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**Principles for Evaluating
Public Sector Projects**

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2 Principles for Evaluating Public Sector Projects

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2.1 INTRODUCTION: PURPOSE OF CHAPTER

A most important function of the public sector of any mixed economy is to plan, organize, and finance projects of various kinds. Such projects may be civilian, like dams, irrigation systems, hydroelectric power plants, schools, parks, universities, hospitals, railways, roads, and communication satellites. Or they may be military. Virtually all military projects produce for the public sector for obvious reasons, although much of the production occurs in the private sector. And, of course, there is a large - arguably excessively large - international market for privately supplied armaments. Whether civilian projects should be undertaken by the public or by the private sector is a matter of great controversy. But only fanatical *laissez-faire* economists will rule out *a priori* the possibility that some projects or parts of projects should be undertaken in the public sector; others, with more open minds, will at least wish to consider the possibility of there being some desirable projects which private sector corporations and entrepreneurs have overlooked. The extent of public involvement in the execution of a project then becomes an important decision concerning the form which that project should take.

The evaluation of public sector projects is therefore an important component of economic policy-making. It is a subject to which economists have properly paid a great deal of attention, particularly since the development of *cost-benefit analysis* in the 1950s and 1960s. That began with the work of Eckstein (1958) and others on water resources, and progressed through the rival manuals of the OECD (Little and Mirrlees, 1968) and UNIDO (Dasgupta, Marglin, and Sen, 1972) in the late 1960s or early 1970s. A good sampling of early work up to about 15 years ago is Layard's (1972) selection of articles. A fairly compre-

hensive and very enlightening recent survey is provided by Dreze and Stern (1985). An associated literature is that on *shadow pricing*, concerning what prices of the inputs to and the outputs from a public sector should be used to calculate the costs and benefits of the project. More recently, as the enormous risks associated with such projects as nuclear power stations (Chernobyl) and large chemical plants (Bhopal) have unfortunately become only too apparent, a closely related methodology of *risk-benefit analysis* has been developed.

This chapter will discuss the theoretical principles which should underlie public sector project evaluation. It will not seek to provide excuses for methods such as shadow pricing and cost-benefit analysis, which have been widely used in the past, although it will, of course, consider under what restrictive circumstances such methods can be valid. Rather, it will look forward to methods which, in this writer's opinion, have much more widespread validity, and which we may hope to see more widely used in the future. If this chapter serves to make some future evaluators of public projects much more thoughtful about the methods they use, it will have succeeded admirably

2.2 PUBLIC SECTOR PROJECTS, SHADOW PRICES, AND COST-BENEFIT ANALYSES

Public sector projects affect the commodity outputs and inputs of the public sector. The vector of *net outputs* of the public sector will be written as z . Each component z_g indicates the net output of good g – the output of g produced by the public sector, minus the input of g demanded by the public sector. The economy is viewed as starting with given net output vector z^0 which is then altered to z^1 as a result of the project. The vector $\Delta z = z^1 - z^0$ measures the net output of the project itself, and will actually be referred to as the *project*.

A *project evaluation* is a real number assigning a value $v(\Delta z)$ to a project Δz . Thus v is really a real valued function on the space of possible projects, Δz . One normalizes so that $v(0) = 0$. The purpose of project evaluation, one presumes, is to identify projects which increase the well-being of individuals in the community affected by it. This leads one to require an evaluation function $v(\Delta z)$ to be positive if and only if the project Δz is favourable. One may also require $v(\cdot)$ to indicate how much the community benefits from a project, so that $v(\Delta z^A) > v(\Delta z^B)$ if and only if the project Δz^A is better than Δz^B . Further, one may want $v(\cdot)$ to indicate how valuable the project is to the community, as measured by how much the community should be willing to pay to be

allowed to undertake the project. Care is needed in the latter case, because the community's willingness to pay is likely to depend upon which of its members are doing the paying. Poor individuals who benefit little, if at all, will be unwilling to pay much; whereas the rich individuals who benefit greatly will be willing to pay a lot.

If there is a differentiable evaluation function, and if the project Δz is small, the first order approximation $v'(0) \cdot \Delta z$ to $v(\Delta z)$ may serve as an evaluation. Here $v'(0)$ denotes the gradient vector of v evaluated at $\Delta z = 0$. In this case there is a *shadow price vector* $s = v'(0)$ with the property that a small project Δz is favourable if and only if $s \cdot \Delta z > 0$. This is a particularly convenient and popular form of evaluation. Much of the theory of project evaluation is concerned with the principles which determine s , and much applied work with practical procedures for estimating s .

A particular shadow pricing rule for goods traded on international markets has received much attention, following the advocacy of Little and Mirrlees (1968, 1974). This is to value all such goods at their actual border prices. Indeed, Little and Mirrlees see this as a special case of a more general rule, of using as shadow prices the prices for private producers that would obtain in an economy with optimal commodity taxes. The theory behind this approach is developed in Diamond and Mirrlees (1971, 1976). There is no disputing that this approach gives valid answers when the government is known to be maintaining an optimal combination of commodity taxes, income taxes, and tax allowances. Since, however, most of the economics profession, let alone those who determine tax systems, have not yet mastered the subtleties of Diamond and Mirrlees' theory, we cannot and should not presume that governments set optimal commodity taxes. Moreover, what constitutes an optimal system of commodity taxes depends upon highly contentious interpersonal comparisons of marginal gains and losses. One might retort that, even so, one should fix shadow prices as if taxes were optimal, because fixed shadow prices are easier to use and otherwise one would have to change them with every change in the economy or improvement in the tax system. Again, this might be right if one were confident that ultimately we were going to reach an optimal tax system, but I would prefer an approach which does not rely on such a heroic assumption.

Thus I shall revert to evaluations $v(\Delta z)$ which indicate likely benefits here and now, rather than in an ideal world of optimal tax systems. At first this approach seems to imply that shadow prices should be closer to consumer than to producer or border prices, as in Hammond (1980). Later work (Hammond, 1986) suggests, however, following

Sen (1972), that it is important to consider policy responses made necessary by a project, and that border prices may receive a new justification as shadow prices.

Valuing net outputs at shadow prices is a particular method of *cost-benefit analysis*. More generally, the costs of a project arise from the inputs - the negative components of Δz - whereas the benefits arise from the outputs - the positive components of Δz . A perverse project in a peculiar economy might produce outputs of negative value. An example might be a project which adds to the butter mountain of the European Economic Community, as a result of which more resources become needed to store and ultimately to sell off the unwanted butter, because there seem to be impossible obstacles to simply throwing it away as quickly as possible. Neglecting such perversities, the *output vector* of the project can be defined as the vector Δz^+ whose components are $\Delta z_g^+ = \max\{z_g, 0\}$, and the *input vector* as Δz^- whose components are $\Delta z_g^- = \max\{-z_g, 0\}$. Thus Δz^+ and Δz^- both have all non-negative components, and $\Delta z = \Delta z^+ - \Delta z^-$. A measure of benefit is then $B(\Delta z^+)$ while a measure of cost is $C(\Delta z^-)$. Typically, both B and C will be increasing functions of all their arguments. The evaluation measure is then usually taken as $v(\Delta z) = B(\Delta z^+) - C(\Delta z^-)$, which is a measure of *net benefit*, in effect. In the special case of shadow pricing, B and C become linear functions $B(\Delta z^+) \equiv s \cdot \Delta z^+$ and $C(\Delta z^-) \equiv s \cdot \Delta z^-$.

There is no particular reason, however, to assume that benefits and costs can be separated from each other like this except in linear approximations. So I shall discuss only general evaluation measures $v(\Delta z)$, linear approximations $s \cdot \Delta z$ with shadow prices, and also quadratic approximations $s \cdot \Delta z + (\frac{1}{2})\Delta z \cdot H \Delta z$ where H is the symmetric Hessian matrix of second-order partial derivatives $v''(0)$ evaluated at $\Delta z = 0$. Whereas linear approximations identify only directions dz in which a small project $\Delta z = \lambda dz$ is favourable, quadratic approximations enable us to discuss also the effect of the scale λ of the project and, indeed, to broach the subject which is usually avoided, of how to evaluate projects which are not small.

2.3 PLAN OF CHAPTER

A public project has been defined as a change Δz in the net output vector z of the public sector. Such a change, except insofar as it involves public goods or aspects of the 'public environment', does not affect individuals directly. Instead its effects are indirect, through price changes induced by the need to sell the outputs and purchase the

inputs of the project, and resulting quantity changes. In an open economy there may also be effects on the balance of trade. Whether the economy is open or closed, the government's budget is also affected, and any imbalance has to be corrected ultimately by a change in fiscal policy.

Such 'general equilibrium' effects of a project will be much more fully considered in the latter half of the chapter. The first half, from sections 2.4 to 2.6, takes as given the effects of the project on consumers' prices and quantities, and discusses how to measure the effect on social welfare. Thus it considers the evaluation of price and quantity changes in general, and defers till the second part the calculation of what changes in policy are induced by a public project.

Section 2.4 begins with an ideal money metric measure for a single consumer, similar to Hicks' equivalent variation. It also points out the importance of the value judgement that consumers' tastes are 'sovereign'. First-order linear approximations are considered. In a closed economy with just one consumer, so that the project Δz is equal to the change in that consumer's net demand vector, the consumer price vector serves as a vector of shadow prices. Section 2.5 then presents some second-order approximations developed recently by the author.

