Selecting the wisdom of an expert

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Abstract

We study endogeneous information acquisition before communication when there is uncertainty about an expert's bias as well as about his information. Investment in information increases credibility while communicating to a decision maker. We define the *signaling* and the *intrinsic value* of information and find the conditions under which separation in information acquisition arises. We find that a more informed expert often ends up transmitting less information. Indeed, communication is most informative with an initially uninformed expert at an intermediary information cost, while the decision maker's preferred level of expertise, which is non monotone in cost, ensures that a separating equilibrium arises.

Keywords: information acquisition; cheap talk; communication; signaling; credibility.

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1 Introduction

In markets and organizations, decision makers often rely on the advice of experts. However, it is difficult to assess the credibility of experts and the quality of their information. The literature on strategic information transmission pioneered by Crawford and Sobel (1982) often assumes that the expert's bias is common knowledge and that the expert has access to information for free. These assumptions have been relaxed in different setups. Uncertainty about bias has been explored in the literature on the reputation of experts in repeated cheap talk initiated by Sobel (1985) and Morris (2001). Costly information acquisition before cheap talk was first considered by Austen-Smith (1994) and recently Sobel (2013) described this area as an open question[1]. In order to study the signaling role of information acquisition, in this paper we consider an expert who may or may not be biased and may or may not be informed plus has the option to make an observable investment into information. These ingredients make the problem one of costly signaling and cheap talk.

To understand the strategic consideration, contemplate a politician consulting a policy advisor about the effects of pollution on the occurrence of lung disease. The advisor may be receiving kickbacks from a polluting industry and hence be biased. Maybe the advisor has no conflict of interest, but already has access to information due to his experience. Finally, it may be that the advisor, although unbiased, isn't informed on the issue and may pay a team of researchers to acquire information. Complication arises because a biased advisor could mimic this behavior only to ensure his advice appears more credible but not change his initially biased advice. There are instances when experts invest into their credibility in order to push their personal agenda, while in reality they are being backed by third parties. For example, Andrew Wakefield, a former gastroenterologist and medical researcher, was found guilty of misconduct in his research paper which claimed the MMR vaccine was linked to autism and bowel disease, although years after his research was published and already had impact. It was later claimed that he had received payment from lawyers who were trying to prove that the vaccine was unsafe.[2]

[1]Some recent work on this topic include Pei (2015), Argenziano et al. (2016) and Deimen and Szalay (2016)

[2]"MMR doctor given legal aid thousands" *The Sunday Times*, 31 December 2006. The motivation for seeing

Similar issues arise when a division manager advices the headquarters about the optimal level of funding in a new project. The preferences of division managers usually differ from the headquarters as the manager often does not internalize the effect of this project on the profitability of other divisions. Furthermore, the manager may estimate the profitability of the project due to his expertise or may ask his team to provide a forecast, the cost being the time spent by his employees. It is known that division managers may knowingly use biased forecasts as they tend to focus on the profits of their own division rather than the whole company. A lawyer may know whether it is optimal to settle a lawsuit or not, but may also have a bias towards fighting in court in order to boost the fees. Although a lawyer who is very experienced may make a recommendation right away by relying on his expertise, it is natural to imagine that a lawyer who spends some time to study the case in detail before making a suggestion appears more credible. Motivated by these examples where experts can overtly spend money, time or resources in order to get informed as well as to increase their credibility before communicating to a decision maker (DM), this paper studies the question of how bias affects incentives for endogenous information acquisition. Under what conditions does the information acquisition behavior also credibly reveal whether a recommendation is coming from a biased or a non biased expert? Furthermore, given that the DM cannot identify the bias of the expert, what type of expert, in terms of how informed they are at the outset, would provide the most accurate information?

We find that an expert who is more informed to begin with usually does not end up providing the best advice. The reason is this type, relying on his prior information, doesn't have enough incentives to separate himself from a biased expert via costly signalling through information acquisition. On the other hand, an uninformed expert faces sufficient uncertainty that he is willing to acquire information and whilst doing so may simultaneously separate himself from a biased type. Hence, acquired information often turns out easier to transmit than ex-ante information when investment serves a dual purpose of getting informed and revealing the preferences of the expert. Hence, our model emphasizes the value of an expert who is uninformed at the outset but willing to acquire information. Furthermore, if the decision maker

this as cheap talk is due to the fact that for a lon time the flaws in the research published wasn't figured out.

could pick an expert according to his level of expertise, which in our setup is the probability that the expert is informed at the outset, it is optimal to do so in a way that separation in investment behavior arises and no wasteful investment happens. It turns out that this optimal level of expertise is non-monotone in the cost of information acquisition. Specifically, it is decreasing in this cost over a certain region. Finally, we show that a higher cost of information acquisition leads to higher overall welfare in case it leads to separation.

To better explain our results, we will sketch the model. There is an unknown state of the world, $\omega \in \{0, 1\}$ and the DM's action space is $y \in [0, 1]$. The expert is biased with probability β and with $1 - \beta$ shares the same preferences as the DM. In addition, the expert is perfectly informed about the state of the world with probability α, and with $1 - \alpha$ shares the same prior as the DM. The DM and the unbiased type of expert want the action to match the state of the world, while the biased type always wants the highest action regardless of the state. Hence, whether the biased expert is informed or not is irrelevant, and whenever we talk of informed expert we imply the unbiased type. We focus on equilibria in which an unbiased expert initially informed of low state always truthfully communicates to the DM without investment. On the other hand, when an unbiased expert is informed of high state, he shares the same incentives as a biased expert as both want the highest action. At the outset, the expert has the option to make a costly investment in information acquisition, which is observable although its outcome is private to the expert. As there is a probability that the expert is already informed, that an expert does not invest in information does not reveal that he is uninformed, neither that he is biased.

