Estimating nonbalanced growth paths: three-factor model and structural change

Qianxue Zhang*

Abstract: This paper proposes a new method of estimating non-stationary non-linear dynamic general equilibrium models with an application in the structural transformation context. By combining different sectoral growth rates with different factor intensities, I construct a three-sector general equilibrium model that generates a hump-shaped path for the size of the manufacturing sector while replicating the observed pattern of increasing and decreasing shares in services and agriculture. Using data from the United States and Sweden over the past decades, I implement a relaxation algorithm to solve the model for non-stationary paths of sectoral shares for given parameter values and then estimate the best fit for the model's parameters via two-stage nonlinear least squares. A counterfactual exercise for China shows the importance of trade in explaining structural change. This method can be easily generalized to solve other similar problems.

Keywords: Nonbalanced growth, structural transformation, relaxation method

JEL: L16, O13, O14, O40, O41

*Department of Economics, University of Warwick. Email: qianxue.zhang@warwick.ac.uk
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1 Introduction

The structural transformation literature is growing rapidly. Since Acemoglu and Guerrieri (2008)'s influential paper, more and more attention has been paid to the non-balanced growth paths in different sectors. Previously, studies about growth models generally focused on balanced growth paths, where all variables grow at a constant rate in equilibrium. The traditional method is to transform all variables by dividing their growth rates to turn the problem into a stationary environment and find the saddle point. However, in the structural transformation context like Acemoglu and Guerrieri (2008), the economy only has an asymptotic steady state and will never reach it. During the whole transition process, the growth rates in each sector are different. Thus, we face a non-stationary environment and are not able to proceed with conventional methods.

This paper contributes to the literature by providing a new method of estimating non-stationary nonlinear dynamic general equilibrium models, with the application of structural transformation in several countries over the past decades. The non-linearity and non-stationarity of the model make the standard technique infeasible. We cannot linearize the system due to the lack of a steady state or take out the trend due to the different and varying sectoral growth rates. To tackle these problems, I combine a relaxation method with nonlinear two-stage least-square regression to estimate the model parameters and conduct statistical inference.

I deal with the non-stationarity with the relaxation method, which is an iterative method to find the solution to a two-point boundary problem. Given the initial and endpoint conditions, the relaxation algorithm can find the solution to a system of nonlinear equations (our dynamic equations) very fast. This method has been used in many contexts to find the transition path towards the steady state. For example, Trimborn et al. (2008) have applied this method to continuous growth models, and they show that it is powerful and efficient for dealing with complicated dynamic systems. After finding the transition paths of certain variables, we want to estimate the parameters of the model such that the model can generate similar patterns as the observed data. In this stage, I apply the standard two-stage nonlinear least-square technique and use bootstrap to construct confidence bands.

I combine the model of Acemoglu and Guerrieri (2008) and Ngai and Pissarides (2007) into a three-sector growth model with different factor intensities and technological growth. With land, labor and capital as inputs, I show that the model is able to generate hump-shaped manufacturing labor shares under certain conditions while keeping track of the changing patterns in the other two sectors. I collect post-WWII data for the US and one century's data for Sweden from various sources, and I use the relaxation algorithm to find the equilibrium paths. Then I estimate the best-fit parameters by the non-linear least squares method. The growth rates for the US agriculture, manufacturing and service sectors are 2.19%, 2.23% and 0.03%, respectively. The utility weights are largely placed on the service goods and the elasticity of substitution across the three sectors is low. As the post-WWII data for the US from the Bureau of Economic Analysis (BEA) has been consistently revised, I also use the same data as in Herrendorf et al. (2015) to ensure that the estimated differences are mainly from the changes in data itself.

For Sweden, the growth rate in the three sectors is 4.33%, 1.39% and 0.01%. The model performs well in generating the hump-shaped labor share in the manufacturing sector while tracing the change in the other two sectors. Different from the US, manufacturing goods have the highest utility weight. As the utility weight is also related to the share of the sectoral investment, I conclude that a similar model assuming investment only comes from the manufacturing sector is not a good assumption for Sweden. Overall, the results show that the estimation method is useful in studying structural transformation under different settings.

I demonstrate the usefulness of this method in conducting counterfactual analysis. I use China as a case study, as structural changes in the country have been closely linked to international trade. The question I aim to answer is: What would have been the sectoral allocations in China over the past four decades if trade was not present? To answer this question, I first estimate the model parameters for China from 1978 to 2019, taking into account the impact of trade.

Next, I run the model using those parameters until the transversality condition is met. Comparing the results of both simulations highlights the significant influence that trade has had on China's labor and GDP allocations over time.

Related literature: This paper is closely related to the structural transformation literature. There are broadly two recognized driving forces for the structural change in the literature: the non-homothetic preference on the demand side (e.g. Kongsamut et al. (2001); Foellmi and Zweimüller (2008); Duarte and Restuccia (2010); Boppart (2014)) and the technological progress on the supply side (e.g. Ngai and Pissarides (2007); Acemoglu and Guerrieri (2008); Herrendorf et al. (2015); Alvarez-Cuadrado et al. (2018)). Although a lot of efforts have been paid to understand the driving forces behind it, the generation of the structural transformation among all three sectors remains challenging and is still under intensive investigation. In a recent survey paper, Van Neuss (2019) finds mixed evidence of the relative importance of supply-side and demand-side effects, especially when considering all three sectors. He claims it is relatively easy to replicate the declining labor share in the agricultural sector and the increasing share in the non-agricultural sector. However, it is more difficult to generate the hump-shaped labor and GDP share in the manufacturing sector. I develop a three-factor, three-sector growth model with different sectoral technological growth and factor intensities and show that the supply-side forces can generate the hump-shaped labor and value-added share in the manufacturing sector as well as match the opposite trends in the other two sectors.

This paper also contributes to the structural transformation literature by introducing a new method of estimating non-stationary general equilibrium models. The structural change literature relies on calibration technique intensively due to the complexity of the estimation procedure (e.g., Herrendorf et al. (2021), Herrendorf et al. (2015), Herrendorf et al. (2013),Porzio et al. (2021)). Only limited papers estimate the model structurally (e.g., Fajgelbaum and Redding ((2022)). Additionally, it is common practice to estimate demand and supply systems separately or in a sequential manner. However, by utilizing a combination of the relaxation method and the nonlinear two-stage least-squares technique, it is possible to estimate the entire system simultaneously with a reasonable level of computational efficiency. This approach not only facilitates the estimation process but also allows for standard statistical inference to be performed. Furthermore, it is also easier to conduct counterfactuals.

In this regard, this paper is an application of the relaxation method. The relaxation method is explained in Press et al. (2007) comprehensively. It has been used in some research but has not been widely applied in growth models. Trimborn et al. (2008) write an algorithm using the relaxation method to solve standard growth models in continuous time. Recently applications of the algorithm include Dalgaard and Strulik (2017), Grossmann et al. (2013), Groth and Madsen (2016) etc. I apply the relaxation method to a discrete-time nonstationary model, where the solution is different from continuous models but more straightforward as each grid implies one year in my algorithm. The code is freely available and can be generalized to similar applications directly.

The paper is organized as follows. Section 2 presents the model. Section 3 describes the path-finding algorithm and the estimation method. This section is of independent interest as the method can be easily extended to other similar problems. Section 4 describes the data used for this study. The estimation strategy and the results are displayed in Section 5. Section 6 concludes.

2 A closed-economy model

This section presents the three-sector model of structural change in a closed economy and derives the equilibrium. I show that the model can generate a hump-shaped labor share in the manufacturing sector under certain conditions. Note in a closed economy, value-added is equal to GDP by definition. I will use them interchangeably in the paper.

2.1 The consumer's problem

The total labor is fixed to be one for the whole economy in each period and the problem can be formed as a representative agent problem. The agent owns a fixed quantity of land H, one unit of labor L and capital K_t in each period.