With many consumers the ethical value judgement of consumer sovereignty has to be supplemented with judgements regarding what weights to attach to different individuals' gains and losses. Section 2.6 discusses 'welfare weighted' measures of net gain. These measures are inadequate when a project is large enough to change the relevant welfare weights, and the appendix to the chapter develops accurate second-order approximations to a 'social equivalent variation' measure of welfare change.

After the lengthy but necessary prior discussion of measures of welfare change, the second part of the paper is relatively brief. Section 2.7 presents a fairly general description of a competitive market economy, and uses it to discuss the comparative static effects of a project. The need for a *balancing policy* is shown, because a project is likely to upset the government's budget. Then section 2.8 introduces first-order approximations to the comparative static effect of a project, which can be used to calculate linear approximations to evaluations, incorporating shadow prices, and also a kind of second-order approximation. Section 2.9 concludes the second part with a different justification from usual for using the border prices of traded goods as shadow prices. This is based on the presumption that any balancing policy need depend upon net outputs of traded goods from the project only

through their value at border prices. Of course, other balancing policies are possible, in which case this rule would no longer be valid.

The last section of the chapter contains brief discussions of important topics otherwise omitted. Thus it treats public goods, rationing, and merit goods; just mentions non-convexities and indivisibilities; and also comments on time and uncertainty.

2.4 A SINGLE CONSUMER

In this section the simplest case will be considered first, with only one consumer, whose money measure of net benefit is given by:

$$\Delta u = E(q^0, x^1) - m^0 = E(q^0, x^1) - E(q^0, x^0) \quad (1)$$

Here u is a 'money metric' measure of utility (Samuelson, 1974), q is the price vector faced by the consumer, x denotes the net consumption vector (in which goods supplied have negative components), m is the (net) *unearned* income available to the consumer after taxes, and the superscripts 0 and 1 denote values before and after the reform (in this case, project), respectively. Earned income is excluded from m because all kinds of labour supply are treated as commodities. Thus m includes net government transfers, dividends, etc. The price vector q includes price gross of any tax paid for commodities purchased by the consumer, and net of any tax paid for commodities - notably labour - sold by the consumer. Finally, E denotes the *expenditure* (or cost) function of the consumer, according to which $E(q, u)$ is the level of unearned income that the consumer needs in order to achieve a utility level u at prices q . Formally, it is defined by:

$$E(q, \bar{x}) = \min_x \{qx \mid x R \bar{x}\} \quad (2)$$

where R denotes the consumer's weak preference relation. Thus (1) defines Δu as the extra amount of income (net) which the consumer would need to achieve a situation no worse than the actual final consumption vector x^1 if prices remained fixed at q^0 and the consumer started with the original level of net unearned income m^0 . The measure of net benefit (1) depends crucially on the original price vector q^0 , as well as on m^0 , in general. Like the closely related measure of 'equivalent variation' (Hicks, 1939, 1940, 1981), it has the merit that the measure really does increase as x^1 becomes better for the consumer, unlike a measure such as $m^1 - E(q^1, x^0)$ based on the 'compensating variation' (see Chipman and Moore, 1980).

One should realize right away that, in assuming $m^0 = E(q^0, x^0)$, the measure (1) already embraces an ethical value judgement. It is presumed that the consumer's behaviour reveals a preference ordering or utility corresponding to the consumer's welfare. Here utility – if an ordinal measure exists which is maximized by the consumer's behaviour – is a 'positive' concept, whereas welfare is a 'normative' or ethical concept. The judgement that the consumer maximizes what should be maximized is an ethical value judgement generally referred to as 'consumer sovereignty'. Sometimes, as when people take addictive drugs, make decisions based on misleading or poor information, or are unduly influenced by advertising or peer pressure, it is ethically legitimate to deny consumer sovereignty and to argue, paternalistically, that the consumer in question would benefit by behaving differently. Here, however, I accept consumer sovereignty provisionally – not because I think it is always right, but because there are many other ways economists have found to make fundamental mistakes in project evaluation. Where deviations from consumer sovereignty are important they will modify the measures of benefit to be considered in an obvious way, as discussed in section 2.10, although the precise effect will usually be impossible to quantify in an agreed manner.

In order to determine Δu – even its sign, to see whether the change is favourable or not – in general one has to know the precise form of the function E . Much econometric work goes into uncovering E , as discussed by Blundell in this volume, for instance (see chapter 8). The subject of this chapter, however, is project evaluation, and most practical project evaluators use approximations to (1) in an effort to circumvent econometric complications.

With consumer sovereignty, $m^1 = E(q^1, x^1)$, and so (1) becomes:

$$\begin{aligned}\Delta u &= E(q^0, x^1) - m^1 = E(q^0, x^1) - E(q^1, x^1) + m^1 - m^0 \\ &= [E(q^0, x^1) - E(q^1, x^1)] + \Delta m\end{aligned}\quad (3)$$

in which the term in square brackets is exactly Hicks' measure of equivalent variation for the change in price alone. The approximations to be considered come from using a Taylor series approximation to the function $E(q, x^1)$ of q , in the neighbourhood of $q = q^1$.

From the definition (2) of the expenditure function, one has:

$$E(q, x^1) \leq qx^1 \quad (4)$$

for all price vectors q , because of course $x^1 R x^1$ and generally there is an alternative x with $x R x^1$ for which $qx \leq qx^1$. Thus $E(q, x^1) - qx^1 \leq 0$ for all q , and there is equality when $q = q^1$ because then $E(q^1, x^1) = q^1 x^1$,

and m^1 are all equal. It follows that the point $q = q^1$ is a global maximum of the function.

$$F(q) = E(q, x^1) - qx^1 \quad (5)$$

because $F(q^1) = 0$ but $F(q) \leq 0$ everywhere else. Assuming that E is differentiable with respect to q at $q = q^1$, it follows that F is differentiable with respect to q at q^1 . The usual first-order condition for a maximum must be satisfied, which is that the gradient vector of F must be zero. But then, at $q = q^1$:

$$\text{grad}_q F = \text{grad}_q E - x^1 = 0 \quad (6)$$

So the gradient of E with respect to q at q^1 is equal to x^1 , the consumer's net demand vector. This gives immediately the first-order approximation:

$$E(q^0, x^1) - E(q^1, x^1) = (q^0 - q^1) \cdot x^1 = -\Delta q \cdot x^1 \quad (7)$$

which, substituted into (3), gives:

$$\Delta u(1) = \Delta m - \Delta q \cdot x^1 \quad (8)$$

as a first-order approximation to Δu . Imposing the budget constraints $m^1 = q^1 \cdot x^1$, $m^0 = q^0 \cdot x^0$ leads to:

$$\begin{aligned} \Delta u(1) &= q^1 \cdot x^1 - q^0 \cdot x^0 - (q^1 - q^0) \cdot x^1 \\ &= q^0 \cdot (x^1 - x^0) = q^0 \cdot \Delta x \end{aligned} \quad (9)$$

as an alternative first-order approximation. Equation (9) is hardly surprising, because the first-order condition for utility maximization subject to a budget constraint is that $\text{grad}_x u = \lambda q$ where λ is the marginal utility of income; thus $\Delta u = \lambda q^0 \cdot \Delta x$ and in (9) $\lambda = 1$ because the money metric utility function is being used.

Equation (9) gives us our first rule for project evaluation, even though it relies on uninteresting assumptions. As a first-order approximation, if $q \cdot \Delta x > 0$ then we know that for all small enough positive scalars α , a project which produces a change $\alpha \Delta x$ in the single consumer's net demand vector is beneficial, whereas if $q^0 \cdot \Delta x < 0$, then such a project is unfavourable. This rule is silent when $q^0 \cdot \Delta x = 0$; then the sign of Δu , even for small projects, depends on second-order effects.

2.5 SECOND-ORDER APPROXIMATIONS FOR THE CONSUMER

First-order approximations like (8) and (9) are the basis of most cost-benefit analyses, as will be seen in later sections. They only give

an indication of what directions of change are favourable, however, and are only suitable for evaluating small projects which, for example, have no significant effect on the price vector q^0 in (9). Yet most projects are not small in this sense. Even deepening a village well, though a small project for a nation, is not always a small project for our purpose. One of the prices included in the vector q^0 is that of well-water. If the project succeeds in making well-water more abundant, it is almost certain to lower the price of well-water significantly. A linear measure like $q^0 \cdot \Delta x$ in (9) may seriously overstate the benefits of deepening the well, by using the original high price of water to value all the extra supply even though water becomes less valuable at the margin as more becomes available.

