Individual and Social Measures of Economic Wellbeing

Peter J. Hammond

3rd March 2018
Prelude: Last of Four Main Themes

A quartet of themes, all discussed over two weeks from both a philosopher’s and an economist’s perspectives:

1. **Individual Choice and Rationality:** (weeks 1–3)
   When faced with a finite decision tree, is an action right if and only if its consequences are right?

2. **Social Choice, Justice, and Welfare:** (weeks 4–5)
   Exploring the possible links between:
   - what is ethically appropriate for individuals;
   - what is ethically appropriate for society as a whole.

3. **Markets and their Ethical Limitations:** (weeks 7–8)
   Do markets always promote economic agents’ well-being?

4. **Measuring Individual and Social Wellbeing:** (weeks 9–10)
   How to measure the effects of economic policy?
Ethical Decisions Should Maximize Expected Utility

In weeks 1–2 we considered the argument that a policy choice is ethical if and only if its consequences maximize a complete and transitive preference ordering over the set of feasible consequences.

Moreover, when consequences are risky, preferences over lotteries should satisfy the independence axiom.

Suppose in addition that the choice of lotteries varies continuously with respect to the probabilities of different consequences. With this extra assumption, it follows that ethical policy choices are those yielding consequence lotteries that maximize the expected value of each utility function in a unique cardinal equivalence class of von Neumann–Morgenstern utility functions.
The Impartial Spectator’s View of Individual Utility

In week 4 we considered the argument that, in a society of many individuals, ethical decisions are those that an impartial actor would make in the Vickrey–Harsanyi original position. That is, from behind a veil of ignorance, when facing an equal chance of becoming any individual. Harsanyi preferred the term “impartial observer”.

Upon emerging from behind the veil of ignorance, where the impartial actor has become a particular individual, the decision to be made is extremely partial. It maximizes the particular individual’s expected utility, perhaps based on Adam Smith’s idea of the “impartial spectator”.

Measuring Individual and Social Wellbeing

In this last lecture we will consider attempts to determine what empirical evidence might be relevant in helping to determine ethically relevant measures of:

1. individual wellbeing, which it would be right for an extremely partial actor to maximize, treating all but that one individual as if they are robots;

2. social wellbeing, which it would be right for an extremely impartial actor to maximize, giving all individuals equal weight.
Outline

Measuring Individual Well-Being
  Nominal Income
  Real Income
  Price Indices

Indirect and Money Metric Utility

Indirect Social Welfare Functional

Life Metric Utility

Subjective Well-Being
Measures of Nominal Income

A traditional and objective measure of well-being has been *nominal income* per head per year (or per week, or per month).

The unit is dollars, or pounds, or euros per person per unit of time. This may be useful as a rather rough first approximation to a measure of individual well-being.

Over a given period, we can compare and even add the incomes of different individuals. Total income, or income per head, without any regard for distribution, is highly questionable as even an approximate measure of social well-being.
Outline

Measuring Individual Well-Being
   Nominal Income
   Real Income
   Price Indices

Indirect and Money Metric Utility

Indirect Social Welfare Functional

Life Metric Utility

Subjective Well-Being
Relevance of Price Changes

Prices vary over time.
They may also be different for different individuals, especially those who live in different countries.
When individuals face different prices, measures of individual or social well-being based on income require nominal incomes to be corrected for price differences or variations.
To do this, usually nominal incomes are converted to real incomes simply by dividing by a price index or price deflator.
In the case of international comparisons, this price deflator is often based on exchange rates corrected for purchasing power (dis)parity.
The Economist’s Big Mac® Index

Subtitle: Global exchange rates, to go
Source: http://www.economist.com/content/big-mac-index

The Big Mac index was invented by The Economist in 1986 as a lighthearted guide to whether currencies are at their “correct” level. . . .

It is based on the theory of purchasing-power parity (PPP), the notion that in the long run exchange rates should move towards the rate that would equalise the prices of an identical basket of goods and services (in this case, a burger) in any two countries.
Limitations of Burgernomics

For example, the average price of a Big Mac in America in January 2018 was $5.28; in China it was only $3.17 at market exchange rates. So the “raw” Big Mac index says that the yuan was undervalued by 40% at that time.

[Check: $3.17/5.28 \approx 0.60 = 1 - 0.40$]
Example of Burgernomics

*Burgernomics was never intended as a precise gauge of currency misalignment, merely a tool to make exchange-rate theory more digestible.* Yet the *Big Mac index has become a global standard, included in several economic textbooks and the subject of at least 20 academic studies.* For those who take their fast food more seriously, we have also calculated a gourmet version of the index.
Income Deflators

A standard measure of real income comes from dividing nominal income by a consumer price index or deflator. This index will be an objective measure provided that

1. it is the value of an observable fixed commodity bundle or “market basket”;

2. or it is based on some observable aggregates like mean expenditure shares and aggregate expenditure.

In principle one could even divide personal income by a different price index for each different consumer. See, for example, the discussion in Boskin et al. (1996, 1998) and associated articles in the *Journal of Economic Perspectives* devoted to the Boskin commission.
Boskin Commission in 1996

*Toward A More Accurate Measure Of The Cost Of Living*
finance.senate.gov/imo/media/doc/Prt104-72.pdf


Prices and Quantities

Consider a finite set $G$ of $n = \# G$ commodities.

Suppose an individual faces an $n$-dimensional price vector $\mathbf{p} = (p_g)_{g \in G} \in \mathbb{R}_+^G$.

Let $\mathbf{p} \cdot \mathbf{x}$ denote the sum $\sum_{g \in G} p_g x_g$ of $n$ terms, representing the consumer’s (net) expenditure if the net consumption quantity vector is $\mathbf{x} = (x_g)_{g \in G} \in \mathbb{R}^G$.

The nominal income level $m$ and the price vector $\mathbf{p}$ jointly determine the budget constraint $\mathbf{p} \cdot \mathbf{x} \leq m$.

