead she distributes food across two cohorts of children. For $m \in (0, 1)$, the amounts of food she allocates to her first-period and second-period children are thus given respectively by $mR$ and $(1-m)R$, where $R = x_2 + y_2$ is the total amount of food she receives from her partner(s) in period 2.

The model also abstracts from other features which are not essential for our results to hold, such as economies of scale in food production.

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14For simplicity, expression (2) assumes the possibility of free storage and borrowing. However, provided that men are less productive in their last adult period, only saving but not borrowing will occur in equilibrium.
We next specify the payoff structure for each convention separately. The ‘hat’-notation is used to indicate variables that are not under the player’s control.

C. Payoffs

Adult Man in the Monogamous Convention — A monogamous man is programmed to select \(g_1, k_1, x_1, g_2, k_2, x_2, y_2,\) and \(y_3\) to maximize

\[
\frac{1}{2} \Pi(g_1, \hat{k}_1) F(x_1, \hat{m}(x_2 + y_2)) + \frac{1}{2} \Pi(g_2, \hat{k}_2) F((1 - \hat{m})(x_2 + y_2), y_3)
\]

subject to (2) and nonnegativity constraints \(0 \leq g_1, k_1, x_1, g_2, k_2, x_2, y_2, y_3\). The first term in (3) indicates the contribution to the man’s payoff from first period offspring. These children are genetically related to the adult male with probability \(\frac{1}{2} \Pi(g_1, \hat{k}_1)\), while \(F(x_1, \hat{m}(x_2 + y_2))\) represents the expected number of children surviving the first and second periods of childhood. When casual sex is arbitrarily close to zero, then \(\Pi(g_1, \hat{k}_1) = 1\) and the degree of genetic relatedness with own child will boil down to the usual coefficient of \(\frac{1}{2}\). The same considerations apply to the second term which captures the payoff obtained by a man through his second period offspring.

The last two terms in (3) refer to the payoff a male can obtain from casual sex. With probability \(1 - \Pi(\hat{g}_t, k_t), t = 1, 2\), he is the “unofficial” father of other children who will not be supported by him. They instead receive food by the official mate of the mother and the cheating male does not have control over such transfers.

Adult Man in the Serially Monogamous Convention — Taking into account that the female he guards already has (or will eventually have) children with a different male, a serially monogamous man is programmed to allocate \(g_1, k_1, x_1, g_2, k_2, x_2, y_2,\) and \(y_3\) in order to maximize

\[
\frac{1}{2} \Pi(g_1, \hat{k}_1) F(x_1, \hat{m}(x_2 + \hat{y}_2)) + \frac{1}{2} \Pi(g_2, \hat{k}_2) F((1 - \hat{m})(x_2 + \hat{y}_2), y_3)
\]

subject to (2) and the usual nonnegativity constraints. This formulation acknowledges that, from the viewpoint of a given male in period 2, the woman he guarded in the first period receives \(x_2\) from him and \(\hat{y}_2\) from the male who guards her in period 2 and with
whom she begets her second period offspring. The opposite occurs for the mother of his second period children.

Adult Woman — A woman allocates food to her progeny. The food is given to her by her official partner(s), i.e., the men who guard her. She cannot influence the amount of food given in her first adult period, \( \tilde{x}_1 \), nor the amount of food received by children in the third period, \( \tilde{y}_3 \). Her problem is thus to determine the food allocation, \( m \), in the second period to maximize her own payoff

\[
\frac{1}{2} F(\tilde{x}_1, m(\tilde{x}_2 + \tilde{y}_2)) + \frac{1}{2} F((1 - m)(\tilde{x}_2 + \tilde{y}_2), \tilde{y}_3),
\]  

while taking males’ decisions as given. Since she has no uncertainty about her motherhood, the degree of genetic relatedness to each of her children is \( \frac{1}{2} \).

It is instructive to reiterate that individual payoffs (3)–(5) account for the fact that men have a reproductive comparative advantage to access extra-marital sex as opposed to women. Our model reflects this gender asymmetry in the reproductive benefits of extra-marital sex by making casual sex payoff relevant for men and normalizing its net benefits for women to zero.\(^{15}\)

D. Stability

As it has become common in evolutionary models (e.g., Kondrashov 1988; Perry, Reny, and Robson 2015), our analysis of stability combines the notions of individual selection and group selection. A family configuration cannot exist unless a group of opposite-sex individuals follow its norms. We assume therefore that a mutation to a new convention must be done by a small group of individuals (with an equal number of men and women) who separate from the rest of the population (and, say, move to another “island” or form a different “village”) and interact only among themselves. We introduce this assumption only for simplicity. Assuming that mutants interact with incumbents will add complexity to the model but will not affect our main result.

Within a specific family configuration, each individual is programmed to act optimally given the strategies adopted by other agents. That is, the allocation of resources selected by individuals is a Nash equilibrium, and a Nash allocation profile is attached to every family structure. A family convention \( C_j \) is evolutionarily stable if it resists the invasion of a (small) group of deviators who adopt an alternative convention \( C_{j'}, j' \neq j \). In other words, the configuration with the highest population growth is the configuration that is

\(^{15}\)An interpretation of this gender asymmetry is that, even if women did not willingly engage in extra-marital sex, men resorted to rape (Thornhill and Palmer 2000). Rape might have been pervasive in ancestral populations. In a study of 186 contemporary societies in the Standard Cross-Cultural Sample, Broude and Greene (1976) show that in one-quarter of societies, rape is accepted or ignored and in another 30 percent is only mildly disapproved or ridiculed.
evolutionarily stable, precisely because it will take over the other configuration in the long run.

4. Monogamy and Serial Monogamy

A. Equilibria

Let us now consider one family convention at a time, and compute the corresponding survival rate. We focus on pure Nash equilibria and adopt the following definition.

**Definition of Equilibrium.** An equilibrium \((C_j, \sigma^*_j)\), \(j = M, S\), is a family convention \(C_j\) and a strategy \(\sigma^*_j\), with \(\sigma^*_j = (m^*, g_1^*, k_1^*, x_1^*, g_2^*, k_2^*, x_2^*, y_2^*, y_3^*)_j\), such that \(\sigma^*_j\) is a pure Nash equilibrium. That is, given \(C_j\), the equilibrium strategy is a fixed point in actions, whereby each man maximizes his own payoff and each woman maximizes hers, given the partner’s actions as well as the actions of everyone else.