To take account of price changes induced by the project one needs to go beyond first-order approximations. This is easily done in principle, using a second-order approximation to the difference in expenditures in (7) above. Thus:

$$E(q^0, x^1) - E(q^1, x^1) = -\Delta q \cdot x^1 + (\frac{1}{2})\Delta q \cdot H\Delta q \quad (10)$$

where H denotes the Hessian matrix of second-order partial derivatives $\partial^2 E / \partial q_i \partial q_j$ evaluated at (q^1, x^1) . But (6) says that $\partial E / \partial q_i = x_i$ for each good i , and the Hessian matrix H is therefore equal to the Jacobian matrix $x_q^1 = x_q(q^1, x^1)$ of the vector demand function which expresses net demands for each commodity as a function of the price vector q , evaluated at q^1 , holding fixed the real income level u^1 . This is the Hicksian compensated demand function of the consumer, in vector form. So one has, to first order:

$$\Delta x = x_q^1 \cdot \Delta q + x_u^1 \Delta u \quad (11)$$

where x_u^1 denotes the response of the net demand vector to the change in real income, and Δu is the change of real income. Recognizing that $H = x_q^1$ and substituting in (10) gives:

$$E(q^0, x^1) - E(q^1, x^1) = -\Delta q \cdot x^1 + (\frac{1}{2})\Delta q \cdot [\Delta x - x_u^1 \Delta u] \quad (12)$$

which is a valid *second-order* approximation, because the first-order approximation (11) is multiplied by Δq . Moreover, (12) remains valid to second order if the first-order approximation $\Delta u(1)$ to Δu is used. So, using (8) to substitute for Δu in (12), and then substituting the result in (3), leads to the second-order approximation:

$$\begin{aligned} \Delta u(2) &= \Delta u(1) + (\frac{1}{2})\Delta q \cdot [\Delta x - x_u^1 \Delta u(1)] \\ &= \Delta m - \Delta q \cdot x^1 + (\frac{1}{2})\Delta q \cdot [\Delta x - x_u^1(\Delta m - \Delta q \cdot x^1)] \end{aligned} \quad (13)$$

It has been all too common for applied economists to neglect the term

in x_u^1 above and to use an approximation which is more accurate than $\Delta u(1)$ but less accurate than $\Delta u(2)$, namely:

$$\begin{aligned}\Delta u(1\frac{1}{2}) &= \Delta u(1) + (\frac{1}{2})\Delta q \cdot \Delta x = \Delta m - \Delta q \cdot x^1 + (\frac{1}{2})\Delta q \cdot \Delta x \\ &= \Delta m - (\frac{1}{2})\Delta q \cdot (x^0 + x^1)\end{aligned}\quad (14)$$

The term $-(\frac{1}{2})\Delta q \cdot (x^0 + x^1)$ represents *Marshallian consumer surplus*, though its origins date back to Dupuit if not earlier. Imposing the budget constraints $m^0 = p^0 \cdot x^0$ and $m^1 = q^1 \cdot x^1$, (14) becomes:

$$\Delta u(1\frac{1}{2}) = (\frac{1}{2})(q^0 + q^1) \cdot \Delta x \quad (15)$$

which suggests simply replacing the initial price vector q^0 in (9) by the average of the initial and final price vectors. However, (14) is applied in *partial equilibrium* analyses, where price and quantity changes are considered only for a small range of goods directly affected by the project. Often, indeed, only one good is considered, in which case one can consider a diagram such as figure 2.1. The approximation $\Delta u(1)$ of (8) corresponds to the sum of Δm and the area ABHJ of figure 2.1 – so ABHJ is a rectangular approximation to the consumer's 'surplus' from the price fall and quantity gain. The approximation $\Delta u(1\frac{1}{2})$ uses the Marshallian demand curve DD in the diagram, when income is held fixed, and corresponds to the sum of Δm and the area ABHG as an approximate measure of surplus – the curve GH is approximated by the straight line between the same two points. The true second-order approximation $\Delta u(2)$, however, considers the *compensated* demand curve CC, where it is *real* rather than nominal income which is held fixed, and $\Delta u(2)$ is the sum of Δm with the *variation* measure ABKG – again, the curve KG is approximated by the line segment joining the same two points.

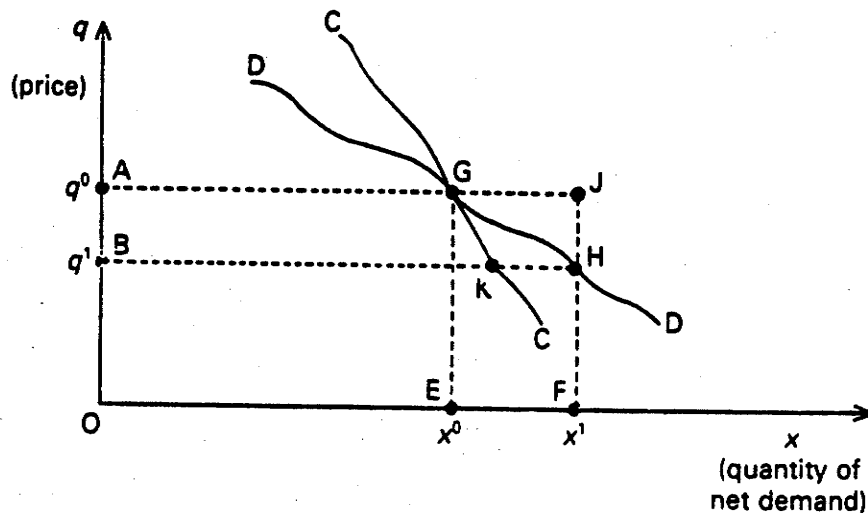


Figure 2.1 Approximations to consumer surplus

Generally, the uncompensated demand change Δx in (14) is corrected in (13) by the subtraction of $x_u^1 \Delta u(1)$, which is the effect on demand of the change in real income, to arrive at the compensated demand change in (13). The correction requires knowledge of the vector x_u^1 . In fact, because the approximation is only meant to be valid to second order, one can replace x_u^1 by x_u^0 - in other words, evaluate derivatives at (q^0, m^0) in effect - without introducing any error except of third or higher order. Also, because u is being measured by a money metric utility function, x_u^0 is really just the vector x_m^0 of demand responses $\partial x_g / \partial m$ for each good g to income changes, evaluated at (q^0, m^0) . This is an extra data requirement, but hardly an onerous one, since income responses have been extensively studied in the past. Thus, (13) becomes:

$$\begin{aligned}
 \Delta u(2) &= \Delta u(1) + \left(\frac{1}{2}\right) \Delta q \cdot [\Delta x - x_m^0 \Delta u(1)] \\
 &= \Delta m - \Delta q \cdot x^1 + \left(\frac{1}{2}\right) \Delta q \cdot [\Delta x - x_m^0 (\Delta m - \Delta q \cdot x^1)] \\
 &= q^0 \cdot \Delta x + \left(\frac{1}{2}\right) \Delta q \cdot \Delta x - \left(\frac{1}{2}\right) (\Delta q \cdot x_m^0) (q^0 \cdot \Delta x) \\
 &\quad (\text{imposing the budget constraints } m^0 = q^0 \cdot x^0, m^1 = q^1 \cdot x^1) \\
 &= \left(\frac{1}{2}\right) [(1 - \Delta q \cdot x_m^0) q^0 + q^1] \cdot \Delta x \\
 &= \Delta u(1 \frac{1}{2}) - \left(\frac{1}{2}\right) (\Delta q \cdot x_m^0) (q^0 \cdot \Delta x), \text{ by (15)} \tag{16}
 \end{aligned}$$

Thus the condition for (15) to be valid is that $\Delta q \cdot x_m^0 = 0$ - i.e. the income effect correction is indeed zero. Because $q^0 \cdot x^0(q, m) = m$ for all m , one has $q^0 \cdot x_m^0 = 1$ and so:

$$\begin{aligned}
 \Delta u(2) &= q^0 \cdot \Delta x + \left(\frac{1}{2}\right) [q^1 - (q^1 \cdot x^0) q_m^0] \cdot \Delta x \\
 &= q^0 \cdot \Delta x + \left(\frac{1}{2}\right) q^1 \cdot [\Delta x - x_m^0 (q^0 \cdot \Delta x)] \tag{17}
 \end{aligned}$$

Apart from some increased accuracy, the second-order approximation $\Delta u(2)$ also tends to limit the scale of an acceptable project. For (16) can also be written as:

$$\begin{aligned}
 \Delta u(2) &= [1 - \left(\frac{1}{2}\right) (\Delta q \cdot x_m^0)] q^0 \cdot \Delta x + \left(\frac{1}{2}\right) \Delta q \cdot \Delta x \\
 &= q^0 \cdot \Delta x + \left(\frac{1}{2}\right) \Delta q \cdot [I - (x_m^0) (q^0)^T] \Delta x \tag{18}
 \end{aligned}$$

where $(x_m^0) (q^0)^T$ denotes the matrix product of the column vector x_m^0 and the row vector $(q^0)^T$. So, to second order, one has:

$$\Delta u(2) = q^0 \cdot \Delta x + \left(\frac{1}{2}\right) \Delta q \cdot S^0 \Delta q$$

where S^0 denotes the Slutsky matrix of compensated demand responses to price changes. Because the Slutsky matrix is negative semi-definite, the quadratic term is non-positive and typically negative.

Thus, if $\Delta x = \alpha \Delta x$, $\Delta q = \alpha \Delta q$, then $\Delta u(2)$ is a quadratic function of α in which the coefficient of α^2 is non-positive and typically negative. So, if $q^0 \cdot \Delta x > 0$, $\Delta u(2)$ will be positive for small α but will achieve a maximum for some finite α - except in the freak case when the coefficient of α^2 is zero because $S^0 \Delta q = 0$. This generally requires Δq to be proportional to q^0 .

