Inter-bank Network Formation – From Heterogeneity to Systemic Risk

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Abstract

We propose a first computational model of endogenous network formation for the overnight inter-bank lending market. In this model the primitive object is the portfolio problem, solutions of which map directly into banks' optimal behaviour. As all the banks are large and their trading affects prices of risky assets, the costs of price slippage breaks the symmetry of the portfolio, providing incentive for inter-bank borrowing and lending. Having known banks' reservation interest rates, our protocol approximates the unique set of inter-bank transactions (linking decisions, volumes, prices) such that no bank is better off by severing its existing link, and no two banks have incentive to form a link with each other. Hence the inter-bank market structure emerges from mutual interactions of heterogeneous agents who are endowed with assets, liabilities and take into account investment risk. Our model yields a vertex distribution that is approximately scale-free. It depicts three different sources of contagion: liquidity erosion, fire sales, and bankruptcy cascades. The model is fully dynamic, valid for any number of banks of arbitrary sizes and it admits learning and forming expectations. This framework is next employed to compare stability of the endogenously generated banking system where the banks are endowed with either the same, or heterogeneous volumes of assets, and to investigate the aftermath of a crisis where banks incur stochastic investment losses.

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This paper develops an endogenous network formation model for the overnight inter-bank lending network.

1 Introduction

Preventing the meltdown of financial system was one of the crucial issues after the collapse of Lehman Brothers in September 2008. A realistic model of banking system is a key component, necessary to verify both the efficiency of different financial regulations and the robustness of alternative market structures. Banks in this model should have assets, liabilities, take into account investment risk and be diversified (heterogeneous) enough to depict a real world diversity of the system. Simultaneously, the banks need to make make their borrowing and lending decisions in a way which is in some sense optimal and depends just on the characteristics of their own and their counterparts (is endogenous). While diversity of market participants is necessary to make the model realistic, endogenous network formation is crucial if we intend to investigate stability of the entire system. If the inter-bank linkages are not endogenous but random or semi-random, the outcome of any stability analysis depends on network configurations that may never arise in practice. Endogenous network formation is also vital in quantifying the implications of endogenous bankruptcies in a distressed banking system. The reason is that the systemic impact of such event to large extent results from the characteristics of both creditors and debtors of the insolvent institution.

We propose a first computational model of endogenous network formation for the overnight inter-bank lending market. In this model the primitive object is the portfolio problem, solutions of which map directly into banks' optimal behaviour. As all the banks are large and their trading affects prices of risky assets, the costs of price slippage breaks the symmetry of the portfolio, providing incentive for inter-bank borrowing and lending. Having known banks' reservation interest rates, our protocol approximates the unique set of inter-bank transactions (linking decisions, volumes, prices) such that no bank is better off by severing its existing link, and no two banks have incentive to form a link with each other. Hence the inter-bank market structure emerges from mutual interactions of heterogeneous agents who are endowed with assets, liabilities and take into account investment risk. Our model yields a vertex distribution that is approximately scale-free. It depicts three different sources of contagion: liquidity erosion, fire sales, and bankruptcy cascades. The model is fully dynamic, valid for any number of banks of arbitrary sizes and it admits learning and forming expectations. It yields (approximately) scale-free vertex distribution and thus conforms with the main empirical feature of inter-bank loan networks. This framework is next employed to compare stability of the endogenously generated banking system where the banks are endowed with either the same, or heterogeneous volumes of assets, and to investigate the aftermath of a crisis where banks incur stochastic investment losses.

Section two of this paper reviews the literature. The third section presents the overview of the model. Section four section proposes the endogenous network formation algorithm and discusses its implementation requirements. The fifth section describes the choices of borrower and lender from an institutional perspective where the agents know their trade affects market prices. Section six is dedicated to calibration of the model. The seventh section presents the results. The final section concludes.

2 Literature review

In the literature on systemic risk and banking system stability two different approaches to modelling financial system are considered. Theoretical papers allow for strategic (endogenous) network formation, but in highly stylized settings. Computational papers depict more realistic pictures of banking system, but at the expense of tractability. In the latter complex models of individual bank behaviour are typically embedded into random network formation schemes. Basic findings of the both strands are as follows.

In their seminal theoretical paper Allen and Gale (2001) demonstrate how contagion (in the form of cascades of defaults) arises in the network of four identical banks. The same phenomenon occurs when banks on periphery are connected to money-centre banks, but not to each other (Freixas et al., 2000). For six identical banks unclustered asset structure of individual institutions entails lower funding costs and thus lower bankruptcy costs and higher welfare (Allen et al., 2012). Simultaneously the effect of banks holding similar assets as well as shares in each other on cascades of defaults is non-monotonic (Elliott et al., 2014). Bankruptcy cascades may also trigger joint collapse of the entire banking system (Gai and Kapadia, 2010). The extent of contagion depends crucially on the patter of interconnectedness in the network (Vivier-Lirimont, 2006), no network structure yields maximum system resilience[1] regardless of the circumstances (Acemoglu et al., 2015). While banks may form costly links to insure against liquidity risk, these links in turn expose the system to a small risk of bankruptcy cascades (Babus, 2009). If large bank acts as insurance provider, its default may trigger a run on the insuring peripheral bank (Zawadowski, 2013).

While these papers shed new light on systemic stability, theoretic analysis is severely constrained. Due to tractability issues banks in these works face limited types of risk and are endowed with only rudimentary assets and liabilities. Market cardinality (4 or 8 banks in Allen and Gale (2001), 6 in Allen et al. (2012)) and market structure (complete, ring, hub network) are fixed. Banking system consists of banks with either identical, or at most two different asset volumes (Babus, 2009; Zawadowski, 2013). None of these models admits any dynamics. These issues contribute to increasing popularity of simulations in the systemic risk research.

In one of the earliest computational approaches to modelling financial systems Eisenberg and Noe (2001) provide the algorithm that clears mutual claims in a (possibly cyclic) network, a metric of vertex systemic exposure is a by-product of this procedure. Elsinger et al. (2006a) combined risk management tools with a (computational) network model of inter-bank loans. The authors identified correlations in banks' portfolios as the main source of systemic risk and found that while insolvency cascades are rare, they might nonetheless wipe out the major part of the Austrian banking system. In subsequent work Elsinger et al. (2006b) conducted stress tests of the UK banking system, results of which were not conditional on all the banks remaining solvent.

The first simulation of insolvency cascades in a system of heterogeneous (in size or liquidity) banks endowed with assets and liabilities was undertaken by Iori et al. (2006). While in their model banks do not optimize investments decisions (assets allocations are stochastic), the authors demonstrate that in heterogeneous system incomplete inter-bank network structures are more robust than complete networks, what reverses[2] a classical result of Allen and Gale (2001). In a recent extension of this work Krause and Giansante (2012) identified network topology and tiering[3] as the two most important factors that determine probability of contagion. The impact of inter-bank

[1] Similar result was earlier obtained by Ladley (2013).

[2] Haldane and May (2011) on p. 353 point out that while excessive homogeneity minimizes the risk for each individual bank, it maximizes the probability of collapse of the entire system.

[3] Neglected by all the quoted theoretical papers, with an exception of Zawadowski (2013).

network connectivity and capital requirements on system resilience is non-monotone and non-linear (Nier et al., 2007). Also the stabilizing effect of a central bank on a banking system is non-linear with respect to fraction of banks's asset acceptable as collateral (Georg, 2013). For a banking system in the (static) evolutionary equilibrium no (random) network structure yields maximum system resilience under all feasible conditions (Ladley, 2013). The impact of heterogeneous asset volumes on erosion of confidence and long term inter-bank lending relationships was established via simulation by Arinaminpathy et al. (2012), extreme heterogeneity (in terms of market concentration) of the US CDS market also plays a pivotal role in the design of super-spreader tax, proposed by Markose et al. (2012). Vallascas and Keasey (2012) demonstrated that a cap imposed on a bank size may be the most efficient tool to reduce its default risk given a systemic event. Martínez-Jaramillo et al. (2010) found that fragility was determined by probabilities of default of individual institutions, their correlations and the number of banks that are instantaneously insolvent if their debtors default. Gai et al. (2011) verified how concentration of linkages under geometric and Poisson network affects liquidity hoarding and systemic crises, he also investigated the inerdependence between market liquidity and haircuts. Bluhm et al. (2014) simulate measures of systemic risk in a complex system of interacting banks. However, their network formation heuristics[4] does not arise from optimal agents' behaviour and thus is not endogenous. Cohen-Cole et al. (2013) depict inter-bank overnight market via Cournot quantity competition model embedded into scale-free networks, they also provide a measure of contribution of individual vertices to systemic risk. Their network formation protocol is not endogenous as it involves an arbitrary probability of forming a link. In all the quoted papers inter-bank market is formed by equating supply and demand and next matching potential creditors and debtors in a random or semi-random manner.

The deficiency of endogenous network formation in the computational models of banking system has serious implications. If the inter-bank linkages are random, the fact that two banks are in a lending relationship does not depend on any of their characteristics other than liquidity demand. The network configurations that thus come into being might never arise had the agents been allowed to choose their counterparts. As the effect of insolvencies on the entire system crucially depends on the topology of the inter-bank lending network, lack of endogenous network formation affects the outcomes of any systemic stability analysis. This distortion is most severe when we measure the impact of endogenous bankruptcies on a distressed banking system. The reason is that the systemic implications of such event result from characteristics of both creditors and debtors of the insolvent institution. During crisis banks often learn that they past decisions were not optimal, decide to reallocate their resources and change their linking patterns. To capture the dynamics of this process with exogenous network the underlying vertex distribution needs to be re-calibrated in an attempt to catch up with the market. If this network is instead endogenous, the agents take the change of environment into account in their (optimal) lending decisions. Thus exogenous lending networks are not appropriate to depict banking systems during crisis.

This works bridges both strands of literature, the model presented here is the first computational framework for endogenous network formation on the inter-bank lending markets. Just as in the work of Arinaminpathy et al. (2012), it admits three channels of contagion, the importance of which was stressed by Haldane and May (2011): (i) erosion of liquidity, where banks cut lending in fear of counterparty default (Brunnermeier, 2009; Gai et al., 2011), (ii) fire sales deteriorating market liquidity (Coval and Strafford, 2007; Adrian and Shin, 2010; Shleifer and Vishny, 2011;

[4]In their paper the agents look for the *closest matching partner* in terms of trading volume, see p. 9 in Bluhm et al. (2014). The only mechanism which could justify heuristics which minimizes number of connections is fixed transaction costs. However, under fixed transaction costs the agents using this rule would in some circumstances prefer a cost of nc over $2c$ for arbitrarily large n. A numerical counterexample is available upon request.

Battiston et al., 2012), (iii) bankruptcy cascades due to counterparty credit risk (Elsinger et al., 2006a; Nier et al., 2007; Upper, 2011; Gai et al., 2011). The proposed approach is valid for arbitrary number of banks with heterogeneous (Iori et al., 2006; Krause and Giansante, 2012) volumes of assets. The banks which dynamically update their assessment of investment risk and probability of counterparty default may shorten their positions in the asset they consider too risky, triggering a flight to quality episode (Caballero and Krishnamurthy, 2008). The model depicts the mutual dependence of market and funding liquidity (Brunnermeier, 2009) that can plunge the system into downward spiral of fire sales and credit denials. It allows for the study of endogenous bankruptcies and takes into account different behaviour of the banks under varying condition, thus addressing the criticism of the systemic stability literature, voiced by Upper (2011). The model allows returns on risky investment of different banks to be correlated, but this dependence is implemented neither via elasticities of demand (Cifuentes et al., 2005; Bluhm et al., 2014), asset commonality (Allen et al., 2012) or common shocks (Georg, 2013), but rather through the demand related component in asset prices.

There are two possible approaches to evaluating a computational model. The first criterion is variety of interacting mechanisms that the model is able to depict and plausibility of the assumptions behind it. My framework covers three different channels of contagion, its foundations are far less restrictive than the assumptions of theoretic models featuring endogenous network formation. The second criterion is ability of the model to reproduce empirical features of both asset structure of individual banks and the inter-bank lending networks. The networks generated by the model display degree distribution that is close to scale-free, which matches the empirical findings of Soramäki et al. (2007) and Cohen-Cole et al. (2013). The density of the simulated networks lies in the plausible range (Becher et al., 2008; Müller, 2006). Lending is persistent Cocco et al. (2009) and disassortative (Becher et al., 2008), as small banks are (on average) creditors of larger institutions (Müller, 2006). Furthermore, size of inter-bank market, distribution of bank sizes and strength of mutual dependence of asset returns in the system all closely match the corresponding empirical values.

3 Model overview

The economy consists of N regions, indexed with $k \in \{1, \cdots, N\}$. Each region harbours n identical consumers whose total mass[5] amounts to h_k. Each region is endowed with a single local bank k. There are T periods, indexed with $t \in \{1, \cdots, T\}$. In every period all local consumers approach bank k to place their deposits of h_k/n. Each consumer, after placing her deposit at time t, liquidates it at $t+1+S$ where S is a Poisson distributed random variate with intensity $\lambda - 1 > 0$. The deposits are being held for λ periods on average, expected net value of deposits placed at k amounts to λh_k. The volume of deposits placed in every region is in the long run stationary.

The banking system constitutes of N regional banks. Banks accept consumer deposits and do not compete with each other on deposit market. All the banks are required to keep a fraction ρ of deposits held as obligatory reserves. The remaining part is invested in the two assets – inter-bank loans and *risky* asset. Inter-bank loans cover transient liquidity shortages and are perceived as almost riskless. They last only one period and can not be extended, but the debtor may obtain a new loan next period on competitive basis. The risky asset replicates portfolio structure of given bank. It consists of the components that are regarded as more risky than inter-bank loans. The expected volume of assets held by each bank is initially proportional to the size of the corresponding region. Banks vary in lending needs, which arise from variability of net deposits. They also display different:

[5] The weight of consumers in all regions adds up to N.

attitudes towards risk and risk perception. Hence the banking system is heterogeneous, with bank characteristics varying along four dimensions.

