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NATURAL SELECTION AND THE ORIGIN OF ECONOMIC GROWTH*

ODED GALOR AND OMER MOAV

This research develops an evolutionary growth theory that captures the interplay between the evolution of mankind and economic growth since the emergence of the human species. The theory suggests that the struggle for survival that had characterized most of human existence generated an evolutionary advantage to human traits that were complementary to the growth process, triggering the takeoff from an epoch of stagnation to sustained economic growth.

> "*It is not the strongest of the species that survive, nor the most intelligent, but the one most responsive to change*" [Charles Darwin].

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I. Introduction

This research develops an evolutionary growth theory concerning the interplay between the evolution of mankind and economic growth since the emergence of the human species. This unified approach captures the intricate evolution of population, technology, and the standard of living throughout human history. The theory suggests that the struggle for survival that had characterized most of human existence generated an evolutionary advantage to human traits that were complementary to the growth process, triggering the takeoff from an epoch of stagnation to sustained economic growth.

For the major part of human existence mankind was subjected to persistent struggle for existence. Diminishing returns to labor along with a positive effect of the standard of living on population growth held income per capita in the proximity of a subsistence level. "The perpetual struggle for room and food" [Malthus 1798, p. 48] left limited resources for child rearing. Improvements in the technological environment or in the availability of land lead to larger but not richer population. The Malthusian pressure, conceivably, affected the composition of the population as well. Lineages of individuals whose characteristics were complementary to the technological environment gained an evolutionary advantage. They generated higher income, devoted more resources for child rearing, and their fraction in the population had gradually risen.

During the Malthusian epoch the growth rate of output per capita had been negligible, and the standard of living had not differed greatly across countries. For instance, the average growth rate of output per capita in Western Europe was nearly zero in the years 0–1000 and 0.14 percent in the years 1000–1820 [Maddison 2001]. Similarly, population growth over this era followed the Malthusian pattern. The average annual rate of population growth in Western Europe was about 0 percent in the years 0–1000 and 0.1 percent in the years 1000–1820 [Maddison 2001], and world population grew at an average pace of less than 0.1 percent per year from the year 1 to 1750 [Livi-Bacci 1997], reflecting the slow pace of resource expansion and technological progress. Fluctuations in population and wages exhibited the

Malthusian pattern as well.[1] Negative shocks to population, such as the Black Death, generated higher real wages and faster population growth. Finally, differences in technological levels across countries were reflected in population density but not in the standard of living (e.g., Mokyr [1985], Easterlin [1981], and Pomerantz [2000]).[2]

The theory suggests that the evolutionary pressure during the Malthusian era gradually increased the representation of individuals whose characteristics were complementary to the growth process, triggering a positive feedback between technological progress and education that ultimately brought about the Industrial Revolution and the takeoff from a Malthusian epoch to sustained economic growth. In contrast, waves of rapid technological progress in the Pre-Industrial Revolution era had not generated sustained economic growth due to the incompletion of the necessary evolutionary process in the population. As depicted in Figure I, the European takeoff from the Malthusian regime was associated with the Industrial Revolution. The average growth rates of output per capita in Western Europe over the period 1820–1870 rose to an annual rate of 1.0 percent, along with an impressive increase in education.[3] During this time interval, the Malthusian mechanism linking higher income to higher population growth continued to function. Fertility rates increased in most of Western Europe until the second half of the nineteenth century, peaking in England and Wales in 1871 and in Germany in 1875 [Dyson and Murphy 1985; Coale and Treadway 1986]. The acceleration in technological progress stimulated the accumulation of human capital and brought about a demographic transition in which fertility rates declined rapidly, paving the way to an era of sustained economic growth with an average

1. Lee [1997] reports positive income elasticity of fertility and negative income elasticity of mortality from studies examining a wide range of pre-industrial countries. Similarly, Wrigley and Schofield [1981] find a strong positive correlation between real wages and marriage rates in England over the period 1551–1801.

2. China's sophisticated agricultural technologies, for example, allowed high per-acre yields, but failed to raise the standard of living above subsistence. Similarly, the introduction of the potato in Ireland in the middle of the seventeenth century had generated a large increase in population over the century without an improvement in the standard of living. The destruction of this productive technology by fungus generated a massive decline in population due to the Great Famine and mass migration [Mokyr 1985].

3. For example, the average years of schooling in England and Wales rose from 2.3 for the cohort born between 1801 and 1805 to 5.2 for the cohort born 1852–1856 and 9.1 for the cohort born 1897–1906. [Matthews, Feinstein, and Odling-Smee 1982].

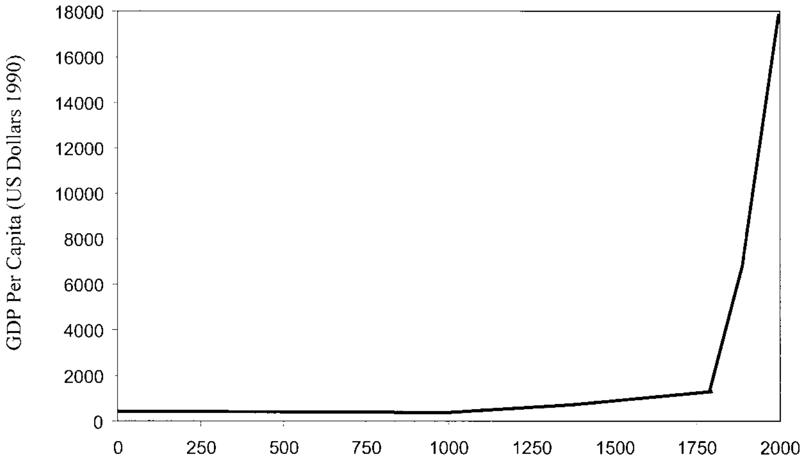

FIGURE I
Output per Capita in Western Europe in the Years 0–2000
Data source: Maddison [2001].

annual increase in income per capita of about 2 percent over the twentieth century.[4]

Evidence suggests that evolutionary processes in the composition of genetic traits may be rather rapid and even the time period between the Neolithic Revolution and the Industrial Revolution that lasted nearly 10,000 years is sufficient for significant evolutionary changes.[5] There are numerous examples of rapid evolutionary changes among various species.[6] In particular, evidence establishes that evolutionary changes oc-

4. The reduction in fertility was most rapid in Europe around the turn of the century. In England, for example, live births per 1000 women aged 15–44 fell from 153.6 in 1871–1880 to 109.0 in 1901–1910 [Wrigley 1969]. The exception was France, where fertility started to decline in the early nineteenth century.

5. In particular, severe adverse shocks to the size of the population over this period (e.g., the Black Death that eliminated one-third of the European population in the fourteenth century) may have expedited the evolutionary process, if it affected the population selectively.

6. The color change that peppered moths underwent during the nineteenth century is a classic example of the speed of evolution in nature (see Kettlewell [1973]). Before the Industrial Revolution light-colored English peppered moths blended with the lichen-covered bark of trees. By the end of the nineteenth century a black variant of the moth, first recorded in 1848, became far more common than the lighter varieties in areas in which industrial carbon killed the lichen and changed the background color. Hence, a dramatic evolutionary change occurred within a few hundred generations. Evidence from Daphne Major in the Galapagos suggests that major evolutionary changes in Darwin's Finches occurred within a few generations [Grant and Grant 1989]. See also the survey by

curred in Homo Sapiens within the time period that is the focus of our analysis. For instance, lactose tolerance was developed among European and Near Easterners since the domestication of dairy animals in the course of the Neolithic Revolution, whereas in regions that were exposed to dairy animals in later stages the population does not retain the ability to digest lactose into adulthood. Similar patterns are reflected in Gluten tolerance (i.e., an ability to tolerate a protein present in wheat, rye, barley, and some oats, first domesticated in the course of the agricultural revolution). Further, genetic immunity to malaria provided by the sickle cell trait has developed among Africans whose engagement in agriculture improved the breeding ground for mosquitoes and thereby raised the incidence of malaria. This genetic trait is absent in nearby populations that have not made the transition to agriculture.[7]

The fundamental hypothesis of this research is based on four central elements. The first consists of the main ingredients of a Malthusian world. The economy is characterized by a fixed factor of production—land, a subsistence consumption constraint below which individuals cannot survive, and an (endogenous) positive effect of income on fertility rates. If technological progress permits output per capita to exceed the subsistence level, population rises, the land-labor ratio falls, and in the absence of further technological progress income falls back to the subsistence level. In the Malthusian era therefore, if differences in income across individuals reflected differences in intergenerationally transmitted traits, then the positive-effect of income on fertility rates would have affected the composition of traits in the population as a whole.

The second element of the model incorporates the main ingredients of Darwinian evolution (i.e., variety, intergenerational transmission of traits, and natural selection) into the economic environment. It demonstrates the impact of the Malthusian pressure on the evolution of the human species. Although individuals do not operate consciously so as to assure the evolutionary advantage of their type (i.e., their variant within the species), those whose operations are most suitable to the environment would dominate the distribution.

Endler [1986]—in particular, the examples of color patterns of guppies that changes dramatically within fifteen generations.

7. See Durham [1982], Livingstone [1958], and Wiesenfeld [1967].

Inspired by fundamental components of the Darwinian theory [Darwin 1859, 1871], individuals' preferences are defined over consumption above a subsistence level as well as over the quality and the quantity of their children.[8] These simple and commonly employed preferences capture the Darwinian survival strategy and represent the most fundamental trade-offs that exist in nature. Namely, the trade-off between resources allocated to the parent and the offspring, and the trade-off between the number of offspring and resources allocated to each offspring.[9]

The subsistence consumption constraint assures the mere physiological survival of the parent and hence increases the likelihood of the survival of the lineage (dynasty). Resources allocated to parental consumption beyond the subsistence level may be viewed as a force that raises parental labor productivity and resistance to adverse shocks (e.g., famine and disease), generating a positive effect on the fitness of the parent and the survival of the lineage. This positive effect, however, is counterbalanced by the implied reduction in resources allocated to the offspring, generating a negative effect on the survival of the lineage.

The significance that the individual attributes to child quantity as well as to child quality reflects the well-known variety in the quality-quantity survival strategies that exists in nature (e.g., MacArthur and Wilson [1967]). Human beings, like other species, confront the basic trade-off between offspring's quality and quantity in their implicit Darwinian survival strategies. Although a quantity-biased preference has a positive effect on fertility rates and may therefore generate a direct evolutionary advantage, it adversely affects the quality of offspring, their income, and their fitness and may therefore generate an evolutionary disadvantage. "*Increased bearing is bound to be paid for by less efficient caring*" [Dawkins 1989, p. 116].

8. Given the time perspective of the current study, we abstract from the old age security hypothesis (e.g., Caldwell [1976]). Although this mechanism appears to capture aspects of the demographic transition (but not the takeoff from Malthusian stagnation), it is apparent, observing other species, that old age support had no relevance for fertility decisions at least in early stages of human existence. Further, in the currently developed world it appears to have limited relevance as well. Some recent empirical studies (e.g., Kaplan [1994]) do not support Caldwell's theory.

9. Resources allocated to quality of offspring in different stages of development take different forms. In early stages of development it is manifested in investment in the durability of the offspring via better nourishment, and parental guidance, whereas in mature stages, investment in quality may capture formal education. The existence of a trade-off between quantity and quality of children is supported by Rosenzweig and Wolpin [1980] and Hanushek [1992].

The theory argues that hereditary human traits, physical or mental, that generate higher earning capacity, and thereby potentially a larger number of offspring, would generate an evolutionary advantage and would dominate the population in the long run. Although the transition to Homo Sapiens in which human intelligence has increased significantly is an integral component of the entire evolutionary process, the model focuses on changes in the composition of types within Homo Sapiens. In particular, the model focuses on the evolution of the fundamental trade-off between resources allocated to the quantity and the quality of offspring. As is well established in the evolutionary biology literature since the seminal work of Lack [1954], the allocation of resources between offspring "caring" and "bearing" is subjected to evolutionary changes.[10]

The economy consists of a variety of types of individuals distinguished by the weight given to child quality in their preferences.[11] "*The smallest grain in the balance, in the long run, must tell on which death shall fall and which shall survive*" [Darwin 1859]. This trait is assumed to be transmitted intergenerationally.[12] The household chooses the number of children and their quality in the face of a constraint on the total amount of resources that can be devoted to child rearing and consumption.[13]

10. Lack [1954] suggests that clutch sizes (e.g., number of eggs per nest), among owls and other predatory vole-eating birds, for instance, are positively related to food abundance. He argues that the clutch size is selected such that under any feeding conditions fertility rates ensure the maximal reproductive success. Furthermore, Cody [1966] documents the existence of significant differences between clutch sizes of the same bird species on islands and nearby mainland localities of the same latitude. In temperate regions where food is more abundant on the mainland than on islands, the average clutch size is smaller on the islands. For instance, for *Cyanoramphus novaezelandeae,* the average mainland clutch is 6.5, whereas the average in the island is 4.

11. The analysis abstracts from heterogeneity in the degree of the trade-off between resources allocated to parent and offspring. As will become apparent, the introduction of this element would not alter the qualitative results.

12. Recent molecular and behavioral genetic research across historical and modern data from the United States and Europe suggests that fertility behavior has a significant hereditary component [Rodgers et al. 2001a]. For instance, as established recently by Kohler, Rodgers, and Christensen [1999] and Rodgers et al. [2001b] based on the comparison of fertility rates among identical and fraternal twins born in Denmark during the periods 1870–1910 and 1953–1964, slightly more than one-quarter of the variance in completed fertility is attributable to genetic influence. These findings are consistent with those of Rodgers and Doughty [2000] based on kinship data from the United States.

13. Anthropological evidence suggests that fertility control was indeed exercised even prior to the Neolithic Revolution. Reproductive control in hunter-gatherer societies is exemplified by "pacing birth" (e.g., birth every four years) conducted by the Pygmies (tribes who live in small, semi-nomadic bands in Africa, Southeast Asia, and New Guinea) in order to prevent the burden of carrying

The distribution of preferences in the population evolves over time due to differences in fertility rates across types. The economic environment determines the type with the evolutionary advantage (i.e., the type characterized by higher fertility rates).

The evolution of the human brain in the transition from Homo Erectus to Homo Sapiens and the complementarity between brain capacity and the reward for human capital has increased the evolutionary optimal level of investment in offspring's quality, and the distribution of valuation for quality thereby lagged behind its optimal level. In a predemographic transition era, when fertility rates are positively associated with income levels, the Malthusian pressure generates an evolutionary advantage to individuals whose preferences are biased toward child quality, increasing their representation in the population. High quality individuals (born to parents with a quality bias) generate higher income and have more resources for a larger number of offspring of higher quality.

The third element of the model links the evolution of the human species to the process of economic growth. Following the well-documented and commonly employed hypothesis, human capital is assumed to have a positive effect on technological progress and therefore on economic growth.[14] Hence, the Malthusian pressure that increases the representation of individuals whose preferences are biased toward child quality, has a positive effect on investment in human capital and therefore ultimately on the rate of technological progress.[15]

The fourth element links the rise in the rate of technological

several children while wandering. The Pygmies abstained from sexual intercourse for a three-year period after each birth. Similarly, Nomadic women of the Kung (a group of the San people of Southern Africa), use no contraceptives but nurse their babies frequently, suppressing ovulation and menstruation for two to three years after birth, and reaching a mean interval between births of 44 months.

14. This link between education and technological change was proposed by Nelson and Phelps [1966]. For supportive evidence see Easterlin [1981] and Doms, Dunne, and Troske [1997]. In order to focus on the role of the evolutionary process, the model abstracts from the potential positive effect of the size of the population on the rate of technological progress. Adding this scale effect would simply accelerate the transition process (e.g., Galor and Weil [2000]).

15. Mokyr [2002] argues that the effect of human capital accumulation on technological progress became significant only in the course of the Scientific Revolution that preceded the Industrial Revolution. The theory is perfectly consistent with this observation. The effect of human capital accumulation on the rate of technological progress need not be significant prior to the scientific revolution, and the qualitative results are unaffected as long as the effect of human capital on technological progress is significant prior to the Industrial Revolution. See Mokyr [1990] as well.

progress to the demographic transition and to sustained economic growth. A rise in the rate of technological progress increases the rate of return to human capital.[16] It induces parents to substitute child quality for child quantity, increasing the average quality in the population and thereby future technological progress.[17]

The interaction between these four fundamental elements generates a pattern that is consistent with the evolution of the world economy and of the human population from the Malthusian epoch to sustained economic growth. The Malthusian pressure generates an evolutionary process that raises the quality of the population, triggering a significant interaction between education and technology that paves the way to sustained economic growth.