2.6 WELFARE WEIGHTED MEASURES FOR MANY CONSUMERS

The measures of welfare change derived in sections 2.3 and 2.4 were for a single consumer. Obviously, however, practicality requires measures which apply when there are many consumers, indexed by the subscript i , for $i \in I$ - a finite set. Each consumer has a net demand vector x_i , but a market system for allocating goods faces each consumer with a common price vector q . In this case each individual i experiences a net gain of welfare, based on i 's own expenditure function E_i , given by:

$$\Delta u_i = E_i(q^0, x_i^1) - E_i(q^0, x_i^0) \quad (19)$$

As in (1), and the first-order approximation is:

$$\Delta u_i(1) = q^0 \cdot \Delta x_i \quad (20)$$

as in (9), and the second-order approximation is

$$\Delta u_i(2) = q^0 \cdot \Delta x_i + \left(\frac{1}{2}\right)[q^1 - (q^1 \cdot x_{mi}^0)q^0] \cdot \Delta x_i \quad (21)$$

as in (17), where x_{mi}^0 is i 's vector of demand responses to income changes, evaluated at (q^0, m_i^0) . Such disaggregated measures of individual gain and loss are about as far as the value judgement of consumer sovereignty can take us on its own. Of course, if a project produces changes such that $\Delta u_i > 0$ for all $i \in I$, that indicates a Pareto improvement and a project which benefits all. The reverse is true if $\Delta u_i < 0$ for all $i \in I$. And if the approximations $\Delta u_i(1)$ or $\Delta u_i(2)$ are all positive, that also indicates that the project is likely to be a Pareto improvement if it is on a small enough scale to make the approximations valid at least as regards their sign. Generally, however, projects benefit some consumers but harm others, in the sense that some Δu_i values are positive but others are negative. Then there is no way of deciding whether or not the project is favourable without making 'interpersonal comparisons' of different individuals' gains and losses.

A simple procedure which is often recommended in this case is to consider the weighted sum $\sum_{i \in I} \beta_i \Delta u_i$ of different individuals' gains

and losses, with corresponding approximations $\sum_{i \in I} \beta_i \Delta u_i(1)$ and $\sum_{i \in I} \beta_i \Delta u_i(2)$. The numbers β_i ($i \in I$) are *welfare weights* reflecting the value to society of the monetary gains and losses of each individual. They are presumed to be positive. They reflect interpersonal comparisons, of course, and so reflect ethical value judgements of a kind economists are much more reluctant to make than they are to postulate consumer sovereignty. Without such interpersonal comparisons, however, there is no procedure for deciding whether projects are favourable or not, except for the very special and unlikely cases of Pareto improvements or their reverse – ‘Pareto deteriorations’. Just as ‘it’s an ill wind that blows nobody any good’, it’s a very good project indeed that causes nobody any harm. Especially given that appropriate lump-sum redistribution is often infeasible because information is lacking, as discussed in Hammond (1979, 1985, 1987).

Some economists have advocated using unweighted sums of the form $\sum_{i \in I} \Delta u_i$. The corresponding first-order approximation is:

$$\sum_{i \in I} \Delta u_i(1) = q^0 \cdot \sum_{i \in I} \Delta x_i = q^0 \cdot \Delta x \quad (22)$$

where now Δx is the change in the aggregate net demand vector $\sum_{i \in I} x_i$. This can in principle be calculated using just aggregate price and quantity data. So can unweighted total surplus, which from (15) is:

$$\sum_{i \in I} \Delta u_i(1\frac{1}{2}) = (\frac{1}{2})(q^0 + q^1) \cdot \sum_{i \in I} \Delta x_i = (\frac{1}{2})(q^0 + q^1) \cdot \Delta x \quad (23)$$

However, the true second-order approximation, with unweighted sums, is:

$$\sum_{i \in I} \Delta u_i(2) = q^0 \cdot \Delta x + (\frac{1}{2}) q^1 \cdot \Delta x - (\frac{1}{2}) \sum_{i \in I} (q^1 \cdot x_{mi}^0) (q^0 \cdot \Delta x_i) \quad (24)$$

from (17), and this requires knowledge of disaggregated data in order to calculate the correlation between the income responses x_{mi}^0 and the quantity changes Δx_i induced by the project.

Because (22) can be calculated using aggregate data of a form economists are accustomed to having before them, Harberger (1971) strongly advocated (22), or (23) as a more exact approximation. Posner (1981) produced somewhat more sophisticated defences. But all these approximations to the unweighted sum $\sum_{i \in I} \Delta u_i$ rely for their significance upon the ethical value judgements that β_i is independent of i . This is the judgement that all people’s monetary gains and losses are equally valuable, even if the rich gain at the expense of the poor or the disadvantaged. It does not take very strongly egalitarian values – or even very strong sympathy for the poorest people affected by the project under consideration – to make such a value judgement ethically unacceptable. Indeed, were $\sum_{i \in I} \Delta u_i$ the evaluation of any project, then

no programme of poverty relief would ever pass the test unless the monetary losses of taxpayers were more than matched, pound for pound, by the monetary gains of the worthy recipients. A severe test indeed, given the usual cost of administering tax collection and welfare programmes of poverty relief.

Thus, while economists are unlikely to agree what precise welfare weights β_i are ethically appropriate, one can hope that they may at least become sufficiently enlightened to realize that taking unweighted sums (with $\beta_i = 1$, all $i \in I$ implicitly) is unacceptable.

Unfortunately, the use of such a weighted welfare sum is not correct when a project is large enough to change the welfare weights themselves. Measures that are appropriate to that situation are developed in the appendix to this chapter.

2.7 COMPARATIVE STATICS EFFECTS AND BALANCING POLICIES

Up to now the effects of the project on quantities x_i , consumer prices q , and unearned incomes m_i have been treated as given, while attention has been focused on welfare measurement. But finding Δx_i , Δq , Δm_i , etc. is no less important a part of project evaluation, since otherwise the formulae of the previous sections are useless. To fill the gap, one typically needs some kind of comparative static analysis of a model describing the economy as accurately as possible. Here I shall present such a model which is essentially of an economy in static competitive equilibrium, except that the government levies taxes and undertakes production.

Indeed, suppose that private producers face prices given by the vector p , which determine their aggregate net output vector $y = y(p)$. Suppose that the tax system implies that the consumer price vector q is a well-defined function $q(p)$ of this producer price vector. For example, if there is a specific tax at rate $100 \theta_g\%$ on commodity g , then $q_g(p) = (1 + \theta_g) p_g$. Write z for the vector of net outputs of the public sector, including the project(s) being evaluated, whose primary effect is to change z . As in section 2.2, a public project can be regarded as a change Δz whose positive components are outputs and whose negative components are inputs. If s is a suitable shadow price vector, one has $s \cdot \Delta z \geq 0$ according as the project $\alpha \Delta z$ is favourable or unfavourable, for all small $\alpha > 0$.

Bearing in mind that consumers' net demands x_i ($i \in I$) depend upon consumer prices q and (net) unearned incomes m_i ($i \in I$), one might be tempted to write down the set of market equilibrium equations:

$$\sum_i x_i(q(p), m_i) = y(p) + z \quad (25)$$

There are as many equations here as unknown components of p , so one might hope to solve (25) for p as a function of z , with m fixed, and so derive Δq , Δx_i to be used in measures of welfare change for evaluating the project Δz . There is a fundamental problem, however, which prevents (25) having a solution in general. For, if (25) is satisfied, one must have:

$$q(p) \cdot \sum_i x_i(q(p), m_i) = q(p)[y(p) + z]$$

and yet $q(p) \cdot x_i(q(p), m_i) = m_i$ for each $i \in I$, because consumers exhaust their budgets. So (25) can only be solved for p as a function $p(z)$ of z if:

$$\sum_i m_i = q(p(z))[y(p(z)) + z] \quad (26)$$

In particular, the right-hand side of (26) has to be a constant, independent of z , which is very unlikely. Indeed, (26) represents the aggregate budget equation of the economy; if the left-hand side exceeds the right, for instance, then consumers have more to spend in total than the value of the net supplies of the economy. To maintain (26) the government has to balance its budget, and also firms must pay out their (net) profits $py(p)$ as dividends. Notice that the government's net budget surplus is given by:

$$-[\sum_i m_i - py(p)] + qz + (q - p)y(p) \quad (27)$$

For $\sum_i m_i - py(p)$ is consumers' net income from government transfers (unearned income less dividends), qz is the government's earnings from selling public output to consumers, and $(q - p)y(p)$ is tax revenue from the net output of private producers. Of course, (26) says that (27) should be zero.

Just as consumers have to respond to increases in the prices of the goods they buy if they are not to exceed their budget allowances, so the government has to respond to exogenous changes in order to maintain budget balance. Now z must be treated as exogenous in project evaluation, precisely because the aim is to evaluate specified changes in z . So the government needs a *balancing policy* which maintains (26) even as z changes. Since the government can determine m and $q(p)$, they must become functions of z . Indeed, m should also be a function of p to ensure that firm's profits are distributed. Thus (40) should be written in the form:

$$\sum_i x_i(q(p, z), m_i(p, z)) = y(p) + z \quad (28)$$

where

$$\sum_i m_i(p, z) = q(p, z)[y(p) + z] \quad (29)$$

Because of (29), there are only $(n - 1)$ independent equations in (28). Assuming that $y(p)$ and all the $x_i(q, m_i)$ are homogeneous of degree zero, and also that q and m_i are homogeneous of degree one in p , there are $(n - 1)$ independent prices. Then, choosing any one good as *numeraire*, (28) has a unique solution $p(z)$ in some neighbourhood of the initial allocation in the economy (provided that the functions x_i , q , m_i , and y are differentiable and the conditions for the implicit function theorem are satisfied).