Each bank has a portfolio which represents its long term risky investments. Let $P_{k,t}$ stand for a value of a unit of long term investment of bank k in period t. Given $P_{k,-1}, P_{k,0} := 1$, the dynamics of $\Delta \ln P_{k,t} := \ln P_{k,t} - \ln P_{k,t-1}$ is for every k driven by the following ARMA-GARCH process:

$$\Delta \ln P_{k,t} = \alpha_0 + \alpha_1 \Delta \ln P_{k,t-1} + Z_{k,t} + \alpha_2 Z_{k,t-1} + I_t, \quad I_{t+1} = \nu(D_t - S_t)/(D_t + S_t), \tag{1}$$

$$\sigma^2_{k,t} = \beta_0 + \beta_1 \sigma^2_{k,t-1} + \beta_2 Z^2_{k,t-1}, \quad Z_{k,t} \sim \text{NID}(0, \sigma^2_{k,t}).$$

Here α_0 represents trend while α_1 is the intertemporal price elasticity. Variable σ_t stands for volatility of risky asset log-returns, and is driven by GARCH(1,1) with parameters β_0, β_1 and β_2. $Z_{k,t}$'s are independent normal r.vs. and represent idiosyncratic components of price dynamics. I_t captures the liquidity effect, common to all the portfolios. D_t and S_t denote, respectively, the aggregate demand for and supply of the risky assets in the entire banking system. Constant ν maps excess relative demand or supply into prices. This common component of asset returns may be motivated by three mechanisms. First, as all the banks are large, the aggregate quantities of risky assets they trade always affect the market. Next, in a competitive environment all the large banks face similar investment opportunities. Finally, the fact that all the market participants use the same risk evaluation and portfolio optimization tools may cause spontaneous dependence of investment outcomes. Each bank is allowed to trade only the units of its own portfolio. The banks are oblivious of the form of risky asset pricing formula, but they know the realized sample mean and standard deviation. Diverse price trajectories represent the effects of individual portfolio composition on the bank's overall performance.

Assets of each bank consist of: investment portfolio $a_{k,t}$, reserves $r_{k,t}$, net cash $c_{k,t}$ and loans $l_{k,t}$ from k to other banks in the system. Liabilities constitute of: equity $e_{k,t}$, customer deposits $d_{k,t}$ and loans $b_{k,t}$ to k from other institutions. Hence for each bank k at every point in time $t \in \{1, \cdots, t\}$ it holds that

$$a_{k,t} + r_{k,t} + c_{k,t} + l_{k,t} = d_{k,t} + e_{k,t} + b_{k,t}.$$

While long term investment and loans to other institutions are both risky, the term *risky asset* will be further reserved for the units of portfolio of given bank. If a ratio of equity to risk weighted assets for any bank falls below 2%, this bank becomes bankrupt. Banks learn unconditional distribution of risky asset prices from past data. They also form expectations on probabilities of counterparty default.

Banks are risk averse (Ratti, 1980; Koppenhaver, 1985) and maximize conditional utility derived from their net profit tomorrow. To optimize its investment each bank first decides how much money to lend (or borrow), and at what interest rate. Next, it identifies a willing counterpart with whom the desired transaction could be concluded.

4 Network formation algorithm

This section introduces the network formation protocol, discusses its assumptions and implementation requirements. The basic idea is that the two groups of banks who would surely trade with each other are *best* borrowers and *best* lenders. The optimal network of lending relations may be approximated by, sequentially: allowing the most desirable counterparts to borrow or lend sufficiently small amounts and updating the both groups.

4.1 Assumptions and implementation

Our algorithm builds on the following assumptions.

(A1) *Banks joint beliefs on the probability of counterparts bankruptcy is given by p, which is collectively learned and time dependent.*

(A2) *All inter-bank lending is concluded at the midpoints of reservation interest rates of borrowers and lenders.*

(A3) *Banks are able to foresee all the stages of the proposed network formation protocol.*

Assumption one is probably the most controversial. It implies that probability of bankruptcy does not vary with counterpart level of equity or reserves. (A1) may be motivated by the fact that bankruptcies are rare events. Thus if a bank evaluates probabilities of tomorrow default for each single bank in the system, all the elicited values would typically small and of similar magnitude. Assuming they are equal might be a reasonable abstraction. A fringe benefit of this assumption is that matching algorithm, presented in further part of this section, needs to keep track of just a single bid and ask price for each market participant.

The second assumption provides an incentive for the proponents of the best bid and ask offers to trade together. Any inter-bank lending which happens in the consecutive steps of our algorithm takes place between parties who are each other's first choices, either in terms of asked, or offered interest rate. The aggregate outcome of this trade is necessary stable, that is: no bank is better off by severing an existing lending relationship, and no two banks have an incentive to form a link with each other. There is also no reason to *a priori* assume that any side of the market – either borrowers or lenders – has a bargaining power over the other. (A2) may be easily relaxed, as it affects neither the link formation process, nor transaction volumes. It is therefore irrelevant to topology of the emergent network.

Assumption three implies that the banks possess full information on characteristics of other agents and the network formation algorithm. It performs a dual role of both rationality and consistency condition – each bank attempts to optimize its actions given the information available at a time of forming links.

What is required to implement the algorithm, presented below, are reservation interest rates of borrower and lender that become less favourable as volume of a loan increases. We will also need a way to map constraints in terms of volume of a loan into constraints in terms of reservation interest rates. Two more necessary results are formulas for reservation interest rates that either clears the market of generates supply (or demand) of desired volume.

4.2 Algorithm

The network formation protocol, introduced in this section, constitutes of three consecutively repeated *stages*. First, the algorithm identifies the sets of *best* prospective: borrowers and lenders. Only the members of these two sets are allowed to trade. Next, for every bank it determines the largest admissible loan volume which obeys certain conditions. This quantity is split among all the bilateral transactions and the common interest rate is set. Finally, all these transactions are concluded and the protocol updates reservation rates and net cash. The proposed algorithm iterates through these three stages until no two banks are willing to trade with each other. The detailed account of each stage is given below.

Stage I consists of the following steps. (1) For every non-bankrupt bank we compute the largest interest rate it is able to accept as a borrower and the lowest interest rate it is able to accept as a lender. (2) All the solvent prospective borrowers are listed in descending order according to the largest interest rate they are willing to accept. All the solvent prospective lenders are listed in ascending order according to the lowest interest rate they are able to accept. Hence each bank is listed twice, once on every side of the market. These two hierarchies introduce a natural ordering among both debtors and creditors – it is clear which offer is the *best* on given side of the market. In consequence we can distinguish the two cliques, consisting of the best prospective borrowers and the best prospective lenders. (3) The banks with, respectively, the highest bid and the lowest ask price, are allowed to trade with each other. During the remaining two stages these groups are termed *active* borrowers and *active* lenders.

Stage II determines the volume of trade between active: debtors and creditors. It is defined as the largest quantity which satisfies the following conditions.

(i) If there is a second best bid interest rate, then the total volume of loans granted to any prospective debtor can not be larger than the amount that makes his bid interest rate equal to the second best bid rate. If there is a second best ask interest rate, then the total volume of loans granted by any prospective creditor can not be larger than the amount that makes his ask interest rate equal to the second best ask rate.

(ii) In the course of trade bid reservation rate of any active borrower can not become smaller then ask reservation rate of any active lender. Similarly, ask reservation rate of any active lender can not become larger than bid reservation rate of any borrower.

(iii) If any active borrower experiences a deficit of net cash, then the maximum total volume of the loans he is allowed to accept exactly offsets this deficit. If any active lender runs a surplus of net cash, then the maximum total volume of the loans he is allowed to grant exactly offsets this surplus.

(iv) Lending is to be financed either by selling risky asset or with cash.

(v) Aggregate volumes of borrowing and lending have to be equal. If no other constraint binds, this condition is equivalent to setting the market clearing interest rate, derived in Proposition 3 (i).

The short discussion of the five postulated conditions is as follows.

Condition (i) is an optimality constraint, imposed on either borrower or lender by his counterpart. It may be triggered if in the course of trading the highest (lowest) interest rate, acceptable to active borrowers (lenders), starts equating the second best interest rate. Then active lenders (borrowers) would also want to trade with all the non-active borrowers (lenders) who display this reservation rate. Trading with more counterparts implies either the same amount of credit at more favourable price, or more credit with no deterioration of price. Hence no rational bank would trade beyond this point.

Condition (ii) is an optimality condition of both borrower and lender. If it is violated, the last quant of credit was granted at the interest rate larger than the highest rate, acceptable to the borrower, and simultaneously smaller than the lowest interest rate, acceptable to the lender.

Condition (iii) is a consistency condition, required when trading risky asset is costly. A bank with a positive cash surplus does not incur a cost of selling risky asset when it either grants a loan to or declines to borrow from another bank. Hence both its reservation interest rates are higher. If surplus of net cash of any bank becomes exactly equal to zero, its reservation interest rates need to be recalculated.

Condition (iv) is a lender budget constraint.

Condition (v) is a market clearing condition. While each borrower (lender) needs to have a counterpart willing to lend to (borrow from) him the agreed amount of money, no other condition guarantees that the desired volumes of borrowing and lending are equal. If supply exceeds demand (or demand exceeds supply), we need to find the aggregate creditor (debtor) reservation rate for which it becomes equal to a given fixed value (Proposition 3, points (ii) and (iii)).

To complete Stage II the total volume that each debtor (creditor) has committed to borrow (lend) is distributed among all his active counterparts. It follows from (A1) that banks are indifferent with whom they trade, as long as their partner on the opposite side of the market offers them the most favourable interest rate. Hence if all the banks were of the same size (in terms of the total assets), all trade would be symmetric. By (A2) the price the banks agree upon is a midpoint of reservation prices that characterize each side of the market.

A simple way to implement restrictions (i)-(v) is to map these conditions into reservation interest rates. For each of these restrictions we can find the largest (or the smallest) interest rate acceptable for active debtors (creditors) which guarantees that the given condition is fulfilled. The maximal (respectively, minimal) reservation rate for all active borrowers (lenders) constitutes a single restriction which binds all the active debtors (creditors). The proposed implementation relies on Propositions 1 and 2.

We might consider networks of lending relations which arise during Stage 2. The vertices in such an interim network are active borrowers and active lenders. Borrower b is connected to lender l if and only if both parties agreed on a loan during given stage. In all such networks every borrower is connected to every lender, and every lender is connected to every borrower (networks are *complete*). This follows from the fact that all active borrowers offer the same (maximal available) interest rate to all active lenders. Simultaneously, all active lenders ask the same (minimal available) interest rate from all active borrowers. As transactions are concluded at the midpoint of reservation prices (A2), each active borrower is strictly better off by borrowing from all the active lenders, which implies either larger loan at the same interest rate, or more favourable rate on the loan of the same size. By the same token, each active lender is strictly better off by lending to all the active borrowers. Simulations in section 7 demonstrate that aggregation of the interim networks leads to complicated network topologies.

In Stage III the reservation rates of active borrowers and lenders are updated with formulas from point (iii) in Propositions 1 and 2, net cash reserves are also recalculated. The algorithm iterates through all the three stages until there are no pairs of creditor and debtor willing to lend to/borrow from each other. Once the algorithm stops, deposits and risky assets prices are updated for each bank. Solvent debtors pay back inter-bank loans with an interest. Creditor of each insolvent debtor receive fraction θ of the principal, then equity and reserves are recalculated. Next section describes the portfolio problem, optimal solution of which is used to implement the network formation algorithm.

5 Portfolio problem

Throughout this section it is assumed that the volume of risky asset traded by every bank is always considered *large* by the market. Banks know that trading risky asset adversely affects transaction prices due to price slippage. When they sell risky asset, they face instantaneous loss and relinquish future stochastic profits. When they buy risky asset, they enjoy uncertain profits but at the expense of instantaneous losses. Hence a bank with transient liquidity shortage might prefer taking loan on the inter-bank market to selling risky asset. By a similar token, a bank with transient liquidity surplus might prefer lending money on inter-bank market to buying risky asset.

In order to depict investment choices of large banks this section approaches portfolio selection problem from institutional perspective, in which market participants are aware of price distortions, caused by their size. In the first subsection basic notation is introduced. Next we characterize the behaviour of borrower and lender and derive the formulas for market aggregates. The final passage explains how the solution of individual portfolio problem affects endogenous network formation.

5.1 Notation

Index $k \in \{1, \cdots, N\}$ is reserved to denote any bank. Every bank learns the mean $\mu_k \equiv \mu_{k,t+1}$ and the standard deviation $\sigma_k \equiv \sigma_{k,t+1}$ of its risky asset returns from past data. All the banks dynamically update their joint beliefs p_{t+1} on the probability of counterparts' default. Let index b denote a bank that is a borrower while l denotes a lender. As the gross inter-bank interest rate is always agreed between both parties, for the sake of brevity we set $i \equiv i_{bl}$. Individual demand and supply characterize the behaviour of, respectively, borrower and lender, thus: $d \equiv d_b$, $s \equiv s_l$.

Assume $B_{b,t+1}$ is equal to one if b defaults at $t+1$ and is equal to zero otherwise. In case b defaults bank l expects to recover only a fraction θ of its loan to b. If b does not default, it pays back the principal plus agreed interest rate i. $R_{t+1} \equiv R_{k,t+1}$ represents gross (multiplicative) returns on risky asset at $t+1$, banks treat it as independent of $B_{b,t+1}$. Let c stand for multiplicative cost of trading, either in terms of price slippage, or incurred due to unfavourable timing of liquidating or purchasing the risky asset. As banks are large enough to affect market prices, these costs are incurred by all the traders[6].

Let $v \equiv v_k$ denote net cash surplus after the bank paid out due deposits and complied with reserve requirements. Assume $\underline{w} = \underline{w}_k$ and $w \equiv w_k$ stand for the share of risky asset in portfolio, respectively, at the beginning of current period and desired at the end of the period. If v is positive, $w = \underline{w}$ implies no necessity to trade risky assets, and hence no extra costs. If the bank is not able either to meet reserve requirements or to pay out the deposits at the beginning of the current period, v is negative. Banks are allowed to keep cash between consecutive periods only to satisfy obligatory reserve requirements.

Assume $\hat{w} \equiv \hat{w}_k$ stands for the aggregate net loans granted to k by other members of the banking system prior current transaction, $\hat{i} \equiv \hat{i}_k$ is the aggregate interest rate on these loans. The quantities v, $\hat{w}$ and $\hat{i}$ that appear in borrower problem describe characteristics of the borrower, the same notation in lender problem pertains to the lender. For the sake of brevity denote $\chi_b := c^{-1}\mathbb{1}_{\{w \geq \underline{w}\}}(w) + c\mathbb{1}_{\{w < \underline{w}\}}(w)$ for borrower and $\chi_l := c^{-1}\mathbb{1}_{\{w > \underline{w}\}}(w) + c\mathbb{1}_{\{w \leq \underline{w}\}}(w)$ in case of the lender. Both threshold functions differ in $\underline{w}$.

5.2 Optimal behaviour of market participants

Now we will characterize the optimal behaviour of both the prospective: borrower and lender.