Our first set of results are about the characterisation of equilibria. The equilibrium investment strategies of different expert types determine the DM's posterior and hence the credibility of communicating the high state with or without investment. The expert cares about his credibility only insofar as it influences the final action: reputation is instrumental and not intrinsic. Hence, a biased expert, as well as an unbiased expert who is already informed of high state, may wastefully invest in order to pool with the uninformed unbiased types who value information. We identify the *intrinsic value of information* for the unbiased uninformed expert as the value of getting informed when his communication is taken at face value, and the *signaling value* for the biased and unbiased informed expert which is the value of deviating to invest and pool with the unbiased uninformed type. We focus on the case in which the signaling value of information is less than its intrinsic value, ensuring the existence of a separating equilibrium for some cost values. In a separating equilibrium, the only type that invests in information is the uninformed and unbiased expert, while a biased expert pools with the unbiased type informed of high state. Hence, as well as being the most informative pure strategy equilibrium, the separating equilibrium is the one in which there is no wasteful investment, i.e. investment that only has signaling value.

That the unbiased informed expert pools with the biased type whenever the state is high means that an unbiased expert can separate himself from the biased one in both states of the world only if he is initially uninformed. This type communicates more precisely than an expert who is already informed: as upon investment, in addition to getting informed he can also identify himself as unbiased whenever a separating equilibrium arises. At the same time, for the separating equilibrium to arise, the probability that the expert is already informed should be sufficiently high, i.e. α high enough, in order to make communication without investment sufficiently credible to prevent pooling by the biased expert. In other words, the presence of the informed type of expert allows the biased type to avoid facing too much prejudice [3] when sending a high message without investment, hence prevents pooling by the biased types. Conditional on the separating equilibrium arising, the DM's payoff is decreasing in α, due to the loss of communication with the informed types who pool with the biased type. Hence, in an ex-ante sense, the DM's payoff is maximized when α is just high enough for a separating equilibrium to arise but not more, and we define this optimal expertise level as α^*. In an ex-post sense, the DM is better off if matched with an unbiased expert initially uninformed who ends up investing.

For lower cost of information acquisition, a pooling equilibrium exists in which all types except the unbiased one with low signal invest in information. In this equilibrium, the expert is always informed but the DM cannot distinguish between a biased and an unbiased type whenever

[3] Che and Kartik (2009) use this expression in their disclosure game to explain the skepticism of the decision maker when a sender doesn't disclose information. We use the term to explain the skepticism towards a sender which doesn't invest.

the high state realizes. Hence, the DM is strictly worse off in this equilibrium compared to the separating one in which all unbiased types are informed as well but an expert who invests in information can be identified as unbiased. The game also admits interesting mixed strategy equilibria in which the biased types and unbiased types informed of high state are indifferent but play different probabilistic investment strategies. As the cost of information acquisition decreases, proportionately more biased types than unbiased types must invest, decreasing the credibility of communication upon investment, so that the indifference condition of these types is satisfied. We show that for any given cost, the DM's highest payoff among mixed strategy equilibria is identical to that of the separating equilibrium with the minimal amount of informed experts. This is the unique α^* which gives the highest possible payoff to the DM among any equilibria.

Our second set of results concern the optimal expertise level, which turns out to be non-monotone in the cost of information acquisition. If the cost is low enough that a pooling equilibrium arises even for $\alpha = 1$, expertise doesn't matter for the DM: it is so cheap to mimic information acquisition behavior that pooling cannot be avoided. When cost is higher, α^* is just high enough to ensure that a separating equilibrium arises. The intuition is as follows: it is optimal to have the expertise level high enough to achieve a separating equilibrium and not more, due to the loss of communication with the informed types which will pool with biased ones. Moreover, this optimal level of expertise decreases in the cost of information acquisition over a certain interval. As the cost of information increases, a lower credibility upon no investment (fewer informed types) is sufficient to prevent wasteful investment by the biased types. When the cost of information acquisition is high enough that a separating equilibrium arises even when the expert is uninformed for sure, then $\alpha^* = 0$. In this case, the biased types find it too costly to invest even if the DM identifies an expert who does not invest as a biased one. Hence, the unbiased expert can perfectly transmit acquired information and it is optimal that this type is uninformed for sure at the outset. Finally, when the cost is high enough that no type would invest, a perfectly informed expert is optimal ($\alpha^* = 1$): when the DM cannot distinguish between a biased and an unbiased expert through investment behavior, he prefers the expert to be informed for sure.

In terms of the welfare, we show that less wasteful investment more than compensates for the higher cost of information acquisition when the cost increases to move the equilibrium from the pooling to the separating region, while the total welfare relating to communication and decision making remains constant. Hence, total welfare is non-monotone in the cost of information: it decreases as cost increases in a given equilibrium, but if this increase shifts the equilibrium to a separating one, then total welfare increases.

Finally, we contrast outcomes in our overt information acquisition setting with the outcomes that would emerge with covert information acquisition when the investment choice of the agent is not observable. In this case, there is no wasteful investment as informed types can pool with unbiased types without incurring any cost, but separation between biased and unbiased types is not possible whenever the state is high. Overall welfare is higher with overt information acquisition when costs are such that a separating equilibrium arises, due to more precise communication. In the overt case, some unbiased types can separate from biased types through the observability of investment which is not the case in covert information acquisition. On the other hand, when pooling in investment arises with overt information acquisition, covert information acquisition always leads to higher welfare because it results in less wasteful investment as there is no costly signaling. The DM is weakly better off in overt information acquisition in any case, as he doesn't internalize the cost of investment plus he has the chance to identify an unbiased expert only in the overt case. The paper is organized as follows: in Section 2 we discuss other related literature, Section 3 describes the model, Section 4 characterizes Equilibria, Section 5 discusses overall welfare, Section 6 studies the DM's optimal expertise level, Section 7 considers an extension to overt information acquisition, and Section 8 concludes.