We start with a characterization of the equilibrium in the life long monogamy and then present the serially monogamous case.\(^{16}\)

**Proposition 1.** In the monogamous family configuration, there exists a unique interior Nash equilibrium \((C_M, \sigma^*_M)\) with the equilibrium strategy \(\sigma^*_M\) characterized by \(m^* = \frac{1}{2}\), \(g_1^* = k_1^* = g_2^* = k_2^*\), and \(x_1^* = y_3^* = \frac{x_2^* + y_2^*}{2}\).

Proposition 1 shows that, in equilibrium, a monogamous mother splits resources equally between the two cohorts of her children and both cohorts will receive equal amounts of food in each of their childhood periods, regardless of birth order. As a result of the concavity and symmetry of the \(F\) function, this is in fact the only efficient allocation of the total amount of food provided by males. An adult male will devote an equal fraction of his resource endowment to guarantee paternity of both cohorts of his children in periods 1 and 2, and this also coincides with the fraction of resources spent on casual sex in both periods. Repeating the same exercise for the individuals who form families in the serial monogamy convention leads to the next result.

**Proposition 2.** In the serial monogamy configuration, there exists a unique interior Nash equilibrium \((C_S, \sigma^*_S)\) with the equilibrium strategy \(\sigma^*_S\) given by \(m^{**} = \frac{1}{2}\), \(g_1^{**} = k_1^{**} = g_2^{**} = k_2^{**}\), \(x_1^{**} = y_3^{**}\) and \(x_2^{**} = y_2^{**}\), with \(x_1^{**} > x_2^{**}\) and \(y_2^{**} < y_3^{**}\).

There are three similarities between the two equilibria: (a) mothers split their resources equally among their children; (b) males spend as much on guarding as they do on cheating;

\(^{16}\)We should point out that in the monogamous family convention as well as in the serial monogamy configuration, there will always exist another equilibrium in which parents provide food only to one cohort of children, starving the other cohort to death. For instance, \(m^* = 0\) is the mother’s optimal response to \(x_1^* = 0\), and similarly \(x_1^* = 0\) is optimal given \(m^* = 0\). It is easy to verify that such an equilibrium is not evolutionarily stable and we thus ignore it.
and (c) children from different cohorts receive an equal total amount of resources over their entire childhood.

There is however a crucial difference in the amount of the transfers by birth order. In the serially monogamous convention, first-born children receive more food in their first period of life, while second-born children receive more food in their second stage of childhood. The reason for this difference is simple, and yet crucial. In period 2, when both official fathers of the children born to the same woman have to transfer food to her, they free ride on each other and supply less food than what they would have done if they were the sole providers. In the second period, food provision is less efficient from the male’s viewpoint because he does not have control over the allocation decision of his (current or former) mate, who could use part of his food provision to care not for his offspring but for the children of the other man. Hence, every man finds it more productive to shift more resources to casual sex, which in turn induces greater guarding in equilibrium.

As casual sex and guarding are socially wasteful activities, the superiority of the monogamous convention therefore rests on its greater efficiency in allocating resources. This is formally stated in the following proposition.

**Proposition 3.** Survivorship in the monogamous convention is always greater than survivorship in the serially monogamous convention.

The mechanism underpinning this result is the absence of free riding in the monogamous convention. Compared to serial monogamy, food provision in the monogamous family is more efficient. This comes about through two channels. The first is that the absence of free riding has a direct effect on the individual male incentive to hunt and supply food, because all his food transfers are received by his own official children. Conversely, in the presence of free riding, the serially monogamous man shifts more resources to casual sex and, since everyone does the same, guarding must go up in equilibrium.

The second channel is a more subtle aspect of the mechanism identified in Propositions 1 and 2. Monogamous men and women distribute food to their offspring equally in such a way that the two childhood periods are perfectly symmetric.\(^{17}\) This guarantees an equal food distribution for both cohorts in both periods. Since \(F\) is concave and symmetric, this food distribution turns out to be the most efficient intrahousehold allocation. If greater food provision is efficiency enhancing, each man will have an incentive to shift resources away from casual sex into hunting, a shift that in turn will reduce the need for mate guarding. As cheating and guarding are socially wasteful, the equilibrium outcome is efficient in the monogamy family.

Although we cannot readily extend this result to the contemporary family, it is interesting to note that it lines up well with much of the recent empirical evidence available

\(^{17}\)Recall that the free riding behavior in \(C_S\) occurs only in period 2, when different cohorts of children of different ages overlap.
for advanced economies, according to which children who live part of their childhood in a blended (serially monogamous) family have lower educational attainment and experience worse outcomes later in life (e.g., McLanahan and Sandefur 1994; Duncan and Brooks-Gunn 1997; Ginther and Pollak 2004).

B. Example

We illustrate the results found so far for the special case in which both \( \Pi \) and \( F \) are power functions. In particular, they take the following specifications:\(^{18}\)

\[
\Pi(g, k) = \left( \frac{g}{g + k} \right)^\alpha \quad \text{and} \quad F(u, v) = (uv)^\beta, \tag{6}
\]

where the parameters \( \alpha \in (0, 1) \) and \( \beta \in (0, \frac{1}{2}) \) are measures of efficiency of the guarding technology and the food provision technology, respectively. With the functions given in (6) and letting \( \alpha/\beta = \rho \), it is easy to show that Proposition 1 holds with \( m^* = \frac{1}{2}; g_1^* = k_1^* = g_2^* = k_2^* = \frac{\omega}{4(2 + \rho)}; \) and \( x_1^* = x_3^* = (x_2^* + y_2^*)/2 = \frac{\omega}{2(2 + \rho)} \). Similarly, Proposition 2 holds with \( m^{**} = \frac{1}{2}; g_1^{**} = k_1^{**} = g_2^{**} = k_2^{**} = \frac{\omega}{2(2 + 3\omega)}; \) \( x_1^{**} = y_3^{**} = \frac{\omega}{3 + 2\rho}; \) and \( x_2^{**} + y_2^{**} = \frac{\omega}{3 + 2\rho} \). With such equilibrium values, the probability of successful guarding is identical in the two conventions and equal to \( \left( \frac{1}{2} \right)^\alpha \). The expected number of surviving children in the serially monogamous family is \( 2 \left( \frac{\omega}{2(2 + \rho)} \right)^\alpha \left( \frac{\omega}{3 + 2\rho} \right)^\alpha \), while the corresponding number in the life long monogamy family is \( 2 \left( \frac{\omega}{2(2 + \rho)} \right)^{2\alpha} \). In the case in which \( \alpha = \beta \) and \( \omega = 1 \), it is straightforward to verify that survivorship in \( C_M \) is \( 2 \left( \frac{1}{50} \right)^\alpha \), which, irrespective of \( \alpha \), is always greater than survivorship in \( C_M \), where it is equal to \( 2 \left( \frac{1}{50} \right)^\alpha \). This shows that child survivorship is greater under monogamy than under serial monogamy.