The prices of the endowments are wage w_t, rent for land R_t and rent for capital r_t. The agent maximizes discounted lifetime utility by consuming three sectoral goods,

$$\max \sum_{t=0}^{\infty} \beta^t log(C_t) = \max \sum_{t=0}^{\infty} \beta^t log \left(\left[\gamma_1 c_{1t}^{\frac{\sigma-1}{\sigma}} + \gamma_2 c_{2t}^{\frac{\sigma-1}{\sigma}} + \gamma_3 c_{3t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right) \tag{2.1}$$

subject to the budget constraint

$$(I_{1t} + c_{1t})p_{1t} + (I_{2t} + c_{2t})p_{2t} + (I_{3t} + c_{3t})p_{3t} = w_t L_t + R_t H_t + r_t K_t. \tag{2.2}$$

where the subscripts $i = 1, 2, 3$ represent the agriculture, manufacturing and service sectors. The intertemporal elasticity of substitution is one and the within-period CES utility function has taste parameters γ_i ($\sum_i \gamma_i = 1$) and the elasticity of substitution σ between consumption categories. The agent needs to decide her savings I_{it} and consumption c_{it} in each period. The sectoral savings equal investments and are then combined into final investment goods in a CES way same as the utility function.[1]

Thus, the capital accumulates according to the following equation,

$$K_{t+1} = (1-\delta)K_t + \left[\gamma_1 I_{1t}^{\frac{\sigma-1}{\sigma}} + \gamma_2 I_{2t}^{\frac{\sigma-1}{\sigma}} + \gamma_3 I_{3t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{2.3}$$

Define λ_t as the Lagrangian multiplier and take FOCs for consumption in each sector we have,

$$\lambda_t p_{1t} = C_t^{\frac{1-\sigma}{\sigma}} \gamma_1 c_{1t}^{\frac{-1}{\sigma}} \tag{2.4}$$

$$\lambda_t p_{2t} = C_t^{\frac{1-\sigma}{\sigma}} \gamma_2 c_{2t}^{\frac{-1}{\sigma}} \tag{2.5}$$

$$\lambda_t p_{3t} = C_t^{\frac{1-\sigma}{\sigma}} \gamma_3 c_{3t}^{\frac{-1}{\sigma}} \tag{2.6}$$

If we calculate each sectoral consumption as a function of the aggregate consumption and the prices, by adding them together, we get

$$\frac{1}{C_t} = \lambda_t \left[\gamma_1^\sigma p_1^{1-\sigma} + \gamma_2^\sigma p_2^{1-\sigma} + \gamma_3^\sigma p_3^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{2.7}$$

[1] It is the same if we assume there is one final good that can be either consumed or invested.

We can define the aggregate price index as

$$P_t \equiv \left[\gamma_1^\sigma p_1^{1-\sigma} + \gamma_2^\sigma p_2^{1-\sigma} + \gamma_3^\sigma p_3^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{2.8}$$

Therefore, the expenditure share of each sector is given by

$$\frac{p_{it}(c_{it}+I_{it})}{P_t(C_t+I_t)} = \frac{p_{it}c_{it}}{P_tC_t} = \gamma_i c_{it}^{\frac{\sigma-1}{\sigma}} C_t^{\frac{1-\sigma}{\sigma}} = \left[1 + \frac{\gamma_j}{\gamma_i}\left(\frac{c_{jt}}{c_{it}}\right)^{\frac{\sigma-1}{\sigma}} + \frac{\gamma_k}{\gamma_i}\left(\frac{c_{kt}}{c_{it}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{-1}. \tag{2.9}$$

Note here the sectoral consumption share is the same as the sectoral expenditure share due to the assumption of the investment goods' production function.

Use (2.4) - (2.6) again to replace the consumption ratios with the price ratios,

$$\frac{p_{it}c_{it}}{P_tC_t} = \left[1 + \left(\frac{\gamma_j}{\gamma_i}\right)^\sigma \left(\frac{p_{it}}{p_{jt}}\right)^{\sigma-1} + \left(\frac{\gamma_k}{\gamma_i}\right)^\sigma \left(\frac{p_{it}}{p_{kt}}\right)^{\sigma-1}\right]^{-1}. \tag{2.10}$$

Here the elasticity of substitution, relative prices and taste parameters determine the consumption share of each sector.

2.2 The firm's problem

Now we move to the firm's problem in order to solve the relative prices. The agriculture, manufacturing and service sectors produce output according to the following functions respectively,

$$f_t = A_{1t}L_{1t}^{1-\alpha_1}H^{\alpha_1}, g_t = A_{2t}L_{2t}^{1-\alpha_2}K_{2t}^{\alpha_2}, s_t = A_{3t}L_{3t}^{1-\alpha_3}K_{3t}^{\alpha_3}.$$

All sectors use labor as input and $L_{1t} + L_{2t} + L_{3t} = L_t = 1$.The agriculture sector also uses land, and the manufacturing and service sectors use capital as inputs. Each sector has its own factor intensity α_i and Hicks-neutral technology A_{it}, which are potentially different from others. A_{it} grows at a constant and exogenous rate T_i. Workers are mobile across sectors and the wages are the same. Similarly, the returns to capital are the same in sectors two and three.

Equalizing the factor prices we get,

$$w_t = \frac{p_{1t}(1-\alpha_1)f_t}{L_{1t}} = \frac{p_{2t}(1-\alpha_2)g_t}{L_{2t}} = \frac{p_{3t}(1-\alpha_3)s_t}{L_{3t}}. \tag{2.11}$$

$$r_t = \frac{p_{2t}\alpha_2 g_t}{K_{2t}} = \frac{p_{3t}\alpha_3 s_t}{K_{3t}}. \tag{2.12}$$

Thus, from (2.11) we can derive the price ratios as below,

$$\frac{p_{2t}}{p_{1t}} = \frac{1-\alpha_1}{1-\alpha_2}\frac{A_{1t}}{A_{2t}}\left(\frac{H_t}{L_{1t}}\right)^{\alpha_1}\left(\frac{L_{2t}}{K_{2t}}\right)^{\alpha_2}, \tag{2.13}$$

$$\frac{p_{2t}}{p_{3t}} = \frac{1-\alpha_3}{1-\alpha_2}\frac{A_{3t}}{A_{2t}}\left(\frac{K_{3t}}{L_{3t}}\right)^{\alpha_3}\left(\frac{L_{2t}}{K_{2t}}\right)^{\alpha_2}. \tag{2.14}$$

Moreover, 2.11 and 2.12 show the relationship between the sectoral capital allocation and labor,

$$\frac{1-\alpha_2}{\alpha_2}\frac{K_{2t}}{L_{2t}} = \frac{1-\alpha_3}{\alpha_3}\frac{K_{3t}}{L_{3t}}. \tag{2.15}$$

Define $\theta = \frac{\alpha_3(1-\alpha_2)}{\alpha_2(1-\alpha_3)}$ and use the condition $K_t = K_{2t} + K_{3t}$, we can derive the capital allocations as functions of total capital stock and sectoral labor allocations,

$$K_{2t} = \frac{K_t L_{2t}}{L_{3t}\theta + L_{2t}}, \tag{2.16}$$

and

$$K_{3t} = \frac{K_t L_{3t}\theta}{L_{3t}\theta + L_{2t}}. \tag{2.17}$$

The last challenge is to solve the labor allocations as a function of capital stock. We obtain the solution by equalizing the ratio of marginal utility to the relative price,

$$\frac{L_{1t}(1-\alpha_2)g_t}{L_{2t}(1-\alpha_1)f_t} = \frac{p_{1t}}{p_{2t}} = \frac{\gamma_1}{\gamma_2}\left(\frac{c_{2t}}{c_{1t}}\right)^{\frac{1}{\sigma}} = \frac{\gamma_1}{\gamma_2}\left(\frac{g_t}{f_t}\right)^{\frac{1}{\sigma}}. \tag{2.18}$$

Note that the last equation holds only when the aggregate investment is combined in the same way as consumption so that

$$\frac{c_{2t}}{c_{1t}} = \frac{g_t - I_{2t}}{f_t - I_{3t}} = \frac{g_t}{f_t}.$$

Similarly,

$$\frac{L_{1t}(1-\alpha_3)s_t}{L_{3t}(1-\alpha_1)f_t} = \frac{p_{1t}}{p_{3t}} = \frac{\gamma_1}{\gamma_3}\left(\frac{c_{3t}}{c_{1t}}\right)^{\frac{1}{\sigma}} = \frac{\gamma_1}{\gamma_3}\left(\frac{s_t}{f_t}\right)^{\frac{1}{\sigma}}. \tag{2.19}$$

Now, given model parameters, (2.18) and (2.19) show that labor allocation in each sector is a function of total capital stock, land endowment and sectoral technologies,[2]

$$\frac{L_{1t}^{1+(1-\alpha_1)(\frac{1}{\sigma}-1)}}{L_{3t}^{1+(1-\alpha_3)(\frac{1}{\sigma}-1)}} = \frac{\gamma_1(1-\alpha_1)}{\gamma_3(1-\alpha_3)}\left(\frac{A_{3t}}{A_{1t}}\right)^{\frac{1-\sigma}{\sigma}}\left(\frac{K_{3t}^{\alpha_3}}{H^{\alpha_1}}\right)^{\frac{1-\sigma}{\sigma}} \tag{2.20}$$

$$\frac{L_{2t}^{1+(1-\alpha_2)(\frac{1}{\sigma}-1)}}{L_{1t}^{1+(1-\alpha_1)(\frac{1}{\sigma}-1)}} = \frac{\gamma_2(1-\alpha_2)}{\gamma_1(1-\alpha_1)}\left(\frac{A_{1t}}{A_{2t}}\right)^{\frac{1-\sigma}{\sigma}}\left(\frac{H^{\alpha_1}}{K_{2t}^{\alpha_2}}\right)^{\frac{1-\sigma}{\sigma}} \tag{2.21}$$

$$\frac{L_{2t}^{1+(1-\alpha_2)(\frac{1}{\sigma}-1)}}{L_{3t}^{1+(1-\alpha_3)(\frac{1}{\sigma}-1)}} = \frac{\gamma_2(1-\alpha_2)}{\gamma_3(1-\alpha_3)}\left(\frac{A_{3t}}{A_{2t}}\right)^{\frac{1-\sigma}{\sigma}}\left(\frac{K_{3t}^{\alpha_3}}{K_{2t}^{\alpha_2}}\right)^{\frac{1-\sigma}{\sigma}}. \tag{2.22}$$

Now we find the sectoral labor share in each period, I will show how the model can generate a hump-shaped labor share in sector two. I follow the literature by assuming that $\sigma \in (0,1)$.