2 Literature

This paper relates to several strands of the long-standing cheap talk and costly signaling literatures. First, it relates to the literature on costly information acquisition before cheap talk, starting with Austen-Smith (1994) in which costly and private information acquisition leads to perfect information and there is uncertainty about the cost of information acquisition. As the expert can prove he is informed but can feign ignorance, low types pool with uninformed types to achieve a higher outcome, which improves communication for higher types. Only recently has there been more work on strategic communication with endogenous information acquisition. Pei (2015) modifies the setup of Crawford and Sobel (1982) by endogenising information acquisition and finds that the expert truthfully transmits all the information he acquires, in other words the expert doesn't acquire information that he will not transmit. Argenziano et al. (2016) consider a similar problem and in addition compare centralisation to delegation. Deimen and Szalay (2016) consider a setup with endogeneous information where a biased expert can choose on which issues to gather information and show that communication dominates delegation.[4] Suurmond et al. (2004) consider the effect of reputation in a delegation setup with information acquisition and an unbiased expert with unknown ability. They show that reputational concerns may help by incentivizing the expert to acquire information when he doesn't know his ability. In contrast if the expert privately knows his ability, he may take inefficient actions in order to mimic an efficient type. Dur and Swank (2005) study the tradeoff between the incentive to acquire information versus the precision of communication as a function of expert's bias. There is no uncertainty about bias and hence no signaling motive for the expert in these papers. In that sense, our setup is novel in considering the signaling motive in acquiring information in presence of uncertainty about the bias of the agent.

The paper also relates to the literature in cheap talk with uncertainty about expert's bias and reputational concerns. Morgan and Stocken (2003) consider strategic communication as in Crawford and Sobel (1982) with added uncertainty about the expert's type. They show that truthful communication cannot arise even with an unbiased analyst whenever the state of the world is sufficiently high. Sobel (1985) and Morris (2001) consider repeated cheap talk, where reputation is instrumental as in our setup, but communication and decision making process is repeated. Outside of the communication literature, Ely and Valimaki (2003) consider a long run player facing short run players who takes a payoff relevant action and highlight the distortional

[4]Esö and Szalay (2010) consider a game in which the expert has no bias and endogenous information acquisition, and show that restricting the message space can induce the sender to acquire information more often.

consequences today of the incentives to avoid bad reputations in the future. In our setting the decision is taken only once but the observable investment in information affects the credibility of the expert in the communicating stage. Hence, messages are never distorted, but distortion takes the form of wasteful investment in information acquisition. Meng (2015) considers a two period setup as in Morris (2001) and endogenizes the precision of the expert's information, where investment is not a signal as it is not observable and finds that reputation building enhances the incentives to invest in information for both types in the first period to build credibility for the second period. There is also a literature (e.g. Ottaviani and Sorensen and Ottaviani and Sorensen (2006b)) which shows that reputation concerns may lead to inefficient herding when experts bias their recommendation in order to appear more informed, where there is an intrinsic value of reputation. While our setup features neither dynamics nor an intrinsic value of reputation, similar effects to those in the literature on the reputation of experts arises nonetheless due to endogenous information acquisition hence the signaling incentives which arise.

Unlike the literature on cheap talk communication, there is a developed literature on the acquisition and disclosure of hard information. Earlier examples include Matthews and Postlewaite (1985), Shavell, in which self interested and uninformed agents invest in information prior to disclosure. Che and Kartik (2009) consider a setup of strategic disclosure where the decision maker and agent have different initial priors. As effort in information acquisition not observable, it is optimal for the DM to hire an advisor with a sufficiently different prior. The reason is due to the prejudice of the DM against an advisor who doesn't disclose information, which is stronger for an advisor with prior farther from hers, which in return induces the advisor to exert more effort. Of course, disclosure from a more biased advisor is more strategic as well and the optimal type of advisor is determined by the tradeoff between these two forces. In our setup, in contrast, we talk about the prejudice towards an expert that doesn't invest in information and the best equilibrium is such that the credibility of an expert that doesn't invest is just high enough to prevent inefficient pooling. In addition, there is uncertainty about expert's bias and the tradeoff is between lower precision of communication with the informed types and prevention of inefficient pooling by biased types. We focus on the optimal level of expertise, taking their bias as uncertain. Kartik et al. (2017) also consider investment in verifiable information by competing experts with opposite biases, in which the investments of experts are strategic substitutes.

Finally, the wasteful investment in our setup which serves the purpose of signalling is reminiscent of *burning money* as identified by Austen-Smith and Banks (2000), where cheap talk is not the only way to communicate but senders may also incur a cost in order to enhance their communication. However, contrary to pure money burning, the investment by the uninformed type of experts is not wasteful.

3 Model

There is a decision maker (DM, she) and an expert (he). There is a state of the world $\omega \in \{0, 1\}$ and a commonly known prior $\Pr(\omega = 1) = \rho$. We assume that $\rho \geq 0.5$ in the main part of the paper, and discuss the case with $\rho < 0.5$ in Appendix A. At the beginning of the game, the expert learns his two dimensional private type. With commonly known probability $(1 - \beta)$ the expert is unbiased and shares the same payoff as the DM which is $-(\omega - y)^2$ and with probability β he is biased and always wants the highest possible action, with payoff $-(1 - y)^2$, where $y \in [0, 1]$ is the decision maker's eventual action.[5] The expert's type is denoted by $\theta \in \{u, b\}$ corresponding to unbiased or biased. Second, with commonly known probability α the expert is perfectly informed about the state of the world ex-ante; while with $(1 - \alpha)$ the expert is uninformed and shares the same prior as the DM. We interpret α as an expertise parameter[6], which may come from the experience that the expert has derived from working on similar issues in the past. The decision maker does not observe whether the expert is biased or not, but knows α and β. In addition, the expert has the option to invest in information by incurring cost c in order to get a private signal that perfectly reveals the state. The DM

[5] This type of assumption about the expert type and utility is made by others (see e.g. in Morris (2001)).

