Ability Bias, Skewness and the College Wage Premium

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Abstract
Changes in educational participation rates across cohorts are likely to imply changes in the ability-education relationship and thereby to impact on estimated returns to education. We show that skewness in the underlying ability distribution is a key determinant of the impact of graduate expansion on the college wage premium. Calibrating the model against the increased proportion of university students in Britain, we find that changes in the average ability gap between university students and others are likely to have mitigated demand-side forces.

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1. Introduction

Changes over cohorts in the proportions of young people at given levels of educational attainment will impact on average ability gaps by educational level, influencing the bias in estimated returns to education associated with the omission of ability. If the magnitude of bias is changing over time, it is correspondingly more difficult to draw legitimate inferences on the trends in returns to education from a series of cross-section snap-shots.

Consider the case of the college wage premium, defined as the difference in wage rates between college and high school graduates. Suppose that all individuals are either college graduates \((c = 1)\) or high school graduates \((c = 0)\) and that we can write wages, \(w\), as a function of ability, \(a\), and \(c\):

\[
w = \beta_0 + \beta_1 c + \beta_2 a + \epsilon ,
\]

(1)

where \(\epsilon\) is a stochastic error term. Ability is typically not observed in the data and hence the OLS estimator, \(\hat{\beta}_1\), will be upward biased. Differentiating (1) with respect to \(c\):

\[
\frac{dw}{dc} = \beta_1 + \beta_2 \left[ \frac{da}{dc} \right].
\]

(2)

The total effect of college on wages comprises the true effect plus an omitted variable bias term which depends on the magnitude of both (i) the true effect of ability on wages, \(\beta_2\), and (ii) the average ability gap between those with and those without a college education, \(da / dc\).

The substantial literature on how the college wage premium, in the US and elsewhere, has changed over time focuses on changes in \(\beta_1\) and \(\beta_2\) (see Cawley et al., 2000, and Taber, 2001); instead, we focus on the role of changes in \(da / dc\). Blackburn
and Neumark (1991, 1993) addressed the issue of whether increases in college matriculation in the US in the 1980s could explain a rising college wage premium through changes in ability composition by educational level. They found that the likely effects were in the opposite direction to the empirical evidence. In contrast, Rosenbaum (2003) finds evidence that ability composition changes are capable of explaining a substantial proportion of the increase in the US college wage premium between 1969 and 1989.

This paper attempts to make two contributions: first, we extend the existing theoretical analysis by identifying the crucial role of the skewness of the underlying ability distribution and, second, we provide a calibration of the model to offer insights into the behaviour over time of the college wage premium in the UK.

2. Skewness in the ability distribution

Blackburn and Neumark (1991, 1993) have shown that an increase in the proportion of college graduates in the population will lead to a reduction in the college wage premium under either a normal distribution or a symmetric triangular distribution of ability, so long as college graduates are in a minority in the population. In extending the Blackburn-Neumark model, we highlight the importance of skewness in the ability distribution for the impact on ability bias arising from an increase in the number of college graduates.

Consider the triangular distribution, on the unit support, characterized by different degrees of skewness, $m$. As $m < 1/2$, $m > 1/2$ or $m = 1/2$, the distribution is positively-skewed, negatively-skewed or symmetric, respectively; see Figure 1, for $m < 1/2$, where:

$$f(a) = \begin{cases} 
\frac{2(1-a)}{(1-m)} & a \geq m \\
\frac{2a}{m} & a < m 
\end{cases}.$$ (3)
(i) **Case 1.** First we consider the case in which the distribution is positively skewed $(m < 1/2)$ with $g < 1 - m$ and $\bar{a}_H > m$.

![Figure 1](image)

**Figure 1** Case 1: Positive skewness with $g < 1 - m$ and $\bar{a}_H > m$.

$\hat{a}_c$ denotes the ability of the marginal investor in college education; $\bar{a}_c$ ($\bar{a}_H$) is the average ability of college (non-college) graduates; and $g$ is the proportion of the cohort who graduate from college. By construction, $m$ denotes the proportion of the distribution below the mode. The difference between the average ability of college and non-college graduates is given by:

$$
\frac{da}{dc} = \bar{a}_c - \bar{a}_H. \quad (4)
$$

The median ability of college graduates, $\bar{a}_c$, is such that:

$$
\int_{\hat{a}_c}^{1} f(a)da = \int_{\hat{a}_c}^{1} \frac{2(1-a)}{(1-m)}da = \frac{g}{2}. \quad (5)
$$

It follows that:
\[ \frac{1}{1-m}(1-\bar{a}_c)^2 = \frac{g}{2}. \]  

(6)

and hence:

\[ \bar{a}_c = 1 - \sqrt{\frac{g(1-m)}{2}}. \]  

(7)

