Supplementary Material

Let \( c_i(\xi, \alpha) \) and \( w'_i(\xi, \alpha, \mu)(\xi') \) be the maximisers in problem (6) - (10) and let \( \lambda_i(\xi, \alpha, \mu) \) be the Lagrange multiplier associated to constraint (8). Let

\[
\tilde{u}_i(\xi, \alpha, \mu) = u_i(c_i(\xi, \alpha)) + \beta(\xi, \mu_i) \sum_{\xi'} \pi_{\mu_i}(\xi' | \xi) w'_i(\xi, \alpha, \mu)(\xi').
\]

**Claim 1.** \( \tilde{u}_i(\xi, \alpha, \mu) \) is nondecreasing in \( \alpha_i \) for all \( \alpha \in \mathbb{R}_+^I \).

**Proof.** Let \( \tilde{\alpha}, \alpha \in \mathbb{R}_+^I \) be such that \( \tilde{\alpha}_i > \alpha_i \) and \( \tilde{\alpha}_j = \alpha_j \) for every \( j \neq i \). To get a contradiction, suppose \( \tilde{u}_i(\tilde{\alpha}, \tilde{\alpha}, \mu) < \tilde{u}_i(\alpha, \alpha, \mu) \). Since the constrained set is independent of the welfare weights, then

\[
\sum_h \tilde{\alpha}_h (\tilde{u}_h(\tilde{\alpha}, \tilde{\alpha}, \mu) - \tilde{u}_h(\alpha, \alpha, \mu)) \geq 0 \quad \text{and} \quad \sum_h \alpha_h (\tilde{u}_h(\tilde{\alpha}, \tilde{\alpha}, \mu) - \tilde{u}_h(\alpha, \alpha, \mu)) \geq 0
\]

and so, on the one hand,

\[
\sum_h (\tilde{\alpha}_h - \alpha_h) (\tilde{u}_h(\tilde{\alpha}, \tilde{\alpha}, \mu) - \tilde{u}_h(\alpha, \alpha, \mu)) \geq 0
\]

But, on the other hand,

\[
\sum_h (\tilde{\alpha}_h - \alpha_h) (\tilde{u}_h(\tilde{\alpha}, \tilde{\alpha}, \mu) - \tilde{u}_h(\alpha, \alpha, \mu)) = (\tilde{\alpha}_i - \alpha_i) (\tilde{u}_i(\tilde{\alpha}, \tilde{\alpha}, \mu) - \tilde{u}_i(\alpha, \alpha, \mu)) < 0
\]

a contradiction. \( \square \)

Let \( \overline{u}_i(\xi, \alpha) \) and \( \overline{w}'_i(\xi, \alpha, \mu)(\xi') \) be the maximisers of the relaxed problem where (8) is ignored. Let

\[
\overline{u}(\xi, \alpha, \mu) = u_i(\overline{u}_i(\xi, \alpha)) + \beta(\xi, \mu_i) \sum_{\xi'} \pi_{\mu_i}(\xi' | \xi) \overline{w}'_i(\xi, \alpha, \mu)(\xi')
\]

**Claim 2.** Let \( \alpha \in \mathbb{R}_+^I \). If \( \alpha_i < \tilde{\alpha}_i \) and \( \alpha_h = \tilde{\alpha}_h \) for all \( h \neq i \), then \( \overline{u}_i(\xi, \alpha, \mu) < \overline{u}_i(\xi, \tilde{\alpha}, \mu) \).