[6] Bhattacharya et al. (2018) also make the assumption that the expert is either perfectly informed or uninformed and interpret this as an expertise parameter, when looking at optimal composition of expert panels without information acquisition.

observes the investment decision but not its outcome.[7] Indeed, any type of expert could invest in information, including a biased or an informed one for whom investment purely serves the purpose of signaling. We call such signaling wasteful investment. As the expert is already perfectly informed with probability α, the fact that he doesn't invest doesn't reveal that he is uninformed, nor that he is biased. Finally, communication happens through cheap talk following the investment decision. Below is a summary of the stages of the game:

1. the state of the world and the expert's type is realized.
2. the expert decides whether to acquire a perfectly revealing signal by incurring c, a decision denoted $x \in [0, 1]$.
3. the expert sends a message $m \in M$ to the DM.
4. the DM takes an action, $y \in [0, 1]$.

The decision maker interprets the expert's message as a function of her prior and the equilibrium information acquisition behavior of the different types. As the biased expert always wants the highest action regardless of the state, there is only one type of biased expert and whether he is informed or not has no relevance.[8] An equivalent assumption is that the biased expert is never informed. We can then summarize the types of experts at the beginning of the game into four:

1. biased
2. unbiased and uninformed
3. unbiased and informed of state 1
4. unbiased and informed of state 0

Call $\Phi \in \{0, 1, \varnothing\}$ the information structure of the expert at the communication stage as a result of his initial information and information acquisition. The expert's communication strategy denoted $m(\Phi, \theta, x)$ will be pure as each type of expert strictly prefers one of the

[7]That the information acquisition process is **observable** to the decision maker but not its **outcome** is a common assumption in the literature, see e.g. Fischer and Stocken (2010), Argenziano et al. (2016), and Deimen and Szalay (2016) who also consider information acquisition before cheap talk in different setups with known bias of the sender.

[8]This would be different were the biased type's payoff not state independent, as in Morgan and Stocken (2003)

three messages, while the investment strategy x may be mixed. The equilibrium concept is Perfect Bayesian Equilibrium (PBE). A strategy profile $\langle x, m, y \rangle$ along with the DM's posterior $\mu(x, m) = Pr(\omega = 1|x, m)$ forms a PBE if and only if:

- The DM's action maximizes her payoff given her posterior:

  $$y^*(x, m) = \arg\max_y -\mu(x, m)(1 - y)^2 - (1 - \mu(x, m))y^2$$

  leading to $y^*(x, m) = \mu(x, m)$.

- The DM's posterior $\mu(x, m)$ is consistent with the expert types' investment and message strategies.

- The expert's strategy, (x, m) maximizes his payoff given the DM's belief updating and action strategy.

4 Equilibrium Analysis

As this is a game of signaling followed by cheap talk, there are multiple equilibria for some parameter values. As usual, there always exists a babbling equilibrium. In our setup, this is one in which no one invests in information and the DM interprets any message as babbling. Other than this, depending on the cost of information acquisition, there exist equilibria with information acquisition. In the following, we always focus on the **most informative equilibrium**. When there exists multiple informative equilibria we use the **Intuitive Criterion** (Cho and Kreps (1987)) to establish the most reasonable out-of-equilibrium belief. The next lemma summarizes the set of messages that are sent in any equilibria we focus on.

Lemma 1. *As the expert is either perfectly aligned or extremely biased, he has a uniquely optimal communication strategy following investment or no investment. The set of messages sent in equilibrium after investment are $m^i \in \{0, 1\}$ and after no investment they are $m^n \in \{0, 1, \varnothing\}$.*

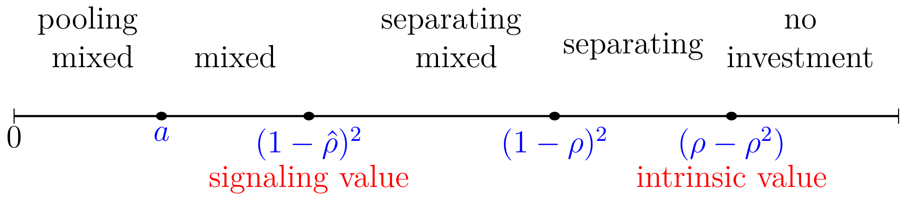
If there is investment, it is sure that the expert is informed and wants to induce either the highest or the lowest action. If there is no investment, there is a possibility that the expert is uninformed and if he is unbiased, he wants to send $\varnothing$. If the expert is biased he wants to send the high message independent of being informed or not. Given that the messages sent conditional on information set is understood, we now study the equilibrium investment behavior. First, we establish the behavior of the unbiased expert informed of low state.

Lemma 2. *The equilibria in which the unbiased type informed with signal 0 sends message* $m = 0$ *without investing after which the DM chooses* $y = 0$ *are the only ones that survive the Intuitive Criterion.*

We restrict attention to equilibria in which communication by the unbiased type of expert informed of low state happens without any investment and it is taken at face value by the DM. This is rather trivial: as neither the biased type nor the uninformed unbiased type would ever want to mimic this type, communicating signal 0 without investing perfectly reveals the state as well as his preferences. Hence, from now on, we only have to deal with the equilibrium behavior of the remaining expert types. In addition, the unbiased type of expert always sends a truthful message: whenever uninformed, this type sends $m = \varnothing$ and whenever informed he sends $m = \omega$. The only uncertainty about this expert type is whether he will invest or not. As the biased and unbiased types informed of high state share the same preferences, they follow the same information acquisition strategies in strict Nash Equilibria and always send message $m = 1$ regardless of their investment strategy. That these types have the same incentives underlines many interesting results, in particular that an uninformed expert who becomes informed later on can separate and reveal himself as unbiased while an expert who is already informed pools with the biased type when the state is high. When these types are indifferent, they can play different strategies, which we will define as mixed strategy equilibria. Even though these types have the same payoff, what they do is significant for the DM: the unbiased type is communicating truthfully while the biased type sends an uninformative message. We define "*pooling*" and "*separating*" in this setup as a function of the investment decisions of the two groups of expert types:

Group 1: biased type and unbiased type informed of high state.