Suppose that in the early era of the human species, the population of the world consisted of two types of individuals: individuals of the "quantity type"—who place a lower weight on the quality of their offspring, and a small fraction of individuals of the "quality type"—who place a higher weight on the quality of their offspring.[18] The existence of a small fraction of individuals of the quality type generated a slow rate of technological progress. Investment in quality was negligible, and resources above subsistence were devoted to child rearing. The Malthusian mechanism therefore generated a proportional increase in output and population, and the economy was in the vicinity of a temporary, locally stable Malthusian steady-state equilibrium.

The model suggests, therefore, that in this Malthusian era when humans merely struggle for survival, the income of individuals of the quantity type was near subsistence, and fertility rates were near replacement level. Individuals of the quality type,

16. See Schultz [1964] and Nelson and Phelps [1966]. If the return to education rises with the *level* of technology rather than with the *rate* of technological progress, the qualitative analysis would not be affected. However, this alternative would imply that changes in technology were skill-biased throughout human history in contrast to those periods in which technological change was skilled-saving (most notably, in the first phase of the Industrial Revolution). Goldin and Katz [1998] provide evidence regarding technology-skill complementarity in the twentieth century that is consistent with the two approaches.

17. Unlike Becker's [1981] hypothesis that a high level of income induces parents to switch to having fewer, higher quality children, the substitution of quality for quantity in this paper is in response to technological progress. The fact that demographic transitions occurred around the same period in Western European countries that differed in their income per capita, but shared a similar pattern of future technological progress, supports our technological approach.

18. As argued earlier in the Introduction, the evolution of the human brain caused the distribution of valuation for quality to lag behind its optimal evolutionary level, and a sequence of mutations therefore generated only a small fraction of individuals of "the quality type" whose valuation is closer to the optimal evolutionary level.

who were wealthier, allocated more resources to child rearing, and therefore had higher fertility rates (of higher quality offspring). Hence, in the early stages of development the Malthusian pressure provided an evolutionary advantage to individuals whose preferences were biased toward quality of offspring raising their representation in the population. As the fraction of individuals of the quality type increased, technological progress intensified. The positive feedback between technological progress and the level of education reinforced the growth process, setting the stage for an industrial revolution that facilitated an endogenous takeoff from the Malthusian trap.

The increase in the rate of technological progress, according to the theory, brought about two effects on the evolution of population and its quality. On the one hand, improved technology eased households' budget constraints and provided more resources for quality as well as quantity of children. On the other hand, it induced a reallocation of these increased resources toward child quality. Hence, the increase in the rate of technological progress raised the average quality in the population, further accelerating technological progress. In the early stages of the transition from the Malthusian regime, the effect of technological progress on parental income dominated, and population growth rate as well as the average quality increased. Ultimately, the subsistence consumption constraint was no longer binding, and further increases in the rate of technological progress induced a reduction in fertility rates, generating a demographic transition in which the rate of population growth declined along with an increase in the average level of education. The economy converged to a steady-state equilibrium with sustained growth of output per worker.

During the transition from the Malthusian epoch to the sustained growth regime, once the economic environment improved sufficiently, the model suggests that the evolutionary pressure weakened, the significance of quality for survival (fertility) declined, and individuals of the quantity type gained the evolutionary advantage. Namely, as technological progress brings about an increase in income, the Malthusian pressure relaxes, and the domination of wealth in fertility decisions diminishes. The inherent advantage of the quantity type in reproduction dominates, and individuals whose preferences are biased toward child quantity gain the evolutionary advantage. Nevertheless, the growth rate of output per worker remains positive since the high rate of technological progress sustains an attractive return to investment in human capital even from the viewpoint of individuals whose valuation for quality is low.

The proposed unified evolutionary growth theory develops hypotheses about the nature of the evolution of the human species as well as the origin of the takeoff from stagnation to sustained economic growth. It contributes to the understanding of the evolution of the human species, the process of economic growth, and the interplay between the two processes.

Unlike the existing evolutionary economics literature, the theory explores the evolution of preferences over the fundamental trade-off that exists in nature between quality and quantity of offspring.[19] Moreover, fertility decisions which are based on the optimization of the household generate a nonmonotonic relationship between population growth and income, in contrast to the positive relationship between population growth and income (i.e., fitness) that is postulated in the existing literature. These fundamental distinctions enable the theory to capture the monotonic evolution of population growth and income per capita until the nineteenth century as well as the demographic transition that generated a reversal in this relationship.

In the context of the transition from stagnation to growth, Galor and Weil [2000] argue that the inherent positive interaction between population and technology during the Malthusian regime had increased the rate of technological progress sufficiently so as to induce investment in human capital that lead to further technological progress, a demographic transition, and sustained economic growth.[20] Unlike Galor and Weil [2000] in

19. The Darwinian methodology has been employed in the study of human behavior within the sociobiology literature (e.g., Wilson [1975]) and in a sequence of studies about the evolution of preferences surveyed by Bowles [1998] and Robson [2001], within the economics literature. The focus of these models is fundamentally different. They are primarily designed to explain the determination of preference (e.g., Becker [1976], Hansson and Stuart [1990], and Bergstrom [1995]). In particular, Hansson and Stuart demonstrate that in a Malthusian environment, from which the economy never escapes, evolution selects individuals with time preference which is closest to the golden rule. The Malthusian setting, however, has no importance in the determination of the type with an evolutionary advantage. It is simply designed to fix the size of the population (for a given level of technology) and hence to eliminate types with an evolutionary disadvantage.

20. Recent growth models that focus on some of the elements of the long transition from stagnation to growth also include Kremer [1993], Goodfriend and McDermott [1995], Galor and Weil [1996], Acemoglu and Zilibotti [1997], Dahan and Tsiddon [1998], Galor and Weil [1999], Galor and Moav [1999], Kelly [2001], Morand [2001], Lucas [2002], Hansen and Prescott [2002], Zak [2002], and Lagerlof [2003] among others.

which the adverse effect of limited resources on population growth delays the process of development, in the proposed theory the Malthusian constraint generates the necessary evolutionary pressure for the ultimate takeoff. Furthermore, this takeoff is brought about by a gradual change in the composition of the population rather than by the size of the population. Hence, waves of rapid technological progress in the Pre-Industrial Revolution era did not generate sustained economic growth due to the shortage of preferences for quality in the population, rather than due to an insufficient population size. Although technological progress increased the return to quality temporarily in these previous episodes, the level of human capital that was generated by the response of the existing population was insufficient to sustain technological progress and economic growth. Further, unlike the existing literature, investment in human capital increased gradually in the Pre-Industrial Revolution era due to a gradual increase in the representation of individuals who have higher valuation for offspring's quality.[21]

Finally, in contrast to the existing literature, the incorporation of heterogeneous individuals generates cross-section predictions regarding the relationship between fertility and income. The theory captures the reversal in the cross-section relationship between income and fertility in the course of the transition from the Malthusian epoch to the regime of sustained economic growth. In the Malthusian regime there is a positive cross-section correlation between income and fertility rates whereas in the Modern Growth Regime this cross-section correlation is negative.[22]

II. The Basic Structure of the Model

Consider an overlapping generations economy in which economic activity extends over infinite discrete time. In every period the economy produces a single homogeneous good using land and

21. Given the current state of historical data, it is difficult to confirm this hypothesis based on a careful empirical analysis. However, anecdotal evidence appears consistent with this hypothesis. In particular, in the Pre-Industrial Revolution era, the increase in the number and size of universities in Europe since the establishment of the first university in Bologna in the eleventh century had significantly outpaced the growth rate of population.

22. The existence of a hump-shaped cross-section relationship between fertility and income per capita is surveyed by Lee [1997]. Further, Boyer [1989] argues that in early nineteenth century England, agricultural laborers' income had a positive effect on fertility: birthrates increased by 4.4 percent in response to a 10 percent increase in annual income.

efficiency units of labor as inputs. The supply of land is exogenous, whereas the supply of efficiency units of labor is determined by households' decisions in the preceding period regarding the number and the level of human capital of their children.

II.A. Production of Final Output

Production occurs according to a constant-returns-to-scale technology that is subject to endogenous technological progress. The output produced at time t, Y_t, is

$$Y_t = H_t^{1-\alpha}(A_t X)^{\alpha}, \tag{1}$$

where H_t is the aggregate quantity of efficiency units of labor at time t, X is land employed in production, which for simplicity is fixed over time, A_t represents the endogenously determined technological level at time t, and $\alpha \in (0,1)$. The multiplicative form in which the level of technology, A_t, and land X appear in the production function implies that the relevant factor for the output produced is the product of the two, defined as "effective resources."

Suppose that there are no property rights over land. The return to land is therefore zero, and the wage per efficiency unit of labor, w_t, is therefore equal to the output per efficiency unit of labor produced at time t. Hence,

$$w_t = x_t^{\alpha}, \tag{2}$$

where $x_t \equiv A_t X/H_t$ denotes effective resources per efficiency unit of labor at time t.

The modeling of the production side is based on two simplifying assumptions. First, capital is not an input in the production function, and second the return to land is zero.[23]

23. Alternatively one could have assumed that the economy is small and open to a world capital market in which the interest rate is constant. In this case, the quantity of capital will be set to equalize its marginal product to the interest rate, while the price of land will follow a path such that the total return on land (rent plus net price appreciation) is also equal to the interest rate. Capital has no role in the mechanism that is underlined in this paper, and the qualitative results would not be affected if the supply of capital were endogenously determined. Allowing for capital accumulation in a closed economy framework, or property rights over land, would complicate the model to the point of intractability.

II.B. Preferences and Budget Constraints

In each period a new generation of individuals is born. Each individual has a single parent.[24] Members of generation t (those who join the labor force in period t) live for two periods. In the first period of their life (childhood), $t - 1$, individuals consume a fraction of their parental unit time endowment. The required time increases with children's quality. In the second period of life (parenthood), t, individuals are endowed with one unit of time, which they allocate between child rearing and labor force participation. They choose the optimal mixture of quantity and quality of children and supply their remaining time in the labor market, consuming their wages.

Every generation t consists of a variety of individuals (type i of generation t) distinguished by the trade-off between child quality and quantity in their preferences. Individuals *within* a dynasty are of the same type. That is, preferences are hereditary, and they are transmitted without alteration from generation to generation within a dynasty. The distribution of types evolves over time due to the effect of natural selection on the relative size of each dynasty. The type with the evolutionary advantage (i.e., the type characterized by higher fertility rates) is determined by the economic environment, and it may be replaced due to the endogenous evolution in this environment.

Individuals' preferences are represented by a utility function defined over consumption above a subsistence level $\tilde{c} > 0$, as well as over the quantity and quality (measured by human capital) of their children:[25]

$$u_t^i = (1 - \gamma) \ln c_t^i + \gamma[\ln n_t^i + \beta^i \ln h_{t+1}^i]; \quad \gamma \in (0,1), \tag{3}$$

where c_t^i is the household consumption of a type i individual of generation t, n_t^i is the number of children, $\beta^i \in (0,1]$ is the relative weight given to quality in the preferences of dynasty i, and h_{t+1}^i is the level of human capital of each child. The quality-

24. For simplicity, the model abstracts from marriages, assuming implicitly that marriages are largely assortative.

25. Alternatively, the utility function could have been defined over consumption above subsistence rather than over a consumption set that is truncated from below by the subsistence consumption constraint. Under this formulation, $u_t^i = (1 - \gamma)\ln(c_t^i - \tilde{c}) + \gamma[\ln n_t^i + \beta^i \ln h_{t+1}^i]$. As will become apparent, the adoption of this formulation would not affect the qualitative analysis, but would greatly add to the complexity of the dynamical system. Under each specification, the subsistence consumption constraint generates the Malthusian effect of income on population growth at low income levels. The effect of higher income on infant mortality and natural fertility would generate a similar effect.

parameter β^i is transmitted from generation to generation within a dynasty and remains stationary across time.[26] The utility function is strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions that assure, for sufficiently high income, the existence of an interior solution for the utility maximization problem. However, for a sufficiently low level of income, the subsistence consumption constraint is binding, and there is a corner solution with respect to the consumption level.

Individuals choose the number of children and their quality in the face of a constraint on the total amount of time that can be devoted to child-raising and labor market activities. For simplicity, only time is required in order to produce child quantity and quality. Let $\tau + e^i_{t+1}$ be the time cost for a member i of generation t of raising a child with a level of education (quality) e^i_{t+1}. That is, τ is the fraction of the individual's unit time endowment that is required in order to raise a child, regardless of quality, and e^i_{t+1} is the fraction of the individual's unit time endowment that is devoted for the education of each child. τ is assumed to be sufficiently small so as to assure that population can have a positive growth rate. That is, $\tau < \gamma$.

Consider a member i of generation t who is endowed with h^i_t efficiency units of labor at time t. Define potential income z^i_t as the potential earning if the entire time endowment is devoted to labor force participation, earning the competitive market wage w_t per efficiency unit:

$$z^i_t \equiv w_t h^i_t = x^\alpha_t h^i_t \equiv z(x_t, h^i_t). \tag{4}$$

Potential income is divided between consumption c^i_t and child rearing (quantity as well as quality), evaluated according to the individual's opportunity cost $w_t h^i_t[\tau + e^i_{t+1}]$ per child. Hence, in the second period of life (parenthood), the individual faces the budget constraint:

$$w_t h^i_t n^i_t(\tau + e^i_{t+1}) + c^i_t \le w_t h^i_t \equiv z^i_t. \tag{5}$$

26. The distribution of β^i changes due to the effect of natural selection on the distribution of types. Furthermore, although β^i is stationary over time within a dynasty, the optimization of individuals changes over time due to changes in the economic environment. For simplicity, it is assumed that the subsistence consumption constraint and the weight given to consumption in the utility function are homogeneous across individuals and hence they are not subjected to natural selection. The later specification is discussed in subsection II.E.

II.C. The Production of Human Capital

Individuals' level of human capital is determined by their quality (education) as well as by the technological environment. Technological progress is assumed to raise the value of education in the production of human capital.[27] Technological progress reduces the adaptability of existing human capital for the new technological environment (the "erosion effect"). Education, however, lessens the adverse effects of technological progress. That is, skilled individuals have a comparative advantage in adapting to the new technological environment. In particular, the time required for learning the new technology diminishes with the level of education and increases with the rate of technological change.

The level of human capital of children of a member i of generation t, h^i_{t+1}, is an increasing strictly concave function of their parental time investment in education, e^i_{t+1}, and a decreasing strictly convex function of the rate of technological progress, g_{t+1}:

$$h^i_{t+1} = h(e^i_{t+1}, g_{t+1}), \tag{6}$$

where $g_{t+1} \equiv (A_{t+1} - A_t)/A_t$.

Education lessens the adverse effect of technological progress. That is, technology complements skills in the production of human capital (i.e., $h_{eg}(e^i_{t+1}, g_{t+1}) > 0$). Furthermore, even in the absence of investment in quality, each individual has a positive level of human capital. In a stationary technological environment this level of basic skills is normalized to 1 (i.e., $h(0,0) = 1$). Finally, in the absence of investment in education, for sufficiently rapid technological progress, the erosion effect renders the existing human capital obsolete (i.e., $\lim_{g \to \infty} h(0, g_{t+1}) = 0$).

Although the potential number of efficiency units of labor is diminished due to the transition from the existing technological state to a superior one—the "erosion effect," each individual operates with a superior level of technology—the "productivity effect." Once the *rate* of technological progress reaches a positive steady-state level, the erosion effect is constant, whereas productivity grows at a constant rate.

27. Schultz [1975] cites a wide range of evidence in support of this assumption. In addition, Foster and Rosenzweig [1996] find that technological change during the green revolution in India raised the return to schooling, and that school enrollment rate responded positively to this higher return. The effect of technological transition on the return to human capital is at the center of the theories of Galor and Moav [2000] and Hassler and Rodriguez Mora [2000].

