Does High Inequality Attract High Skilled Immigrants?

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Abstract

This paper examines how the sources and levels of income inequality affect how a country attracts and retains high skilled workers. With parameter values that yield realistic levels of emigration, our model shows that emigration rates increase with education when the returns to education are higher abroad. However, the relationship between unobservable skills and emigration can display an inverse u-shaped pattern, if unobservable skills are composed of both “general” and “country-specific” skills. Using data on Israeli emigrants before they decide to emigrate, we find strong empirical evidence in support of the model’s predictions.

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1 Introduction

This paper examines whether income inequality within a country attracts highly skilled workers from foreign countries, as well as affecting incentives for highly skilled natives to move abroad. In particular, we analyze how these incentives depend on the sources of inequality, not just the levels. A seminal paper by Borjas (1987) demonstrates that skilled workers are more likely than less-skilled workers to leave a country with a low return to skill and move to a country with a higher return to skill.\footnote{Borjas (1987) builds on the work of Roy (1951) and Sjaastad (1962).} The reverse is also true – a lower skilled individual is more likely than a highly skilled worker to leave a country with a high return to skill and move to a country with a lower return to skill. These predictions are quite intuitive – skilled individuals benefit from high returns to skill since they are at the top of the skill distribution, while less-skilled individuals benefit from lower returns to skill since they are at the bottom of the skill distribution.

Given that the returns to skill are an important component of overall inequality, a more compressed wage distribution tends to encourage the most skilled individuals to leave (“positive selection”), while a more dispersed wage distribution entices lower skilled individuals to leave (“negative selection”). These insights show how incentives for highly skilled individuals to emigrate depend on a country’s relative level of inequality.

Using unique data on Israeli emigrants before they move abroad, we present new evidence about the patterns of emigrant selection along different dimensions of skill. Similar to the case in most countries, the propensity to emigrate from Israel increases with education, which is consistent with the idea that educated individuals are attracted to the higher returns to education in the US, which is the main destination for emigrant Israelis. However, emigration rates do not rise with wages or residual wages (controlling for education, age, occupation, and other controls), despite the higher levels of residual wage inequality in the US. In fact, Israelis in the middle of the residual wage distribution appear the most likely to emigrate – leading to an inverse u-shaped pattern between residual wages and the propensity to emigrate. This finding suggests that the most talented individuals, according to their residual wages, are not necessarily the ones most tempted to move to a country with high wage inequality like the United States.

To examine how the patterns of emigrant selection in terms of education versus residual wages can be different from the same country, we develop and test a model based on the idea that inequality is generated by different types of skills. Some skills are “general”
in the sense that they can be easily transported and rewarded in a foreign country, and some skills are “country-specific” in nature, and therefore, are not easily transferred to another country. Education and intelligence are examples of “general skills” which are likely to be remunerated in any country, while examples of “country-specific” skills include personal connections, local knowledge of market conditions and regulations, language-specific skills, licenses which are country-specific, rents from political ties or labor market rigidities, firm-specific skills (unless the firm operates in both countries), and certain instances of luck (being at the right place at the right time). Although “luck” is typically not considered a form of human capital, it affects a person’s wage and potentially plays an important role in determining how much of an income loss is associated with moving abroad.

While some skills, like education, are observable to the econometrician in many data sets, information about the skill composition of a person’s residual wage is not. As a result, workers with the same residual wage could have very different types of skills, and therefore, face different incentives to emigrate. For example, one worker may have a high residual wage that is due to abundant general skills that could be rewarded in a foreign country, which creates a strong incentive to move to a country with a larger reward for general skills. Meanwhile, a worker with a high residual wage due to abundant country-specific skills will lose most of his success in a foreign country, and thus is unlikely to move abroad.

This framework produces familiar predictions about emigrant selection in terms of an observable general skill like education – countries with a relatively low return to education will lose educated workers to countries with income inequality based on larger returns to education. Our main contribution is to see how this finding interacts with the selection process based on the residual wage. The model shows that the relationship between residual wages and the probability to emigrate can be monotonic or non-monotonic. When the return to unobservable general skills is lower abroad than in the home country, the probability of emigrating declines with residual wages since both components of the residual wage (the general and the country-specific components) are rewarded better in the home country. In other words, even if income inequality in country A is very high, a person with a high residual wage in country B will not be enticed to move there if Country A’s high level of inequality is due to skills which cannot be acquired before moving there.

The more interesting case, however, is when the return to general skills is higher abroad but the parameters are restricted so that emigration rates are strictly positive but not very high, which is consistent with our empirical findings. Under these restrictions, the relationship between residual wages and emigration is an inverse u-shaped function.
That is, inequality in Country A due to higher returns to unobserved general skills will attract individuals from Country B that are likely to be in the middle of the residual wage distribution.

The intuition is straightforward – a higher return to unobservable general skills in a foreign country implies that the benefits of emigration increase with a person’s level of unobservable general skill, but a higher level of unobserved country-specific skill raises the costs of emigration, since these skills will be rendered obsolete. Therefore, a larger amount of unobserved general skill relative to country-specific skill raises the probability that a person emigrates. Furthermore, a larger proportion of individuals have a high relative level of general to country-specific skills in the middle of the residual wage distribution than in the tails. This is due to the fact that those at the bottom of the residual wage distribution have very little of both types of skills, while those at the top have high levels of both. Thus, individuals who are most likely to emigrate, those that have a high level of unobservable general skills versus country-specific skills, are more likely to be in the middle of the residual wage distribution than in the tails.

The basic patterns in our Israeli data support the main predictions of our model. Specifically, the probability of emigrating increases with education, which is consistent with a higher return to education in the US relative to Israel. Also, the overall relationship between residual wages and emigration displays the inverse u-shaped pattern described above. However, we also test the model further by seeing whether the patterns of selection vary systematically with variation across different sectors (industries and occupations) in the differences between Israel and the United States in the returns to observable (education) and unobservable skills. Our findings suggest that this is indeed the case: a lower return to unobservable general skill (proxied by the sector’s residual wage variance) in Israel versus the US entices Israelis with higher residual wages to leave the country. Also, we find that emigrants are more positively selected in terms of their education in industries with a lower relative return to education in Israel versus the US.

Most of the existing literature on immigrant selection examines the selection of immigrants according to levels of education, with a large share dedicated to the study of Mexican immigrants to the US. According to the Borjas model, Mexican emigrants should be negatively selected on education since the return to education is higher in Mexico than the United States. The most prominent paper in this literature is Chiquiar and Hanson (2005), who find that Mexican immigrants to the US come from the middle of the education distribution, which they explain by adding migration costs which decline with education to
the model. As a result, Mexican workers at the bottom of the distribution are less likely to move due to their higher costs of moving, while those at the top of the distribution are less likely to move due to the higher returns to education in Mexico.

In contrast to the Mexican case, Israeli emigrants display “positive selection”, which is similar to the experience of most other countries (Grogger and Hanson (2011)). Therefore, our data is consistent with the basic model with or without the assumption that migration costs decline with education, and this additional assumption cannot help to explain the inverse u-shaped pattern regarding selection on residual wages.

The literature on the selection of emigrants from countries other than Mexico presents a mixed picture in terms of being consistent with Borjas (1987). This is largely due to the general pattern whereby highly educated individuals leave less developed countries with high returns to education and move to developed countries with lower returns to education. This pattern is not consistent with the predictions of the basic model in Borjas (1987). However, there may be many confounding factors in this type of empirical analysis, since there is large variation across countries in many factors which may influence the size and direction of the selection – such as language barriers, proximity, moving costs, immigration policy, visa requirements, etc. This may be one reason why the evidence in favor of the Borjas (1987) model is stronger in studies looking at internal migration (Borjas, Bronars, and Trejo (1992) and Dahl (2002)).

Our study is not affected by cross-country differences in factors which influence the

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2 They argue that migration costs decline with education due to the effect of education on the ability to overcome bureaucratic requirements, the lower time costs required to earn enough money to pay the fixed-costs of moving, and fewer credit constraints on educated individuals. Chiswick (1999) also argues that moving costs decline with education, which tends to produce positive selection in emigrants. Orrenius and Zavodny (2005) find similar results for illegal immigrants from Mexico. McKenzie and Rapoport (2010) show that the selection of Mexican immigrants becomes more negative from areas in Mexico which have lower overall migration costs (i.e. stronger migration networks).

3 This literature includes Borjas (1987), who uses data on U.S. immigrants from 41 countries in the 1970 and 1980 U.S. Censuses and finds weak evidence that the source country’s income inequality is negatively related to immigrant wages. Cobb-Clark (1993) finds similar results for female immigrants. Feliciano (2005) examines 32 immigrant groups in the United States, and finds that all but one group (immigrants from Puerto Rico) are positively selected in terms of education. However, Feliciano (2005) finds an insignificant relationship between inequality and the degree of positive selection from the source country. Grogger and Hanson (2011) examine the sorting of immigrants to 15 OECD countries from 102 source countries and find that immigrants in host countries are positively selected in relation to the source country when the education gap in wages between the host and source countries increases. However, when the education wage gap is measured in logs, they find evidence in favor of negative selection when the return to education is higher in the source country. Belot and Hatton (2012) examine immigrants in 29 OECD countries from 80 source countries and find little evidence in favor of the main prediction in Borjas (1987). Only after considering the poverty constraints in poor countries do they find evidence in support of Borjas (1987).
selection of emigrants, since we exploit variation in the patterns of selection across sectors within one country. Moreover, we examine emigrant selection based on education and unobservable skill (residual wages), and not just on education which is the focus of the studies mentioned above. To do this requires information on the wages of immigrants before they move, which is rare to find.