Figure 1: Equilibria as a function of cost, for fixed α and β



$\hat{\rho} = \mu(0, 1)$ in the separating equilibrium.
$\tilde{\rho} = \mu(1, 1)$ in the pooling equilibrium.
$a = (1 - \hat{p})^2 - (1 - \tilde{p})^2$

Group 2: unbiased uninformed type.

We characterize all equilibria in which there is information acquisition as a function of cost level (see Appendix A for the details):

1. **Separating equilibrium:** Only group 2 type invests hence the DM takes communication after investment at face value. Group 1 types send $m = 1$ without investing upon which the DM chooses $y = \hat{\rho} > \rho$.

2. **Pooling equilibrium:** Both groups invest. The unbiased type sends $m = \omega$ while the biased type sends $m = 1$. Upon $m = 1$, the DM chooses $y = \tilde{\rho} > \hat{\rho}$.

3. **Mixed strategy (semi-pooling) equilibria:** Group 2 types invest and group 1 types play different mixed investment strategies. As cost decreases, the portion of biased types who invest increases compared to unbiased types.

Figure 1 shows the regions for different equilibria as a function of the cost of information acquisition, where multiple equilibria exist in some regions. When $c \geq \rho - \rho^2$, the unique equilibrium has no investment. As a result we focus on the region $c < \rho - \rho^2$ as this is where different equilibria exist. Our primary focus will be the separating equilibrium as it is the most informative one and we will identify the conditions under which this equilibrium can arise. First, we study the types of equilibria as a function of cost, and in section 6 we focus on the question of optimal expertise level that the DM would pick.

Separating Equilibrium: In a separating equilibrium, the only types who invest are those in group 2 while group 1 types send high message without investing. This equilibrium exists only when the **signaling value** is lower than the **intrinsic value** of information and the cost of information acquisition is intermediary. The **signaling value** of information is the gain in payoff for group 1 types of deviating to invest and sending message $m = 1$, while the decision maker believes this message is coming from an unbiased type. This gain is given by $-(1 - \mu(1, 1))^2 + (1 - \mu(0, 1))^2$, where $-(1 - \mu(1, 1))^2$ is the payoff upon investing and $-(1 - \mu(0, 1))^2$ is the payoff upon not investing in a separating equilibrium, and sending $m = 1$ in either case. In the separating equilibrium, $\mu(0, 1)$ is given by:

$$\hat{\rho} = \frac{\alpha(1-\beta)\rho + \beta\rho}{\alpha(1-\beta)\rho + \beta}$$

as the DM infers that the high message either comes from a biased agent (with probability $\beta\rho$) or from an unbiased informed agent (with probability $\alpha(1 - \beta)\rho$), and optimally chooses $y = \hat{\rho}$. The payoffs to both the biased and unbiased informed expert types are $-(1 - \hat{\rho})^2$. Upon deviating to invest, they can induce the DM to choose $y = \mu(1, 1) = 1$ and obtain a payoff of 0. Thus, $(1 - \hat{\rho})^2$ is the signaling value of investment. The condition for this deviation not to be profitable for group 1 types is:

$$c \geq (1-\hat{\rho})^2 = \left[\frac{\beta(1-\rho)}{\alpha(1-\beta)\rho + \beta}\right]^2 \tag{1}$$

The **intrinsic value** of information is the payoff gain for the uninformed unbiased types from getting informed and obtaining the maximum payoff of 0 after investment, rather than deviating to not invest and sending $m = \varnothing$ upon which the DM would choose $y^* = \rho$ leading to a payoff of $-(\rho - \rho^2)$[9]. Then, group 2 type invests if and only if $c \leq \rho - \rho^2$. Hence, the cost values for which a separating equilibrium exists is:

$$(1-\hat{\rho})^2 \leq c \leq \rho - \rho^2 \tag{2}$$

A separating equilibrium exists when c falls in between the signaling and intrinsic value of information, in other words it exists when cost is neither too high nor too low. The lower

[9]To see this, realise that the expert's expected payoff when $y = \rho$ is given by: $-\rho(1-\rho)^2 - (1-\rho)\rho^2$ which leads to $-(\rho - \rho^2)$

threshold decreases as ρ and α increase and increases as β increases. High α and ρ lower the biased type's incentives to invest by making communication upon no investment more credible. On the other hand, the uninformed type is willing to invest whenever $c \leq \rho - \rho^2$, which is more likely to be satisfied when ρ is closer to 0.5, in other words when uncertainty is higher hence the intrinsic value of information is higher. Now, we can explain why we make the following assumption throughout the rest of the paper:

Assumption 4.1. $\rho_0 > 0.5$

The signaling value of information can be at most $(1-\rho)^2$ which arises for $\alpha = 0$.[10] Then, for $\rho > 0.5$, $(1-\rho)^2 < (\rho - \rho^2)$ and for any α there exists a region of cost values for which the separating equilibrium arises. This assumption gives us the richest set of equilibria, while our results do not depend on it. If instead $\rho < 0.5$, a separating equilibrium region of cost exists only for α high enough and it never exists for $\alpha = 0$. If the signaling value of information is above the intrinsic value, then separation is never possible in pure strategies as a biased expert has an incentive to invest whenever the uninformed unbiased one does. We consider the case with $(1-\rho)^2 > \rho - \rho^2$ in appendix A.

Now, consider the region $c < (1-\hat{\rho})^2$. There exists no separating equilibrium as the biased type would have an incentive to deviate and invest if the DM attributes the investment to an unbiased type. However, just below this threshold it cannot be that the group 1 types invest with probability 1 either, as in equilibrium this would mean $\mu(1,1) < 1$ and the biased types informed of high state would not find it profitable to invest. Then, only mixed strategy equilibria exist in this region where group 1 types play mixed investment strategies.