**Proof.** Note that \( \overline{u}_i(\xi, \alpha) \) is the unique solution to

\[
\begin{align*}
&c_i + \sum_{h \neq i} \left( \frac{\partial u_h}{\partial c_h} \right)^{-1} \left( \frac{\alpha_i}{\alpha_h} \frac{\partial u_i(c_i)}{\partial c_i} \right) = y(\xi),
\end{align*}
\]

and so it is strictly increasing in \( \alpha_i \). Therefore, \( \overline{u}_i(\xi, \alpha) > \overline{u}_i(\xi, \tilde{\alpha}) \). Note that

\[
\overline{w}'_i(\xi, \alpha, \mu)(\xi') = \frac{\alpha_i}{\sum_h \alpha_h} \int \frac{\pi(\xi' | \xi) \mu'_i(\xi, \mu)(\xi') (d\pi)}{\int \pi(\xi' | \xi) \mu'_h(\xi, \mu)(\xi') (d\pi)}
\]

Thus, \( \overline{w}'_i(\xi, \alpha, \mu)(\xi') \) is nondecreasing in \( \alpha_i \). Since \( \overline{w}'_i(\xi, \alpha, \mu)(\xi') \) satisfies (9) and (10), it follows by Lemma A.1 and Theorem 1 that \( \overline{w}'_i(\xi, \alpha, \mu)(\xi') = \overline{u}_i(\xi', \overline{w}'(\xi, \alpha, \mu)(\xi'), \mu'(\xi, \mu)(\xi')) \). Thus, Claim 1 implies that \( \overline{w}'_i(\tilde{\alpha}, \mu)(\xi') \geq \overline{w}'_i(\xi, \alpha, \mu)(\xi') \) for all \( \xi' \). We conclude that \( \overline{u}_i(\xi, \alpha, \mu) < \overline{u}_i(\xi, \tilde{\alpha}, \mu) \), as desired. \( \square \)
Proof of Proposition 2. (i) Suppose \( \alpha_i \in \Delta(\xi, \mu) \). Consider first the case where \( \alpha_i > \alpha_i(\xi, \mu) \) for all \( i \). By the definition of \( \hat{u}_i(\xi, \alpha, \mu) \), we have that \( \hat{u}_i(\xi, \alpha, \mu) \geq U_i(\xi, \mu) \) and \( \sum_i \alpha_i \hat{u}_i(\xi, \alpha, \mu) = v^*(\xi, \alpha, \mu) \). It follows by Lemma A.1, that \( (\hat{u}_1(\xi, \alpha, \mu), \ldots, \hat{u}_I(\xi, \alpha, \mu)) \in \mathcal{U}_E(\xi, \mu) \). Since \( \sum_i \alpha_i \hat{u}_i(\xi, \alpha, \mu) = v^*(\xi, \alpha, \mu) \), it is easy to see that \( (u_1(\xi, \alpha, \mu), \ldots, u_I(\xi, \alpha, \mu)) \in \mathcal{U}_E(\xi, \mu) \). Then, \( \hat{u}_i(\xi, \alpha, \mu) > U_i(\xi, \mu_i) \) for all \( i \) by definition of \( \alpha_i(\xi, \mu) \). Thus, \( \lambda_i(\xi, \alpha, \mu) = \lim_{n \to \infty} \alpha^i_n > \alpha_i(\xi, \mu) \) for all \( i \) and \( n \) and \( \alpha^n \to \alpha \). It follows that
\[
\lambda_i(\xi, \alpha, \mu) = \lambda_i(\xi, \mu) = \lim_{n \to \infty} \alpha^i_n = \lim_{n \to \infty} \lambda_i(\xi, \alpha, \mu) = 0,
\]
where the second equality follows by continuity of \( \lambda_i(\xi, \alpha, \mu) \) in \( \alpha \) and the last one because weak inequalities are preserved under limits. It follows that, \( \hat{u}_i(\xi, \alpha, \mu) = \pi_i(\xi, \alpha, \mu) \) and so \( c_i(\xi, \alpha) = \tau_i(\xi, \alpha) \), i.e. \( c_i(\xi, \alpha) \) solves the relaxed problem.

(ii) Let \( \alpha \in \mathbb{R}^I_+ \) and \( \alpha^* \equiv \left( \sum_{i=1}^I \alpha_i^* \right) \). If \( \alpha^* \in \Delta(\xi, \mu) \), then \( c_i(\xi, \alpha) = c_i(\xi, \alpha^*) \) because \( \hat{u}_i(\xi, \alpha, \mu) \) is homogeneous of degree zero in \( \alpha \). If \( \alpha^* \notin \Delta(\xi, \mu) \), there is \( i \) such that \( \alpha_i^* > \alpha_i(\xi, \mu) (\alpha_i^{*}) \).