Assume bank b is a prospective borrower. The value of b's portfolio at $t+1$ in case of $w > \underline{w}$ is

$$\underline{w}R_{t+1} + \mathbb{1}_{\{w > \underline{w}\}}(w)(w - \underline{w})R_{t+1} - i(c^{-1}\mathbb{1}_{\{w > \underline{w}\}}(w)(w - \underline{w}) - v - \hat{w}) - \hat{i}\hat{w},$$

if $w = \underline{w}$ it amounts to

$$\underline{w}R_{t+1} - i(-v - \hat{w}) - \hat{i}\hat{w},$$

while for $w < \underline{w}$ we obtain

$$\underline{w}R_{t+1} + \mathbb{1}_{\{w < \underline{w}\}}(w)(w - \underline{w})R_{t+1} - i(c\mathbb{1}_{\{w < \underline{w}\}}(w)(w - \underline{w}) - v - \hat{w}) - \hat{i}\hat{w}.$$

[6]See i.e. Coval and Strafford (2007).

If we denote the value of borrower's portfolio at $t+1$ by V_{t+1}, combining the three cases above yields

$$V_{t+1}(w) = wR_{t+1} - i(\chi_b(w - \underline{w}) - v - \hat{w}) - \hat{i}\hat{w}.$$

Note that b is a prospective borrower iff it can not finance its desired portfolio choice with either available cash by selling risky asset. Thus he is a borrower for $w \in I_b$ where

$$I_b = \{w: \ w \geq \underline{w} + \chi_b^{-1}(v + \hat{w})\} = \{w: \ w \geq e_b\}.$$

Here e_b is an edge where bank b does not want to borrow any more. The utility that b expects to obtain at $t+1$ from a portfolio, consisting of $w \in I_b$ units of risky assets and a loan from l, is

$$\mathbb{E}u(V_{t+1}(w)|B_{b,t+1} = 0) = \mathbb{E}u(i(v + \hat{w} + \chi_b\underline{w}) - \hat{i}\hat{w} - i\chi_b w + wR_{t+1}).$$

Next we will assume that the distribution of asset returns is Gaussian, in result log-derivatives of the corresponding moment generating function are linear. In such a setting it is possible to derive analytic formulas for the reservation interest rates of borrower and lender. From these equations we will obtain: demand and inverse demand, supply and inverse supply of individual banks which are consistent with the assumptions, necessary for the network formation algorithm. These four equations are the main result of this section. Their full derivation is relegated to Appendix A. Optimal behaviour of the borrower may be characterized as follows.

Proposition 1 (Borrower's behaviour) *Assume borrower b displays CARA with risk-aversion parameter $\gamma_b > 0$ and the gross (multiplicative) returns fulfil $R_{t+1} \sim N(\mu_{t+1}, \sigma_{t+1}^2)$.*

*(i) Borrower's **f.o.c.** is equivalent to*

$$w = \frac{1}{\gamma_b \sigma_{t+1}^2}(\mu_{t+1} - i\chi_b), \quad \text{where } w \geq e_b, \quad w \neq \underline{w}.$$

*(ii) The set of **admissible corners** of the borrower problem E_b is equal to $\{\max\{0, e_b\}\}$ for $v + \hat{w} > 0$ and to $\{\max\{0, e_b\}, \underline{w}\}$ otherwise. Let $E_b' := E_b \cup \{w^*\}$ if w^* fulfils point (i) above and $E_b' := E_b$ otherwise. Borrower's expected conditional utility attains maximum in E_b'.*

*(iii) Borrower's **reservation interest rate** $\bar{i}_b$ is the smallest interest rate at which the borrower would not borrow any money and is given by*

$$\bar{i}_b = \chi_b^{-1}\big(\mu_{t+1} - \gamma_b\sigma_{t+1}^2[\underline{w} + \chi_b^{-1}(v + \hat{w})]\big).$$

*(iv) The **maximum volume of a loan** $\tilde{w}$ that borrower b would be willing to accept at the interest rate $\tilde{i}$ is*

$$\tilde{w} = \gamma_b^{-1}\chi_b^2\sigma_{t+1}^{-2}(\bar{i}_b - \tilde{i}).$$

Now we will characterize the behaviour of a lender.

Assume bank l is a prospective lender. The value of l's portfolio at $t+1$ in case of $w > \underline{w}$ is

$$\underline{w}R_{t+1} + \mathbb{1}_{\{w > \underline{w}\}}(w)(w - \underline{w})R_{t+1} +$$

$$+(\theta B_{b,t+1} + i(1 - B_{b,t+1}))(v + \hat{w} - c^{-1}\mathbb{1}_{\{w > \underline{w}\}}(w)(w - \underline{w})) - \hat{i}\hat{w},$$

if $w = \underline{w}$ it amounts to

$$\underline{w}R_{t+1} + (\theta B_{b,t+1} + i(1 - B_{b,t+1}))(v + \hat{w}) - \hat{i}\hat{w},$$

while for $w < \underline{w}$ we obtain

$$\underline{w}R_{t+1} + \mathbb{1}_{\{w<\underline{w}\}}(w)(w - \underline{w})R_{t+1} +$$

$$+(\theta B_{b,t+1} + i(1 - B_{b,t+1}))(v + \hat{w} - c\mathbb{1}_{\{w<\underline{w}\}}(w)(w - \underline{w})) - \hat{i}\hat{w}.$$

After combining the three cases above and substituting χ_l, the formula for $V_{t+1}(w)$ is

$$V_{t+1}(w) = wR_{t+1} + (\theta B_{b,t+1} + i(1 - B_{b,t+1}))(v + \hat{w} - \chi_l(w - \underline{w})) - \hat{i}\hat{w}.$$

Note that l is a prospective lender iff its desired portfolio implies it has still available cash. Therefore it is a lender for $w \in I_l$ where

$$I_l = \{w : \ w \leq \underline{w} + \chi_l^{-1}(v + \hat{w})\} = \{w : \ w \leq e_l\}.$$

Above e_l is an edge where bank l does not want to lend any more. The utility that l expects to obtain at $t + 1$ from a portfolio, consisting of $w \in I_l$ units of risky asset and a loan to b (from the rule of iterated expectations) is

$$\mathbb{E}u(V_{t+1}(w)) = \mathbb{E}[\mathbb{E}u(V_{t+1}(w)|B_{b,t+1})] =$$

$$= p \cdot \mathbb{E}u(\theta(v + \hat{w} + \chi_l\underline{w}) - \hat{i}\hat{w} - \theta\chi_l w + wR_{t+1}) + (1 - p) \cdot \mathbb{E}u(i(v + \hat{w} + \chi_l\underline{w}) - \hat{i}\hat{w} - i\chi_l w + wR_{t+1}).$$

Optimal behaviour of the lender may be characterized as follows.

Proposition 2 (Lender's behaviour) *Assume lender l displays CARA with risk-aversion parameter $\gamma_l > 0$ and the gross (multiplicative) returns fulfil $R_{t+1} \sim N(\mu_{t+1}, \sigma^2_{t+1})$.*

(i) Define the following constants

$$c_1 := -\gamma_l(i - \theta)(v + \hat{w} + \chi_l\underline{w}) - \ln\frac{p_{t+1}}{1 - p_{t+1}}, \ c_2 := -(i - \theta)\chi_l,$$

$$c_3 := \gamma_l(i - \theta)\chi_l, \ c_4 := \mu_{t+1} - i\chi_l, \ c_5 := -\gamma_l\sigma^2_{t+1},$$

*and assume that $c_2 < 0$. Then lender's **f.o.c.** is equivalent to*

$$c_1 + c_3\, w - \ln\big(\frac{c_2}{c_4 + c_5 w} - 1\big) = 0, \quad \textit{where} \ \ w < (c_2 - c_4)/c_5, \ w \leq e_l, \ w \neq \underline{w}.$$

*(ii) The set of **admissible corners** of the lender problem E_l is equal to $\{0, \max\{0, e_l\}\}$ for $v + \hat{w} < 0$ and to $\{0, \max\{0, e_l\}, \underline{w}\}$ otherwise. Let $E_l^{'} := E_l \cup \{w^*\}$ if w^* fulfils point (i) above and $E_l^{'} := E_l$ otherwise. Lender's expected utility attains maximum in $E_l^{'}$.*

*(iii) Lender's **reservation interest rate** $\underline{i}_l$ is the largest interest rate at which the lender would not lend any money and is given by*

$$\underline{i}_l = \bar{i}_l\frac{1}{1 - p_{t+1}} - \theta\frac{p_{t+1}}{1 - p_{t+1}}.$$

(iv) The ***maximum volume of a loan*** $\underset{\sim}{w}$ *that lender* l *would be willing to accept at the interest rate* $\underset{\sim}{i}$ *is*

$$\underset{\sim}{w} = \frac{1}{\gamma_l {\chi_l}^{-2} \sigma_{t+1}^2} (\underline{i}_l - \underset{\sim}{i})(1 - p_{t+1}).$$

Points (iii) and (iv) in Propositions 1 and 2 represent, respectively, demand and inverse demand for, supply and inverse supply of inter-bank loans, obtained for an individual bank k. These formulas are recovered from reservation interest rates of borrower and lender which are perturbed and inverted in the limit. In case of borrower b this approach approximates the solution of borrower's f.o.c. This approximation is exact if an offer interest rate $\tilde{i}$ is sufficiently close to $\bar{i}_b$ or, alternatively, if $\tilde{w} > 0$ is sufficiently small. Thus the behaviour of a debtor who borrows $\tilde{w}$ at an interest rate $\tilde{i}$ is optimal. No such reverse engineering is possible in case of lender l. This is because non-linearity of credit supply equation vanishes in the limit for $s(i) \nearrow 0^+$. While $\tilde{w}$ is approximate solution to (borrower) f.o.c., $\underset{\sim}{w}$ is just a solution to approximate (lender) f.o.c. These two concepts do not need to coincide, lender's behaviour is not optimal by itself. However, it becomes optimal due to (A2) in the previous section.

Note that points iii) in Propositions 1 and 2 imply satiation effect as both parties become more reluctant to trade when their volume of either borrowing or lending increases. No lender ever lends at an interest rate it would accept as a borrower. Furthermore, no bank $k \in \{b, l\}$ would ever trade with itself as it always holds that $\underline{i}_k > \bar{i}_k$.

We may also obtain the formulas for reservation interest rates which either clear the market, or guarantee that credit demand (or supply), generated by a group of banks, is equal to certain value. Both formulas are useful if one side of the market is willing to trade less than the other.

Proposition 3 (Market aggregates) *Let* B *and* L *denote, respectively, the sets of active borrowers and lenders. For* $k \in B \cup L$ *define* $c_k := \gamma_k^{-1} \chi_k^2 \sigma^{-2}$, *let* f_k *stand for the size of the portfolio of bank* k.

(i) If no other constraint binds, then credit demand equates credit supply at the interest rate

$$i_{eq} = \frac{\sum_{b \in B} f_b c_b \bar{i}_b + (1-p) \sum_{l \in L} f_l c_l \underline{i}_l}{\sum_{b \in B} f_b c_b + (1-p) \sum_{l \in L} f_l c_l}.$$

(ii) A bid interest rate which generates a monetary demand for credit equal to V *is*

$$i_c = \frac{\sum_{b \in B} f_b c_b \bar{i}_b - V}{\sum_{b \in B} f_b c_b}.$$

(iii) An ask interest rate which (if no other constraints bind) generates a monetary supply of credit equal to V *is*

$$i_c = \frac{\sum_{l \in L} f_l c_l \underline{i}_l + \frac{1}{1-p} V}{\sum_{l \in L} f_l c_l}.$$

5.3 From portfolio selection to network formation

Given the solution of portfolio problem, there are two main reasons why inter-bank market exists. First, there are trading costs and funding costs. Banks who finance a deficit of net deposits by selling risky asset suffer the costs of portfolio adjustment and opportunity lost. Thus they may prefer to take a loan on the inter-bank market and repay it with interest. Simultaneously, banks who invest a surplus of net deposits in the risky asset face instant loss due to price slippage, while their future profits remain uncertain. Hence they may prefer to grant a loan on the inter-bank market and collect a (seemingly) certain interest. Financing a new loan by selling a risky asset is more expensive than financing it with net deposits. Therefore banks who enjoy a surplus of net deposits have stronger incentives to become lenders, banks who run a deficit have motivation to become borrowers. In this model transaction costs play the role of *symmetry-breaking* mechanism, as market participants assume that buying and then selling a unit of risky asset is not neutral to their financial situation. Next, banks are heterogeneous. Different degree of risk aversion, diverse risk perception and liquidity needs of individual banks map to reservation interest rates. If these rates are diversified enough, the banks find it desirable to trade.

6 Calibration

This section describes how we set the values of fifteen parameters, required to run the simulation. The figures quoted below refer to 1st July 2016 and were often rounded.

The system simulated in this paper comprises of $N := 40$ banks whose relative sizes match the top US-chartered commercial banks having total assets of at least 300M USD. According to the Board of Governors of the Federal Reserve System large commercial banks data, the share of 40 largest institutions in the aggregate assets of all 1784 banks amounted (on 31st December 2015) to 72.39%.This network should be thus sufficient to model the bulk of the industry without excessive computational burden. High concentration of banking system is also typical for many European countries. In example, Blåvarg and Nimander (2002) report that four large Swedish banks constitute 80% of the entire domestic banking sector.

According to the Federal Reserve Requirements (Regulation D) *reserve rate* of 10% applies to all deposits above 71 M USD. As all the banks are *large*, the deposits in the simulation are all subject to 10% reserve rate. For each bank initial *equity to assets ratio* amounts to 13%, which is approximated by the share of asset minus liabilities residual in the total assets of the US banking system according to H.8 statement of the Board of Governors of the Federal Reserve System (pp. 10-11).

Loss Given Default (LGD) is calibrated to $1 - \theta := 0.05$. LGD of 5% was reported by Kaufman (1994) in the case of Continental Illinois, the same calibrated value was used by Georg (2013) in his simulation. Since banks become insolvent if their equity to risk-weighted assets ratio falls below 2%, this level corresponds to an overnight loss of approximately 3.5% of lender's total assets. As risky assets display high persistence and low volatility, this value may be regarded as conservative.

The multiplicative *cost of trading* is set to $c := 0.997$. Risk-averse banks assume that each transaction involving risky asset entails a small additional cost due to price slippage. A priori *probability of counterparty default* is set to $p := 8.5 \cdot 10^{-5}$. If US banking system consists of 1745 banks open 260 days per year, approximately this number may be obtained from the failed bank list of the Federal Deposit Insurance Corporation (FDIC). Thus inter-bank loans are perceived as (virtually) riskless.