Mixed strategy (semi-pooling) equilibria: In this type of equilibrium, biased types and unbiased types informed of high state (group 1 types) play different probabilistic investment strategies, determining the posterior for the decision maker and hence the incentives of these types themselves, while the uninformed type (group 2) still invests with probability 1. Specifically, in this equilibrium the strategies of group 1 players must be consistent with their indifference

[10] Without investment, the biased experts can at worse get the outcome $y = \rho$, when they are the only ones to not invest and the DM can identify them as biased.

condition. We use σ and γ to denote respectively the probability that the biased type and the unbiased informed type invest, while both send $m = 1$ in any case. When these types are indifferent, the unbiased uninformed type strictly prefers to invest. In this equilibrium, the DM's posterior upon investment is:

$$\mu(1,1) = \frac{(1-\beta)(\alpha\gamma + (1-\alpha))\rho + \beta\sigma\rho}{(1-\beta)(\alpha\gamma + (1-\alpha))\rho + \beta\sigma} > \hat{\rho}$$

while upon no investment, it is:

$$\mu(0,1) = \frac{(1-\beta)\alpha(1-\gamma)\rho + \beta(1-\sigma)\rho}{(1-\beta)\alpha(1-\gamma)\rho + \beta(1-\sigma)}$$

The indifference condition that should be satisfied for group 1 types in any mixed strategy equilibrium is:

$$c = (1-\mu(0,1))^2 - (1-\mu(1,1))^2$$

From the equation above we can verify that σ and γ are complements and there are infinitely many pairs of $\{\sigma, \gamma\}$ that form a mixed strategy equilibrium for a given c.

When $c \geq (1-\rho)^2$, there exists no mixed strategy equilibria, as even when $\mu(1,1) = 1$, group 1 types strictly prefer not to invest, and separating equilibrium is unique in this region.[11] Mixed strategy equilibria exist for $c \in ((1-\tilde{\rho})^2, (1-\rho)^2)$. In the region $c \in [(1-\hat{\rho})^2 - (1-\tilde{\rho})^2, (1-\hat{\rho})^2]$, mixed strategy equilibria are the unique equilibria while outside this region, they co-exist with pure strategy equilibria and survive the Intuitive Criterion.[12]

Focus on the region $c \in [(1-\hat{\rho})^2, (1-\rho)^2]$ in which mixed and separating equilibria coexist.

[11] $(1-\rho)^2$ is the highest cost level at which it is possible to have a mixed strategy equilibrium, as above this value it is impossible to have an equilibrium in which group 1 types invest, given that they prefer the prior which gives them $-(1-\rho)^2$ rather than incurring the cost and inducing the decision maker to choose the highest action 1. $(1-\tilde{\rho})^2$ is defined as the lowest value of cost for which there exists mixed strategy equilibria, and it is shown in the Appendix.

[12] To see why mixed strategy equilibria always survive the Intuitive Criterion, realize that in a mixed strategy equilibrium, both investment and no investment can take place, hence there is no out of equilibrium action in terms of investment choice and also in terms of messages sent. The only out of equilibrium belief can arise if a message $\varnothing$ is sent, however given that this would lead to a choice $y = \rho$, this deviation can never be optimal for any type.

Mixed strategy equilibria in this region have $\mu(0,1) < \hat{\rho}$: as group 1 types would strictly prefer not to invest if $\mu(1,1) = 1$ and $\mu(0,1) = \hat{\rho}$, and given that $\mu(1,1)$ cannot increase further, $\mu(0,1)$ must decrease to make these types indifferent. In other words, the no investment payoff must be made worse to keep the indifference condition of the biased types. Then it should be that $\gamma > \sigma$: more unbiased informed types investing is higher than biased types.

Lemma 3. *For any cost value $c \in [(1-\hat{\rho})^2, (1-\rho^2)]$ where mixed strategy and separating equilibria co-exist, there exists a unique mixed strategy equilibrium which maximizes both the DM's and the expert's welfare which has $\gamma > 0$ and $\sigma = 0$.*

Hence, there is agreement between expert types and the DM on the best mixed strategy equilibria and from now on, when we talk about mixed strategy equilibrium in this region, we will refer to this specific one. This is the one in which only some of the unbiased informed types wastefully invest, keeping $\mu(1,1) = 1$ and decreasing $\mu(0,1)$. Intuitively, given that investment is costly, it is optimal if as a result of investment there is more differentiation between the biased and unbiased types which happens when only unbiased types invest although they are already informed.

When $c < (1-\hat{\rho})^2$, mixed strategy equilibria should have $\sigma > 0$ as in any other case $\mu(1,1) = 1$ and $\mu(0,1) \leq \hat{\rho}$, hence the group 1 types will strictly prefer to invest. Hence, some biased types must be investing in this region. Given that investment is wasteful and some biased types will be investing, the best mixed strategy equilibrium in this region has $\gamma = 0$ and $\sigma > 0$. Plus, as c increases, $\frac{\mu(1,1)}{\mu(0,1)}$ should increase: communication after investment should become relatively more credible. Realize that group 2 types keep investing with probability 1 in any mixed strategy equilibria given that $c \leq (\rho - \rho^2)$.[13]

Proposition 1. *The region in which a separating equilibrium exists is $(1-\hat{\rho})^2 \leq c \leq (\rho - \rho^2)$ and:*

- *Given β, ρ, and c, the minimum α for which a separating equilibrium exists satisfies $(1-\hat{\rho})^2 = c$ where $\hat{\rho}$ is an increasing function of α.*

[13] If $\rho > 0.5$, then $(1-\rho)^2 > \rho - \rho^2$ and hence this equilibrium couldn't arise for any value of c.

