# Movers and Shakers

Robert Akerlof and Richard Holden*

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**Abstract**

Most projects, in most walks of life, require the participation of multiple parties. While it is difficult to unite individuals in a common endeavor, some people, whom we call "movers and shakers," seem able to do it. The paper specifically examines moving and shaking of an investment project, whose return depends both on its quality and the total capital invested in it. We analyze a model with two types of agents: managers and investors. Managers and investors initially form social connections. Managers then bid to buy control of the project and the winning bidder puts effort into making investors aware of it. Finally, a subset of aware investors are given the chance to invest and they decide whether to do so after receiving private signals of the project's quality. We first show that connections are valuable since they make it easier for a manager to "move and shake" the project (i.e., obtain capital from investors). When we endogenize the network, we find that, while managers are identical *ex ante*, a single manager emerges as most connected; he consequently earns a rent. In extensions, we move away from the assumption of *ex ante* identical managers to highlight forces that lead one manager or another to become a mover and shaker. Our theory sheds light on a range of topics including: entrepreneurship, venture capital, and anchor investments.

*Akerlof: University of Warwick. email: r.akerlof@warwick.ac.uk. Holden: UNSW Australia Business School. email: richard.holden@unsw.edu.au. We are grateful to George Akerlof, Wouter Dessein, Rosalind Dixon, Bhaskar Dutta, Bob Gibbons, Oliver Hart, Bengt Holmstrom, Rachel Kranton, Hongyi Li, Michael Powell and especially Andrea Prat for helpful discussions, and to seminar participants at: ANU, the Spring 2015 NBER Organizational Economics meetings, MIT, LMU Munich, Oxford University, Paris 2e, QUT, UMass Amherst, UNSW, and Yale. Holden acknowledges support from the Australian Research Council (ARC) Future Fellowship FT130101159.

# 1 Introduction

Most projects – in business, politics, sports, and academia – require the participation of *multiple parties*. In business, they usually involve, among other things, raising capital from disparate sources. Many projects fail – or do not even get off the ground – because of the difficulty of bringing together the relevant parties. While it is not easy to unite individuals in a common endeavor, some people – often called ”movers and shakers” – seem able to do it. This paper develops an equilibrium theory regarding who these movers and shakers will be and why they receive outsize compensation for their endeavors.

Skill, of course, helps in obtaining participation since people are more inclined to participate in skillfully run projects. But another attribute – social connectedness – can also make someone a mover and shaker. Someone who is well-connected can increase participation not only by making agents aware of a project but, even more importantly, by making agents aware that *others are aware* and are considering participating. Expressed differently, connections help both in raising awareness and in making that awareness *common knowledge*.

In our baseline model, there are a number of potential managers of a project – all equally skilled – and a number of potential investors. Initially, there are no connections between managers and investors. The model has four stages. In stage 1, investors form connections with managers. For simplicity, we assume each investor can link to one manager. In stage 2, managers bid to buy an asset. The asset is necessary for undertaking the project and entitles the owner to the project's return. For instance, if the project were the construction of a shopping mall, the asset might be the plot of land on which the mall is to be built. In stage 3, the winning bidder puts effort into raising awareness of the project among investors and gives a subset of the aware investors the chance to invest. In stage 4, investors given the chance to invest decide whether to do so after receiving private signals of the project's quality.

We first analyze the model taking the social network between managers and investors as exogenous (i.e., we exclude stage 1). Connections increase a manager's valuation of the asset: since they make it easier to raise capital for the project (i.e., move and shake it). Consequently, in equilibrium, the manager – or one of the managers –

who is most connected wins the auction and puts effort into moving and shaking the project. Furthermore, provided the auction winner is strictly more connected than other managers, he receives a higher expected payoff.

When we endogenize the social network (i.e., add stage 1), we find that all investors link to one particular manager, whom we may refer to as M. Therefore, even though managers are identical *ex ante*, one manager (M) emerges as most connected. M wins the auction, moves and shakes the project, and earns a higher payoff than other managers. Investors link to the same manager in equilibrium because they have a preference to link to whichever manager is most connected. The most connected manager ends up controlling the project; unless an investor connects to the manager who controls the project, he will not have an opportunity to invest.

We later extend the model by making managers heterogeneous along several dimensions: (1) their skill at running the project; (2) their talent at communicating with investors; and (3) how much capital they have personally. We assume that managers can use their personal capital as "seed money" for the project. Taking the social network as exogenous, we find that these characteristics affect how much managers value the project and which one becomes mover and shaker. When we endogenize the network, we find that these characteristics are also predictive of who emerges as most connected.

It is useful, in thinking about movers and shakers, to have a concrete example in mind. To that end, consider William Zeckendorf, who was, in the 1950s and 60s, the United States' preeminent real estate developer. He undertook a variety of ambitious projects including Mile High Center in downtown Denver, Place Ville-Marie in Montreal, and L'Enfant Plaza in Washington, D.C. He was also famous for his role in bringing the United Nations to New York.[1] Key to Zeckendorf's success (and his ability to move and shake) were his social connections, as he recognized himself: "the greater the number of...groups...one could interconnect...the greater the profit."[2] He knew all the important real estate brokers, bankers, and insurance agents; he served on numerous corporate boards; and he was a fixture of New York society. Zeckendorf also owned a

[1] Upon learning of the United Nations' difficulty finding a suitable New York site – and their intention, in consequence, to locate in Philadelphia – he realized he could help. He offered them a site he had assembled on the East River for a large development.

[2] Zeckendorf and McCreary (1970), p. 42.

nightclub, the Monte Carlo, where he would hold court several nights a week, entertaining friends and business acquaintances.

His Montreal project, Place Ville-Marie, provides an excellent example of his talents as a mover and shaker. Since the 1920s, the Canadian National Railway (CNR) had been attempting, without success, to develop a 22-acre site in downtown Montreal, adjacent to the main train station: "a great, soot-stained, angry-looking, open cut where railway tracks ran out of a three-mile tunnel."[3] While the site had enormous potential, Canadian developers shied away, considering the challenges too daunting. Desperate, CNR approached Zeckendorf in 1955. He was immediately enthusiastic, appreciating that: "a sort of Rockefeller Center-cum-Grand Central Station could create a new center of gravity and focal point for the city."[4] But, making this vision a reality would require the participation of two constituencies. First, he would need to raise large sums from investors: one hundred million dollars for the tower he proposed to build as the site's centerpiece. Second, and even more vexing, was the challenge of leasing office space. Every major company had its offices on St. James Street. "The very idea of a shift to center-town offices struck many as dangerously radical."[5] Zeckendorf initially faced a freeze, unable to get anyone to lease space. As he put it, "nobody...believed we would ever put up a project as big as we said we would."[6] But, through his tireless efforts, the freeze began to thaw. The first crack came when he convinced the Royal Bank of Canada to move into the new building and become its prime tenant. He had been introduced to the CEO, James Muir, by his friend John McCloy, chairman of Chase; Zeckendorf set out to woo Muir, making him his Canadian banker. With RBC lined up, he managed, with considerable pressing, to obtain a fifty million dollar loan from Met Life – half of what was needed. Also with considerable pressing, he lined up a second big tenant: Aluminium Limited. At that point, it became clear to all that the project would indeed become a reality. Other companies – which had previously turned him down – agreed to take space, and he was able to obtain the additional capital he needed.

Our theory sheds light on a range of topics, one of which is entrepreneurship. Founding a business often requires moving and shaking. One can think of real estate develop-

[3]Ibid., p. 167.
[4]Ibid., p. 170.
[5]Ibid., p. 174.
[6]Ibid., p. 174.

ers such as Zeckendorf as a type of entrepreneur. A number of ideas have been advanced regarding entrepreneurs' function. Schumpeter (1934), for instance, stresses their role as innovators involved in "creative destruction"; Knight (1921) sees them primarily as risk-takers; Rajan and Zingales (1998) highlight their role in regulating access to resources. Others, such as Baumol (2010), bemoan that, despite economists' longstanding interest, "[entrepreneurs] are almost entirely excluded from our standard theoretical models."[7] Our theory offers a new perspective on their role. The aspect of entrepreneurship captured by our model is new to economics, but it is related to theoretical perspectives in sociology. Ronald Burt, for instance, argues that entrepreneurs exploit network position. In his terminology, they bridge "structural holes." He writes that "bringing together separate pieces [of a network] is the essence of entrepreneurship."[8]

Our theory also speaks to the role of venture capitalists. According to Kaplan and Schoar (2005), venture capital funds, on average, yield roughly the same return, net of fees, as the S&P 500; however, certain fund managers consistently outperform the market, achieving higher risk-adjusted returns. The standard interpretation of this finding is that these fund managers are particularly skilled at originating investment ideas. While this is a possibility, the model suggests a novel explanation. Such fund managers may instead earn high returns by moving and shaking. Such VC firms take an equity stake in a startup; then, they move and shake on the company's behalf (in particular, helping the startup find additional investors). For example, Andreessen Horowitz, one of the preeminent Silicon Valley VC firms, "maintains a network of twenty thousand contacts and brings two thousand established companies a year to its executive briefing center to meet its startups." According to Marc Andreessen, "we give our founders...networking superpower."[9,10]

Additionally, seeding of projects – or, "anchor investments" – seems to be empirically

[7]Baumol (2010), p. 2.

[8]Burt (2001), p. 210.

[9]Friend, Tad, "Tomorrow's Advance Man," *The New Yorker* 18 May 2015, Retrieved from `http://www.newyorker.com`.

[10]It is not a matter of indifference to a startup, of course, which VC firm invests. A startup would rather take money from a VC who is better at moving and shaking. Lower-ranked VCs, in consequence, find it hard to compete. Andreesen puts it this way: "Deal flow is everything...If you're a second-tier firm, you never get a chance at that great company." (Friend, Tad, "Tomorrow's Advance Man," *The New Yorker* 18 May 2015, Retrieved from `http://www.newyorker.com`)

important. Movers and shakers are often independently wealthy and use their own funds to seed projects. In other instances, a mover and shaker might obtain help in seeding a project from a large investor. Our model speaks to this topic as well, and we discuss this briefly in the conclusion.

Our paper relates to a number of different literatures. At a formal level, the problem we analyze is a global game and thus relates to the now large literature pioneered by Carlsson and van Damme (1993) and Morris and Shin (1998).

The model we analyze also relates to large theoretical and empirical literatures in finance. A natural benchmark for thinking about investments and returns is, of course, Q-theory.[11] Investors, in Q-theory, earn the same rate of return whether they invest one dollar or one million. By contrast, investment is lumpy in our model. Agents invest in projects; projects yield a poor rate of return unless they are well capitalized. An important consequence is that the rate of return to a project/asset depends upon the social network that exists among agents. We predict, moreover, that agents with a privileged position in the network will earn outsize returns, because they can move and shake contributions from others.[12] Our model is, to the best of our knowledge, the first to emphasize the importance of network structure for investment.