2.8 SHADOW PRICES AND OTHER APPROXIMATIONS

The ideal, of course, is to solve (28) exactly for p before and after the change in z due to the project, and deduce q and each m_i from the government's tax policy, followed by each x_i from the demand equation. That gives exact measures for Δq and Δx_i , etc. But one may have to resort to first-order approximations, on which shadow pricing formulae are based. Indeed, the differential form of (28) is:

$$\sum_i [x_{qi}^0(q_p^0 dp + q_z^0 dz) + x_{mi}^0(m_{pi}^0 dp + m_{zi}^0 dz)] = y_p^0 dp + dz \quad (30)$$

Here x_{qi}^0 , q_p^0 , q_z^0 , y_p^0 denote the obvious Jacobian matrices of partial derivatives evaluated at the original equilibrium, x_{mi}^0 the vector of income responses, and (m_{pi}^0, m_{zi}^0) the gradient vector of m_i evaluated at (p^0, z^0) also. There is one redundant equation in (30), and one can keep $p_{n^*} = 1$ for a *numeraire* good n^* while solving for the other prices. With slight abuse of notation, after dropping the *numeraire* good and its price, (30) has a solution in differential form:

$$dp = [\sum_i (x_{qi}^0 q_p^0 + x_{mi}^0 m_{pi}^0) - y_p^0]^{-1} [I - \sum_i (x_{qi}^0 q_z^0 + x_{mi}^0 m_{zi}^0)] dz \quad (31)$$

provided that the $(n - 1) \times (n - 1)$ inverse matrix exists. Then (31) can be substituted into the differential expressions for dq and dm_i :

$$dq = q_p^0 dp + q_z^0 dz = \bar{q}_z^0 dz \quad (32)$$

$$dm_i = m_{pi}^0 dp + m_{zi}^0 dz = \bar{m}_{zi}^0 dz \quad (33)$$

where the bars represent the *total* effect of the project. Also:

$$dx_i = x_{qi}^0 dq + x_{mi}^0 dm_i = \bar{x}_{zi}^0 dz \quad (34)$$

This leads directly to the following differential measure of each individual's net gain in money metric utility:

$$du_i = q^0 \cdot dx_i + dm_i - dq \cdot x_i = u_{zi}^0 \cdot dz = s_i \cdot dz \quad (35)$$

where, from (32), (33) and (34):

$$x_i = u_{zi}^0 = q^0 x_{zi} = m_{zi}^0 - q_z^0 \cdot x_i \quad (36)$$

The vector s_i is i 's personal 'shadow price vector'. The first-order approximation to i 's net benefit, based upon (35), is therefore:

$$\Delta u_i(1) = s_i \cdot \Delta z \quad (37)$$

For society as a whole, (A.2) and (37) together give:

$$\Delta W(1) = \sum_{i \in I} \beta_i^0 \Delta u_i(1) = \sum_{i \in I} \beta_i^0 (s_i \cdot \Delta z) = s \cdot \Delta z \quad (38)$$

where $s = \sum_{i \in I} \beta_i^0 s_i$ is the social shadow price vector, a welfare weighted sum of the individual shadow price vectors. Generally s is sensitive to the welfare weights $(\beta_i^0)_{i \in I}$, except in the very special case when all the vectors $(s_i)_{i \in I}$ are proportional.

Notice that each component of each shadow price vector $(s_i)_{i \in I}$ and s may be completely unrelated to the corresponding component of either the consumer price vector q or the producer price vector p ; only in special cases will it be equal to either. It could even happen that any or all of these shadow price vectors s_i could have negative components if more production would simply add to the surplus of an unwanted commodity which then had to be disposed of at large cost.

Equations (37) and (38) give first-order approximations. Rather more accurate approximations based upon (36) together with (15) and (A.4) are the following measures of consumer and of social 'surplus':

$$\begin{aligned} \Delta u_i(1\frac{1}{2}) &= s_i \cdot \Delta z + (\frac{1}{2}) \Delta q \cdot \Delta x_i \\ &= s_i \cdot \Delta z + (\frac{1}{2}) \Delta z \cdot (q_z^0 x_{zi}^0) \Delta z \end{aligned} \quad (39)$$

and

$$\begin{aligned} \Delta W(1\frac{1}{2}) &= \sum_i [\beta_{0i} \Delta u_i(1\frac{1}{2}) + (\frac{1}{2}) \Delta \beta_i (q^1 \cdot \Delta x_i)] \\ &= [\sum_i \beta_i^0 \Delta u_i(1\frac{1}{2}) + (\frac{1}{2}) \Delta \beta_i (q^0 \cdot \Delta x_i)] \text{ (to second order)} \\ &= \sum_i [(\frac{1}{2}) (\beta_i^0 + \beta_i^1) (s_i \cdot \Delta z) + (\frac{1}{2}) \beta_{0i} \Delta z \cdot (q_z^0 x_{zi}^0) \Delta z] \end{aligned} \quad (40)$$

because (35) implies that $q^0 \cdot \Delta x_i = s_i \cdot \Delta z$ for all $i \in I$. An even better approximation involves correcting for the income effects $(x_{Mi}^0)_{i \in I}$ as in (A.5) or, in the additively separable case, (A.15). Indeed, substituting (40) into (A.15) gives:

$$\begin{aligned} \Delta W(2) &= \sum_i [(\frac{1}{2}) \{ \beta_i^0 [1 - (\beta_i^1 q^1 - \beta_i^0 q^0) \cdot x_{Mi}^0] + \beta_i^1 \} (s_i \cdot \Delta z) \\ &\quad + (\frac{1}{2}) \beta_i^0 \Delta z \cdot (q_z^0 x_{zi}^0) \Delta z] \end{aligned} \quad (41)$$

This is still not a true second-order approximation, of course, because only first-order approximations to the equilibrium equations (28) and (29) are incorporated. Nevertheless, considerable allowance is made for second-order effects.

2.9 BORDER PRICES AS SHADOW PRICES

Diamond and Mirrlees (1971), followed by Hahn (1973), demonstrated how optimal commodity taxation would entail weakly efficient production. Roughly speaking, the argument is as follows. If production were not weakly efficient one could find a new aggregate net output which is strictly greater in every component, by definition of weak efficiency. Then one could also adjust commodity taxes, or a uniform lump-sum allowance to all individuals, in a way which guarantees feasibility – if necessary after disposing of some extra production – while also making all consumers strictly better off. So weak efficiency is a necessary implication of optimal taxation. This result lies behind Little and Mirrlees' (1968, 1974) advocacy of producer prices as shadow prices. For weak efficiency of production implies being at point z^0 on the production frontier. If the production frontier is smooth at z^0 it has a unique normal whose direction is indicated by the producer price vector p^0 at which profits are being maximized. And, with $s^0 = p^0$, the condition $s^0 \cdot \Delta z > 0$ is sufficient, when Δz is small, for z^1 to be outside the original production frontier, and also better than z^0 . Conversely when $s^0 \cdot \Delta z < 0$.

In the special case of goods for which the country is small in world markets, foreign trade in such goods can be regarded as the activity of a firm facing fixed border prices w . These border prices are the producer prices of this firm. So the equality between shadow prices and producer prices translates, for such traded goods, into equality between shadow and border prices. Diamond and Mirrlees (1976) present a rather more careful argument.

The above reasoning presumed, in effect, completely optimal tax systems both before and after the project, such as we are all too unlikely to see in actual economies. In the absence of optimal taxes after the project, Dasgupta and Stiglitz (1974) and Blitzer, Dasgupta, and Stiglitz (1981), have shown how the government's policy responses could make the appropriate shadow price vector differ from the border price vector. Recently, Bruce and Harris (1982), together with Diewert (1983) have shown how, in the absence of optimal commodity taxes, shadow prices could differ from producer prices. Nevertheless, Diewert (1983), followed by Hammond (1986), finds an interesting

case when border prices are appropriate shadow prices even in the absence of optimal commodity taxes. This case is greatly generalized in the rest of this section.

Suppose that the model of section 2.7 describes demands and supplies for domestic, 'non-traded', commodities. Suppose too that traded goods have fixed border prices w . Write e_i for the net demand vector of individual i for traded goods, f for the private producers' aggregate net supply vector, and g for public sector net outputs. Let b denote the net balance of trade.

Suppose that the balancing policy $q(p, z), (m_i(p, z))_{i \in I}$ remains constant. For traded goods, w remains constant, by hypothesis; suppose too that tariff inclusive producer prices r and tax inclusive consumer prices v remain unchanged. Then equation (28) for equilibrium in markets for domestic goods remains unchanged, and the budget balance equation (29) is replaced by:

$$b = w.[f(p) + g - \sum_i e_i(q(p, z), m_i(p, z))] \quad (42)$$

Notice too that the government's net budget surplus, from paying total transfers equal to total unearned income $\sum_i m_i$ less aggregate private profits $py + rf$, from its own profits $pz + rg$ at producer prices, from commodity taxes on domestic and traded goods, and from tariffs, is given by:

$$\begin{aligned} & -(\sum_i m_i - py - rf) + (pz + rg) + (q - p)(y + z) + (v - r)\sum_i e_i \\ & \quad + (r - w)(\sum_i e_i - f - g) \\ & = -\sum_i m_i + q(y + z) + v\sum_i e_i - w(\sum_i e_i - f - g) = b \end{aligned}$$

where the last equality follows from (42), (28) and the individual budget equations $m_i = qx_i + ve_i$.