A notable exception is Akee (2010) who exploits information from a sample of individuals from Micronesia who were found to be immigrants in the US. Akee (2010) finds that immigrants to the US are positively selected in terms of education and log wages, but does not examine whether the relationship is non-linear. However, the results in Akee (2010) demonstrate the importance of using information on immigrants before they move to evaluate the direction and size of immigrant selection – by showing that a conventional approach in the literature that compares the wages of immigrants in the US to the wages of non-immigrants in Micronesia yields the opposite conclusion about immigrant selection. In addition, we build on Akee (2010) by examining whether variation in the returns to education and unobservable skill across sectors can explain the patterns of emigrant selection on each dimension across sectors.

The paper is organized as follows. The next section develops a theoretical model of emigration based on general and “country-specific” skills. Section 3 presents the data on Israeli emigration and the aggregate selection patterns which are consistent with the model. The model’s predictions are tested further in Section 4 by exploiting variation across sectors in the returns to education and unobservable skill in Israel versus the US. Section 5 concludes the analysis.

2 A Model of Emigration with Country-Specific Skills

Building on Borjas (1987), we develop a model that analyzes how the incentives to emigrate interact with the different sources of income and inequality. The model is based on the decision by each individual living in the home country (country 0) whether to emigrate to the host country (country 1). The wage in the home country, \( w_0 \), is determined by the individual’s level of education, \( e \), and by his unobservable skills. Unobservable skills are composed of skills which are country-specific, \( s \), and skills which are more general, \( g \). Country-specific skills are rendered obsolete if the person emigrates, while general skills are rewarded according to the return to general skill in the host country. Education is observable, and following the literature, is considered a general skill. An individual’s wage,
The wage, \( w_0 \), in the source country is modeled by:

\[
w_0 = a_0 + e + g + s,
\]

where \( a_0 \) is the intercept of the wage function and is constant across individuals. Without loss of generality, the returns to each type of skill \( (e, g, \text{and } s) \) are normalized to one in the home country. The two unobservable components, \( s \) and \( g \), are distributed independently with a uniform distribution over the unit interval:

\[
s \sim U[0, 1]; \\
g \sim U[0, 1].
\]

Using the uniform distribution simplifies the analysis, but is not crucial for the qualitative results. Appendix Figures 1-3 present simulations that show that the main results hold for the normal distribution and when we allow for a negative or positive (but not perfect) correlation between \( s \) and \( g \).

In the host country (country 1), the individual’s wage (net of the direct cost of moving) would be:

\[
w_1 = a_1 + \beta_1 e + \gamma_1 g - f,
\]

if the individual decides to emigrate, where \( a_1 \) is the intercept of the wage function in country 1, \( \beta_1 \) and \( \gamma_1 \) are the returns to \( e \) and \( g \) in country 1, and \( f \) is the direct cost of relocating which is considered identical across all individuals.\(^4\) The key assumption is that some skills are valued more in the home country versus the host country, which we simplify by setting the return to unobserved country-specific skills, \( s \), to zero in country 1. If \( \gamma_1 > 1 \), the return to general skills is higher in the host country, otherwise it is lower. If \( \gamma_1 \) is much lower than 1, then “general skills” are essentially country-specific skills. Therefore, the cases of interest that are analyzed below are for a sufficiently large \( \gamma_1 \).

Following the framework developed by Roy (1951) and Borjas (1987), the decision to emigrate is based on income maximization.\(^5\) Therefore, individuals emigrate if and only

\(^4\)Migration costs include costs associated with the move and the time costs of assimilation (Hamermesh and Trejo 2013). Several papers model the cost of moving as a decreasing function of education (Chiswick 1999 and Chiquiar and Hanson 2005), in order to obtain a positive selection of emigrants, despite lower returns to skill in the host country. Adding this assumption to the model does not qualitatively alter the results, and does not help us understand the observed inverse u-shaped pattern of emigration rates in terms of residual wages.

\(^5\)This assumption is consistent with the evidence in Gould and Moav (2007) that Israeli emigrants are motivated largely by economic considerations, and less by political considerations or schooling decisions.
if $w_1 > w_0$. Based on equations (1) and (2), this condition holds if and only if:

$$\beta e + \gamma g > a + s,$$

where $a = (a_0 - a_1) + f$ is the total fixed-cost of emigration (the difference in the constant plus the direct cost of relocating), $\beta = \beta_1 - 1$, and $\gamma = \gamma_1 - 1$. The parameters $\beta$ and $\gamma$ are the differences in the returns between country 1 and country 0 to education and general unobserved skills, respectively. (Note that since $\beta_1 \geq 0$ and $\gamma_1 \geq 0$, it follows that $\beta \geq -1$ and $\gamma \geq -1$). Hence, the right hand side of equation (3), $a + s$, is the total emigration cost including the loss of specific skills and the fixed cost of emigrating (which could be negative), whereas the left hand side, $\beta e + \gamma g$, is the gain from emigration. Naturally, an individual decides to emigrate if the gain is greater than the costs. If $\beta$ or $\gamma$ are negative, then education or general skills respectively contribute to the cost of moving rather than to the gain of moving. For the sake of brevity and simplicity, we restrict the return to general skills in the host country to be less than twice the return in the home country:

$$\gamma < 1.$$  \hspace{1cm} (A1)

This assumption is not crucial and does not limit the model’s range of possible results.

As stated above, an individual’s unobservable component of income in country 0 is represented by $s + g$. This sum is called the individual’s “residual wage,” and is denoted by $\tilde{w}$, i.e.,

$$\tilde{w} = s + g.$$  

Since both $s$ and $g$ are distributed uniformly over the unit interval, it follows that the residual wage, $\tilde{w}$, is distributed between 0 and 2, and under the assumption that the two distributions are independent, the probability density function of $\tilde{w}$ is:

$$f(\tilde{w}) = \begin{cases} \frac{\tilde{w}}{2} & \text{for } \tilde{w} \leq 1; \\ 2 - \tilde{w} & \text{for } \tilde{w} > 1. \end{cases}$$  \hspace{1cm} (4)

We now derive the probability of emigration for any individual with a given level of education $e$ and a given residual wage, $\tilde{w}$. We condition on these two variables since $e$ and $w_0$ are observable to the econometrician, and thus $\tilde{w}$ can be calculated. In contrast, the individual components of the residual wage, $g$ and $s$, are not observable to the econometrician. Since the distributions of $g$ and $s$ are uniform and independent, it follows that for any given $\tilde{w}$, $g$ is uniformly distributed in its feasible range. In particular, the conditional distribution of $g$, given $\tilde{w}$, is:
These properties allow for a straightforward calculation of the probability that an individual with any given $\tilde{w}$ and $e$ will emigrate, which we denote by $p(\tilde{w}, e)$. From equation (3), and noting that $s = \tilde{w} - g$, it follows that:

$$p(\tilde{w}, e) = \mathbb{P}(\gamma g + \beta e > \alpha + s | \tilde{w}).$$

In the following two propositions, we study the properties of the probability of emigration, $p(\tilde{w}, e)$, with respect to $\tilde{w}$. In the first proposition, we focus on the case that $\gamma$ is positive – the return to general skills is higher in the host country. The second proposition examines the opposite case where $\gamma < 0$. Both propositions cover all possible outcomes for any $\beta$ (positive or negative) and any level of education, $e$. In both propositions, the function $p(\tilde{w}, e)$ takes different forms for different levels of $\beta e - a$, which we refer to as the “net education incentive” to emigrate, since it represents the return or cost associated with emigrating that stems only from a person’s education level and fixed-cost of moving. The net education incentive is independent of $\tilde{w}$, and it increases with education if the return to education is higher abroad ($\beta > 0$), and falls with education if the return to education is higher at the home country ($\beta < 0$). Therefore, as the ranges of $\beta e - a$ increase in the propositions below, these ranges refer to increasing levels of education if $\beta > 0$ and decreasing levels of education if $\beta < 0$.

**Proposition 1** *(The properties of $p(\tilde{w}, e)$, when the return to general skills are higher in the host country).* Under Assumption A1 for $\gamma > 0$:

i. For $\beta e - a \leq -\gamma$:

The probability that an individual emigrates is 0.

ii. For $\beta e - a \in (-\gamma, 0)$:

1. The probability that an individual with education $e$ and wage residual $\tilde{w}$ emigrates is:

$$p(\tilde{w}, e) = \begin{cases} 
0 & \text{for } \tilde{w} \leq \frac{a - \beta e}{\gamma} \\
\frac{\gamma \tilde{w} - a + \beta e}{(1 + \gamma) \tilde{w}} & \text{for } \tilde{w} \in \left(\frac{a - \beta e}{\gamma}, 1\right) \\
\frac{1 + \gamma - a + \beta e - \tilde{w}}{(2 - \tilde{w})(1 + \gamma)} & \text{for } \tilde{w} \in [1, \beta e - a + 1 + \gamma) \\
0 & \text{for } \tilde{w} \geq \beta e - a + 1 + \gamma
\end{cases}$$
where:

2. $p(\tilde{\omega}, e)$ is continuous
3. $\frac{a - \beta e}{\gamma} \in [0, 1)$
4. $\beta e - a + 1 + \gamma \in (1, 2)$
5. $p(\tilde{\omega}, e)$ is increasing and concave with respect to $\tilde{\omega}$ for $\tilde{\omega} \in \left(\frac{a - \beta e}{\gamma}, 1\right)$
6. $p(\tilde{\omega}, e)$ is decreasing and concave with respect to $\tilde{\omega}$ for $\tilde{\omega} \in (1, \beta e - a + 1 + \gamma)$

iii. For $\beta e - a \in (0, 1 - \gamma]$

1. The probability that an individual with education $e$ and wage residual $\tilde{\omega}$ emigrates is:

$$p(\tilde{\omega}, e) = \begin{cases} 
\frac{1}{\gamma \tilde{\omega} - a + \beta e} & \text{for } \tilde{\omega} \leq \beta e - a \\
\frac{1}{1 + \gamma - a + \beta e - \tilde{\omega}} & \text{for } \tilde{\omega} \in (\beta e - a, 1] \\
\frac{1}{(2-\tilde{\omega})(1+\gamma)} & \text{for } \tilde{\omega} \in (1, \beta e - a + 1 + \gamma) \\
0 & \text{for } \tilde{\omega} \geq \beta e - a + 1 + \gamma 
\end{cases}$$

where:

2. $p(\tilde{\omega}, e)$ is continuous
3. $\beta e - a \in (0, 1)$
4. $\beta e - a + 1 + \gamma \in (1, 2)$
5. $p(\tilde{\omega}, e)$ is decreasing and convex with respect to $\tilde{\omega}$ for $\tilde{\omega} \in (\beta e - a, 1]$ 
6. $p(\tilde{\omega}, e)$ is decreasing and concave with respect to $\tilde{\omega}$ for $\tilde{\omega} \in (1, \beta e - a + 1 + \gamma)$

iv. For $\beta e - a \in (1 - \gamma, 1)$

1. The probability that an individual with education $e$ and wage residual $\tilde{\omega}$ emigrates is:

$$p(\tilde{\omega}, e) = \begin{cases} 
\frac{1}{\gamma \tilde{\omega} - a + \beta e} & \text{for } \tilde{\omega} \leq \beta e - a \\
\frac{1}{1 + \gamma - a + \beta e - \tilde{\omega}} & \text{for } \tilde{\omega} \in (\beta e - a, 1] \\
\frac{1}{(1+\gamma)(2-\tilde{\omega})} & \text{for } \tilde{\omega} \in (1, (a - \beta e + 1 + \gamma) / \gamma) \\
1 & \text{for } \tilde{\omega} \geq (a - \beta e + 1 + \gamma) / \gamma 
\end{cases}$$

where:

2. $p(\tilde{\omega}, e)$ is continuous
3. $\beta e - a \in (1 - \gamma, 1)$
4. $(a - \beta e + 1 + \gamma) / \gamma \in (1, 2)$
5. $p(\tilde{\omega}, e)$ is decreasing and convex with respect to $\tilde{\omega}$ for $\tilde{\omega} \in (\beta e - a, 1]$ 
6. $p(\tilde{\omega}, e)$ is increasing and convex with respect to $\tilde{\omega}$ for $\tilde{\omega} \in (1, (a + \gamma - \beta e + 1) / \gamma) 

v. For $\beta e - a \geq 1$

The probability that an individual emigrates is 1.
The Proposition follows from (5) and (6).

Proposition 1 states that, for \( \gamma > 0 \), the shape of the function \( p(\tilde{w}, e) \) changes with \( \beta e - a \). For \( \beta > 0 \), an increase in the relative return to education in the host country, \( \beta \), or the level of education, \( e \), will increase the probability of emigration and change the shape of the emigration function with respect to the wage residual, \( \tilde{w} \). For \( \beta < 0 \), the same shift occurs with a decline in education.

In particular, moving from part \( i \) to \( v \) of the proposition as \( \beta e - a \) increases: no one emigrates when \( \beta e - a \leq -\gamma \); the probability to emigrate, \( p(\tilde{w}, e) \), is an inverse u-shaped function with respect to the residual wage, \( \tilde{w} \), for \( \beta e - a \in (-\gamma, 0] \); \( p(\tilde{w}, e) \) monotonically declines with \( \tilde{w} \) for \( \beta e - a \in (0, 1 - \gamma] \); \( p(\tilde{w}, e) \) is u-shaped with respect to \( \tilde{w} \) for \( \beta e - a \in (1 - \gamma, 1) \); and finally, all individuals emigrate for \( \beta e - a \geq 1 \).

To better understand these patterns, consider individuals that are at the extreme levels of the residual wage distribution. For individuals with the lowest possible residual wage (\( \tilde{w} = 0 \)), both unobservable skills are at their minimum values: \( s = g = 0 \). For these individuals, there is no variation in the composition of unobservable skills, so they all make the same decision for any level of education (i.e. for any level of \( \beta e - a \)). There is a discrete jump from a probability zero of emigration to a probability of one when the education level, \( e \), crosses the threshold \( a/\beta \). Similarly, for the highest possible residual wage (\( \tilde{w} = 2 \)), both unobservable skills are at their highest possible values: \( s = g = 1 \). For these individuals, there is again no variation in the composition of unobservable skills, so they all make the same decision: a discrete jump from a probability zero of emigration to one as \( e \) crosses above the threshold \( (a + 1 - \gamma) / \beta \). Changes in the probability of emigrating are more gradual with changes in \( e \) as the residual wage is closer to the middle of the distribution (\( \tilde{w} \) is closer to one). This is the case because there is more variation in the composition of the residual near the center of the distribution.

This helps understand our model’s results, in particular, the inverse u-shape relationship between the residual wage and the emigration probability that is found in our data and predicted by our model for \( \beta e - a \in (-\gamma, 0] \), and \( \gamma > 0 \). A necessary condition for a non-monotonic relationship is that some unobserved skills are rewarded more in the home country while others are rewarded more in the host country, which is the case with \( \gamma > 0 \). In addition, when \( \beta e - a \in (-\gamma, 0] \), the “net education incentive” is not sufficient to induce emigration by itself. To induce emigration, a person’s incentive to emigrate must be augmented by a sufficiently high level of general skill, which is rewarded more in the host country. At the same time, high levels of country-specific skills would work in the
opposite direction, reducing the probability of moving. Therefore, individuals who decide to emigrate are those with relatively high general skills, $g$, and low country-specific skills, $s$. Individuals with a low residual wage have low levels of both general and country-specific skills ($g$ and $s$), and therefore, do not emigrate. Similarly, individuals with a high residual wage have high levels of both general and country-specific skills, and therefore, have low rates of emigration. Individuals with a high $g$ and low $s$ - those who would benefit the most from emigration - are more likely to be found in the middle of the residual wage distribution.

To be more specific, the middle of the residual wage distribution contains individuals with varying levels of $s$ and $g$. Those with similar levels of both unobserved skills, $s$ and $g$, or high levels of $s$ versus $g$, will behave similar to those at the tails of the distribution since the loss of country-specific skills and the fixed-costs of moving will prevent them from leaving. However, in the middle of the distribution, there is also a group of individuals with high levels of $g$ relative to $s$. For those individuals, the return to emigration is high enough to produce the largest rate of emigration within the population. As a result, the rate of emigration is larger in the middle of the residual wage distribution for moderate levels of $\beta e - a$.

It is important to note that the inverse u-shape pattern of emigration crucially depends on our assumption that both types of unobservable skills ($g$ and $s$) play important roles in determining the variation of residual wages. If the variation or magnitudes of country-specific skills are negligible, an increase in the residual wage is largely a product of more general skills. In this case, the emigration probability monotonically increases with the wage residual when $\gamma > 0$ (and decreases if $\gamma < 0$). Thus, the inverse u-shape is one possible outcome, which depends on the net education incentive lying in a moderate range ($\beta e - a \in (-\gamma, 0]$) and the existence of significant variation across individuals in both types of unobservable skills – general and country-specific.

When the net education incentive to emigrate ($\beta e - a$) increases beyond 0, Proposition 1 states (for $\gamma > 0$) that the inverse u-shaped pattern turns into a decreasing function followed by a u-shaped function. This is the case where the net education incentive to

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$^6$It should be noted that the inverse u-shaped relationship between the residual wage and the probability of emigration does not depend on our assumption that the two components of the residual, $s$ and $g$, are not correlated. The u-shape patterns persists unless the two are perfectly, positively correlated (which would produce positive selection). However, the u-shaped pattern becomes flatter when the correlation becomes more positive. Interestingly, a negative correlation will strengthen the inverse u-shape pattern, since a more negative correlation would increase the variance in the distribution of $s$ and $g$ in the middle of the distribution of their sum, and hence a higher probability of $g$ exceeding its threshold level for triggering emigration.
emigrate is relatively strong, and therefore, a high $s$ relative to $g$ is required to prevent emigration. Similar to the logic behind the inverse u-shape, individuals with a high $s$ relative to $g$ are more likely to be found in the middle of the residual distribution relative to the tails, thus creating a u-shaped function. This finding implies that there could exist some levels of education for which a majority of individuals decide to emigrate, in particular those with high or low wage residuals.

The reason for the monotonic decline in the probability of emigration with the wage residual for low, but positive, levels of the net education incentive ($\beta e - a$) follows. For individuals with $\tilde{w} = 0$, there is no loss or gain from unobserved skills when emigrating, and therefore, they decide to move when $\beta e - a > 0$. Individuals at the other end of the distribution, with $\tilde{w} = 2$, do not move as the gain from the higher return on their high general skills is insufficient to compensate for the loss of country specific skills when emigrating. This is an outcome of the fact that an emigrant loses the entire return on specific skills and gains only the difference in the return to general skills between the host country and the home country, which we restrict to less than one.

However, a further increase in $\beta e - a$ will induce individuals with $\tilde{w} = 2$ to emigrate and the pattern of emigration will then become u-shaped. For a sufficiently high $\beta e - a$, such that individuals with the highest residual wage choose to emigrate, only individuals with low general skills and high country specific skills would not emigrate. As mentioned above, these individuals are located at the middle of the residual wage distribution, and therefore, a u-shaped pattern emerges.

Finally, in the last part of Proposition 1, the net education incentive to emigrate is so high ($\beta e - a \geq 1 - \gamma$) that considerations stemming from the residual wage are not important, leading all individuals to emigrate.

The following proposition examines the case where the return to unobserved general skill is higher in the home country ($\gamma < 0$).