5. Extensions and Discussion

In this section we discuss a number of important extensions that build on the results of the previous section. For the sake of space concern we keep the discussion at a minimum. The proofs of the results in subsections A and C are reported in the online appendix.

A. Polygyny

Polygyny is a family convention in which one man has multiple female partners at the same time. While this is a widespread family type among many contemporary societies, it should be emphasized that even in these societies the vast majority of unions are monogamous.\(^{19}\)

\(^{18}\)It is easy to verify that the function \( \Pi \) in (6) does not satisfy restriction (iv) of Assumption 1. The set of conditions in Assumption 1 in fact is not necessary but only sufficient.

\(^{19}\)See, among others, Hartung (1982), Marlowe (2000), Tertilt (2005), and Henrich, Boyd, and Richerson (2012).
Previous studies have typically explained the emergence of polygyny through either differential fecundity between men and women or male inequality in wealth, whereby richer (and generally older) men tend to have multiple wives at the expense of males with fewer resources (Becker 1991).\textsuperscript{20}

Our analysis focuses on the role of endowments. We start by considering the case in which all males have identical resource endowments. This case offers an important benchmark in a steady state evolutionary environment where interpersonal differences are weakened by selection. In this case, the polygynous matching protocol must guarantee that only a random subset of the homogenous men in the population are matched for a lifetime union. Each matched polygynous man will transfer food to all his official mates. The other men instead remain unmatched, and, as a result, they use their entire endowment on casual sex as they do not have the need to hunt or guard. The inefficiency of this convention is easily verified. Not only is a positive mass of the male population unengaged in food production, but those who are engaged in food production devote too much of their resources to guard against unmatched men, who in turn are busy stealing paternity from the matched males. Children of polygynous parents must then rely on fewer resources than their monogamous counterparts.

With male heterogeneity (and identical women), it can be verified that there exists a threshold in male endowments such that, when the inequality in male resources is large enough, polygyny is more efficient than monogamy.\textsuperscript{21} In general, the threshold depends on the technological conditions of a convention. In the context of the example of Section 4, it increases as the food production technology becomes more concave ($\beta \to 0$) and the guarding technology becomes more linear ($\alpha \to 1$).

**B. Group Marriage**

Another possible family configuration is group marriage (or cenogamy), in which a group of women share a group of men as partners throughout their entire adult lives (Ingoldsby and Smith 2006). In this convention, a man does not know which of the women, if any, bear his progeny. It is then natural for men to supply food to all the women who, in turn, pool it together communally and use it to feed their children. As in all other configurations, cenogamous males have an incentive to steal paternity from men in other groups by having casual sex with outside females, and they also need to exert effort into mate guarding. Because each man’s best response is to free ride on the other inside men’s food supply, free riding increases with the size of the group, and eventually food provision will vanish. This is a classic example of the tragedy of the commons, and monogamy continues to be more

\footnotesize{\textsuperscript{20}As mentioned in Section 2, Gould, Moav, and Simhon (2008) develop a model in which higher inequality in female endowments may reduce the prevalence of polygyny.

\textsuperscript{21}A similar relationship between male (income) inequality and type of family organization (polygyny versus monogamy) has also been stressed by Lagerlöf (2005).}
efficient.

One reason why we still observe cases of group marriage in some contemporary traditional societies is possibly due to the existence of complementarities in the food production technology (which are assumed away in our setup), whereby a group of males could supply more food than in the case in which each of them worked alone. The trade off between the advantage given by economies of scale in food production and the disadvantage of free riding will then determine the optimal size of the group.

C. Kinship

Individuals care not only for their own progeny but also for other genetically close relatives (Knight 2008). While a full treatment of kinship transfers and their efficiency impact on any specific family convention is beyond the scope of our paper, we provide a simple example to demonstrate that including kinship in the model enhances the superiority of monogamy through the channels described above.

We consider an environment in which adult men can make food transfers to their presumed younger siblings. Siblings’ birth order now matters, as we allow for older siblings to pass resources to their younger siblings. This means that older males face a different optimization program from that faced by their younger brothers. The Appendix formalizes such programs.

In the context of the example in Section 4 with $\alpha = \beta$ and when food transfers between siblings are possible we can show two results: (a) In serial monogamy, there exists a unique interior Nash equilibrium, in which the optimal food transfer is zero; and (b) In monogamy, there exists a unique interior Nash equilibrium in which the optimal food transfer is positive and total survivorship is greater than without kin ties.

In the serially monogamous convention therefore kin ties cannot emerge in equilibrium (result (a)). The intuition is simple. Because each child has the same likelihood of survival and because any given man is genetically closer to his own children than to any other individual in the population (including his half-siblings), it is optimal for him to provide food only to his own official offspring.

For monogamy, result (b) highlights two points. First, as marginal productivity of food is decreasing, positive kin transfers occur in equilibrium since a man finds it advantageous to provide food to both his offspring and his (still dependent) siblings for these are equally genetically close to him. Greater efficiency in food provision translates into greater food transfers, lower levels of extra-marital sex, and lower need for guarding. Second, firstborn receive strictly more food from their father in both periods of childhood than in the case without kin ties. Later born children instead receive less food from their father, and

---

22 The opposite transfers (from younger to older siblings) and more complex kinship systems involving individuals with lower levels of genetic relatedness (e.g., cousins, nephews, and nieces) are expected to be of second order importance and are thus not considered here.
even after taking into account the positive transfer from their older brothers, their total resources are lower than what they would have received in a context without kin ties. This primogeniture effect emerges because fathers, irrespective of whether they made or received food transfers, internalize the possibility that their firstborn will support their younger siblings. By transferring more food to their first born children, fathers spread their investment across children more equally, which, owing again to the concavity of $F$, increases their payoff.\textsuperscript{23}