Suppose that the public project leaves the net output vector z for domestic goods unaffected, but changes only the net output vector g for traded goods. Suppose that $p, q, m_i (i \in I)$ also remain unchanged. Then equation (28) remains valid, whereas (42) yields $\Delta b = w \cdot \Delta g$. Thus a project which only affects net outputs of traded goods has no effect on domestic prices, or on net demands for domestic or traded goods. Only the balance of trade is changed, and only foreigners' net demands are affected.

A project which uses only traded goods to produce other traded goods may be rare. But more generally, for a project $(\Delta z, \Delta g)$ which affects both domestic and traded goods, domestic prices and all net demands depend only upon Δz , while Δg affects only Δb . Thus, to convert the benefits to the balance of trade of the exports produced by a project into benefits for the individuals in the economy, some policy

response to the project is required. Indeed, an economy in the long run will typically be forced to satisfy $b \geq 0$ and will want to maintain b equal to zero. Thus the balancing policy takes the form $q(p, z, g)$, $m_i(p, z, g)$, $r(p, z, q)$, $v(p, z, g)$, with all taxes and tariffs – including taxes and tariffs on traded goods – responding to changes in g induced by the project. The new equilibrium equations replacing (28) and (29) become:

$$\sum_i x_i \{q(p, z, q), v(p, z, g), m_i(p, z, g)\} = y\{p, r(p, z, g)\} + z \quad (43)$$

$$0 \equiv w \cdot [f\{p, r(p, z, g)\} + g - \sum_i e_i \{q(p, z, g), v(p, z, g), m_i(p, z, g)\}] \quad (44)$$

There are extra arguments in the demand functions x_i , e_i ($i \in I$) and the supply functions y , f because now the relevant prices of traded goods can be affected by tax and tariff changes. One could differentiate this system to arrive at more complicated forms of the equations of section 2.8. It is more instructive, however, to consider the special case in which the changes in q , $(m_i)_{i \in I}$, r , and v depend on changes in g only through the scalar variable $w \cdot g$ representing the value of public net output of traded goods at border prices. Then the formulae (35), (37), and (38) of section 2.8 take forms such as:

$$du_i = s_i \cdot dz + \rho_i w \cdot dg \quad (45)$$

$$\Delta u_i(1) = s_i \cdot \Delta z + \rho_i w \cdot \Delta g \quad (46)$$

$$\Delta W(1) = \sum_{i \in I} \beta_i^0 (s_i \cdot \Delta z + \rho_i w \cdot \Delta g) = s \cdot \Delta z + \rho w \cdot \Delta g \quad (47)$$

where $(s_i)_{i \in I}$ and s denote shadow price vectors for domestic goods, and $(\rho_i)_{i \in I}$ and ρ denote shadow prices of foreign exchange, which are scalars. Of course, what is remarkable about the formulae (46) and (47) is the appropriateness of border prices as shadow prices. Indeed, one can even construct money metric measures of individual utility in terms of foreign exchange as the *numeraire*, so that $\rho_i = 1$ (all $i \in I$) by definition. If the relative welfare weights β_i^0 are also normalized so that they sum to one, then (45), (46), (47) above become:

$$du_i = s_i \cdot dz + w \cdot dg \quad (48)$$

$$\Delta u_i = s_i \cdot dz + w \cdot dg \quad (49)$$

$$\Delta W(1) = s \cdot \Delta z + w \cdot \Delta g, \text{ where } s = \sum_i \beta_i^0 s_i. \quad (50)$$

The extra terms for traded goods are also simple for higher-order approximations, because border prices remain fixed. Thus, corresponding to (39), (40), and (41) one has:

$$\Delta u_i(1\frac{1}{2}) = s_i \cdot \Delta z + (\frac{1}{2}) \Delta q \cdot \Delta z + w \cdot \Delta g \quad (51)$$

$$\Delta W(1\frac{1}{2}) = \sum_i (\frac{1}{2})(\beta_i^0 + \beta_i^1)(s_i \cdot \Delta z + w \cdot \Delta g) + (\frac{1}{2}) \Delta q \cdot \sum_i \beta_i^0 \Delta x_i \quad (52)$$

$$\Delta W(2) = \sum_i (\frac{1}{2}) \{ \beta_i^0 [1 - (\beta_i^1 q^1 - \beta_i^0 q^0) \cdot x_{Mi}^0 - \Delta \beta_i (w \cdot e_{Mi}^0)] + \beta_i^1 \} \\ \times (s_i \cdot \Delta z + w \cdot \Delta g) + (\frac{1}{2}) \Delta q \cdot \sum_i \beta_i^0 \Delta x_i \quad (53)$$

The term e_{Mi}^0 in (53) arises from the responses of i 's net demands for traded goods to changes in the measure M of total income, holding welfare weights $(\beta_j^0)_{j \in I}$ fixed, as in section 2.8.

2.10 FURTHER ISSUES

This chapter has confined itself so far to private goods which are bought or sold by price-taking consumers who are free to transact as they wish within their budget constraint. Also, consumer sovereignty has been assumed. Many projects, however, involve public goods which cannot be bought and sold by consumers. They may also use labour in regions of high unemployment where it is questionable indeed if all potential workers can sell their skills at the prevailing market wage, instead of facing a quantity constraint that rations scarce jobs. And a project to reduce drug-taking, alcoholism, or tobacco consumption may well be judged beneficial precisely because it denies the wishes of some consumers to obtain harmful products, thus violating consumer sovereignty.

Public goods are usually provided by a government agency in a way which puts them outside the control of most consumers. Actually, since public health services can only be used by those who make demands upon them, and even compulsory schools can be changed if the parents move to a different local authority or school district, one should be careful here. It seems most appropriate to discuss the *public environment*, in the sense of the school *system*, the health care *system*, etc., which are beyond the control of individual consumers. An advantage of this view, moreover, is that other aspects of the environment, such as pollution and traffic congestion, can be treated rather similarly; the only difference being that such externalities are caused by private consumers and firms as well as government agencies.

With aspects of the environment, be they public or not, consumers face common quantities (or qualities) and have different prices which reflect their marginal willingness to pay for a quantity change. Examples of quantity changes are more teachers per pupil in elementary schools, more particles of pollutant per cubic centimetre of air, more hours of the night disturbed by noisy aircraft or motor vehicles, etc.

The fact that Δx is independent of i for the environment, but now Δq_i depends on i , makes hardly any difference to approximate evaluations such as (9) and (15), which for individual i remain as:

$$\Delta u_i(1) = q_i^0 \cdot \Delta x_i, \Delta u_i(1\frac{1}{2}) = (\frac{1}{2})(q_i^0 + q_i^1) \cdot \Delta x_i \quad (54)$$

The associated aggregate measures of welfare are:

$$\Delta W(1) = \sum_i \beta_i^0 q_i^0 \cdot \Delta x_i, \Delta W(1\frac{1}{2}) = (\frac{1}{2}) \sum_i (\beta_i^0 q_i^0 + \beta_i^1 q_i^1) \cdot \Delta x_i \quad (55)$$

The second-order approximations $\Delta u_i(2)$ and $\Delta W(2)$ do become rather more complicated; but similar principles apply. The only differences are that the prices attached to aspects of the environment can no longer be inferred from market observations, and that the budget constraint $m_i = q_i \cdot x_i$ no longer applies when such aspects are included in the vectors q_i and x_i .

With second-order approximations such as (17), a correct approximation $\Delta u_i(2)$ involves a different kind of income response for aspects of the public environment. What matters, not surprisingly, is how marginal willingness to pay for each aspect responds to income changes. Appropriate formulae are given in Hammond (1983).

Rationing of private goods can be dealt with similarly. Now both quantities and prices vary from individual to individual. Prices relevant for welfare measurement are, of course, measures of marginal willingness to pay rather than actual market prices facing consumers; the difference – a premium for goods which consumers want to buy but cannot, and a discount for those they want to sell but cannot – reflects the intensity of disequilibrium in the market. Such disequilibrium may be frustrating to individual consumers unable to buy and sell what they please at prevailing prices, but may also be part of a good allocation process in economies with various kinds of imperfection.

Merit goods (or 'demerit bads'), where one wants to depart from consumer sovereignty, also present no fundamental conceptual problems. For these, whether consumers are rationed or not, the appropriate prices q_i depart from market prices. Indeed, appropriate prices even differ from consumers' marginal willingness to pay. Dangerous and addictive drugs may be given a negative social value even if some individuals betray a readiness to pay large amounts for them. All such categories of good or bad are already covered by formulae like (54) and (55) and their full second-order extensions $\Delta u_i(2)$ and $\Delta W(2)$.