2.3 The hump-shaped labor share

To derive the condition for a hump-shaped labor share in the manufacturing sector, we need to write the growth rate for each variable explicitly. Denote the growth of variable x by g_x. Formally, from the production functions we can get

$$g_f = T_1 + (1-\alpha_1)g_{L_1} \tag{2.23}$$

[2] Note every two equations lead to the third. I present all of them for discussion only.

$$g_g = T_2 + \alpha_2 g_{K_2} + (1-\alpha_2) g_{L_2} \tag{2.24}$$

$$g_s = T_3 + \alpha_3 g_{K_3} + (1-\alpha_3) g_{L_3} \tag{2.25}$$

Combine (2.18) and (2.19) we can derive the relationship between g_{L_2} and g_{L_3}

$$g_{L_2} - g_{L_3} = (\frac{1}{\sigma} - 1)(T_3 + \alpha_3 g_{K_3} + (1-\alpha_3) g_{L_3} - T_2 - \alpha_2 g_{K_2} - (1-\alpha_2) g_{L_2}) \tag{2.26}$$

Since (2.15) shows that

$$g_{K_2} - g_{L_2} = g_{K_3} - g_{L_3}, \tag{2.27}$$

we can simplify (2.26) to

$$g_{L_2} = (1-\sigma)(T_3 - T_2 + (\alpha_3 - \alpha_2) g_{K_3}) + [1 - (1-\sigma)(\alpha_3 - \alpha_2)] g_{L_3} \tag{2.28}$$

Suppose $\alpha_3 > \alpha_2$ and $\sigma \in (0,1)$, we know that the second term is positive since L_3 is growing over time. To have the hump shape in sector two, the first term needs to be negative in the end, meaning that $T_3 < T_2 - (\alpha_3 - \alpha_2) g_{K_3}$. As $\frac{K_{3t}}{K_t}$ is increasing overtime due to the growing L_3, the necessary condition is that $T_3 < T_2$.

Similarly, the relationship between L_1 and L_2 is given by (2.18),

$$g_{L_1} - g_{L_2} = (\frac{1}{\sigma} - 1)(T_2 + \alpha_2 g_{K_2} + (1-\alpha_2) g_{L_2} - T_1 - (1-\alpha_1) g_{L_1}). \tag{2.29}$$

Simplification yields

$$g_{L_2} = \frac{1 - \alpha_1(1-\sigma)}{1 - \alpha_2(1-\sigma)} g_{L_1} + \frac{(1-\sigma)(T_1 - T_2 - \alpha_2 g_{K_2})}{1 - (1-\sigma)\alpha_2} \tag{2.30}$$

In this case, we require the second term to be positive at the beginning. Thus $T_1 > T_2 + \alpha_2 g_{K_2}$ and $(1 - \alpha_1(1-\sigma)) g_{L_1} + (1-\sigma)(T_1 - T_2 - \alpha_2 g_{K_2}) > 0$. With time pass by, however, $(1 - \alpha_1(1-\sigma)) g_{L_1} + (1-\sigma)(T_1 - T_2 - \alpha_2 g_{K_2}) < 0$.

Finally,

$$g_{L_3} = \frac{1-\alpha_1(1-\sigma)}{1-\alpha_3(1-\sigma)} g_{L_1} + \frac{(1-\sigma)(T_1 - T_3 - \alpha_3 g_{K_3})}{1-(1-\sigma)\alpha_3}. \tag{2.31}$$

Since L_3 is increasing over time while L_1 decreases, we have $T_1 > T_3 + \alpha_3 g_{K_3}$.

In a special case when σ is very small, two conditions from (2.30) simplify to $(1-\alpha_1)g_{L_1} \lesseqgtr T_1 - T_2 - \alpha_2 g_{k_2}$. At the initial stage of development, g_{L_1} is in general small, thus the second part will be positive as T_1 is much higher.[3] As industrialization accelerates the move out of agriculture, g_{L_1} becomes more negative (along with the speeding up in capital accumulation). The difference between T_1 and T_2 can no longer compensate for the decline, and the manufacturing share starts to fall. On the contrary, in the later stage of development (as in the post-war US), the change in L_1 is almost zero and T_1 is similar to T_2. Even with a mild capital accumulation rate, the growth rate of L_2 will be negative all the time.

Now let us think about the value-added shares. In this closed economy model, the VA shares are the same as the consumption expenditure shares (or output shares) due to the lack of trade. (2.10) shows that the share of value-added is closely related to the sectoral price ratios. (2.18) and (2.19) enable us to rewrite (2.10) as (take sector two as an example)

$$\frac{p_{2t}c_{2t}}{P_t C_t} = \left[1 + \left(\frac{\gamma_1}{\gamma_2}\right)\left(\frac{f_t}{g_t}\right)^{\frac{\sigma-1}{\sigma}} + \left(\frac{\gamma_3}{\gamma_2}\right)\left(\frac{s_t}{g_t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{-1}. \tag{2.32}$$

Moreover, since we know that

$$\left(\frac{f_t}{g_t}\right)^{\frac{\sigma-1}{\sigma}} = \frac{L_{1t}\gamma_2(1-\alpha_2)}{L_{2t}\gamma_1(1-\alpha_1)}$$

and

$$\left(\frac{s_t}{g_t}\right)^{\frac{\sigma-1}{\sigma}} = \frac{L_{3t}(1-\gamma_2)\alpha_2}{L_{2t}(1-\gamma_3)\alpha_3},$$

[3] Since T_1 is a fixed average growth rate, but g_{L_1} is changing over time.

we can formulate (2.32) as

$$\frac{p_{2t}c_{2t}}{P_tC_t} = \left[1 + \frac{(1-\alpha_2)}{(1-\alpha_1)}\frac{L_{1t}}{L_{2t}} + \frac{(1-\alpha_2)}{(1-\alpha_3)}\frac{L_{3t}}{L_{2t}}\right]^{-1} = \frac{L_{2t}}{L_{2t} + \frac{(1-\alpha_2)}{(1-\alpha_1)}L_{1t} + \frac{(1-\alpha_2)}{(1-\alpha_3)}L_{3t}}. \tag{2.33}$$

Therefore, given sectoral labor shares, the value-added shares can be fully pinned down by the factor intensities α_i.

2.4 Equilibrium

To complete the model, the dynamic parts of the model are the Euler equation and the law of motion for capital,

$$\frac{1}{C_t}\left(\frac{I_{2t}}{c_{2t}}\right)^{\frac{1}{\sigma}} = E_t\left[\frac{\beta}{C_{t+1}}\left(\frac{I_{2t+1}}{c_{2t+1}}\right)^{\frac{1}{\sigma}}\left(1-\delta+\alpha_2\gamma_2 I_{2t+1}^{-1/\sigma}A_{2t}\left(\frac{L_{2t+1}}{K_{2t+1}}\right)^{1-\alpha_2}\right)\right], \tag{2.34}$$

and

$$K_{t+1} = (1-\delta)K_t + I_t \tag{2.35}$$

The transversality condition is

$$\lim_{t->\infty}\beta^t\frac{1}{C_t}\left(\frac{c_{2t}}{I_{2t}}\right)^{-1/\sigma}K_{t+1} = 0, \tag{2.36}$$

Note the Euler equation is different from the standard one-sector model. One unit of consumption today will be transformed into one unit of capital tomorrow after depreciation as usual, but then capital is only used in the manufacturing and services sectors. The return of capital is represented by sector two's marginal product of capital in (2.12) since the return of sectors two and three must be the same. Moreover, the production of sectors two and three will be split into sectoral consumption and investment. Then the sectoral consumption needs to be combined in a CES way to yield utility. Therefore, sectoral investment also shows up in (2.34). Nevertheless, the intuition of balancing forgone consumption today and the gain tomorrow is the same.