D. The Risk of Adult Mortality

There are many types of idiosyncratic risk that can be relevant to our model. Here we discuss whether the possibility of adult mortality makes monogamy less efficient than either serial monogamy or group marriage.\textsuperscript{24}

With no risk, Section 4 shows serially monogamous fathers invest more in their firstborn than their monogamous counterparts in their first period of adulthood. If there is a positive probability of adult (parental) mortality, there might be conditions under which serial monogamy is more efficient, for older children could be better equipped to face the risk of death. Indeed, in the presence of unanticipated mortality risk, there are cases in which there exists a critical probability cutoff above which serial monogamy is socially more efficient than monogamy. However, numerical computations applied to a specification of the model similar to the example given in subsection 4.B reveal this can occur only if mortality rates are at least 90%. Our claim is that such a value is implausibly high.\textsuperscript{25} If instead the risk of mortality is anticipated, individuals in any family convention can optimally adjust to the risk. We expect that the monogamous configuration will continue to be more efficient, because of the absence of free-riding among male providers. A formal analysis of this case is a promising area for further work.

In the cenogamous convention with no risk, the free-rider problem is more severe than in the serially monogamous convention, and monogamous couples will always achieve greater fitness. When there is a positive probability of adult mortality, however, the group of surviving men and women may provide an insurance against the unanticipated death of one (or more) of its members. The insurance motive is likely to be weak in large groups, but could be biting in smaller groups. Numerical computations, however, show again that the probability threshold above which group marriage dominates monogamy must be at least 50%, which — as in the previous case — is implausibly high. As before, extending

\textsuperscript{23}Several empirical studies based on contemporary data find evidence that is consistent with this effect, with sizeable negative impacts of higher birth order on intellectual development, education, earnings, employment, and teenage fertility (e.g., Black, Devereux, and Salvanes 2005).


\textsuperscript{25}Hill, Hurtado and Walker (2007) discuss a large body of evidence according to which at least 75 percent of 10-year-olds are expected to survive to an age at which they might become grandparents (age 40, or roughly double the mean age of first birth) in most of the traditional societies for which there are reliable demographic data available.
the model to deal with anticipated risk is likely to give monogamy the usual no-free-riding advantage and increase the probability threshold even further.

Of course a group might insure against other shocks, such as attacks from rival tribes, better than a household. This sort of analysis is interesting and deserves attention in future research.

E. Social Norms

It worth stressing that, even though family conventions are assumed to pass through genetic links, social norms can come into play in our model. Central to our theory in fact is the observation that, in a world where individuals seek to maximize their reproductive success, casual sex and mate guarding are unavoidable, and yet socially wasteful. Because of the inefficiencies they generate, cheating and guarding might induce the development of social norms, beliefs, or institutions that can attenuate their negative effects. One of these is “religion”. Religions generate moral systems that can affect equilibrium outcomes building new norms and beliefs that are evolutionarily advantageous while ruling out alternative goals and beliefs that are not (Wilson 2002).

In this sense our model provides a simple explanation as to why all of the major world religions have openly condemned a wide range of sexual acts, such as adultery, and stigmatized the product of extra-marital sex, illegitimate children (Browning, Green, and Witte 2006), while allowing or encouraging mate guarding, even if this is a socially unproductive activity. According to our model, reducing extra-marital sex has not only the direct effect of freeing up resources for food provision but also the indirect effect of reducing the necessity of guarding. Both effects increase survivorship. On the contrary, forbidding mate guarding, independently of the difficulty of its implementation, will inevitably give rise to increased cheating.26

26Judaism, Christianity, Hinduism, Buddhism, Islam, and Confucianism view the monogamous family as a practice that lies at the foundation of society (Browning, Green, and Witte 2006). Of course there are exceptions. Confucianism and ancient Judaism permitted powerful men to have concubines. But children of concubines were usually stigmatized. Christianity sometimes idealized the sexually abstinent marriage and, along with Buddhism, commanded celibacy for its religious leaders. Islam permitted, sometimes encouraged, polygynous marriages. Interestingly, there are only two Qur’anic verses on polygyny. The first is a permission in a very specific context, i.e., the treatment of orphaned girls whose guardians may want to marry them to appropriate their wealth. The second imposes strong conditions on men who intend to marry more than one woman, making polygyny highly unlikely for a ‘righteous’ man (Rashid Rida 1975). The permission to marry up to four wives is premised only on concerns about the oppression of orphans (Al-Hibri and El Habti 2006). Polygynous marriages within the Mormon Church are a much more recent phenomenon, with the frequency of polygynous men as a fraction of all men in the Utah Population Database reaching a maximum of 18 percent among men born in 1833 and with a reduction of the frequency of such marriages to less than 1 percent over the next few decades (Moorad et al. 2011).
F. Contemporary Family

Caution, of course, should be taken before hastening to interpret our results in the context of the contemporary family. The widespread availability of effective contraceptive methods has made extra-marital sex substantially less hazardous for women (Guinnane 2011). Another dimension of caution refers to parental investment in children that, albeit still crucial in the contemporary family (Cunha and Heckman 2010), has been supplemented or replaced by the state and the market (Becker 1991; Lundberg and Pollak 2007). Likewise, endowment and preference heterogeneity could open up new explanations of why mating strategies other than monogamous bonding may arise over time (Gould, Moav, and Simhon 2008). Yet another dimension is the increased labor force participation of mothers (Goldin 2006). Medical innovations, binding commitments, the market, the state, heterogeneity, and the greater economic independence of women might influence the nature of exchange within families in ways that our model cannot capture.

6. Conclusion

This paper presents a new model of the family with uncertain paternity and paternal investment in overlapping cohorts of children. Despite its simplicity the model delivers interesting insights about the monogamous family and its stability. Our main result is that life long monogamy is the most efficient type of family structure as opposed to serial monogamy and polygyny. The mechanism underpinning this greater efficiency is the existence of free riding between males. The same mechanism is valid when we compare monogamy to group marriage and when we allow for the presence of the risk of adult mortality.