**Proposition 2** *(The properties of $p(\tilde{w}, e)$ when the return to general skills are higher in the home country).* Under Assumption A1 for $\gamma < 0$:

i. For $\beta e - a \leq 0$

The probability that an individual emigrates is 0.

ii. For $\beta e - a \in (0, -\gamma)$
1. The probability that an individual emigrates is:

\[ p(\tilde{w}, e) = \begin{cases} 
1 & \text{for } \tilde{w} \leq \beta e - a \\
\frac{1}{\gamma \tilde{w} + \beta e - a} & \text{for } \tilde{w} \in (\beta e - a, \frac{\beta e - a}{\gamma}) \\
0 & \text{for } \tilde{w} \geq \frac{\beta e - a}{\gamma}
\end{cases} \]

where:

2. \( p(\tilde{w}, e) \) is continuous
3. \( \beta e - a \in (0, -\gamma) \)
4. \( (\beta e - a) / - \gamma \in (\beta e - a, 1) \)
5. \( p(\tilde{w}, e) \) is decreasing and convex with respect to \( \tilde{w} \) for \( \tilde{w} \in (\beta e - a, \frac{\beta e - a}{\gamma}) \)

iii. For \( \beta e - a \in [-\gamma, 1) \)
1. The probability that an individual emigrates is:

\[ p(\tilde{w}, e) = \begin{cases} 
1 & \text{for } \tilde{w} \leq \beta e - a \\
\frac{1}{1 + \gamma + \beta e - a - \tilde{w}} & \text{for } \tilde{w} \in [1, \beta e - a + 1 + \gamma) \\
0 & \text{for } \tilde{w} \geq \beta e - a + 1 + \gamma
\end{cases} \]

where:

2. \( p(\tilde{w}, e) \) is continuous
3. \( \beta e - a \in (0, 1) \)
4. \( \beta e - a + 1 + \gamma \in [1, 2) \)
5. \( p(\tilde{w}, e) \) is decreasing and convex with respect to \( \tilde{w} \) for \( \tilde{w} \in (\beta e - a, 1) \)
6. \( p(\tilde{w}, e) \) is decreasing and concave with respect to \( \tilde{w} \) for \( \tilde{w} \in (1, \beta e - a + 1 + \gamma) \)

iv. For \( \beta e - a \in [1, 1 - \gamma) \)
1. The probability that an individual emigrates is:

\[ p(\tilde{w}, e) = \begin{cases} 
1 & \text{for } \tilde{w} \leq \frac{\beta e - a - 1 - \gamma}{-\gamma} \\
\frac{1 + \gamma - a + \beta e - \tilde{w}}{(2 - \tilde{w})(1 + \gamma)} & \text{for } \tilde{w} \in \left(\frac{\beta e - a - 1 - \gamma}{-\gamma}, \beta e - a + 1 + \gamma\right) \\
0 & \text{for } \tilde{w} \geq \beta e - a + 1 + \gamma
\end{cases} \]

where:

2. \( p(\tilde{w}, e) \) is continuous
3. \( \frac{\beta e - a - 1 - \gamma}{-\gamma} > 1 \)
4. \( \beta e - a + 1 + \gamma > \frac{\beta e - a - 1 - \gamma}{-\gamma} \)
5. \( \beta e - a + 1 + \gamma \leq 2 \)
6. \( p(\tilde{w}, e) \) is decreasing and concave with respect to \( \tilde{w} \) for \( \tilde{w} \in \left(\frac{\beta e - a - 1 - \gamma}{-\gamma}, \beta e - a + 1 + \gamma\right) \)

v. For \( \beta e - a \geq 1 - \gamma \)
The probability that an individual decides to emigrate is 1.
The Proposition follows from (5) and (6).

In the case studied in Proposition 2, where the return to unobserved general skill is lower in the host country \((\gamma < 0)\), the relationship between the residual wage and the emigration probability is (weakly) negative. In this scenario, both types of unobservable skill \((g\) and \(s)\) receive a higher reward in the home country, and thus, larger combinations of these two skills (i.e., a larger residual wage) lowers the emigration probability. Therefore, the highest residual wage will have the lowest probability of emigrating when the return to general skill is higher at home.

As noted above, there is no variation in the combination of unobservable skills \((s\) and \(g)\) when the residual wage is at the minimum or maximum values. In these corners, every individual, given \(\beta e - a\), makes the same decision. Therefore, if the net education incentive \((\beta e - a)\) is high enough to induce the least likely individual (with the maximum residual wage) to emigrate, then all individuals at that education level will emigrate. Similarly, if the net education incentive is too low to induce the most likely person to emigrate (someone with the minimum residual wage), then no one with that level of education will emigrate. Between these two extreme cases, for any given level of education, individuals with the maximum residual wage do not emigrate, those with the minimum residual wage emigrate, and the probability to emigrate monotonically declines with the wage residual, \(\tilde{w}\).

Propositions 1 and 2 indicate that a non-monotonic relationship between the residual wage and the emigration probability can only be obtained if some skills have a higher return in the home country (i.e., “country specific skill”) while other skills are rewarded more in the host country (“general skills”). However, even when \(\gamma < 0\) (Proposition 2), the relationship between total wages and the emigration probability is not necessarily monotonic (for \(\beta > 0\)), as a higher wage that is a result of higher education will increase the emigration probability whereas a higher wage resulting from a higher residual will decrease this probability.

We now examine the effect of education on the shape of the probability function of emigrating, \(p(\tilde{w}, e)\). Clearly, if the return to education is higher in the host country \((\beta > 0)\), the model will generate positive selection on education: the probability of emigration is (weakly) increasing with education. The opposite is true for \(\beta < 0\). The next proposition, however, studies how this pattern interacts with the residual wage. To do this, we examine the derivatives of \(p(\tilde{w}, e)\) with respect to \(e\) and the cross derivative with respect to \(e\) and \(\tilde{w}\). We study these derivatives for \(p(\tilde{w}, e) \in (0, 1)\) since they are zero otherwise.
Proposition 3 (The effect of $e$ on $p(\tilde{w}, e)$). For $p(\tilde{w}, e) \in (0, 1)$:

1. 
\[
\frac{dp(\tilde{w}, e)}{de} = \begin{cases} 
\frac{\beta}{(1+\gamma)\tilde{w}} & \text{for } \tilde{w} < 1 \\
\frac{\beta}{(1+\gamma)(2-\tilde{w})} & \text{for } \tilde{w} > 1
\end{cases}
\]

2. 
\[
\frac{d^2p(\tilde{w}, e)}{ded\tilde{w}} = \begin{cases} 
\frac{-\beta}{(1+\gamma)\tilde{w}^2} & \text{for } \tilde{w} < 1 \\
\frac{\beta}{(1+\gamma)(2-\tilde{w})^2} & \text{for } \tilde{w} > 1
\end{cases}
\]

The proposition follows directly from Propositions 1 and 2.

Proposition 3 states that the probability of emigrating increases with the level of education if $\beta > 0$. In other words, emigrants are positively selected in terms of their education if the return to education is higher in the host country. This result and its mechanism is similar to the prediction in Borjas (1987). However, when $\beta > 0$, the proposition demonstrates that the marginal effect of $e$ on $p(\tilde{w}, e)$ is decreasing with $\tilde{w}$ for $\tilde{w} < 1$, and increasing for $\tilde{w} > 1$. Therefore, for the parameters for which an inverse u-shaped pattern of $p(\tilde{w}, e)$ with respect to $\tilde{w}$ emerges (established in Proposition 1), the probability of emigration is shifting up and flattening out as $e$ increases. Indeed, as shown in Proposition 1, the inverse u-shape turns into a u-shaped pattern for higher levels of $e$.

We now examine how emigration rates across different levels of $\tilde{w}$ change as the return to $g$ increases in the host country relative to the home country. That is, we examine the effect of $\gamma$ on emigration. It is straightforward to infer from Propositions 1 and 2 that $p(\tilde{w}, e)$ is increasing with $\gamma$. However, the next proposition establishes an interesting result that under the conditions that $p(\tilde{w}, e)$ has an inverse u-shape with respect to $\tilde{w}$, the distribution of $p(\tilde{w}, e)$ shifts to the right when $\gamma$ increases while holding constant the probability of emigration for individuals with a mean wage residual (by raising the fixed cost of emigration). Thus, the proposition performs a type of “mean preserving spread” exercise.

Proposition 4 (“mean preserving spread”). Under assumption A1, for $\gamma > a - \beta e > 0$: A rise in the return to unobservable general skills in the host country versus the source country, $\gamma$, and in the fixed cost of emigrating, $a$, such that $p(1, e)$ is held constant, generates a decline in $p(\tilde{w}, e)$ for $\tilde{w} < 1$, and a rise in $p(\tilde{w}, e)$ for $\tilde{w} > 1$.

\[\text{If } \beta > 0 \text{ and } \gamma < 0, \text{ and in the fixed cost of emigrating, } a, \text{ such that } p(1, e) = 0 \text{, then } p(\tilde{w}, e) \text{ is decreasing with respect to } \tilde{w} \text{ (established in Proposition 2), and the absolute value of the slope is increasing for } \tilde{w} < 1 \text{ and decreasing for } \tilde{w} > 1 \text{ as } e \text{ increases. The reverse is true for } \beta < 0 \text{ and a sufficiently large } \gamma \text{ (otherwise } p(\tilde{w}, e) = 0).\]
The proof of the proposition is presented in the Appendix.

It follows from Proposition 4 that a rise in $\gamma$, while holding $p(1; e)$ constant, generates a shift to the right in $p(\tilde{w}; e)$ around the point $p(1; e)$. In other words, a higher relative return to unobservable general skills abroad, while holding $p(1; e)$ constant, maintains the overall inverse u-shaped pattern of emigration according to residual wages, but shifts the whole curve to the right (raising emigration rates for those with high residuals and lowering the rate for low residual wages).