II.D. Optimization

Members of generation t choose the number and quality of their children, and their own consumption, so as to maximize their utility function. Substituting (5) and (6) into (3), the optimization problem of a member i of generation t is

$$\{n_t^i, e_{t+1}^i\} = \text{argmax}\{(1-\gamma)\ln w_t h_t^i[1 - n_t^i(\tau + e_{t+1}^i)] + \gamma[\ln n_t^i + \beta^i \ln h(e_{t+1}^i, g_{t+1})]\} \tag{7}$$

subject to

$$w_t h_t^i[1 - n_t^i(\tau + e_{t+1}^i)] \geq \tilde{c};$$
$$(n_t^i, e_{t+1}^i) \geq 0.$$

The optimization with respect to n_t^i implies that as long as potential income of a member i of generation t is low the subsistence constraint binds. The individual devotes a sufficient fraction of the time endowment for labor force participation so as to assure consumption of the subsistence level, $\tilde{c}$, and uses the rest of the time endowment for child rearing. Once potential income is sufficiently high, the subsistence consumption constraint is no longer binding, the individual devotes a fraction $1 - \gamma$ of the unit time endowment for labor force participation, consuming $c_t^i > \tilde{c}$, and a fraction γ for child rearing.

Let $\tilde{z}$ be the level of potential income above which the subsistence constraint is no longer binding. That is, $\tilde{z} \equiv \tilde{c}/(1-\gamma)$. It follows that for $z_t^i \geq \tilde{c}$

$$n_t^i(\tau + e_{t+1}^i) = \begin{cases} \gamma & \text{if } z_t^i \geq \tilde{z}; \\ 1 - \tilde{c}/z_t^i & \text{if } z_t^i \leq \tilde{z}. \end{cases} \tag{8}$$

If $z_t^i \leq \tilde{c}$, then $n_t^i = 0$ and type i would become extinct.

It should be noted that for a given level of potential income, $z_t^i = x_t^\alpha h_t^i$, the parameter β^i, does not affect the time allocation between child rearing and labor force participation. However, it affects, the division of time between the number of children and their quality. As will become apparent, individuals with a higher β^i spend more time on child quality on account of lower quantity.

As long as the potential income of a member i of generation t, z_t^i, is below $\tilde{z}$, then the fraction of time necessary to assure subsistence consumption $\tilde{c}$ is larger than $1 - \gamma$, and the fraction of time devoted to child rearing is therefore below γ. As the wage per efficiency unit of labor increases, the individual can generate

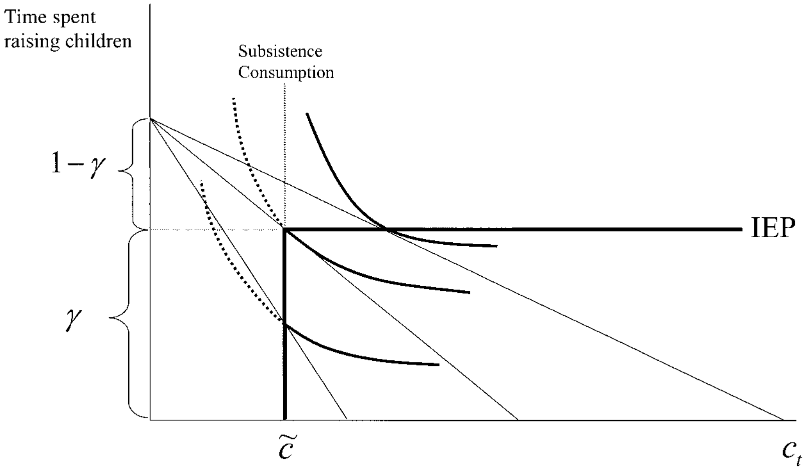

FIGURE II
Preferences, Constraints, and Income Expansion Path

The figure depicts the household's indifference curves, budget constraints, as well as the subsistence consumption constraint. The income expansion path is vertical so long as the subsistence consumption constraint is binding and horizontal once the subsistence consumption constraint is not binding.

the subsistence consumption with lower labor force participation, and the fraction of time devoted to child rearing increases.

Figure II shows the effect of an increase in potential income z_t^i on the individual's choice of total time spent on children and consumption. The income expansion path is vertical until the level of income passes the critical level that permits consumption to exceed the subsistence level. Thereafter, the income expansion path becomes horizontal at a level γ in terms of time devoted for child rearing.[28]

Regardless of whether potential income is above or below $\tilde{z}$, increases in wages will not change the division of child-rearing time between quality and quantity. The division between time spent on quality and time spent on quantity is affected by the rate of technological progress, as well as the preference for quality, β^i. Specifically, using (8), the optimization with respect to e_{t+1}^i im-

28. If the utility function would have been defined over consumption above subsistence rather than over consumption set that is truncated from below by the subsistence consumption constraint, i.e., if $u_t^i = (1 - \gamma)\ln(c_t^i - \tilde{c}) + \gamma[\ln n_t^i + \beta^i \ln w_{t+1}h_{t+1}^i]$, then the income expansion path is a smooth convex approximation of the one depicted in Figure II; for low levels of income it is asymptotically vertical, and for high levels of income it is asymptotically horizontal.

plies that independently of the subsistence consumption constraint the implicit functional relationship between investment in child quality, e^i_{t+1}, and the rate of technological progress, g_{t+1}, is given by

$$(9)\quad G(e^i_{t+1},g_{t+1};\ \beta^i) \equiv \beta^i h_e(e^i_{t+1},g_{t+1}) - \frac{h(e^i_{t+1},g_{t+1})}{(\tau + e^i_{t+1})} \begin{cases} =0 & \text{if } e^i_{t+1} > 0 \\ \leq 0 & \text{if } e^i_{t+1} = 0, \end{cases}$$

where $G(e^i_{t+1},g_{t+1};\ \beta^i)$ is the difference in the benefits from a marginal increase in time investment in quality and a marginal increase in time investment in quantity. For all $g_{t+1} \geq 0$, and $e_{t+1} \geq 0$, $G_e(e_{t+1},g_{t+1};\ \beta^i) < 0$, $G_g(e_{t+1},g_{t+1};\ \beta^i) > 0$, and $G_\beta(e_{t+1},g_{t+1};\ \beta^i) > 0$.

Individuals with a sufficiently low level of β^i do not invest in the human capital of their offspring when the future rate of technological progress is zero. To assure that individuals with a sufficiently high level of β^i would invest in the human capital of their offspring even when the future rate of technological progress is 0, it is sufficient to assume that for individuals with the highest valuation for quality (i.e., $\beta^i = 1$) the benefit from an infinitesimal time investment in quality is larger than an additional infinitesimal investment in quantity; i.e.,

$$(\text{A1})\qquad h_e(0,0) > 1/\tau.$$

As follows from A1 and (9), $G(0,0;\ 0) < 0$, and $G(0,0;\ 1) = \tau h_e(0,0) - h(0,0) > 0$. Let $\underline{\beta}$ be the threshold level of the quality parameter above which individuals of type i of generation t invest in the education of their offspring even when $g_{t+1} = 0$. That is, $G(0,0;\ \underline{\beta}) = 0$. Hence, as follows from the properties of (9), there exists $\underline{g}(\beta^i) \geq 0$ such that $G(0,\underline{g}(\beta^i),\beta^i) = 0$ for all $\beta^i \leq \underline{\beta}$.

LEMMA 1. Under Assumption A1, the quality of children, e^i_{t+1}, chosen by a member i of generation t is an increasing function of the rate of technological progress, g_{t+1}, and the individual's valuation for quality, β^i,

$$e^i_{t+1} = \epsilon(g_{t+1};\ \beta^i) \equiv e^i(g_{t+1}) \begin{cases} =0 & \text{if } g_{t+1} \leq \underline{g}(\beta^i) \text{ and } \beta^i \leq \underline{\beta} \\ >0 & \text{if } g_{t+1} > \underline{g}(\beta^i) \text{ or } \beta^i > \underline{\beta}, \end{cases}$$

where, $\epsilon_g(g_{t+1};\ \beta^i) > 0$ and $\epsilon_\beta(g_{t+1};\ \beta^i) > 0$ $\forall g_{t+1} > \underline{g}(\beta^i)$ and $\forall \beta^i > \underline{\beta}$.

The proof of Lemma 1 follows from A1 and the properties of (6) and (9). As is apparent from (9), $\epsilon_{gg}(g_{t+1};\ \beta^i)$ depends upon the third derivatives of the production function of human capital. A concave reaction of the level of education to the rate of technological progress appears plausible; hence in order to simplify the exposition (without affecting the qualitative results), it is assumed that

$$\text{(A2)} \qquad \epsilon_{gg}(g_{t+1};\ \beta^i) < 0 \quad \forall g_{t+1} > \underline{g}(\beta^i) \text{ and } \forall \beta^i > \underline{\beta}.$$

As follows from Lemma 1, the level of human capital of an individual of type i in period $t + 1$ is therefore

$$\text{(10)} \quad h^i_{t+1} = h(e^i_{t+1}, g_{t+1}) = h(\epsilon(g_{t+1};\ \beta^i),\ g_{t+1}) = h(e^i(g_{t+1}),\ g_{t+1}) \equiv h^i(g_{t+1}).$$

As is apparent from (9) and the properties of (6), $\partial h^i(g_t)/\partial g_t$ can be positive or negative. Since the response of education, e_{t+1}, to g_{t+1} may be viewed as a measure intended to offset the erosion effect of g_{t+1} on the level of human capital, it is natural to assume that

$$\text{(A3)} \qquad \frac{\partial h^i(g_{t+1})}{\partial g_{t+1}} < 0 \quad \forall g_{t+1} > 0.$$

As will become apparent, this assumption simplifies the geometrical analysis of the dynamical system without affecting the qualitative results. Furthermore, substituting $e^i_{t+1} = \epsilon(g_{t+1};\ \beta^i)$ into (8), noting that $z^i_t = x^\alpha_t h(\epsilon(g_t;\ \beta^i),\ g_t) = x^\alpha_t h^i(g_t)$, it follows that for $z^i_t \geq \tilde{c}$,

$$\text{(11)} \qquad n^i_t = \begin{cases} \gamma/[\tau + \epsilon(g_{t+1};\ \beta^i)] & \text{if } z^i_t \geq \tilde{z} \\ (1 - [\tilde{c}/z^i_t])/[\tau + \epsilon(g_{t+1};\ \beta^i)] & \text{if } z^i_t \leq \tilde{z}. \end{cases}$$
$$\equiv n(g_{t+1}, z^i_t;\ \beta^i) = n(g_{t+1}, z(x_t, h^i(g_t));\ \beta^i),$$

where $n(g_{t+1}, z(x_t, h^i(g_t));\ \beta^i)$ is increasing and strictly concave in x_t, as long as x_t is smaller than the level $[\tilde{z}/h^i(g_t)]^{1/\alpha}$ above which the subsistence constraint is no longer binding for individuals of type i, and is independent of x_t otherwise.

The next proposition, which follows directly from Lemma 1 and (11), summarizes the properties of the functions $\epsilon(g_{t+1};\ \beta^i)$, and $n(g_{t+1}, z^i_t;\ \beta^i)$ and their significance for the evolution in the substitution of child quality for child quantity in the process of development.

PROPOSITION 1. Under A1,

1. Technological progress decreases the number of children of individual i and increases their quality (i.e., $\partial n_t^i/\partial g_{t+1} \leq 0$, and $\partial e_{t+1}^i/\partial g_{t+1} \geq 0$).
2. If parental potential income is below $\tilde{z}$ (i.e., if the subsistence consumption constraint is binding), an increase in parental potential income raises the number of children, but has no effect on their quality (i.e., $\partial n_t^i/\partial z_t^i > 0$, and $\partial e_{t+1}^i/\partial z_t^i = 0$ if $z_t^i < \tilde{z}$).
3. If parental potential income is above $\tilde{z}$, an increase in parental potential income does not change the number of children or their quality (i.e., $\partial n_t^i/\partial z_t^i = \partial e_{t+1}^i/\partial z_t^i = 0$ if $z_t^i > \tilde{z}$).

It follows from Proposition 1 that if the subsistence consumption constraint is binding, an increase in the effective resources per worker raises the number of children, but has no effect on their quality, whereas if the constraint is not binding, an increase in effective resources per worker does not change the number of children or their quality. Hence, for a given rate of technological progress, parental type, rather than parental income, is the sole determinant of offspring's quality.

II.E. The Distribution of Types and Human Capital Formation

In period 0 there are a small number L_0^a of identical adult individuals of type a—"the quality type"—with a high valuation for quality, $\beta^a > \underline{\beta}$, and L_0^b identical adult individuals of type b—"the quantity type"—with a low valuation for quality, $\beta^b < \underline{\beta}$.[29] Since the quality parameter is transmitted without alteration within a dynasty, and since Proposition 1 implies that given the rate of technological progress parental type is the sole deter-

29. The existence of a large number of types would not affect the qualitative analysis. The presence of two types of individuals simplifies the exposition considerably and permits the analysis of the effect of a single quality-parameter on the evolution of this complex three-dimensional system. Since the process of evolution is inherently associated with an improvement in the fitness and hence the evolutionary advantage of certain mutants, the underlying assumption is that when mutation starts affecting the economy in period 0, it introduces a type that at least temporarily has an evolutionary advantage (i.e., a type with a parameter β that is closer to the evolutionary optimal level relative to the preexisting type—b). One way of justifying the existence of only a small fraction of individuals of high valuation for quality is that the evolution of the human brain caused the distribution of valuation for quality to lag behind its optimal evolutionary level, and a sequence of mutations therefore generated only a small fraction of individuals of "the quality type" whose valuation is closer to the optimal evolutionary level. Further discussion appears in the Concluding Remarks.

minant of offspring education, it follows that in each period t, the population of generation t, L_t, consists of two homogeneous groups of type a and b, whose size is L_t^a and L_t^b, respectively. That is, $L_t = L_t^b + L_t^a$.[30]

It should be noted that the parameter γ that reflects the trade-off between resources allocated to parent and offspring is assumed for simplicity to be identical across individuals. As follows from the optimization of each individual, independently of β^i, the parameter γ, that measures the weight given to resources allocated to offspring, has no effect on the distribution of resources between quality and quantity of offspring. Heterogeneity in γ would be reflected in the height of the vertical portion of the income expansion path in Figure II. The incorporation of heterogeneity, therefore, would not alter the process of development so long as the economy is in the Malthusian regime and the subsistence constraint binds for the entire population. However, once the Malthusian pressure is relaxed, a high value of γ would generate an evolutionary advantage. Nevertheless, it is unlikely that the population would be dominated by individuals with $\gamma = 1$ once the model would reflect the evolutionary trade-off that is associated with this parameter. Resources allocated to parental consumption beyond subsistence may be viewed as a force that raises parental resistance to adverse shocks (famine, disease, etc.), generating a positive effect on parental fitness and the survival of the lineage. This positive effect, however, is counterbalanced by the implied reduction in the resources allocated to the offspring, generating a negative effect on the survival of the lineage. Provided that the resistance to shocks is an increasing and concave function of consumption, consumption would increase in income, although the average propensity to consume would decline.

The optimal investment in child quality by members of each dynasty of type i is affected by their attitude toward child quality and the rate of technological progress.

30. Until period $t = -2$, the population of the world is homogeneous, and it consists of type b individuals. In period $t = -2$, however, a very small fraction of the adult population gives birth to mutants of type a, whose quality parameter β^a is higher than that in the existing adult population. In period $t = -1$, the mutants are adults who make fertility decisions. Their income is identical to that of type b individuals, but their fertility rate is nevertheless lower due to their higher preference for child quality. In period $t = 0$, the mutants are "regular" individuals of type a whose potential income is higher than that of type b individuals. Finally, in all periods $t \geq 0$, all individuals of type a have parents who are of type a as well.

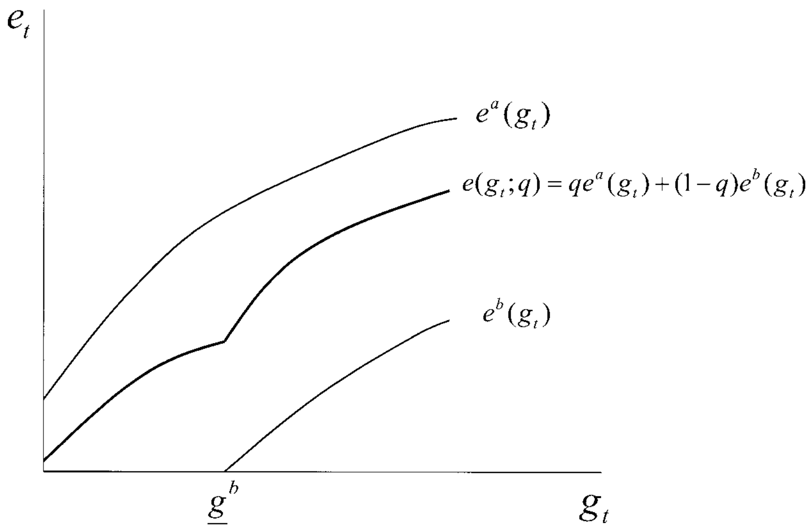

FIGURE III
Investment in Quality as a Function of the Rate of Technological Progress

The figure depicts the effect of the rate of technological progress, g_t, on offspring's quality chosen by parents with high valuation for quality, e_t^a, and low valuation for quality, e_t^b, and on the average quality of the population, e_t, weighted according to the fraction q_t of individuals with high valuation for quality.