In every simulation for every bank $k \in \{1, \cdots, N\}$ *risk aversion* parameter γ_k is drawn from a uniform distribution $U(\gamma_{\min}, \gamma_{\max})$ with $\gamma_{\min} := 1$ and $\gamma_{\max} := 2$. Parameter $\lambda := 11.58$ represents the expected duration of deposits and is elicited from saving rates. Assume bank customers spend money with constant intensities and save only on current accounts. Given 21 working day per month the customers who are expected to collect their deposits after exactly 10.5 working days have on average no savings. Those who collect their deposits after 11.58 days run a surplus of $1.08/21 \approx$

5.17%, which is the forecasted savings rate of US households.[7] If each region hosts $n := 100$ bank clients, then for given λ the long term deposit variance (see Appendix B) attains a moderate value of 0.82%. As the model does not depict consumer behaviour, the only role of deposits is to cause variability of net cash that provides incentive for the inter-bank lending. For $\lambda = 12$ monthly saving rate increases on average to $1.5/21 \approx 7.14\%$, which is more appropriate in case of the Eurozone (forecasted household saving rate of 6.74%).

Price process parameters are set to $\alpha_0 := 1.4 \cdot 10^{-4}$, $\alpha_1 := -0.14$, $\alpha_2 := 0.2$ $\beta_0 := 3 \cdot 10^{-7}$, $\beta_1 := 0.28$, $\beta_2 := 0.35$ and $\sigma_0 := 6.5 \cdot 10^{-4}$. These values are estimated by fitting formula (1) to synthetic index that replicates the relative aggregate asset composition of large US-chartered banks, revealed in H.8 statement of the Board of Governors of the Federal Reserve System. Mortgages and mortgage backed securities, residential real estate loans and loans secured by multifamily properties that constitute 82% of the index are assumed to be risk-free and to yield an annual interest rate of 4%. US treasury bonds, which contribute 8.6% to the weight of the index, are represented by 10-year benchmark US bonds. Foreign bonds drive the dynamics of 9.3% of the synthetic index and are replicated with Markowitz risk-minimizing portfolio of 10-year benchmark bonds obtained for Japan, Germany, UK, China, Australia and Switzerland. While only bond components entail risk, the index represents a portfolio which is far less diversified and thus more volatile, than the portfolios of US banks. The index does not account for inter-bank loans.

Any bank becomes insolvent if its ratio of equity to risk weighted assets falls below 2%. Given the construction of the index that replicates the asset composition of US-chartered banks, risk weighted assets are computed as $0.35 \cdot 0.82 \cdot a_{k,t} + 0.2 \cdot l_{k,t}$. Here $a_{k,t}$ and $l_{k,t}$ are the volumes of risky asset and loans to other banks (held by bank k at time t), 0.2 is the mandatory weight of short-term unsecured loans, 0.35 is the risk weight that applies to mortgages[8], 82% is the share of mortgages and mortgage backed securities in the risky asset. The remaining 18% of the risky asset volume consists of sovereign debt, treated as risk-free.

To obtain $\nu := 0.0027$ assume for a moment that banks hold only risky asset. If all the banks have the same assets to equity ratio, then risky asset returns are proportional to equity growth rates. If financial markets are efficient, then equity growth rates are equal to stock price growth rates. In eq. (1) the term I_t stands for a component of risky asset returns common to all the banks. Under these two assumptions to replicate the average correlation rate of individual banks' portfolios it is enough to select ν such that the average correlations of banks' stock growth rates and simulated risky asset returns are equal. The average 5-year correlation rate of daily stock returns, obtained for 10 largest publicly listed US-chartered banks, amounts to 70.5%. The selected parameter value yields average correlation of returns equal to 70.2%.

The calibrated tuple $(\alpha_0, \alpha_1, \beta_0, \beta_1, \beta_2, \sigma_0; \nu)$ implies that prices of risky assets replicate three empirical characteristics of the synthetic index – they are persistent, dependent on the excess demand or supply generated by the other market participants and display moderate volatility clustering. This second feature enables positive feedback mechanism where excess demand for or supply of risky assets affects financial position of given bank. The next section presents the simulated results.

7 Results

This section describes the topology of the inter-bank market, obtained with the proposed network formation protocol. It also investigates the stability of the entire banking system in two scenarios.

[7]See: OECD (2016), Household savings forecast (indicator).

[8]Provided they account for less than 60% of the value of the underlying property.

The first scenario assumes that all the banks are initially endowed with the same expected amount of assets. In the second scenario the expected assets of consecutive banks are calibrated to reflect heterogeneity, similar to the US banking system. In both cases all the observed bankruptcies arise endogenously. All the figures depicting networks were plotted with Pajek software package (de Nooy et al., 2011). Unless stated otherwise, all the simulation parameters are calibrated with values from section 6.

The following network terminology is employed in the remaining part of this section. An *inter-bank network* consists of set of vertices (banks) connected by *links* or *connections* (overnight loans). In every transaction there is a borrower and a lender, hence all the links are *directed*. A convention followed here is that links run from creditor to debtor, just as the corresponding money flow. The *weight* of a link (volume of a loan) indicates significance of given link. *Degree* of a vertex is the number of links that either originate (*in degree*) or terminate (*out degree*) in the given node. As the network formation protocol delivers net lending between two parties, there can be no more than one link between each pair of banks. The ratio of all links formed in a network to all potential connections is termed network *connectivity*. *Assortativity* coefficients indicate how probable vertices of different types are to form links with each other.

To investigate the topology of inter-bank lending 1000 networks were generated, each depicting the development of financial linkages after one month (21 working days) of trading. As all the relevant quantities are normalized each turn (the model is stationary), inter-bank network characteristics stabilize after a short period in which all the banks are allowed to adjust their portfolios. A limited time span of this simulation rules out insolvencies that affect[9] the maximum feasible number of connections. To obtain sample correlates of systemic risk the simulation was run 250 times, the duration of each experiment amounted to 6 months (126 working days).

7.1 Homogeneous bank sizes

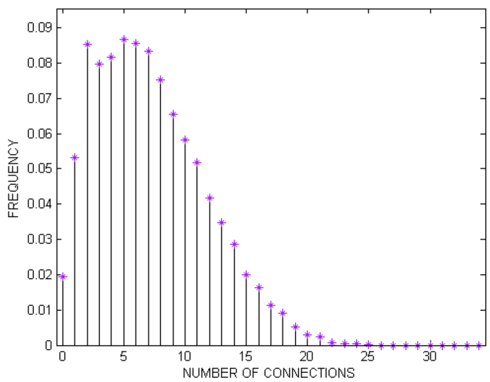
The first investigated case is the benchmark where the expected assets of each bank are equal and amount to a single unit.

The simulated degree distribution is depicted by Figure 4 (a). While this chart takes into account the total number of connections, it does not distinguish between lenders and borrowers. The fraction of unconnected vertices amounts to 1.93%. Mean vertex degree (including non-connected nodes whose degree is zero) is equal to 7.18, average ratio of formed links to all feasible links is 10.56%. If sample degree distribution followed Erdös and Rény (1959) random attachment model, then for given connectivity the mean vertex degree should be less then 3.6. Conversely, connectivity (which in this model corresponds to the sample mean vertex degree) is 21.12%. Hence the simulated networks entail a degree distribution with heavier tails than the one implied by purely random networks. In result few banks trade with many counterparts while majority trades with few.

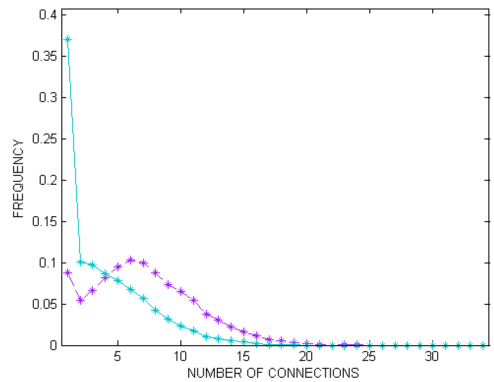
Connectivities of complete, ring, and star-shaped networks of N vertices amount to, respectively 1, $1/(N-1)$ and $1/N$. In a network of 40 banks these are: 100, 2.56 and 2.5 percent. The simulated lending networks are therefore less interconnected than complete, but markedly more than ring or star-shaped networks. The degree distribution generated by network formation protocol is hump-shaped and appears to be unimodal. Each simulation returns one connected graph.

The aggregate ratio of loans to assets in the simulated banking system amounts to 2.89%. This is more thrice the volume of inter-bank lending expressed as a fraction of total consolidated assets of US-chartered banks, which amounts to approximately 0.87% (16-th March 2016, only asset categories present in the model are being included).

[9] Bankrupted banks do not trade.



(a) Simulated degree frequency.



(b) Borrowers (solid cyan line) vs. lenders (dash- ed violet line).

Figure 1: Degree frequencies simulated for homogeneous bank sizes. Plots obtained: (a) including non connected vertices, (b) conditional on bank k being either a borrower or a lender.

Figure 4 (b) presents the differences in simulated degree distributions between borrowers and lenders. In the sample 4.74% of the vertices are exclusively lenders, 46.63% are only borrowers while 46.70% of all the banks are simultaneously creditors and debtors. As this last group is particularly large, instantaneous bankruptcy cascades are both possible and likely. The considerable amount of lenders who are also borrowers may arise when numerous banks in the course of trading either cover their entire net cash deficit or exhaust all their cash surplus. As hitting a boundary implies their reservation interest rates are updated (in a non-linear fashion), these banks may become most desirable counterparts on the opposite side of the market in one of the following interim stages. These figures also suggest that the depicted inter-bank market is a lender market with large excess credit demand. Mean borrower degree amounts to 3.85 while maximum borrower degree is 19. Mean lender degree is 6.98, maximum lender degree is 24. Hence in the entire banking system there is relatively more debtors with smaller number of connections and relatively fewer more interconnected creditors. Within the sample the lenders are relatively more likely to have 1-4 connections while the borrowers more often have 5-20 links.

A typical characterization of debtor-creditor pair is one of network features that may affect stability of the entire system. As expected asset volumes of all the banks are initially equal, the main factors that distinguish different market participants from each other are their risk aversion and risk perception. While for given bank the former may be expressed with CARA coefficient, that latter is captured with sample variance of returns. Each of those characteristics may be termed either *low* or *high*, depending on whether its value falls below or above sample median. The entries of matrices A_g and A_s below are empirical frequencies, computed form sample of $71,189$ links formed during the simulation. They describe how often pairs of banks that differ with respect to risk aversion and risk perception form links with each other

$$A_g = \begin{pmatrix} 0.195 & 0.375 \\ 0.164 & 0.267 \end{pmatrix}, \qquad A_s = \begin{pmatrix} 0.236 & 0.266 \\ 0.249 & 0.245 \end{pmatrix}.$$

In A_g the intersection of first row and first column contains probability that a pair of connected banks consists of a borrower and lender who both have low (i.e. below sample median) gamma

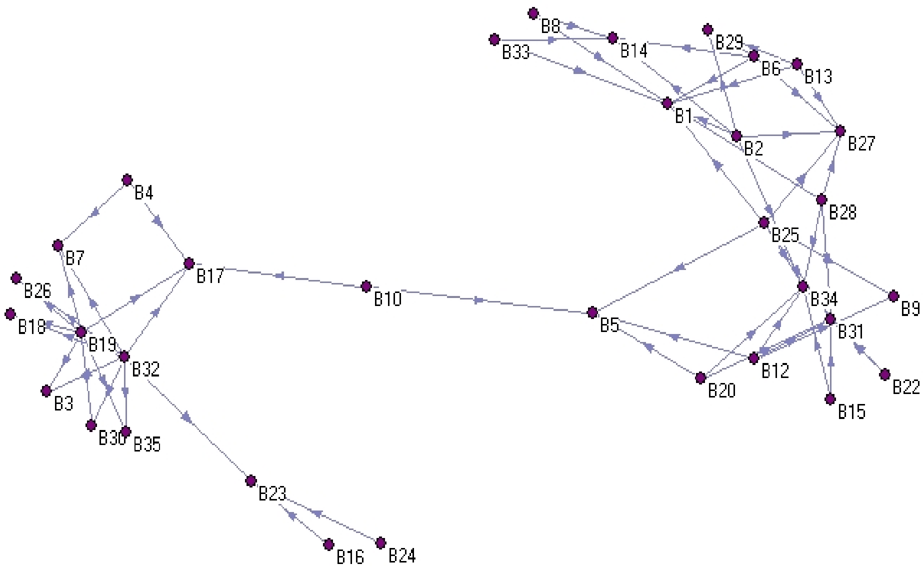


Figure 2: Inter-bank overnight lending network plotted with Pajek. All banks are solvent, non-connected banks are omitted. The layout of the vertices was determined by Fruchterman-Reingold (2D) energy minimization algorithm.

parameter. The intersection of first row and second column conveys the probability that borrower has low while lender has high (above sample median) level of gamma. The intersection of second row and first column contains probability that borrower has high while lender has low gamma parameter. Finally, the intersection of second row and second column conveys the probability that both borrower and lender have hight level of gamma. Matrix A_s carries a similar information for risk perception.

Assortativity reveals the propensity of the agents' to conclude transactions with the counterparts whose characteristics are different to their own. It captures the important feature of real world trade networks where involved parties differ in a much more fundamental way than just with the respective value of demand, supply or initial endowment. This is because the latter three quantities arise endogenously and stem from the fact that the agents are heterogeneous.

In order to assess significance of sample assortativity matrices, their largest entries may be compared against the simulated quantiles of matrix supremum norms. Critical values for the largest matrix entry simulated for a population of 10,000 links at 10, 5, 1 and 0.1% levels amount to 25.85, 25.97, 26.22 and 26.52%. As the simulated critical levels decrease with a number of links in each sample, these values may be considered as conservative. Both A_g and A_s matrices are significant at 0.1% level. Note that since the entries are dependent (sum up to one), significance of empirical frequencies has to be tested jointly.

In case of both risk aversion and risk perception it is more likely that borrower and lender are of a different than of same types. The most common pairing consists of a borrower who is less risk averse and perceives lower level of risk than his counterpart. This result is consistent with basic economic intuition: the risk averse banks with conservative assessment of investment risk regard inter-bank loans as desirable asset and are thus more likely to grant loans on the inter-bank market. Simultaneously, the risk-prone institutions who believe that the risk is low are more inclined to purchase the risky asset. This also means that the risk-prone debtor expects higher benefits in favourable scenarios. The least probable pair consists of highly risk averse borrower and less risk averse lender who both expect low variance of unconditional risky asset returns.

The fact that agent's linking decisions are assortative has two important implications. First, it implies that financial linkages are dependent on characteristics of both involved parties, and thus are not random. Second, if a risk-prone bank which perceives investment risk as low suddenly defaults, then the institutions hit most by its collapse are likely to be risk-averse and to perceive investment risk as high. This is precisely the group of banks that is worth saving by financial authorities in case of major financial crisis.

Figure 2 depicts an example of a network of inter-bank connections, created endogenously by the proposed algorithm. Network formation protocol delivers a complex system of financial linkages in which creditors typically reside in the centre of star-shaped formations and lend to banks, located on the periphery. Furthermore, the borrowers form fewer links than the lenders.