* First, we show that \( \lambda_i(\xi, \alpha, \mu) > 0 \). To get a contradiction, suppose \( \lambda_i(\xi, \alpha, \mu) = 0 \). It follows that
\[
\hat{u}_i(\xi, (\alpha_i, \ldots, \alpha_i), \mu) = \hat{u}_i(\xi, (\alpha_i^*, \ldots, \alpha_i^*), \mu)
\]
\[
= \pi_i(\xi, (\alpha_i^*, \ldots, \alpha_i^*), \mu)
\]
\[
= \pi_i \left( \xi, \left( \frac{\alpha_i^*}{\alpha_i(\xi, \mu)} (\alpha_i^*) \right), \left( \frac{\alpha_i^*}{\alpha_i(\xi, \mu)} (\alpha_i^*) \right) \right), \mu
\]
\[
< \pi_i \left( \xi, \left( \frac{\alpha_i^*}{\alpha_i(\xi, \mu)} (\alpha_i^*) \right), \left( \frac{\alpha_i^*}{\alpha_i(\xi, \mu)} (\alpha_i^*) \right) \right), \mu
\]
\[
= \pi_i \left( \xi, (\alpha_i(\xi, \mu) (\alpha_i^{*}) \right), \mu)
\]
\[
= U_i(\xi, \mu),
\]
where the first equality follows because \( \hat{u}_i \) is homogeneous of degree zero in \( \alpha \), the second one is due to the assumption that \( \lambda_i(\xi, \alpha, \mu) = 0 \) and the homogeneity of degree zero of \( \lambda_i(\xi, \alpha, \mu) \) in \( \alpha_i \), the third and fifth follows by homogeneity of degree zero of \( \pi_i(\cdot) \) in \( \alpha_i \), the inequality follows by Claim 2 and the last equality follows by definition of the minimum enforceable weights. But then, \( \hat{u}_i(\xi, (\alpha_i, \ldots, \alpha_i), \mu) < U_i(\xi, \mu) \) which contradicts constraint (8).

* Second, note that problem (6) - (10) is equivalent to maximising
\[
\sum_{i=1}^I \left( \alpha_i + \lambda_i \right) \left\{ u_i(c_i) + \beta(\xi, \mu_i) \sum_{\xi'} \pi_{\mu_i}(\xi|\xi') w_i'(\xi') \right\},
\]
subject to constraints (7), (9) and (10).

* Finally, the latter is equivalent to the relaxed problem with welfare weights \( \hat{\alpha} \) given by
\[
\hat{\alpha}_i = \frac{\alpha_i + \lambda_i(\xi, \alpha, \mu)}{\sum_{h=1}^I (\alpha_h + \lambda_h(\xi, \alpha, \mu))},
\]
Thus, \( \pi_i(\xi, \hat{\alpha}, \mu) = \hat{u}_i(\xi, \alpha, \mu) \geq U_i(\xi, \mu) = \pi_i(\xi, \alpha_i, \mu) \). It follows by Claim 2 that \( \hat{\alpha}_i \geq \alpha_i \). Therefore, \( \hat{\alpha} \in \Delta(\xi, \mu) \) and \( c_i(\xi, \alpha) = \tau_i(\xi, \hat{\alpha}) = c_i(\xi, \hat{\alpha}) \) as desired.
Now we prove Theorem 11. We begin with some results on Markov Processes.

**Lemma 7.1.** Let \( \{z_t\}_{t=0}^{\infty} \) be a two-state time homogeneous Markov process with transition function \( F \) on \((Z, \mathcal{Z})\) and invariant distribution \( \psi : Z \to [0, 1] \), \( P^F \) be the probability measure on \((Z^\infty, \mathcal{Z}^\infty)\) uniquely induced by \( F \) and \( \psi \) and let \( R : Z \times Z \to \mathbb{R} \). Suppose there exists \( z_+ \in Z \) such that

(a) \( E^{P^F} (R(z_1, z_2)) = 0 \).
(b) \( R(z, z_+) > 0 \) for all \( z \).
(c) \( E^{P^F} (R(z_0, z_1) R(z_1, z_2)) > 0 \).