Our paper connects to the economic literature on networks – particularly work on network formation. In the endogenous network version of our model, investors have a preference to link to the most important manager. This feature of our model is referred to as "preferential attachment." Two classic papers, Jackson and Wolinksy (1996) and Bala and Goyal (2000), show that this force will, in general, lead to the emergence of a star network.[13]

[11] A host of papers have documented departures from Q-theory and highlighted the implications of such departures. Liquidity constraints are important (see, among others, Fazzari et al. (1988), Hoshi et al. (1991), Blanchard et al. (1994), Kashyap et al. (1994), Sharpe (1994), Chevalier (1995), Kaplan and Zingales (1997), Lamont (1997), Peek and Rosengren (1997), Almeida et al. (2004), and Bertrand and Schoar (2006)) as are short-term biases (Stein (1988, 1989)). Moreover, there is compelling evidence that there are real consequences of such inefficiencies (see, for instance, Morck et al. (1988) and the large ensuing literature on the equity channel of investment).

[12] Another, quite distinct, form of "lumpiness" has been well studied: adjustment costs (see Uzawa (1969), Lucas and Prescott (1971), Hayashi (1982), and for a recent dynamic analysis, Miao and Wang (2014)). It is well known that such lumpiness can have significant macroeconomic implications (see, for instance, Lucas (1967), Prescott (1986), and Caballero et al. (1995)).

[13] Other papers that predict the emergence of star networks include Galeotti et al. (2006), Goyal and Vega-Redondo (2007), Feri (2007), Hojman and Szeidl (2007), Bloch and Dutta (2009) and Galeotti and

While our focus is an investment setting, our model also relates to a literature on attention within organizations (see especially Dessein (2002), Dessein and Santos (2006), Alonso et al. (2008), Rantakari (2008), Calvo-Armengol et al. (2014), Dessein and Santos (2014), and Dessein et al. (2014)). Agents in these models, as in our own, wish to coordinate their actions. In Calvo-Armengol et al. (2014), agents decide whom to pay attention to; attention is dispersed in equilibrium, in contrast to our model in which attention is concentrated on a mover and shaker.[14] Dessein and Santos (2014) and Dessein et al. (2014) consider a setting in which a principal decides the allocation of attention. They find that it is optimal for there to be some concentration of attention, since it aids coordination. Attention is also concentrated in our model, but it is not necessarily optimally placed. In particular, we obtain equilibria in which the mover and shaker is more or less skilled, resulting respectively in a more or less efficient outcome.[15] Another related paper, Hellwig and Veldkamp (2009), examines attention in a trading rather than an organizational setting. Somewhat analogous to the coordination of attention in our setting, they find traders may coordinate attention on one piece of information or another.

Our model relates to the economic literature on leadership since a mover and shaker is arguably a type of leader. It particularly relates to work examining how leaders persuade followers. Several papers consider signaling by leaders as a means of persuasion (see, for instance, Prendergast and Stole (1996), Hermalin (1998), and Majumdar and Mukand (2004)). There is also work on leaders creating cascades to influence followers (see Caillaud and Tirole (2007)). In our paper, the mover and shaker persuades investors by publicizing the project. This feature of our model bears some relation to Dewan and Myatt (2007, 2008), who have explored how public speeches by politicians can influence followers. Chwe (2001) also emphasizes the role of public announcements in acting as coordination devices in a variety of settings such as advertising. In addition, there is work on the use of authority by leaders in settings where agents, as in our model, have

Goyal (2010). A recent paper that is particularly relevant is Herskovic and Ramos (2015).

[14] In Calvo-Armengol et al. (2014), agents' attention is dispersed in equilibrium because there are neither increasing costs nor decreasing benefits of listening to multiple agents.

[15] Intuitively, investors coordinate on linking to a particular manager, whose skill may be higher or lower. This finding suggests the possibility of constructing a mover-and-shaker model, similar to our own, in which there are persistent performance differences across firms. Some firms get stuck paying attention to the wrong people. Persistent performance differences have been shown to be ubiquitous (see Gibbons and Henderson (2012)). There is considerable interest in understanding what drives these productivity differences (see Gibbons (2006), Chassang (2010), and Ellison and Holden (2014)).

a desire to coordinate. For instance, Bolton et al. (2013) argue that resoluteness is an important quality in a leader because a leader who is overly responsive to new information can undermine coordination.

The paper proceeds as follows. Section 2 contains the setup of our model and the analysis of equilibrium. We first take the network structure as exogenous; subsequently, we endogenize it. In Section 3 we consider a number of extensions of the basic model analyzed in Section 2. Section 4 concludes. Proofs of all formal results are contained in the Appendix.

# 2 The model

## 2.1 Statement of the problem

Consider a setting with an investment project and two types of agents: managers and investors. Managers have skills needed to run the project; investors each have one unit of capital they can contribute to the project. We assume there are at least two managers; the total number of managers and investors is finite. Let $N_M$ denote the set of managers and $N_I$ denote the set of investors.

A network $g$ exists between agents. $g_{ij} = 1$ if agent $i$ and agent $j$ are connected; $g_{ij} = 0$ otherwise. For now, we take the network as exogenous; we will endogenize it in Section 2.4.

The model has four periods. All choices made by agents are observable. In the first period, managers bid in a second-price auction for an asset A. The asset is needed to undertake the project and entitles the owner to the project's return. The asset might, for instance, be a parcel of land; the project might be the construction of a building on that parcel. The project yields a return $R$ at the end of the game that depends both upon the project's underlying quality ($\theta$) and the amount of capital raised for the project ($K$). More specifically, $R = \theta + v \cdot K$, where $v > 1$ parameterizes the return to raising capital (i.e., the return to "moving and shaking"). The agents have a common prior that $\theta$ is distributed $N(\mu, \tau^2)$, with $\mu, \tau > 0$. Let $b_i$ denote manager $i$'s bid in the auction, let $b_{(2)}$ denote the second highest bid, and let $M$ denote the winning bidder. In the event of a tie in the auction, the manager of lowest index wins. We assume managers do not following bidding strategies that are weakly dominated.

In the second period, the auction winner, $M$, decides how much effort $e_M \in [0,1]$ to exert to make investors aware of the project. An investor's chance of becoming aware of the project depends upon $e_M$ and upon his degree of separation from $M$. Specifically, investor $j$ becomes aware with probability $\delta^{(\text{length path } j \to M)-1} \cdot e_M$, where (length path $j \to M$) denotes the length of the shortest path between investor $j$ and $M$ that does not include other managers, (length path $j \to M$) $= \infty$ when no such path exists, and $\delta \in (0,1)$. We assume mutual independence of investors' awareness of the project. The cost to $M$ of exerting effort is $c(e_M)$, where $c'(0) = 0$ and $c'(e) > 0$ for $e > 0$. Let $n$ denote the number of investors who become aware of the project.

In the third period, $M$ can offer aware investors equity in the project in exchange for contributing their capital. $M$ chooses how much equity, $\beta_M$, to offer and the number, $m \leq n$, of equity offers he will make. The $m$ investors who receive equity offers are randomly drawn from the pool of aware investors. Let $S$ denote the set of investors who receive equity offers. $S$, once drawn, is commonly known to investors in set $S$.

Investors in set $S$ then receive private signals of the project's quality: $x_j = \theta + \varepsilon_j$, where the $\varepsilon_j$'s are distributed iid $N(0, \sigma^2)$. We will focus on the case where $\sigma \to 0$ as this results in closed-form solutions.

In the final period, investors who received equity offers decide whether to take them. Let $a_j \in \{0,1\}$ denote investor $j$'s decision. Observe that the total capital raised for the project is $K = \sum_{j \in S} a_j$.

The project is then undertaken, yields return $R = \theta + v \cdot K$, and players receive the shares of the return due to them. We can write players' payoffs at the end of the game as follows. Investors receive a payoff of $\beta_M R$ if they invest in the project and $1$ otherwise. The auction winner receives a payoff of $(1 - \beta_M K)R - c(e_M) - b_{(2)}$ while other managers receive $0$.

It is useful to summarize the timing: (1) managers simultaneously place bids ($b_i$) for asset A and the winning bidder (M) acquires the asset; (2) the auction winner (M) decides how much effort to exert ($e_M$) to make investors aware of the project; (3) M offers equity shares ($\beta_M$) to $m \leq n$ of the aware investors; (4) investors who receive equity offers then acquire private signals of the project's quality and simultaneously decide whether to invest ($a_i$), after which the project is undertaken, its return $R$ is realized, and players receive the share of the return due to them.

## 2.2 Discussion of the model

We now pause briefly to discuss a number of the modeling choices we have made.

First, our game has four periods and, at first inspection, might seem complicated in this respect. In fact, this is the simplest formulation that captures all the economics we wish to convey. It is important to us to highlight that, in equilibrium, more connected players value asset A more than less connected players. The simplest way to demonstrate this is through the auction we consider at time 1. Similarly, the effort choice is indispensable to our story since this is what moving and shaking *is* – hence time 2. Finally, we need two periods to address investment since it necessarily involves the equity offer and the choice of whether to invest.

Second, we model the project's return as increasing in the amount of capital invested. This results in strategic complementarities and captures our basic story about the importance of participation. Note that it is important that $R$ is increasing in $K$ over some range; it is not important that $R$ is increasing in $K$ indefinitely; we have only made this assumption for simplicity.

Third, the set $S$ is commonly known to investors in set $S$. This assumption reflects the idea that the mover and shaker not only raises awareness of the project; he also makes the existence of a pool of potential investors common knowledge.

Fourth, we assume the marginal cost of effort is equal to zero at $e_M = 0$ ($c'(0) = 0$). This ensures that it is optimal for $M$ to exert positive effort when he has social connections and the returns to moving and shaking, $v$, are large. More importantly, it means that more connected managers value asset A strictly – rather than weakly – more than less connected managers when $v$ is large.

Fifth, we consider a particular form of financial contracting: equity. The benefit of focusing on equity contracting is that it results in closed-form solutions; but this is not the most general contracting space one could consider, to be sure. However, we conjecture that our main results hold in a more general contracting space. [16]

Sixth, we adopt a common assumption regarding how information diffuses within the network.[17] Under this assumption, a manager's ability to raise awareness depends upon how many connections he has of each degree. [18]

Seventh, we assume that $S$, the set of investors who receive equity offers, once drawn, is commonly known. If one assumes that this information can only be conveyed by M then the issue of strategic information transmission may arise. We have sidestepped this issue by assuming that the information is simply observable. An alternative approach would be to assume that M conveys the information, but that it is "hard" information.