7.2 Calibrated bank sizes

Next investigated case is when sizes (in terms of total assets) of the banks in the system where calibrated to match top 40 banks, active on the US market.

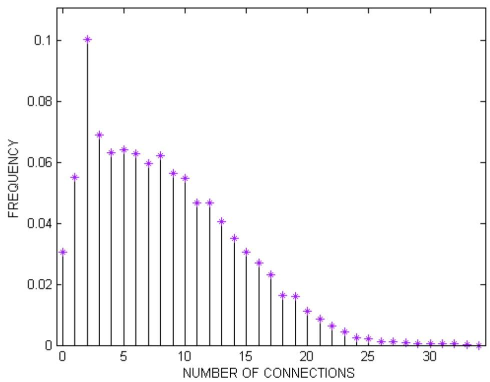
The simulated degree distribution is depicted in Figure 3 (a). While this chart takes into account the total number of connections, it does not distinguish between lenders and borrowers. The fraction of unconnected vertices amounts to 3.05%. Mean vertex degree (including non-connected nodes degree of which is zero) is equal to 8.25, average ratio of formed links to all feasible links is 12.13%. Connectivities of complete, ring, and star-shaped networks for 40 vertices amount to, approximately, 100%, 2.56% and 2.5%. The networks obtained in the heterogeneous case are again far less interconnected than complete networks, but markedly more than ring or star-shaped networks. Each simulation returns one connected graph.

Although the magnitude of the simulated networks connectivity is too large for entire banking systems, it lies in the range typical for subnetworks or clusters of these systems. In example, Müller (2006) reports that while Swiss inter-bank network connectivity amounts to only 3%, it soars to 27% for the subnetwork of cantonal banks. Average connectivities of CHAPS Sterling and *Giant Strongly Connected Components*[10] of CHAPS Sterling and Fedwire overnight inter-bank networks provided by Becher et al. (2008) amount, respectively, 88%, 5.1% and 0.3%.

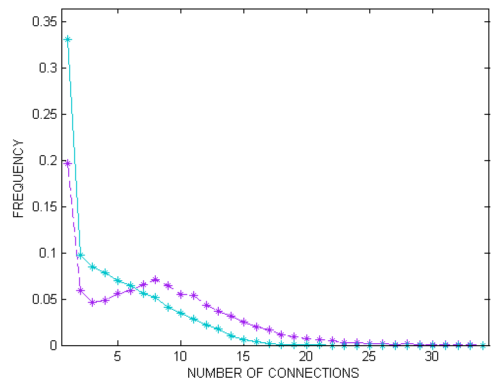
Figure 4 presents the simulated degree distributions for the both investigated cases, conditional on vertex having at least one connection. This empirical density is plotted against *scale-free* and conditional *purely random* degree distributions, fitted to match the simulated sample mean. Estimated values of characteristic exponent for scale-free density amount to 1.11 for homogeneous and 0.97 for calibrated bank assets. Both numbers are low and imply fat-tails. As scale-free distribution does not allow for vertices with no connections, purely random density is conditioned on vertex having at least one counterpart.

The degree distribution generated by the network formation protocol is not scale-free. Empirical densities depicted on Figure 4 (a) and (b) are not monotonously decreasing, they also have lighter tails than the scale-free distributions with the same average vertex degrees. Sample density obtained for homogeneous banks sizes appears to be hump–shaped, while the distribution simulated for the banks sizes calibrated to the US data seems to be multimodal. Furthermore, the densities generated by the network formation algorithm do not comply with Erdös and Rény (1959) random attachment model. Both simulated distributions are not symmetric around mean vertex degree, their tails are also heavier than in binomial density. Connectivity of purely random network is identical to the estimated probability of forming a link. Hence if the network simulated for the calibrated bank assets was purely random, its connectivity would be 24.26%, which is exactly two times more

[10] See Becher et al. (2008) for more details.



(a) Simulated degree frequency.



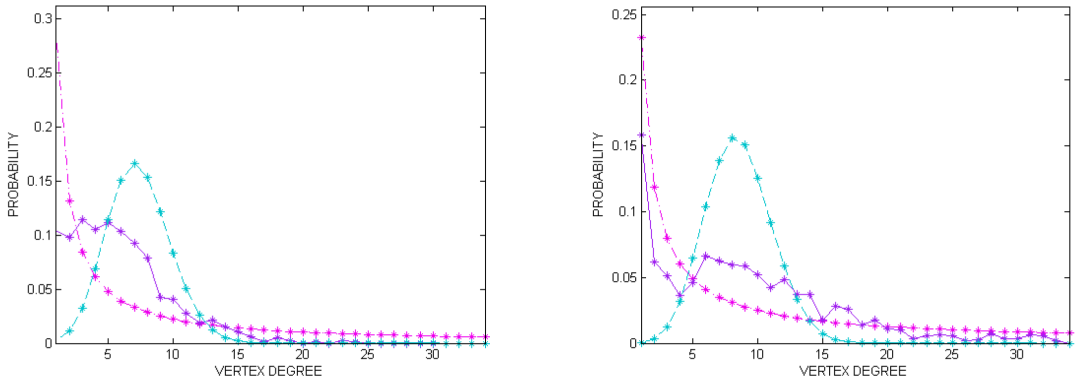
(b) Borrowers (solid cyan line) vs. lenders (dashed violet line).

Figure 3: Degree frequencies simulated for bank sizes calibrated to US market. Plots obtained: a) including non connected vertices, b) conditional on bank k being either a borrower or a lender.

than the actual sample value. The chart indicates that both simulated inter-bank networks are somewhere in between purely random and scale-free networks. Sample density obtained for the case with the calibrated bank assets oscillates around scale-free pdf and intersects with it four times. Hence it may be regarded as approximately scale-free.

The shape of degree distribution presented above conforms with the findings of Iori et al. (2008), who observed that vertex degree distribution on Italian segment of the European e-MID market is neither scale-free, nor purely random. It contrasts with a more recent picture of the entire e-MID market (250 banks) provided by Cohen-Cole et al. (2013), who found that degree density of European inter-bank market is approximately scale-free and thus (approximately) monotonously decreasing. The similar results were obtained by Soramäki et al. (2007) who demonstrated that vertex distribution for US Fedwire market was close to scale-free with characteristic coefficient equal to approximately 2.2. The difference between these empirical results and the shape of the simulated densities may be attributed to two factors. First, the investigated system consists of only 40 major banks. As noted by Blåvarg and Nimander (2002), in more concentrated systems large banks, who typically trade significant volumes of assets, have fewer alternative counterparties. Thus a network, consisting only of large banks, is necessarily much more interconnected than the system, where majority of banks is either small (eMID with 250 banks) or negligible (Fedwire, 1784 banks). When small banks which typically form only few links are being omitted, low degree vertices are under-represented and relative frequency of vertices with large number of links is weighted up. Next, in the proposed algorithm all the market participants are endowed with full information about the other members of the banking system. In result they find it optimal to conclude more transactions of smaller volume. These two factors lead to endogenous formation of highly interconnected networks with distribution that may be approximately humped-shaped (just as in homogeneous case).

The simulated average aggregate volume of loans as a fraction of total borrower assets amounts to 4.46%. The same quantity expressed as a percentage of total lender assets is equal to 8.02%. Hence in the investigate system it is the smaller banks who are crediting their larger counterparts. Empirically this is demonstrated by Müller (2006). The average total share of loans in the aggregate assets equates to 3.44%, which is four times more than the share of inter-bank loans in consolidated



(a) Homogeneous assets. (b) Calibrated assets.

Figure 4: Simulated degree distribution, conditional on vertex having at least one connection (violet solid line). Degree distributions: scale-free (magenta dash-dotted line) and random (cyan dashed line).

assets of US-chartered banks (0.87% on 16th March 2016). However, while inter-bank market plays secondary role in the US, its importance is much more pronounced in some European countries. In example, Degryse and Nguyen (2007) report that in their data inter-bank loans amount to 20-30% of assets and 30-40% of Belgian bank liabilities.

Figure 3 (b) presents the differences between borrowers and lenders in simulated degree distributions. In the sample 5.09% of the vertices are exclusively lenders, 42.29% are only borrowers, while 49.58% of all the banks are simultaneously creditors and debtors. As this last group is particularly large, instantaneous bankruptcy cascades are both possible and likely. Mean borrower degree amounts to 4.49 while maximum borrower degree is 21. Mean lender degree is 7.55, maximum lender degree is 33. Hence in the entire banking system there is relatively more debtors with smaller number of connections and relatively fewer more interconnected creditors. Within the sample the borrowers are relatively more likely to have 1-5 connections while the borrowers more often have 6-24 links.

A typical characterization of debtor-creditor pair is one of network characteristics that may affect stability of the entire system. In this model the main factors that distinguish different market participants from each other are their: size (in terms of total assets), risk aversion (CARA coefficient) and risk perception (sample variance of returns). Each of those characteristics may be termed either *low* or *high*, depending on whether its value falls below or lies above sample median. Empirical frequencies in the matrices

$$A_h = \begin{pmatrix} 0.226 & 0.303 \\ 0.212 & 0.258 \end{pmatrix}, \qquad A_g = \begin{pmatrix} 0.199 & 0.335 \\ 0.191 & 0.270 \end{pmatrix}, \qquad A_s = \begin{pmatrix} 0.231 & 0.267 \\ 0.246 & 0.249 \end{pmatrix},$$

describe how often in the generated networks pairs of banks that differ with respect to size, risk aversion and risk perception form links with each other. The frequencies summarize the characteristics of $81,115$ inter-bank links, formed during the simulation. In A_h the intersection of first row and first column contains probability that a pair of connected banks consists of a borrower and lender who both have low level of assets. The intersection of first row and second column conveys the probability that borrower has low while lender has high level of assets. The intersection of sec-

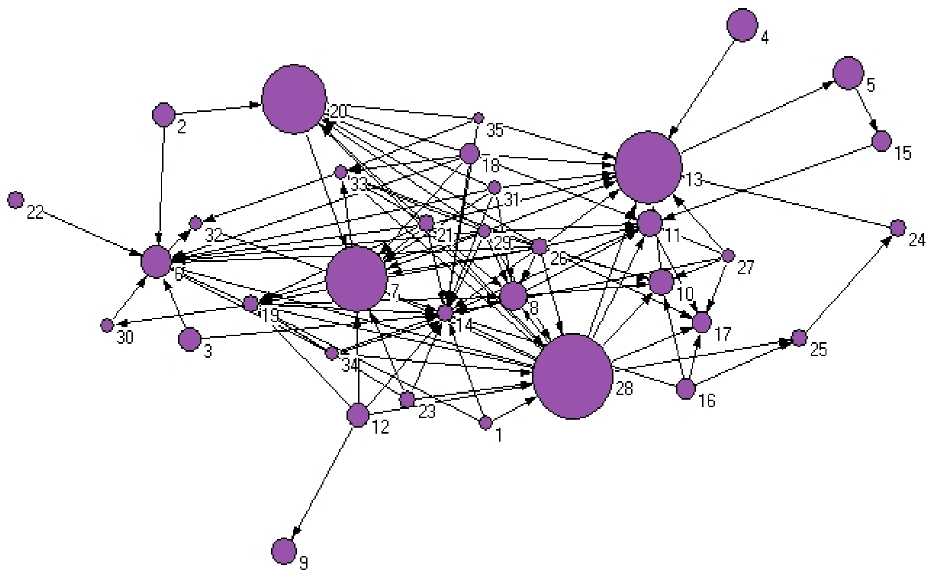


Figure 5: Example of inter-bank overnight lending network plotted with Pajek. Non-connected and insolvent banks are omitted.

ond row and first column contains probability that borrower has high while lender has low level of assets. Finally, the intersection of second row and second column conveys the probability that both borrower and lender have hight level of assets. Matrices A_h and A_s carry a similar information for risk aversion and risk perception.

The largest entries of all three matrices are significant at 0.1% level. The most important factor that differentiates banks' linking patterns is their risk aversion and next volume of their assets, risk perception parameter is somehow less important.

In the simulated networks the lenders are more likely to perceive investment risk as *high*. The borrowers who participate in the inter-bank trade more often are *small* and display *high* risk aversion while the lenders are *large* and display *high* risk aversion. In line with the empirical results by Cocco et al. (2009) banks tend to lend or borrow from the institutions of different size to themselves. A typical debtor-creditor pair consists of a borrower with low level of assets, low risk aversion and low risk perception and a lender with high level of assets, high risk aversion and risk perception. The least probable pairing consists of small risk-loving lender who perceives risk as low and large risk-averse borrower who perceives investment risk as low.

Just as in the homogeneous case, financial linkages are dependent on characteristics of both involved parties and thus are not random. Again if a risk-prone bank which perceives investment risk as low suddenly defaults, then the institutions hit most by its collapse are likely to be risk-averse and to perceive investment risk as high. By tempering the exuberance of the first group, a prudent banking system supervisor would protect the latter.

Figure 5 depicts an example of a network of inter-bank connections, created endogenously by the proposed algorithm. Network formation protocol delivers a complex system of financial linkages in which banks with more assets (typically of lower indices) reside on periphery of star-shaped formations and borrow from smaller banks, located in the centre. The relatively less numerous lenders form more links and are endowed with a smaller volume of assets. They are more exposed to counterparty default, as the volume of credit they grant constitutes more significant portion of their total assets.

	Bank assets					
	Homogeneous			Calibrated		
Variable	Eq.	Ins.	Sh.	Eq.	Ins.	Sh.
Deposits	-0.527****	-0.334****	-0.333****	0.565****	-0.925****	-0.937****
Risky asset	0.737****	-0.975****	-0.975****	0.852****	-0.994****	-0.975****
Reserves	-0.527****	-0.334****	-0.334****	0.565****	-0.925****	-0.937****
Loans	-0.685****	0.985****	0.985****	-0.840****	0.995****	0.977****
Cash	-0.297****	0.501****	0.501****	-0.297****	0.369****	0.373****
Borrower share	-0.943****	0.695****	0.696****	-0.908****	0.972****	0.950****
Lender share	-0.924****	0.805****	0.806****	-0.924****	0.888****	0.869****
Interest rate	0.814****	-0.886****	-0.886****	0.900****	-0.884****	-0.829****
Leverage	-0.973****	0.756****	0.756****	-0.977****	0.926****	0.884****
Funding liquidity	-0.067	-0.021	-0.021	-0.226****	-0.090*	-0.116**
Market liquidity	-0.394****	0.602****	0.602****	-0.668****	0.914****	0.918****
Prob. of def.	-0.614****	1.000****	1.000****	-0.826****	1.000****	0.989****
Equity	1.000****	-0.614****	-0.615****	1.000****	-0.826****	-0.771****

Table 1: Simulated correlation rates of aggregates. *Eq.* stands for equity, *Ins.* for total number of observed insolvencies, *Sh.* for insolvencies as a share of assets of the banking system. A number of 1-4 stars denote significance levels of, respectively, 10, 5, 1 and 0.1%.