Then \( E^{P^F} (R(z_2, z_3)| z_1 = z_+ < 0 < E^{P^F} (R(z_2, z_3)| z_1 = z_-) \iff F(z_+|z_+) < \psi(z_+) \).

**Proof.** Hypothesis (a) and the Markov property implies that \( E^{P^F} (R(z_k, z_{k+1})) = 0 \) for any \( k \). Thus,

\[
\psi(z-) E^{P^F} (R(z_{k'}, z_{k'+1})| z_k = z_-) = -\psi(z+) E^{P^F} (R(z_{k'}, z_{k'+1})| z_k = z_+)
\]

where \( z_- \neq z_+ \). Note also that

\[
E^{P^F} (R(z_0, z_1) R(z_1, z_2)) = E^{P^F} (R(z_0, z_1) E^{P^F} (R(z_1, z_2)| z_1))
\]

\[
= [P^F (z_+, z_+) R(z_+, z_+) + P^F (z_-, z_+) R(z_-, z_+)] E^{P^F} (R(z_1, z_2)| z_1 = z_+)
\]

\[
+ [P^F (z_+, z_-) R(z_+, z_-) + P^F (z_-, z_-) R_+(z_-, z_-)] E^{P^F} (R(z_1, z_2)| z_1 = z_-).
\]

By hypothesis (a) and (b), \( R(z, z_-) < 0 \) for all \( z \). Therefore,

\[
P^F (z_+, z_-) R(z_+, z_-) + P^F (z_-, z_-) R(z_-, z_-) < 0,
\]

\[
P^F (z_+, z_+) R(z_+, z_+) + P^F (z_-, z_+) R(z_-, z_+) > 0.
\]

It follows from (34) evaluated at \( k = 1 \) and \( k' = 1 \), hypothesis (c) and (35) that

\[
E^{P^F} (R(z_1, z_2)| z_1 = z_-) < 0 < E^{P^F} (R(z_1, z_2)| z_1 = z_+)
\]

and the Markov Property implies

\[
E^{P^F} (R(z_2, z_3)| z_2 = z_-) < 0 < E^{P^F} (R(z_2, z_3)| z_2 = z_+).
\]

Condition (34), evaluated at \( k = 1 \) and \( k' = 2 \), implies that

\[
E^{P^F} (R(z_2, z_3)| z_1 = z_-) < 0 < E^{P^F} (R(z_2, z_3)| z_1 = z_+) \iff E^{P^F} (R(z_2, z_3)| z_1 = z_+) > 0.
\]

In addition,

\[
E^{P^F} (R(z_2, z_3)| z_1 = z_+) = E^{P^F} (R(z_2, z_3)| z_1 = z_+) - E^{P^F} (R(z_2, z_3))
\]

\[
= (F(z_2 = z_+|z_1 = z_+) - \psi(z_+)) E^{P^F} (R(z_2, z_3)| z_2 = z_+)
\]

\[
+ (F(z_2 = z_-|z_1 = z_+) - \psi(z_-)) E^{P^F} (R(z_2, z_3)| z_2 = z_-)
\]

\[
= (F(z_2 = z_+|z_1 = z_+) - \psi(z_+)) \times
\]

\[
(E^{P^F} (R(z_2, z_3)| z_2 = z_+) - E^{P^F} (R(z_2, z_3)| z_2 = z_-))
\]

where the first line follows by the definition of unconditional expectation and (a). (36) implies that

\[
E^{P^F} (R(z_2, z_3)| z_1 = z_+) < 0 \iff F(z_2 = z_+|z_1 = z_+) - \psi(z_+ < 0.
\]