The intuition for this result is as follows. For a given education level, $e$, the decision to emigrate depends on both $g$ and $s$. The benefit of emigration increases with $g$, while the costs increase with $s$. Therefore, as $\tilde{w}$ increases, the sum of $s$ and $g$ increases, which implies that the threshold level of $g$, above which individuals choose to emigrate, increases with $\tilde{w}$. To derive this formally, the condition for a person with any given $\tilde{w}$ to decide to emigrate, given by equation (3), can be re-written as,

$$\beta e + \gamma g > a + \tilde{w} - g,$$

where $\tilde{w} - g = s$. Re-arranging implies that $\beta e + (1 + \gamma) g > a + \tilde{w}$. Therefore, for any $\tilde{w}$ and $e$, there exists a threshold level of $g$, denoted by $\hat{g}(\tilde{w}, e)$, such that equation (3) holds with equality:

$$\hat{g}(\tilde{w}, e) = \frac{a - \beta e + \tilde{w}}{1 + \gamma}.$$

Hence, the threshold level of $g$ is increasing with $\tilde{w}$, and therefore, a rise in the difference in the return to general skills, $\gamma$, has a stronger impact on individuals with larger residual wages, $\tilde{w}$. This can also be seen from equation (3), where the marginal benefit of emigrating with respect to $\gamma$ is equal to $g$. Therefore, those with higher $g$ are more sensitive to changes in $\gamma$. This general result is independent of the specific “mean preserving spread” exercise performed in the proposition.8

Finally, we examine how the selection of emigrants is affected by changes in the relative returns to education between the two countries. To do this, we derive the probability of emigrating as a function of education:

$$p(e) = \int_0^2 p(\tilde{w}, e) f(\tilde{w}) d\tilde{w}. \quad (7)$$

8A rise in the fixed cost, $a$, has a negative effect on the emigration probability. However, this effect declines (in absolute value) with $\tilde{w}$ for $\tilde{w} < 1$, in particular for $\tilde{w} \in \left(\frac{a - \beta e}{\gamma}, 1\right)$, $\frac{d^2 p(\tilde{w}, e)}{d a d\tilde{w}} = \frac{1}{(w^{\gamma+1})} > 0$. For $\tilde{w} > 1$ the opposite is true. For $\tilde{w} \in (1, 1 + \gamma - (a - \beta e))$, $\frac{d^2 p(\tilde{w}, e)}{d a d\tilde{w}} = -\frac{1}{(\gamma+1)w^{\gamma+1}} < 0$. For $\tilde{w} < 1$, the effect of a rise in $a$ on $\hat{g}(\tilde{w}, e)$ is not monotone.
We start with the case studied in Proposition 1 in which the return to general unobservable skills is higher in the host country, $\gamma > 0$.

**Proposition 5** (The properties of $p(e)$ when the return to general skills are higher in the host country). Under assumption A1, for $\gamma > 0$:

1. The probability of emigrating as a function of education is:

   $$
p(e) = \begin{cases} 
   0 & \text{for } \beta e - a \leq -\gamma \\
   \frac{(\gamma - a + \beta e)^2}{2\gamma} & \text{for } \beta e - a \in (-\gamma, 0] \\
   \frac{\gamma - 2 - a + \beta e}{2\alpha - a - 2a - \beta^2 + 2\alpha + 2\gamma - 1} & \text{for } \beta e - a \in (0, 1 - \gamma] \\
   \frac{1}{2\gamma} & \text{for } \beta e - a \geq 1
   \end{cases}
$$

where:

2. $p(e)$ is continuous in $e$.

3. The probability of emigration is increasing with education if $\beta > 0$ and decreasing if $\beta < 0$; it is convex with respect to education for low levels of education and concave for high levels of education:

   $$
p'(e) = \begin{cases} 
   = 0 & \text{for } \beta e - a < -\gamma \\
   > 0 & \text{for } \beta e - a \in (-\gamma, 1); \beta > 0 \\
   < 0 & \text{for } \beta e - a \in (-\gamma, 1); \beta < 0 \\
   = 0 & \text{for } \beta e - a > 1
   \end{cases}
$$

   $$
p''(e) = \begin{cases} 
   = 0 & \text{for } \beta e - a < -\gamma \\
   > 0 & \text{for } \beta e - a \in (-\gamma, 0) \\
   = 0 & \text{for } \beta e - a \in (0, 1 - \gamma) \\
   < 0 & \text{for } \beta e - a \in (1 - \gamma, 1) \\
   = 0 & \text{for } \beta e - a > 1
   \end{cases}
$$

The proposition follows directly from equations (4) and (7) and Proposition 1.

Proposition 5 states, once again, that emigrants are positively selected in terms of education if $\beta > 0$ and negatively if $\beta < 0$. It further shows that the shape of the relationship between education and the probability of emigrating is convex for low levels of education and concave for high levels of education.

We now study the case of a lower return to general skills in the host country, $\gamma < 0$.

**Proposition 6** (The properties of $p(e)$, when the return to general skills are higher in the home country). Under Assumption A1, for $\gamma < 0$:  

17
1. The probability of emigrating as a function of education is:

\[
p(e) = \begin{cases} 
0 & \text{for } \beta e - a \leq 0 \\
\frac{-\alpha^2}{\gamma^2} & \text{for } \beta e - a \in (0, -\gamma) \\
\gamma/2 + \beta e - a & \text{for } \beta e - a \in [-\gamma, 1) \\
\frac{a^2 - 2\alpha e - 2\alpha^2 + 2e\gamma^2 + 2e\beta e - \gamma^2 + 1}{\gamma^2} & \text{for } \beta e - a \in [1, 1 - \gamma) \\
1 & \text{for } \beta e - a \geq 1 - \gamma 
\end{cases}
\]

where:

2. \( p(e) \) is continuous in \( e \)

3. The probability of emigration is increasing with education if \( \beta > 0 \) and decreasing if \( \beta < 0 \); it is convex with respect to education for low levels of education and concave for high levels of education:

\[
p'(e) = \begin{cases} 
= 0 & \text{for } \beta e - a < 0 \\
> 0 & \text{for } \beta e - a \in (0, 1 - \gamma); \beta > 0 \\
< 0 & \text{for } \beta e - a \in (0, 1 - \gamma); \beta < 0 \\
= 0 & \text{for } \beta e - a > 1 - \gamma 
\end{cases}
\]

\[
p''(e) = \begin{cases} 
= 0 & \text{for } \beta e - a < 0 \\
> 0 & \text{for } \beta e - a \in (0, -\gamma) \\
= 0 & \text{for } \beta e - a \in (-\gamma, 1) \\
< 0 & \text{for } \beta e - a \in (1, 1 - \gamma) \\
= 0 & \text{for } \beta e - a > 1 - \gamma 
\end{cases}
\]

The proposition follows directly from equations (4) and (7) and Proposition 2.

Proposition 6 shows that the qualitative shape of \( p(e) \) is independent of the sign of \( \gamma \). As in Proposition 5, also when \( \gamma < 0 \), emigrants are positively selected in terms of education if \( \beta > 0 \) and negatively if \( \beta < 0 \), and the relationship between education and the probability of emigrating is convex for low levels of education and concave for high levels of education.

Overall, the model developed in this section shows that the combination of education and unobservable skills (general and country-specific) can generate rich patterns of emigrant selection, including a non-monotonic effect of the residual wage on emigration rates. Specifically, the model produces an inverse u-shape pattern between the wage residual and emigration (Proposition 1) for moderate levels of the net education incentive to emigrate: \( \beta e - a \in (\gamma, 0] \). In that range, a higher return to education in the host country (\( \beta > 0 \)) is not sufficient for (most) educated individuals to emigrate. High unobserved general skills and low unobserved country-specific skills are also required. This gives rise to the inverse u-shaped relationship between emigration and residual wages. In addition, for the same
range of $\beta e - a \in (-\gamma, 0]$ and when $\gamma > 0$ and $\beta > 0$, Proposition 5 predicts an increasing and convex relationship between education and emigration. Furthermore, the probability of emigration increases with the relative return to education in the host country, $\beta$, and so does the marginal effect of education: the cross derivative of $p(e)$ with respect to $e$ and $\beta$ is positive.

These results demonstrate how individuals with the same residual wage can face very different incentives to emigrate, depending on the skill composition of their unobservable skills. Moreover, the model shows how there may be a non-monotonic relationship between emigration and skill based on many factors, but the basic premise of the Borjas model is still relevant – an increase in the returns to skill abroad shifts the emigration curve in a way that intensifies the level of positive selection for both education and the residual wage. In the next section, we examine the empirical relevance of these predictions using unique Israeli data on potential emigrants before they move.

Our focus, however, will be on the range of parameters that give rise to the inverse u-shaped relationship between emigration and residual wages, which is what we observe in the data. In addition, this range of parameters yields additional predictions that are empirically supported: (1) emigration rates are not zero for any particular education level or residual wage; (2) emigration rates are not very high for any particular education level or residual wage; (3) the return to education is higher in the foreign country relative to the host country; and (4) the return to unobserved general skill is higher in the foreign country relative to the host country.

As we show in the next section, support for the last two predictions come from our findings that the return to education is estimated to be higher for the US relative to Israel, and the residual variance is higher in the US as well. The latter finding is indicative of higher returns to unobserved general skill in the US versus Israel – an inference which is consistent with the idea that the returns to observable (education) and unobservable general skills behave similarly across countries. For all these reasons, the range of parameters which yield the inverse u-shape relationship between emigration and residual wages appears to be the most relevant for our empirical analysis.