Finally, one could imagine modeling movers and shakers in a different way. Imagine an investment game with a good equilibrium (with a high level of investment) and a bad equilibrium (with a low level of investment). The mover and shaker might serve as a coordination device that makes the good equilibrium focal. While it is certainly plausible that movers and shakers play such a role, there are three reasons it is not so appealing to model them in this way. First, Schelling-type focal points are interesting but not micro-founded and raise more questions than they answer. Second, the global games approach was developed precisely to provide more rigorous answers to the multiple equilibrium problem. Perhaps most importantly, the global games approach is more fruitful in generating predictions. It yields the prediction that social connections matter for moving and shaking. It also allows us, in extensions to the baseline model, to describe characteristics associated with movers and shakers.

## 2.3 Equilibrium

Our focus will be on pure-strategy Perfect Bayesian Equilibria, which henceforth, we will refer to simply as the equilibria of the game.

[16]For instance, we believe our results hold when the project is debt financed rather than equity financed.

[17]For instance, Jackson and Wolinksy (1996) make a similar assumption.

[18]Depending upon how information diffuses within a network, different network properties are important. For a discussion, see Banerjee et al. (2013). Banerjee et al. (2013) provide empirical evidence that an agent's "diffusion centrality" is an important determinant of ability to diffuse information (a concept that nests degree centrality, eigenvector centrality, and Katz-Bonacich centrality).

Some notation will be useful for characterizing the equilibria. Let $d_i^k$ denote the number of connections of degree $k$ that manager $i$ has to investors.[19] Let $\mathbf{d}_i$ denote the corresponding vector: $\mathbf{d}_i = (d_i^k)_{k=1}^{\infty}$. We will define a partial ordering over the $\mathbf{d}_i$'s as follows.

**Definition 1.** *We will say that:*

*(1) Manager i is weakly more connected than manager j ($\mathbf{d}_i \geq \mathbf{d}_j$) if:*
$\sum_{k=1}^{l} d_i^k \geq \sum_{k=1}^{l} d_j^k$ *for all* $l$.

*(2) Manager i is strictly more connected than manager j ($\mathbf{d}_i > \mathbf{d}_j$) if:*
$\sum_{k=1}^{l} d_i^k \geq \sum_{k=1}^{l} d_j^k$ *for all* $l$ *and* $\sum_{k=1}^{l} d_i^k > \sum_{k=1}^{l} d_j^k$ *for some* $l$.

Notice that $\mathbf{d}_i > \mathbf{d}_j$ when manager $i$ has more connections of every degree ($d_i^k > d_j^k$ for all $k$). In addition, $\mathbf{d}_i > \mathbf{d}_j$ when managers $i$ and $j$ have the same total number of connections but $i$'s connections are all direct while some of $j$'s are not.

Proposition 1 characterizes the equilibria of the game. The proof is discussed in detail below.

**Proposition 1.** *In equilibrium:*

*(1) Managers bid their valuations of asset A in the auction: $b_i = V_i$.*

*(2) Manager $i$'s valuation of asset A is a function of his social connections: $V_i = V(\mathbf{d}_i)$.*

*(3) $V(\mathbf{d}_i)$ is weakly increasing in $\mathbf{d}_i$.*

*(4) There exists $\hat{v}$ such that, whenever the returns to moving and shaking exceed $\hat{v}$ ($v > \hat{v}$):*

   *(i) $V(\mathbf{d}_i)$ is strictly increasing in $\mathbf{d}_i$.*

   *(ii) Provided the manager who wins the auction has some social connections ($\mathbf{d}_M > \mathbf{0}$), he exerts positive effort ($e_M > 0$).*

According to the proposition, more connected managers value the project more than less connected managers. Provided the returns to moving and shaking are large ($v > \hat{v}$),

[19]By "a connection of degree $k$," we mean an investor $j$ for whom length path $j \rightarrow i = k$. We will also refer to connections of degree $1$ as "direct connections."

they value the project *strictly* more. The formal proof is given below, but the intuition is straightforward: more connected managers value the project more because they are more able to move and shake the project (i.e., raise capital).

Proposition 1 implies that, if the returns to moving and shaking exceed $\hat{v}$ and one manager is more connected than his peers, he becomes the project's mover and shaker and earns a positive rent from control of the project. This is stated below as a corollary.

**Corollary 1.** *Provided the returns to moving and shaking are sufficiently large ($v > \hat{v}$): if one of the managers is strictly more connected than other managers, he wins the auction, exerts positive effort to move and shake the project, and earns a higher expected payoff than his peers.*

Let us now consider the proof of Proposition 1.

**Proof of Proposition 1.**

We can use backward induction to solve for the equilibria of the game. First, consider time 4. The time 4 game is a global game. As is standard in such games, in equilibrium, investors invest if and only if their private signals exceed a cutoff: $x_j > \kappa$. Lemma 1, the proof of which is given in the Appendix, characterizes the cutoff $\kappa$ for the case where $\sigma \to 0$.

**Lemma 1.** *As $\sigma \to 0$, the cutoff $\kappa \to \frac{1}{\beta_M} - v(\frac{m+1}{2})$.*

According to Lemma 1, investors are more inclined to invest ($\kappa$ is lower) when: (1) they are offered more equity ($\beta_M$ is higher); and (2) there are more investors who receive equity offers ($m$ is higher). It is intuitive that investors are more inclined to invest when they are offered more equity. They are more inclined to invest when $m$ is higher because they expect greater total investment in the project, leading to a higher overall return $R$.

Turning to time 3, we can write the auction winner's expected payoff as $\Pi_M(m, \beta_M) - c(e_M) - b_{(2)}$, where $\Pi_M(m, \beta_M)$ denotes $M$'s expected share of the project's return when $m$ investors are offered equity shares of size $\beta_M$. $M$ will choose $\beta_M$ to maximize $\Pi_M$: $\beta_M^*(m) = \arg\max_\beta \Pi_M(m, \beta)$. $M$ will also choose $m$ to maximize $\Pi_M$ subject to the constraint that $m$ is less than or equal to the number of aware investors ($n$): $m^*(n) = \arg\max_{m \leq n} \Pi_M(m, \beta_M^*(m))$. We will denote by $\Pi_M^*(n)$ the value of $\Pi_M(m, \beta_M)$ when $m$ and $\beta_M$ are chosen optimally.

$\Pi^*_M(n)$ must be weakly increasing in $n$ since, as $n$ increases, $M$ is less constrained in his choice of $m$.

We can also show that $\Pi^*_M(n)$ is strictly increasing in $n$ provided the returns to moving and shaking, $v$, are large. Observe that, as $\sigma \to 0$, investors who receive equity offers invest with probability $1$ when $\theta > \kappa$ and with probability zero when $\theta < \kappa$. Consequently, $K = m$ with probability $1$ when $\theta > \kappa$; $K = 0$ with probability $1$ when $\theta < \kappa$. This allows us to write an explicit formula for $\Pi_M(m, \beta_M)$:

$$\begin{aligned}
\Pi_M(m, \beta_M) &= E[(1 - \beta_M K)R] \\
&= E[(1 - \beta_M K) \cdot (\theta + vK)] \\
&= E[\theta] + E[vK \cdot (1 - \beta_M K)] - E[\beta_M K\theta] \\
&= \mu + v \cdot m(1 - \beta_M m) \cdot \Pr(\theta > \kappa) - \beta_M m \cdot E(\theta|\theta > \kappa) \cdot \Pr(\theta > \kappa),
\end{aligned}$$

where $\kappa = \dfrac{1}{\beta_M} - v\left(\dfrac{m+1}{2}\right)$.

From this formula for $\Pi_M$, we can show that $\Pi_M(m, \beta^*_M(m))$ is strictly increasing in $m$ (provided $v$ is sufficiently large). The formal proof is given in the Appendix as part of the proof of Lemma 2 but the intuition is straightforward. The larger is $m$, the easier it is for the auction winner to raise capital. It is easier because there are more potential investors; it is also easier because any given investor's willingness to invest is increasing in $m$. It follows that, if $v$ is large – so that it is particularly valuable to $M$ to raise capital – an increase in $m$ unambiguously raises $M$'s payoff.

Notice that if $\Pi_M(m, \beta^*_M(m))$ is strictly increasing in $m$, it is optimal for $M$ to make equity offers to all aware investors ($m^*(n) = n$). Furthermore, $M$'s expected payoff ($\Pi^*_M(n)$) will be strictly increasing in $n$. Lemma 2, stated below, summarizes.[20]

**Lemma 2.**

*(1)* $\Pi^*_M(n)$ *is weakly increasing in* $n$*.*

*(2) There exists* $\hat{v}$ *such that, whenever* $v > \hat{v}$*:*

[20] We can show that when $v$ is small, there are, in fact, cases where not all aware investors receive equity offers ($m^*(n) < n$).

*(i)* $m^*(n) = n$.

*(ii)* $\Pi^*_M(n)$ *is strictly increasing in* $n$.

Now, consider time 2. Let us denote by $G(\mathbf{d}_M, e_M)$ the distribution from which $n$, the number of aware investors, is drawn when $M$ exerts effort $e_M$ and has connections $\mathbf{d_M}$. $M$'s expected payoff when he exerts effort $e_M$ is: $E[\Pi^*_M(n)|n \sim G(\mathbf{d}_M, e_M)] - c(e_M) - b_{(2)}$. $M$ will choose the level of effort, $e^*_M(\mathbf{d}_M)$, that maximizes this expression. We can write $M$'s resulting payoff as: $V(\mathbf{d}_M) - b_{(2)}$.

Lemma 3, the formal proof of which is given in the Appendix, follows almost immediately from Lemma 2 and from the observation that $G(\mathbf{d}_M, e_M)$ is increasing – in a first-order stochastic dominance sense – in both $\mathbf{d}_M$ and $e_M$.

**Lemma 3.**

*(1)* $V(\mathbf{d}_M)$ *is weakly increasing in* $\mathbf{d}_M$.

*(2) Provided* $v > \hat{v}$:

*(i)* $V(\mathbf{d}_M)$ *is strictly increasing in* $\mathbf{d}_M$.

*(ii)* $e^*_M(\mathbf{d}_M) > 0$ *whenever* $\mathbf{d}_M > \mathbf{0}$.

Finally, let us turn back to time 1. Observe that the value of asset A to manager $i$ is $V(\mathbf{d}_i)$. Consequently, manager $i$ will bid $b_i = V(\mathbf{d}_i)$ in the auction. This completes the proof of Proposition 1.