Often a measure of real income is taken to be $m/P(\mathbf{p})$ for some price index $\mathbb{R}_+^G \ni \mathbf{p} \mapsto P(\mathbf{p}) \in \mathbb{R}_+$ that deflates the measure of nominal income.
Outline

Measuring Individual Well-Being
- Nominal Income
- Real Income
- Price Indices

Indirect and Money Metric Utility

Indirect Social Welfare Functional

Life Metric Utility

Subjective Well-Being
Laspeyres and Paasche Price Indices

We consider an index to measure the change in prices:

1. from a base situation 0 with price vector \( p^0 = \langle p^0_g \rangle_{g \in G} \);
2. to a new situation 1 with price vector \( p^1 = \langle p^1_g \rangle_{g \in G} \).

Given any fixed reference quantity vector \( x^R = \langle x^R_g \rangle_{g \in G} \),
one measure or price index is the ratio

\[
P(p^0, p^1; x^R) = \frac{p^1 \cdot x^R}{p^0 \cdot x^R} = \frac{\sum_{g \in G} p^1_g x^R_g}{\sum_{g \in G} p^0_g x^R_g}
\]

of the cost of \( x^R \) at prices \( p^1 \) to its cost at prices \( p^0 \).

Two special cases: the situations are \( t = 0 \) (base) or \( t = 1 \) (new):

- **Laspeyres index** \( \frac{p^1 \cdot x^0}{p^0 \cdot x^0} \) with base quantities as weights;
- **Paasche index** \( \frac{p^1 \cdot x^1}{p^0 \cdot x^1} \) with new quantities as weights.
Real Income and Quantity Indices, 1

We define an index of real income as an index of incomes divided by a price index.

Suppose that income \( m^t = p^t \cdot x^t \) for \( t = 0 \) and \( t = 1 \).

Dividing the income index \( \frac{m^1}{m^0} \)
by the Laspeyres price index \( \frac{p^1 \cdot x^0}{p^0 \cdot x^0} \) gives the real income index

\[
\frac{m^1}{m^0} \cdot \frac{p^1 \cdot x^0}{p^0 \cdot x^0} = \frac{p^1 \cdot x^1}{p^0 \cdot x^0} \cdot \frac{p^1 \cdot x^0}{p^0 \cdot x^0} = \frac{p^1 \cdot x^1}{p^1 \cdot x^0}
\]

This has the structure of a quantity index in which the base quantities \( x^0_g \) and new quantities \( x^1_g \) of different goods \( g \in G \) are all weighted at the new prices \( p^1_g \).
Suppose that \( \frac{p^1 \cdot x^1}{p^1 \cdot x^0} > 1 \) or equivalently, that \( p^1 \cdot x^1 > p^1 \cdot x^0 \),
or equivalently, that \( \frac{m^1}{m^0} = \frac{p^1 \cdot x^1}{p^0 \cdot x^0} > \frac{p^1 \cdot x^0}{p^0 \cdot x^0} \),
implying that incomes rise by more than the Laspeyres price index.

Given that \( p^1 \cdot x^1 = m^1 > p^1 \cdot x^0 \),
there exists a vector \( z \) such that \( p^1 \cdot z = m^1 \) and \( z \gg x^0 \) — i.e., the components \( z_g \) of \( z \) and \( x^0_g \) of \( x^0 \) satisfy \( z_g > x^0_g \) for every good \( g \).
Revealed Preference, II

When the budget constraint is $p^1 \cdot x = m^1$, note that $x^1$ is bought.

But because $p^1 \cdot z = m^1$, the alternative $z$ could also have been bought.

So the hypothesis of revealed preference implies that $x^1 \succsim z$.

Because $z \gg x^0$, the hypothesis of monotone preferences (requiring that more is always preferred to less) implies that $z \succ x^0$.

Because $x^1 \succsim z$ and $z \succ x^0$, transitivity then implies that $x^1 \succ x^0$. 
Real Income and Quantity Indices, II

Similarly, dividing the income index \( \frac{m^1}{m^0} \)
by the Paasche price index \( \frac{p^1 \cdot x^1}{p^0 \cdot x^1} \) gives the real income index

\[
\frac{m^1}{m^0} \cdot \frac{p^1 \cdot x^1}{p^0 \cdot x^1} = \frac{p^1 \cdot x^1}{p^0 \cdot x^0} \cdot \frac{p^1 \cdot x^1}{p^0 \cdot x^1} = \frac{p^0 \cdot x^1}{p^0 \cdot x^0}
\]

This has the structure of a different quantity index
in which the base quantities \( x^0_g \) and new quantities \( x^1_g \)
of different goods \( g \in G \) are weighted at the base prices \( p^0_g \).
Revealed Preference, III

Suppose that \( \frac{p^0 \cdot x^1}{p^0 \cdot x^0} < 1 \) or equivalently, that \( p^0 \cdot x^1 < p^0 \cdot x^0 \), or equivalently, that \( \frac{m^1}{m^0} = \frac{p^1 \cdot x^1}{p^0 \cdot x^0} < \frac{p^1 \cdot x^1}{p^0 \cdot x^1} \), meaning that incomes rise by less than the Paasche price index.

Given that \( p^0 \cdot x^0 = m^0 > p^0 \cdot x^1 \), there exists a vector \( w \) such that \( p^0 \cdot w = m^0 \) and \( w \succ x^1 \).

The hypothesis of revealed preference implies that, because \( x^0 \) was bought given the budget constraint \( p^0 \cdot x = m^0 \), which is also satisfied by \( w \), one has \( x^0 \succeq w \).

Because \( w \succ x^1 \), the hypothesis of monotone preferences (requiring that more is always preferred to less) implies \( w \succ x^1 \).