In the next section, we describe our unique data on Israeli emigration, and show how the observed patterns of selection are consistent with our model’s findings. We test the model’s predictions further in Section 4 by exploiting variation across sectors in the returns to education and unobserved skill between Israel and the US.
3 The Data

The empirical analysis uses a unique data set composed of the 1995 Israeli Census merged with an indicator for whether each respondent died or left the country as of 2002 and as of 2004. If the person is considered a “mover” (a person who has left Israel), then the data set contains variables indicating the month and year when this person is considered to have left the country permanently. Information on whether the person left the country is obtained by the Israeli border police, which closely monitors who is coming and going at the country’s points of entry and exit. Therefore, one advantage of this data is that we do not have to worry about counting illegal immigrants, which is typically very difficult to do using governmental data in the host country (see Hanson and Spilimbergo (1999) and Hanson (2006)).

Defining who is an emigrant is not straightforward, since many individuals do not know themselves if they will stay abroad for the long-term. As a result, any definition of a “mover” is somewhat arbitrary. In our analysis, we use the official definition used by the Israel Central Bureau of Statistics, which considers any individual as a “mover” if he/she left the country for at least a full year. By design, the variable for being a “mover” is intended to capture a long-term absence from the country. According to the algorithm used by the Central Bureau of Statistics, a short visit back to Israel in the midst of a long-term absence does not change the status from “mover” to being a “non-mover.”

There are a few potential weaknesses of the data worth noting. First, the data set does not indicate why a person leaves and whether the person intends to come back or not. Therefore, although we mainly use the given measure of a “mover” throughout our analysis, we check the robustness of our results by using a more restrictive definition of “mover” – those who were considered a mover in 2002 and 2004. In addition, as we discuss below, there is strong evidence to believe that the measure we received is picking up moves which are indeed long term. A second weakness in the data is that it does not contain information on where the person is living if he/she resides outside of Israel. However, the Global Migrant Origin Database Version 4.0 indicates that the United States is by far the most likely destination for Israeli emigrants. Therefore, we treat the United States as

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9 The Israel Central Bureau of Statistics received information from the Interior Ministry about who is leaving the country, which the Interior Ministry collects at the airports and borders according to the personal identification number.

10 The data in the Global Migrant Origin Database Version 4.0 is problematic since it is based on country of birth, and many Israelis were not born in Israel. Also, it does not distinguish between Jewish and Arab Israelis, while this study focuses on the emigration status of the Jewish population. However, according to the database, 122,591 Israelis moved to the US and the next highest country (excluding Arab countries) is
the “host” country of interest. To the extent that the United States is not the actual destination for a particular emigrant, this should bias the results towards zero.

Descriptive statistics for the main variables of interest from the 1995 Israeli Census are presented in Table 1. Since the focus of the paper is to determine the selection of emigrants in terms of their education and wages, we restrict the analysis to males with a strong attachment to the labor force, who are old enough to finish their schooling (Israelis typically finish their BA studies in their late twenties) but young enough so that they are not leaving for retirement purposes. Specifically, we restrict the sample to Jewish males between the ages of 30 and 45 who were not self-employed (so their income measure is reliable), worked at least 30 hours a week, worked at least six months in the previous year, and are not ultra-orthodox.\textsuperscript{11}

\textbf{TABLE 1 HERE}

Table 1 shows that the overall rate of emigration as of 2004 in this sample stands at 1.6 percent, increasing from 1.3 percent in 2002. Over 67 percent of those characterized as “movers” in 2004 emigrated by the end of 2000. In addition, only two percent of those characterized as a “mover” in 2002 returned to Israel by the end of 2004. So, our measure of a “mover” appears to be picking up longer term stays abroad.

Although the overall rate of emigration appears to be rather low at 1.6 percent, there are stark differences across levels of education and wages. Figure 1 presents the rates of emigration across different levels of education for native-born Israelis. The rate varies from 0.65 percent for the least educated to 2.6 percent for those with an MA degree or higher. Figures 2 shows a similar patterns for non-natives, although the magnitudes are much higher.\textsuperscript{12}

\textbf{FIGURES 1 AND 2 HERE}

\begin{flushleft}
\textsuperscript{11} The self-employed represent about ten percent of the population of interest, and their rate of emigration is 1.2 percent compared to 1.6 percent for the non-self-employed sample. The lower rate of emigration for the self-employed is largely the result of lower emigration rates for those with more than a college degree: 2.9 for the non-self-employed versus 1.6 for the self-employed. This latter result is most likely due to the idea that self-employed people, especially the most educated, possess more country-specific skills versus salaried workers.

\textsuperscript{12} Gould and Moav (2007) show that the rate of emigration is very high for professors, scientists, doctors, and engineers. All of these groups are at least three times higher than teachers or workers in all the rest of the occupations. Additional evidence that Israel suffers from a “Brain Drain” is presented in Ben-David (2008).
\end{flushleft}
The positive relationship between education and the rate of emigration is consistent with the predictions of the previous section and the Borjas model if the returns to education are higher in the US versus Israel. The findings in Table 2 suggest that this is indeed the case. Using a similar sample from the majority population in the US and Israel in the same period, the estimated return to a year of schooling is 10.0 percent for the US and only 7.1 percent for Israel.\footnote{To estimate comparable returns to education in both countries, we restrict the sample to the majority population in both countries. For the US, the sample includes white natives, which is ethnically homogeneous. For Israel, the sample includes the Jewish population. Since this population is characterized by observable ethnic differences which can explain significant wage variation, we control for these differences with the Israeli data in order to produce estimates of the return to education which are not confounded by these factors (native versus non-native, Ashkenazic versus Sephardic).} Using dummy variables for education categories instead of years of schooling, Table 2 shows that the return to a college degree (versus finishing high school but not going to college) was 56.7% versus 39% in the US and Israel respectively. The gap between college graduates and high school dropouts is much larger for the US relative to Israel as well (100.1% versus 57.7%). Therefore, the returns to education appear to be much larger in the US than Israel, which is likely to contribute to the positive relationship between education and emigration in Figures 1 and 2. It is worth noting that this relationship is positive and significant after controlling for a host of other demographic characteristics of the individual, as shown in a regression in the fifth column of Table 2.

TABLE 2 HERE

One potential explanation for the pattern exhibited in the figures could be that individuals with higher education levels are more likely to spend time abroad temporarily (sabbaticals, being stationed abroad by a firm or the government, etc.). However, we find no evidence to support this idea. Figure 3 shows that there is no discernible relationship between education and the propensity to return between 2002 and 2004, given that a person was considered a “mover” in 2002. The last two columns of Table 2 confirm that there is no systematic pattern between “returning” and education levels even after controlling for other demographic characteristics and wages.

FIGURES 3 AND 4 HERE

Another potential explanation for the “positive” selection of immigrants in terms of education could be related to restricting the sample to those above the age of 30. If less-educated Israelis tend to leave in their 20’s while more educated individuals wait until they finish college, the positive relationship in Figures 1 and 2 could be spurious.
However, Figure 4 shows that emigration rates as of 2004 for teenagers in 1995 increase with their father’s education level. Therefore, the evidence suggests that emigrants are indeed positively selected according to observable general skills like education.\textsuperscript{14}

However, the model in the previous section highlights how the patterns of emigrant selection differ for observable general skills like education versus unobservable skills – which are a mixture of general and country-specific skills. Empirical support for this idea is displayed in Figure 5, which presents the rates of emigration according to the decile of the residual log wage distribution (estimated from the regression specified in column 3 of Table 2). This figure shows that the propensity to emigrate increases with residual wages and then declines.\textsuperscript{15} This inverse u-shaped pattern is consistent with the range of parameters in our model which yield small, but positive emigration rates for each level of education and residual wage as depicted in Figures 1, 2, and 5. Columns 5 and 6 of Table 2 show that this relationship persists after controlling for education and demographic characteristics, and also show directly that using a linear specification for wages to explain the probability of emigrating yields misleadingly insignificant results.

\textbf{FIGURE 5 HERE}

In the context of the model above, the inverse u-shaped pattern is consistent with a higher return to unobserved general skills in the US relative to Israel. This cannot be checked directly, but there are two reasons why the return to unobserved skills is likely to be higher in the US. First, the returns to all observable skills (education, age, marital status) in Table 2 are higher in the US relative to Israel, and thus, it is likely to be the case that unobserved skills behave similarly (see Altonji, Elder, and Taber 2005)). This line of reasoning follows from the idea that the labor market in the US is perhaps the freest and most competitive in the developed world, and as such, highly rewards skills regardless of their direct observability in labor force surveys. Secondly, our estimates of the residual variance in both countries (using the specification in column 3 of Table 2) are 0.523 for the US and 0.498 for Israel.\textsuperscript{16} This finding is consistent with a higher return to unobservable skill in the US (see Juhn, Murphy, and Pierce (1993)).

\textsuperscript{14} We abstract from the issue of return migration (see Dustmann (2003) and Dustmann and Weiss (2007)). However, Table 1 shows that only two percent of those that were considered movers in 2002 moved back by 2004. Given this low number of returning Israelis and the finding that they are not disproportionately educated (as noted above), we focus exclusively on the selection of the moving decision.

\textsuperscript{15} This pattern persists even after controlling for twelve industry categories or eight occupation groups.

\textsuperscript{16} Income inequality is high in both countries (Brandolini and Smeeding (2006)).
The overall patterns of emigrant selection based on observables (education) and unobservables (residual wages) appear to be consistent with the model, since the returns to education and unobserved general skills are likely to be higher in the US. However, this might simply be a coincidence. To test the model, we need variation in the returns to education and unobservable skill in the US versus Israel, and then examine whether that variation explains whether the selection becomes more positive or negative in accordance with the model’s predictions. To do this, we estimate the returns to education and the residual variance within sectors in the United States and Israel using the same specifications used for each sample in Table 2 (separate regressions for each country-sector combination). Table 3 presents these estimates when the sectors are defined by industry, and Table 4 presents the estimates when the sector is defined by occupation. In the next section, we will exploit variation across sectors in the relative returns to education and unobservable general skill (Israel versus the US) to formally test whether the aggregate patterns are influenced by the factors highlighted in our model.

