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# Income distribution and macroeconomics: the persistence of inequality in a convex technology framework

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## Abstract

I show that non-convexities in technology, assumed in the capital market imperfection literature on the relationship between income distribution and economic development, can be replaced by an assumption that the bequest function is convex with respect to income. © 2002 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

The literature on the relationship between income distribution and economic development, which follows the capital market imperfection approach, shows that income inequality affects long-run economic performance provided that the production technology of human capital or final output is non-convex.<sup>1</sup> In particular, Banerjee and Newman (1993), Galor and Zeira (1993), Benabou (1996b), Durlauf (1996), Piketty (1997), Maoz and Moav (1999), Ghatak and Jiang (2000) and Mookherjee and Ray (2000), among others, show that credit constraints combined with investment thresholds prevent poor individuals from investing in human or physical capital, thus generating persistence of poverty.<sup>2</sup>

This paper strengthens the empirical plausibility of the capital market imperfection approach to the relationship between income distribution and economic development. It demonstrates that the

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<sup>1</sup>For a survey of the income inequality and growth literature see Benabou (1996a).

<sup>2</sup>In the model developed by Piketty (1997), the effort level, rather than capital investment, is indivisible. Mookherjee and Ray (2000) show that while inequality persists irrespective of the divisibility of human capital, the multiplicity of steady states requires indivisibilities in the return to education.

non-convexity of the technology can be replaced by an assumption that saving, which is bequeathed to the next generation, is a convex function of income. The convexity of the bequest and savings functions are supported by empirical evidence provided by Menchik and David (1983) and Dynan et al. (2000).<sup>3</sup>

I develop an open economy model, in which the evolution of income within each dynasty in society is governed by a dynamical system that generates a poverty trap equilibrium along with a high-income equilibrium. Poor dynasties, those with income below a threshold level, converge to a low income steady state, whereas dynasties with income above the threshold level converge to a high income steady state. Therefore, if initial average income in society is above the threshold level, then in a more equal society more individuals are above the threshold and more dynasties converge to the high income steady state. In poor economies, however, where average income is below the threshold level, inequality may have a positive effect on economic development.<sup>4</sup>

Interestingly, the mechanism of convex savings that generates a negative impact of inequality on economic performance, can also bring about a positive effect of inequality on output, due to the positive effect of inequality on aggregate saving, as shown by Bourguignon (1981). While in Bourguignon inequality in the wealth distribution generates higher income, in this paper I assume that credit is constrained and reach an opposite result. Thus, while inequality can increase the aggregate savings in the economy, it could lead to lower output, since in an imperfect capital markets environment, capital is not allocated efficiently among individuals.<sup>5</sup>

## 2. The model

Consider a small, open, overlapping-generations economy that operates in a one-good world. Economic activity extends over infinite discrete time in a competitive environment. In each period the economy produces a single homogeneous good that can be used either for consumption or for investment. The good is produced by two factors of production: physical capital, and efficiency units of human capital. The supply of production factors is endogenously determined.

### 2.1. Production and prices

Production occurs within a period according to a concave, constant-returns-to-scale technology. The output produced at time  $t$  uses capital,  $K_t$ , and human capital efficiency units,  $H_t$ , in the production process:

$$Y_t = F(K_t, H_t), \quad (1)$$

<sup>3</sup>See also Moav (2000), where a non-convex budget set, due to the trade-off between child quality and quantity, generates a poverty trap.

<sup>4</sup>Evidence regarding the effect of inequality on growth is not conclusive. See the discussion and survey of the literature by Banerjee and Duflo (2000).

<sup>5</sup>Fishman and Simhon (2001) analyze the effect of income distribution on economic growth in a monopolistically competitive economy, characterized by indivisible occupational choice and saving rates that increase with income. They show that equality contributes to long-run growth by increasing the degree of labor division.

where investment in physical and human capital is made one period in advance.

It is assumed that the world capital rate of return remains constant and is equal to  $R$ . Since the competitive small economy permits unrestricted international capital movements, the amount of capital is adjusted in each period such that  $F_K(K_t, H_t) = R$ . As follows from the properties of the production function, the wage per unit of human capital,  $w$ , is uniquely determined given the rate of return to capital,  $R$ , and is therefore, constant over time.