**Distributions of $K$ and $R$.**

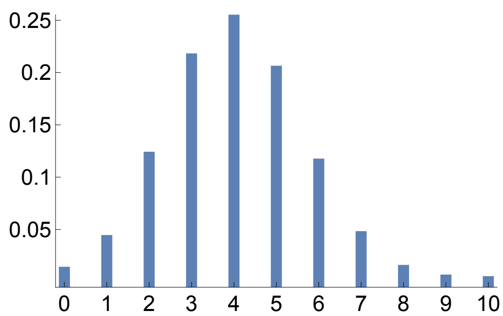
We showed, as part of the proof of Proposition 1, that $K = m$ with probability 1 when $\theta > \kappa$; $K = 0$ with probability 1 when $\theta < \kappa$. Consequently,

$$K = m^*(n) \cdot \mathbb{1}_{\{\theta > \kappa^*(n)\}} \text{ almost surely,} \tag{1}$$

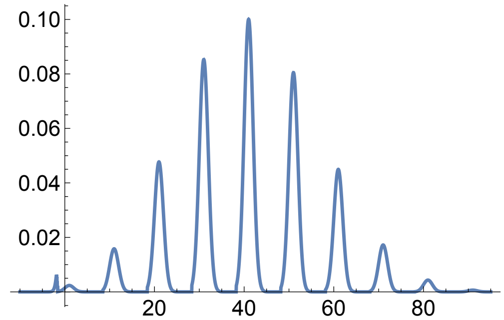
where $\kappa^*(n) = \frac{1}{\beta^*_M(m^*(n))} - v\left(\frac{m^*(n)+1}{2}\right)$. We can also write $R$ as follows:

$$\begin{aligned} R &= \theta + v \cdot K \\ &= \theta + v \cdot m^*(n) \cdot \mathbb{1}_{\{\theta > \kappa^*(n)\}} \text{ almost surely.} \end{aligned} \tag{2}$$

Observe that (1) and (2) express $K$ and $R$ as functions of $\theta$ and $n$. $\theta$ and $n$ are independent random variables, distributed $N(\mu, \tau^2)$ and $G(\mathbf{d}_M, e^*(\mathbf{d}_M))$ respectively. Hence, from (1) and (2), we can derive the distributions of $K$ and $R$ (for more details, see the Appendix). Figure 1 plots these distributions for a particular numerical example.




**(a)** Distribution of $K$




**(b)** Distribution of $R$

**Figure 1** – Figure 1 plots the distributions of $K$ and $R$ for a numerical example. The parametric assumptions are as follows: $\mu = 1$, $\tau = 1$, $v = 10$, $c(e) = (10e)^2$, and the auction winner is directly connected to 10 investors and has no indirect connections ($\mathbf{d}_M = (10, 0, 0, ...)$). Note that the distribution of $K$ is discrete while the distribution of $R$ is continuous.

## 2.4 Endogenizing the Network

Thus far, we have taken the network, $g$, between agents as exogenous. We can endogenize the network by adding an initial period to the game. Assume there are initially no connections between agents. In period 0, each investor chooses one manager to whom he will link. Proposition 2 characterizes the equilibria of this game. The proof is given in the Appendix.

**Proposition 2.** *Suppose there are at least three investors ($card(N_I) \geq 3$) and the returns to moving and shaking are large ($v > \hat{v}$). All equilibria have the following properties, and moreover, such equilibria exist:*

*(1) In period 0, all investors link to one particular manager: $Y$. $Y$ can be any manager.*

*(2) Subsequently, $Y$ wins the auction ($Y = M$) and exerts positive effort ($e_Y > 0$).*

*(3) $Y$ receives a higher expected payoff than other managers.*

According to Proposition 2, even though managers are identical *ex ante*, one emerges as most connected in equilibrium. In fact, all investors link to the same manager. This manager consequently wins the auction, moves and shakes the project, and earns a higher payoff than his peers.

While the proof is left for the Appendix, the intuition is as follows. Investors strictly prefer to link to the most connected manager. They prefer to do so because the most connected manager wins the auction; unless an investor links to the auction winner, he has no opportunity to invest in the project. Since investors strictly prefer to link to the most connected manager, all investors end up linking to the same manager in equilibrium.

Note that Proposition 2 assumes $v > \hat{v}$ because it ensures the auction winner exerts positive effort ($e_M > 0$). If the auction winner exerts zero effort ($e_M = 0$), investors are indifferent over whom to link to, since whomever they choose to link to, there is zero chance of having an opportunity to invest in the project.

# 3 Extensions

We can extend the baseline model by making managers heterogeneous along several dimensions: (1) their skill at running the project; (2) their talent at communicating with investors; and (3) how much capital they have.

Taking the social network as exogenous, we find that these characteristics affect how much managers value the project and which one becomes mover and shaker. When we endogenize the network, we find that these characteristics are also predictive of who emerges as most connected.

**Skill at running the project.** In the baseline model, managers were equally skilled at running the project. We now assume that, if manager $i$ runs the project, it yields a return $R = \theta + v \cdot K + \alpha_i$, where $\alpha_i$ denotes the skill of manager $i$. We assume $\alpha_i \geq 0$.[21]

[21] In some instances, a manager may be able to contract with another party to compensate for lack of skill. To give a concrete example, a manager might be able to hire a consultant. Our definition of skill concerns those aspects for which it is not possible to compensate.

**Ability to communicate.** Managers had the same ability to communicate in the baseline model (i.e., the same ability to raise awareness of the project among social connections). We now assume the cost of effort for manager $i$ is $c(\frac{e_i}{\gamma_i})$, with $\gamma_i > 0$. One can think of $\gamma_i$ as manager $i$'s ability to communicate with investors.

**Seed money.** Managers had no capital of their own in the baseline model. We now assume manager $i$ has an amount $k_i \geq 0$. The auction winner, $M$, can use his capital as "seed money" for the project. Specifically, at time 2, before investors decide whether to contribute capital, $M$ chooses $s_M \leq k_M$: the amount of capital he will put into the project. The auction winner receives a payoff $(1 - \beta_M \cdot \sum_{j \in S} a_j)R - s_M - c(\frac{e_M}{\gamma_M}) - b_{(2)}$, where $K = s_M + \sum_{j \in S} a_j$. Managers who do not win the auction receive a payoff of $0$.

## 3.1 Exogenous Network

When we take the network $g$ between agents as exogenous, we obtain the following analog of Proposition 1.

**Proposition 3.** *In equilibrium:*

*(1) Managers bid their valuations of asset A in the auction: $b_i = V_i$.*

*(2) Manager $i$'s valuation of asset A is a function of his social connections, skill at running the project, communication ability, and capital: $V_i = V(\mathbf{d}_i, \alpha_i, \gamma_i, k_i)$.*

*(3) $V(\mathbf{d}_i, \alpha_i, \gamma_i, k_i)$ is weakly increasing in $\mathbf{d}_i$, $\gamma_i$, and $k_i$, and strictly increasing in $\alpha_i$.*

*(4) There exists $\hat{v}$ such that, whenever $v > \hat{v}$:*

- *(i) $V(\mathbf{d}_i, \alpha_i, \gamma_i, k_i)$ is strictly increasing in $\mathbf{d}_i$, $\alpha_i$, and $k_i$, and strictly increasing in $\gamma_i$ provided $\mathbf{d}_i > \mathbf{0}$.*
- *(ii) Provided the manager who wins the auction has some social connections ($\mathbf{d}_M > \mathbf{0}$), he exerts positive effort ($e_M > 0$).*
- *(iii) The auction winner uses his capital to seed the project ($s_M = k_M$).*

According to Proposition 3, the auction winner will be the manager who values the project most. As in the baseline model, managers value the project more when they are more connected. Managers also value the project more when they are more skilled at running the project, when they are more able communicators, and when they have more capital.

Additionally, Proposition 3 says that, provided the returns to moving and shaking are large ($v > \hat{v}$), the auction winner uses his capital to seed the project ($s_M = k_M$). Furthermore, managers' valuations of the project are strictly increasing in the amount of seed capital ($k_i$) they have.

The formal proof is left for the Appendix, but the logic is as follows. Managers value the project more when they have more seed capital because seeding is worthwhile. Seeding is worthwhile for two reasons. First, it directly contributes an amount $s_i$ to the project. Second, and more importantly, seeding indirectly contributes to the project by increasing investors' willingness to provide capital.

## 3.2 Endogenous Network

We can endogenize the network $g$ in the same manner as in Section 2.4: by assuming there are initially no connections between agents and that each investor, in period 0, chooses one manager to whom he will link. Proposition 4 characterizes the equilibria of this game.

**Proposition 4.** *Suppose there are at least three investors ($card(N_I) \geq 3$) and the returns to moving and shaking are large ($v > \hat{v}$).*

*(1) All equilibria have the following properties, and moreover, such equilibria exist:*

*(i) In period 0, all investors link to one particular manager: $Y$.*

*(ii) Subsequently, $Y$ wins the auction ($Y = M$), exerts positive effort ($e_Y > 0$), and uses his capital to seed the project ($s_Y = k_Y$).*

*(iii) $Y$ receives a higher expected payoff than other managers.*

*(2) An equilibrium does not exist in which manager $i = Y$ if $(\alpha_i, \gamma_i, k_i)$ is small. Specifically, an equilibrium does not exist if:*

*$V(\mathbf{d}_{max}, \alpha_i, \gamma_i, k_i) < \max_{j \in N_M} V(\mathbf{0}, \alpha_j, \gamma_j, k_j)$, where $\mathbf{d}_{max}$ denotes the case where a manager is directly connected to all investors.*

*(3) An equilibrium exists in which manager $i = Y$ if $(\alpha_i, \gamma_i, k_i)$ is large. Specifically, an equilibrium exists if:*

*$V(\mathbf{d}_{max} - \hat{\mathbf{d}}, \alpha_i, \gamma_i, k_i) > \max_{j \in N_M} V(\hat{\mathbf{d}}, \alpha_j, \gamma_j, k_j)$, where $\hat{\mathbf{d}} = (1, 0, 0, ...)$ denotes the case where a manager has one direct connection and no indirect connections.*

Proposition 4 closely mirrors Proposition 2. Once again, we find that all investors link to one particular manager $Y$; $Y$ subsequently wins the auction, exerts effort to move and shake the project, and earns a higher expected payoff than his peers. However, in Proposition 2, any manager could emerge as most connected and as the project's mover and shaker. In this case, we find that a manager must be sufficiently able and have sufficient capital in order to do so (that is, $(\alpha_i, \gamma_i, k_i)$ must be sufficiently large).

