Asset Prices, Market Selection and Belief Heterogeneity

Ambiguity Aversion

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April, 2016

References

- This lecture is based on:
  1. Guerdjikova and Sciubba (2015): "Survival with Ambiguity", *Journal of Economic Theory*, 155, 50 - 94.

Motivation

- Models with a representative expected utility maximiser cannot account for the historically observed prices in financial markets for reasonable values of the parameters.
- This has created the interest for models with alternative preferences specifications.
- Models where agents display aversion towards ambiguity capture behaviour observed in experiments and can account for some financial market anomalies like home bias (Uppal and Wang, JoF 2003), equity premium puzzle (Epstein and Schneider, JoF 2008) etc.
- The limitation of this approach is that it replaces one representative agent for another without considering the impact of heterogeneity.
- Since many models where agents have non-SEU preferences can be represented as SEU with incorrect beliefs, the MSH suggests these preferences have no long-run impact in the presences of some agents withe SEU and correct beliefs.

Previous Results on Deviations from SEU

- Condie (ET, 2008) shows that max-min expected utility maximisers survive in the presence of SEU maximisers with correct beliefs only if (i) the true distribution is in the interior of his set of priors and (ii) they are completely insured. Thus, they never affect prices in the long run.
- Easley and Yang (JET, 2012) show that loss-averse agents cannot survive in the presence of investors with Epstein-Zin preferences who do not exhibit loss-aversion.

Guerdjikova and Sciubba (JET, 2015)

- The market exhibits two levels of uncertainty:
  1. Uncertainty about the investors' endowments (risk).
  2. Uncertainty about the probability distribution determining the evolution of endowments (ambiguity).
- Ambiguity is described by the set of probability distributions which can govern the endowment process and by the probability distribution over these distributions.
- The main difference between ambiguity and risk is that the realisation of the risky state is verifiable while the realisation of the ambiguous state is not. Hence asset payoffs can only depend on the risky state.
- They consider the Klibanoff, Marinacci and Mukerji (KMM) smooth ambiguity-averse investor model.
- It allows to separate the objective ambiguity in the market, to which all investors are exposed, from the subjective attitude towards ambiguity.
- They assume at date zero the economy has a complete Arrow-Debreu market to trade consumption contingent on the risky state.
- All agents are risk averse, some are ambiguity-averse and some are ambiguity-neutral (SEU maximisers).

Results

- If there is no aggregate risk, all investors are fully insured against risk, and thus also against ambiguity. Ambiguity-averse investors with correct beliefs survive but they do not affect prices.
- If investors can learn the probability distribution governing the risky state (i.e. ambiguity is not persistent), ambiguity-averse investors with correct beliefs survive. Since ambiguity vanishes, they do not affect asset prices in the long-run.
- If ambiguity is large and persistence so that the investors cannot insure against ambiguity, survival depend on the attitude towards ambiguity.
- Consumers with decreasing absolute ambiguity aversion whose prudence with respect to ambiguity exceeds twice their absolute ambiguity aversion survive in the presence of SEU maximisers with correct beliefs.
- If the economy exhibits aggregate risk, they drive SEU maximisers with correct beliefs out of the market and determine asset prices in the limit.
- Consumers with increasing or constant absolute ambiguity aversion only survive in the absence of aggregate risk and have no impact on prices in the limit.

Intuition

- A smooth ambiguity-averse agent with correct beliefs and a constant discount effectively behaves as a SEU maximiser with incorrect beliefs and a history-dependent stochastic discount factor.
- His effective beliefs and effective discount factor depend on the decision maker's equilibrium comsumption and on his attitude towards ambiguity.
- The effective discount factor is equal, smaller or greater than the actual discount factor if the agent displays absolute ambiguity aversion that is constant (CAAA), increasing (IAAA) or decreasing (DAAA), respectively.
- For CAAA and IAAA, the effect of ambiguity on the discount factor does not compensate the effect on beliefs. Thus, they survive only if they are fully insured against ambiguity.
- For DAAA, ambiguity makes the discount factor larger than the actual discount factor.

The Model

- $\{\pi^n\}_{n=1}^N$ is a family of probability distribution on $(S^\infty, \mathcal{F})$ with disjoint support.
- $\Theta \equiv \{s_0\} \times \left\{ \{\pi^n\}_{n=1}^N \times S \right\}^\infty$ with $\theta = (s_0, (\pi_1, s_1), (\pi_2, s_2), ..., (\pi_t, s_t), ...)$ an element of Θ.
- μ denotes the "true" probability measure on $\left(\Theta, \mathcal{F}^\Theta\right)$.

Definition

In an economy with vanishing ambiguity, $\mu\left(\pi^n \middle| \mathcal{F}_1\right) > 0$ and for any $t > 1$, $\mu\left(\pi^n \middle| \mathcal{F}_t\right) = 1$ if and only if $\pi_t = \pi_{t-1}$.