\[ \square \]
Proof of Theorem 11(a). Consider any CE of an arbitrary baseline growth economy. Since the allocation is PO, it follows by Theorem 8 that (15) holds and the marginal distribution of $\psi_{po}$ over welfare weights is a point mass on $\alpha_\infty$. By standard arguments, there exists $\overline{R}_{po} : \{l, h\} \times \{l, h\} \to \mathbb{R}$ such that for any $\tau \in \{1, 2\}$ and $\omega \in \Omega$

$$
\overline{R}_{\tau, po} (\omega) = \begin{cases} 
\overline{R}_{po} (l, l) & \text{if } \xi_{\tau-1}(\omega) \in \{1, 3\} \text{ and } \xi_{\tau}(\omega) \in \{1, 3\} \\
\overline{R}_{po} (l, h) & \text{if } \xi_{\tau-1}(\omega) \in \{1, 3\} \text{ and } \xi_{\tau}(\omega) \in \{2, 4\} \\
\overline{R}_{po} (h, l) & \text{if } \xi_{\tau-1}(\omega) \in \{2, 4\} \text{ and } \xi_{\tau}(\omega) \in \{1, 3\} \\
\overline{R}_{po} (h, h) & \text{if } \xi_{\tau-1}(\omega) \in \{2, 4\} \text{ and } \xi_{\tau}(\omega) \in \{2, 4\}
\end{cases}
$$

and

$$
\overline{R}_{po} (\xi, l) < 0 < \overline{R}_{po} (\xi, h) \text{ for all } \xi \in \{l, h\}.
$$

(37)

Let $Z = \{l, h\}$, $\mathcal{Z}$ be its finest partition, $\tilde{\pi}^*$ be the transition function on $(Z, \mathcal{Z})$ defined as the restriction of $\pi^*$ to $(Z, \mathcal{Z})$ and let $\tilde{\psi}_{po}$ be the restriction of the invariant measure $\psi_{po}$ to $(Z, \mathcal{Z})$. Let $Z^\infty$ be the set of infinite sequences with elements in $Z$ and $Z_0 \subset Z_1 \subset ... \subset Z_t \subset ... Z^\infty$ be the standard filtration. $P^{\tilde{\pi}^*}$ is the probability measure over $(Z^\infty, Z^\infty, P^{\tilde{\pi}^*})$ uniquely induced by $\tilde{\pi}^*$ and $\tilde{\psi}_{po}$. Let $z_t : Z^\infty \to Z$ be $Z_t$–measurable. The collection $\{z_t\}_{t=0}^\infty$ on the probability space $(Z^\infty, Z^\infty, P^{\tilde{\pi}^*})$ is a two state time-homogeneous Markov process with transition function $\tilde{\pi}^*$ on $(Z, \mathcal{Z})$ and invariant distribution $\tilde{\psi}_{po} : Z \times Z \to [0, 1]$ satisfying

$$
E^{P^{\tilde{\pi}^*}} (\overline{R}_{po} (z_1, z_2)) = 0.
$$

(38)

First note that (38) and (37) are conditions (a) and (b), respectively, in Lemma 7.1. Second, since the asset displays short-term momentum,

$$
0 < E^{P_{po}} (\overline{R}_{1, po} \overline{R}_{2, po}) = E^{P^{\tilde{\pi}^*}} (\overline{R}_{po} (z_0, z_1) \overline{R}_{po} (z_1, z_2))
$$

and so condition (c) in Lemma 7.1 also holds. By Lemma 7.1, we conclude that

$$
E^{P^{\tilde{\pi}^*}} (\overline{R}_{po} (z_2, z_3) | z_1 = h) < 0 < E^{P^{\tilde{\pi}^*}} (\overline{R}_{po} (z_2, z_3) | z_1 = l) \iff \tilde{\pi}^* (h | h) < \tilde{\psi}_{po} (h).
$$

(39)