LEMMA 2. Suppose that $\beta^b < \underline{\beta} < \beta^a$. Under A1, as depicted in Figure III, investment in child quality in each dynasty of type i, $i = a, b$, is

$$e_t^a = e^a(g_t) > 0 \quad \text{for all } t$$
$$e_t^b = e^b(g_t) > 0 \quad \text{for } g_t > \underline{g}(\beta^b) \equiv \underline{g}^b > 0$$
$$e^a(g_t) > e^b(g_t) \quad \text{for all } t.$$

The argument behind Lemma 2, which follows from Lemma 1 and the definition of $\underline{\beta}$, is straightforward. For individuals of type a, $\beta^a > \underline{\beta}$, where $\underline{\beta}$ denotes the threshold level of the quality parameter above which individuals of generation t invest in the education of their offspring even if $g_{t+1} = 0$. Hence, within a dynasty with high valuation for quality (type a) investment in child quality, e_t^a, is strictly positive for all t. For individuals with low valuation for quality (type b), however, $\beta^b < \underline{\beta}$, and investment in child quality takes place if and only if the rate of technological change and hence the return to quality is sufficiently large. Hence, as follows from (6), the level of human capital within dynasties with high valuation for quality is higher; i.e., $h_t^a > h_t^b$ for all t.

Let q_t be the fraction of individuals of high valuation for quality (type a) in generation t:

$$q_t \equiv L_t^a/L_t. \tag{12}$$

The average level of education, e_t, as depicted in Figure III, is therefore a weighted average of the level of education of the two types of individuals:

$$e_t = q_t e^a(g_t) + (1 - q_t)e^b(g_t) \equiv e(g_t,q_t). \tag{13}$$

As depicted in Figure III, following Lemmas 1 and 2, and Assumption A2, the function $e(g_t,q_t)$ is increasing in both arguments and is piecewise strictly concave with respect to g_t.[31]

The aggregate supply of efficiency units of labor in period t, H_t, is

$$H_t = L_t^a f_t^a h_t^a + L_t^b f_t^b h_t^b = L_t[q_t f_t^a h_t^a + (1 - q_t) f_t^b h_t^b], \tag{14}$$

where f_t^i is the fraction of time devoted to labor force participation by an individual of type $i = a, b$. As follows from (8), noting that $z_t^i = x_t^{\alpha} h^i(g_t)$,

$$f_t^i = \begin{cases} 1 - \gamma & \text{if } z_t^i \geq \tilde{z} \\ \tilde{c}/z_t^i & \text{if } z_t^i \leq \tilde{z} \end{cases} \equiv f^i(g_t,x_t), \tag{15}$$

where as follows from (4) and Assumption A3, $f_x^i(g_t,x_t) < 0$ and $f_g^i(g_t,x_t) > 0$ for $z_t^i \leq \tilde{z}$.

III. The Time Path of the Macroeconomic Variables

III.A. Technological Progress

Suppose that technological progress, g_{t+1}, which takes place from periods t to period $t + 1$, depends upon the average quality (education) among the working generation in period t, e_t:

$$g_{t+1} \equiv (A_{t+1} - A_t)/A_t = \psi(e_t), \tag{16}$$

31. It should be noted that although the kink in the function $e(g_t,q_t)$ is an artifact of the existence of two types of individuals, the inflection in the curve would emerge under a wide range of continuous distributions.

where the rate of technological progress, g_{t+1}, is an increasing, strictly concave function of the average level of education of the working generation at time t, e_t, and $\psi(0) = 0$.[32]

The level of technology at time $t + 1$, A_{t+1}, is, therefore,

$$A_{t+1} = [1 + g_{t+1}]A_t = [1 + \psi(e_t)]A_t, \tag{17}$$

where the technological level at time 0 is historically given at a level A_0.

Hence as follows from (13), (16), and Lemmas 1 and 2, g_{t+1} is uniquely determined by g_t and q_t:

$$g_{t+1} = \psi(e(q_t,g_t)) \equiv g(g_t,q_t), \tag{18}$$

where $g_q(g_t,q_t) > 0$, $g_g(g_t,q_t) > 0$, and $g_{gg}(g_t,q_t) < 0$.

III.B. Population and Fertility Rates across Types

The evolution of the working population over time is given by

$$L_{t+1} = n_t L_t, \tag{19}$$

where $L_t = L_t^b + L_t^a$, is the population size of generation t; L_0^a, L_0^b, and therefore L_0 are given, and n_t is the average fertility rate in the population. That is,

$$n_t \equiv q_t n_t^a + (1 - q_t)n_t^b, \tag{20}$$

where as defined in (12), $q_t \equiv L_t^a/L_t$ is the fraction of adult individuals of type a in generation t (born to type a individuals), and n_t^i is the number of children of each individual of type $i = a,b$. Given that $g_{t+1} = g(g_t,q_t)$, it follows from (11) that

$$n_t^i = n^i(g_t,x_t,q_t), \quad i = a,b. \tag{21}$$

The evolution of the fraction, q_t, of individuals with high valuation for quality (type a), is governed by the evolution of the population of the two types over time. Since for $i = a, b$, $L_{t+1}^i = n_t^i L_t^i$, where L_t^i is the size of the population of type i in generation t, it follows from (12), (18), and (21) that

$$q_{t+1} = \frac{n_t^a}{n_t} q_t \equiv q(g_t,x_t,q_t). \tag{22}$$

32. The abstraction from the complementary role of the scale of the economy (i.e., the size of the population) in the determination of technological progress is designed to sharpen the focus on the role of the evolutionary process in the transition to modern growth.

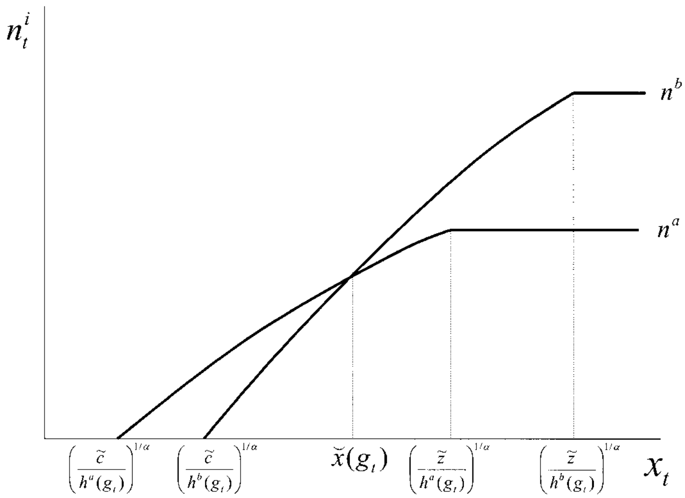

FIGURE IV

Differential Fertility Rates across Types in the Process of Development

The figure depicts the reversal in the evolutionary advantage in the process of development, as effective resources per worker, x_t, increase over time. In the predemographic transition era, fertility rates of individuals with high valuation for quality, n_t^a, are higher than those with low valuation for quality, n_t^b, and the quality trait increases its representation in the population, whereas in the post-demographic transition era this evolutionary advantage is reversed.

The analysis of the relationship between the economic environment and the evolutionary advantage of different types of individuals, as derived in the following lemma, indicates that in the early Malthusian era, when humans merely struggle for survival, individuals of type a (i.e., individuals with a preference bias toward offspring's quality) have an evolutionary advantage over individuals of type b. That is, the fraction of individuals of type a, q_t, rises in the population, despite their preference bias against quantity. However, once the economic environment improves sufficiently the evolutionary pressure weakens, the significance of quality for survival (fertility) declines, and type b individuals—the quantity type—gain the evolutionary advantage.

LEMMA 3. Under A1, for any given $g_t \geq 0$, as depicted in Figure IV, there exists a unique $\breve{x}_t \in ([\tilde{c}/h^b(g_t)]^{1/\alpha}, [\tilde{z}/h^b(g_t)]^{1/\alpha}) \equiv$

$\breve{x}(g_t;\, q)$ such that $\forall x_t > [\tilde{c}/h^b(g_t)]^{1/\alpha}$ (i.e., $\forall z_t^b > \tilde{c}$),

$$n_t^a \begin{cases} > n_t^b & \text{for } x_t < \breve{x}_t \\ = n_t^b & \text{for } x_t = \breve{x}_t \\ < n_t^b & \text{for } x_t > \breve{x}_t. \end{cases}$$

Proof. As follows from (11), $n_t^a > n_t^b = 0$ for $x_t = [\tilde{c}/h^b(g_t)]^{1/\alpha}$, and $n_t^b > n_t^a$ for $x_t \geq [\tilde{z}/h^b(g_t)]^{1/\alpha}$. Hence, since $\forall x_t \in ([\tilde{c}/h^b(g_t)]^{1/\alpha}, [\tilde{z}/h^b(g_t)]^{1/\alpha})$ (i.e., for the range under which $\partial n^b(g_t,x_t;\, q)/\partial x_t > 0$) $\partial n^b(g_t,x_t;\, q)/\partial x_t > \partial n^a(g_t,x_t;\, q)/\partial x_t$. Noting that as follows from Lemma 2 $e_t^a > e_t^b$ $\forall t > 0$, the lemma follows from the Intermediate Value Theorem. □

Figure IV depicts the fertility rates, n_t^b and n_t^a, of individuals of the two types as a function of effective resources per efficiency unit of labor x_t, given the rate of technological progress g_t. In the early stages of development, effective resources per efficiency unit of labor are low (less than $\breve{x}(g_t;\, q)$) and the fraction of individuals with high valuation for quality (type a) increases. However, as the level of effective resources per efficiency unit of labor increases sufficiently (i.e., $x_t > \breve{x}(g_t;\, q)$) and the Malthusian pressure relaxes, the rate of population growth among individuals of type b—the quantity type—overtakes the rate among type a—the quality type.[33] It should be noted, that as established in Proposition 1, the increase in the rate of technological progress that brings about the increase in effective resources generates initially an increase in fertility rates of both types of individuals, but ultimately, due to the substitution of quality for quantity a demographic transition takes place, and fertility rates decline.[34]

If the economy is populated only with individuals of the quantity type (type b), as is derived from the following lemma, fertility rates of individuals of type b are at replacement level.

LEMMA 4. For g_t and q_t such that $g_{t+1} = g(g_t,q_t) \leq \underline{g}^b$, there exists a unique level of effective resources per efficiency unit of labor, $\dot{x}(g_t,q_t) \in (0,[\tilde{z}/h^b(g_t)]^{1/\alpha})$, such that the fertility

33. Fertility rates of type b individuals exceed those of type a, when type b individuals are still constrained by subsistence consumption. However, for type a the constraint may not be binding. Figure IV is drawn for the case in which the constraint is binding for both types.

34. An increase in g_t shifts the curves $n^a(g_t,x_t;\, q)$ and $n^b(g_t,x_t;\, q)$ in Figure IV rightward and downward.

rate of type b individuals is at a replacement level; i.e.,

$$n^b(g_t,\grave{x}(g_t,q_t),q_t) = 1 \quad \text{for } g(g_t,q_t) \leq \underline{g}^b.$$

Proof. As follows from (11),

$$n_t^b \begin{cases} = 0 & \forall x_t \leq [\tilde{c}/h^b(g_t)]^{1/\alpha} \\ > 1 & \forall x_t \geq [\tilde{z}/h^b(g_t)]^{1/\alpha} \end{cases} \text{for } g(g_t,q_t) \leq \underline{g}^b.$$

Hence, since n_t^b is continuous and monotonically increasing in x_t, the lemma follows from the Intermediate Value Theorem. □

Suppose that the entire population in the economy is of type b, i.e., $q = 0$, and the economy is in a steady-state equilibrium where the rate of technological progress is 0 (and accordingly $e = 0$). Furthermore, since n_t^b increases in x_t, and x_t decreases when $n_t^b > 1$ and increases when $n_t^b < 1$, it follows from Lemma 4 that in this steady-state equilibrium fertility rate is precisely at a replacement level, i.e., $n_t^b = 1$, and effective resources per efficiency unit of labor is $\grave{x}$.

The evolution of the human brain in the transition to Homo Sapiens and the complementarity between brain capacity and the reward for human capital has increased the evolutionary optimal investment in offspring's quality. The distribution of valuation for quality lagged behind the evolutionary optimal level, and dynasties characterized by higher valuation for quality had therefore an evolutionary advantage.[35] They generated higher income, and in the Malthusian epoch when income is positively associated with aggregate resources allocated to child rearing, a larger number of offspring, and therefore an evolutionary advantage.

Hence, it is assumed that when g and q are infinitesimally small, the return to quality is sufficiently high so as to assure that the income of type a individuals—the quality type—is sufficient to permit their fertility rates to be above replacement. Namely,

$$\text{(A4)} \qquad \grave{x}(0,0) < \breve{x}(0;\, 0),$$

where $\grave{x}(0,0) = \tilde{c}/[1 - \tau])^{1/\alpha}$ is the level of effective resources under which fertility rates of individuals of type b are at a replacement level, and $\breve{x}(0;\, 0)$ is the level of effective resources under which fertility rates of individuals of both types are equal.

Since the size of the population who has high valuation for

35. The Concluding Remarks further discuss the delay in the evolution of the valuation for quality.

quality (type a) is assumed to be very small, it has a negligible effect on the size of x_0 and therefore in period 0, $x_0 = \breve{x}(0; 0) < \breve{x}_0$. Hence, as follows from Lemmas 3 and 4, the fertility rate of individuals of the quality type in period 0, n_0^a, is above replacement, and it exceeds the fertility rate of individuals of the quantity type in this period, n_0^b; i.e.,

$$n_0^a > n_0^b = 1. \tag{23}$$

Thus, in early stages of development the Malthusian pressure provides an evolutionary advantage for the quality type. The income of individuals of the quantity type is near subsistence and fertility rates are therefore near the replacement level. In contrast, the wealthier, quality type, can afford higher fertility rates (of higher quality offspring). As technological progress brings about an increase in income, the Malthusian pressure relaxes, and the domination of wealth in fertility decisions diminishes. The inherent advantage of the quantity type in reproduction gradually dominates, and fertility rates of the quantity type ultimately overtake those of the quality type.

III.C. Human Capital and Effective Resources

The growth rate of efficiency units of labor, μ_{t+1}, as follows from (14), is

$$\mu_{t+1} \equiv \frac{H_{t+1}}{H_t} - 1 = \frac{q_t n_t^a f_{t+1}^a h_{t+1}^a + (1-q_t) n_t^b f_{t+1}^b h_{t+1}^b}{q_t f_t^a h_t^a + (1-q_t) f_t^b h_t^b} - 1. \tag{24}$$

LEMMA 5. Under A1 and A3, $\forall x_t > [\tilde{c}/h^b(g_t)]^{1/\alpha}$ (i.e., $\forall z_t^b > \tilde{c}$),[36]

$$\mu_{t+1} = \mu(g_t, x_t, q_t),$$

where $\forall z_t^b \geq \tilde{z}$, $\mu_g(g_t, x_t, q_t) < 0$, and

$$\mu_x(g_t, x_t, q_t) \quad \begin{cases} >0 & \text{if } x_t < [\tilde{z}/h^b(g_t)]^{1/\alpha} \\ =0 & \text{otherwise} \end{cases}$$

$$\mu_q(g_t, x_t, q_t)|_{g_{t+1}=g_t} \gtreqless 0 \text{ if and only if } n_t^a \gtreqless n_t^b.$$

Proof. Substituting (11) and (18) into (24), noting (15), $\mu_{t+1} = \mu(g_t, x_t, q_t)$, and the properties follow, noting Proposition 1. □

36. This is the range in which individuals of type b (and hence of type a) do not become extinct.

The evolution of effective resources per efficiency unit of labor, $x_t \equiv A_tX/H_t$, as follows from (18) and (24), is

$$x_{t+1} = \frac{1 + g_{t+1}}{1 + \mu_{t+1}} x_t \equiv x(g_t, x_t, q_t). \tag{25}$$

It depends, therefore, on the rate of technological progress and the growth rate of efficiency units of labor.