Specifically, an equilibrium does not exist in which manager $i = Y$ if:

$$V(\mathbf{d}_{max}, \alpha_i, \gamma_i, s_i) < \max_{j \in N_M} V(\mathbf{0}, \alpha_j, \gamma_j, s_j),$$

where $\mathbf{d}_{max}$ denotes the case where a manager is directly connected to all investors. In this case, even if manager $i$ is socially connected ($\mathbf{d}_i = \mathbf{d}_{max}$) and his peers are not, he will be outbid in the auction. Since investors have a preference to link to the eventual auction winner, manager $i$ cannot be socially connected in equilibrium.[22]

Proposition 4 rules out manager $i$ becoming mover and shaker if $(\alpha_i, \gamma_i, k_i)$ is sufficiently low. However, manager $i$ can potentially become mover and shaker even if he is less skilled at running the project than some other manager $j$, has lower communication ability, and has less capital. Furthermore, if manager $i$ emerges as mover and shaker rather than $j$, he receives a strictly higher expected payoff.

Movers and shakers are good from an efficiency point of view – in the sense that there would be no investment without the mover and shaker's effort; but the outcome will be more or less efficient depending upon which manager emerges as mover and

[22]Note that, if investors are more willing to link to some managers than others, this would also affect who can emerge as mover and shaker. We could model this by assuming a cost to investors, $l_i$, of linking to manager $i$. A high linking cost does not directly affect a manager's valuation of asset A; but it could indirectly affect it since it might prevent investors from connecting to him.

shaker. Intuitively, investors may coordinate on a manager who is more or less suited to run the project.

# 4 Concluding remarks

We have analyzed a model with two types of agents – managers and investors – and an investment project, whose return is a function both of its underlying quality and aggregate investment. Managers and investors form social connections. Managers then bid to buy control of the project and the winning bidder puts effort into making investors aware of it. Finally, a subset of aware investors are given the chance to invest and they decide whether to do so after receiving private signals of the project's quality.

We first analyze the model taking the social network as exogenous. Connections increase a manager's ability to raise capital. Consequently, the most connected manager wins the auction, exerts effort to move and shake the project, and, provided he is strictly most connected, earns a positive rent. When we endogenize the network, we find that all investors link to one particular manager. Therefore, even though all managers are *ex ante* identical, one emerges as most connected in equilibrium, becomes the project's mover and shaker, and receives a higher expected payoff than his peers.

We also extend our baseline model by making managers heterogeneous along several dimensions: (1) their skill at running the project; (2) their talent at communicating with investors; and (3) how much capital they have. These characteristics affect how much managers value the project. Consequently, when we take the network as exogenous, they affect who emerges as mover and shaker. When we endogenize the network, these characteristics are also predictive of who emerges as most connected.

There are a number of implications of our theory and potential avenues for future work. Here, we briefly sketch five.

One notable feature of our model is that rents earned by managers do not correspond to their "marginal product" – at least not in the conventional usage of that term. In our setting, rents are derived from social position. The mover and shaker is socially useful, to be sure, but can derive "outsize" rewards. Furthermore, the model suggests that it is easy to misattribute a mover and shaker's success to his skill at running the project. In fact, a mover and shaker may succeed in spite of – rather than because of – his skill. The broad debate about rising inequality (see Piketty (2014) for a notable recent contribution) has focused to a large degree on returns to capital versus labor, but relatively little on what might be termed "returns to social position." Our theory differs from existing accounts of the drivers of inequality because technological factors play a secondary role. Empirical tests of the relative importance of network position versus marginal product may be informed by the structure of our model.

Second, our model suggests that having capital to seed projects can be valuable. This raises the possibility that, in the absence of having such capital oneself, one may wish to contract with a large investor to play such a role. In serving as anchor investors for projects they may earn higher rates of return than small investors. In other words, such investors may receive compensation not just for the capital they personally provide to projects but also for the additional capital their investments help attract.[23] To give an example, when Blackstone was raising its first private equity fund its cofounders, Steve Schwarzman and Pete Peterson, found it enormously challenging to raise money.[24] Prudential became an anchor investor, putting in $100 million, but extracted very positive terms. According to Carey and Morris (2010):

> "Prudential insisted that Blackstone not collect a dime of the profits until Prudential and other investors had earned a 9 percent compounded annual return on every dollar they'd pledged to the fund...Prudential also insisted that Blackstone pay investors in the fund 25 percent on the net revenue...from its M&A advisor work, even on deals not connected to the fund...In the end,

[23] We should mention that there is an existing literature on anchor stores. For instance, Gould et al. (2005) demonstrate empirically that shopping mall store contracts are written to take account of the positive externality that "national brand" stores generate in driving traffic to smaller stores. Bernstein and Winter (2012) derive the structure of the optimal contract in the presence of heterogeneous externalities. Our theory, adapted to such a setting, suggests that anchor stores may receive preferable terms, but for rather different reasons than given in this strand of literature, which typically assumes that only the anchor store imposes (positive) externalities on other stores. By contrast, our model (as applied to stores), involves all stores imposing externalities on one another; these externalities being proportional to size. The argument in, say, Gould et al. (2005) or Bernstein and Winter (2012) as to why there should be a better rental rate for a large store does not apply in our environment. Our theory nonetheless suggests that anchor stores might obtain a better rate: the reason being that their participation helps to secure other stores' participation.

[24] The Blackstone Group now has around $30 billion in funds under management and more than 1500 employees.

> these were small prices to pay for the credibility the Pru's backing would give Blackstone."

Third, political campaigns have many of the features of our model. They are "projects"; people make contributions (financial and non-financial); and there are strong complementarities. Moreover, beliefs about what others will do seem to matter a lot. Donors often worry about what other donors are contributing, and it is common wisdom that voters typically like to vote for winning candidates. The strong momentum effects in, for example, US Presidential Primaries (see Knight and Schiff (2010) for persuasive empirical evidence) may be explained, in part, by considerations present in our theory.

Fourth, there is a burgeoning literature on "persistent performance differences" in organizations. Most models seeking to rationalize differences among otherwise identical organizations involve some kind of equilibrium theory where *ex ante* identical organizations end up in different positions ex post. In, for example, Chassang (2010) and Ellison and Holden (2014) this wedge is due to dynamics. Our model suggests an alternative explanation for persistent performance differences that does not involve dynamics. In our theory, agents/investors focus their attention on one particular manager; that manager may be more or less skilled.

Finally, one might be tempted to take a benign view of moving and shaking, given the coordinating role of movers and shakers. It is worth remembering there may be externalities associated with the outcomes they effect. For instance, the heads of organized crime syndicates may be movers and shakers; so, too, lobbyists for nefarious special interests. To draw appropriate welfare conclusions it is necessary to take these externalities into account.

# 5 Appendix

## 5.1 Proofs

*Proof of Lemma 1.* Consider the global game investors in set $S$ play at time 4. In Section 3 of Morris and Shin (2003), it is shown that, in such a game, provided $\frac{\sigma^2}{\tau^4}$ is sufficiently small, there is a unique equilibrium in which investors follow strategies of the form: invest if and only if $\overline{\theta}_j > \kappa$, where $\overline{\theta}_j = \frac{\sigma^2 \cdot \mu + \tau^2 \cdot x_j}{\sigma^2 + \tau^2}$ denotes investor $j$'s posterior on $\theta$. We are focused on the case where $\sigma \to 0$, so $\frac{\sigma^2}{\tau^4}$ is indeed small. Furthermore, $\overline{\theta}_j \to x_j$ as $\sigma \to 0$, so in the limit, the cutoff rule becomes: invest if and only if $x_j > \kappa$.

Let us now solve for the cutoff ($\kappa$). If investor $j$ invests, his expected payoff will be:

$$
\begin{aligned}
\text{Payoff from investing} &= \beta_M \cdot E(R|x_j, \text{j invests}) \\
&= \beta_M \cdot E(\theta + vK|x_j, \text{j invests}) \\
&= \beta_M \cdot [\overline{\theta_j} + v(1 + (m-1) \cdot \Pr(\overline{\theta}_k > \kappa|x_j))] \\
&= \beta_M \cdot \left[\overline{\theta_j} + v\left(1 + (m-1) \cdot \Pr\left(x_k > \kappa + \left(\frac{\sigma^2}{\tau^2}\right)(\kappa - \mu) \;\middle|\; x_j\right)\right)\right].
\end{aligned}
$$

Observe that investor $j$'s posterior on $\theta$ is that it is distributed normally with mean $\overline{\theta_j}$ and with variance $\frac{\sigma^2\tau^2}{\sigma^2+\tau^2}$. Since $x_k = \theta + \varepsilon_k$, $j$'s posterior on $x_k$ is that it is distributed normally with mean $\overline{\theta_j}$ and with variance $\frac{\sigma^2\tau^2}{\sigma^2+\tau^2} + \sigma^2$ (or, simplifying, with variance $\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}$). Consequently:

$$
\begin{aligned}
\Pr\left(x_k > \kappa + \left(\frac{\sigma^2}{\tau^2}\right)(\kappa - \mu) \;\middle|\; x_j\right) &= \Pr\left(\frac{x_k - \overline{\theta}_j}{\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}} > \frac{(\kappa - \overline{\theta}_j) + \left(\frac{\sigma^2}{\tau^2}\right)(\kappa - \mu)}{\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}} \;\middle|\; x_j\right) \\
&= 1 - \Phi\left(\frac{(\kappa - \overline{\theta}_j) + \left(\frac{\sigma^2}{\tau^2}\right)(\kappa - \mu)}{\sqrt{\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}\right),
\end{aligned}
$$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution.

At the cutoff (that is, when $\overline{\theta}_j = \kappa$), investor $j$ should be indifferent between investing and not. Hence, the payoff from investing should be equal to $1$. This gives us the following formula:

$$\beta_M \cdot \left[\kappa + v\left(1 + (m-1)\cdot\left(1 - \Phi\left(\frac{\kappa-\mu}{\sqrt{\left(\frac{\tau^4}{\sigma^4}\right)\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}\right)\right)\right)\right] = 1.$$

As $\sigma \to 0$, $\Phi\left(\frac{\kappa-\mu}{\sqrt{\left(\frac{\tau^4}{\sigma^4}\right)\frac{2\sigma^2\tau^2+\sigma^4}{\sigma^2+\tau^2}}}\right) \to \Phi(0) = \frac{1}{2}$. Hence, the formula simplifies to:

$$\beta_M \cdot \left[\kappa + v\left(\frac{m+1}{2}\right)\right] = 1.$$

Rearranging terms, we find:

$$\kappa = \frac{1}{\beta_M} - v\left(\frac{m+1}{2}\right).$$

This completes the proof.