Definition

In an economy with Markov persistent ambiguity, $\mu\left(\pi^n \middle| \mathcal{F}_t^\Theta\right) = \mu\left(\pi^n \middle| s_t\right) > 0$ and $\pi^n\left(s_t \middle| \mathcal{F}_{t-1}\right) = \pi^n\left(s_t \middle| s_{t-1}\right)$ for all $n \geq 1, t \geq 1$

Preferences

- There is a single good.
- Agents' preferences can be represented by:

$$V^i_{s^t}\left(c^i\right) = u_i\left(c^i\left(s^t\right)\right) + \beta\phi_i^{-1}\left[\sum_{n=1}^{N}\phi_i\left(\sum_{\xi\in S} V^i_{\left(s^t,\xi\right)}\left(c^i\right)\pi^n\left(\xi|\, s^t\right)\right)\mu^i\left(\pi^n|\, s^t\right)\right]$$

- $\beta \in (0,1)$, $u_i : \Re_+ \mapsto \Re$ and $\phi_i : \Re \mapsto \Re$.
- **Assumption 1**: u_i are C^2, strictly concave, $u(0) = 0$ and satisfies Inada conditions.
- **Assumption 2**: ϕ_i is either linear or strictly concave, C^2 and $\lim_{y\to 0}\phi'(y) > 0$
- **Assumption 3**: Endowments are uniformly bounded away from zero and above.
- **Assumption 4**: There is $\overline{\delta} > 0$ such that $\pi^n\left(s_{t+1}|\, s^t\right) > \delta$ for all $s_{t+1} \in S$, $s \in S^\infty$ and $t \geq 1$.
- **Assumption 5**: For every s^t, $\left\{\pi^n\left(\cdot|\, s^t\right)\right\}_{n=1}^{N}$ are linearly independent vectors.

Arrow-Debreu Equilibrium

- **Proposition 4.2**: Under assumptions 1-4, the equilibrium satisfies:

$$\frac{\beta_i u_i'\left(c_i\left(s^t, s_{t+1}\right)\right) \frac{\sum_{n=1}^{N} \phi'\left[E_{\pi^n}\left(V^i_{\left(s^t,\xi\right)}\left(c^i\right)\right)\right] \pi^n\left(s_{t+1}|s^t\right)\mu_i\left(\pi^n|s^t\right)}{\phi_i'\left(\phi_i^{-1}\left(\sum_{n=1}^{N} \phi_i\left[E_{\pi^n}\left(V^i_{\left(s^t,\xi\right)}\left(c^i\right)\right)\right]\mu_i\left(\pi^n|s^t\right)\right)\right)}}{u_i'\left(c_i\left(s^t\right)\right)} = \frac{p\left(s^t, s_{t+1}\right)}{p\left(s^t\right)}$$

- The expression in red can be rewritten as:

$$\underbrace{\frac{\sum_{n=1}^{N} \phi'\left[E_{\pi^n}\left(V^i_{\left(s^t,\xi\right)}\left(c^i\right)\right)\right]\mu_i\left(\pi^n|s^t\right)}{\phi_i'\left(\phi_i^{-1}\left(\sum_{n=1}^{N} \phi_i\left[E_{\pi^n}\left(V^i_{\left(s^t,\xi\right)}\left(c^i\right)\right)\right]\mu_i\left(\pi^n|s^t\right)\right)\right)}}_{\hat{\beta}_i\left(c^i, s^t\right) \rightarrow \text{"effective discount rate"}} \underbrace{\frac{\sum_{n=1}^{N} \phi'\left[E_{\pi^n}\left(V^i_{\left(s^t,\xi\right)}\left(c^i\right)\right)\right]\pi^n\left(s_{t+1}|s^t\right)\mu_i\left(\pi^n|s^t\right)}{\sum_{n=1}^{N} \phi'\left[E_{\pi^n}\left(V^i_{\left(s^t,\xi\right)}\left(c^i\right)\right)\right]\mu_i\left(\pi^n|s^t\right)}}_{\hat{\pi}_i\left(s_{t+1}|c^i, s^t\right) \rightarrow \text{"effective beliefs"}}$$

- Therefore

$$\hat{\beta}_i\left(c^i, s^t\right)\hat{\pi}_i\left(s_{t+1}\middle| c^i, s^t\right)\frac{u_i'\left(c_i\left(s^t, s_{t+1}\right)\right)}{u_i'\left(c_i\left(s^t\right)\right)} = \frac{p\left(s^t, s_{t+1}\right)}{p\left(s^t\right)}$$