Given the initial capital stock K_0, the equilibrium of this economy is a set of sequences $\{c_{2t}, K_t, L_{1t}, L_{2t}\}_{t=0}^{\infty}$ which satisfies equations (2.18), (2.19), (2.23), (2.24) and (2.25).

3 Finding the equilibrium path

This section presents the path-searching algorithm used in this project, which can be generalized to any similar nonstationary path-finding problem without a steady state (or with a very far asymptotic steady state). For the economic growth models, we are in general interested in balanced growth paths. After dividing the variables of interest by their growth rates, we are able to find balanced growth paths and apply the standard techniques. However, in the structural change literature, one interesting feature is the nonblanaced growth in Acemoglu and Guerrieri (2008), where different sectors display distinct growth patterns. In these cases, we are not able to have stationary transformations for all variables in one shot. Moreover, Acemoglu and Guerrieri (2008) have proved that although there is a unique constant growth path where the sector with the slowest growth rate will finally dominate the whole economy, the asymptotic steady state of the economy will not be reached in 3000 years. Therefore, the transition path is more important and interesting than the steady state. Besides, we should focus more on the specific observed path instead of the whole path, given the empirical relevance.

To generate the transition path, we need to first obtain the model parameters. One commonly used technique is calibration and simulation. It involves matching certain moments with the data to calibrate parameters and compare the observed data with the simulated ones (e.g.Uy et al. (2013), Gabardo et al. (2020)). This approach, however, either takes parameter values from other papers or often only takes the long-run mean for choosing the parameters while ignoring other features of the data. Thus the results may be misleading. On the other hand, some papers use a proper econometric method for estimation, but they are generally reduced-form or only use partial information instead of estimating the whole general equilibrium system (e.g., Herrendorf et al. (2021), Herrendorf et al. (2015), Herrendorf et al. (2013), Porzio et al. (2021)). One example is Fajgelbaum and Redding ((2022), which estimates the parameters sequentially, and each following step relies on the last one. Overall, the difficulty of structurally estimating the whole model simultaneously is, at first, the high computational burden. Moreover, for nonlinear dynamic models without a steady state, a proper econometric technique is necessary for estimation. I propose a new estimating strategy that resolves these problems by combining a relaxation algorithm with a nonlinear two-stage least-squares.

3.1 The relaxation method: solve for the equilibrium path

We want to first solve for the equilibrium path given any parameter values. Here I reply on the relaxation method. The relaxation method is a very powerful tool for solving two-point boundary value problems, which is described comprehensively in section 18.3 of Press et al. (2007). Given a trivial initial guess of the whole path, the algorithm will adjust all the values on the path in each iteration to bring them closer to the desired path while maintaining the boundary condition. While the algorithm is widely used in estimating systems of ordinary differential equations via approximation, it is naturally suitable for finite difference equations as our discrete-time model here. This method is particularly useful in solving our optimal growth paths since the optimal solutions need to satisfy our transversality condition given the initial capital stock value. This question is naturally a two-point boundary value problem. Although we do not have a stationary environment, we can set the final boundary condition to be our observed consumption. In this regard, we can generate the non-balanced optimal growth paths very fast given any parameter sets.

3.1.1 A simple example

I will illustrate this method with a very simple example. Suppose we want to solve a three-period optimization problem and we only have two variables, consumption and capital c_t, k_t. Define $X_t = [k_t; c_t]$. The initial condition $F_0(X_0)$ and final condition $F_3(X_3)$ must be met, which are the initial capital stock level and the final transversality condition. In the running periods, two equations

$F_t = [F_{kt}; F_{ct}] = 0$ (the capital accumulation rule and the Euler equation) must be satisfied. In order to find the roots of $F_t = 0$ for all t, the algorithm implements Gauss-Newton's method. Recall that given an initial guess of a vector X^0, Newton's method updates the guess in each iteration n via $X^{n+1} = X^n - J_F(X^n)^{-1}F(X^n)$. Therefore, given the initial guess $X^0 = [X_1^0, X_2^0, X_3^0]$, we can write the whole system in matrix form as[4],

$$\begin{bmatrix} X_1^{n+1} \\ X_2^{n+1} \\ X_3^{n+1} \end{bmatrix} = \begin{bmatrix} X_1^{n} \\ X_2^{n} \\ X_3^{n} \end{bmatrix} - \begin{bmatrix} J_0^n & 0 & 0 \\ J_{11}^n & J_{12}^n & 0 \\ 0 & J_{22}^n & J_{23}^n \\ 0 & 0 & J_3^n \end{bmatrix}^{-1} \begin{bmatrix} F_0(k_1^n) \\ F_1(X_1^n, X_2^n) \\ F_2(X_2^n, X_3^n) \\ F_3(X_3^n) \end{bmatrix}$$

where

$$J_0^n = \frac{\partial F_0}{\partial X_1^n}, J_3^n = \frac{\partial F_3}{\partial X_3^n}, J_{ij}^n = \frac{\partial F_i}{\partial X_j^n}, i, j = 1, 2.$$

The whole process works as the following:

1. Given some initial guesses for X^0, we can compute the value for all dynamic equations $F(X)$ and the corresponding Jacobian matrix.

2. We use the Gauss–Jordan elimination and solve for X_3^{n+1} first. Backsubstitution toward the top gives us X_1^{n+1} and X_2^{n+1}. When the difference between X^{n+1} and X^n is small enough, the algorithm will stop. Otherwise, the next iteration begins with the new guess X^{n+1}.

The reader may find the example trivial as optimization problems typically involve very long periods, which may dramatically increase the computation effort. However, we can always divide the whole system into multiple blocked structures as above and thus only need to consider one block at a time.

Suppose we have n_1 initial and n_2 endpoint conditions, with $N = n_1 + n_2$ constraints that need to be satisfied each period. We can treat the first $n_1 + N$ rows

[4]Note the superscript indicates iteration number while the subscript means the time.

as one block. For example, here we have

$$\begin{bmatrix} \Delta X_1 \\ \Delta X_2 \end{bmatrix} = \begin{bmatrix} J_0 & 0 & 0 \\ J_{11} & J_{12} & 0 \end{bmatrix}^{-1} \begin{bmatrix} F_0 \\ F_1 \end{bmatrix}$$

as the first block. Diagonalization of this block leads to

$$\begin{bmatrix} \Delta X_1' \\ \Delta X_2' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & A \\ 0 & 1 & 0 & B \\ 0 & 0 & 1 & C \end{bmatrix}^{-1} \begin{bmatrix} F_0' \\ F_1' \end{bmatrix}$$

Then we take the last row of this block with the next N rows for the next operation,

$$\begin{bmatrix} \Delta X_2' \\ \Delta X_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & C \\ 0 & 0 & J_{22} & J_{23} \end{bmatrix}^{-1} \begin{bmatrix} F_1' \\ F_2 \end{bmatrix}$$

where we diagonalize this block again and maintain the last row for the next block. Note the variables with a prime means after the Gauss-Jordan elimination. We repeat the same procedure until we hit the final boundary condition and start the back substitution. In this regard, the algorithm is highly efficient and the computation time will not increase significantly if only adding more blocks.

3.1.2 Equilibrium path for the model

Now we are ready to study our structural transformation model. We want to find the equilibrium paths for six variables, the labor and the value-added shares in the agriculture, manufacturing and services sectors. We find those series by first searching for the optimal paths for $\{c_{2t}, K_t, L_{1t}, L_{2t}\}_{t=0}^{T}$ and then calculating the relevant shares.[5] The basic idea is the following: the equilibrium path satisfies the initial condition (capital stock is given by K_0) and the endpoint condition, which is restricted by our observed consumption. Given our initial guess of variables for T periods, the algorithm generates the paths for $\{c_{1t}, K_t, L_{1t}, L_{2t}\}_{t=0}^{T}$ using the equilibrium conditions (2.18), (2.19), (2.23)-(2.25). Then it checks if

[5]We only have manufacturing consumption c_{2t} here as it appears in the Euler equation directly and the consumption in the other two sectors is a function of it.

the endpoint condition is satisfied. When the difference between the simulated and observed values is larger than the critical value set by the user, the algorithm updates the path from the endpoint recursively by adding a fraction of the difference to the original variable path. Then it repeats the procedure until the difference is small enough.