Let $\omega^+$ and $\omega^-$ be such that $\overline{R}_{1, po} (\omega^+) > 0$ and $\overline{R}_{1, po} (\omega^-) < 0$. Then,

$$
E^{P_{po}} (\overline{R}_{3, po} | \overline{R}_{1, po}) (\omega^+) = E^{P^{\tilde{\pi}^*}} (\overline{R}_{po} (z_2, z_3) | z_1 = h),
$$

$$
E^{P_{po}} (\overline{R}_{3, po} | \overline{R}_{1, po}) (\omega^-) = E^{P^{\tilde{\pi}^*}} (\overline{R}_{po} (z_2, z_3) | z_1 = l).
$$

It follows from (39), $\tilde{\pi}^* (h | h) = \pi^* (2 | 2) + \pi^* (4 | 2)$ and $\tilde{\psi}_{po} (h) = \psi_{po} (2) + \psi_{po} (4)$ that

$$
E^{P_{po}} (\overline{R}_{3, po} | \overline{R}_{1, po}) (\omega^+) < 0 < E^{P_{po}} (\overline{R}_{3, po} | \overline{R}_{1, po}) (\omega^-) \iff \pi^* (2 | 2) + \pi^* (4 | 2) < \psi_{po} (2) + \psi_{po} (4)
$$

that is, $E^{P_{po}} (\overline{R}_{3, po} | \overline{R}_{1, po})$ reverts to the mean if and only if $\pi^* (2 | 2) + \pi^* (4 | 2) < \psi_{po} (2) + \psi_{po} (4)$. By Proposition 9, the asset displays long-term reversal if $\pi^* (2 | 2) + \pi^* (4 | 2) < \psi_{po} (2) + \psi_{po} (4)$. To show the converse, suppose that $\pi^* (2 | 2) + \pi^* (4 | 2) \geq \psi_{po} (2) + \psi_{po} (4)$. Then by the argument above, $E^{P_{po}} (\overline{R}_{3, po} | \overline{R}_{1, po})$ trends and it follows by Proposition 9 that the 2nd-order autocorrelation is positive and so long-run reversal fails.

□
Proof of Theorem 11(b). Consider any CESC of an arbitrary baseline growth economy. The price of an asset at state \((\xi, \alpha)\) must satisfy the Bellman equation:

\[
p (\xi, \alpha) = \sum_{\xi'} Q (\xi, \alpha) (\xi') (p (\xi', \alpha' (\xi, \alpha) (\xi')) + d (\xi')) \psi_{\text{cpo}} - \text{a.s.}
\]

It is easy to see that the invariant distribution places positive mass only on points \((\xi, \alpha)\) such that \(\alpha \in \Delta \cap \Delta (\xi, \mu^\ast)\) where \(\Delta = \{(\alpha_1, \alpha_2) \in \Delta : \exists \xi \in S \text{ such that } \alpha_1 = \alpha_1 (\xi) \text{ or } \alpha_2 = \alpha_2 (\xi)\}\). The hypothesis \(\alpha_1 (1) = \alpha_1 (2)\) and symmetry implies that \(\alpha_2 (3) = \alpha_2 (4)\). If \(p_\xi\), \(q_{\xi \xi'}\) and \(d_\xi\) denotes \(p (\xi, \alpha (\xi)), Q (\xi, \alpha (\xi)) (\xi')\) and \(d (\xi), \) respectively, then the Bellman equation becomes

\[
p_\xi = \sum_{\xi'} q_{\xi \xi'} (p_{\xi'} + d_{\xi'}) \quad \text{for all } \xi
\]

which can be written as \((I - Q) P = QD\) where \(Q\) is the \(4 \times 4\) matrix with entries \(q_{\xi \xi'}\), \(P\) is the \(4 \times 4\) vector with entries \(p_\xi\) and \(D\) is the \(4 \times 1\) vector with entries \(d_\xi\). Note that

\[
c_1, (1) = c_2 (3, \alpha (3)) \quad \text{and} \quad c_1, (2, \alpha (2)) = c_2 (4, \alpha (4))
\]

and so

\[
q_{\xi 1} = \beta (\xi, \mu) \pi (1 | \xi) \frac{\partial}{\partial \xi_1} c_1 (1, \alpha (1)) = \beta (\xi, \mu) \pi (1 | \xi) \frac{\partial}{\partial \xi_1} c_2 (3, \alpha (3)) = q_{\xi 3},
\]

\[
q_{\xi 2} = \beta (\xi, \mu) \pi (2 | \xi) \frac{\partial}{\partial \xi_2} c_1 (2, \alpha (2)) = \beta (\xi, \mu) \pi (2 | \xi) \frac{\partial}{\partial \xi_2} c_2 (4, \alpha (4)) = q_{\xi 4}.
\]