- *For* $(1-\hat{\rho})^2 \leq c \leq (1-\rho)^2$*, the separating equilibrium coexists with mixed strategy equilibria. In this region, the separating equilibrium is the expert optimal and welfare maximizing equilibrium while the mixed strategy equilibrium is the DM optimal one.*
- *In the region* $(1-\rho)^2 < c < (\rho-\rho^2)$*, the separating equilibrium the unique equilibrium.*

Finally, $c = (1-\rho)^2$ is the highest cost value for which a mixed strategy equilibrium exists, and in this equilibrium the unbiased type invests for sure and biased type do not invest, hence $\sigma = 0$ and $\gamma = 1$. In this equilibrium, we have $\mu(1,1) = 1$ and $\mu(0,1) = \rho$ and a group 1 types are indifferent between investing or not as $c = (1-\rho)^2$.[14] It is easy to see that this equilibrium is the best one for the decision maker, as he can perfectly identify biased types. However, given that a very specific condition c or ρ needs to be satisfied for this type of equilibrium, it will not be our focus.

Pooling Equilibrium: When the cost of information acquisition is low enough, a pooling equilibrium exists in which all types except the unbiased type informed of low state invest in information. When cost is even lower, this is the unique equilibrium.

Lemma 4. *A pooling equilibrium exists when:*

$$c \leq (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$$

and the DM's posterior takes the values $\mu(1,1) = \tilde{\rho} = \frac{\rho}{\beta+(1-\beta)\rho}$ *and* $\mu(0,1) = \hat{\rho}$*.*

As β increases, so does the incentive of the biased type to invest making pooling more attractive. When $c \leq (1-\tilde{\rho})^2$, this is the unique equilibrium.

5 Overall Welfare

We now compare the overall welfare among equilibria. The DM's payoff depends on how indicative communication is about the state of the world. Hence, she gets the lowest surplus

[14] Where $-(1-\rho)^2$ is the payoff for the group 1 type from not investing, as $\mu(0,1) = \rho$ and payoff from investment is $-c$ as $\mu(1,1) = 1$.

in the no investment equilibrium in which no type invests plus it is impossible to distinguish amongst biased and unbiased types. Among equilibria in which there is investment, the DM gets the lowest surplus in pooling equilibrium where all expert types are informed but she cannot differentiate between a biased and unbiased expert. It is not surprising that the DM's surplus increases when cost increases to move the equilibrium from pooling into the separating one as there is then some separation between biased and unbiased types. In the separating equilibrium, all the unbiased types are informed as well plus the DM can identify the types who invest as unbiased. However, the same might not be true for the expert payoff when cost of information increases, although the equilibrium becomes more informative. Indeed, it turns out the expert welfare also increases as the next proposition shows..

Proposition 2. *Total welfare increases when cost of information increases from* $c = (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$ *to the separating region* $c = (1-\hat{\rho})^2$*. Hence, welfare is non-monotone in cost: in a given type of equilibrium it decreases as cost of information acquisition increases while it increases if the increase in cost moves the equilibrium from pooling to separating.*

To understand this result, it's useful to separate the overall welfare into two parts: the first part is related to decision making and second one related to investment cost. When cost increases from the pooling region to the separating one, the welfare related to decision making does not change while the total cost of information acquisition does. The group 1 types lose while the uninformed unbiased type and the DM gain in terms of value from the decision making, which as it turns out exactly cancel out. There is more investment in the pooling equilibrium at a lower cost, while there is less investment in the separating equilibrium at a higher cost which in overall turns out more than compensates and the cost of investment is lower in the separating equilibrium. This is an intuitive but non-trivial result. In both pooling and separating equilibria, the uninformed unbiased expert is the only one to invest efficiently, so that investment strategies of group 1 types do not affect the total amount of information present: the biased type doesn't use the information acquired while the unbiased type doesn't learn more than what he already knew before investment. However, when moving to the separating equilibrium, some decision related value is transferred from the group 1 types to the group 2 type and the DM. Hence, the only difference is indeed due to the total cost incurred in information acquisition, which turns out to be lower in the separating equilibrium.

6 DM's Optimal Expertise Level

We now study the relation between the expert's type and the DM's welfare, and specifically the question of what type of expert the DM should choose, given the option, as a function of the cost of information acquisition. The parameter we focus on is α, which we called the **expertise** parameter, denoting the probability that an expert is already informed on this specific issue from his prior experience.

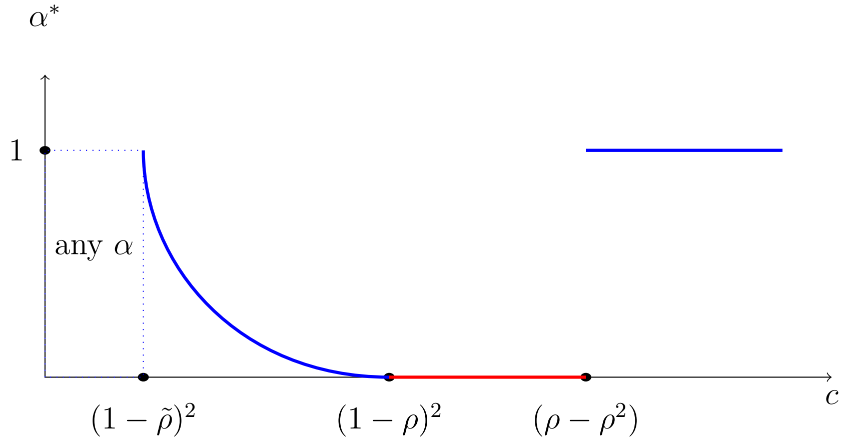
Proposition 3. *For any $c \in [(1-\tilde{\rho})^2, (1-\rho)^2]$[15], there is a unique expertise level α^* which satisfies $c = (1-\hat{\rho})^2$ and provides the DM her highest possible payoff among any other equilibrium at any α. Moreover, at this optimal level, the separating equilibrium and the best mixed strategy equilibrium coincide*[16]*. This α^* is a decreasing function of cost hence as information acquisition cost rises it is optimal to have a less informed expert.*