The networks simulated for relative quantities of assets calibrated to the US market have a degree density that displays heavier tails than the networks obtained for homogeneous case. Their sample vertex distribution is also much closer to scale-free density and thus more realistic. Both instances yield similar market structures, with few smaller lenders granting numerous loans and numerous larger borrowers taking fewer loans. As the networks generated in heterogeneous case are more interconnected, for a sufficiently mild crisis they are expected to be more robust than the market configurations, obtained in homogeneous case. However, heterogeneous system would be much more vulnerable to cascades of insolvencies if the crisis was sufficiently severe. It could be also conjectured it would display much more vehement transition from the state where all the banks are solvent to the state where a significant fraction of the inter-bank market is bankrupt.

7.3 Correlates of systemic risk

This section investigates the correlates of systemic risk. Contrary to other the bulk of literature on inter-bank network simulation, the bankruptcies investigated here are all endogenous events. A single insolvency signals that the entire system is distressed and thus it is more likely to be followed by a cascade of bankruptcies. In order to generate sufficient number of insolvencies the equity to asset ratio was set to 4% of all the banks. This figure corresponds to the average relative equity of US banks in the eve of 2007 financial crisis. Furthermore, the parameter α_0 in formula (1) was reset to -0.0037. Hence what is modeled in this exercise is a *moderate* crisis where the banks in the short term (simulation time span) on average loose money on their investment, but the returns on investment itself do not become more volatile.

Table 7.3 contains sample correlation rates of system aggregates with three indicators of banking system stability. These indicators are: aggregate equity (denoted as *Eq.*), number of insolvent banks (*Ins.*) and a share of assets of bankrupted banks in total assets of the banking system (*Sh.*). The components of both the assets and the liabilities are normalized to one. Funding liquidity is approximated in the model by an average volume of inter-bank loans, expressed as a fraction of banking system assets. Market liquidity is approximated with an average relative contribution to the risky asset price of the excess supply or demand for this asset, generated by the banks. Note that probability of default may be estimated on the sample as a proportion of bankrupted banks to all banks. Hence correlation rate between any quantity and a number of insolvent banks is (for sufficiently large sample) identical to correlation rate between given quantity and the estimated probability of default. One to four stars denote rejection of null hypothesis that sample correlation rate is insignificant at significance levels of, respectively, 10, 5, 1 and 0.1 percent. The table indicates that most of the estimated correlation coefficients is significant at 0.1% level. As normality assumptions underlying Pearson's test are not fulfilled, these values may be only treated as reference. The results obtained were as follows.

The aggregate results allow us to identify two important subsets of aggregates. The quantities that are positively correlated with equity and negatively with a number of insolvencies and share of bankrupted banks in total assets are the variables whose high values characterize a system that is more robust. These aggregates are: share of equity and deposits in liabilities, share of risky asset and reserves in assets and inter-bank interest rate. Deposits converted to risky asset constitute the main source of profits. Retained profits contribute to equity and thus make the banks less susceptible to insolvency. Reserves are by definition proportional to deposits. High levels of the aggregate interest rate make inter-bank lending more profitable and thus increase equity of the lenders. Both reserves and equity provide a safety buffer, high levels of which make the bank more resilient to crisis. The quantities that are negatively correlated with equity and positively with a number of insolvencies and share of bankrupted banks' assets in total assets are the variables whose high levels characterize a system that is more fragile. These aggregates are: share of loans in assets of borrowers and lenders, the ratio of loans to aggregate assets, net cash, leverage, market liquidity and the estimate of the probability of counterparty default common to all the banks in the system. If risky asset is profitable, large amount of loans result in high costs of lost opportunity which are detrimental to equity of the entire system. Net cash is invested in the risky assets at the end of current period and it decreases equity due to slippage costs. High levels of leverage are associated with high risk and larger losses if the investment will not be profitable. Significant market liquidity is associated with higher volumes of trade and may decrease equity due to trade frictions. High assessment of the probability that the counterpart defaults is characteristic for the systems where large number of bankruptcies has already occurred and thus the aggregate equity is low.

Table 7.3 contains results simulated for 7000 individual banks. For each of the two investigated cases the first column contains correlates of consecutive indicators with bank equity. The second column presents correlation of given quantity with a binary variable which denotes that bank is solvent. The latter is equivalent to negative correlation rate with probability of default estimated within the sample. Both the asset and the liability size of each bank's balance sheet is normalized to one. A size of a bank is defined as the amount of assets the bank holds and is treated as *numéraire*. Again the majority of the estimated correlation coefficients is significant at 0.1% level.

Vertex in degree, bank size (total volume of assets), equity and equity ratio are positively correlated with both equity and the binary index which denotes that the bank remains solvent. Thus a larger bank, especially a one borrowing from numerous sources, is more resilient to crisis. The high relative amount of risky asset, accumulated reserves or deposits held (in heterogeneous case) coincide with high level of equity and lower probability of remaining solvent. As both assets and liabilities are normalized to one, the higher volume of deposits (and thus the higher obligatory reserves) typically implies the lower equity. Volume of loans both to and from given bank, vertex out degree and the risk perception parameter correspond to higher level of equity and lower level

	Bank assets			
	Homogeneous		Calibrated	
Variable	Equity	Solvent	Equity	Solvent
In degree	0.076****	0.163****	0.022**	0.183****
Out degree	-0.350****	0.107****	-0.324****	0.128****
Gamma	0.013	0.028***	-0.047****	-0.074****
Sigma	-0.204****	-0.054****	-0.007	0.023**
Deposits	-0.213****	-0.137****	0.125****	-0.163****
Equity	1.000****	0.360****	1.000****	0.389****
Loans to	0.222****	0.129****	-0.158****	0.151****
Risky asset	0.485****	0.004	0.579****	-0.021**
Reserves	-0.339****	-0.173****	0.085****	-0.182****
Loans from	-0.432****	0.091****	-0.527****	0.119****
Net cash	-0.134****	-0.160****	-0.179****	-0.345****
Equity ratio	0.993****	0.363****	0.991****	0.396****
Leverage	-0.952****	-0.499****	-0.929****	-0.516****
Size	0.040****	0.005	0.070****	0.059****

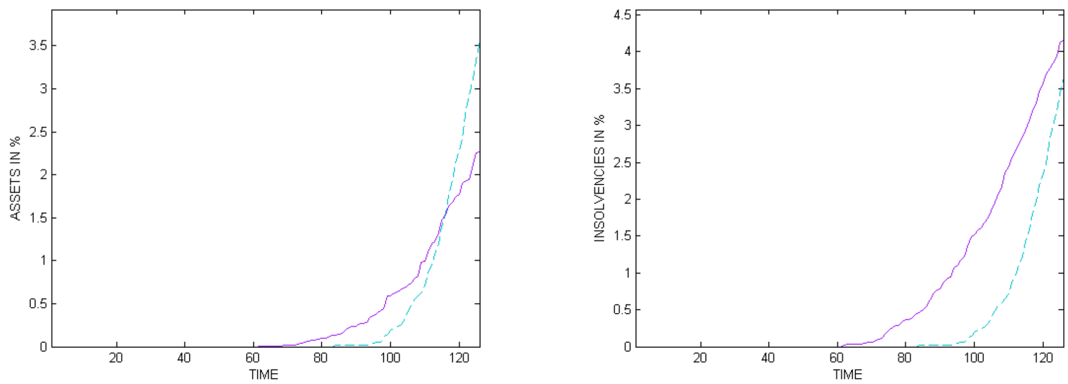
Table 2: Correlation rates simulated for individual banks. A number of 1-4 stars denote significance levels of, respectively, 10, 5, 1 and 0.1.

of the binary variable which indicates that the bank remains solvent. Thus while inter-bank lending during the simulated crisis decreases equity, it also makes the banks less likely to go bankrupt. Leverage, net cash and risk aversion coefficient gamma are negatively correlated with equity and positively correlated with probability of default.

At the level of individual banks we may distinguish two subsets of variables. The quantities that are positively correlated both with equity and the binary variable equal to one when bank remains solvent contribute to resilience of given bank. These indicators are: vertex in degree, bank size (total volume of assets), equity and equity ratio. The quantities negatively correlated both with equity and the binary variable equal to one when bank remains solvent are the ones whose high levels characterize banks prone to failure. These variables are: leverage, amount of net cash and risk aversion coefficient.

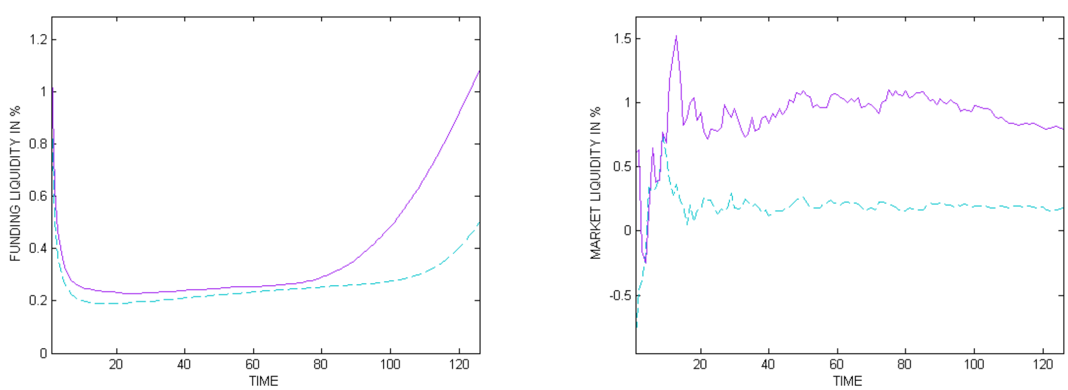
Figure 6 depicts as functions of time: sample mean number of insolvent banks and mean share of insolvent banks' assets in the total assets of the banking system. It reveals that after quiet spell of 80 working days the system with homogeneous bank assets enters a much more turbulent phase when a fraction the total number of banks becomes bankrupt. These insolvent institutions represent 3.5% of both the total number of banks and the aggregate assets of the banking system. In the case of calibrated bank assets the crisis starts taking its toll earlier (60 working days), but it is also less harmful. While more banks become bankrupt (4.1%), their total assets account for only 2.2% of the aggregate assets of the entire system. Hence heterogeneity of bank assets makes the entire system more stable and resilient to ruptures.

Figures 7 (a) and (b) represent the mean dynamics of the indicators of funding liquidity and market liquidity, recorder during the simulated crisis. Figure 7 (a) reveals that after a short period of decline in the amount of inter-bank loans caused by initial portfolio adjustments, the volume of the overnight loan market stabilizes and then slowly increases. This gradual process is abruptly interrupted some time after the first insolvency is observed. Once the banks learn that their portfolios incur losses, this sudden realization causes a flight to quality episode (Caballero and Krish-



(a) Assets of the system. (b) Number of insolvencies.

Figure 6: Expected: (a) loss of banking system assets, (b) number of insolvent banks for homogeneous (dashed cyan line) and calibrated (solid violet line) bank sizes.



(a) Funding liquidity. (b) Market liquidity.

Figure 7: The moderate crisis dynamics of the indicators of: (a) funding liquidity and (b) market liquidity, plotted for homogeneous (cyan dashed line) and calibrated (solid violet line) bank sizes.

namurthy, 2008) where they rapidly increase involvement in the less risky inter-bank market. This effect in the case of homogeneous bank assets is triggered somehow later, it is also less pronounced. Figure 7 (b) indicates that the contribution of the banking system to the risky asset price peaks in between 10th (homogeneous assets) or 15th (calibrated assets) working day. This effect decays over time and may be also attributed to the cumulated portfolio adjustments. Next the indicators of market liquidity in both investigated cases start to oscillate around a certain fixed level. It amounts to approximately 0.8% for the calibrated bank assets and only 0.2% when bank sizes are homogeneous. Thus when the banks substantially differ in the amount of assets they hold, risky asset market is much more liquid. This fact may contribute to the higher overall systemic resilience.

7.4 Parameter sensitivity

The results presented in this chapter were obtained for one combination of model parameters. The objective of the following two series of exercises is to verify the impact of parameter perturbation on both the generated networks and the correlates of systemic risk. In the tests summarized below it is always a single parameter that is being altered, while all the remaining model characteristics are calibrated as in section 6.

In order to obtain results comparable with the ones, presented in subsection 7.2, the first series of tests was run for 21 periods and repeated 600 times. The qualitative results thus obtained were as follows. Setting $c := 0.999$ slightly decreases connectivity of generated networks. The share of loans in the assets of, respectively, borrowers and lenders falls by 0.29% and 0.9%. Setting $c := 0.995$ has exactly the opposite effect. It marginally increases the connectivity, while boosting the share of the loans in the assets of debtors and creditors by 0.31% and 1.1%. In this second scenario borrowers and lenders not only conclude transactions of larger volume, but they also form more (by 1% and 2%, respectively) links. The source of this effect are larger incentives to become involved in the inter-bank market, which stem from higher transaction costs when the agents invest in the risky asset. When lenders' assessment of the probability of their counterparty overnight default goes up to $p := 0.00001$, the size of the inter-bank market declines by 11% while the network connectivity falls by almost 15%. Further growth of this probability to $p := 0.0001$ has even more devastating effects as the inter-bank market dwindles by total 48% and the connectivity drops by 66%. Simultaneously, mean network degree falls by two thirds while the average vertex degrees of borrower or lender decline by, respectively 15% and 35%. When p is high, the inter-bank loans are perceived as a more risky (and thus less desirable) asset. Assuming $\lambda := 12$ marginally decreases the total inter-bank loan volume and has negligible effect on all the other model characteristics. Larger λ implies lower volatility of the deposits, and thus smaller demand for loans, triggered by transient liquidity shortages. If $\gamma \sim U(1, 2)$ then the connectivity of the generated networks drops by 90.1%. The banks no longer differ with respect to risk aversion, their size and risk perception become the two main factors that determine their linking decisions. If the dispersion of risk aversion parameters is low, the banks are more similar to each other and thus lack incentives to trade. Finally, setting $\gamma \sim U(0, 0.5)$ increases the size of the inter-bank loans market by 216%, while the connectivity of the generated networks increases by 37%. As the utility functions in this parameter region are more steep, the banks much more fundamentally differ in their reservation interest rates, and thus find it optimal to lend or borrow more on the inter-bank market.