Because \( x^0 \succeq w \succ x^1 \), transitivity then implies that \( x^0 \succ x^1 \).
Laspeyres and Paasche Quantity Indices, III

We have proved that:

1. if \( \frac{m^1}{m^0} > \frac{p^1 \cdot x^0}{p^0 \cdot x^0} \), meaning that incomes rise by more than the Laspeyres price index, then \( x^1 \succ x^0 \);

2. if \( \frac{m^1}{m^0} < \frac{p^1 \cdot x^1}{p^0 \cdot x^1} \), meaning that incomes rise by less than the Paasche price index, then \( x^0 \succ x^1 \).

But if neither is true because \( \frac{p^1 \cdot x^0}{p^0 \cdot x^0} \leq \frac{m^1}{m^0} = \frac{p^1 \cdot x^1}{p^0 \cdot x^1} \leq \frac{p^1 \cdot x^0}{p^0 \cdot x^0} \), then there is too little information to tell which of \( x^0 \) and \( x^1 \) is preferred to the other.
**Divisia Chain Indices**

In successive time periods \( t = 0, 1, 2, \ldots \), suppose one observes:
- price vectors \( \mathbf{p}^t \);
- quantity vectors \( \mathbf{x}^t \);
- associated income levels \( m^t = \mathbf{p}^t \cdot \mathbf{x}^t \).

One can have either
- fixed weight price indices \( \frac{\mathbf{p}^t \cdot \mathbf{x}^0}{\mathbf{p}^0 \cdot \mathbf{x}^0} \) and quantity indices \( \frac{\mathbf{p}^0 \cdot \mathbf{x}^t}{\mathbf{p}^0 \cdot \mathbf{x}^0} \);
- chain indices with regularly updated weights taking the form

\[
P = \frac{\mathbf{p}^1 \cdot \mathbf{x}^0}{\mathbf{p}^0 \cdot \mathbf{x}^0} \cdot \frac{\mathbf{p}^2 \cdot \mathbf{x}^1}{\mathbf{p}^1 \cdot \mathbf{x}^1} \cdot \frac{\mathbf{p}^3 \cdot \mathbf{x}^2}{\mathbf{p}^2 \cdot \mathbf{x}^2} \cdots \frac{\mathbf{p}^t \cdot \mathbf{x}^{t-1}}{\mathbf{p}^{t-1} \cdot \mathbf{x}^{t-1}} = \prod_{s=0}^{t-1} \frac{\mathbf{p}^{s+1} \cdot \mathbf{x}^s}{\mathbf{p}^s \cdot \mathbf{x}^s}
\]

\[
Q = \frac{\mathbf{p}^0 \cdot \mathbf{x}^1}{\mathbf{p}^0 \cdot \mathbf{x}^0} \cdot \frac{\mathbf{p}^1 \cdot \mathbf{x}^2}{\mathbf{p}^1 \cdot \mathbf{x}^1} \cdot \frac{\mathbf{p}^2 \cdot \mathbf{x}^3}{\mathbf{p}^2 \cdot \mathbf{x}^2} \cdots \frac{\mathbf{p}^{t-1} \cdot \mathbf{x}^t}{\mathbf{p}^{t-1} \cdot \mathbf{x}^{t-1}} = \prod_{s=0}^{t-1} \frac{\mathbf{p}^s \cdot \mathbf{x}^{s+1}}{\mathbf{p}^s \cdot \mathbf{x}^s}
\]
Outline

Measuring Individual Well-Being

Indirect and Money Metric Utility
   Indirect Utility
   Money Metric Utility
   Surplus Economics

Indirect Social Welfare Functional

Life Metric Utility

Subjective Well-Being
Direct versus Indirect Utility

One way to measure an individual’s living standard is through a direct utility function $U(x)$. The disadvantage is that, in addition to knowing the consumer’s preferences, one must also observe the consumer’s own demand vector $x$.

An alternative that may work better is the indirect utility function $V(p, m)$. It represents the consumer’s preferences over different budget constraints $p \cdot x \leq m$. In a competitive market system, the price vector $p$ should be the same for many consumers.

Of course, each consumer’s income $m$ still has to be observed.
The passage from direct utility $U(x)$ to indirect utility $V(p, m)$ is straightforward in principle. Suppose that, for each price vector $p$ and nominal income $m$, the consumer chooses the demand quantity vector $x = h(p, m)$ to maximize $U(x)$ subject to the budget constraint $p \cdot x \leq m$.

Then the consumer’s indirect utility satisfies the functional identity $V(p, m) \equiv U(h(p, m))$ derived by inserting the vector demand function $x = h(p, m)$ into the utility function $U$. 
Homogeneity of Degree Zero

**Definition** A real-valued function \( f(x_1, x_2, \ldots, x_n) \) of \( n \) non-negative variables is homogeneous of degree \( k \) just in case

\[
f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) = \lambda^k f(x_1, x_2, \ldots, x_n)
\]
for all scalars \( \lambda > 0 \).

Suppose that all prices \( p_g \) and income \( m \) are multiplied by the same scalar \( \lambda > 0 \).

This has no effect on the budget constraint \( p \cdot x = m \), which is equivalent to \( \lambda p \cdot x = \lambda m \).

Hence \( h_g(\lambda p, \lambda m) = h_g(p, m) \) for each good \( g \in G \), and also

\[
V(\lambda p, \lambda m) \equiv U(h(\lambda p, \lambda m)) \equiv U(h(p, m)) \equiv V(p, m)
\]

So the functions \( (p, m) \mapsto h_g(p, m) \) and \( (p, m) \mapsto V(p, m) \) are all homogeneous of degree 0. (Recall that the power \( \lambda^0 = 1 \).)
Roy’s Identity

Another important implication is Roy’s identity

$$\frac{\partial}{\partial p_g} V(p, m) \div \frac{\partial}{\partial m} V(p, m) = -h_g(p, m)$$

Roy’s identity is named after the French economist René Roy, so Roy should be pronounced accordingly (very roughly, like “rwa”).