TABLES 3 AND 4 HERE

4 Empirical Results

4.1 Selection on Observables (Education)

The goal of this section is to test whether a lower relative return to education in Israel versus the US intensifies the positive relationship between education and the propensity to leave Israel (Propositions 5 and 6). With information on each individual in Israel before he makes the decision to leave the country or not, the basic regression specification explains the probability that person \( i \) who works in sector \( j \) (before leaving Israel) decides to emigrate from Israel by the following probit specification:

\[
\Pr(Mover_{ij}) = \Phi(\gamma_0 + \gamma_1 x_i + \gamma_2 educ_i + \gamma_3(residual\ wage)_{ij} + \gamma_4(residual\ wage)_{ij}^2 \\
+ \gamma_5(Israel\ ROR\ Educ)_j + \gamma_6(US\ ROR\ Educ)_j \\
+ \beta_1(Israel\ ROR\ Educ)_j \cdot educ_i + \beta_2(US\ ROR\ Educ)_j \cdot educ_i \\
+ \alpha_j)
\]

where:

\( Mover_{ij} \) is an indicator equal to one if the person emigrates from Israel and is zero otherwise; \( x_i \) is a vector of personal characteristics (age, marital status, number of children

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in the household, an indicator for being a native Israeli or not, and age that the person
took to Israel if he is not a native), and dummy variables for ethnicity (European descent
or Middle Eastern descent); \( educ_i \) is the number of completed years of schooling by person
\( i \); \( residual wage_{ij} \) is the individual’s residual from a standard Mincer-like wage regression
from the 1995 Israel Census using observations of workers in sector \( j \) (regressing wages on
education, age, age squared, marital status, and indicators for ethnic status and immigrant
status); \( Israel ROR Educ_j \) is the estimated return to education from the 1995 Israel
Census in sector \( j \) in the regression described above for estimating the residual wage for
each person in sector \( j \); \( US ROR Educ_j \) is the estimated return to education from the US
CPS (combining 1994, 1995, and 1996) within workers in sector \( j \) using the specification in
the first column of Table 2; \( \alpha_j \) is a fixed-effect for sector \( j \); and \( \Phi \) is the cdf of the normal
distribution.

In the context of the model, the main coefficients of interest are \( \beta_1 \) and \( \beta_2 \). The
model predicts that a lower (higher) relative return to skill in Israel versus the US will
entice higher (lower) skilled workers to leave Israel. Formally, this prediction is represented
by \( \beta_1 < 0 \) and \( \beta_2 > 0 \). These parameters are identified by exploiting variation across sectors
in the difference between Israel and the US in the returns to education within each sector,
and testing for whether the relationship between education and emigration becomes more
(less) positive in sectors with a lower (higher) return to education in Israel versus the
United States.

It is important to emphasize that we are not assuming that all emigrants move to
the US, although as indicated previously, the US is by far the most likely destination for
Israeli emigrants. Nor do we assume that Israeli emigrants do not change sectors. Our
specification is motivated by the idea that there are substantial switching costs between
occupations due to previous investments in occupation-specific human capital, and that
existing evidence does indicate substantial persistence in the occupational choices of immi-
grants after immigration in other contexts (Friedberg (2001)). If Israelis do not consider
the wage differential between the US and Israel within their own sector, then the causal
effect of these variables should be zero \( (\beta_1 = 0 \) and \( \beta_2 = 0 \)), and the estimates should be
insignificant. In other words, a lot of sector switching or moving to other countries should
only attenuate our estimates.

In terms of defining the sectors, we use either the twelve industrial sectors depicted in
Table 3 or the nine occupations in Table 4. Sectors have to be defined rather broadly so that
we can obtain reasonable estimates of the returns to education and residual variation (used
in the next section) within each sector. Tables 3 and 4 present the returns to education within each sector in Israel and the US. Differences in these estimates across sectors are the source of the “treatment variation” that is exploited in the empirical analysis.17

Table 5 presents our results for emigrant selection in terms of education. The coefficients represent the estimated marginal effect of each explanatory variable (evaluated at the means of all the explanatory variables) from a probit regression. The left panel defines sectors by industry while the right panel repeats the analysis using occupation categories as sectors.

TABLES 5 HERE

The first column of Table 5 uses an interaction between education and the return to education in sector \( j \) in Israel only (no interaction between education and sector \( j \)’s return to education in the US). The estimate is not significant. In the second column, we include only the interaction with the return to education in the US. This coefficient turns out to be positive and significant, indicating that higher educated Israelis tend to leave Israel more if they work in a sector which has a high return to education in the United States. The direction of this effect is consistent with the predictions of the model – positive selection is larger in a sector with a higher return to general skills abroad.

The third column of Table 5 includes interactions with the returns to education in the US and Israel. Interestingly, both of the main parameters of interest are now significant, and the magnitudes are much larger than in the first two columns. This pattern is an important result in the context of the model, since it illustrates the model’s prediction that the relative returns to skill between the two countries is what matters, not necessarily the level of inequality or returns to skill in the home or host country alone.18

The fourth column of Table 5 uses a specification which interacts a person’s education level with the difference in the returns to education between Israel and the US in his sector. Using this specification, the main parameter of interest is highly significant and consistent with the model: the probability of emigrating for Israelis with higher levels of education

17 Although there is a large literature which examines the bias in the estimated returns to schooling using a standard Mincer-like regression, this issue should only affect our results if the biases across sectors and between the US and Israel are somehow systematically related to the selection of emigrants within a sector.

18 In addition, the fact that the results are highly sensitive to the inclusion of the US measures of the returns to skill indicate that our measured returns to skill in the US and Israel are not simply noisy measures which are fraught with biases and measurement error. If this were the case, they should not be significant determinants of the probability to emigrate, and they should not be influencing the other coefficients to such a high degree.
declines in sectors with a higher relative return to education in Israel versus the US. The fifth column in Table 5 estimates the same model, but expands the sample to include individuals between the ages of 25 and 50 (instead of 30 to 40). The coefficient is smaller but still significant. The reduction in magnitude is to be expected, since many Israelis in their 20’s have not finished their BA degree, and emigrants who leave Israel after the age of 40 are probably placing a larger weight on non-labor market considerations. The last column in the left panel of Table 5 uses a more conservative measure of “mover” by considering an individual an emigrant if he was out of the country in both 2002 and 2004. The results are robust to this change in definition of mover status.

The right panel of Table 5 presents a similar analysis, but defines sectors according to occupations not industries. Overall, the results using occupations as sectors are similar to those using industries in one respect, but differ in a few important ways. They are similar in the sense that higher educated Israelis tend to leave occupations with a low return to education in Israel. However, higher educated Israelis also tend not to leave Israel if the return to education in their occupation in the US is higher. The former result is consistent with the model but the later one is not. As a result, the specification which uses the difference in the returns to education between Israel and the US yields an insignificant coefficient, since the individual coefficients (in the first three columns of the right panel) are of a similar magnitude and have the same sign. That is, the difference specification masks the two individual effects.

Even though one of the two main coefficients has an unexpected sign, it is worth noting that the sizes of the coefficients in the occupation level analysis are considerably smaller than the ones obtained in the industry-level analysis. The difference in the size of the coefficients, and the direction of one of them, could be due to the idea that occupational status is already a proxy for education. Much of the variation in education is between occupations, and therefore, testing for selection by education within occupations is weakened by the lack of variation. The larger selection effects obtained in the industry-level analysis are most likely a product of exploiting a broader range of education levels within each industry. Evidence for this idea is found by the much lower estimated returns to education within occupations versus industries, as seen in Tables 3 and 4. In addition, there is naturally more variation in the industry level analysis which exploits 12 sectors versus 8 in the occupation level results.
4.2 Selection on Unobservables (Residual Wages)

This section analyzes the prediction of Proposition 1 that the rate of emigration should be an increasing and then decreasing function of the residual wage, and Proposition 4’s prediction that a higher relative return to unobservable skill in Israel versus the US decreases the probability that individuals with higher unobserved skill will emigrate (i.e. shift the inverse u-shaped relationship between emigration and residual wages to the right). The basic probit specification explains the probability that person \(i\) who works in sector \(j\) (before leaving Israel) decides to emigrate from Israel by the following equation:

\[
Pr(Move_{ij}) = \Phi(\gamma_0 + \gamma_1 x_i + \gamma_2 educ_i + \gamma_3 (residual\ wage)_{ij} + \gamma_4 (residual\ wage)_{ij}^2 \\
+ \beta_3 (Israel\ residual\ SD)_{jk} \cdot (residual\ wage)_{ij} \\
+ \beta_4 (US\ residual\ SD)_{jk} \cdot (residual\ wage)_{ij} \\
+ \alpha_{jk})
\]

where each variable is defined as before except: \(Israel\ residual\ SD_{jk}\) is the standard deviation of the residuals within sector \(j\) and education group \(k\) (\(k=1\) if person \(i\) is a high school dropout, \(k=2\) if person \(i\) completed only high school, and \(k=3\) if person \(i\) completed college) from the wage regression described in Table 2 using the 1995 Israel Census; \(US\ residual\ SD_{jk}\) is the standard deviation of residuals within sector \(j\) and education group \(k\) from the wage regression described in Table 2 using the US CPS data; and \(\alpha_{jk}\) is a fixed-effect for education group \(k\) in sector \(j\). The analysis uses a person’s residual wage within a sector as a measure of his unobservable skill, and the residual variation in sector \(j\) for education group \(k\) as a proxy for the return to unobservable skill in sector \(j\) and education group \(k\).\(^{19}\)

Similar to the previous section, the main coefficients of interest are \(\beta_3\), and \(\beta_4\), and the model predicts that \(\beta_3 < 0\) and \(\beta_4 > 0\). Table 6 presents our estimates for these coefficients after defining sectors by industry or occupation. The pattern of results in Table 6 is similar to the analysis of selection on education in Table 5. All of the individual interaction coefficients have the expected sign, and most are significant. The difference specification yields highly significant results that are consistent with the model: a higher return to unobservable skill in Israel induces individuals with high levels of unobservable

\(^{19}\)There are likely to be differences across education groups within a given sector in the returns to unobservable skills, so the regression explicitly considers this possibility.
skill to stay, while higher returns to unobservable skill in the US increase the chances for individuals with higher levels of unobservable skill to leave Israel.