Similarly, for \( \bar{a}_H \):

\[ \int_{\bar{a}_H}^{1} f(a)da = \frac{1}{1-m}(1-\bar{a}_H)^2 = g + \frac{1-g}{2} = \frac{1+g}{2}, \]  

which implies that:

\[ \bar{a}_H = 1 - \sqrt{\frac{(1+g)(1-m)}{2}}. \]  

(8)

(9)

From (7) and (9), it follows that:

\[ \frac{da}{dc} = \bar{a}_c - \bar{a}_H = \left( \sqrt{1+g} - \sqrt{g} \right) \sqrt{\frac{1-m}{2}}, \]  

(10)

in which case:

\[ \frac{d}{dg} \left[ \frac{da}{dc} \right] = \frac{1}{2} \left( \sqrt{g} - \sqrt{1+g} \right) \sqrt{\frac{1-m}{2}} < 0. \]  

(11)

Hence, with relatively strong positive skewness such that \( g < 1-m \) and \( \bar{a}_H > m \), a rise in the proportion of college graduates within the cohort causes a fall in the premium attaching to a degree.

(ii) Case 2 Consider now the case depicted in Figure 2, where the ability distribution is sufficiently negatively skewed that \( g > 1-m \) and \( \bar{a}_c < m \). In contrast to Case 1, a rise in \( g \) will increase the average ability gap between college and high school graduates.
Figure 2  Case 2: Negative skewness with \( g > 1 - m \) and \( \hat{a}_u < m \).

For Case 2, we can use similar methods to those outlined for Case 1 in order to obtain:

\[
\tilde{a}_c = \sqrt{\frac{(2-g)m}{2}} \tag{12}
\]

and

\[
\tilde{a}_h = \sqrt{\frac{(1-g)m}{2}}. \tag{13}
\]

From (12) and (13), it follows that:

\[
\frac{da}{dc} = \tilde{a}_c - \tilde{a}_h = \left( \sqrt{2-g} - \sqrt{1-g} \right) \sqrt{\frac{m}{2}} \tag{14}
\]

and hence that:

\[
\frac{d}{dg} \left[ \frac{da}{dc} \right] = \frac{1}{2} \left( \frac{\sqrt{2-g} - \sqrt{1-g}}{\sqrt{(2-g)(1-g)}} \right) \sqrt{\frac{m}{2}} > 0. \tag{15}
\]
Thus, for this case of a sufficiently negatively skewed distribution, a rise in $g$ causes an increase in the premium for a degree.

(iii) Case 3 We now consider the intermediate case in which $g$ and $m$ are such that $ar{a}_u < m < ar{a}_c$. In this case, $\bar{a}_c$ will be given by equation (7), while the value of $\bar{a}_u$ will be equal to that shown in equation (13). Combining these, it follows that:

$$\frac{da}{dc} = \bar{a}_c - \bar{a}_u = \frac{1}{\sqrt{2}} \left\{ \sqrt{2} - \sqrt{(1-m)g} - \sqrt{(1-g)m} \right\} ,$$

(16)

and hence:

$$\frac{d}{dg} \left[ \frac{da}{dc} \right] = \frac{1}{2\sqrt{2}} \left[ \frac{\sqrt{gm} - \sqrt{(1-g)(1-m)}}{\sqrt{g(1-g)}} \right] \begin{cases} <0 & \text{if } g < 1-m \\ >0 & \text{if } g > 1-m \end{cases} .$$

(17)

Together with (11) and (15) for Cases 1 and 2, respectively, (17) establishes the result captured in the following proposition.

Proposition 1 In the case of the uni-modal triangular distribution, the premium for the possession of a degree is decreasing (increasing) in the proportion with a degree if $g < 1-m$ ($g > 1-m$).

In other words, whether the degree premium is falling or rising in the proportion, $g$, depends solely on the relative size of $g$ and $1-m$. For given $m$, the premium will fall (rise) as $g$ rises if $g$ is relatively small (large). For given $g$, the premium is more likely to be falling in $g$, at the margin, the smaller is $m$: that is, the more positively skewed is the ability distribution. Notice one corollary of the analysis, which may have particular empirical relevance; while small increases in $g$ might be associated with a falling college wage premium – consistent with the Becker Woytinsky lecture hypothesis.
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(Becker, 1975) – this need not always hold as, if \( g \) rises beyond a critical point, given by \( g = 1 - m \), further increases in \( g \) will lead to a rising premium for a degree. This is more plausible the larger is \( m \): that is, the less positively skewed is the ability distribution.

Our analysis identifies the extent of the skewness of the underlying ability distribution as a key determinant of the behaviour of the college wage premium, showing the knife-edge sensitivity to the extent of skewness relative to the size of \( g \). It is likely that in more general single-peaked distributions, for which the uni-modal triangular distribution is just a linear approximation, second and higher order derivatives will influence over the properties of the model – though these are likely to be of lower order importance compared to the significance of the skewness property we have isolated.