Roy’s identity requires equality between:

- minus the demand for good $g$
  at the price vector $p$ and income $m$;
- the MRS between increases in:
  1. the price $p_g$ of good $g$;
  2. income $m$. 
Example: Cobb–Douglas Preferences

Suppose that a consumer’s preferences can be represented by the Cobb–Douglas utility function defined whenever $x \gg 0$ by

$$U(x) = \prod_{g \in G} x_g^{\alpha_g}$$

where $(\alpha_g)_{g \in G}$ are positive taste parameters with $\sum_{g \in G} \alpha_g = 1$.

Often, it is convenient to consider the ordinally equivalent function found by taking natural logarithms.

$$\ln U(x) = \sum_{g \in G} \alpha_g \ln x_g$$

Using the chain rule to take the partial derivative of the last equation w.r.t. $x_g$ gives

$$\frac{U'_g(x)}{U(x)} = \frac{\alpha_g}{x_g}. $$
Marginal Rates of Substitution and Prices

The consumer’s **marginal rate of substitution** \( \text{MRS}_{g,h}(x) \) between any pair of goods \( g, h \in G \) is then given by the ratio

\[
\frac{\partial U}{\partial x_g} \div \frac{\partial U}{\partial x_h} = \frac{U'_g(x)}{U'_h(x)} = \text{MRS}_{g,h}(x) = \frac{\alpha_g}{x_g} \div \frac{\alpha_h}{x_h} = \frac{\alpha_g x_h}{x_g \alpha_h}
\]

of marginal utilities of the two goods.

Equating each \( \text{MRS}_{g,h}(x) \) to the corresponding price ratio \( p_g/p_h \) yields the equations \( \frac{\alpha_g x_h}{x_g \alpha_h} = \frac{p_g}{p_h} \), or equivalently \( \alpha_g p_h x_h = \alpha_h p_g x_g \).

This equation can be rewritten as \( \frac{p_g x_g}{\alpha_g} = \frac{p_h x_h}{\alpha_h} \).

It implies that there exists a single positive constant \( c \), independent of \( g \), such that \( p_g x_g = \alpha_g c \) for all goods \( g \in G \).
Homogeneous Linear Expenditure System

The equation \( \frac{p_g x_g}{\alpha_g} = c \) implies that expenditure \( p_g x_g \) on each good \( g \) is \( \alpha_g c \).

Because of the budget constraint \( \mathbf{p} \cdot \mathbf{x} = m \) and the assumption \( \sum_{g \in G} \alpha_g = 1 \), the equations \( p_g x_g = \alpha_g c \) imply that

\[
m = \sum_{g \in G} p_g x_g = c \sum_{g \in G} \alpha_g = c
\]

So expenditure \( p_g x_g \) on each good \( g \) is \( \alpha_g m \), a constant proportion of income, with the proportion \( p_g x_g / m \) equal to the taste parameter \( \alpha_g \).

The resulting demand functions are then \( x_g(p, m) = \alpha_g \frac{m}{p_g} \).
Price and Income Elasticities of Demand

Consider a system of demand functions \( x_g(p, m) \), one for each good \( g \in G \).

The **own price elasticity** of demand for good \( h \in G \) is defined as

\[
\text{El}_{p_h} x_h(p, m) = \frac{p_h}{x_h} \frac{\partial x_h}{\partial p_h} = \frac{\partial \ln x_h}{\partial \ln p_h}
\]

The **cross price elasticity** of demand for good \( h \in G \) is defined as

\[
\text{El}_{p_g} x_h(p, m) = \frac{p_g}{x_h} \frac{\partial x_h}{\partial p_g} = \frac{\partial \ln x_h}{\partial \ln p_g} \quad \text{for all } g \neq h
\]

The **income elasticity** of demand for good \( h \in G \) is defined as

\[
\text{El}_{m} x_h(p, m) = \frac{m}{x_h} \frac{\partial x_h}{\partial m} = \frac{\partial \ln x_h}{\partial \ln m}
\]
Elasticities
in the Homogeneous Linear Expenditure System

For the system with $x_g(p, m) = \alpha_g \frac{m}{p_g}$ for all $g \in G$:

1. each own price elasticity $El_{p_h}x_h = -1$;
2. each cross price elasticity $El_{p_g}x_h = 0$;
3. each income elasticity $El_{m}x_h = 1$. 
A Cobb–Douglas Price Index

Inserting the demand functions \( x_g(p, m) = \alpha_g \frac{m}{p_g} \)
into the direct utility function \( U(x) = \prod_{g \in G} x_g^{\alpha_g} \), one obtains

\[
V(p, m) = \prod_{g \in G} \left( \alpha_g \frac{m}{p_g} \right)^{\alpha_g} = \prod_{g \in G} \alpha_g^{\alpha_g} \frac{m}{P(p)} = \alpha^* \frac{m}{P(p)}
\]

because \( \sum_{g \in G} \alpha_g = 1 \) implies that \( \prod_{g \in G} m^{\alpha_g} = m \sum_{g \in G} \alpha_g = m \).

Here \( \alpha^* := \prod_{g \in G} \alpha_g^{\alpha_g} \) is a positive constant,
and \( P(p) \) is the \textbf{Cobb–Douglas price index} defined by

\[
P(p) = \prod_{g \in G} p_g^{\alpha_g}
\]

The function \( p \mapsto P(p) \) is \textit{homogeneous of degree 1},
meaning that \( P(\lambda p) = \lambda P(p) \) for all \( \lambda > 0 \).
Outline

Measuring Individual Well-Being

Indirect and Money Metric Utility
  Indirect Utility
  Money Metric Utility
  Surplus Economics

Indirect Social Welfare Functional

Life Metric Utility

Subjective Well-Being
Money Metric Utility for Private Goods

One way to try measuring welfare changes makes use of the consumer’s willingness to pay for those changes.

This relies on fixing a reference price vector $p^R$.