To deliver results comparable with subsection 7.3, the second series of tests was run for 126 periods and repeated 200 times. In the seven investigated exercises one model parameter always assumed either different value or range of values ($c := 0.999$, $c =: 0.995$, $p := 0.00001$, $p := 0.0001$, $\nu := 0.05$, $\gamma \sim U(1, 2)$, $\gamma \sim U(0, 0.5)$), while all the remaining parameters were set as in section 6. The qualitative results thus obtained for the banking system calibrated to the US market were the following. At the level of the entire banking system the correlates of: loans, cash, share of the loans in either borrowers' or lenders' assets, aggregate interest rate, systemic leverage estimated probability of default and aggregate system equity were qualitatively stable. Stable was also the correlation rate of market liquidity and the share of risky asset in banks' portfolio with a number of insolvent banks. At the level of individual institutions the correlates of: in degree, out degree, net cash equity ratio and leverage were qualitatively stable. Stable were also the correlates with bank equity of: the share of risky asset in total assets and risk aversion parameter gamma (the latter with the exception of the case when $\nu := 0.05$) and the correlates with a binary variable which indicates that the bank remains solvent of: equity, loans to given bank and reserves (again with the exception of $\nu := 0.05$). The fact that not all correlates of systemic risk prove to be stable is not surprising, since some of the considered exercises are rather extreme (e.g. parameter p is multiplied by a hundred).

8 Conclusions

This chapter made the following contributions.

First, the paper proposes a computational model of endogenous network formation designed for the inter-bank market. The algorithm relies on the solution of portfolio problem where banks displaying constant absolute risk aversion maximize their expected utility while simultaneously taking into account price slippage, costs of financing and investment risk. The banks who differ with respect to size, risk aversion and risk perception, form links with their most preferred counterparts. The emergent market structure arises due to banks being heterogeneous. The network formation protocol yields simultaneously the optimal: choice of a counterpart, volume of a loan and agreed interest rate. The generated system may be analysed as a whole, at the level of individual banks or separate transactions. The outcome of network formation procedure is deterministic and pairwise-stable. According to our best knowledge, this is the only fully endogenous (depending solely on agent's characteristics) computational model featuring link formation that simultaneously incorporates investment risk and takes into accounts assets and liabilities of commercial banks.

Second, the proposed model is calibrated to the subnetwork of 40 largest US banks and run to simulate network topologies. In the generated networks banks with more assets typically reside on the periphery of star-shaped formations and borrow from smaller banks, located in the centre. The lenders form more links than the borrowers. Thus they are more exposed to counterparty default as the volume of credit they grant constitutes more significant portion of their total assets. Just as in real world financial networks, degree distribution in the generated networks displays a tail gravity of which is between that of purely random and scale-free network. If the expected initial bank assets are homogeneous, the factor which affect linking decisions of the agents most is risk aversion. If the assets are calibrated to the US market, the factors that affects bank's linking behaviour are first risk aversion and next volume of assets while risk perception seems least important. As there is a fraction of banks who are simultaneously borrowers and lenders, the topology of the resulting networks allows for instantaneous bankruptcy cascades.

Third, the correlates of systemic stability are being investigated. The quantities that on aggregate level characterize more robust system are: share of equity and deposits in liabilities, share of risky asset and reserves in assets and inter-bank interest rate. Thus one of the ways to make system more resilient to crisis is to incite the banks to rise additional capital. As high inter-bank interest rates may contribute to equity via the retained profits, keeping interest rates artificially high might be detrimental to systemic stability. At the level of individual banks the factors which contribute to resilience of given bank are: vertex in degree, bank size (total volume of assets), equity and equity ratio. These results indicate that the lender of the last resort could mitigate the investigated crisis, by providing loans to the distressed banks. They also suggest one of the reasons behind high banking system concentration might be that the concentrated systems are more robust. The simulation also shows that size matters - for the considered crisis scenario the case where bank assets are calibrated to US market generates a more robust system with less common bankruptcies than the homogeneous case.

The model presented in this work has some interesting extensions. First, the proposed framework could be used to analyse robustness of the banking system under different crisis scenarios. Next, it could be utilized to investigate the impact of fire sales and asset price erosion on systemic stability. Third, the model maybe employed as a simulation tool to quantify the impact of different banking system regulations (such as Basel III accord) on the emergent market structure.

A Agents' behaviour

A.1 Proof of Proposition 1 (Borrower's behaviour)

Proof: **(i)** Let $M_{t+1}(u)$ stand for the moment generating function of R_{t+1}. Then we have

$$\mathbb{E}u(V_{t+1}(w)|B_{b,t+1}=0) = 1 - \mathbb{E}e^{-\gamma_b(a-bw+wR_{t+1})} =$$

$$= 1 - e^{-\gamma_b(a-bw)} \cdot M_{t+1}(-\gamma_b w).$$

Thus the optimal portfolio is given by

$$w_b^* = \underset{w\in I_b}{\operatorname{argmax}} \left\{1 - e^{-\gamma_b(a-bw)} \cdot M_{t+1}(-\gamma_b w)\right\} = \underset{w\in I_b}{\operatorname{argmin}} \left\{e^{-\gamma_b(a-bw)} \cdot M_{t+1}(-\gamma_b w)\right\},$$

and the f.o.c of **borrower** problem is equivalent to

$$\frac{d}{dq} \ln M_{t+1}(q)\big|_{q=-\gamma_b w} - b = 0 \ \equiv \ \mu - \gamma_b \sigma^2 w - i\chi_b = 0 \ \equiv$$

$$\equiv \ w = \frac{1}{\gamma_b \sigma^2}(\mu - i\chi_b).$$

To validate consistency of the assumptions we need to check if $w_b^* \in I_b$.

(ii) Denote the set of admissible corners of the borrower problem as E_b, set $e_b = \underline{w} + \chi_b^{-1}(v+\hat{w})$. If $v+\hat{w} > 0$ then $E_b = \{\max\{0, e_b\}\}$, otherwise $E_b = \{\max\{0, e_b\}, \underline{w}\}$. Hence the borrower problem has at most two corner solutions. Any admissible internal solution that satisfies the borrower's f.o.c is a strict (local) maximum.

Fact 1 (Local strict concavity) *Under CARA with Gaussian asset returns borrower's expected conditional utility* $\mathbb{E}u(V_{t+1}(w)|B_{b,t+1}=0)$ *is strictly concave on any real interval* $I \subset (e, +\infty) \setminus E_b$.

Proof: First consider an auxiliary function $f(x) = e^{c_1+c_2x+c_3x^2}$ with $c_1, c_2, c_3 \in \mathbb{R}$. We have

$$f^{''}(x) = ([c_2 + 2c_3 x]^2 + 2c_3)e^{c_1+c_c x+c_3x^2}, \ f^{''}(x) > 0 \equiv (c_2 + 2c_3 x)^2 + 2c_3 > 0.$$

To demonstrate $f(x)$ is strictly convex on any interval $I \in \mathbb{R}$ it is enough to show that $c_3 > 0$. To show that $\mathbb{E}u(V_{t+1}(w)|B_{k,t+1} = 0)$ is strictly concave on any interval $I \subset (e, +\infty) \setminus E$ it is enough to prove that

$$e^{-\gamma_b(a-bw)} \cdot M_{t+1}(-\gamma_b w)$$

is strictly convex on I. If risky asset returns are normally distributed, this expression reads as

$$e^{-\gamma_b(a-bw)} \cdot e^{-\gamma_b \mu w + \frac{1}{2}\gamma_b^2 \sigma^2 w^2} = e^{\frac{1}{2}(\gamma_b \sigma)^2 w^2 + \ldots}$$

where lower order terms were omitted. As $(\gamma_b\sigma)^2 > 0$, this function is strictly convex. □

Fact 1 allows us to simplify the utility maximization problem. Define $E_b^{'}$ as the set of (potential) expected utility maximizing portfolios for the borrower, initially set $E_b^{'} := E_b$. Denote borrower's optimal portfolio choice as $w_b^* \equiv w_b^*(i_{bl})$, let w satisfy the borrower's f.o.c. If either $w > \max\{\underline{w}, e_b\}$ or $e_b < w < \underline{w}$, then $E_B^{'} := E_B^{'} \cup \{w\}$. The optimal borrower's portfolio is given by

$$w_b^* = \operatorname*{argmax}_{w \in E_b^{'}} \{\mathbb{E}u(V_{t+1}(w)|B_{b,t+1} = 0)\}.$$

Having characterized the set of feasible corners and the interior solution we may derive demand for credit as a functions of interest rate.

(iii) Monetary **demand** for credit d depends on the difference between the optimal portfolio choice and the largest position that b could finance without additional funding. For sufficiently small $d > 0$ the optimal portfolio choice w_b^* is an internal solution of the borrower problem and lies above $e_b := \underline{w} + \chi_b^{-1}(v + \hat{w})$. There are two distinct cases. If $v + \hat{w} \geq 0$ then $w_b^* \geq \underline{w}$. Bank b utilizes credit to increase the amount of risky assets, held at the end of the period. As buying large quantities of assets causes price slippage, for every extra unit of asset it has to pay c^{-1} units of money. Demand for credit is given by $c^{-1}(w_b^* - e_b)$. By the similar token, if $v + \hat{w} < 0$ and if $d > 0$ is sufficiently small, then we have $w_b^* < \underline{w}$. Bank b takes credit in order to avoid selling risky assets it already has in its portfolio. As disposing of large quantities of assets incurs cost, for every c units of money b can refrain from selling one unit of asset. Demand for credit is given by $c(w_b^* - e_b)$. In this case credit demand is sufficiently small if only $d < -v - \hat{w}$. Since threshold function χ_b depends on w_b^*, this additional restriction guarantees that borrower's actions are indeed optimal. In our notation credit demand in both cases may be jointly expressed as

$$d(i) = \chi_b(w_b^* - e_b) = \chi_b(w_b^* - \underline{w}) - (v + \hat{w}).$$

The solution of borrower's problem w_b^* may be thus treated as given for each offered interest rate i. In both cases we may determine $\bar{i}_b$ as a limit interest rate for $d \nearrow 0^+$.

Now we may find the largest interest rate $\bar{i}_b$ that **borrower** b would be willing to accept. The monetary demand for credit as a function of the offered interest rate is

$$d(i) = \chi_b\Big(\frac{1}{\gamma_b\sigma^2}[\mu - i\chi_b] - \underline{w}\Big) - (v + \hat{w}) \equiv \chi_b^{-1}(v + \hat{w} + d(i)) = \frac{1}{\gamma_b\sigma^2}(\mu - i\chi_b) - \underline{w} \equiv$$

$$\equiv \gamma_b\sigma^2(\underline{w} + \chi_b^{-1}[v + \hat{w} + d(i)]) = \mu - i\chi_b \equiv i(d) = \chi_b^{-1}\Big(\mu - \gamma_b\sigma^2[\underline{w} + \chi_b^{-1}(v + \hat{w} + d)]\Big).$$

Now we have

$$\bar{i}_b = \lim_{d \nearrow 0^+} i(d) = \chi_b^{-1}\Big(\mu - \gamma_b\sigma^2[\underline{w} + \chi_b^{-1}(v + \hat{w})]\Big).$$

Formula for $\bar{i}_b$ implies that the more debt the borrower has, the more reluctant she is to borrow.

(iv) Note that the maximum size of a loan that borrower is eager to take at interest rate $\tilde{i}$ is the volume that makes $\tilde{i}$ her reservation interest rate. Given interior solution, this volume may be obtained as

$$\tilde{i} = \chi_b^{-1}\Big(\mu - \gamma_b\sigma^2[\underline{w} + \chi_b^{-1}(v + \hat{w} + \tilde{w})]\Big) = \bar{i}_b - \gamma_b\chi_b^{-2}\sigma^2\tilde{w} \equiv$$

$$\equiv \gamma_b\chi_b^{-2}\sigma^2\tilde{w} = \bar{i}_b - \tilde{i} \equiv \tilde{w} = \gamma_b^{-1}\chi_b^2\sigma^{-2}(\bar{i}_b - \tilde{i}).$$

This completes the reasoning. □

A.2 Proof of Proposition 2 (Lender's behaviour)

Proof: **(i)** For the **lender** problem we obtain

$$\mathbb{E}u(V_{t+1}(w)) = p\cdot\mathbb{E}u(e - fw + wR_{t+1}) + (1-p)\cdot\mathbb{E}u(a - bw + wR_{t+1}) =$$
$$= p(1 - \mathbb{E}e^{-\gamma_l(e-fw+wR_{t+1})}) + (1-p)(1-\mathbb{E}e^{-\gamma_l(a-bw+wR_{t+1})}) =$$
$$= 1 - p\cdot e^{-\gamma_l(e-fw)}\cdot M_{t+1}(-\gamma_l w) - (1-p)\cdot e^{-\gamma_l(a-bw)}\cdot M_{t+1}(-\gamma_l w). \quad (2)$$

Thus the optimal unit portfolio is given by

$$w_l^* = \underset{w\in I_L}{\operatorname{argmax}}\left\{1 - \Big(p\cdot e^{-\gamma_l(e-fw)} + (1-p)\cdot e^{-\gamma_l(a-bw)}\Big)\cdot M_{t+1}(-\gamma_l w)\right\} =$$

$$= \underset{w\in I_L}{\operatorname{argmin}}\left\{\Big(p\cdot e^{-\gamma_l(e-fw)} + (1-p)\cdot e^{-\gamma_l(a-bw)}\Big)\cdot M_{t+1}(-\gamma_l w)\right\},$$

where the interval $I_L = \{w:\ w \le e_l\}$. The lender's f.o.c. may be written as

$$\frac{d}{dq}\ln M_{t+1}(q)\big|_{q=-\gamma_l w} - \frac{fp\cdot e^{-\gamma_l(e-fw)} + b(1-p)\cdot e^{-\gamma_l(a-bw)}}{p\cdot e^{-\gamma_l(e-fw)} + (1-p)\cdot e^{-\gamma_l(a-bw)}} = 0 \equiv$$

$$\equiv \Big(\frac{d}{dq}\ln M_{t+1}(q)\big|_{q=-\gamma_l w} - f\Big)p\cdot e^{-\gamma_l(e-fw)} + \Big(\frac{d}{dq}\ln M_{t+1}(q)\big|_{q=-\gamma_l w} - b\Big)\cdot(1-p)\cdot e^{-\gamma_l(a-bw)} = 0 \equiv$$

$$\equiv \Big(\frac{d}{dq}\ln M_{t+1}(q)\big|_{q=-\gamma_l w} - b + (b-f)\Big) +$$
$$+\Big(\frac{d}{dq}\ln M_{t+1}(q)\big|_{q=-\gamma_l w} - b\Big)\cdot e^{-\gamma_l[(a-e)-(b-f)w]-\ln\frac{p}{1-p}} = 0.$$

Define the following constants

$$c_1 := -\gamma_l(a-e) - \ln\frac{p}{1-p} = -\gamma_l(i-\theta)(v + \hat{w} + \chi_l\underline{w}) - \ln\frac{p}{1-p},$$

$$c_2 := -(b-f) = -(i-\theta)\chi_l,\ c_3 := -\gamma_l c_2 = \gamma_l(i-\theta)\chi_l,$$

$$c_4 := \mu - i\chi_l,\ c_5 := -\gamma_l\sigma^2,$$

which for Gaussian distribution of gross (multiplicative) returns R_{t+1} implies

$$\frac{d}{dq}\ln M_{t+1}(q)\big|_{q=-\gamma_l w} - b = c_3 + c_4 w.$$

Assume in addition that $w \neq -c_4/c_5$. Then the lender's f.o.c. may be written as

$$e^{c_1+c_3w} = -\frac{c_4+c_5w-c_2}{c_4+c_5w} = \frac{c_2}{c_4+c_5w} - 1.$$

Note that if $c_2/(c_4+c_5w) \leq 1$, then the equality above has no solution as its right hand side is non-positive. Thus we need to have $c_2/(c_4+c_5w) > 1$ which is equivalent to $w < (c_2-c_4)/c_5$. For plausible parameter values c_2 is always negative, hence the latter also implies that $w \neq -c_4/c_5$. Therefore given $c_2 < 0$ the lender's f.o.c. is equivalent to

$$c_1 + c_3w - \ln\left(\frac{c_2}{c_4+c_5w} - 1\right) = 0.$$

if and only if $w < (c_2-c_4)/c_5$. Otherwise there is no interior solution.