Dynamics

- Hence

$$\frac{\hat{\beta}_i\left(c^i, s^t\right)}{\beta_j} \frac{\hat{\pi}_i\left(s_{t+1} \middle| c^i, s^t\right)}{\sum_{n=1}^N \pi^n\left(s_{t+1} \middle| s^t\right) \mu\left(\pi^n \middle| s^t\right)} \frac{u_i'\left(c_i\left(s^t, s_{t+1}\right)\right)}{u_j'\left(c_j\left(s^t, s_{t+1}\right)\right)} = \frac{u_i'\left(c_i\left(s^t\right)\right)}{u_j'\left(c_j\left(s^t\right)\right)}$$

$$\frac{u_i'\left(c_i\left(s^T, s_{T+1}\right)\right)}{u_j'\left(c_j\left(s^T, s_{T+1}\right)\right)} = \frac{u_i'\left(c_i\left(s_0\right)\right)}{u_j'\left(c_j\left(s_0\right)\right)} \prod_{t=0}^T \frac{\beta_j}{\hat{\beta}_i\left(c^i, s^t\right)} \frac{\sum_{n=1}^N \pi^n\left(s_{t+1} \middle| s^t\right) \mu\left(\pi^n \middle| s^t\right)}{\hat{\pi}_i\left(s_{t+1} \middle| c^i, s^t\right)}$$

- **Lemma 5.3** Suppose Assumptions 1-4 hold. If i is ambiguity averse and j is an expected utility maximiser, then

$$\lim_{T\to\infty} \frac{1}{T+1} \ln \frac{u_i'\left(c_i\left(s^T, s_{T+1}\right)\right)}{u_j'\left(c_j\left(s^T, s_{T+1}\right)\right)} = \lim_{T\to\infty} \frac{1}{T+1} \sum_{t=1}^T \ln\left(\beta_j\right) - \ln\left(\hat{\beta}_i\left(c^i, s^t\right)\right)$$

$$+ \lim_{T\to\infty} \frac{1}{T+1} \sum_{t=1}^T \ln\left(\sum_{n=1}^N \pi^n\left(s_{t+1} \middle| s^t\right) \mu\left(\pi^n \middle| s^t\right)\right) - \ln\left(\hat{\pi}_i\left(s_{t+1} \middle| c^i, s^t\right)\right)$$

Survival with Vanishing Ambiguity

- **Proposition 5.1**: Suppose that Assumptions 1-4 hold. Consider an economy with vanishing ambiguity and $\beta_i = \beta_j$ for all i, j. Suppose for some agent i:
  1. $\mu^n > 0$ implies $\mu_i^n > 0$
  2. The function $G_i\left(\mu\left(\pi^1 \middle| s^t\right), \ldots, \mu\left(\pi^n \middle| s^t\right)\right) = \frac{\sum_{n=1}^N \phi'(y^n)\pi^n(s_{t+1}|s^t)\mu_i(\pi^n|s^t)}{\phi_i'\left(\phi_i^{-1}\left(\sum_{n=1}^N \phi_i(y^n)\mu_i(\pi^n|s^t)\right)\right)}$ where $y^n \in \left[0, \frac{1}{1-\beta} u(m)\right]$ is C^1 and its total derivative is uniformly bounded on $\left[0, \frac{1}{1-\beta} u(m)\right]$.

  Then i survives almost surely. In particular, he survives if ϕ_i exhibits HAAA.

Survival with Aggregate Risk

- **Proposition 5.2**: Suppose that Assumptions 1-4 hold. Suppose all consumers have the same discount factors and correct beliefs. In an economy with persistent ambiguity but no aggregate risk, all consumers survive a.s.

Survival with Persistent Ambiguity and Aggregate Risk

- **Proposition 5.12**: Let Assumptions 1-4 hold. Consider an economy with persistent ambiguity where j is an expected utility maximiser and i is a smooth ambiguity-averse with DAAA such that $-\frac{\phi_i'''}{\phi_i''} \geq -2\frac{\phi_i''}{\phi_i'}$. Suppose that both i and j have correct beliefs and identical discount factors. Then i survives a.s. Furthermore, j vanishes a.s.

- The ambiguity precautionary premium is Φ_A that solves:

$$\sum_{n=1}^{N} \phi_i'(y^n)\,\mu_i\left(\pi^n \middle| s^t\right) = \phi'\left(\sum_{n=1}^{N} y^n \mu_i\left(\pi^n \middle| s^t\right) - \Phi_A\right)$$

- The ambiguity premium is P_A defined by:

$$\phi_i^{-1}\left(\sum_{n=1}^{N} \phi_i(y^n)\,\mu_i\left(\pi^n \middle| s^t\right)\right) = \sum_{n=1}^{N} y^n \mu_i\left(\pi^n \middle| s^t\right) - P_A$$

- $\Phi_A \geq P_A \Leftrightarrow -\frac{\left(\ln \phi_i'\right)''}{\left(\ln \phi_i'\right)'} \geq -\frac{\phi_i''}{\phi_i'} \Leftrightarrow -\frac{\phi_i'''}{\phi_i''} \geq -2\frac{\phi''}{\phi'}$