Given the indirect utility function $(p, m) \mapsto V(p, m)$, assumed to be strictly increasing in $m$, one can define an ordinally equivalent money metric utility function $(p, m) \mapsto \mu(p, m)$ by

$$V(p^R, \mu(p, m)) \equiv V(p, m)$$

It is the amount of income which, if made available to the consumer at the reference price vector $p^R$, would leave the consumer just as well off as when facing the budget constraint $p \cdot x = m$. 

Special Case: Quasi-Linear Utility

Many applied economists consider quasi-linear utilities of the form $U(x, y) = w(x) + y$, where:

- $x \mapsto w(x)$ is an increasing concave function that measures willingness to pay;
- $y = m - p^\top x$ denotes the part of initial income $m$ that is not spent on the commodity bundle $x \in \mathbb{R}^G$.

The problem of maximizing $w(x) + y$ subject to the budget constraint $p \cdot x + y = m$ can be solved by considering the Lagrangian

$$
\mathcal{L}_\lambda(x, y) = w(x) + y - \lambda \left( \sum_{g \in G} p_g x_g + y - m \right)
$$
First-Order Conditions

Differentiating the Lagrangian

\[ \mathcal{L}_\lambda(x, y) = w(x) + y - \lambda \left( \sum_{g \in G} p_g x_g + y - m \right) \]

partially:

▶ w.r.t. \( y \) gives \( 1 - \lambda \);

▶ w.r.t. each component \( x_g \) of \( x \) gives \( \frac{\partial}{\partial x_g} w(x) - \lambda p_g \).

Setting these partial derivatives equal to zero gives \( \lambda = 1 \) as one first-order condition.

Using the equality \( \lambda = 1 \) gives, for each good \( g \in G \), the additional first-order condition \( \frac{\partial}{\partial x_g} w(x) = p_g \).

That is, the marginal willingness to pay for each good \( g \) must equal its price \( p_g \).
Zero Income Elasticities

The first-order conditions \( \frac{\partial}{\partial x_g} w(x) = p_g \) typically have a unique solution in the form of demand functions \( x_g = h_g(p) \) that are \textit{independent} of income \( m \).

Hence each income elasticity of demand \( El_m x_g(p, m) \) is 0.

From the budget constraint, the demand for the other good is

\[
y(p, m) = m - \sum_{g \in G} p_g h_g(p) = m - p \cdot x(p)
\]

Then indirect utility is

\[
V(p, m) \equiv U(x(p), y(p, m)) \equiv w(x(p)) + m - p \cdot x(p)
\]

Note that this is \textit{not} homogeneous of degree zero in \((p, m)\).

This is a symptom of a fundamentally flawed approach.
Individual Consumer Surplus

Given the indirect utility $V(p, m) = w(x(p)) + m - p \cdot x(p)$, define the consumer surplus function

$$p \mapsto s(p) := w(x(p)) - p \cdot x(p)$$

So surplus = willingness to pay − actual payment.

Then indirect utility takes the form $V(p, m) = s(p) + m$. 
Surplus as Money Metric Utility

In this special case, consider the equation

\[ V(p^R, \mu(p, m)) \equiv V(p, m) \]

that defines the money metric utility function \((p, m) \mapsto \mu(p, m)\).

It reduces to

\[ s(p^R) + \mu(p, m) \equiv s(p) + m \]

implying that

\[ \mu(p, m) = s(p) - s(p^R) + m \]

Hence money metric utility is the sum of income and the change in surplus from the reference price vector \(p^R\).
Total Consumer Surplus

Consider a society of several individuals \( i \in N \).

Suppose each individual \( i \in N \) has a quasi-linear utility function \( U^i(x^i, y^i) = w^i(x^i) + y^i \) of their own consumption bundle \( x^i \) and residual income \( y^i \).

Then each individual has their own demand function \( x^i_g = h^i_g(p) \), independent of \( i \)'s income \( m^i \).

Suppose individuals trade in a competitive market where they face the common price vector \( p \) and a distribution of income \( m^N = \langle m^i \rangle_{i \in N} \).

Then the measure of total indirect utility is

\[
W(p, m^N) = \sum_{i \in N} V^i(p, m^i) = \sum_{i \in N} [w^i(x^i(p)) + m^i - p \cdot x^i(p)]
\]
Total Consumer Surplus and Total Demand

Given, the measure of total indirect utility

$$W(p, m^N) = \sum_{i \in N} [w^i(x^i(p)) + m^i - p \cdot x^i(p)]$$

define the total consumer surplus function

$$p \mapsto S(p) := \sum_{i \in N} w^i(x^i(p)) - p \cdot \sum_{i \in N} x^i(p)$$

whose value, very conveniently, is total willingness to pay minus the value of total demand at the price vector $p$.

Then total indirect utility takes the form

$$W(p, m^N) = S(p) + \sum_{i \in N} m^i$$

This is the sum of total surplus and total income.
Outline

Measuring Individual Well-Being

Indirect and Money Metric Utility

Indirect Social Welfare Functional

Life Metric Utility

Subjective Well-Being
Bergson Social Welfare Functionals

A **Bergson social welfare function** is a mapping \( x^N \mapsto W(x^N) \in \mathbb{R} \) defined on the interpersonal profile or allocation \( x^N = \langle x^i \rangle_{i \in N} \) of individual demand vectors \( x^i \).

Consider the **Bergson social ordering** defined by \( x^N \succeq y^N \iff W(x^N) \geq W(y^N) \).

Assume this **Bergson social ordering** is determined by a **Sen social welfare functional** \( U^N \mapsto F(U^N) \) of the interpersonally comparable profile \( U^N = \langle U^i \rangle_{i \in N} \) of individual utility functions \( \mathbb{R}^G \ni x^i \mapsto U^i(x^i) \mapsto \mathbb{R} \).