Notice here the path is a function of model parameters. Therefore, given different sets of parameters, we will get different equilibrium paths. Our goal is to find the parameters that generate the path that matches the observed data the best.

3.2 Two-stage nonlinear least squares: estimating the parameters

Given this is a perfect foresight model, the functional form is known, thus the standard nonlinear least squares technique can be implemented. The only problem is that the error terms are autocorrelated and the least squares will be biased in a finite sample. Thus I need to use a two-stage nonlinear least squares method as in Cochrane and Orcutt (1949). I first minimize the differences between the simulated and observed series $\min \sum_{t=1}^{T} [y_t - f(\theta, x_t)]^2$. Then I record the residuals $\hat{e}_t$ from the first step. I conduct an Augmented Dickey–Fuller test for all residuals to ensure stationarity and then run AR(1) model on the residuals to get the auto-regressive coefficients ρ. In the second step, I re-estimate the model by

$$\min \sum_{t=1}^{T} [y_t - \hat{\rho} y_{t-1} - f(\theta, x_t) - \hat{\rho} f(\theta, x_{t-1})]^2.$$

To conduct the statistical inference, I simulate 1000 error terms and add them to the estimated series of the first stage and re-estimate the model 1000 times to get the mean and variance of all parameters.

4 Data

I collect data for employment by industry, sectoral value-added shares, real GDP, capital stock and real consumption for the US and Sweden over different time horizons.

For the US, I collect data from 1950-2019. The sectoral employment, compensation of employees, labor shares and GDP shares are from the Bureau of Economic Analysis (BEA) for the US. There was a change in the BEA calculation method around the year 2000 and the data for 1998-2000 has been revised. The revision leads to some gaps between the total employees and sectoral distribution before and after. The land size for agriculture and forestry in 1950 is obtained from Table 1 in the Major Uses of Land in the United States published by the Bureau of Agricultural Economics in 1953, which is 1159 million acres. The capital stock and the real consumption at the 2017 constant national price are from the Penn World Table 10.0. The nominal value-added and real value-added are also obtained from BEA. However, the BEA only provides real VA in 2012 chained price. In order to convert the base year into 2017 to match the capital data, I use the Cyclical Expansion Method as documented in Herrendorf et al. (2015). The main procedures are first calculating the real value using nominal VA and chain-weighted price and then changing the chain-weighted quantity index into fixed-base index using the following equation,

$$\frac{Q_t}{Q_{t-1}} = \sqrt{\frac{\sum_i P_{it-1} y_{it}}{\sum_i P_{it-1} y_{it-1}} \frac{\sum_i P_{it} y_{it}}{\sum_i P_{it} y_{it-1}}} \tag{4.1}$$

Then we can renormalize the quantity index in 2017 to be 1. Then the real value can be calculated as $y_t = Q_t Y_{2017}$.[6]

Another adjustment needs the relative price of investment to consumption from the PWT. As the model assumes consumption and investment have the same price each period, while in the data the prices are not only very different but changing over time. I renormalize the investment by dividing its relative price. Note this will not change other parts of the model.

[6]For more details, see Herrendorf et al. (2015) Appendix A.

For Sweden, I collect data from 1910 to 2010. The data mainly comes from Swedish Historical National Accounts (SHNA) by Schön et al. (2016). I do not extend the data series after 2010 since the SHNA has a different system from most of the other sources, such as the World Bank database, due to the difficulty of obtaining data centuries ago. Changing the source will lead to a very large discrepancy. The SHNA database provides detailed information on sectoral employment, nominal, real value-added and nominal consumption.[7] The manufacturing sector includes building and construction, while the services sector includes transport and communication, private services, public services and services for dwellings. As there is no real consumption data, I calculate the series assuming that the consumer price indices are the same as the value-added indices. The capital stock data is collected from Waldenström (2017). I change the base year to 1910 for consistency with the SHNA data. Data for agricultural land is found in Historical Statistics of Sweden II Table E2, published by Statistics Sweden in 1955.

5 Estimation and Results

I first calculate the labor income shares $1 - \alpha_i$ for each country by dividing the sectoral compensation of employees by the total value-added in each year and then I take the averages. The shares of land and capital are thus assumed to be α_i. This is a standard method in the literature to first calculate the income shares before estimating the production functions. For the US, the labor income shares are 0.25, 0.58 and 0.53 for the three sectors, indicating land and capital shares are 0.75, 0.42 and 0.47. I set the depreciation rate δ to 0.2 for the US. BEA does not report the aggregate depreciation rate but has detailed information for each type of capital, varying from 0.01 to 0.3. I choose 0.2 and also test that changing it to 0.1 or 0.3 does not change the results significantly. β is set to 0.96. There are 6 model parameters left to be estimated, i.e. $\Theta = [T_1, T_2, T_3, \gamma_1, \gamma_2, \sigma]$. Note the initial technology rates are assumed to be 1 in all sectors. The Matlab function *fmincon* is used to find the solution. This algorithm may lead to a local

[7]Real value-added is at the constant price of the average of 1910-1912.

minimum depending on the starting points. I try different starting points and choose the estimation with the minimized objective functions.

The estimation results for the United States are presented in Table 1. The estimated taste parameter γ is 0.011 for agriculture goods and 0.06 for manufacturing goods, meaning that service has the highest utility weight at 0.93. The elasticity of substitution is 0.257, showing that the sectoral goods are poor substitutes. Our main focus is the sectoral technological growth rates. The agriculture sector has the highest growth rate, while the services has the lowest one. This is consistent with the findings in Herrendorf et al. (2014) and Herrendorf et al. (2015).

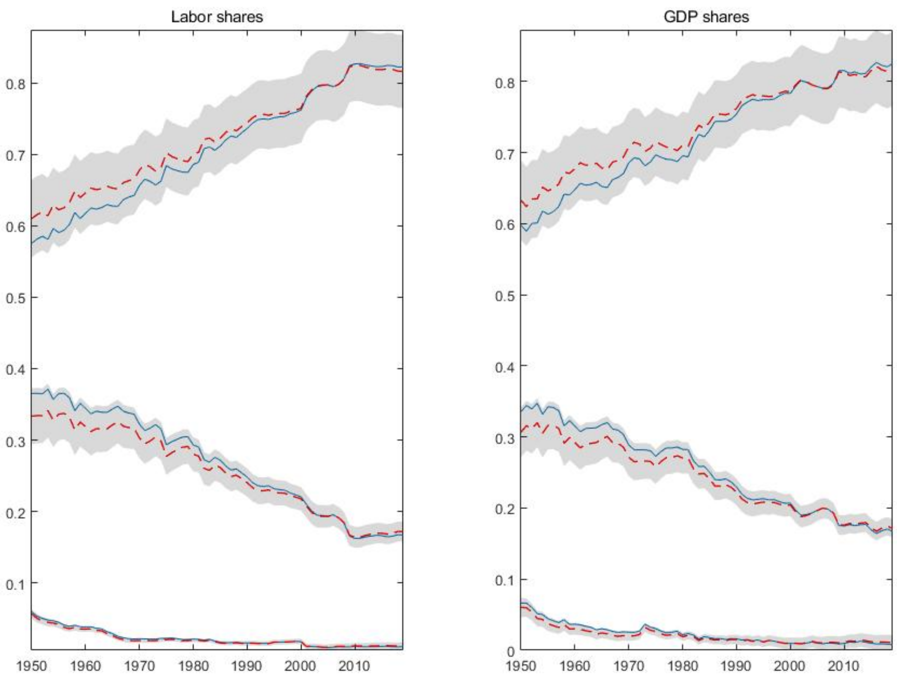
Compared with the results in Herrendorf et al. (2015), the agricultural sectoral growth is halved. But this is largely due to the revised data by BEA instead of the estimation method (see section 5.4 for more details). Notice here the growth rate of the service sector is very low (0.35%). Another reason, except for the changes in data, is that in the model, we assume the returns of capital are the same in the manufacturing and service sectors. Moreover, there is a fixed relationship between capital per labor in these two sectors (Eq. 2.15). This restriction causes the capital in services to be growing too fast compared with data. Thus the TFP will be lower than observed. The bootstrap standard errors are very low in general, meaning that the estimated parameters are fairly accurate.

Figures 1-2 present the estimation results (red lines) and the 95% confidence intervals. The results are reasonably good in capturing the post-WWII structural changes in the US except for the agricultural GDP share. In our closed economy model, the sectoral GDP share is closely related to the labor share (Eq. 2.24), while in the data this is not true since the assumption that consumption shares are the same as VA shares are obviously violated. Another reason for the underestimated GDP and labor share in the manufacturing sector is that I am also matching the real GDP per capita (see section 5.2). Figure 3 presents the model simulated and observed real GDP per capita. To decrease the discrepancy between sectors, I use GDPpc in level in agriculture and log of GDPpc in the other two sectors. As the model matches the data in agriculture and service pretty well, the algorithm does not adjust to match the manufacturing sector.