□

*Proof of Lemma 2.* $\Pi^*_M(n)$ must be weakly increasing in $n$ since $M$ is less constrained in his choice of $m$ as $n$ increases. To prove the remainder of the lemma, it is sufficient to show that $\Pi_M(m, \beta^*(m))$ is strictly increasing in $m$ for $v$ large. Recall that:

$$\Pi_M(m,\beta) = \mu + v\cdot m(1-\beta m)\cdot \Pr(\theta>\kappa) - \beta m \cdot E(\theta|\theta>\kappa)\cdot\Pr(\theta>\kappa),$$

where $\kappa = \frac{1}{\beta} - v\left(\frac{m+1}{2}\right)$.

Observe that $E(\theta|\theta > \kappa)\cdot\Pr(\theta>\kappa)$ is bounded between $0$ and $E(\theta|\theta>0)\cdot\Pr(\theta > 0)$. Therefore, $\frac{1}{v}\Pi_M(m,\beta) \to m(1-\beta m)\cdot\Pr(\theta>\kappa) \equiv \tilde{\Pi}_M(m,\beta)$ as $v\to\infty$. To show $\Pi_M(m,\beta^*(m))$ is strictly increasing in $m$ for $v$ large, it will be sufficient to show $\tilde{\Pi}_M(m,\tilde{\beta}^*(m))$ is strictly increasing in $m$ for $v$ large, where $\tilde{\beta}^*(m) = \arg\max_\beta \tilde{\Pi}_M(m,\beta)$.

We can apply the Envelope Theorem to differentiate $\tilde{\Pi}_M(m,\tilde{\beta}^*(m))$ with respect to $m$:

$$\frac{d\tilde{\Pi}_M(m,\tilde{\beta}^*(m))}{dm} = m(1-\tilde{\beta}^*(m)m)\cdot\frac{vf(\kappa)}{2} + (1-2\tilde{\beta}^*(m)m)\cdot\Pr(\theta>\kappa),$$

where $\kappa = \frac{1}{\tilde{\beta}^*(m)} - v\left(\frac{m+1}{2}\right)$ and $f(\cdot)$ denotes the pdf of the $N(\mu, \tau^2)$ distribution. We see that, provided $\tilde{\beta}^*(m)m < \frac{1}{2}$, $\frac{d\tilde{\Pi}_M(m,\tilde{\beta}^*(m))}{dm} > 0$.

We can show, by contradiction, that $\tilde{\beta}^*(m)m < \frac{1}{2}$ for $v$ large. In fact, we can show $\tilde{\beta}^*(m)m < \rho$ for any $\rho > 0$. Suppose $\tilde{\beta}^*(m)m \geq \rho$. The first-order condition for $\tilde{\beta}^*$ is:

$$m(1 - \tilde{\beta}^* m) f(\kappa) \left(\frac{1}{\tilde{\beta}^*}\right)^2 - m^2 \Pr(\theta > \kappa) = 0,$$

where $\kappa = \frac{1}{\tilde{\beta}^*} - v\left(\frac{m+1}{2}\right)$. Observe that $\kappa \to -\infty$ as $v \to \infty$ if $\tilde{\beta}^* m \geq \rho$. But, when $\kappa \to -\infty$, the left-hand side of the first-order condition converges to $-m^2$. The first-order condition is consequently violated, which is a contradiction. This proves the lemma.

□

*Proof of Lemma 3.* For now, let us take it as given that $G(\mathbf{d}_M, e_M)$ is weakly increasing in $e_M$ (in a first-order stochastic dominance sense) and strictly increasing in $e_M$ whenever $\mathbf{d}_M > \mathbf{0}$. Let us also take it as given that $G(\mathbf{d}_M, e_M)$ is weakly increasing in $\mathbf{d}_M$ and strictly increasing in $\mathbf{d}_M$ whenever $e_M > 0$.

From Lemma 2, we know that $\Pi^*_M(n)$ is weakly increasing in $n$. Since $\Pi^*_M(n)$ is weakly increasing in $n$ and $G(\mathbf{d}_M, e_M)$ is weakly increasing in $\mathbf{d}_M$, $E[\Pi^*_M(n)|n \sim G(\mathbf{d}_M, e_M)] - c(e_M) - b_{(2)}$ is weakly increasing in $\mathbf{d}_M$. It follows from the Envelope Theorem that $V(\mathbf{d}_M)$ is weakly increasing in $\mathbf{d}_M$. This establishes part (1) of the lemma.

Now, assume $v > \hat{v}$. From Lemma 2, we know that $\Pi^*_M(n)$ is strictly increasing in $n$. Since $\Pi^*_M(n)$ is strictly increasing in $n$ and $G(\mathbf{d}_M, e_M)$ is strictly increasing in $\mathbf{d}_M$ when $e_M > 0$, $E[\Pi^*_M(n)|n \sim G(\mathbf{d}_M, e_M)] - c(e_M) - b_{(2)}$ is strictly increasing in $\mathbf{d}_M$ when $e_M > 0$. It follows from the Envelope Theorem that $V(\mathbf{d}_M)$ is strictly increasing in $\mathbf{d}_M$ provided $e^*(\mathbf{d}_M) > 0$. While it remains to be shown, $e^*(\mathbf{d}_M) > 0$ whenever $\mathbf{d}_M > \mathbf{0}$; this establishes that $V(\mathbf{d}_M)$ is strictly increasing in $\mathbf{d}_M$.

Now, let us show that $e^*(\mathbf{d}_M) > 0$ when $\mathbf{d}_M > \mathbf{0}$ and $v > \hat{v}$. If $\mathbf{d}_M > \mathbf{0}$ and $v > \hat{v}$, $G(\mathbf{d}_M, e_M)$ is strictly increasing in $e_M$ and $\Pi^*_M(n)$ is strictly increasing in $n$. It follows that $E[\Pi^*_M(n)|n \sim G(\mathbf{d}_M, e_M)]$ is strictly increasing in $e_M$. Therefore, the marginal benefit of exerting effort is greater than zero; at $e_M = 0$, the marginal cost of exerting effort is equal to zero ($c'(0) = 0$). Hence, it must be the case that $e^*(\mathbf{d}_M) > 0$.

It remains to show that $G(\mathbf{d}_M, e_M)$ is stochastically increasing in its arguments. Let $B_{lk}$ denote a random variable that equals $1$ when an investor $l$, connected to $M$ with degree $k$, becomes aware of the project and $0$ otherwise. We can write $n$, the total number of aware investors, as follows:

$$n = \sum_{k=1}^{\infty} \sum_{l=1}^{d_M^k} B_{lk}$$

Observe that the $B_{lk}$'s are independent random variables; $B_{lk}$ follows a Bernoulli distribution with parameter $\delta^{k-1} \cdot e_M$. Hence $n$, which follows a $G(\mathbf{d}_M, e_M)$ distribution, is the sum of independent Bernoulli distributions.

The Bernoulli distribution is strictly increasing in its parameter (in a first-order stochastic dominance sense). An increase in $e_M$ increases $\delta^{k-1} \cdot e_M$: the parameter of $B_{lk}$'s distribution. Therefore, provided $\mathbf{d_M} > \mathbf{0}$, $G(\mathbf{d}_M, e_M)$ is strictly increasing in $e_M$. When $\mathbf{d_M} = \mathbf{0}$, $n = 0$; so we can say that $G(\mathbf{d}_M, e_M)$ is weakly increasing in $e_M$ for all $\mathbf{d}_M$.

Now, let us show that if $\mathbf{d}_i > \mathbf{d}_j$ and $e > 0$, $G(\mathbf{d}_i, e) >_{FOSD} G(\mathbf{d}_j, e)$. This will show that $G(\mathbf{d}_M, e_M)$ is strictly increasing in $\mathbf{d}_M$ when $e_M > 0$. When $e_M = 0$, $n = 0$; so we can say that $G(\mathbf{d}_M, e_M)$ is weakly increasing in $\mathbf{d}_M$ for all $e_M$.

Let us define $\mathbf{d}_{i1}$ as follows: $\mathbf{d}_{i1} = (d_j^1, [d_i^1 - d_j^1] + d_i^2, d_i^3, d_i^4, ...)$. Notice that $\mathbf{d}_i > \mathbf{d}_j$ implies $d_i^1 - d_j^1 \geq 0$. Observe that, if $n$ follows a $G(\mathbf{d}_{i1}, e)$ distribution rather than a $G(\mathbf{d}_i, e)$ distribution, the difference is that $(d_i^1 - d_j^1)$ of the Bernoullis switch from parameter $e$ to parameter $\delta \cdot e$. Hence $G(\mathbf{d}_i, e) \geq_{FOSD} G(\mathbf{d}_{i1}, e)$ and $G(\mathbf{d}_i, e) >_{FOSD} G(\mathbf{d}_{i1}, e)$ if $d_i^1 - d_j^1 > 0$.

We can similarly define $\mathbf{d}_{i2}$ as follows: $\mathbf{d}_{i2} = (d_j^1, d_j^2, [(d_i^1 + d_i^2) - (d_j^1 + d_j^2)] + d_i^3, d_i^4, d_i^5, ...)$. Notice that $\mathbf{d}_i > \mathbf{d}_j$ implies $[(d_i^1 + d_i^2) - (d_j^1 + d_j^2)] \geq 0$. Observe that, if $n$ follows a $G(\mathbf{d}_{i2}, e)$ distribution rather than a $G(\mathbf{d}_{i1}, e)$ distribution, the difference is that $[(d_i^1 + d_i^2) - (d_j^1 + d_j^2)]$ of the Bernoullis switch from parameter $\delta \cdot e$ to parameter $\delta^2 \cdot e$. Hence $G(\mathbf{d}_{i1}, e) \geq_{FOSD} G(\mathbf{d}_{i2}, e)$ and $G(\mathbf{d}_{i1}, e) >_{FOSD} G(\mathbf{d}_{i2}, e)$ if $[(d_i^1 + d_i^2) - (d_j^1 + d_j^2)] > 0$.

More generally, define $\mathbf{d}_{il} = (d_j^1, ..., d_j^l, [\sum_{k=1}^l d_i^k - \sum_{k=1}^l d_j^k] + d_i^{l+1}, d_i^{l+2}, d_i^{l+3}, ...)$. By the same logic, $G(\mathbf{d}_{il}, e) \geq_{FOSD} G(\mathbf{d}_{i(l+1)}, e)$ and $G(\mathbf{d}_{il}, e) >_{FOSD} G(\mathbf{d}_{i(l+1)}, e)$ if $\sum_{k=1}^l d_i^k > \sum_{k=1}^l d_j^k$.