TABLE 6 HERE

In contrast to the results in Table 5 regarding selection on education, the results in Table 6 are very similar in terms of selection on unobservable skill in the industry and occupation level analyses. Again, one possible explanation for this could be that occupation is already a proxy for education, so this is likely to affect the results for selection on education levels in a way that does not affect selection on unobservables (since residuals are computed after controlling for occupation and education).

4.3 The Magnitude of the Coefficients

So far, we found statistically significant evidence in favor of increased positive selection on observable and unobservable skill in sectors with a low relative return to skill in Israel versus the US. However, the magnitudes of the coefficients are not easy to interpret. Therefore, we now use our estimates to present the relationship between levels of skill and the propensity to emigrate under alternative scenarios. Specifically, we compute the predicted values of the regression after substituting different relative returns to skill between Israel and the US, and show how the relationship between skill and the probability of emigration changes under each scenario.

For example, Figure 6 compares the actual rates of emigration by education level with the predicted rates (using the estimated parameters in the fourth column of Table 5) after increasing the relative return to education in Israel versus the US by 0.03 (which roughly makes the average return equal in Israel and the US). Overall, Figure 6 shows that almost the entire relationship between education and emigration can be eliminated if the relative returns to skill were similar to those in the US. This simulation suggests that the estimated selection parameters are not only significant statistically, but are considerably large in magnitude.

FIGURES 6, 7, AND 8 HERE

In order to demonstrate the magnitude of selection on unobservable skill, Figure 7 compares the actual emigration rates versus the predicted rates after increasing the residual variation in each sector by 0.025, which is the overall gap in residual variation between the two countries. Consistent with the model’s predictions, equalizing the residual variance
across the two countries shifts the inverse u-shaped pattern to the left, so that the selection of emigrants becomes less positive. However, the size of the shift is not noticeably large, which may be due in part to the small difference in the residual variance between the two countries as well as the size of the coefficient itself. This is seen in Figure 8 which performs a similar exercise using the coefficients from the occupation-level analysis, which tended to be larger than those in the industry-level analysis on unobservable skills. When the residual variance is equalized using these estimates, a much larger shift to the left occurs, thus reducing the positive selection based on unobservable skills in a noticeable way, while keeping the overall u-shaped pattern intact. Overall, these figures show that the parameters governing the selection process on observable and unobservable skills are significant statistically and also in magnitude.

5 Conclusion

This paper develops a model which describes how a country’s level of income inequality can sometimes serve as a way to attract high skilled immigrants from other countries, or retain its own high skilled individuals from leaving. The model highlights the idea that the sources of inequality, not just the levels, are important determinants of who is likely to want to leave a given country. In particular, inequality that is determined by general types of human capital which can be transported to other countries serves to attract high skilled individuals from home and abroad, while inequality stemming from country-specific factors tends to keep high-earning individuals at home, but does not serve as an attraction for high earning individuals abroad.

Using a rare data set which contains information on the labor market outcomes of individuals before they decide to move or not, we provide empirical support for all the main predictions of the model. In particular, we observe low but positive rates of emigration for all levels of education and wages, and provide evidence that the returns to general skills are likely to be higher in the US versus Israel. The range of parameters in our model which yield these patterns also predict positive emigrant selection in terms of education and an inverse u-shaped relationship between emigration and residual wages. Our data is consistent with both of these predictions, and we test the model further by exploiting differences across sectors in the relative returns to observable and unobservable skills between Israel and the United States. This analysis reveals how differences in inequality across sectors can explain variation in the patterns of emigrant selection in a
manner which is consistent with our model. Namely, a higher relative return to general human capital abroad makes the selection of emigrants more positive in terms of education or residual wages, while keeping the overall shape of the function intact.

These findings support the idea in Borjas (1987) that a higher return to skill abroad not only creates higher inequality there, but also makes it a more attractive destination for high skill immigrants from all other countries. However, we show that an important distinction between general and “country-specific” skills is necessary to make this prediction consistent with the observed non-monotonic relationship between emigration and residual wages. In this manner, our paper highlights the idea that a country’s level and sources of inequality will determine how a country competes internationally to attract high skill immigrants from abroad, and to keep their own high skilled workers at home.
References


6 APPENDIX

PROOF OF PROPOSITION 4:

For any given $\tilde{w}$ and $\epsilon$, there exists a threshold level of $g$ above which individuals decide to emigrate. In particular, for $\tilde{w} = 1$, it follows that $g \sim U[0,1]$, and hence this threshold level of $g$ is represented by $g \hat{\gamma}(1; \epsilon) = 1 - p(1; \epsilon) = 1 - \frac{\gamma}{\gamma + 1} = \frac{1 + (a - \beta \epsilon)}{\gamma + 1}$. Since $s + g = \tilde{w}$, it follows that an individual at the threshold level of $g$, given that $\tilde{w} = 1$, will have specific skills equal to $s = 1 - \hat{g} = \frac{\gamma}{\gamma + 1}$. For this individual, by definition of being at the threshold, $w_0 = w_1$, and hence $\beta e + \gamma g = a + s$. Suppose there is a rise in $\gamma$ and $a$ such that this individual remains indifferent, and hence, $p(1, \epsilon)$ is unchanged. In this case, the rise in $a$ is given by $da = g d\gamma = \frac{1 + (a - \beta \epsilon)}{\gamma + 1} d\gamma$, where $d\gamma$ is the change in $\gamma$.

For $\tilde{w} \in \left(\frac{a - \beta e}{\gamma}, 1\right)$, $p(\tilde{w}, \epsilon) = \frac{\gamma}{\gamma + 1} - \frac{a - \beta e}{(1 + \gamma) \tilde{w}}$. Therefore, the change in $p(\tilde{w}, \epsilon)$, $dp(\tilde{w}, \epsilon)$, holding $p(1, \epsilon)$ constant, is

$$dp(\tilde{w}, \epsilon) = \left(\frac{\gamma + d\gamma}{\gamma + d\gamma + 1} - \frac{a + da - \beta e}{(1 + \gamma + d\gamma) \tilde{w}}\right) - \left(\frac{\gamma}{\gamma + 1} - \frac{a - \beta e}{(1 + \gamma) \tilde{w}}\right).$$

Replacing $da$ by $\frac{1 + (a - \beta \epsilon)}{\gamma + 1} d\gamma$,

$$dp(\tilde{w}, \epsilon) = \left(\frac{\gamma + d\gamma}{\gamma + d\gamma + 1} - \frac{a + \frac{1 + (a - \beta \epsilon)}{\gamma + 1} d\gamma - \beta e}{(1 + \gamma + d\gamma) \tilde{w}}\right) - \left(\frac{\gamma}{\gamma + 1} - \frac{a - \beta e}{(1 + \gamma) \tilde{w}}\right),$$

and re-arranging:

$$dp(\tilde{w}, \epsilon) = \frac{1}{\tilde{w}} \frac{d\gamma}{\gamma + 1} \frac{\tilde{w} - 1}{\gamma + d\gamma + 1}.$$

Hence, for $d\gamma > 0$ and $\tilde{w} < 1$, $dp(\tilde{w}, \epsilon) < 0$.

Similarly, for $\tilde{w} \in (1, 1 + \gamma - (a - \beta e))$, $p(\tilde{w}, \epsilon) = \frac{1 + \gamma - (a - \beta e) - \tilde{w}}{(2 - \tilde{w})(1 + \gamma)}$, and the change in $p(\tilde{w}, \epsilon)$, $dp(\tilde{w}, \epsilon)$, holding $p(1, \epsilon)$ constant is,

$$dp(\tilde{w}, \epsilon) = \frac{1 + \gamma + d\gamma - (a + da - \beta e) - \tilde{w}}{(2 - \tilde{w})(1 + \gamma + d\gamma)} - \frac{1 + \gamma - (a - \beta e) - \tilde{w}}{(2 - \tilde{w})(1 + \gamma)},$$

replacing $da$ by $\frac{1 + (a - \beta \epsilon)}{\gamma + 1} d\gamma$,

$$dp(\tilde{w}, \epsilon) = \frac{1 + \gamma + d\gamma - \left(a + \frac{1 + (a - \beta \epsilon)}{\gamma + 1} d\gamma - \beta e\right) - \tilde{w}}{(2 - \tilde{w})(1 + \gamma + d\gamma)} - \frac{1 + \gamma - (a - \beta e) - \tilde{w}}{(2 - \tilde{w})(1 + \gamma)},$$

and re-arranging:

$$dp(\tilde{w}, \epsilon) = \frac{d\gamma}{\gamma + 1} \frac{1 - \tilde{w}}{(\tilde{w} - 2)(\gamma + d\gamma + 1)}.$$

Hence, for $d\gamma > 0$ and $\tilde{w} > 1$, $dp(\tilde{w}, \epsilon) > 0$. □