Another finding is that altruism between siblings emerges within monogamy. This is because every man, who is uncertain about his paternity and yet cares about his reproductive success, values his siblings in part because they provide him with the assurance that some of his genes will survive into future generations. Although our framework is based on genetic transmission of family conventions, the model could be reinterpreted to allow for cultural diffusion of family norms.
Appendix

Proofs

This Appendix reports the proofs of the propositions of the paper. We begin with the proof of three technical results that will be used to prove the propositions stated in the paper. Throughout the Appendix, the notation $F_i$, $i=1,2$, indicates the first derivative of $F$ with respect to its $i$-th argument; similarly, $F_{ij}$, with $i,j=1,2$, indicates the derivative of $F_i$ with respect to the $j$-th argument.

**Lemma 1.** $F_1(x_1, x_2) = F_2(x_2, x_1)$.

**Proof.** The symmetry condition on $F$, so that $F(x_1, x_2) = F(x_2, x_1)$, implies that

\[
F_1(x_1, x_2) = \lim_{h \to 0} \frac{F(x_1 + h, x_2) - F(x_1, x_2)}{h} = \lim_{h \to 0} \frac{F(x_2, x_1 + h) - F(x_2, x_1)}{h} = F_2(x_2, x_1).
\]

**Lemma 2.** If $F_1(x_1, x_2) = F_2(x_1, x_2)$, then $x_1 = x_2$.

**Proof.** Strict concavity of $F$ implies that for any two points $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ we have

\[
F(x_1, x_2) > F(y_1, y_2) + (y_1 - x_1, y_2 - x_2) \nabla F(x_1, x_2)
\]

whenever $X \neq Y$. Assume $x_1 \neq x_2$ and let $Y = (y_1, y_2) = (x_2, x_1)$. Then

\[
F(x_1, x_2) > F(x_2, x_1) + (x_2 - x_1, x_1 - x_2) \nabla F(x_1, x_2) = F(x_2, x_1) + (x_2 - x_1)[F_1(x_1, x_2) - F_2(x_1, x_2)]
\]

which is impossible because $F(x_1, x_2) = F(x_2, x_1)$ and because of the assumption $F_1(x_1, x_2) = F_2(x_1, x_2)$. Then $x_1 = x_2$.

**Lemma 3.** If $x_1 + x_2 > y_1 + y_2$, then either $F_1(x_1, x_2) < F_1(y_1, y_2)$, or $F_2(x_1, x_2) < F_2(y_1, y_2)$, or both inequalities are true.

**Proof.** Because $x_1 + x_2 > y_1 + y_2$, then $x_1 - y_1 > 0$ and/or $x_2 - y_2 > 0$. Let:

\[
\begin{align*}
\Phi(x_1, x_2) &= F_1(x_1, x_2) \\
\Psi(x_1, y_2) &= F_2(x_1, x_2).
\end{align*}
\]

An infinitesimal increase in the value of the images of $\Phi$ and $\Psi$, denoted by $\Delta \Phi$ and $\Delta \Psi$, corresponding to an increase $\Delta_1$ in $x_1$ and $\Delta_2$ in $x_2$ can be written as

\[
\begin{bmatrix}
\Delta \Phi \\ 
\Delta \Psi
\end{bmatrix} =
\begin{bmatrix}
F_{11} & F_{12} \\
F_{21} & F_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta_1 \\
\Delta_2
\end{bmatrix}
\]

By contradiction, assume

\[
\begin{bmatrix}
F_{11} & F_{12} \\
F_{21} & F_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta_1 \\
\Delta_2
\end{bmatrix} >
\begin{bmatrix}
0 \\
0
\end{bmatrix}.
\]

We now consider the three possible cases.

Case 1. $\Delta_1 > 0, \Delta_2 > 0$.  

19
The system of equations
\[
\begin{align*}
F_{11}\Delta_1 + F_{12}\Delta_2 &> 0 \\
F_{21}\Delta_1 + F_{22}\Delta_2 &> 0
\end{align*}
\]
implies that
\[
\begin{align*}
F_{12}\Delta_2 &> -F_{11}\Delta_1 \\
F_{21}\Delta_1 &> -F_{22}\Delta_2.
\end{align*}
\]
Because all terms are positive, term-wise multiplication of the two equations leads to \((F_{12})^2 > F_{11}F_{22}\), which contradicts the assumption that \(F\) is concave.

Case 2. \(\Delta_1 > 0, \Delta_2 < 0\).

Since Assumption 2(iv) guarantees that \(F_{12} > 0\), the inequality \(F_{12}\Delta_2 > -F_{11}\Delta_1\) can never be satisfied because the left-hand term is negative and the right-hand term positive.

Case 3. \(\Delta_1 < 0, \Delta_2 > 0\).

The inequality \(F_{21}\Delta_1 > -F_{22}\Delta_2\) can never be satisfied because the left-hand term is negative and the right-hand term positive.

Notice the supermodularity condition \(F_{12} > 0\) is sufficient but not necessary. In addition, the inequalities obtained are for infinitesimal increments \(\Delta_1\) and \(\Delta_2\). However, because they hold everywhere, the effect of a change from \(x_1\) to \(y_1\) and from \(x_2\) to \(y_2\) can be obtained by integrating \(\Delta\Phi\) and \(\Delta\Psi\) over the path from \((x_1, x_2)\) to \((y_1, y_2)\). The corresponding changes \(\Phi(y_1, y_2) - \Phi(x_1, x_2)\) and \(\Psi(y_1, y_2) - \Psi(x_1, x_2)\) keep the same signs as those implied by the infinitesimal increments \(\Delta\Phi\) and \(\Delta\Psi\). Hence, the contradiction holds. □

Proof of Proposition 1 (Monogamy)

Adult woman. The monogamous woman’s is programmed to select \(z_2\) to maximize
\[
\frac{1}{2} [F(\hat{x}_1, z_2) + F(\hat{R} - z_2, \hat{y}_3)]
\]
subject to \(0 \leq z_2 \leq \hat{R}\), where \(z_2\) is the amount of food she gives to her first period children in period 2, \(\hat{R} = \hat{x}_2 + \hat{y}_2\), and the terms denoted by a ‘hat’ are exogenous to her decision problem. Because of strict concavity of \(F\) the solution is unique. The first order necessary condition associated to an interior solution is
\[
F_2(\hat{x}_1, z_2) = F_1(\hat{R} - z_2, \hat{y}_3)
\]
(A.1)

For \(\hat{x}_1 > 0\) and \(\hat{y}_3 > 0\) this condition is necessary and sufficient. In this case we denote the solution as \(z_2 = m\hat{R}\), where \(0 < m < 1\). Now, Lemma 1 guarantees that if \(\hat{x}_1 = \hat{y}_3\), then \(m = 1/2\).