Parameter	Meaning	Estimation	Standard error
γ_1	Agriculture utility weight	0.0110	0.0047
γ_2	Manufacturing utility weight	0.0645	0.0355
σ	Elasticity of substitution	0.2571	0.0916
T_1	Agriculture technological growth	0.0219	0.0014
T_2	Manufacturing technological growth	0.0223	0.0013
T_3	Service technological growth	0.0035	8.63e-04

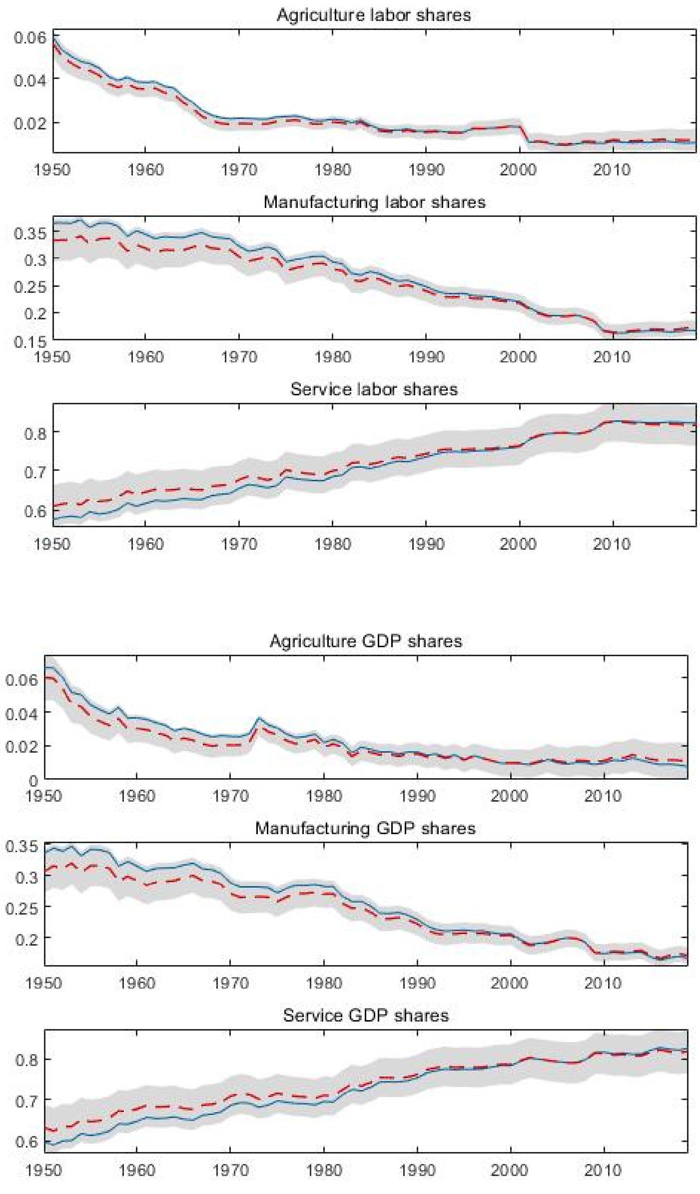
Table 1: Estimated parameters for the US

Figure 1: Estimation results for the US



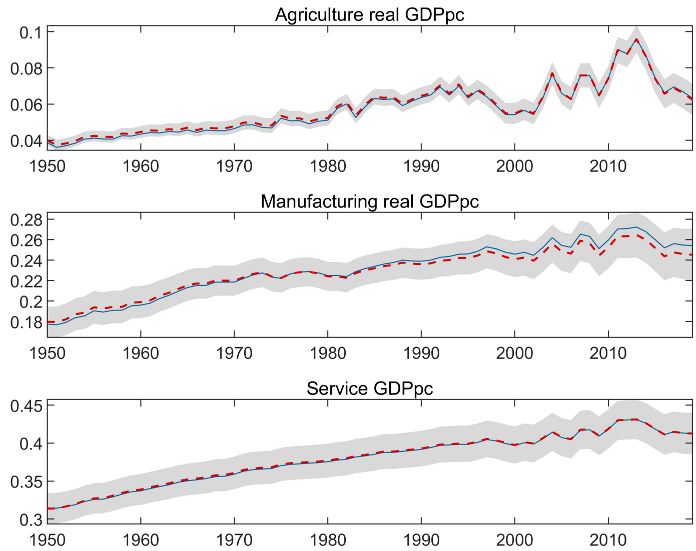
Note: Blue lines are the observed data. Red lines are the average estimation based on 1000 simulations. 95% confidence bands are in grey.

Figure 2: Results for sectoral labor and VA shares



Note: Same as Figure 1.

Figure 3: Results for real GDP per capita



Note: Same as Figure 1.

5.1 Results for Sweden

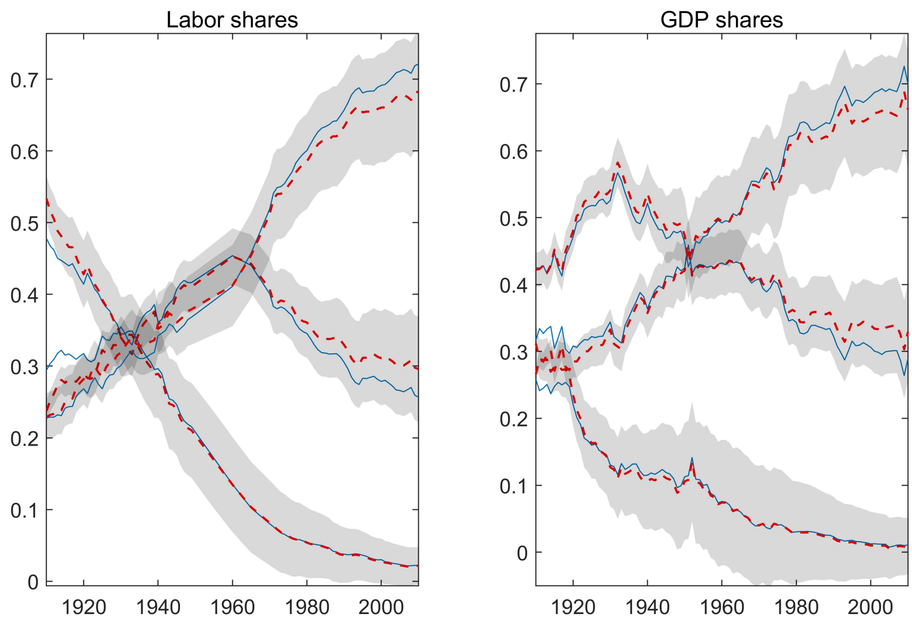
Sweden presents different patterns from the US and is a good source for studying the hump-shaped labor share in the manufacturing sector during the past century. For Sweden, I take the depreciation rate to be 0.03 and β again to be 0.96. I follow a similar process as before and the estimation results are presented in Table 2.

Parameter	Meaning	Estimation	Standard error
γ_1	Agriculture utility weight	0.7874	0.0603
γ_2	Manufacturing utility weight	0.1904	0.0398
σ	Elasticity of substitution	0.1872	0.0992
T_1	Agriculture technological growth	0.0433	0.0070
T_2	Manufacturing technological growth	0.0139	0.0035
T_3	Service technological growth	0.0012	0.0016

Table 2: Estimated parameters for Sweden

Sweden has a much higher growth in the agriculture sector than the US, although the standard error is much larger. The other two sectors have lower growth rates than the US. One striking difference is that almost no utility weight is now put on the service sector, which is the opposite of the US. The agricultural sector contributes 80% to the aggregate utility. Note this is also the weight of how much production is invested. Thus, the assumption that all investment is from the manufacturing sector is not proper for both the US and Sweden. Overall, the model performs well in generating sectoral labor shares.

Figure 4: Sectoral labor shares for Sweden



Note: Blue lines are the observed data. Red lines are the average estimation based on 1000 simulations. 95% confidence bands are in grey.

5.2 Identification

Although the key of this paper is not GDP levels, to identify the parameters, it is crucial to include real GDP series in the nonlinear least squares's objective function. This is because we have fixed the initial capital stock and the final consumption, which determines the whole capital path. Therefore, real GDP and labor shares can pin down the sectoral growth rates if the series themselves are determined. The VA shares are one-to-one related to sectoral labor shares and thus are also pinned down. Thus we can identify the remaining taste parameters and the elasticity of substitution as in (2.20)-(2.22) since they are constructed using the assumptions that consumption ratios are the same as production ratios in (2.23). Without the level of real sectoral GDP, we cannot identify technological progress.