The complications created by rationing and public goods may actually come rather in comparative static analyses like those of sections 2.7 and 2.8. With rationing, equilibrium is no longer described just by equations like (25). The comparative statics are correspondingly more

complicated. With public goods, balancing policies may involve changes in the public environment. But I shall not discuss these further. The interested reader should turn to Dreze and Stern's (1985) discussion of shadow pricing in a 'messy' economy to get some idea of the difficulties and of how one can begin to treat them.

Throughout this chapter so far it has been presumed that small changes of quantities are possible, and correspond to small changes in prices. Yet in reality consumers find it hard to live in more than one place at a time, or to supply many different kinds of labour service to many different employers. When labour supply shifts from one place to another in response to a change in wages, it is very unlikely to be because many workers each spend a few more hours each week working where the wage has risen and a few less working where it has fallen. Rather, several workers change their jobs entirely. Of course, if the wage change was small, those who move are close to being indifferent anyway, so the effect on their welfare is also small. But that is not obviously well captured by the approximations in this chapter. Some work related to this problem is by Small and Rosen (1981).

More serious is when changes occur not from individual choice but through aspects of the environment. A deterioration in the health service, or in road safety, is likely to have a dramatic effect on the lives (or deaths even) of a few unfortunate people. The changes become too large for even second-order approximations to carry any conviction whatsoever. *Ex ante*, of course, it is likely that many individuals find themselves exposed to a marginally increased probability of personal catastrophe, in which case some of this difficulty may disappear. It becomes possible to think in terms of how much an individual should be willing to pay for marginal improvements in safety or health care, and the marginally reduced probability of catastrophe. However, even this approach has been challenged.

Another major limitation of this chapter has been its apparent concern only with static measures of welfare change and static models of the economy. A standard defence would be that the limitation is really only apparent because one can always differentiate commodities by date and treat uncertainty by considering probability distributions and commodities contingent upon the state of the world. And in principle this defence is certainly valid.

Time and uncertainty introduce many new complications, however. Project evaluation has to compare alternative probability distributions of possible entire future courses of events in the economy. Consumer sovereignty is more questionable when consumers lack foresight, and may be misinformed anyway. More fundamentally still, there is to date

no very satisfactory model of a sequence economy which can be used to carry out comparative static analyses as in sections 2.7 and 2.8. Indeed, the workings of even a 'perfect' market economy remain rather a mystery when borrowers may be able to default. That seems to be far removed from project evaluation, of course. Yet until it is resolved we shall be reduced to various expedients in comparative static analysis which avoid the fundamental problems. For some of the more sensible possibilities, the interested reader should consult Lind et al. (1982).

APPENDIX

A.1 APPROXIMATE MONEY MEASURES OF SOCIAL WELFARE CHANGE

So far, I have not questioned the use of a weighted sum $\sum_{i \in I} \beta_i \Delta u_i$ as a measure of welfare change. It is now time to do so. Suppose a project is intended to redistribute income significantly from rich to poor. If such a project is successful, the welfare weights after the project, written as β_i^1 , should differ from those before the project, β_i^0 . A successful redistribution project, after all, makes further projects of that kind less desirable, otherwise one would go on redistributing from the existing rich to the existing poor indefinitely, regardless of the new gross inequalities thereby created, with rich and poor simply interchanged.

It will turn out that $\sum_{i \in I} \beta_i^0 (q^0 \cdot \Delta x_i)$ is an acceptable first-order approximation to a monetary measure of welfare change. But second-order approximations will have to recognize the change from β_i^0 to β_i^1 in the welfare weight of each individual.

Really, money metric measures of welfare will be constructed for a whole society rather as they were for individual consumers. The trick, following Sen (1976, 1979), is to find a way of treating the whole society as one grand consumer. This involves considering consumption of the same good by different individuals as different commodities. The relevant net demand vector becomes $x = (x_i)_{i \in I}$, a complete list of net demand vectors, one for each individual. If there are l physical commodities and I individuals, then x has $l \cdot I$ components, and is a complete description of the net demands for all goods and all individuals. There is a corresponding price vector $q = (q_i)_{i \in I}$ with $l \cdot I$ components also.

In an economy with market clearing, where all consumers face the same price vector q for the l physical goods, the vector q takes the form $(\lambda_i q)_{i \in I}$ where each λ_i is a scalar, which is usually positive because commodities consumed by an individual who desires them are usually

desirable for society as well. That is, $q_i = \lambda_i q$ for each individual i . The ratio λ_i/λ_j for any pair i, j of individuals reflects the relative values to society of i 's and j 's consumption of any physical good. The first-order approximation corresponding to (9) to the measure of welfare change $\Delta W(1)$ can then be written as:

$$\begin{aligned}\Delta W(1) &= q^0 \cdot \Delta x = \sum_{i \in I} (q_i^0 \cdot \Delta x_i) = \sum_{i \in I} \lambda_i^0 (q \cdot \Delta x_i) \\ &= \sum_{i \in I} \lambda_i^0 \Delta u_i(1)\end{aligned}\quad (\text{A.1})$$

Here, then, the scalars λ_i^0 play the role of the welfare weights β_i in the sum $\sum_{i \in I} \beta_i \Delta u_i(1)$ considered in section 2.6. This shows that in a market economy one has $q_i = \beta_i q$ for each individual i - that is, the 'social' price vector associated with i 's net demand vector is the welfare weighted consumer price vector. So (A.1) is really:

$$\Delta W(1) = \sum_{i \in I} \beta_i^0 \Delta u_i(1) = \sum_{i \in I} \beta_i^0 (q \cdot \Delta x_i) \quad (\text{A.2})$$

- a first-order approximate measure of welfare change. An approximation neglecting income effects, like (15), is:

$$\begin{aligned}\Delta W(1\frac{1}{2}) &= (\frac{1}{2})(q^0 + q^1) \cdot \Delta x = \sum_{i \in I} (\frac{1}{2})(q_i^0 + q_i^1) \cdot \Delta x_i \\ &= \sum_{i \in I} (\frac{1}{2})(\beta_i^0 q^0 + \beta_i^1 q^1) \cdot \Delta x_i\end{aligned}\quad (\text{A.3})$$

This approximation takes some account of the change in welfare weights resulting from the redistribution induced by the project. It can be rewritten in the form:

$$\begin{aligned}\Sigma W(1\frac{1}{2}) &= \sum_{i \in I} (\frac{1}{2}) \beta_i^0 (q^0 + q^1) \cdot \Delta x_i + \sum_{i \in I} (\frac{1}{2}) \Delta \beta_i (q^1 \cdot \Delta x_i) \\ &= \sum_{i \in I} [\beta_i^0 \Delta u_i(1\frac{1}{2}) + (\frac{1}{2}) \Delta \beta_i (q^1 \cdot \Delta x_i)]\end{aligned}\quad (\text{A.4})$$

which makes clear the difference from the weighted sum $\sum_{i \in I} \beta_i^0 \Delta u_i(1\frac{1}{2})$ of individual measures of surplus gain.

Turning next to true second-order approximations, which allow for income effects as in (16) or (17), a new problem arises. What are the 'social' income responses $x_M^0 = (x_{Mi}^0)_{i \in I}$ in the counterpart:

$$\begin{aligned}\Delta W(2) &= (\frac{1}{2})(q^0 + q^1) \cdot \Delta x - (\frac{1}{2})(\Delta q \cdot x_M^0) q^0 \cdot \Delta x \\ &= (\frac{1}{2}) \sum_i (q_i^0 + q_i^1) \cdot \Delta x_i - (\frac{1}{2}) [\sum_i (\Delta q_i \cdot x_{Mi}^0)] [\sum_i (q_i^0 \cdot \Delta x_i)] \\ &= \Delta W(1\frac{1}{2}) - (\frac{1}{2}) [\sum_i (\beta_{1i} q^1 - \beta_i^0 q^0) \cdot x_{Mi}^0] [\sum_i \beta_i^0 (q^0 \cdot \Delta x_i)]\end{aligned}\quad (\text{A.5})$$

to the approximation (16)? What, indeed, is the exact measure of welfare change to which (A.1), (A.3), and (A.5) are successive approximations?

A.2 MEASURES BASED ON THE SOCIAL EXPENDITURE FUNCTION

The counterparts for a society, of equations (1) and (2) for a single individual, are the *exact* money measure of social net benefit:

$$\Delta W = E(q^0, x^1) - E(q^0, x^0) \quad (\text{A.6})$$

and the definition of the social expenditure function:

$$E(q, \bar{x}) = \min_x \{q \cdot x \mid x R \bar{x}\} \quad (\text{A.7})$$

where now R denotes a *social* preference or welfare ordering on the l -dimensional space of vectors x . One can then perform all the steps of sections 2.3 and 2.4, resulting in the approximations $\Delta W(1)$ of (A.2), $\Delta W(1\frac{1}{2})$ of (A.3), and $\Delta W(2)$ of (A.5) in the case when $q_i^0 = \beta_i^0 q^0$ and $q_i^1 = \beta_i^1 q^1$ for all individuals $i \in I$. These steps rely for their validity, however, on the presumption that x^0 and x^1 are minimizing vectors for (A.7) when $(q, \bar{x}) = (q^0, x^0)$ and (q^1, x^1) respectively. This presumption corresponds to the hypothesis of consumer sovereignty that underlies the earlier analysis for a single consumer. Whereas q^0 and x^0 are observable for a single consumer, however, here q includes unobservable welfare weights which reflect ethical views. So (A.7) does *not* assume that either x^0 or x^1 is being chosen optimally, nor that value judgements are based on the 'revealed preferences' of the government. Indeed, there is not even a presumption that the government has coherent preferences. Instead, the ordering R reflects the ethical views of the project evaluator. All that matters about R in (A.7) are the marginal rates of substitution between different components of x at x^0 and x^1 , since these must correspond to the price vectors q^0 and q^1 respectively.