Adult man. The monogamous man’s problem is to select \(x_1, g_1, k_1, x_2, y_2, g_2, k_2, y_3\) to maximize his payoff function \(\phi\) given by
\[
\frac{1}{2}\left\{\Pi(g_1, \hat{k}_1)F(x_1, \hat{m}\hat{R}) + \{1 - \Pi(g_1, k_1)\}F(\hat{x}_1, \hat{m}\hat{R}) + \Pi(g_2, \hat{k}_2)F((1 - \hat{m})\hat{R}, \hat{y}_3) + \{1 - \Pi(g_2, k_2)\}F((1 - \hat{m})\hat{R}, \hat{y}_3)\right\},
\]
where \( R = x_2 + y_2 \), subject to \( \omega = x_1 + R + y_3 + g_1 + g_2 + k_1 + k_2 \) and \( 0 \leq x_1, x_2, y_2, y_3, g_1, g_2, k_1, k_2 \), and where, for now, \( 0 < \hat{m} < 1 \). As before, the terms with a ‘hat’ are exogenous. The following first order conditions are necessary but not sufficient for an interior maximum:

\[
\frac{\partial \phi}{\partial x_1} = F_1(x_1, \hat{m}R)\Pi(g_1, \hat{k}_1) + \lambda = 0 \tag{A.2}
\]

\[
\frac{\partial \phi}{\partial R} = \hat{m}F_2(x_1, \hat{m}R)\Pi(g_1, \hat{k}_1) + (1 - \hat{m})F_1((1 - \hat{m})R, y_3)\Pi(g_2, \hat{k}_2) + \lambda = 0 \tag{A.3}
\]

\[
\frac{\partial \phi}{\partial g_1} = F(x_1, \hat{m}R)\Pi_y(g_1, \hat{k}_1) + \lambda = 0 \tag{A.4}
\]

\[
\frac{\partial \phi}{\partial k_1} = -F(\hat{x}_1, \hat{m}\hat{R})\Pi_k(\hat{g}_1, \hat{k}_1) + \lambda = 0 \tag{A.5}
\]

\[
\frac{\partial \phi}{\partial y_2} = F_2((1 - \hat{m})R, y_3)\Pi(g_2, \hat{k}_2) + \lambda = 0 \tag{A.6}
\]

\[
\frac{\partial \phi}{\partial k_2} = -F((1 - \hat{m})\hat{R}, \hat{g}_3)\Pi_k(\hat{g}_2, \hat{k}_2) + \lambda = 0. \tag{A.8}
\]

Next, let us evaluate (A.2)–(A.8) at the fixed point, which we denote with \( (\hat{m}, \hat{k}_1, \hat{g}_1, \hat{k}_2, \hat{g}_2, \hat{R}, \hat{y}_3) \). Now, equations (A.4) and (A.5) together with (A.7) and (A.8) imply that \( \Pi_y(g^*_1, k^*_1) = -\Pi_k(g^*_2, k^*_2) \) and \( \Pi_y(g^*_3, k^*_3) = -\Pi_k(g^*_2, k^*_2) \). Assumption 1 implies that \( g^*_1 = k^*_1 \) and \( g^*_2 = k^*_2 \), which then lead to \( \Pi(g^*_1, k^*_1) = \Pi(g^*_2, k^*_2) \). From (A.1) we obtain

\[
F_2(x^*_1, m^*R^*) = F_1((1 - m^*)R^*, y^*_3), \tag{A.9}
\]

and expression (A.3) becomes

\[
F_2(x^*_1, m^*R^*)\Pi(g^*_1, k^*_1) + \lambda = 0. \tag{A.10}
\]

From (A.2) and (A.6) evaluated at the fixed point and (A.10) we obtain

\[
F_1(x^*_1, m^*R^*) = F_2(x^*_1, m^*R^*) = F_2((1 - m^*)R^*, y^*_3). \tag{A.11}
\]

Combining (A.9) and (A.11) we get

\[
F_1(x^*_1, m^*R^*) = F_2(x^*_1, m^*R^*) = F_2((1 - m^*)R^*, y^*_3) = F_1((1 - m^*)R^*, y^*_3).
\]

Using Lemma 2 we find \( x^*_1 = mR^* \) and \( (1 - m)R^* = y^*_3 \), while Lemma 3 applied to \( F_1(x^*_1, x^*_1) = F_2(x^*_1, x^*_1) = F_2(y^*_3, y^*_3) = F_1(y^*_3, y^*_3) \) yields

\[
x^*_1 = y^*_3.
\]

With (A.9) this solution leads to

\[
m^* = 1/2 \text{ and } x^*_1 = mR^* = (1 - m)R^* = y^*_3.
\]

The unique symmetric fixed point of the first order conditions (A.2)–(A.8) is then characterized by \( x^*_1 = y^*_3 = R^*/2, m^* = 1/2, g^*_1 = k^*_1 = g^*_2 = k^*_2 = g^* \) and \( \omega = 2R^* + 4y^* \).

**Existence of equilibrium.** To establish that the fixed point \( \sigma^* \) is an equilibrium we need to show that, after fixing the strategies of all players but agent \( j \) at \( \sigma^* = (m^*, x^*_1, R^*, y^*_3, g^*_1, g^*_2, k^*_1, k^*_2), \sigma^* \)
is in fact the best response for \( j \). This is not trivial because the problem is not concave. Before getting into the proof, two remarks are in order. First, the allocation implied by \( \sigma^* \) is the only interior solution for (A.2)–(A.8) for player \( j \). This implies that \( \sigma^* \) is the only candidate for an interior equilibrium. Second, we stress that the constraints of the optimization problem define a compact set for each player \( i \) and the objective is continuous. As the exogenous parameters, the terms denoted with a ‘hat’, are fixed at the \( * \)-values and \( \hat{k}_t > 0 \) and \( \hat{g}_t > 0 \), the ratios \( g/(g + \hat{k}) \) and \( \hat{g}/(\hat{g} + k) \) are well behaved. The optimization problem then must admit a solution. This is either the interior solution \( \sigma^* \) or a corner solution in \( m \), which we now rule out but which we shall turn to at the end of this proof. Below we consider all other possible non-interior alternatives.