It follows that \(Q\) has rank 2. Therefore, \(p_1 = p_3\) and \(p_2 = p_4\).

Let \(\tilde{\pi}^*\) and \((Z^\infty, Z^\infty)\) be the transition matrix and the measurable space, respectively, introduced in the proof of Theorem 11(a). \(P^{\pi^*}\) is the probability measure over \((Z^\infty, Z^\infty)\) uniquely induced by \(\tilde{\pi}^*\) and \(\psi_{\text{cpo}}\). Let \(z_t : Z^\infty \rightarrow Z\) be \(Z_t\)-measurable. The collection \(\{z_t\}_{t=0}^\infty\) on the probability space \((Z^\infty, Z^\infty, P^{\pi^*})\) is a two state time-homogeneous Markov process with transition function \(\tilde{\pi}^*\) on \((Z, Z)\) and invariant distribution \(\psi_{\text{cpo}} : Z \times Z \rightarrow [0, 1]\).

Let \(p (l) = p_1, p (h) = p_2, R_{\text{cpo}} (z, z') \equiv \frac{p_{z' + d_{z'}}}{p_z}\) for all \(z \in \{l, h\}\) and \(\overline{R}_{\text{cpo}} : \{l, h\} \times \{l, h\} \rightarrow \mathbb{R}\) be such that

\[
\overline{R}_{\text{cpo}} (\omega) = \begin{cases} R_{\text{cpo}} (l, l) & \text{if } \xi_{t-1} (\omega) \in \{1, 3\} \text{ and } \xi_{t} (\omega) \in \{1, 3\} \\ R_{\text{cpo}} (l, h) & \text{if } \xi_{t-1} (\omega) \in \{1, 3\} \text{ and } \xi_{t} (\omega) \in \{2, 4\} \\ R_{\text{cpo}} (h, l) & \text{if } \xi_{t-1} (\omega) \in \{2, 4\} \text{ and } \xi_{t} (\omega) \in \{1, 3\} \\ R_{\text{cpo}} (h, h) & \text{if } \xi_{t-1} (\omega) \in \{2, 4\} \text{ and } \xi_{t} (\omega) \in \{2, 4\} \end{cases}
\]

Moreover,

\[
\overline{R}_{\text{cpo}} (z, l) < 0 < \overline{R}_{\text{cpo}} (z, h) \text{ for all } z \in \{l, h\}
\]

and

\[
E^{P^{\pi^*}} (\overline{R}_{\text{cpo}} (z_1, z_2)) = 0.
\]

It follows from (40) that for any \(k \in \{2, 3\}\)

\[
E^{P_{z_0}} (\overline{R}_{l, \text{cpo}} \overline{R}_{k, \text{cpo}}) = E^{P^{\pi^*}} (\overline{R}_{\text{cpo}} (z_0, z_1) \overline{R}_{\text{cpo}} (z_1, z_k)).
\]

Note that (42) and (41) are conditions (a) and (b) in Lemma 7.1. Since the asset displays short-term momentum,

\[
E^{P^{\pi^*}} (\overline{R}_{\text{cpo}} (z_0, z_1) \overline{R}_{\text{cpo}} (z_1, z_2)) = E^{P_{z_0}} (\overline{R}_{l, \text{cpo}} \overline{R}_{2, \text{cpo}}) > 0,
\]

and so (c) in Lemma 7.1 also holds. The rest of the proof is identical to that in Theorem 11(a). □