## 2.2. Individuals

Individuals, within as well as across generations, are identical in their preferences and their technology of human capital formation. They may differ, however, in their initial wealth, inherited from their parents. Hence, due to capital markets imperfection, their investment in human capital could differ as well. In particular, I assume that individuals cannot borrow in order to finance investment in human capital.<sup>6</sup>

An individual lives for two periods, has a single parent, and bears a single child; the parent-child connection creates a dynasty. When an individual is in her first life period, her parent is in the second. In their first life period, individuals invest in their own human capital and supply inelastically their labor in the second period. Individual  $i$ , born in period  $t$ , allocates her second life period income,  $I_{t+1}^i$ , between household consumption,  $c_{t+1}^i$ , and a bequest to the offspring,  $b_{t+1}^i$ . Hence, the budget constraint is given by,

$$I_{t+1}^i = c_{t+1}^i + b_{t+1}^i. \quad (2)$$

Preferences are defined by the utility function:

$$u_t^i = (1 - \beta) \log c_{t+1}^i + \beta \log (\bar{\theta} + b_{t+1}^i), \quad (3)$$

where  $\beta \in (0, 1)$  and  $\bar{\theta} > 0$ . This utility function is designed to generate the convex bequest function. Hence the optimal, non-negative, transfer of individual  $i$  born in period  $t$  is given by,

$$b_{t+1}^i = b(I_{t+1}^i) = \begin{cases} 0 & \text{if } I_{t+1}^i \leq \theta; \\ \beta(I_{t+1}^i - \theta) & \text{if } I_{t+1}^i > \theta; \end{cases} \quad (4)$$

where  $\theta \equiv \bar{\theta}(1 - \beta)/\beta$ .

## 2.3. The formation of human capital

In the first period of their lives, period  $t$ , individuals devote their entire time for the acquisition of human capital (measured in efficiency units). The acquired level of human capital increases if their time investment is supplemented with investment in education. However, even in the absence of real expenditure, individuals acquire one efficiency unit of labor-basic skills.

<sup>6</sup>It is sufficient to assume that the interest rate for borrowers is larger than the marginal return to education,  $\gamma w$ .

In particular, the level of human capital of an individual  $i$ ,  $h_{t+1}^i$ , is an increasing concave function of real resources invested in education,  $e_t^i$ ,<sup>7</sup>

$$h_{t+1}^i = h(e_t^i) = \begin{cases} 1 + \gamma e_t^i & \text{if } e_t^i < \bar{e}; \\ 1 + \gamma \bar{e} & \text{if } e_t^i \geq \bar{e}. \end{cases} \quad (5)$$

It is assumed that the marginal return to human capital, for  $e_t^i < \bar{e}$ , is larger than the marginal return to physical capital,

$$w\gamma > R, \quad (A1)$$

assuring that individuals invest in human capital. Noting that  $w$  is a decreasing function of  $R$ , Assumption (A1) implies that  $R$  is sufficiently low.

#### 2.4. Optimization and the evolution of income

Under Assumption (A1), second life period income,  $I_{t+1}^i$ , is uniquely determined by first life period bequest,  $b_t^i$ ,

$$I_{t+1}^i = I(b_t^i) \equiv \begin{cases} w(1 + \gamma b_t^i) & \text{if } b_t^i < \bar{e}; \\ w(1 + \gamma \bar{e}) + R(b_t^i - \bar{e}) & \text{if } b_t^i \geq \bar{e}. \end{cases} \quad (6)$$

Hence, as follows from (4) and (6), the evolution of income within a dynasty is uniquely determined. That is, as depicted in Fig. 1,  $I_{t+1}^i$  is uniquely determined given  $I_t^i$  by the following dynamical system,

$$I_{t+1}^i = \psi(I_t^i) = \begin{cases} w & \text{if } \beta(I_t^i - \theta) < 0; \\ w(1 + \gamma\beta(I_t^i - \theta)) & \text{if } \beta(I_t^i - \theta) \in [0, \bar{e}]; \\ w(1 + \gamma\bar{e}) + R(\beta(I_t^i - \theta) - \bar{e}) & \text{if } \beta(I_t^i - \theta) > \bar{e}, \end{cases} \quad (7)$$

where  $I_0^i$  is given. Note that,  $I_{t+1}^i = \psi(I_t^i) \geq w$  for all  $I_t^i$ .

Additional restrictions on the parameter values are required in order for the dynamical system to generate multiple income level steady states. First, it is assumed that the income level below which individuals choose a zero bequest,  $\theta$ , is larger than the wage rate,  $w$ ,

$$\theta > w. \quad (A2)$$

Under (A2),  $I_{t+1}^i = \psi(I_t^i) = w$ , for all  $I_t^i \in [w, \theta]$ , assuring that there exist a low income, locally stable, poverty trap steady state,  $\bar{I} = w$ .