(ii) Denote the set of admissible corners of the lender problem as E_L, set $e_l = \underline{w} + \chi_l^{-1}(v+\hat{w})$. If $v + \hat{w} < 0$ then $E_L = \{0, \max\{0, e_l\}\}$, otherwise $E_L = \{0, \max\{0, e_l\}, \underline{w}\}$. Hence the lender problem has at most three corner solutions. Any admissible internal solution that satisfies the lender's f.o.c. is a strict (local) maximum.

Fact 2 (Local strict concavity) *Under CARA with Gaussian asset returns lender's expected utility $\mathbb{E}u(V_{t+1}(w))$ is strictly concave on any real interval $I \subset (0,e) \setminus E_L$.*

Proof: Note that $f(w) \equiv 1$ is linear, hence both convex and concave. By previous point the expected utility is eq. (2) is a weighted average of one concave function and two strictly concave functions. The outcome is strictly concave. □

Fact 2 allows us to simplify the utility maximization problem. Define $E_L^{'}$ as the set of (potential) expected utility maximizing portfolios for the lender problem, initially set $E_L^{'} := E_L$. Denote lender's optimal portfolio choice as $w_l^* \equiv w_l^*(i_{bl})$. Let $E_L^{'} := E_L \cup S$ where S is a set of internal solutions of lender's f.o.c's. A number of internal solutions $\#S$ is at most two. The optimal lender's portfolio is given by

$$w_l^* = \underset{w \in E_L^{'}}{\text{argmax}} \left\{\mathbb{E}u(V_{t+1}(w))\right\}.$$

Having characterized the set of feasible corners and the interior solution we may derive supply of credit as a functions of interest rate.

(iii) Monetary **supply** of credit s depends on the difference between the optimal portfolio choice and the largest position that l could finance without additional funding. For sufficiently small credit supply $s > 0$ the optimal portfolio choice w_l^* is an internal solution of the lender problem and lies below $e_l := \underline{w} + \chi_l^{-1}(v+\hat{w})$. There are two distinct cases. If $v + \hat{w} \geq 0$ and if in addition credit supply $s < v + \hat{w}$, then $w_l^* \geq \underline{w}$. Bank l finances current loan with surplus cash. As buying large quantities of risky assets causes price slippage, for every extra unit of asset that l does not to buy it can grant a loan of c^{-1} units of money. Supply of credit is given by $c^{-1}(e_l - w_l^*)$. By the similar token, if $v + \hat{w} < 0$ then we have $w_l^* < \underline{w}$. Bank l finances the loan by selling speculative asset. As disposing of large quantities of assets incurs cost, for every unit of asset l sells it obtains c units of money that can be lent on the inter-bank market. Supply of credit is $c(e_l - w_l^*)$. In our notation credit supply in both cases may be jointly expressed as

$$s(i) = \chi_l(e_l - w_l^*) = \chi_l(\underline{w} - w_l^*) + (v + \hat{w}).$$

Again we may determine $\underline{i}_l$ as a limit interest rate implied by $s \nearrow 0^+$.

The solution of lender's problem satisfies f.o.c at $w_l^* = \underline{w} + \chi_l^{-1}(v + \hat{w} - s(i))$. To find $\underline{i}_l$ it is enough to substitute w_l^* to lender's f.o.c. and solve for i in the limit. First compute

$$c_1 + c_3 w_l^* = -\gamma_l(i-\theta)(v + \hat{w} + \chi_l \underline{w}) - \ln\frac{p}{1-p} + \gamma_l(i-\theta)\chi_l(\underline{w} + \chi_l^{-1}(v + \hat{w} - s(i)) =$$

$$= -\gamma_l(i-\theta)s(i) - \ln\frac{p}{1-p},$$

$$c_4 + c_5 w_l^* = \mu - i\chi_l - \gamma_l\sigma^2(\underline{w} + \chi_l^{-1}(v + \hat{w} - s(i)) = (\bar{i}_l - i)\chi_l + \gamma_l\sigma^2\chi_l^{-1}s(i).$$

Since we already have $c_2 = -(i-\theta)\chi_l$, in the limit it holds that

$$\lim_{s\nearrow 0^+} \frac{c_2}{c_4 + c_5 w_l^*} - 1 = -\frac{(i-\theta)\chi_l}{(\bar{i}_l - i)\chi_l} - 1 = \frac{(i - \bar{i}_l) + (\bar{i}_l - \theta)}{i - \bar{i}_l} - 1 = \frac{\bar{i}_l - \theta}{i - \bar{i}_l}.$$

After inserting these results into f.o.c. we obtain

$$\lim_{s\nearrow 0^+}\left\{c_1 + c_3 w_l^* - \ln\left(\frac{c_2}{c_4 + c_5 w_l^*} - 1\right)\right\} = 0 \equiv -\ln\frac{p}{1-p} - \ln\frac{\bar{i}_l - \theta}{i - \bar{i}_l} = 0 \equiv$$

$$\equiv \ln\frac{p(\bar{i}_l - \theta)}{(1-p)(i - \bar{i}_l)} = 0 \equiv \frac{p(\bar{i}_l - \theta)}{(1-p)(i - \bar{i}_l)} = 1 \equiv$$

$$\equiv \bar{i}_l - p\theta = (1-p)i \equiv i = (\bar{i}_l - p\theta)/(1-p).$$

Therefore the smallest interest rate that the lender is willing to accept is

$$\underline{i}_l = \bar{i}_l\frac{1}{1-p} - \theta\frac{p}{1-p}.$$

The more loans the lender l has, the more she is reluctant to lend. The lender never lends at the interest rate it would accept as a borrower. Hence no bank l would ever trade with itself as $\underline{i}_l > \bar{i}_l$.

(iv) The maximum size of a loan that lender is eager to grant at interest rate $\underset{\sim}{i}$ is the volume that makes $\underset{\sim}{i}$ her reservation interest rate. Given interior solution, this volume may be obtained as

$$\underset{\sim}{i} \approx (\bar{i}_l - \gamma_l\chi_l^{-2}\sigma^2\underset{\sim}{w})\frac{1}{1-p} - \theta\frac{p}{1-p} = \underline{i}_l - \gamma_l\chi_l^{-2}\sigma^2\underset{\sim}{w}\frac{1}{1-p} \equiv$$

$$\equiv \underset{\sim}{w} \approx \frac{1}{\gamma_l\chi_l^{-2}\sigma_{t+1}^2}(\underline{i}_l - \underset{\sim}{i})(1 - p_{t+1}). \tag{3}$$

□

A.3 Proof of Proposition 3 (Market aggregates)

Proof: **(i)** Given the agreed interest rate i, supply equates demand if

$$\sum_{b\in B} f_b w_b = -\sum_{l\in L} f_l w_l.$$

After substituting the formulas from parts iv), Propositions 1 and 2,we obtain

$$\sum_{b\in B} f_b c_b(\bar{i}_b - i) = -\sum_{l\in L} f_l c_l(\underline{i}_l - i)(1-p) \equiv \sum_{b\in B} f_b c_b \bar{i}_b + (1-p)\sum_{l\in L} f_l c_l \underline{i}_l = i\big[\sum_{b\in B} f_b c_b + (1-p)\sum_{l\in L} f_l c_l\big] \equiv$$

$$\equiv \; i_{eq} = \frac{\sum_{b\in B} f_b c_b \bar{i}_b + (1-p)\sum_{l\in L} f_l c_l \underline{i}_l}{\sum_{b\in B} f_b c_b + (1-p)\sum_{l\in L} f_l c_l}.$$

The interest rate i_{eq} implies that the desired volumes of trade on both sides of the market are equal.

(ii) To obtain an interest rates which generates demand for credit equal to V observe that

$$i = (\bar{i}_b - c_b^{-1} w_b) \;\equiv\; i c_b = c_b \bar{i}_b - w_b \;\equiv\; i f_b c_b = f_b c_b \bar{i}_b - f_b w_b \Rightarrow i\sum_{b\in B} f_b c_b = \sum_{b\in B} f_b c_b \bar{i}_b - \sum_{b\in B} f_b w_b \equiv$$

$$\equiv \; i_c = \frac{\sum_{b\in B} f_b c_b \bar{i}_b - V}{\sum_{b\in B} f_b c_b}$$

where $V = \sum_{b\in B} f_b w_b$ is the demand for credit that corresponds to reservation interest rate i_c.

(iii) An interest rates which generates demand for credit equal to V may be obtained from

$$i = \underline{i}_l - c_l^{-1} w_l \frac{1}{1-p} \;\equiv\; i(1-p)c_l = (1-p)c_l \underline{i}_l - w_l \;\equiv$$

$$\equiv\; i(1-p) f_l c_l = (1-p) f_l c_l \underline{i}_l - f_l w_l \Rightarrow i(1-p)\sum_{l\in L} f_l c_l = (1-p)\sum_{l\in L} f_l c_l \underline{i}_l - \sum_{l\in L} f_l w_l \;\equiv$$

$$\equiv\; i_c = \frac{\sum_{l\in L} f_l c_l \underline{i}_l + \frac{1}{1-p} V}{\sum_{l\in L} f_l c_l},$$

where $V = -\sum_{l\in L} f_l w_l$ is the supply of credit that corresponds to reservation interest rate i_c. □

B Long-run stationarity of deposits

In every period n local consumers approach bank k to place their deposits of $n^{-1}h_k$. Each consumer, after placing her deposit at time t, liquidates it at $t+1+S_t$ where each S_t is a Poisson distributed random variables with intensity $\lambda - 1 > 0$. Banks hold deposits for λ periods on average. In the long-run the expected volume of deposits held by k is

$$\lim_{t+1\to+\infty} n\Big(n^{-1}h_k \cdot \mathbb{P}(S_t \geq 0) + \ldots + n^{-1}h_k \cdot \mathbb{P}(S_1 \geq t-1)\Big) = h_k \sum_{i=0}^{+\infty}\sum_{j=i}^{+\infty} (\lambda-1)^j \frac{e^{-(\lambda-1)}}{j!} =$$

$$= h_k \sum_{j=0}^{+\infty} (1+j)(\lambda-1)^j \frac{e^{-(\lambda-1)}}{j!} = [1 + (\lambda-1)]h_k = \lambda h_k.$$

Let H_k be a random variable, depicting net deposits placed in bank k at given point in time. In the long-run $\mathbb{E}H_k = 0$, as the expected volume of deposits is constant in the limit. Define random variable $Q \equiv 1 - \mathbb{1}_{\{S_t=0\}} - \ldots - \mathbb{1}_{\{S_1=t-1\}}$ that counts the net number of deposits placed by a single customer. Deposits placed by the same customer across different periods of time are independent, therefore

$$\lim_{t+1\to+\infty} \mathbb{V}ar\, Q = \mathbb{P}(S_t = 0)(1 - \mathbb{P}(S_t = 0)) + \mathbb{P}(S_{t-1} = 1)(1 - \mathbb{P}(S_{t-1} = 1)) + \ldots =$$

$$= 1 - \sum_{j=0}^{+\infty} \mathbb{P}(S_{t-j} = j)^2.$$

Deposits placed by n different customers are independent and worth $n^{-1}h_k$ each, at sufficiently large $t+1$ variance of net deposits is approximately equal

$$\mathbb{V}ar\, H_k = \frac{h_k^2}{n}\Big(1 - e^{-2(\lambda-1)} \sum_{j=0}^{+\infty} \frac{(\lambda-1)^{2j}}{(j!)^2}\Big). \tag{4}$$

In the long-run the volume of deposits held by k is stationary, standard deviation of H_k depends on both λ and n.

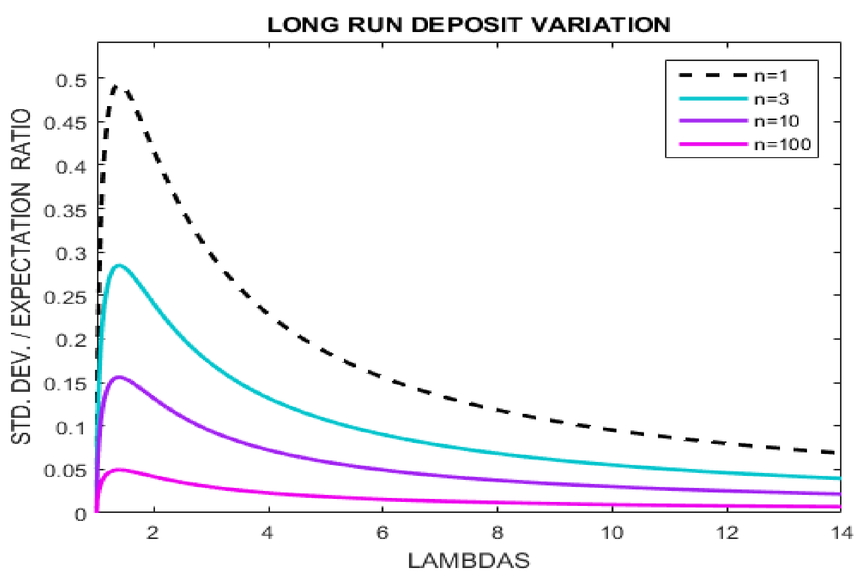


Figure 8: Long-run relative variation of deposits as a function λ, plotted for $n \in \{1, 3, 10, 100\}$ agents in the region.

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