5.3 Compare the results using different data

As the US Bureau of Economic Analysis (BEA) has revised the data substantially, there is a discrepancy between the newly available data and the old one used in previous studies as Herrendorf et al. (2015). I have calculated the changes in quantity indices and sectoral real GDPs using the same method but a different dataset (see Appendix A Figures 8 and 9). To validate the method and ensure the differences are mainly from the data, I first estimate the model using the old dataset provided by Herrendorf et al. (2015). The data required are the sectoral labor income shares, sectoral real GDP, real capital stock, real consumption, sectoral labor and value-added shares and the land size. The only missing data is the real consumption as the authors only estimate the production functions. I collect the consumption data from the PWT 10.0. As PWT's data is based on the 2017 price, I change the base year to 2005 using the price index to ensure all variables are measured in the 2005 constant price. I set $\alpha_1 = 0.61, \alpha_2 = 0.29, \alpha_3 = 0.34$ as them and I assume δ and β are the same as before. The estimation results are as follows:

Parameter	Meaning	Baseline	Use old data	Herrendorf et al.
T_1	Agriculture growth	0.0219	0.0267	0.033
T_2	Manufacturing growth	0.0223	0.0222	0.015
T_3	Service growth	0.0035	0.0036	0.010

Table 3: Estimation result for the US

The growth rates are higher than the baseline when using the old data, which are in line with the steeper lines in Figures 8 and 9. The agricultural growth is lower than their estimation, largely due to the assumption of using land instead of capital as input. Moreover, the fact that I assume the sectoral capital prices are the same in the manufacturing and service sectors causes different estimations from Herrendorf et al. (2015). However, the sum of the growth in the manufacturing and service sector is 2.58%, which is close enough to the 2.5% estimation by them. This means that although the estimation is biased toward the manufacturing sector, this is only due to the distribution assumption of capital stock instead of other issues.

6 A counterfactual exercise: results for China

This section performs a counterfactual exercise using the method described above, assuming there is no international trade in China from 1978 to 2019. China's rise has been studied intensively, and literature reaches a broad consensus that trade plays a significant role in the process. Therefore, this exercise compares the structural change in China with the simulated scenario when there is no trade in China during the sample period.

To perform the exercise, we first need to estimate the parameters using the same procedures as before but change the closed economy model to an open one. To simplify the problem, I treat the net exports as exogenous as given in the data. Therefore, the only difference is the inequality of consumption and production, indicating equations (2.18) and (2.19) are now,

$$\frac{L_{1t}(1-\alpha_2)g_t}{L_{2t}(1-\alpha_1)f_t} = \frac{p_{1t}}{p_{2t}} = \frac{\gamma_1}{\gamma_2}\left(\frac{c_{2t}}{c_{1t}}\right)^{\frac{1}{\sigma}} = \frac{\gamma_1}{\gamma_2}\left(\frac{g_t + NX_2}{f_t + NX_1}\right)^{\frac{1}{\sigma}} \tag{6.1}$$

and

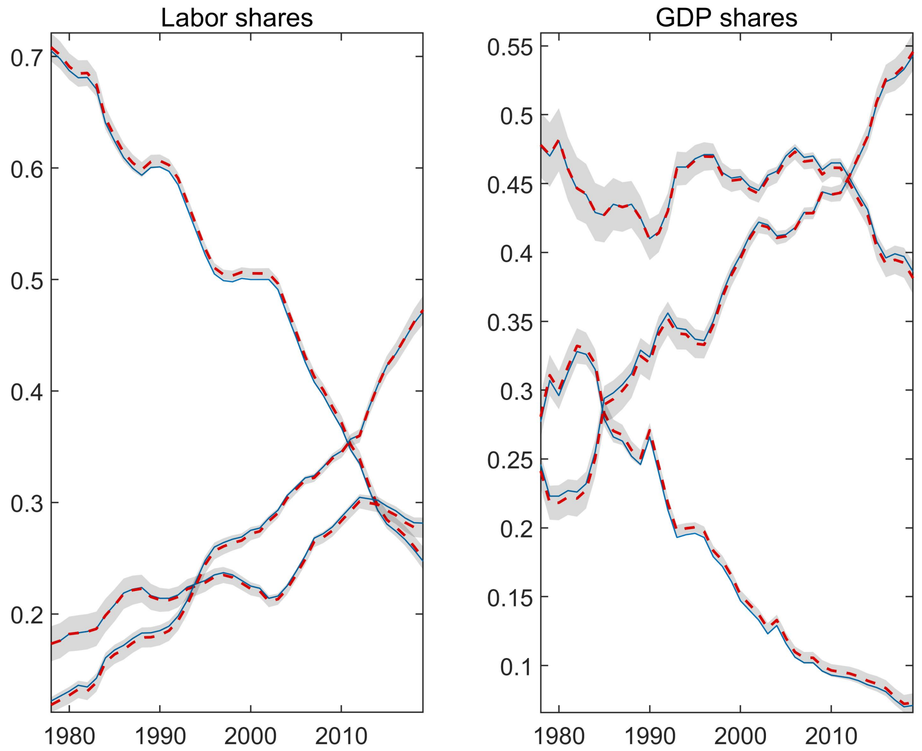
$$\frac{L_{1t}(1-\alpha_3)s_t}{L_{3t}(1-\alpha_1)f_t} = \frac{p_{1t}}{p_{3t}} = \frac{\gamma_1}{\gamma_3}\left(\frac{c_{3t}}{c_{1t}}\right)^{\frac{1}{\sigma}} = \frac{\gamma_1}{\gamma_3}\left(\frac{s_t + NX_3}{f_t + NX_1}\right)^{\frac{1}{\sigma}}, \tag{6.2}$$

where NX means net exports. The sectoral real GDP is calculated using the nominal GDP and GDP deflator from the National Bureau of Statistics of China, where I also obtained trade flows. Then the data is changed to 2017 constant USD using the annual exchange rate from the PWT. The compensation of employees is obtained from National Input-output tables (2002-2020) and I use the average. Agricultural land is from the World Bank Database. The real consumption and capital are from the PWT directly. The estimated parameters are presented in Table 4.

Parameter	Meaning	Estimation	Standard error
γ_1	Agriculture utility weight	0.1977	0.0181
γ_2	Manufacturing utility weight	0.5599	0.0178
σ	Elasticity of substitution	0.3747	0.0159
T_1	Agriculture technological growth	0.0609	0.0081
T_2	Manufacturing technological growth	0.0192	0.0019
T_3	Service technological growth	0.0142	0.0024