(a) \( x_1 = R = y_3 = 0 \) and \( g_1 > 0 \) and/or \( g_2 > 0 \) is suboptimal because it yields a payoff which is lower than \( x_1 = R = y_3 = g_1 = g_2 = 0 \) and \( k_t = \frac{\alpha}{2} \), \( t = 1, 2 \).

(b) \( x_1 = R = y_3 = g_1 = g_2 = 0 \) and \( k_t = \frac{\alpha}{2} \) is suboptimal because it yields a payoff of \( 2[1 - \Pi(k_t^*, \frac{\alpha}{2})]F(x_1^*, x_2^*) \) which is lower than the payoff obtained following the strategy \( \sigma^* \), which is \( 2F(x_1^*, x_2^*) \).

(c) \( R = 0 \) and \( x_1 > 0 \) and/or \( y_3 > 0 \) is suboptimal because \( x_1 = y_3 = R = 0 \) will yield higher utility (recall that by assumption \( F(u, 0) = F(0, v) = 0 \)).

(d) \( R > 0 \) and \( x_1 = 0 \) and/or \( y_3 = 0 \) is suboptimal because \( \lim_{x \to 0} F'(x, R/2) = \infty \).

(e) From (a)–(d), we infer that as a best response to \( \sigma^* \) each individual must set \( x_1 > 0 \); \( R > 0 \); \( y_3 < 0 \).

(f) Recall that, by Assumption 1(iv), \( \Pi(0, \hat{k}_t) = 0 \), with \( \hat{k}_t \geq 0 \). Point (e) implies that in man’s \( j \)’s best response to \( \sigma^* \) we must have \( g_t \geq 0 \), \( t = 1, 2 \).

(g) Assumption (iv), which imposes \( \Pi_k(\hat{g}_t, k_t) = -\infty \) for \( \hat{g}_t > 0 \), rules out the possibility that \( k_t = 0 \), \( t = 1, 2 \). Indeed, for \( k_t \) small enough and for all \( x_1, R, y_3, g_1, g_2, k_t > 0 \), the value of \( -\Pi_k(g_t, \hat{k}_t) \) is large enough so that increasing \( k_t \) (and decreasing some of the other variables) increases individual \( j \)’s payoff.

(h) Finally, as \( x_1^* = y_3^* \) the optimal response of the mother is \( m^* = 1/2 \).

(i) Selecting \( \sigma^* \) is male \( j \)’s best response when all other individuals (men and women) select \( \sigma^* \). The same argument holds true for a woman’s best response.

From (a)–(i) we conclude that \( \sigma^* \) is an equilibrium. QED

As mentioned before, there are always two other equilibria in which \( m \) is on the boundary. Consider the case in which the mother sets \( m = 0 \). It is easy to verify that the best response of each man is \( x_1 = g_1 = k_1 = 0 \). Likewise \( m = 0 \) is the best response to \( x_1 = g_1 = k_1 = 0 \). Similarly, \( m = 1 \) and \( x_2 = g_2 = k_2 = 0 \) deliver an equilibrium. As emphasized in the text, these equilibria are such that parents provide food only to one cohort of children, starving the other cohort to death. Given the purpose of the paper, we ignore such equilibria.

**Proof of Proposition 2** (Serial Monogamy)

**Adult woman.** The first order condition associated to the woman’s problem is

\[
F_2(\tilde{x}_1, m(\tilde{x}_2 + \tilde{y}_2)) = F_1((1 - m)(\tilde{x}_2 + \tilde{y}_2), \tilde{y}_3).
\]

For \( \tilde{x}_1 > 0 \) and \( \tilde{y}_3 > 0 \), this condition is necessary and sufficient. Let \( z_2 \) be the amount of food a mother gives to her first period children. Then her optimal response is

\[
z_2 = m(\tilde{x}_2 + \tilde{y}_2),
\]
where $0 < m < 1$.

**Adult man.** Assuming $0 < \hat{m} < 1$, the serially monogamous man is programmed to select $x_1, g_1, k_1, x_2, y_2, y_3, g_2, k_2$ to maximize his payoff function $\varphi$ given by

$$\frac{1}{2}\left\{\Pi(g_1, \hat{k}_1) F(x_1, \hat{m}(x_2 + \hat{y}_2)) + [1 - \Pi(\hat{g}_1, k_1)] F(\hat{x}_1, \hat{x}_2)
+ \Pi(g_2, \hat{k}_2) F(1 - \hat{m})(\hat{x}_2 + y_2), y_3) + [1 - \Pi(\hat{g}_2, k_2)] F(\hat{y}_2, \hat{y}_3)\right\},$$

subject to $\omega = x_1 + x_2 + y_2 + y_3 + g_1 + g_2 + k_1 + k_2$ and $0 \leq x_1, g_1, k_1, x_2, y_2, y_3, g_2, k_2$.

The first order conditions of this problem are:

$$\frac{\partial \varphi}{\partial x_1} = F_1(x_1, \hat{m}(x_2 + \hat{y}_2)) \Pi(g_1, \hat{k}_1) + \lambda = 0 \quad (A.12)$$

$$\frac{\partial \varphi}{\partial x_2} = \hat{m} F_2(x_1, \hat{m}(x_2 + \hat{y}_2)) \Pi(g_1, \hat{k}_1) + \lambda = 0 \quad (A.13)$$

$$\frac{\partial \varphi}{\partial y_1} = F(x_1, \hat{m}(x_2 + \hat{y}_2)) \Pi_g(g_1, \hat{k}_1) + \lambda = 0 \quad (A.14)$$

$$\frac{\partial \varphi}{\partial k_1} = -F(\hat{x}_1, \hat{m}(x_2 + \hat{y}_2)) \Pi_k(\hat{g}_1, k_1) + \lambda = 0 \quad (A.15)$$

$$\frac{\partial \varphi}{\partial y_2} = (1 - \hat{m}) F_1((1 - \hat{m})(\hat{x}_2 + y_2), y_3) \Pi(g_2, \hat{k}_2) + \lambda = 0 \quad (A.16)$$

$$\frac{\partial \varphi}{\partial y_3} = F_2((1 - \hat{m})(\hat{x}_2 + y_2), y_3) \Pi(g_2, \hat{k}_2) + \lambda = 0 \quad (A.17)$$

$$\frac{\partial \varphi}{\partial y_2} = F((1 - \hat{m})(\hat{x}_2 + y_2), y_3) \Pi_g(g_2, \hat{k}_2) + \lambda = 0 \quad (A.18)$$

$$\frac{\partial \varphi}{\partial k_2} = -F((1 - \hat{m})(\hat{x}_2 + y_2), \hat{y}_3) \Pi_k(\hat{g}_2, k_2) + \lambda = 0. \quad (A.19)$$