Then there exists a **Bergson social welfare functional** \( \mathbb{R}^N \ni u^N \mapsto G(u^N) \in \mathbb{R} \) for aggregating utility level vectors \( u^N \in \mathbb{R}^N \) with the property that \( W(x^N) = G(U^N(x^N)) \).
Two Examples of Social Welfare Functionals

Two obvious examples of Bergson social welfare functionals:

1. In the Rawlsian case, one has $G(u^N) := \min_{i \in N} u^i$ and so

$$W(x^N) = G(U^N(x^N)) = \min_{i \in N} U^i(x^i)$$

2. In the utilitarian case, one has $G(u^N) := \sum_{i \in N} u^i$ and so

$$W(x^N) = G(U^N(x^N)) = \sum_{i \in N} U^i(x^i)$$
The Indirect Social Welfare Functional

Suppose each individual $i$’s direct utility function $x^i \mapsto U^i(x^i)$ corresponds to the indirect utility function $(p, m^i) \mapsto V^i(p, m^i)$.

Consider the Bergson social welfare functional $\mathbb{R}^N \ni u^N \mapsto G(u^N)$ with the property that $W(x^N) = G(U^N(x^N))$.

Then the same Bergson social welfare functional induces a Bergson indirect social welfare function $(p, m^N) \mapsto W^*(p, m^N)$ of the price vector $p$ and the interpersonal income distribution $m^N$.

This is given by using the same Bergson social welfare functional $G$ in order to aggregate the levels $\langle V^i(p, m^i) \rangle_{i \in N}$ of all the different individual $i$’s indirect utility functions $V^i(p, m^i)$.

The result can be written as

$$W^*(p, m^N) := G(\langle V^i(p, m^i) \rangle_{i \in N})$$
Two Examples of Indirect Social Welfare Functionals

Back to our two obvious examples of the Bergson social welfare functional $\mathbb{R}^N \ni u^N \mapsto G(u^N)$.

1. In the Rawlsian case when $G(u^N) := \min_{i \in N} u^i$, one has

$$W^*(p, m^N) = \min_{i \in N} V^i(p, m^i)$$

In the special case when $V^i(p, m) \equiv \phi(p, m)$, independent of $i$, this becomes $W^*(p, m^N) = \phi(p, \min_{i \in N} m^i)$, which depends only on the price vector and the minimum income.

2. In the utilitarian case when $G(u^N) := \sum_{i \in N} u^i$, one has

$$W^*(p, m^N) = \sum_{i \in N} V^i(p, m^i)$$
Uniform Money Metric Social Welfare

We defined the money metric utility function \((p, m) \mapsto \mu(p, m)\) as the unique solution of the equation \(V(p^R, \mu(p, m)) \equiv V(p, m)\), which relies on fixing a reference price vector \(p^R\).

For a society, consider in addition a reference income level \(m^{iR}\) for each individual \(i \in N\).

Now define the function \((p, m^N) \mapsto \mu(p, m^N)\) as the unique solution of the equation

\[
W^*(p^R, \langle m^{iR} + \mu(p, m^N) \rangle_{i \in N}) \equiv W^*(p, m^N)
\]

This defines \(\mu(p, m^N)\) as the amount of extra income which, if paid to all \(i \in N\) in the reference situation \((p^R, \langle m^{iR} \rangle_{i \in N})\), is just as good as moving from this reference situation to the price vector \(p\) and income distribution \(m^N\).
Outline

Measuring Individual Well-Being

Indirect and Money Metric Utility

Indirect Social Welfare Functional

Life Metric Utility

Subjective Well-Being
Quality Adjusted Life Years

QALY — quality adjusted life year

This treats the additional years of a longer life as less beneficial, and less worth paying for, to the extent that they involve pain and suffering.

How far should the UK use National Health Service (NHS) funds in order to treat patients near the end of life?

National Institute for Health and Clinical Excellence (NICE) *Guide to the methods of technology appraisal 2013*

NICE suggests a monetary value of between £20,000 and £30,000 per individual QALY.
Life and Well Being

Canning, David (2013)
“Axiomatic Foundations for Cost-Effectiveness Analysis”
*Health Economics* 22 (12) 1405–1416.

Another way to try measuring welfare changes makes use of the consumer’s own willingness to exchange money for QALYs.

Suppose a consumer’s well-being can be measured using the ordinal utility function $V(p, m, \ell)$ whose arguments are:

1. the price vector $p \in \mathbb{R}^G$;
2. the income or wealth level $m$;
3. a measure $\ell$ of QALYs (though Canning writes of “life span”).
First fix a reference price–income pair \((p^R, m^R)\).

Given the indirect utility function \((p, m) \mapsto V(p, m, \ell)\), assumed to be strictly increasing in \(\ell\), one can define an ordinally equivalent life metric utility function \((p, m, \ell) \mapsto \lambda(p, m, \ell)\) by

\[V(p^R, m^R, \lambda(p, m, \ell)) \equiv V(p, m, \ell)\]

It is the amount of QALYs which, if experienced by the individual along with the reference price–income pair \((p^R, m^R)\), would leave the consumer just as well off as when

1. facing the budget constraint \(p \cdot x = m\);
2. enjoying \(\ell\) QALYs.
Valuing Lives Equally

Consider a society of individuals $i \in N$ with life metric utility functions $(p, m, \ell) \mapsto \lambda_i(p, m, \ell)$ defined by

$$V_i(p^R, m^R, \lambda_i(p, m, \ell)) \equiv V_i(p, m, \ell)$$

for a reference price vector $p^R$ and income distribution $m^R = \langle m_i^R \rangle_{i \in N}$.

As an ethical value judgement, assume that each QALY has the same ethical value, regardless of who experiences it.

Then QALYs offer an interpersonally comparable measure of individual well-being.

People who live longer, when their life span is measured in QALYs, are judged to be better off.