Since $\mathbf{d}_i > \mathbf{d}_j$, we know that $\sum_{k=1}^{l} d_i^k > \sum_{k=1}^{l} d_j^k$ for some $l$. So, $G(\mathbf{d}_{il}, e) >_{FOSD} G(\mathbf{d}_{i(l+1)}, e)$ for some $l$. We also know that $\lim_{l \to \infty} \mathbf{d}_{il} = \mathbf{d}_j$. It follows that $G(\mathbf{d}_i, e) >_{FOSD} G(\mathbf{d}_j, e)$. This completes the proof.

□

*Proof of Proposition 2.* As a first step in the proof, observe that if investor $j$ does not receive an equity offer, he earns a payoff of $1$ for sure. An investor $j$ who does receive an equity offer earns an expected payoff strictly greater than $1$. The reason is as follows. Recall from the proof of Lemma 1 that, when $x_i \leq \kappa$, $j$ earns a payoff of $1$ but when $x_i$ is strictly greater than $\kappa$, $j$ earns an expected payoff strictly greater than $1$. Since we assumed $\sigma \to 0$, $\Pr(x_i > \kappa) = \Pr(\theta > \kappa)$. $\Pr(\theta > \kappa) > 0$ since $\theta$ is distributed $N(\mu, \tau^2)$.

*Uniqueness.* Suppose there is an investor $j$ who does not link at time 0 to the eventual auction winner, $Y$, but instead links to another manager. Investor $j$ earns a payoff of $1$ since there is no chance of receiving an equity offer. Now suppose $j$ deviates and links to $Y$. Investor $j$'s deviation makes $Y$ more connected than he was previously, so $Y$ still wins the auction. Consequently, in deviating, $j$ has a nonzero chance of receiving an equity offer so receives an expected payoff strictly greater than $1$. Since the deviation is profitable, it follows that all players must link to the eventual auction winner in equilibrium. This establishes uniqueness.

*Existence.* To prove existence, we need to show that, in linking to manager $Y$ at time 0, investors are best-responding. Observe that, when all investors link to $Y$, they receive an expected payoff strictly greater than $1$ since there is a chance of receiving an equity offer. Suppose investor $j$ deviates at time 0 and links to another manager. There are at least 3 investors, so even when $j$ deviates, $Y$ is still the most connected manager and wins the auction. Hence, in deviating, $j$ links to a manager who loses the auction. Investor $j$ receives a payoff of $1$ from this deviation since there is no chance of receiving an equity offer. Therefore, $j$'s deviation is not profitable. We conclude that players are indeed best responding in linking to $Y$.

□

*Proof of Proposition 3.* We can use backward induction to analyze the game.

Consider time 4. Let $\Delta_M = \alpha_M + v s_M$. Observe that the project yields a return

$R = \tilde{\theta} + v \cdot \sum_{j \in S} a_j$, where $\tilde{\theta} = \theta + \Delta_M$. The time 4 game is therefore analogous to the time 4 game in the baseline model, with $\tilde{\theta}$ substituted for $\theta$. Consequently, we conclude that investors invest with probability 1 when $\tilde{\theta} > \kappa$; investor do not invest with probability 1 when $\tilde{\theta} < \kappa$. As before, $\kappa = \frac{1}{\beta_M} - v\left(\frac{m+1}{2}\right)$.

Now, consider time 3. Previously, we wrote $M$'s expected share of the project's return as $\Pi_M(m, \beta_M)$. In this case, $M$'s share also depends upon $\Delta_M$. Therefore, we will write $M$'s expected share of the project's return as $\Pi_M(m, \beta_M, \Delta_M)$. The following is a formula for $\Pi_M(m, \beta_M, \Delta_M)$ (which we obtain by substituting $\tilde{\theta}$ for $\theta$ in our previous formula for $\Pi_M(m, \beta_M)$):

$$\Pi_M(m, \beta_M, \Delta_M) = (\mu + \Delta_M) + vm(1 - \beta_M m) \Pr(\tilde{\theta} > \kappa) - \beta_M m E(\tilde{\theta} | \tilde{\theta} > \kappa) \Pr(\tilde{\theta} > \kappa),$$

where $\kappa = \frac{1}{\beta_M} - v\left(\frac{m+1}{2}\right)$ and $\tilde{\theta} = \theta + \Delta_M$.

As before, $M$ will choose $\beta_M$ and $m$ to maximize $\Pi_M$ subject to the constraint that $m \leq n$. We will write the value of $\Pi_M$ when $\beta_M$ and $m$ are optimally chosen as $\Pi^*(n, \Delta_M)$. Observe that Lemma 2 carries over, since all we have done is substitute $\tilde{\theta}$ for $\theta$. So, $\Pi^*(n, \Delta_M)$ is weakly increasing in $n$ and strictly increasing in $n$ for $v$ large.

There are two further results that are useful to establish: (1) $\Pi^*(n, \Delta_M)$ is strictly increasing in $\Delta_M$, and (2) $\frac{\partial \Pi^*(n, \alpha_M + v s_M)}{\partial s_M} > 1$ for $v$ large.

First, let us establish (1). We would like to show that $\Pi^*(n, \Delta_1) > \Pi^*(n, \Delta_0)$ when $\Delta_1 > \Delta_0$. Let $m_0^*$ and $\beta_0^*$ denote the optimal choices of $m$ and $\beta_M$ when $\Delta_M = \Delta_0$. Let $m_1 = m_0^*$ and let $\beta_1 = \frac{1}{\frac{1}{\beta_0^*} + (\Delta_1 - \Delta_0)}$. It will be sufficient to show that $\Pi_M(m_1, \beta_1, \Delta_1) > \Pi^*(n, \Delta_0)$.

Three observations will be useful. First, notice that $\kappa_1$ – the value of $\kappa$ corresponding to $m_1$ and $\beta_1$ – is equal to $\kappa_0 + (\Delta_1 - \Delta_0)$. Second, $\beta_1 < \beta_0^*$. Finally, we know that $m_0^* \beta_0^* < 1$. The reason is that it is never optimal to choose $m\beta_M \geq 1$: one could always do better by choosing $m\beta_M = 0$ instead. Consequently:

$$\begin{aligned}
\Pi_M(m_1, \beta_1, \Delta_1) =& (\mu + \Delta_1) + vm_1(1 - \beta_1 m_1) \Pr(\tilde{\theta}_1 > \kappa_1) \\
& - \beta_1 m_1 E(\tilde{\theta}_1 | \tilde{\theta}_1 > \kappa_1) \Pr(\tilde{\theta}_1 > \kappa_1) \\
=& (\mu + \Delta_1) + vm_0^*(1 - \beta_1 m_0^*) \Pr(\tilde{\theta}_0 > \kappa_0) \\
& - \beta_1 m_0^* E(\tilde{\theta}_0 | \tilde{\theta}_0 > \kappa_0) \Pr(\tilde{\theta}_0 > \kappa_0) - \beta_1 m_0^*(\Delta_1 - \Delta_0) \Pr(\tilde{\theta}_0 > \kappa_0) \\
>& (\mu + \Delta_1) + vm_0^*(1 - \beta_0^* m_0^*) \Pr(\tilde{\theta}_0 > \kappa_0) \\
& - \beta_0^* m_0^* E(\tilde{\theta}_0 | \tilde{\theta}_0 > \kappa_0) \Pr(\tilde{\theta}_0 > \kappa_0) - (\Delta_1 - \Delta_0) \\
=& \Pi_M(m_0^*, \beta_0^*, \Delta_0) = \Pi_M^*(n, \Delta_0).
\end{aligned}$$

This establishes (1).

Now, let us show (2):

$$\begin{aligned}
\Pi_M(m, \beta_M, \alpha_M + vs_M) =& (\mu + \alpha_M + vs_M) + vm(1 - \beta_M m) \Pr(\tilde{\theta} > \kappa) \\
& - \beta_M m E(\tilde{\theta} | \tilde{\theta} > \kappa) \Pr(\tilde{\theta} > \kappa) \\
=& (\mu + \alpha_M + vs_M) + [vm(1 - \beta_M m) \\
& - \beta_M m(\alpha_M + vs_M)] \Pr(\theta > \kappa - \alpha_M - vs_M) \\
& - \beta_M m E(\theta | \theta > \kappa - \alpha_M - vs_M) \Pr(\theta > \kappa - \alpha_M - vs_M),
\end{aligned}$$

where $\kappa = \frac{1}{\beta_M} - v\left(\frac{m+1}{2}\right)$.

Observe that $E(\theta|\theta > \kappa - \alpha_M - vs_M) \Pr(\theta > \kappa - \alpha_M - vs_M)$ is bounded between $0$ and $E(\theta|\theta > 0) \Pr(\theta > 0)$. Therefore, $\frac{1}{v}\Pi_M(m, \beta_M, \alpha_M + vs_M) \to s_M + m(1 - \beta_M m - \beta_M s_M) \cdot \Pr(\theta > \kappa - \alpha_M - vs_M)$ as $v \to \infty$. So, in the limit as $v \to \infty$, $M$'s problem becomes one of choosing $m$ and $\beta_M$ to maximize $\tilde{\Pi}_M(m, \beta_M) = s_M + m(1 - \beta_M m - \beta_M s_M) \cdot \Pr(\theta > \kappa - \alpha_M - vs_M)$. Let $\tilde{m}^*$ and $\tilde{\beta}_M^*$ denote the maximizing choices. Applying the Envelope Theorem, we find:

$$\begin{aligned}
\frac{\partial \tilde{\Pi}_M(\tilde{m}^*, \tilde{\beta}_M^*)}{\partial s_M} = 1 + \tilde{m}^* v(1 - \tilde{\beta}_M^* \tilde{m}^* - \tilde{\beta}_M^* s_M) f(\kappa - \alpha_M - vs_M) \\
- \tilde{\beta}_M^* \tilde{m}^* \cdot \Pr(\theta > \kappa - \alpha_M - vs_M),
\end{aligned}$$

where $\kappa = \frac{1}{\beta_M^*} - v\left(\frac{\tilde{m}^*+1}{2}\right)$ and $f$ denotes the pdf of the $N(\mu, \tau^2)$ distribution. Recall from the proof of Lemma 2 we showed, for $v$ sufficiently large, $\tilde{\beta}_M^* \tilde{m}^* < \rho$ for any $\rho > 0$. Or, put another way, $\tilde{\beta}_M^* \tilde{m}^* \to 0$ as $v \to \infty$. The same argument applies here. Therefore, the second term is positive for $v$ large. The third term converges to zero as $v \to \infty$. Hence, for $v$ large, $\frac{\partial \tilde{\Pi}_M(\tilde{m}^*, \tilde{\beta}_M^*)}{\partial s_M} \geq 1$.