IV. The Dynamical System

The development of the economy is characterized by the trajectory of output, population, technology, education, and human capital. The dynamic path of the economy, is fully determined by a sequence $\{x_t, g_t, q_t\}_{t=0}^{\infty}$ which describes the time path of effective resources per efficiency unit of labor, x_t, the rate of technological progress, g_t, and the fraction, q_t, of individuals of the quality type in the population. It is governed by a three-dimensional, first-order, autonomous dynamical system given by equations (18), (22), and (25):

$$\begin{cases} x_{t+1} = x(g_t, x_t, q_t); \\ g_{t+1} = g(g_t, q_t); \\ q_{t+1} = q(g_t, x_t, q_t). \end{cases} \tag{26}$$

The analysis of the dynamical system is greatly simplified since as analyzed in the following subsection, the evolution of g_t and therefore of $e_t = e(g_t; q)$ is determined independently of x_t, provided that q_t is held constant.

IV.A. Conditional Dynamics of Technology and Education

The conditional dynamical subsystem, $g_{t+1} = g(g_t; q)$, which describes the time path of the rate of technological change, for a given q, is a one-dimensional system. The geometrical analysis is more revealing, however, if the equation of motion $g_{t+1} = \psi(e(g_t, q_t)) \equiv g(g_t, q_t)$, is decomposed into the joint evolution of technology, $g_{t+1} = \psi(e_t)$, and education, $e_t = e(g_t, q_t)$.

The evolution of the rate of technological progress and education, conditional on a given fraction q of individuals with high valuation for quality, is characterized by the sequence $\{g_t, e_t; q\}_{t=0}^{\infty}$

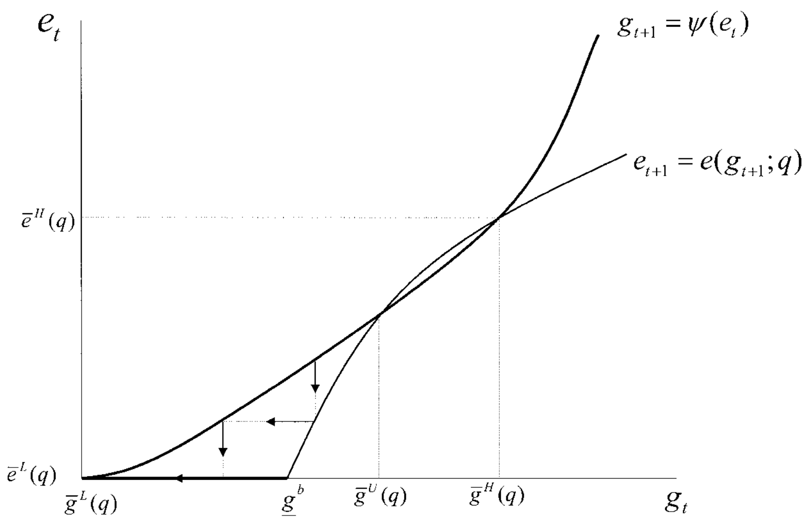

FIGURE Va
The Evolution of Education and Technological Progress in the Absence of Individuals of the Quality Type in the Population

$q = 0$

The figure describes the evolution of education, e_t, and the rate of technological progress, g_t, when the fraction q of individuals with high valuation for quality is zero. The curve labeled $g_{t+1} = \psi(e_t)$ shows the effect of the average quality of the population on the rate of technological progress. The curve labeled $e_{t+1} = e(g_{t+1}; q)$ shows the effect of expected technological change on the average investment in offspring quality. The dynamical system is characterized by multiple steady-state equilibria. The point of intersection between the two curves, at the origin, is a Malthusian locally stable temporary steady-state equilibrium. In early stages of development, given the initial conditions, the economy is in the vicinity of this steady-state where investment in education and the rate of technological progress are zero.

that satisfies in every period t the conditional two-dimensional system

$$(27) \qquad \begin{cases} e_t = e(g_t; q); \\ g_{t+1} = \psi(e_t). \end{cases}$$

In light of the properties of the functions $e_t = e(g_t; q)$ and $g_{t+1} = \psi(e_t)$, given by (13) and (16), it follows that in any time period this conditional dynamical subsystem may be characterized by one of the two *qualitatively* different configurations, which are depicted in Figures Va–Vc and derived formally in Appendix 1. The economy shifts endogenously from one configuration to another as the frac-

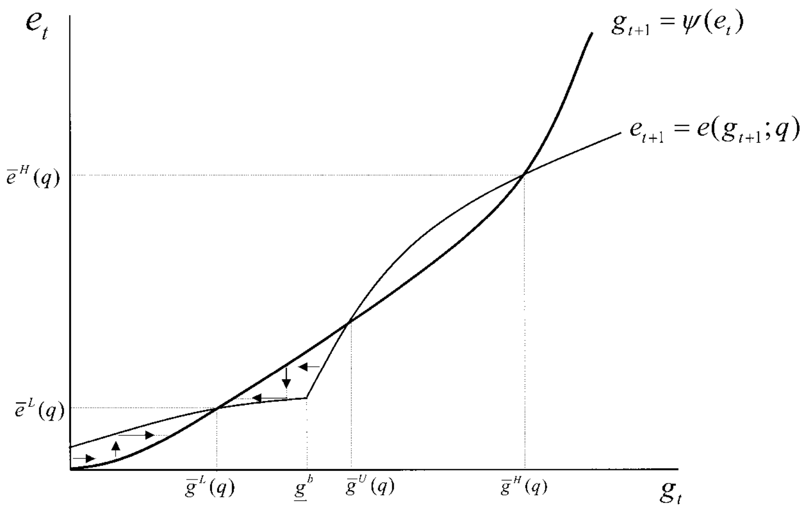

FIGURE Vb
The Evolution of Education and Technological Progress for a Small Fraction of Individuals of the Quality Type

$q \in (0,\hat{q})$

The figure describes the evolution of education, e_t, and the rate of technological progress, g_t, for a fraction of individuals of the quality type that is below a threshold level that would trigger the takeoff from the Malthusian epoch (i.e., $q \in (0,\hat{q})$). Given the initial conditions, in the absence of large shocks the economy remains in the vicinity of a low steady-state equilibrium ($\bar{g}^L$,$\bar{e}^L$), where both the level of education and the rate of technological progress are low but positive. Due to the evolutionary advantage of individuals of the quality type, their fraction in the population, q, increases over time, and consequently the curve $e_t = e(g_t; q)$ shifts upward.

tion of individuals with high valuation for quality q increases and the curve $e_t = e(g_t; q)$ shifts upward to account for the positive effect of an increase in q on the average investment in quality, e_t.

As depicted in Figures Va–Vc, the set of steady-state equilibria of the conditional dynamical system (27) changes qualitatively as the value of q passes a threshold level $\hat{q}$ (which is formally derived in Appendix 1). That is for all $q < \hat{q}$ the system is characterized by two locally stable steady-state equilibria, $[\bar{g}^L(q),\bar{e}^L(q)]$ and $[\bar{g}^H(q),\bar{e}^H(q)]$, and an unstable equilibrium, $[\bar{g}^U(q),\bar{e}^U(q)]$, whereas for all $q > \hat{q}$ by a unique globally stable steady-state equilibrium, $[\bar{g}^H(q),\bar{e}^H(q)]$:

$$(28)\qquad \begin{array}{ll} \{[\bar{g}^L(q),\bar{e}^L(q)],[\bar{g}^U(q),\bar{e}^U(q)],[\bar{g}^H(q),\bar{e}^H(q)]\} & \text{for } q < \hat{q} \\ [\bar{g}^H(q),\bar{e}^H(q)] & \text{for } q > \hat{q}, \end{array}$$

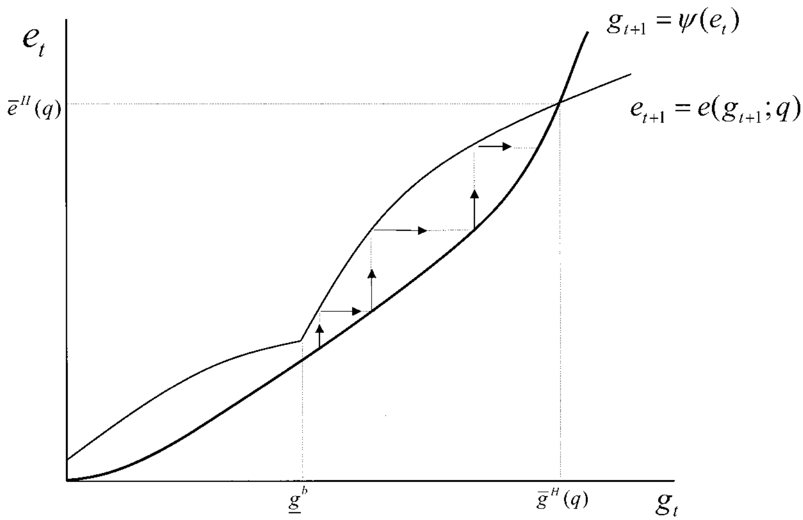

FIGURE Vc

The Evolution of Education and Technological Progress for a Large Fraction of Individuals of the Quality Type

$q > \hat{q}$

The figure describes the evolution of education, e_t, and the rate of technological change, g_t, once the fraction of individuals with high valuation for quality type in the population has grown sufficiently, due to the evolutionary process, and is above the threshold level that triggers the takeoff from the Malthusian epoch (i.e., $q > \hat{q}$). The dynamical system is characterized by a unique globally stable steady-state equilibrium $(\bar{g}^H, \bar{e}^H)$. The economy converges to this steady state characterized by high levels of education and technological progress and sustained economic growth.

where $\bar{g}^L(q) < \underline{g}^b < \bar{g}^H(q)$ and $\partial[\bar{g}^j(q), \bar{e}^j(q)]/\partial q \geqslant 0$ for $j = L, H$.

As depicted in Figure Va (for $q = 0$) and Figure Vb (for $0 < q < \hat{q}$), if the fraction of individuals with high preference for quality is low (i.e., $q < \hat{q}$), the economy is characterized by multiple locally stable steady-state equilibria: a low steady-state equilibrium $[\bar{g}^L(q), \bar{e}^L(q)]$, where $\bar{g}^L(q) < \underline{g}^b$ and therefore only individuals with high valuation for quality invest in human capital, and a high steady-state equilibrium $[\bar{g}^H(q), \bar{e}^H(q)]$, where $\bar{g}^H(q) > \underline{g}^b$ and therefore both types of individuals invest in human capital. As the fraction q of individuals with high valuation for quality increases the rate of technological progress and the average level of education in each of the two stable steady-state equilibria increase as well.[37]

As depicted in Figure Vc, for a sufficiently high fraction of individuals with high preference for quality (i.e., as long as $q \geq \hat{q}$), the economy is characterized by a unique globally stable steady-state equilibrium $[\bar{g}^H(q),\bar{e}^H(q)]$ where both types of individuals invest in human capital.

IV.B. Conditional Dynamics of Technology and Effective Resources

The evolution of the rate of technological progress, g_t, and effective resources per efficiency unit of labor, x_t, for a given fraction, q, of individuals of the quality type, is characterized by the sequence $\{g_t,x_t;\ q\}_{t=0}^{\infty}$ that satisfies in every period t the conditional two-dimensional system:

$$\begin{cases} g_{t+1} = g(g_t;\ q); \\ x_{t+1} = x(g_t,x_t;\ q). \end{cases} \tag{29}$$

The phase diagrams of this conditional dynamical system, depicted in Figures VIa–VIc, contain three loci: The *CC* locus, the *GG* locus, and the *XX* locus. The properties of these loci and the dynamics of (g_t,x_t) in relation to these loci are derived in Appendix 2.

The CC locus—The Subsistence Consumption Frontier. This locus is the set of all pairs (g_t,x_t) such that the income of the low income individuals (the quantity type) is at a level above which the subsistence consumption constraint is no longer binding (i.e., $z_t^b = \tilde{z}$). The locus separates the regions in which the subsistence constraint is binding for at least individuals of type b from those regions in which it is not binding for both types. As depicted in Figure VIa–VIc, the *CC* locus, is an upward sloping curve in the plane (g_t,x_t) with a positive vertical intercept.

The GG locus. This locus is the set of all pairs (g_t,x_t) such that, for a given level of q, the rate of technological progress, g_t, is at a steady state (i.e., $g_{t+1} = g_t$). As follows from (18), along the *GG* locus, $g_{t+1} = g(g_t;\ q) = g_t$. The *GG* locus is therefore not affected by the effective resources per efficiency unit of labor,

37. In the knife-edge case in which $q = \hat{q}$, the system is characterized by multiple steady-state equilibria. However, only the upper one, $[\bar{g}^H(q),\bar{e}^H(q)] \gg [\underline{g}^b,0]$, is locally stable.

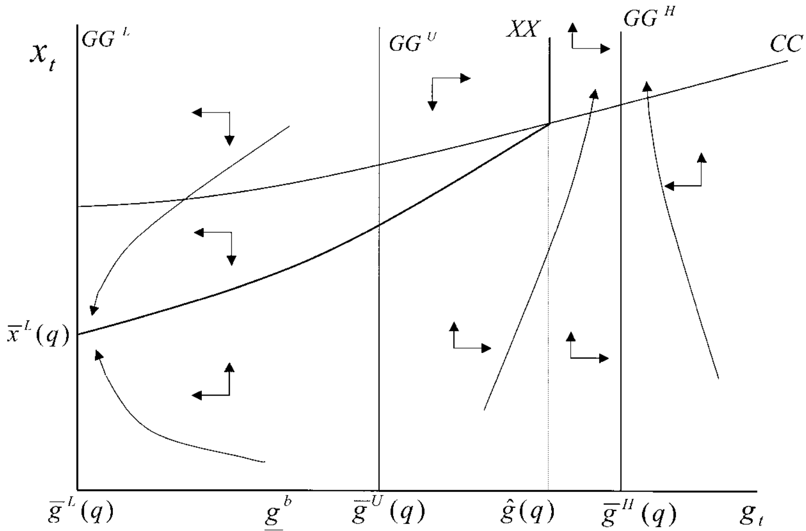

FIGURE VIa

The Evolution of Technological Progress and Effective Resources in the Absence of Individuals of the Quality Type in the Population

$q = 0$

The figure describes the evolution of the rate of technological progress, g_t, and effective resources per efficiency unit of labor, x_t, when the fraction, q of individuals with high valuation for quality is zero. The *CC* locus is the set of all pairs (g_t,x_t) below which the subsistence constraint is binding for at least part of the population. The *GG* locus is the set of all pairs (g_t,x_t), for which the rate of technological progress is constant over time. The *XX* locus is the set of all pairs (g_t,x_t) for which effective resources per efficiency units of labor are constant over time. The point of intersection between the *GG* locus and the *XX* locus, at $g_t = 0$, is a Malthusian locally stable temporary steady-state equilibrium. In the early stages of development, given the initial conditions, the economy is in the vicinity of this steady state where investment in education and the rate of technological progress are zero, and the level of resources per efficiency unit of labor places the fertility rate at a replacement level, and output per capita is constant.

x_t, and as depicted in Figures VIa–VIc, the *GG* locus consists of vertical line(s) at the steady-state level(s) of g, shown in (28) and depicted in Figures Va–Vc. There are two qualitatively different configurations. For $q < \hat{q}$, as depicted in Figures VIa and VIb (and corresponding to Figures Va and Vb), the *GG* locus consists of three vertical lines at the conditional steady-state level of g: $\{\bar{g}^L(q),\bar{g}^U(q),\bar{g}^H(q)\}$. For $q > \hat{q}$, as depicted in Figure VIc (and corresponding to Figure Vc) the *GG* locus

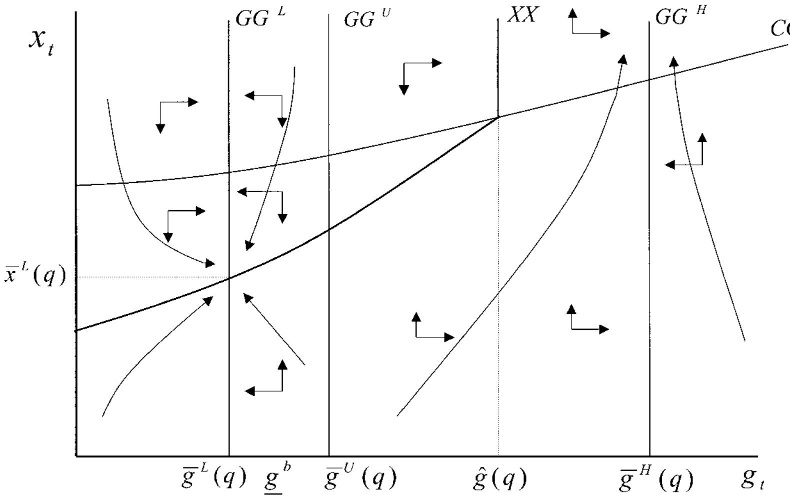

FIGURE VIb
The Evolution of Technological Progress and Effective Resources for a Small Fraction of Individuals of the Quality Type

$q \in (0,\hat{q})$

The figure describes the evolution of the rate of technological progress, g_t, and effective resources per efficiency unit of labor, x_t, for a fraction of individuals of the quality type that is below a threshold level that would trigger the takeoff from the Malthusian epoch (i.e., $q \in (0,\hat{q})$). Given the initial conditions, in the absence of large shocks the economy remains in the vicinity of the low steady-state equilibrium $(\bar{g}^L,\bar{x}^L)$, where both the rate of technological progress and education are low, and the level of effective resources permits fertility rates above replacement. Due to the evolutionary advantage of individuals of the quality type their fraction in the population, q, increases over time, and consequently the GG^L locus and the GG^U locus shift toward one another.

consists of a unique vertical line at the conditional steady-state level of g, $\bar{g}^H(q)$.[38]

The XX locus. This locus is the set of all pairs (g_t,x_t) such that the effective resources per efficiency unit of labor, x_t, for a given level of q, is in a steady state (i.e., $x_{t+1} = x_t$). As follows from (25), along the XX locus, $x_{t+1} = [(1 + g_{t+1})/(1 + \mu_{t+1})]x_t \equiv x(g_t,x_t;\, q) = x_t$. Hence, along the XX locus the growth rate of efficiency units of labor, μ_t, and the rate of technological progress, g_t, are equal. As depicted in Figures VIa–VIc, the XX locus has a positive vertical intercept at $g = 0$, it increases monotonically

38. For the knife-edge case of $q = \hat{q}$ and $\bar{g}^L(\hat{q}) = \bar{g}^U(\hat{q}) = \hat{g}^b$, the GG locus consists of two vertical lines at the steady-state level of g: $\{\hat{g}^b,\bar{g}^H(\hat{q})\}$.