Policies that increase total QALYs are judged to be better.
Outline

Measuring Individual Well-Being

Indirect and Money Metric Utility

Indirect Social Welfare Functional

Life Metric Utility

Subjective Well-Being
  Four Income Distributions
  Dominance
  Mean Life Satisfaction as Utility of Income
  British Household Panel Survey (BHPS) Data
Discussion at World Bank Conference

Peter Hammond, Federica Liberini, and Eugenio Proto

Revision available at:
Raw Income Data


Waves 1–5 between 1981 and 2007, totalling 117,876 individual observations.

Each interviewee reported income within one of a range of specified intervals that vary by country and year.
Interpersonally Comparable Income Data

In order to construct a world distribution of income, we need one interpersonally comparable income measure.

To achieve it, Federica:

- transformed the lower and upper bounds of each reported income range into annual income \( y \), measured in local currency for the appropriate year;
- used an interval regression to estimate for each nation \( k \) a continuous probability density function \( y \mapsto p_k(y) \) of income \( y \), measured in the local currency of nation \( k \);
- used data from World Development Indicators (WDI) 2010 to correct for exchange rates and price changes, and express the incomes in year 2000 US dollars.
Income Distribution: Blue
Income Distribution: Green
Income Distribution: Comparing Blue and Green
Income Distribution: Yellow
Income Distribution: Red
Four Income Distributions Together
Income Correlates

What is signified by the colours we have used to mark these (stochastically dominating) rightward shifting curves describing the conditional income distributions?

The World Values Survey also has people report a happiness level, which we use as a proxy of subjective well-being (SWB):

1. blue for “not at all happy” (2.55%);
2. green for “not very happy” (15.16%);
3. yellow for “quite happy” (53.94%);
4. red for “very happy” (28.35%).

Income distribution conditional on a higher happiness level stochastically dominates (fewer people have lower incomes) income distribution conditional on a lower happiness level.
Measuring Individual Well-Being

Indirect and Money Metric Utility

Indirect Social Welfare Functional

Life Metric Utility

Subjective Well-Being
   Four Income Distributions
   Dominance
   Mean Life Satisfaction as Utility of Income
   British Household Panel Survey (BHPS) Data
Psychological Precursors


Easterlin Paradox


Study of the US during the period between 1946 and 1970. Economic growth appeared not to have enhanced mean SWB. Though US income per person rose steadily, average reported happiness showed no long-term trend, and actually declined between 1960 and 1970.
Stevenson, Wolfers, and Our Earlier Paper


Joint Distribution of SWB and Income

Two variables $s \in S$ and $y \in \mathbb{R}_+$, where:

1. $s$ denotes self-reported subjective well-being (or SWB) measured on a finite scale running through successive integers from a minimum of $s$ to a maximum of $\bar{s}$;
2. $y \geq 0$ denotes self-reported household income.

Assume that the discrete data of pairs $(s, y)$ can be smoothed and then well approximated by a continuous joint density function $S \times \mathbb{R}_+ \ni (s, y) \mapsto f(s, y) \in \mathbb{R}$

That is, for all $s \in S$ and $a, b \in \mathbb{R}$ with $0 \leq a < b$, the proportion of individuals whose reported SWB is $s$ and whose income $y$ belongs to each interval $(a, b]$ is approximately $\int_a^b f(s, y) \, dy$. 
The Conditional Distribution of SWB Given Income

The joint density \( S \times \mathbb{R}^+ \ni (s, y) \mapsto f(s, y) \in \mathbb{R} \)
satisfies the normalization condition \( \sum_{s=s}^\bar{s} \int_0^\infty f(s, y) \, dy = 1 \).

Define \( y \mapsto \psi_f(y) := \sum_{s=s}^\bar{s} f(s, y) \) as the income density function,
which satisfies the normalization condition \( \int_0^\infty \psi(y) \, dy = 1 \).

Next, for each \( y \geq 0 \) such that \( \psi_f(y) > 0 \),
define \( s \mapsto f(s|y) := f(s, y)/\psi_f(y) \)
as the conditional distribution of subjective well-being given the income level \( y \).

The definitions of \( \psi(y) \) and \( f(s|y) \) evidently imply
the normalization condition \( \sum_{s=s}^\bar{s} f(s|y) = 1 \).
Mean Subjective Well-Being as Social Welfare

The proportion of individuals reporting the SWB level \( s \) is given by

\[
F(s) := \int_0^\infty f(s, y) \, dy
\]

One possible measure of social welfare is mean SWB, defined as

\[
W(f) := \sum_{s=\bar{s}}^{\bar{s}} s F(s) = \sum_{s=\bar{s}}^{\bar{s}} \int_0^\infty s f(s, y) \, dy
\]

Define \( u(y) := \sum_{s=\bar{s}}^{\bar{s}} s \, f(s|y) = \mathbb{E}[s|y] \) as the conditionally expected SWB given the income level \( y \).

**Theorem**

These definitions imply that

\[
W(f) = \int_0^\infty u(y) \, \psi_f(y) \, dy.
\]
Conditionally Expected SWB as Utility of Income

Proof.
Reversing integration and summation in the definition of $W(f)$ implies that $W(f) = \int_{0}^{\infty} \left[ \sum_{s=\bar{s}}^{\tilde{s}} s f(s, y) \right] dy$.

But then the definition of $f(s|y)$ implies that

$$W(f) = \int_{0}^{\infty} \left[ \sum_{s=\bar{s}}^{\tilde{s}} s \frac{f(s, y)}{\psi(y)} \right] \psi(y) dy = \int_{0}^{\infty} u(y) \psi(y) dy$$

where $u(y) := \sum_{s=\bar{s}}^{\tilde{s}} s f(s|y) = \mathbb{E}[s|y]$. 

\[ \square \]
Marginal Utility of Income

“The Marginal Utility of Income”

Definition
The marginal utility of income could be defined as

\[ u'(y) = \frac{d}{dy} \mathbb{E}[s|y] = \sum_{s=s}^{\bar{s}} s \frac{d}{dy} f(s|y) \]

provided that the derivatives all exist.
The key **MLRP** (monotone likelihood ratio property) hypothesis is that, whenever \( s, s' \) are two satisfaction levels in \( S \) with \( s' > s \), then the conditional likelihood ratio \( p(s'|y)/p(s|y) \) given \( y \) is an increasing function of \( y \).