3. Calibration

Evidence for the UK suggests that despite increases in the relative demand for more highly educated workers, the estimated college wage premium showed little, if any, tendency to increase during the late 1980s and 1990s (see Walker and Zhu, 2008). Similarly, Bratti et al. (2008) find that, for men, the college wage premium for those born in Britain in 1970 (and typically graduating in the early 1990s) is no different to that for those born in 1958 (and graduating around 1980): for women, the premium fell considerably. Over time, the college participation rate\(^1\) was rising dramatically: from about 15% for the 1958 cohort to 30% for the 1970 cohort (source: DfES, 2003). An explanation for the absence of a clear skill-biased demand-side influence on the estimated college wage premium is the possibility that the increase in the proportion of the cohort attending college produced changes in relative ability composition and hence affected the

\(^1\) More usually referred to as the higher education participation rate in the UK
extent of ability bias. Our analysis suggests that if \( g < 1 - m \), then a rise in \( g \) will lead to a reduction in ability bias, *ceteris paribus*, thus producing a lower estimate for the size of the college wage premium and hence offsetting any positive demand-side forces. We now develop numerical predictions for the change in \( da/dc \) from a calibrated version of the model, inputting values of \( g \) for the 1958 and 1970 British birth cohorts and considering various values of \( m \).\(^2\)

We set \( g = 0.15 \) for the 1958 birth cohort and \( g = 0.30 \) for the 1970 birth cohort. In Table 1, we calibrate \( da/dc \) for Case 1 and find that, for all values of \( m \) which satisfy this case, the average ability gap between those with and those without a college education falls by about 14%. This is not a trivial change, though the extent to which this might impact on estimates of the college wage premium will depend on the return to ability. In Table 2, we consider a calibration for Case 3, the intermediate case.\(^3\) In this case, when the distribution is symmetric the doubling in \( g \) is associated with an 11% fall in the average ability gap – similar to that in Case 1. However, as the distribution becomes increasingly negatively-skewed, the extent of the fall in the gap diminishes until, for \( m = 0.8 \), the impact of the increase in \( g \) is a (small) rise in the gap. As \( g \) grows further – as has been the case in the UK – it becomes more likely that a rise in \( g \) might lead to an increase in the average ability gap.

**3. Conclusions**

We have demonstrated the critical role of skewness in the distribution of ability in determining the impact of changing educational participation on the relationship between

\(^2\) In further work, we address the issue of whether differential levels and changes in university participation by gender might explain observed differences in the college wage premium by gender over time.

\(^3\) Note that, given the values of \( g \), no values of \( m \) satisfy the conditions under which Case 2 is feasible.
ability and education and hence on estimated returns to education, focusing on the college wage premium and graduate expansion. We have also examined a calibration of the model for the British birth cohorts of 1958 and 1970 and shown how the extent of changes in the average ability gap between university students and others varies under alternative assumptions regarding skewness. We find that unless the distribution is quite strongly negatively-skewed, the observed increase in the proportion of the cohort graduating is capable of generating a reduction in the average ability gap of at least ten percent, thereby potentially mitigating the effects of demand-side forces on the college wage premium.
References


### Table 1: Calibration based on Case 1, $g < 1 - m$ and $\bar{\alpha}_H > m$.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>$g$ 1958</th>
<th>$g$ 1970</th>
<th>$m$</th>
<th>$da / dc$</th>
<th>$\Delta da / dc$</th>
<th>$\Delta da / dc$ (%)</th>
</tr>
</thead>
</table>
| 0.15   | 0.1      | 0.1      | 0.46
| 0.15   | 0.2      | 0.2      | 0.43
| 0.15   | 0.3      | 0.3      | 0.41
| 0.3    | 0.1      | 0.40     | -0.06 | -14.0
| 0.3    | 0.2      | 0.37     | -0.06 | -14.0
| 0.3    | 0.3      | 0.35     | -0.05 | -14.0

### Table 2: Calibration based on Case 3, $\bar{\alpha}_H < m < \bar{\alpha}_C$.

<table>
<thead>
<tr>
<th>Case 3</th>
<th>$g$ 1958</th>
<th>$g$ 1970</th>
<th>$m$</th>
<th>$da / dc$</th>
<th>$\Delta da / dc$</th>
<th>$\Delta da / dc$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>0.5</td>
<td>0.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>0.6</td>
<td>0.32</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>0.7</td>
<td>0.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>0.8</td>
<td>0.29</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 0.3    | 0.5      | 0.31     | -0.04 | -10.9
| 0.3    | 0.6      | 0.30     | -0.03 | -7.8
| 0.3    | 0.7      | 0.29     | -0.01 | -3.8
| 0.3    | 0.8      | 0.30     | 0.00  | 1.1

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