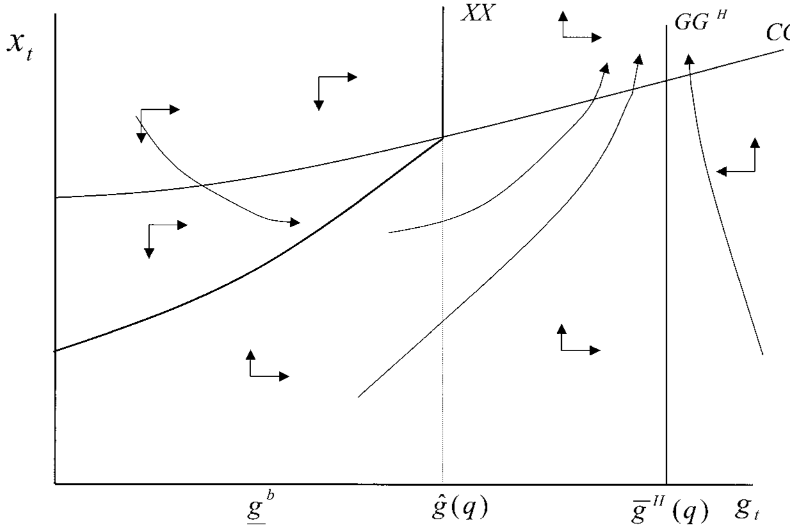

FIGURE VIc
The Evolution of Technological Progress and Effective Resources for a Large Fraction of Individuals of the Quality Type

$q > \hat{q}$

The figure describes the evolution of the rate of technological progress, g_t, and effective resources per efficiency unit of labor, x_t, once, due to the evolutionary process, the fraction of the quality type in the population has grown sufficiently and is above the threshold level that triggers the takeoff from the Malthusian epoch (i.e., $q > \hat{q}$). The economy converges to a steady-state equilibrium which is characterized by a high level of education, a high rate of technological progress, and positive growth rates of effective resources per efficiency unit of labor and output per capita.

with g_t, and it becomes vertical once the XX locus intersects the CC locus (at $g_t = \hat{g}(q)$). Furthermore, as q increases, the value of $\hat{g}(q)$ declines.

IV.C. Conditional Steady-State Equilibria

This subsection describes the properties of the conditional steady-state equilibria of the conditional dynamical system (29) that governs the evolution of $\{g_t, x_t;\ q\}_{t=0}^{\infty}$ based on the phase diagrams depicted in Figures VIa–VIc.

The set of steady-state equilibria of this dynamical system is characterized by a constant growth rate of the technological level and a constant growth rate of effective resources per efficiency unit of labor, and therefore a constant growth rate of output per capita. Let χ_t denote the growth rate of effective resource per worker. As follows from (25),

$$(30)\qquad \chi_t \equiv \frac{x_{t+1} - x_t}{x_t} = \frac{g_{t+1} - \mu_{t+1}}{1 + \mu_{t+1}} \equiv \chi(g_t, x_t; q).$$

As depicted in Figures VIa–VIc, the set of steady-state equilibria of the conditional dynamical system (29) changes qualitatively as the fraction of individuals with high valuation for quality passes the threshold level $\hat{q}$. That is, for all $q < \hat{q}$ the system is characterized by two locally stable steady-state equilibria, $[\bar{g}^L(q), \bar{\chi}^L(q)]$ and $[\bar{g}^H(q), \bar{\chi}^H(q)]$, and an unstable equilibrium $[\bar{g}^U(q), \bar{\chi}^U(q)]$, whereas for all $q > \hat{q}$ by a unique globally stable steady-state equilibrium $[\bar{g}^H(q), \bar{\chi}^H(q)]$, where the growth rate of output per capita is equal to the growth rate of effective resources per efficiency unit of labor:[39]

$$(31)\qquad \begin{array}{ll} \{[\bar{g}^L(q), \bar{\chi}^L(q)], [\bar{g}^U(q), \bar{\chi}^U(q)], [\bar{g}^H(q), \bar{\chi}^H(q)]\} & \text{for } q < \hat{q}; \\ [\bar{g}^H(q), \bar{\chi}^H(q)] & \text{for } q > \hat{q}, \end{array}$$

where $\bar{g}^L(q) < \underline{g}^b < \bar{g}^H(q)$, $\bar{\chi}^H(q) > \bar{\chi}^L(q) = 0$, $\partial\bar{\chi}^L(q)/\partial q = 0$, $\partial\bar{\chi}^H(q)/\partial q > 0$, and for $j = L, H$, $\partial\bar{g}^j(q)/\partial q > 0$.

Hence, in early stages of development, when the fraction q of individuals with high valuation for quality in the population is sufficiently small, the conditional dynamical system, as depicted in Figure VIa and VIb in the space (g_t, x_t), is characterized by two locally stable steady-state equilibria. However, since the initial levels of g and q are sufficiently small, the economy converges to the Malthusian steady-state equilibrium $[\bar{g}^L(q), \bar{x}^L(q)]$, where the rate of technological progress is positive, but output per capita is constant.

In later stages of development as q_t increases sufficiently, the Malthusian conditional steady-state equilibrium vanishes. The dynamical system as depicted in Figure VIc is characterized by a unique steady-state equilibrium where the growth rates of the level of technology and effective resources per efficiency unit of labor are constant at a level $[\bar{g}^H(q), \bar{\chi}^H(q)] \gg 0$. The steady-state growth rate of output per capita is equal to the growth rate of effective resources per efficiency unit of labor.

39. Note that for $q = \hat{q}$, the system is characterized by multiple steady-state equilibria. However, only the upper one is locally stable.

V. HUMAN EVOLUTION AND THE TRANSITION FROM STAGNATION TO GROWTH

This section analyzes the relationship between the evolution of mankind and economic growth since the emergence of the human species. The analysis demonstrates that the inherent evolutionary pressure that has been associated with the Malthusian epoch brought about the transition from Malthusian stagnation to sustained economic growth. The Malthusian pressure generated a process of natural selection in which the representation of individuals with high valuation for child-quality increased, raising the average level of human capital and inducing higher rates of technological progress that ultimately brought about the transition from Malthusian stagnation to sustained economic growth.

Suppose that in the early era of the human species, the population of the world consisted of two types of individuals: individuals of the quantity type—who place a lower weight on the quality of their offspring, and an infinitesimally small fraction of individuals of the quality type—who place a higher weight on the quality of their offspring.[40] Since the fraction q of individuals of the quality type is infinitesimally small, they have no impact on the rate of technological progress. Given the initial conditions, the economy is therefore in a steady-state equilibrium where the rate of technological progress, g_t, is nearly zero, parents of type b have no incentive to invest in the quality of their children and the average quality in the population is therefore nearly zero. Hence, as depicted in Figure Va in the plane (g_t,e_t) for $q = 0$, this conditional system is in a locally stable steady-state equilibrium, where $\bar{g}^L(0) = 0$ and $\bar{e}^L(0) = 0$. As depicted in Figure VIa in the plane (g_t,x_t), the economy is in a locally stable Malthusian steady-state equilibrium where effective resources are constant at a level $\bar{x}^L(0) > 0$, the level of human capital is constant, and hence, output per capita is constant as well. In this steady-state equilibrium the population is constant, and the fertility rate is therefore at replacement level; i.e., $n_t^b = 1$. Furthermore, (small) shocks to population or resources would be undone in a classic Malthusian fashion.

40. The existence of only a small fraction of individuals of high valuation in the initial period is discussed in the Introduction and in the Concluding Remarks.

As long as the fraction of individuals of the quality type is sufficiently small, (i.e., $q_t < \hat{q}$), as depicted in Figure Vb in the space (g_t, e_t), the economy is in the vicinity of a conditional locally stable steady-state equilibrium $[\bar{g}^L(q), \bar{e}^L(q)]$, where $\bar{g}^L(q) < \underline{g}^b$. As established in Lemma 2, as long as the rate of technological progress is below $\underline{g}^b$—the threshold level of the rate of technological progress above which individuals of type b start investing in the quality of their children—the quality chosen by type b individuals is $e_t^b = 0$, and the quality chosen by type a individuals is $e_t^a > 0$. Since the fraction of individuals of type a is small, the average level of education, e_t, is therefore positive but small (i.e., $g_{t+1} = \psi(e_t) < \underline{g}^b$). Furthermore, as depicted in Figure VIb in the space (g_t, x_t), this conditional locally stable steady-state equilibrium corresponds to a locally stable conditional Malthusian steady-state equilibrium, $[\bar{g}^L(q), \bar{x}^L(q)]$, where $\bar{g}^L(q) < \underline{g}^b$. The existence of a small fraction of individuals of the quality type generates a slow rate of technological progress. Investment in quality is negligible, and resources above subsistence are devoted to child rearing. The Malthusian mechanism therefore generates a proportional increase in output and population, and the economy is in the vicinity of a temporary locally stable Malthusian steady-state equilibrium.

In this early Malthusian era, individuals of type a (i.e., individuals with a preference bias toward quality of offspring) have an evolutionary advantage over individuals of type b. That is, the fraction of individuals of type a rises in the population, despite their preference bias against the quantity of their offspring. Hence, in early stages of development the Malthusian pressure provides an evolutionary advantage to the quality type. The income of individuals of the quantity type is near subsistence, and fertility rates are therefore near replacement level. In contrast, the wealthier, quality type, as depicted in Figure VII, can afford higher fertility rates (of higher quality offspring). As depicted in Figure IV (and analyzed in Appendix 2), in the Malthusian epoch $n_t^a > n_t^b$ for all $q_t < \hat{q}$, and hence the fraction q_t of individuals of the quality type in the population increases monotonically over this Malthusian regime. As q_t increases, the locus $e(g_t, q_t)$ in Figure Vb shifts upward, and the corresponding conditional steady-state equilibrium reflects higher rate of technological progress along with higher average quality.

Eventually, as q_t crosses the threshold level $\hat{q}$, the condi-

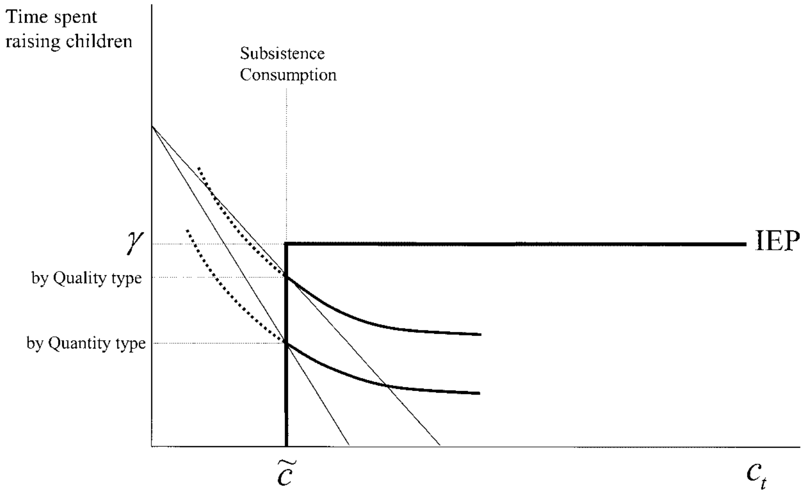

FIGURE VII
The Evolutionary Advantage of Individuals of the Quality Type in the Malthusian Epoch

The figure shows that in the Malthusian epoch there is a positive relationship between income and resources allocated for child rearing. Individuals of the quality type have higher income, and thereby more children of higher quality, and their fraction in the population increases during this epoch.

tional dynamical system changes qualitatively. The $e(g_t,q_t)$ locus in Figure Vb shifts sufficiently upward so as to eliminate the lower intersection with the locus $g_{t+1} = \psi(e_t)$, and the loci GG^L and GG^U depicted in Figure VIb vanish, whereas the GG^H locus shifts rightward, and the XX locus above the Subsistence Consumption Frontier shifts leftward. As depicted in Figures Vc and VIc, the Malthusian conditional steady-state equilibrium vanishes, and the economy is no longer trapped in the vicinity of this equilibrium. The economy converges gradually to a unique globally stable conditional steady-state equilibrium $[\bar{g}^H(q),\bar{e}^H(q),\bar{\chi}^H(q)] \gg [\underline{g}^b,0,0]$, where both types of individuals invest in human capital, the rate of technological progress is high, and the growth rate of effective resources per efficiency unit of labor is positive. Once the rate of technological progress exceeds $\underline{g}^b$—the threshold level of the rate of technological progress above which individuals of type b start investing in the quality of their children—the growth rate of the average level of education increases, and consequently there is an acceleration in the rate of technological progress that may be associated with the Industrial Revolution. The positive feedback between the rate of technological progress and the level of education reinforces the growth process, the economy ultimately crosses the Subsistence Consumption Frontier, setting the stage for a demographic transition in which the rate of population growth declines and the average level of education increases. The economy converges to the unique, stable, conditional steady-state equilibrium above the Subsistence Consumption Frontier with a positive growth rate of output per worker.[41]

Technological progress has two effects on the evolution of population, as shown in Proposition 1. First, by inducing parents to give their children more education, technological progress, ceteris paribus, lowers the rate of population growth. But, second, by raising potential income, technological progress increases the fraction of time that parents devote to raising children. Initially, while the economy is in the Malthusian region of Figure VIb, the effect of technology on the parental budget constraint dominates, and the growth rate of the population increases. As the economy eventually crosses the Subsistence Consumption Frontier, further improvements in technology no longer have the effect of changing the amount of time devoted to child rearing. Faster technological change therefore raises the quality of children while reducing their number.

During the transition from the Malthusian epoch to sustained economic growth, once the economic environment improves sufficiently, the evolutionary pressure weakens, the significance of quality for survival (fertility) declines, and type b individuals—the quantity type—gain the evolutionary advantage. Namely, as technological progress brings about an increase in income, the Malthusian pressure relaxes, and the domination of wealth in fertility decisions diminishes. The inherent advantage of the quantity type in reproduction gradually dominates, and fertility rates of the quantity type ultimately overtake those of the quality type (as the level of effective resources exceeds $\breve{x}$). Hence, the fraction q_t of individuals who have high valuation for quality starts declining as the economy approaches the Subsistence Consumption Frontier. The model predicts therefore that

41. It should be noted that once the fraction of individuals of the quality type exceeds $\hat{q}$ and therefore $g_t > \underline{g}^b$, the demographic transition occurs regardless of the evolutionary process.