Not that the possible prices q^0 and q^1 are entirely unrestricted. If $q^0 \cdot x^1 < q^0 \cdot x^0$, that implies $x^0 P x^1$, and so to avoid a contradiction one must have $q^1 \cdot x^0 > q^1 \cdot x^1$. In other words, as with Samuelson's (1947) theory of revealed preference, one must not have both $q^0 \cdot x^1 < q^0 \cdot x^0$ and $q^1 \cdot x^0 < q^1 \cdot x^1$ - both $q^0 \cdot \Delta x < 0$ and $q^1 \cdot \Delta x > 0$. This, however, is the only restriction on the possible values of q^0 and q^1 .

With market clearing, so that consumer prices are q , the presumption that $q_i = \beta_i q$ for all i amounts to consumer sovereignty, and a social welfare ordering satisfying the Pareto principle that $x_i R_i x_i$ (all i) implies $x R \bar{x}$, and in fact implies x is strictly preferred to \bar{x} unless it happens that x_i and \bar{x}_i are indifferent for all i .

Realizing that one is approximating (A.6) provides the basis for calculating responses x_{Mi}^0 . For society is to be thought of as maximizing the ordering R subject to a budget constraint $q \cdot x < M$ or $\sum_i \beta_i q \cdot x_i \leq M$. Given consumer sovereignty and the Pareto principle, this amounts to

having each consumer i maximize R_i subject to a budget constraint $q \cdot x_i \leq m_i$, and then choosing the income distribution $m = (m_i)_{i \in I}$ to maximize the resultant allocation x subject to $\sum_i \beta_i m_i \leq M$. Then each x_{Mi}^0 is the product $\theta_i^0 x_{mi}^0$ of the response $\theta_i^0 = \partial m_i / \partial M$ of the optimal income distribution to a change in (weighted) total income m , and of x_{mi}^0 , i 's own income responses at (q^0, m_i^0) . If consumer sovereignty is judged to be ethically appropriate, x_{mi}^0 can in principle be estimated from individual consumer behaviour. The responses θ_i^0 , however, depend upon distributional value judgements regarding the best response of the income distribution m to a change in M , faced with the budget constraint $\sum_i \beta_i^0 m_i \leq M$ involving the welfare weights β .

To examine these responses further, assume that the social ordering R can be represented by a Bergson social welfare function of the additively separable form:

$$\Omega(x) \equiv \sum_i \Omega_i(x_i) \quad (\text{A.8})$$

There is a corresponding 'indirect' Bergson social welfare function of consumer prices q and the income distribution m which is also additively separable in m for each fixed q :

$$V(q, m) \equiv \sum_i V_i(q, m_i) \quad (\text{A.9})$$

Fix q at q^0 and write:

$$v(m) = V(q^0, m), v_i(m_i) = V_i(q^0, m_i) \quad (\text{all } i) \quad (\text{A.10})$$

Then the welfare weights β_i^0 are proportional to the derivatives $v_i'(m_i^0)$. The responses θ_i^0 are the derivatives at M^0 of the functions $m_i(M)$ ($i \in I$) which together maximize $v(m) \equiv \sum_i v_i(M_i)$ subject to $\sum_i \beta_i^0 m_i \leq M$ with β_i^0 ($i \in I$) fixed. The first-order conditions for the maximum are that $v_i' = \lambda \beta_i^0$ for each i . Differentiating these conditions with respect to M gives $v_i''(dm_i/dM) = \lambda' \beta_i^0$ for each i . Assume $v_i'' < 0$ for all i . This is the familiar assumption of diminishing marginal utility of income. It follows that $dm_i/dM = \lambda' \beta_i^0 / v_i'' = \lambda' \beta_i^0 / v_i'' = \lambda' v_i' / \lambda v_i''$. But the budget equation gives

$$\sum_i \beta_i^0 (dm_i/dM) = 1 \quad \text{or} \quad \lambda' v \sum_i [(\beta_i^0)^2 / v_i''] = 1 \quad \text{or} \quad \lambda' \sum_i [(v_i')^2 / \lambda^2 v_i''] = 1$$

so that $\lambda' / \lambda = \lambda \sum_i [(v_i')^2 / v_i'']$. Ultimately one has:

$$\begin{aligned} dm_i/dM &= \lambda (v_i' / v_i'') / [\sum_j (v_j')^2 / v_j''] \\ \beta_i^0 dm_i/dM &= [(v_i')^2 / v_i''] / [\sum_j (v_j')^2 / v_j''] \end{aligned} \quad (\text{A.11})$$

To interpret (A.11), consider fixing the incomes m_j ($j \neq i$) of all individuals except i , and adjusting m_i so that i 's welfare weight becomes β_i .

This involves m_i being a function of β_i with:

$$v'_i(m_i(\beta_i)) = \lambda^0 \beta_i \quad (\text{A.12})$$

Differentiating (A.12) with respect to β_i gives:

$$v''_i \cdot dm_i/d\beta_i = \lambda^0$$

and so the response of m_i to a change in $1/\beta_i$, when $\beta_i = \beta_i^0$, is given by:

$$\begin{aligned} \eta_i^0 &= dm_i/d(1/\beta_i) = (dm_i/d\beta_i)/[d(1/\beta_i)/d\beta_i] \\ &= -(\beta_i^0)^2 \lambda^0 v''_i = -(v'_i)^2/\lambda^0 v''_i \end{aligned}$$

Because of the assumption that $v''_i < 0$, one has $\eta_i^0 > 0$. Thus (A.11) becomes:

$$\beta_i^0 dm/dM = \eta_i^0/\sum_j \eta_j^0 \quad (\text{A.13})$$

Finally:

$$x_{Mi}^0 = \theta_i^0 x_{mi}^0 = \eta_i^0 x_{mi}^0/\beta_i^0 \sum_j \eta_j^0 \quad (\text{A.14})$$

which needs to be substituted into (A.5) to give the following (after realizing that $\sum_i \beta_i^0 q^0 \cdot x_{Mi}^0 = 1$):

$$\begin{aligned} \Delta W(2) &= \Delta W(1\frac{1}{2}) - \frac{1}{2} [\sum_i \beta_i^1 (q^1 \cdot x_{mi}^0) (\eta_i^0/\beta_i^0 \sum_j \eta_j^0) - 1] [\sum_i \beta_i^0 (q^0 \cdot \Delta x_i)] \\ &= \sum_i \beta_i^0 (q^0 \cdot \Delta x_i) + (\frac{1}{2}) \sum_i \beta_i^1 q^1 \cdot \left[\Delta x_i - x_{mi}^0 \frac{\eta_i^0 \sum_j \beta_j^0 (q^0 \cdot \Delta x_j)}{\beta_i^0 \sum_j \eta_j^0} \right] \end{aligned} \quad (\text{A.15})$$

This should be compared with (17) for a single consumer. Notice that whereas (16) becomes the consumer surplus approximation $\Delta u(1\frac{1}{2})$ when $\Delta q \cdot x_m^0 = 0$, (A.15) does not become $\Delta W(1\frac{1}{2})$ when $\Delta q \cdot x_{mi}^0 = 0$ for every i . Instead, when individual income effects are zero, $q^1 \cdot x_{mi}^0 = 1$ for all $i \in I$, and (A.15) becomes:

$$\begin{aligned} \Delta W(1\frac{1}{2}) + \frac{1}{2} \left[1 - \sum_i \frac{\beta_i^1 \eta_i^0}{\beta_i^0 (\sum_j \eta_j^0)} \right] [\sum_i \beta_i^0 q^0 \cdot \Delta x_i] \\ = \sum_i \beta_i^0 (q^0 \cdot \Delta x_i) + \frac{1}{2} \sum_i \beta_i^1 q^1 \cdot \Delta x_i - \frac{1}{2} \sum_i \frac{\beta_i^1 \eta_i^0}{\beta_i^0 \sum_j \eta_j^0} [\sum_j \beta_j^0 (q^0 \cdot \Delta x_j)] \end{aligned}$$

Only when $\sum_i (\beta_i^1 q^1 - \beta_i^0 q^0) \cdot x_{mi}^0 = 0$ do we get $\Delta W(2) = \Delta W(1\frac{1}{2})$, as is obvious from (A.5) and (A.3).

It remains to be seen how well a formula like (A.15) can be applied in practice. There is no way of avoiding its complications, however, if an accurate second-order approximation is desired.

One step remains to make (A.6) a meaningful money measure of social welfare change to which (A.2), (A.3), and (A.15) are approximations. This is to make M , which satisfies the budget constraint $M = \sum_i \beta_i^0 m_i$ meaningful - i.e. a measure of total or average income. This merely involves normalizing the welfare weights β_i^0 ($i \in I$) so that, for total income, $\sum_i \beta_i^0 m_i^0 = M^0 = \sum_i m_i^0$; or for mean income per head, $\sum_i \beta_i^0 m_i^0 = M^0 = \sum_i m_i^0 / I$.

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