That is, higher SWB levels become relatively more likely as income increases.
First-Order Stochastic Dominance

The MLRP implies first-order stochastic dominance — meaning that for the every pair $s, s'$ with $s < s'$, the inequality $F(y|s) > F(y|s')$ holds for all $y \geq 0$.

This is precisely the condition that the conditional CDF $y \mapsto F(y|s')$ for the higher satisfaction value $s'$ must first-order stochastically dominate the conditional CDF $y \mapsto F(y|s)$ for the lower satisfaction value $s$.

Data from the World Values Survey supports the conjecture: first-order stochastic dominance holds empirically.

What about other data sets?
Measuring Individual Well-Being

Indirect and Money Metric Utility

Indirect Social Welfare Functional

Life Metric Utility

Subjective Well-Being
  Four Income Distributions
  Dominance
  Mean Life Satisfaction as Utility of Income
  British Household Panel Survey (BHPS) Data
Second Paper

Peter Hammond, Federica Liberini, and Eugenio Proto
“Do Happier Britons Have More Income? First-Order Stochastic Dominance Relations”
CAGE Online Working Paper 165,
Competitive Advantage in the Global Economy, University of Warwick, 2013; revised 2014.

British Household Panel Survey (BHPS)

BHPS data covering the years 1996–2008, ever since the question on life satisfaction was first introduced in 1996.

Longitudinal data — the same individuals interviewed every year, though our analysis does not exploit this feature.

Focus on three data sets:

1. the whole period 1996–2008 (94,011 observations);
2. just the wave for the year 1997 (7,602 observations);
3. just the wave for the year 2005 (8,473 observations).
Two Main Variables

The two main variables we use:

**Self-Reported Life Satisfaction (or SRLS).**
The relevant multiple choice question is

“How dissatisfied or satisfied are you with your life overall?”

Each subject’s answer is coded on a scale running from 1 (not satisfied at all) to 7 (completely satisfied).

**Household income.** Reported figures in £ are converted, using World Bank/World Development indicators, into annual household income in US dollars measured at 2005 constant prices.

Then they are adjusted for family composition.
### Percentages reporting different satisfaction levels

<table>
<thead>
<tr>
<th>Level</th>
<th>Full Sample</th>
<th>Sample of 1997</th>
<th>Sample of 2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>1.45</td>
<td>1.80</td>
<td>1.46</td>
</tr>
<tr>
<td>Level 2</td>
<td>2.39</td>
<td>2.74</td>
<td>2.63</td>
</tr>
<tr>
<td>Level 3</td>
<td>6.53</td>
<td>6.58</td>
<td>6.81</td>
</tr>
<tr>
<td>Level 4</td>
<td>14.79</td>
<td>14.86</td>
<td>15.53</td>
</tr>
<tr>
<td>Level 5</td>
<td>31.24</td>
<td>29.65</td>
<td>33.07</td>
</tr>
<tr>
<td>Level 6</td>
<td>32.83</td>
<td>32.02</td>
<td>32.16</td>
</tr>
<tr>
<td>Level 7</td>
<td>10.77</td>
<td>12.35</td>
<td>8.33</td>
</tr>
<tr>
<td>Sample Size</td>
<td>94,011</td>
<td>7,602</td>
<td>8,473</td>
</tr>
</tbody>
</table>

*Note: The samples include all respondents aged between 18 and 65 who were not full time students at the time of the interview.*
Graphical Tests of Stochastic Dominance

Graphs of all seven conditional income distributions, one for each possible level of SWB or SRLS in the range $S := \{1, \ldots, 7\}$.
All Waves Combined

Household Income conditional on Life Satisfaction

British Household Survey (All Waves)
The Wave of 1997

Household Income conditional on Life Satisfaction

British Household Survey (1997)

Cumulative Conditional Frequency

Annual Household Income

- Life Sat.=1
- Life Sat.=2
- Life Sat.=3
- Life Sat.=4
- Life Sat.=5
- Life Sat.=6
- Life Sat.=7
The Wave of 2005

Household Income conditional on Life Satisfaction

British Household Survey (2005)
Stochastic Dominance Confirmed . . .

For each pair of levels \( s, s' \in S \setminus \{7\} := \{1, \ldots, 6\} \) with \( s > s' \), the level \( s \) income distribution strictly stochastically dominates the level \( s' \) income distribution for the same data set.

That is, the respective conditional CDFs \( y \mapsto F_s(y) \) and \( y \mapsto F_{s'}(y) \) satisfy \( F_s(y) > F_{s'}(y) \) for all \( y \geq 0 \).

So, given any fixed income level \( \bar{y} \), the proportion of level \( s \) households with income \( y \leq \bar{y} \) exceeds the corresponding proportion of level \( s' \) households.
...Except at the Top

But stochastic dominance fails for “fully satisfied” individuals who report the top level 7.

In fact, the level 7 income distribution dominates only those for the two lowest levels — namely, 1 and 2.

A similar top anomaly emerges in all single waves.

Also for data for both the US and Germany.
Conclusions

On the whole, except at the highest SRLS level, the first-order stochastic dominance test is passed. Nevertheless:

1. Linear (or nonlinear) regression results of SRLS on income and control variables may tell a very misleading story.

2. So may ordered probit or logit regressions.

3. In the UK, there does seem to be a “top anomaly” whereby individuals reporting the highest SRLS level have an income distribution that is stochastically dominated by others who report lower SRLS levels.

4. There is some evidence that this may be because more education makes the highest level of SRLS less likely, perhaps due to higher aspirations.

5. There may be similar phenomena within other nations, including the US and especially Germany.