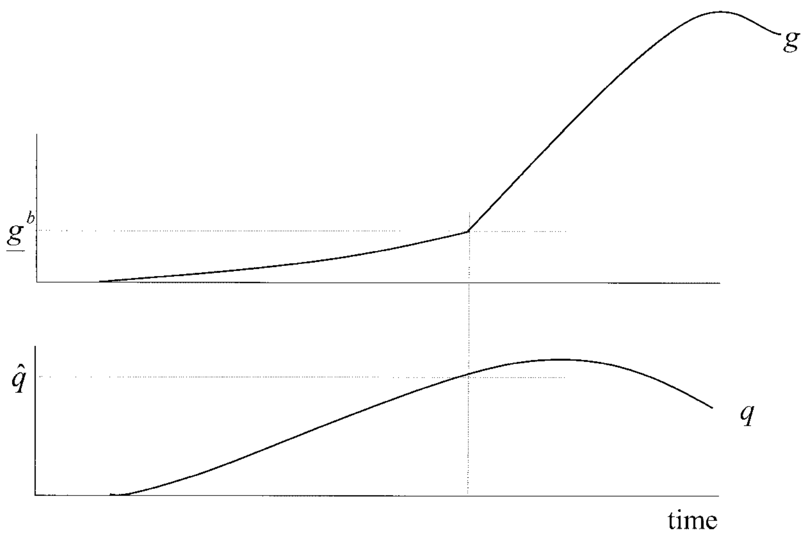

FIGURE VIII
The Dynamics of the Fraction of Individuals of the Quality Type and the Rate of Technological Progress

The figure shows that the fraction of individuals of the quality type in the population, q, increase gradually in the Malthusian epoch, and once it reaches the critical level, $\hat{q}$, it triggers an intensive interaction between technological progress and investment in quality that brings about the Industrial Revolution. The onset of the demographic transition reverses the evolutionary advantage, q declines gradually, and the rate of technological progress declines but remains positive.

the long-run equilibrium is characterized by a complete domination of the quantity type (i.e., $q = 0$). Nevertheless, the growth rate of output per worker remains positive, although at a lower level than the one that existed in the peak of the transition. As the level of q declines below the threshold level $\hat{q}$, depicted in Figure VIII, the conditional dynamical system that describes the economy is once again characterized by multiple locally stable steady-state equilibria, depicted in Figures Va, Vb, VIa, and VIb. However, unlike the situation in early stages of development, the position of the economy prior to the decline in q_t assures that the economy converges to the high steady-state equilibrium.[42] The

42. If mutations reduce the lower bound of the valuation for quality, an additional assumption will be needed in order to assure the existence of a sustained growth steady-state equilibrium. In particular, these mutations would generate an evolutionary disadvantage if for a sufficiently high rate of technological progress, the erosion effect dominates the productivity effect for individuals

incorporation of some additional plausible factors into the analysis, such as environmental effect on preferences (i.e., learning and imitation of the behavior of quality type) would permit heterogeneity of types in the long run. Furthermore, the incorporation of a positive effect of the scale of the population (given quality), on the rate of technological progress might prevent the decline in the growth rate of output per capita, depicted in Figure VIII, in the advanced stages of the evolution of the economy toward the (unconditional) long-run equilibrium.

Finally, fertility differential across income groups evolves nonmonotonically in the process of development. As depicted in Figure IV, in any period within the Malthusian Regime (i.e., as long as $g_t \leq \underline{g}^b$ and therefore $x_t < \check{x}$), fertility rates among richer individuals are predicted to be higher than those among poorer individuals, whereas in any period once the takeoff took place (i.e., once $x_t \geq x^{CC}(g_t)$ and therefore $x_t > \check{x}$), fertility rates among richer individuals are predicted to be lower than those among poorer individuals. Hence, in the course of the transition from the Malthusian Regime to the Modern Growth Regime the *cross-section* relationship between income and fertility is reversed. In the Malthusian Regime there is a positive cross-section correlation between income and fertility rates, whereas in the Modern Growth Regime this cross-section correlation is negative.

VI. Failed Takeoff Attempts

The analysis suggests that the interaction between the composition of the population and the rate of technological progress is the critical factor that determines the timing of the transition from stagnation to growth. In particular, the theory indicates that waves of rapid technological progress in the Pre-Industrial Revolution era had not generated a sustainable economic growth due to the shortage of individuals of the quality type in the population, whereas sustained economic growth in the Post-Industrial Revolution era may be attributed to the presence of a sufficiently high fraction of individuals of the quality type in the population.

As depicted in Figure VIa and VIb, if the fraction of individ-

who do not invest in human capital and consequently available resources for child rearing would not permit fertility rates above replacement.

uals of the quality type is low, the economy is characterized by multiple steady-state equilibria. Two locally stable equilibria: a Malthusian steady-state equilibrium where output per capita is constant near a subsistence level of consumption and a growth steady-state equilibrium where a positive growth rate of output per capita is sustainable.

Initial conditions place the economy in the vicinity of the Malthusian steady-state equilibrium. However, a sufficiently large technological shock would place the economy on a trajectory that leads to sustained growth. The composition of the population determines the effectiveness of a technological shock. The smaller is the fraction of individuals of the quality type in the population, the larger is the necessary size of the shock in order to generate a sustained takeoff from Malthusian stagnation. As the fraction of the quality type in the population increases (i.e., q_t rises), the distance between the loci GG^L and GG^U (depicted in Figure VIb) narrows, and the necessary jump in the rate of technological progress in order to facilitate a sustained takeoff decreases. Ultimately, as depicted in Figure VIc once q crosses the threshold level $\hat{q}$, the dynamical system changes qualitatively. It is characterized by a unique globally stable steady-state equilibrium with sustained economic growth, and the transition from Malthusian stagnation occurs without a need for a technological shock.

The analysis suggests therefore that those nonsustainable growth episodes during the Pre-Industrial Revolution period may be attributed to the presence of a relatively small fraction of individuals of the quality type in the population that would have invested sufficiently in education in response to the change in the technological environment and would have therefore allowed this rapid change in technology to be sustained.[43] Furthermore, one may meaningfully argue that given the finiteness of a technological leap, an adverse composition of the population could have virtually prevented a sustained takeoff from a Malthusian steady state. Unlike the nonsuccessful takeoff attempts in the Pre-Industrial Revolution era, the paper argues that the successful takeoff during the Industrial Revolution that has been attributed largely to the acceleration in the

43. The effect of nonsustainable technological advance on output growth would vanish gradually. It would generate an increase in the average human capital of the population, but at a level that would sustain only slower technological progress. This lower rate, however, would not sustain the return to human capital. The average human capital in the population would decline, leading to a decline in the rate of technological change that would ultimately end in a state of stagnation.

pace of technological progress is at least partly due to the gradual evolution of the composition of the population that generated a sufficiently large mass of quality type individuals on the eve of the Industrial Revolution. This compositional change has allowed the pace of technological progress to be sustained by generating an impressive increase in the average level of education.

VII. Alternative Evolutionary Mechanisms

The theory argues that during the Malthusian epoch hereditary human traits, physical or mental, that generate higher earning capacity, and thereby potentially larger number of offspring, would generate an evolutionary advantage and would dominate the population in the long run. Hereditary traits that stimulate technological progress (e.g., intelligence) or raise the incentive to invest in offspring's human capital (e.g., ability, longevity, and a preference for quality), may trigger the positive feedback loop between investment in human capital and technological progress that would bring about a takeoff from an epoch of Malthusian stagnation, a demographic transition, and sustained economic growth. Hence, the struggle for existence that had characterized most of human history stimulated natural selection and generated an evolutionary advantage to individuals whose characteristics are complementary to the growth process, ultimately triggering a takeoff from an epoch of stagnation to sustained economic growth.

The model focuses on the evolution of the trade-off between resources allocated to the quantity and the quality of offspring. This framework of analysis can be modified to account for the interaction between economic growth and the evolution of other hereditary traits. However, in evaluating the importance of various genetic traits whose evolution contributed to the takeoff from an epoch of Malthusian stagnation, one should be concerned about the possibility that some of these traits may have completed most of their evolutionary change tens of thousands of years before the takeoff, as discussed in the Concluding Remarks.

VII.A. Evolution of Intelligence and Economic Growth

Consider the model described earlier. Suppose that individuals' preferences, as given by (3), are defined over consumption above a subsistent level and over child quality and quantity. Individuals are identical in their preferences, but differ in their hereditary innate ability. Suppose further that offspring's level of human capital is an increasing function of two complementary factors: innate ability and investment in quality. Thus, since the marginal return to investment in child quality increases with ability, higher ability individuals and hence dynasties would allocate a higher fraction of their resources to child quality.

In the Malthusian era individuals with higher ability generate more income and hence are able to allocate more resources for child quality and quantity. High ability individuals, therefore, generate higher income due to the fact that their innate ability as well as their quality are higher. As established in Proposition 1, in the Malthusian era fertility rates are positively affected by the level of income and (under plausible configurations) the high ability individuals therefore have an evolutionary advantage over individuals of lower ability. As the fraction of individuals of the high ability type increases, investment in quality rises, and technological progress intensified. Ultimately, the dynamical system changes qualitatively, the Malthusian temporary steady state vanishes endogenously and the economy takes off from the Malthusian trap. As in the case with the mechanism studied in earlier sections, once the evolutionary process triggers the positive feedback between the rate of technological progress and the level of education, technological progress is reinforced, the return to human capital increases further, setting the stage for the demographic transition and sustained economic growth.

VII.B. Evolution of Life Expectancy and Economic Growth

Suppose that individuals differ in their level of health due to hereditary factors. Suppose further that there exists a positive interaction between the level of health and economic well-being. Higher income generates a higher level of health, whereas a higher level of health increases labor productivity and life expectancy. Parents who are characterized by high life expectancy and thereby expect their offspring to have a longer productive life would allocate more resources toward child quality. In the Malthusian era fertility rates are positively affected by the level of income, and individuals with higher life expectancy, and therefore higher quality and higher income, would have (under plausible configurations) an evolutionary advantage. Natural selection therefore, increases the level of health as well as the quality of the population. Eventually, this process triggers a positive feedback loop between investment in child quality, technological progress and health, bringing about a transition to sustained economic growth with low fertility rates and high longevity.

VIII. Concluding Remarks

This research develops an evolutionary growth theory that captures the interplay between the evolution of mankind and economic growth since the emergence of the human species. The theory suggests that the struggle for survival that had characterized most of human existence stimulated a process of natural selection and generated an evolutionary advantage to human traits that were complementary to the growth process, triggering the takeoff from an epoch of stagnation to sustained economic growth.

The theory argues that during the Malthusian stagnation hereditary human traits, physical or mental, that raised earning capacity, generated an evolutionary advantage. Those hereditary traits that stimulated technological progress or raised the incentive to invest in offspring's human capital (e.g., ability, longevity, and preferences for quality), ultimately triggered a reinforcing interaction between investment in human capital and technological progress that brought about the takeoff from stagnation to growth.[44]

The model focuses on the evolution of one of the most fundamental trade-offs that exist in nature. The trade-off between resources allocated to the quantity and the quality of offspring in the implicit Darwinian survival strategy. As is well established in the evolutionary biology literature, the allocation of resources between offspring caring and bearing is subjected to evolutionary changes. Although a quantity-biased strategy has a positive effect on fertility rates and may therefore generate a direct evolutionary advantage, it adversely affects the quality of offspring and their fitness, and may therefore generate an evolutionary disadvantage. The evolution of the human brain in the transition from Homo Erectus to Homo Sapiens and the complementarity between brain capacity and the reward for human capital has

44. The theory is perfectly applicable for either social or genetic intergenerational transmission of traits. It should be noted that a cultural transmission is likely to be more rapid and may govern some of the observed differences in fertility rates across regions. The interaction between cultural and genetic evolution is explored by Boyd and Richardson [1985] and Cavalli-Sforza and Feldman [1981]. A cultural transmission of preferences was recently explored by Bisin and Verdier [2000].

increased the evolutionary optimal investment in offspring's quality. The distribution of valuation for quality lagged behind its evolutionary optimal level and dynasties characterized by higher but yet suboptimal valuation for quality, generated higher income and, in the Malthusian epoch when income is positively associated with aggregate resources allocated to child rearing, a larger number of offspring. Thus, the trait of high valuation for quality gained the evolutionary advantage.[45]

Evidence suggests that evolutionary processes may occur rather rapidly. In particular, evidence about the evolution of some human traits (e.g., lactose tolerance, gluten tolerance, sickle cell trait) suggests that the time period between the Neolithic Revolution and the Industrial Revolution is sufficient for significant evolutionary changes. Further, the evolutionary process may be influenced significantly by a modest variety in genetic traits. As stated by Darwin: "*What a trifling difference must often determine which shall survive and which perish!*"

In evaluating the importance of various genetic traits whose evolution contributed to the takeoff from an epoch of Malthusian stagnation, one should consider the possibility that some of these traits may have completed most of their evolutionary change tens of thousands of years before the takeoff and may therefore be a precondition for the takeoff rather than the trigger itself. In particular, the conventional wisdom among evolutionary biologists is that intelligence has not evolved markedly since the emergence of Homo Sapiens (i.e., intelligence may have reached a temporary evolutionary optimum that reflects the trade-off between the benefits and the energy cost associated with a larger brain). In contrast, it is unlikely that preferences reflecting quality bias would have reached a complete domination very early in the evolution of mankind. Prior to the Neolithic period, the majority of people lived in tribes where resources as well as child

45. Selection of individuals who are, from a physiological viewpoint, moderately fertile (e.g., individuals with a moderate sperm count) and hence, can potentially invest more in the quality of their offspring, would be consistent with the main thesis that traits that induce investment in quality (and are thus complementary to the growth process) were selected during the Malthusian epoch. This alternative mechanism, that could be incorporated into the existing framework, would lead to a different testable implication regarding the genetic trait that had been selected. As shown by Rodgers et al. [2001a], however, at least part of the genetic influence on fertility differential is associated with human traits and behavior under volitional control.

rearing were shared by the community.[46] Given this tribal structure, the latent attribute of preferences for quality, unlike observable attributes such as strength and intelligence, could not generate a disproportionate access to sexual mates and resources that could affect fertility rates and investment in offspring's quality, delaying the manifestation of the potential evolutionary advantage of individuals with a quality bias. It was the emergence of the nuclear family in the aftermath of the agricultural revolution that fostered intergenerational links, and thereby enhanced the manifestation of the potential evolutionary advantage of individuals with a quality bias.[47]

The theory indicates that waves of rapid technological progress in the Pre-Industrial Revolution era had not generated sustained economic growth due to the shortage of individuals of the quality type in the population. Although the return to quality increased temporarily, the level of human capital that was generated by the response of the existing population was not sufficient to support sustained technological progress and economic growth. In contrast, the era of sustained economic growth in the aftermath of the Industrial Revolution may be attributed to the presence of a sufficiently large fraction of individuals with high valuation for quality whose vigorous response to the rise in the return to human capital has supported sustained technological progress and growth.

The interaction between the composition of the population and the rate of technological progress determines the timing of the transition from stagnation to growth. Although in the deterministic setting of the model the evolutionary process and the ultimate takeoff are an inevitable by-product of the Malthusian epoch, the entire process might be influenced significantly by nondeterministic factors. Further, for a given composition of population, the timing of the transition may differ significantly across

46. Among the Pygmies, for example, women help each other nursing their children, and collective child rearing is common.

47. An alternative explanation for the delay in the evolutionary process of the quality bias relative to the evolution of intelligence is based on the notion of punctuated equilibria [Gould 1977]. A sequence of mutations, which result in a gradual increase in the variance in the distribution of the (latent) quality bias trait, had not affected investment in offspring's quality for a long period due to the low rate of return to human capital. Ultimately, however, mutations increased the variance sufficiently so as to induce investment in offspring's quality, despite the low return, and brought about an evolutionary advantage for the quality type. In contrast, a gradual increase in the variance of nonlatent variables, such as intelligence, would have an immediate effect on the evolutionary process.

countries due to historical accidents, as well as geographical, cultural, social, and institutional factors, that affect the relationship between human capital formation and technological progress.[48]

Appendix 1: Conditional Dynamics of Technology and Education

This appendix derives the properties of the phase diagrams of the conditional dynamical system (27) that describes the dynamics of technology and education, $\{g_t, e_t;\ q\}_{t=0}^{\infty}$, depicted in Figures Va–Vc, for a given level of q.