We conclude that, for $v$ large, $\frac{\partial \Pi^*(n, \alpha_M + v s_M)}{\partial s_M} \geq v$. So, it is certainly true that $\frac{\partial \Pi^*(n, \alpha_M + v s_M)}{\partial s_M} > 1$ for $v$ large. This establishes (2).

Now, consider time 2 of the game. $M$'s expected payoff at time 2 is: $E[\Pi^*(n, \alpha_M + v s_M) | n \sim G(\mathbf{d_M}, e_M)] - c(\frac{e_M}{\gamma_M}) - s_M - b_{(2)}$. $M$ will choose $e_M$ and $s_M$ to maximize this expression subject to the constraint that $s_M \leq k_M$. We can write $M$'s resulting payoff as: $V(\mathbf{d}_M, \alpha_M, \gamma_M, k_M) - b_{(2)}$.

Lemma 3 clearly carries over. So, $V$ is weakly increasing in $\mathbf{d_M}$ and strictly increasing in $\mathbf{d_M}$ provided $v$ is large ($v > \hat{v}$). Furthermore, $e_M^* > 0$ whenever $v$ is large ($v > \hat{v}$) and $\mathbf{d_M} > \mathbf{0}$.

We showed that $\Pi^*(n, \alpha_M + v s_M)$ is strictly increasing in $\alpha_M$. So, by the Envelope Theorem, $V$ is strictly increasing in $\alpha_M$. The Envelope Theorem also implies that $V$ is weakly increasing in $\gamma_M$ and strictly increasing provided $e_M^* > 0$.

As $k_M$ increases, this simply makes $M$ less constrained in his choice of $s_M$. So, it is clear that $V$ is weakly increasing in $k_M$. We showed that, for $v$ large ($v > \hat{v}$), $\frac{\partial \Pi^*(n, \alpha_M + v s_M)}{\partial s_M} > 1$. Hence, when $v > \hat{v}$, the marginal benefit of increasing $s_M$ always exceeds the marginal cost (which is equal to $1$). So, it is optimal for $M$ to choose $s_M = k_M$. Additionally, an increase in $k_M$ strictly increases $M$'s payoff, $V$.

Finally, turning back to time 1, we know that managers will bid their valuations of asset A in the auction: $b_i = V(\mathbf{d_i}, \alpha_i, \gamma_i, k_i)$. This completes the proof.

□

*Proof of Proposition 4.* As in the baseline model, if an investor $j$ does not receive an equity offer, he earns a payoff of $1$ for sure. An investor $j$ who does receive an equity offer earns an expected payoff strictly greater than $1$.

*Uniqueness.* Suppose there is an investor $j$ who does not link at time $0$ to the eventual auction winner, $Y$, but instead links to another manager. Investor $j$ earns a payoff of $1$ since there is no chance of receiving an equity offer. Now suppose $j$ deviates and links to $Y$. Investor $j$'s deviation makes $Y$ more connected than he was previously, so $Y$ still wins the auction. Consequently, in deviating, $j$ has a nonzero chance of receiving an equity offer so receives an expected payoff strictly greater than $1$. Since the deviation is profitable, it follows that all players must link to the eventual auction winner in equilibrium. This establishes uniqueness.

*Existence.* Suppose $V(\mathbf{d}_{max} - \hat{\mathbf{d}}, \alpha_i, \gamma_i, k_i) > \max_{j \in N_M} V(\hat{\mathbf{d}}, \alpha_j, \gamma_j, k_j)$. We need to show that an equilibrium exists in which $i = Y$. One condition that must be met for such an equilibrium to exist is that manager $i$ outbids other managers in the auction when he is connected to all the investors: $V(\mathbf{d}_{max}, \alpha_i, \gamma_i, k_i) > \max_{j \in N_M} V(\mathbf{0}, \alpha_j, \gamma_j, k_j)$. Observe that this follows from $V$ being an increasing in $\mathbf{d}$ and $V(\mathbf{d}_{max} - \hat{\mathbf{d}}, \alpha_i, \gamma_i, k_i) > \max_{j \in N_M} V(\hat{\mathbf{d}}, \alpha_j, \gamma_j, k_j)$. The second condition that must be met for existence is that there is no profitable deviation at time $0$ for investors. Observe that, when all investors link to manager $i$, they receive an expected payoff strictly greater than $1$ since there is a chance of receiving an equity offer. Suppose investor $j$ deviates at time $0$ and links to another manager. Since $V(\mathbf{d}_{max} - \hat{\mathbf{d}}, \alpha_i, \gamma_i, k_i) > \max_{j \in N_M} V(\hat{\mathbf{d}}, \alpha_j, \gamma_j, k_j)$, manager $i$ still wins the auction. Hence, in deviating, $j$ links to a manager who loses the auction. Investor $j$ receives a payoff of $1$ from this deviation since there is no chance of receiving an equity offer. Therefore, $j$'s deviation is not profitable. We conclude that an equilibrium indeed exists in which manager $i = Y$.

Suppose $V(\mathbf{d}_{max}, \alpha_i, \gamma_i, k_i) > \max_{j \in N_M} V(\mathbf{0}, \alpha_j, \gamma_j, k_j)$. An equilibrium does not exist in which $i = Y$ since, when all investors link to manager $i$, he is outbid in the auction. This completes the proof.

$\square$

## 5.2 Distributions of $K$ and $R$

This section provides more detail regarding how to calculate the distributions of $K$ and $R$.

Recall from Section 2.3 that:

$$K = m^*(n) \cdot \mathbb{1}_{\{\theta > \kappa^*(n)\}} \text{ almost surely,}$$

where $\kappa^*(n) = \frac{1}{\beta^*_M(m^*(n))} - v\left(\frac{m^*(n)+1}{2}\right)$. Since $\theta$ and $n$ are independent random variables, distributed $N(\mu, \tau^2)$ and $G(\mathbf{d}_M, e^*(\mathbf{d}_M))$ respectively, we conclude that:

$$\Pr(K = k) = \sum_{\{\hat{n}: m^*(\hat{n}) = k\}} \Pr(n = \hat{n}) \cdot (1 - F(\kappa^*(\hat{n})) \text{ for all } k \geq 1.$$
$$\Pr(K = 0) = 1 - \sum_{k \geq 1} \Pr(K = k).$$

$F$ denotes the cdf of the $N(\mu, \tau^2)$ distribution. $\Pr(n = \hat{n})$ is the probability that $n = \hat{n}$ given that $n$ is distributed $G(\mathbf{d}_M, e^*(\mathbf{d}_M))$.

We can calculate $\Pr(n = \hat{n})$ from the observation that:

$$n = \sum_{k=1}^{\infty} \sum_{l=1}^{d_M^k} B_{lk},$$

where the $B_{lk}$'s are independent random variables with $B_{lk}$ following a Bernoulli distribution with parameter $\delta^{k-1} \cdot e^*(\mathbf{d}_M)$.

In the numerical example from Figure 1, we assume $M$ only has direct connections. When $M$ has $d$ direct connections and no indirect connections, $n$ simply follows a Binomial distribution with parameters $d$ and $e^*_M$. Hence, in this case: $\Pr(n = \hat{n}) = \binom{d}{\hat{n}} (e^*_M)^{\hat{n}} (1 - e^*_M)^{d - \hat{n}}$.

Now, let us characterize the distribution of $R$. We can denote the cdf and pdf of $R$ by $H(r)$ and $h(r)$ respectively. Observe that:

$$\begin{aligned} H(r) &= \Pr(R \leq r) \\ &= \Pr(\theta \leq r - vK) \\ &= \sum_k F(r - vk) \Pr(K = k). \end{aligned}$$

Differentiating, we find that:

$$h(r) = \sum_k f(r - vk) \Pr(K = k).$$

## 5.3 Distributions of $K$ and $R$ (More General)

This section shows how to calculate the distributions of $K$ and $R$ for the generalized version of the model presented in Section 3.

In Section 2.3, we showed that for the baseline model:

$$K = m^*(n) \cdot \mathbb{1}_{\{\theta > \kappa^*(n)\}} \text{ almost surely,}$$

where $\kappa^*(n) = \frac{1}{\beta_M^*(m^*(n))} - v\left(\frac{m^*(n)+1}{2}\right)$.

From the proof of Proposition 3, we obtain the following generalization:

$$K = s_M^* + m^*(n, \alpha_M, s_M^*) \cdot \mathbb{1}_{\{\theta + \alpha_M + v s_M^* > \kappa^*(n, \alpha_M, s_M^*)\}} \text{ almost surely,}$$

where $\kappa^*(n, \alpha_M, s_M^*) = \frac{1}{\beta_M^*(n, \alpha_M, s_M^*)} - v\left(\frac{m^*(n, \alpha_M, s_M^*)+1}{2}\right)$.

Since $\theta$ and $n$ are independent random variables, distributed $N(\mu, \tau^2)$ and $G(\mathbf{d}_M, e^*(\mathbf{d}_M))$ respectively, we conclude that:

For all $k \geq 1$,

$$\Pr(K = k + s_M^*) = \sum_{\{\hat{n} : m^*(\hat{n}, \alpha_M, s_M^*) = k\}} \Pr(n = \hat{n}) \cdot (1 - F(\kappa^*(\hat{n}, \alpha_M, s_M^*) - \alpha_M - v s_M^*)).$$

$$\Pr(K = s_M^*) = 1 - \sum_{k \geq 1} \Pr(K = k + s_M^*).$$

$F$ denotes the cdf of the $N(\mu, \tau^2)$ distribution. $\Pr(n = \hat{n})$ is the probability that $n = \hat{n}$ given that $n$ is distributed $G(\mathbf{d}_M, e^*(\mathbf{d}_M))$.

As before, we can calculate $\Pr(n = \hat{n})$ from the observation that:

$$n = \sum_{k=1}^{\infty} \sum_{l=1}^{d_M^k} B_{lk},$$

where the $B_{lk}$'s are independent random variables with $B_{lk}$ following a Bernoulli distribution with parameter $\delta^{k-1} \cdot e^*(\mathbf{d}_M)$.

Now, let us characterize the distribution of $R$. Again, we can denote the cdf and pdf of $R$ by $H(r)$ and $h(r)$ respectively. Observe that:

$$\begin{aligned} H(r) &= \Pr(R \leq r) \\ &= \Pr(\theta + vK + \alpha_M \leq r) \\ &= \Pr(\theta \leq r - \alpha_M - vK) \\ &= \sum_k F(r - \alpha_M - v(s_M^* + k)) \Pr(K = s_M^* + k). \end{aligned}$$

Differentiating, we find that:

$$h(r) = \sum_k f(r - \alpha_M - v(s_M^* + k)) \Pr(K = s_M^* + k).$$

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