Second, it is assumed that the return to human capital,  $\gamma w$ , and its potential magnitude,  $\bar{e}$ , are sufficiently large, such that an individual who received a bequest  $b_t^i = \bar{e}$ , will transfer to her offspring a higher bequest,  $b_{t+1}^i > b_t^i = \bar{e}$ . Therefore it is assumed that:

$$\beta[w(1 + \gamma\bar{e}) - \theta] > \bar{e}, \quad (A3)$$

<sup>7</sup>At some cost to the model's simplicity, this human capital production function could be replaced by a strictly concave function with no effect on the qualitative results.

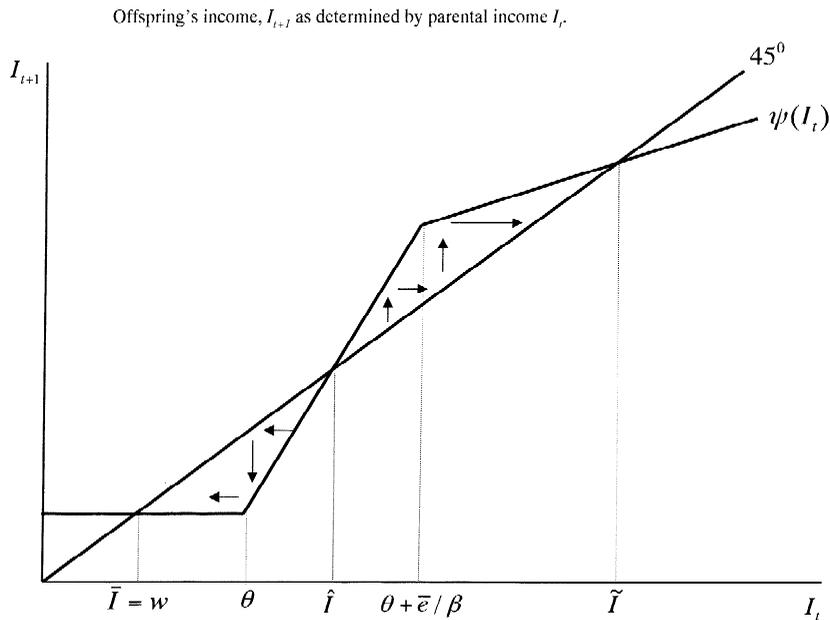


Fig. 1. The evolution of income.

assuring that there exist a range of income, above the poverty trap range of income, in which  $I_{t+1}^i = \psi(I_t^i) > I_t^i$ . Hence, under Assumptions (A2) and (A3) there exists an income threshold,  $\hat{I}$ , such that dynasties with income below the threshold,  $I_t^i < \hat{I}$ , converge to the poverty trap income level, and dynasties with income above the threshold,  $I_0^i > \hat{I}$ , converge to the high income steady state. As follows from the dynamical system in Eq. (7), the threshold is given by,

$$\hat{I} = \frac{(\gamma\beta\theta - 1)w}{\gamma\beta w - 1},$$

where as follows from Assumptions (A2) and (A3)  $\beta\gamma w > 1$  and  $\hat{I} > 0$ . To assure the existence of a high income steady state, rather than a diverging path, it is finally assumed that the return to physical capital,  $R$ , is sufficiently low,

$$R\beta < 1. \tag{A4}$$

Under Assumptions (A2)–(A4),<sup>8</sup> the dynamical system,  $I_{t+1}^i = \psi(I_t^i)$ , depicted in Fig. 1, generates multiple steady states. A poverty trap,  $\bar{I} = w$ , a high income steady state,  $\tilde{I}$ , and a threshold income  $\hat{I} \in (\bar{I}, \tilde{I})$ , where, as follows from (7),  $\tilde{I}$  is given by,

$$\tilde{I} = \frac{w(\gamma\bar{e} + 1) - R(\beta\theta + \bar{e})}{1 - R\beta}.$$

<sup>8</sup> Assumptions (A2)–(A4) assure that  $w\gamma > R$ , and hence they replace Assumption (A1).

Dynasties with initial income below the threshold level,  $\hat{I}$ , converge to a low income steady state,  $\bar{I}$ , characterized by low human capital and a zero bequest. Dynasties with income above the threshold level converge to a high income steady state,  $\bar{I}$  characterized by a high level of human capital and a large bequest. Therefore, initial income inequality persists and, provided that initial average income is above the threshold,  $\hat{I}$ , inequality negatively affects investment in human capital and output in the long run.

Finally, note that the corner solution of zero bequest and thus zero investment in human capital characterizing the poverty trap, is not crucial for the existence of multiple steady states. Consider, for instance, the replacement of the bequest function, given in Eq. (4), with a strictly convex function, such that the bequest level in the poverty trap range, is strictly positive, but low. If the marginal propensity to bequeath in that range is sufficiently low such that its product with the marginal return to human capital,  $\gamma w$ , is less than 1, then the poverty trap may still exist along with a high income equilibrium.

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