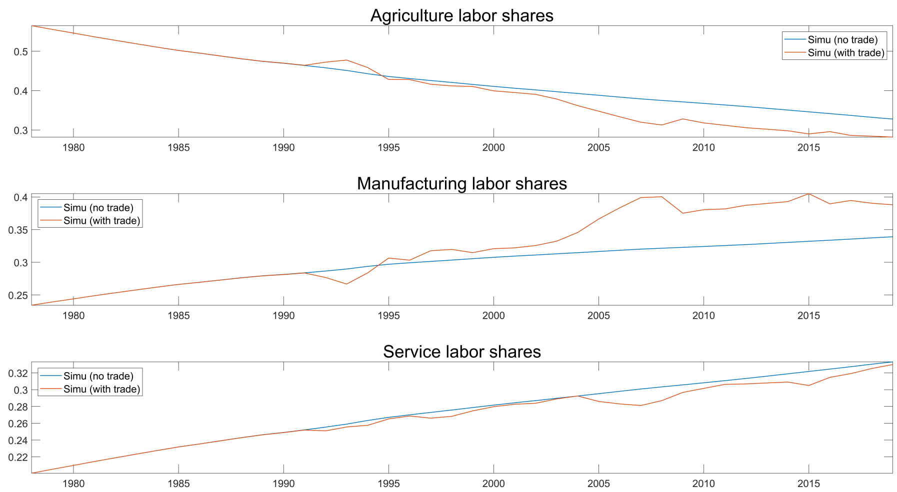
Table 4: Estimated parameters for China

Figure 5: Sectoral labor and GDP shares for China



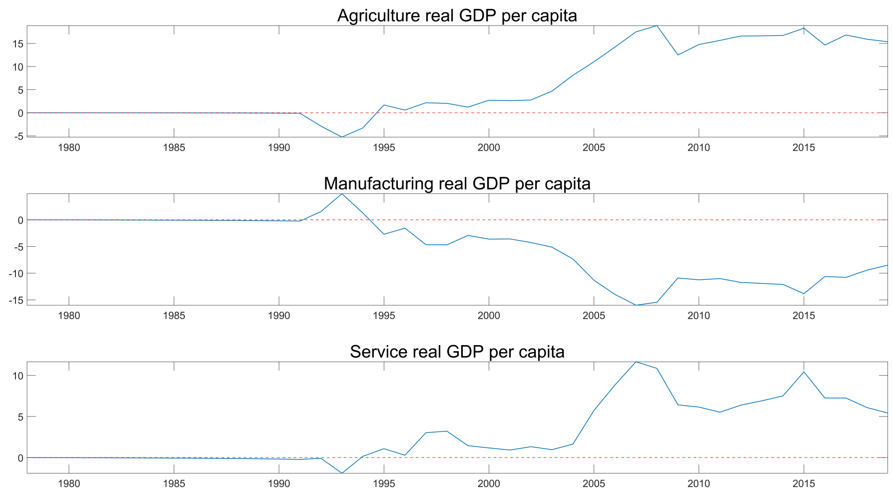
The counterfactual exercise uses the parameters in Table 4 but eliminates all trade flows. The challenge is to choose a proper endpoint condition, as we do not observe the counterfactual consumption level. We can circumvent the problem by simulating the original path until the transversality condition (2.36) is binding and then compare the first 42 periods with what we observe in the data. Figure 6 and 7 illustrate the comparison. Noth as there was no trade data from 1978 to 1991, there is no difference between the two simulations before period 15. Without trade, there would be a significant decline (11.10% on average) in the manufacturing labor share. 8.19% of the labor would stay in agriculture and the rest (2.26%) in service. In terms of GDP, agriculture's and service's real GDP per capita would increase by more than 9% and 4.49%, while manufacturing's decrease by 7.56%. Both figures indicate a considerable shift in the sectoral allocation due to international trade.

Figure 6: Changes in labor shares



Note: The blue lines are the simulations without trade; the red lines are with trade.

Figure 7: percentage changes in real GDP per capita



Note: The blue lines are the simulations without trade; the red lines are with trade.

7 Conclusion

This paper introduces a new method for estimating nonlinear non-stationary dynamic general equilibrium models. By combining the relaxation method with two-stage least-squares regression, I show that this method is useful in estimating structural transformation models, especially among the non-balanced growth paths.

I build a three-sector structural change model based on Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008), where I combine the different sectoral technological progress and factor intensities together. I estimate the model using post-war data for the US and a century's data for Sweden. I find both countries have the highest growth rate in the agriculture sector and the lowest growth in the service sector. The elasticity of substitution is very low in both countries, which justifies the work that studies sectoral growth separately. While almost all of the estimated utility weight is put on the service sector for the US, the agricultural sector contributes 80% to the aggregate utility in Sweden. As the utility weights are the same as the investment shares, I claim it is not suitable to ignore the investment from the service sector in the US and the agricultural sector in Sweden. Overall, the model can replicate the sectoral labor reallocation and value-added reallocation pretty well. I perform a counterfactual analysis for China to examine the impact of trade on structural change. I first simulate the model until the transversality condition is satisfied, then eliminate trade flows and re-run the simulation. By comparing the results of both simulations, I demonstrate the crucial role of trade in shaping China's structural change. This exercise highlights the importance of considering trade when analyzing the evolution of China's sectoral allocations. This estimation method can be generalized to estimate other similar models without steady states, where the traditional solution methods are infeasible.

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Appendix A: Changes in BEA data

Here I present the changes in the quantity indices and relevant calculated real GDPs due to BEA's data revision. The old data is from Herrendorf et al. (2015), where the ultimate source was the BEA before the revision. The new data was collected from BEA in September 2022. The quantity indices are calculated using the method described in section 4, and then the real value-added is calculated at the constant 2005 price. There are clear changes in the data, and the new VA series are much flatter than the old ones, indicating the growth rates will be substantially lower using the new data.

Figure 8: Changes in quantity indices

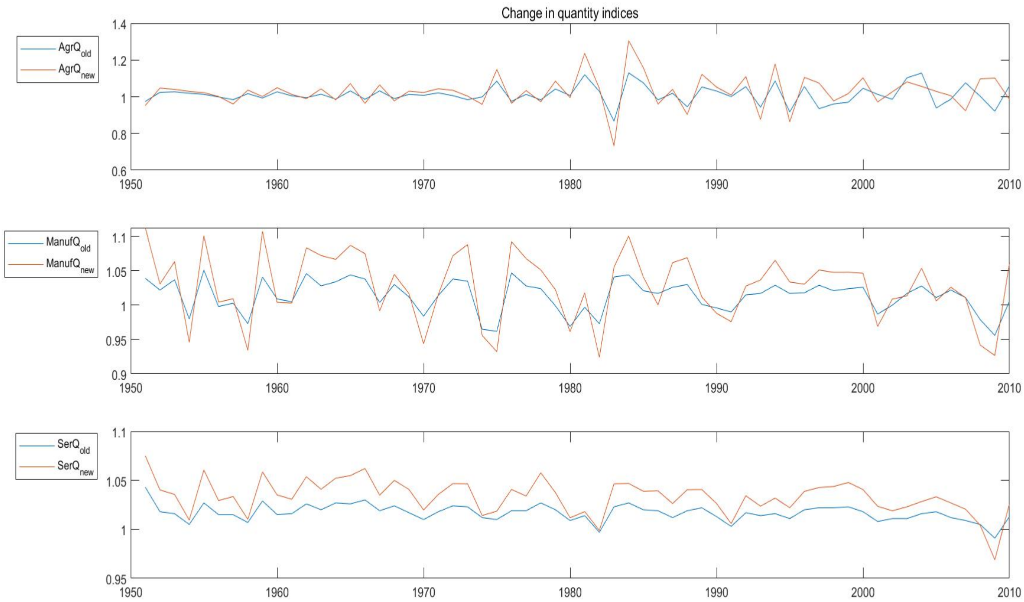
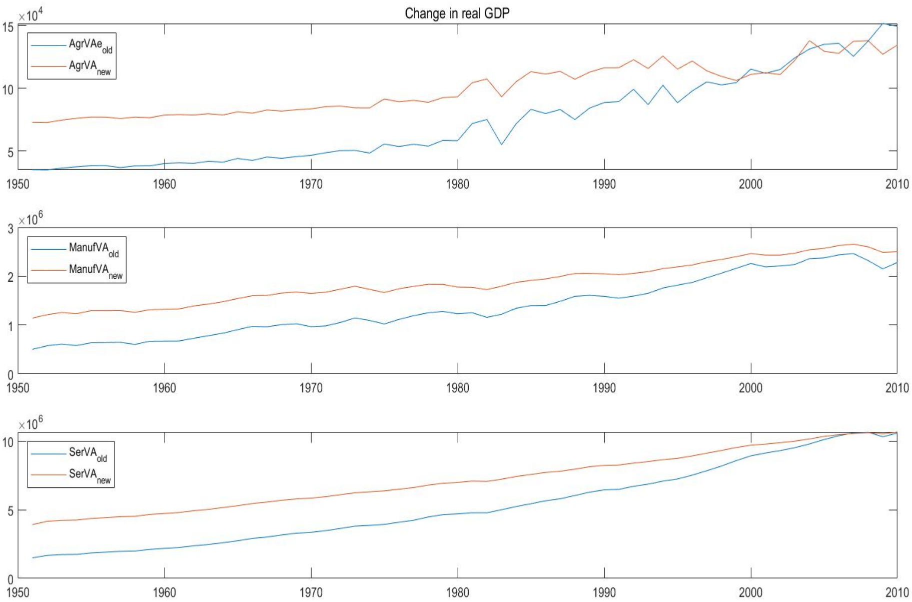


Figure 9: Changes in real value-added



Appendix B: Practical issues in implementation

There are some particle issues in implementing the technique. Firstly, labor shares are bounded between zero and one. Thus I use the standard logistic function transformation. I normalize total labor to be one in each period to comply with the model. However, capital and consumption per capita are very large in the data. Therefore, the production functions will involve labor as a number between 0 and 1 but capital as 20 thousand, for example. The difference in magnitude will cause the algorithm to be infeasible. Thus I normalize capital and consumption by 10^{-3} for the US and Sweden and 10^{-2} for China. Notice this normalization will lead to the simulated production being different from the data. To back out the production, one needs to multiply the estimated production by $10^{3(1-\alpha_i)}$ and $10^{2(1-\alpha_i)}$ in the manufacturing and service sector.

Secondly, as the sectoral technologies are assumed to be one in the initial period, it is necessary to scale the estimated data such that the estimated and observed value-added are equal in the first period. This will ensure that the estimated values accurately reflect the true data.