Let the fixed point of this problem be denoted by $\sigma^* = (m^*, x_1^*, g_1^*, k_1^*, x_2^*, y_2^*, y_3^*, g_2^*, k_2^*)$. From (A.12) and (A.13), owing to the symmetry of $F$, it can be verified that

$$F_1(x_1^*, m^*(x_2^* + y_2^*)) = m^* F_2(x_2^*, m^*(x_2^* + y_2^*)) = m^* F_1(m^*(x_2^* + y_2^*), x_1^*).$$

Provided $m^* < 1$, we obtain $F_1(x_1^*, m^*(x_2^* + y_2^*)) < F_1(m^*(x_2^* + y_2^*), x_1^*)$, which leads to

$$x_1^* \neq m^*(x_2^* + y_2^*).$$

Assumption 2 guarantees that $F_{11}(u, v) < 0$ and $F_{12}(u, v) > 0$ for all $(u, v)$. This implies $F_1(u, v)$ is decreasing in the first argument and increasing in the second and thus

$$m^*(x_2^* + y_2^*) < x_1^*. $$

Similarly, from (A.16) and (A.17) we find that

$$(1 - m^*)(x_2^* + y_2^*) < y_3^*. $$

Expressions (A.14) and (A.15) lead to $F(x_1^*, m^*(x_2^* + y_2^*)) \Pi_g(g_1^*, k_1^*) = -F(x_1^*, m^*(x_2^* + y_2^*)) \Pi_k(g_1^*, k_1^*)$, which implies $g_1^*$ and $k_1^*$ are such that $\Pi_g(g_1^*, k_1^*) = -\Pi_k(g_1^*, k_1^*)$. Similarly, equations (A.18) and (A.19) yield $\Pi_g(g_2^*, k_2^*) = -\Pi_k(g_2^*, k_2^*)$. Using Assumption 1, it is easy
Proof of Proposition 3

Use the same strategy. In this equilibrium children are treated in a semi-symmetric fashion. As a unique equilibrium in which \(0 < m < 1\) and in which all men use the same strategy and all women use the same strategy. In this equilibrium children are treated in a semi-symmetric fashion. Therefore, there exists a unique equilibrium in which \(0 < m < 1\) and in which all men use the same strategy and all women use the same strategy. In this equilibrium children are treated in a semi-symmetric fashion. As in the monogamous convention, also here there are two equilibria in which \(x_t^* = g_t^* = k_t^* = 0\), \(t = 1, 2\). As in the previous convention, we ignore such equilibria. QED

Proof of Proposition 3 (Stability)

Let \(\sigma^*\) denote the equilibrium values in the monogamy convention and \(\sigma^{**}\) the equilibrium values in the serial monogamy convention. By contradiction, assume \(F(x_1^*, x_2^*) + F(y_1^*, y_2^*) < F(x_1^*, m^{**}(x_2^* + y_2^*)) + F((1 - m^{**})(x_2^* + y_2^*), y_3^*)\) or equivalently

\[
F(x_1^*, x_2^*) < F(x_1^*, x_2^{**}).
\]  (A.20)

Inequality (A.20) would imply that the total resources devoted to food provision are larger in the \(C_2\) convention, i.e., \(2x_1^* < x_1^{**} + x_2^{**}\). As a result, \(C_2\) is stable while \(C_M\) is not. We show that this leads to a contradiction. Indeed, the symmetric allocation \((x_1^*, x_1^*)\) is the most efficient way to use the amount of resources \(2x_1^*\). Since \(g_1^{**} = k_1^{**} = g_2^{**} = k_2^{**} = g^*\) and \(g_1 = k_1 = g_2 = k_2 = g^*\), it follows that, under (A.20), \(g^{**} < g^*\).

Using Assumption 1 we obtain that \(\Pi_2(g^{**}, k^{**}) > \Pi_2(g^*, k^*)\). Consequently \(F_1(x_1^{**}, x_2^{**})\Pi(g_1^{**}, k_1^{**}) > F_1(x_1^*, x_1^*)\Pi(g_1^*, k_1^*) = F_1(x_1^*, x_1^*)\Pi(g_1^*, k_1^*)\) and \(\frac{1}{2}F_2(x_1^{**}, x_2^{**})\Pi(g_2^{**}, k_2^{**}) > 2F_2(x_1^*, x_2^*)\Pi(g_2^*, k_2^*) = 2F_2(x_1^*, x_2^*)\Pi(g_2^*, k_2^*)\), and \(F_1(x_1^{**}, x_2^{**}) = F_1(x_1^*, x_1^*)\Pi_2(g_1^{**}, k_1^{**}) > F_1(x_1^*, x_1^*)\Pi_2(g_1^*, k_1^*) = F_1(x_1^*, x_1^*)\Pi_2(g_1^*, k_1^*)\). Since \(\Pi_2(g_2^{**}, k_2^{**}) = \Pi_2(g_2^*, k_2^*)\) we obtain

\[
F_1(x_1^{**}, x_2^{**}) > F_1(x_1^*, x_2^*) = F_1(x_1^*, x_1^*)
\]  
\[
F_2(x_1^{**}, x_2^{**}) > 2F_2(x_1^*, x_2^*) = F_2(x_1^*, x_2^*)
\]

Lemma 3 implies that \(x_1^{**} + y_1^{**} < 2x_1^*\). A contradiction. QED

A Class of Functions Satisfying Assumption 1

The set of functions \(\Pi : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+\) that satisfy Assumption 1 is non-empty and contains open and dense subsets in the appropriate topology. For example, consider

\[
P(z) = \frac{1}{2} \left[ \frac{z^2}{(1 - z)^2} \right] + 1
\]

where \(\alpha \in (0, 1/2)\). In this case, it is easily verified that \(\Pi\) satisfies Assumption 1 for all \(\alpha \in (0, 1/2)\).
References