In order to allow for the existence of a long-run steady state with a positive growth rate, it is necessary to assume that

$$\text{(A5)} \qquad \exists g > 0 \text{ subject to } e(g; 0) > \psi^{-1}(g).$$

(Alternatively, $\exists g > 0$ such that $g(g,0) > g$). That is, in Figure Va, for $q = 0$, there exists a positive rate of technological progress such that, in the plane (g_t, e_t), the curve $e(g,0)$ lies above the curve $\psi(e_t)$.

LEMMA 6. Under A1, A2, and A5, as depicted in Figure Va for $q = 0$, the conditional dynamical system (27) is characterized by two locally stable steady-state equilibria:

$$[\bar{g}^L(q), \bar{e}^L(q)] = [0,0]$$
$$[\bar{g}^H(q), \bar{e}^H(q)] \gg [\underline{g}^b, 0].$$

Proof. Follows from the properties of $e_t = e(g_t;\ q)$ and $g_{t+1} = \psi(e_t)$, given by (13) and (16), Assumption A5, and Lemmas 1 and 2.□

LEMMA 7. Under A1, A2, and A5, there exists a critical level $\hat{q} \in (0,1)$ such that

$$e(\underline{g}^b, \hat{q}) = \psi^{-1}(\underline{g}^b).$$

Proof. It follows from the properties of $e_t = e(g_t;\ q)$ and Lemma 6 that $e(\underline{g}^b; 1) > \psi^{-1}(\underline{g}^b)$ and $e(\underline{g}^b; 0) < \psi^{-1}(\underline{g}^b)$. Therefore, the lemma follows from the continuity of $e(g_t;\ q)$ in q. □

48. Clearly, some of these factors may not be independent of the growth process.

Appendix 2: Conditional Dynamics of Technology and Effective Resources

This appendix derives the properties of the phase diagrams of the conditional dynamical system (29) that describes the dynamics of technology and effective resources, $\{g_t, x_t;\ q\}_{t=0}^{\infty}$, depicted in Figures VIa–VIc, for a given level of q. It derives the properties of the CC locus, the GG locus, the XX locus, and the dynamics of (g_t, x_t) in relation to these loci.

A. The CC Locus

Let the CC locus be the set of all pairs (g_t, x_t) for which $z_t^b = \tilde{z}$. $CC \equiv \{(g_t, x_t): z_t^b = \tilde{z}\}$, where $z_t^b = x_t^\alpha h^b(g_{t+1})$ and $\tilde{z} = \tilde{c}/(1 - \gamma)$.

LEMMA 8. Under A1 and A3, there exists a single-valued strictly increasing function,

$$x_t = (\tilde{c}/[(1 - \gamma)h^b(g_t)])^{1/\alpha} \equiv x^{CC}(g_t),$$

such that for all $g_t \geq 0$,

$$(g_t, x^{CC}(g_t)) \in CC,$$

where

$$x^{CC}(0) = (\tilde{c}/[1 - \gamma])^{1/\alpha};$$
$$\frac{\partial x^{CC}(g_t)}{\partial g_t} > 0.$$

Proof. Follows from Assumption A1 and A3, noting that $h(0,0) = 1$ and $e^b(0) = 0$. □

Hence, as depicted in Figures VIa–VIc, the CC locus is an upward sloping curve in the plane (g_t, x_t) with a positive vertical intercept.

B. The GG Locus

Let the GG locus be the set of all pairs (g_t, x_t) such that, for a given level of q, the rate of technological progress, g_t, is in a steady state. $GG \equiv \{(g_t, x_t;\ q): g_{t+1} = g_t\}$.

As follows from (18), along the GG locus, $g_{t+1} = g(g_t;\ q) = g_t$. The GG locus is therefore not affected by the effective resources per efficiency unit of labor, x_t, and as depicted in Figures VIa–VIc the GG locus consists of vertical line(s) at the steady-state level(s) of g, derived in Lemma 6, stated in (28), and depicted in Figures Va–Vc.

The dynamics of g_t in relation to the GG locus, as follows from the properties of (18), are

$$g_{t+1} - g_t \begin{cases} >0 & \text{if } g_t < \bar{g}^L(q) \text{ or } g_t \in (\bar{g}^U(q), \bar{g}^H(q)), \\ =0 & \text{if } g_t \in \{\bar{g}^L(q), \bar{g}^U(q), \bar{g}^H(q)\} \\ <0 & \text{if } g_t \in (\bar{g}^L(q), \bar{g}^U(q)) \text{ or } g_t > \bar{g}^H(q), \end{cases} \qquad \text{for } q < \hat{q}$$

$$g_{t+1} - g_t \begin{cases} >0 & \text{if } g_t < \bar{g}^H(q), \\ =0 & \text{if } g_t = \bar{g}^H(q), \\ <0 & \text{if } g_t > \bar{g}^H(q). \end{cases} \qquad \text{for } q > \hat{q}.$$

C. The XX Locus

Let the XX locus be the set of all pairs (g_t, x_t) such that, for a given level of q, the effective resources per efficiency unit of labor, x_t, is in a steady state. $XX \equiv \{(g_t, x_t;\, q): x_{t+1} = x_t\}$.

As follows from (25), along the XX locus, $x_{t+1} = [(1 + g_{t+1})/(1 + \mu_{t+1})]x_t \equiv x(g_t, x_t;\, q) = x_t$. Thus, as follows from (18) and Lemma 5, along the XX locus $\mu(g_t, x_t;\, q) = g(g_t;\, q)$.

To simplify the exposition and to assure the existence of the XX locus, it is further assumed that[49]

$$\begin{aligned} &\mu_g(g_t, x_t;\, q) \le 0; \\ \text{(A6)} \qquad &\lim_{g_t \to \infty} \mu(g_t, x^{CC}(g_t);\, q) \le 0; \\ &\mu(0, x^{CC}(0);\, q) > g(0;\, q). \end{aligned}$$

Lemma 9 and Corollary 1 derive the properties of the XX locus.

LEMMA 9. Under Assumptions A3–A6, given q, there exists a critical level of the rate of technological progress; $\hat{g}(q) > 0$, such that the XX locus in the plane (g_t, x_t) is

49. A sufficient condition for the negativity of $\mu_g(g_t, x_t;\, q)$ is a sufficiently small value of $|\partial h^i(g_t)/\partial g_t|$. The second condition is consistent with $\mu_g(g_t, x_t;\, q) \le 0$, given the feasible range of μ; i.e., $\mu \ge -1$. The third condition is satisfied if $g(0,q)$ is sufficiently small, since as follows from Lemma 4, $\mu > 0$ weakly above the Malthusian frontier for $g_t = g_{t+1} = 0$.

1. Vertical at $g_t = \hat{g}(q)$, where $\hat{g}'(q) < 0$ for all x_t above the CC locus, i.e.,

$$(\hat{g}(q), x_t) \in XX \quad \forall\, x_t \geq x^{CC}(\hat{g}(q));$$

2. Represented by a strictly increasing single value function $x_t = x^{XX}(g_t;\, q) > 0$ over the interval $[0,\hat{g}(q))$, i.e.,

$$(g_t, x^{XX}(g_t;\, q)) \in XX \quad \forall\, g_t \in [0,\hat{g}(q));$$

3. Below the CC locus over the interval $[0,\hat{g}(q))$, i.e.,

$$x^{XX}(g_t;\, q) < x^{CC}(g_t;\, q) \quad \forall\, g_t \in [0,\hat{g}(q));$$

4. Empty for $g_t > \hat{g}(q)$, i.e.,

$$(g_t, x_t) \notin XX \quad \forall\, g_t > \hat{g}(q).$$

Proof.

1. If the XX locus is nonempty weakly above the CC frontier, it is necessarily vertical in this range, since as follows from Lemma 5 $\mu_x(g_t,x_t,q) = 0$ above CC. Hence it is sufficient to establish that there exists a unique value $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q)) \in XX$. As follows from Assumption A6, $\mu(0,x^{CC}(0);\, q) > g(0;\, q)$, and $\lim_{g_t\to\infty}\mu(g,x^{CC}(0);\, q) < \lim_{g_t\to\infty} g(g;\, q)$. Hence, since $\mu(g_t,x_t;\, q)$ is monotonically decreasing in g_t and $g(g_t;\, q)$ is monotonically decreasing in g_t and $g(g_t;\, q)$ is monotonically increasing in g_t, there exists a unique value $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q)) \in XX$. Since along the XX locus $\mu(g_t,x^{CC}(g_t);\, q) = g(g_t;\, q)$, it follows from the properties of these function as derived in (18) and Lemma 5 that $\hat{g}'(q) < 0$.
2. Given the existence of a unique value $g_t = \hat{g}(q)$ such that $(\hat{g}(q),\, x^{CC}(\hat{g}(q)) \in XX$, the existence of $x_t = x^{XX}(g_t;\, q)$ follows continuity and the implicit function theorem, noting that along the XX locus, $\mu(g_t,x_t;\, q) = g(g_t;\, q)$ and the positivity of $\mu_x(g_t,x_t,q)$ over the interval $[0,\hat{g}(q))$, as established in Lemma 5. In particular,

$$\frac{\partial x^{XX}(g_t;\, q)}{\partial g_t} = \frac{g_g(g_t;\, q) - \mu_g(g_t,x_t,q)}{\mu_x(g_t,x_t,q)} > 0 \quad \forall g_t \in [0,\hat{g}(q)).$$

(Note that as established in Lemma 5 $\mu_x(g_t,x_t,q) = 0$ for $g_t = \hat{g}(q)$, and the verticality of the XX locus follows.) Furthermore, since $\mu(0,0;\, q) = -1 < g(0;\, q)$, it follows that the vertical intercept of the XX locus is strictly positive. In particular, $x^{XX}(0,0) = (\tilde{c}/[1 - \tau])^{1/\alpha}$.

3. Given the uniqueness of the value $g_t = \hat{g}(q)$ such that $(\hat{g}(q), x^{CC}(\hat{g}(q)) \in XX$, it follows that the XX locus and the CC frontier do not intersect over the interval $[0,\hat{g}(q))$. In addition, the XX locus is vertical above the CC frontier. Hence, the XX locus is below the CC frontier in the range $[0,\hat{g}(q))$. In particular, $x^{XX}(0,0) = (\tilde{c}/[1 - \tau])^{1/\alpha} < x^{CC}(0,0) = (\tilde{c}/[1 - \gamma])^{1/\alpha}$ since $\gamma > \tau$.
4. Given the uniqueness of the value of $g_t = \hat{g}(q)$ such that $(\hat{g}(q),x^{CC}(\hat{g}(q)) \in XX$, it follows that if the XX locus exists over the interval $(\hat{g}(q),\infty)$, then it must lie below the CC frontier. However, since $\mu_x(g_t,x_t,q) > 0$, and since along the XX locus $\mu(g_t,x_t; q) = g(g_t; q)$, it follows that along the CC frontier, over the interval $(\hat{g}(q),\infty)$, $\mu(g_t,x_t; q) > g(g_t; q)$, in contradiction to the fact that over the interval $(\hat{g}(q),\infty)$, $\mu(g_t,x_t; q) < g(g_t; q)$, as follows from Assumption A6 and established in part 1. □

Hence, as depicted in Figures VIa–VIc, the XX locus has a positive vertical intercept at $g = 0$, it increases monotonically with g_t, as long as $g_t \in [0,\hat{g}(q))$, and it becomes vertical at $g_t = \hat{g}(q)$. Furthermore, as q increases, the value of $\hat{g}(q)$ declines.

COROLLARY 1. Given q, there exists a unique pair $g_t = \hat{g}(q)$ and $x_t = x^{XX}(\hat{g}(q),q)$ such that $\{g_t,x_t; q\} \in XX \cap CC$.

The dynamics of x_t in relation to the XX locus, as follows from the properties of (18) and (24) are

$$x_{t+1} - x_t \begin{cases} >0 & \text{if } x_t < x^{XX}(g_t) \text{ or } g_t > \hat{g}(q); \\ =0 & \text{if } x_t = x^{XX}(g_t); \\ <0 & \text{if } x_t > x^{XX}(g_t) \text{ and } g_t < \hat{g}(q). \end{cases}$$

In order to assure the existence of a long-run (unconditional) steady-state equilibrium with sustained economic growth, it is further assumed that[50]

50. As follows from the dynamics of x_t, Assumption A7 holds *if and only if* for all x_t, $x_{t+1} = x(\bar{g}^H(0),x_t; 0) - x_t > 0$, i.e., (noting (25)), *if and only if,* for all x_t $\mu(\bar{g}^H(0),x_t; 0) \leq \bar{g}^H(0)$. As follows from (24), $\mu(\bar{g}^H(0),x_t; 0) = n_t^b - 1$. Hence, it follows from (11) that Assumption A7 holds *if and only if* $\gamma \leq [\bar{g}^H(0) + 1][\tau + e^b(\bar{g}^H(0))]$. Assumption A7 holds, therefore, for sufficiently (i) high preference for quality by individuals of type b, β^b (since e^b and hence $\bar{g}^H(0)$ increase with

$$\hat{g}(0) < \bar{g}^H(0). \tag{A7}$$

Hence, since $\bar{g}^H(q)$ increases in q, as follows from (28), and since $\hat{g}(q)$ decreases in q, as follows from Lemma 9, $\hat{g}(q) < \bar{g}^H(q)\ \forall q$.

Hence, as depicted in Figures VIa–VIc, and as established in the lemma below, Assumption A7 assures that if the economy crosses the CC locus and enters into the sustained growth regime, it would not cross back to the Malthusian regions.

Furthermore, in order to assure that the economy takes off from the Malthusian regime, as is apparent from Figures Va–Vc, it is necessary that the value of q increases sufficiently so as to pass the critical level $\hat{q}$. Hence, it is necessary to assure that the fraction of individuals of type a in the population increases as long as $q \in [0,\hat{q}]$ and $g_t \in [0,\underline{g}^b]$. Since $n_t^a > n_t^b$ as long as $x_t < \check{x}_t$, it is therefore sufficient to assume that

$$x^{XX}(g_t;\, q) < \check{x}(g_t;\, q) \quad \text{for } g_t \in [0,\underline{g}^b] \text{ and } q \in [0,\hat{q}]. \tag{A8}$$

LEMMA 10. Under A2–A6 and A8, $\forall q \in [0,\hat{q}]$

$$\hat{g}(q) > \underline{g}^b.$$

Proof. As follows from (11) and Proposition 1, $n_t^b > n_t^a$ weakly above the CC locus, and therefore $\check{x}(g_t;\, q) < x^{CC}(g_t)$ for all g_t and q. Hence, it follows from Assumption A8 that $x^{XX}(g_t;\, q) < x^{CC}(g_t)$ for $g_t \in [0,\underline{g}^b]$ and $q \in [0,\hat{q}]$. As established in Lemma 9, $x^{XX}(g_t; q) < x^{CC}(g_t; q)\ \forall g_t \in [0,\hat{g}(q))$. It follows therefore that $\hat{g}(q) > \underline{g}^b$. □

Thus, so long as the economy is in the range of a low rate of technological progress, (i.e., as long as $g_t < \underline{g}^b$), individuals of type b do not invest in the quality of their offspring, and the economy cannot take off from the Malthusian regime.

It should be noted that since the dynamical system is discrete, the trajectories implied by the phase diagrams do not necessarily approximate the actual dynamic path, unless the state variables evolve monotonically over time. As shown in subsection IV.A., the evolution of g_t is monotonic, whereas the evolution and convergence of x_t may be oscillatory. Nonmonotonicity may arise only if $g < \hat{g}$. Nonmonotonicity in the evolution of x_t does not affect the qualitative description of the system. Further-

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β^b), (ii) high cost of child raising τ, and (iii) low weight, γ, for children relative to consumption in the utility function.

more, if $\partial x(g_t, x_t; q)/\partial x_t > -1$ for $q \leq q$, the conditional dynamical system is locally nonoscillatory. The local stability of the steady-state equilibrium $(0, \bar{x}(g_t))$ can be derived formally. The eigenvalues of the Jacobian matrix of the conditional dynamical system evaluated at the conditional steady-state equilibrium are both smaller than one (in absolute value) under A1–A3.

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