The intuition for this result is as follows: if α is such that $c < (1-\hat{\rho})^2$, the DM's welfare will increase in α until the point is reached where a separating equilibrium is feasible, given that $(1-\hat{\rho})^2$ is decreasing in α. However, once a separating equilibrium is achieved, starting at the point when $c = (1-\hat{\rho})^2$, the DM's welfare decreases if α is increased further. This implies that it is optimal to have the expert's probability of being informed just high enough to achieve separation but not higher. As the proposition says, at this unique α^*, the DM ensures the highest possible payoff regardless of any equilibrium selection issue, as the best mixed strategy equilibrium (DM optimal) coincides with the separating equilibrium (expert optimal) at this value. We see the region where a separating equilibrium can be achieved by varying α is $(1-\tilde{\rho})^2 < c \leq \rho - \rho^2$. When $c < (1-\tilde{p})^2$, even for α arbitrarily close to 1, a separating equilibrium cannot be achieved hence pooling is the unique equilibrium when the cost of investment is so low. On the other hand, when $c > \rho - \rho^2$, no investment takes place in

[15]Where $(1-\tilde{\rho})^2$ is the cutoff below which pooling is the unique equilibrium for any α

[16]This means we avoid the questions of multiplicity of equilibria; as both the principal's (mixed) and expert types' best equilibrium (separating) coincide.

Figure 2: α^* as a function of cost



equilibrium.

We now summarize in the next proposition the optimal expert type for all possible cost levels.

Proposition 4. *The optimal expertise parameter α^* for the DM is non monotone in c:*

- *When $c > \rho - \rho^2$, $\alpha^* = 1$: as no investment takes place, a perfectly informed expert is optimal.*
- *When $(1-\rho)^2 < c \leq \rho - \rho^2$, $\alpha^* = 0$: as separating equilibrium is the unique equilibrium for any α, an uninformed expert is optimal.*
- *When $(1-\tilde{\rho})^2 < c \leq \rho - \rho^2$, unique α^* solves $(1-\hat{\rho})^2 = c$, hence α^* decreases in c.*
- *When $c < (1-\tilde{\rho})^2$, expertise doesn't matter as the unique equilibrium for any α is the pooling equilibrium.*

As the proposition says, the DM's optimal expert type is non-monotone in the cost of information acquisition. When $c > \rho - \rho^2$, in the unique equilibrium expert doesn't invest. As it is impossible to separate between a biased and an unbiased expert through investment behavior, the DM is better off having a perfectly informed expert. Whenever $(1-\rho)^2 \leq c \leq \rho - \rho^2$, an unbiased uninformed expert strictly prefers to invest while informed or biased types still strictly prefer not to invest even when $\mu(1,1) = 1$ and for any value of α. Then, it is optimal to have

$\alpha = 0$ in order to avoid the loss in communication due to the pooling of informed types with biased types in the high state. Intuitively, when the investment behavior is able to perfectly separate an unbiased type from a biased type, it is optimal not to have any ex-ante informed types. Indeed, this is the best scenario for the DM, if it arises.

However, once the cost of information acquisition is lower, $c < (1-\rho)^2$, there exists no separating equilibrium for $\alpha = 0$, as whenever $\mu(1,1) = 1$ in equilibrium, the biased type strictly prefers to invest. Now, to induce separation, there should be a sufficient portion of unbiased types who are informed, and this α^* is given by the condition $c = (1-\hat{\rho})^2$. A lower α leads to a worse payoff for the DM as some biased types may then invest in any equilibrium, making $\mu(1,1) < 1$. For any α higher than this critical level, the separating equilibrium payoff is lower for the DM while the mixed strategy equilibrium at most leads to the same payoff as the strategy separating equilibrium with α^*. This is because, given the same cost of investment and higher α, in the DM optimal mixed strategy equilibrium, some informed unbiased types reinvest to decrease $\mu(0,1)$ to the same level. At α^*, the best mixed strategy equilibrium coincides with the separating equilibrium and there is no issue of multiplicity of equilibrium. It follows that there is no benefit of having a higher α than this critical level, and it may even hurt the principal depending on the equilibrium played. This means, if it were the case that experts get paid a fee as a function of their expertise parameter, the DM shouldn't hire an expert more informed than this level.

Finally, $c = (1-\tilde{\rho})^2$ is defined as the threshold such that, when cost is lower, a pooling equilibrium is inevitable, even for α arbitrarily close to 1. Then, expertise level has no importance for the DM's welfare as all unbiased experts will end up being informed and also pool with biased in any equilibrium.

Discussion of the results: Our results show that when the bias of an expert is not observable and endogenous information acquisition is possible, a more informed expert often doesn't lead to better decision making. Indeed, the only way the DM can distinguish an unbiased expert from a biased one in both states of the world in equilibrium is via separation in information acquisition behavior. In that case, the DM is lucky if initially matched with an uninformed expert who invests to get informed, of course conditional on the expert being unbiased. However, having some probability that the expert is already informed is also necessary to discourage wasteful investment by biased types, by ensuring that communicating high state without investing is sufficiently credible. This is why, for a region of cost, the optimal expertise parameter decreases in the cost of information acquisition. Hence, **sunk** expertise in this setup is valuable only insofar as it discourages pooling by the biased types by providing them **credibility**, while **acquired** expertise is more valuable as it can be more easily transmitted. When the cost is high enough that separation arises even with a perfectly uninformed expert, then a perfectly uninformed expert is optimal. If the cost is so high that no expert type would invest, then given that investment behavior cannot separate a biased and unbiased expert, a perfectly informed expert is optimal. In overall, our paper highlights the value of an uninformed expert who is motivated to become informed once the DM hires them, as opposed to an expert who is already informed and cannot demonstrate that his preferences are aligned with the DM. Hence, contrary to the common intuition, we study a situation in which hiring a more informed expert may lead to a worse outcome.

7 Extension: Covert information acquisition

Now we consider covert information acquisition, in other words when the decision maker does not observe the investment made by the expert. In this setup, as investment has no signaling value, group 1 types never invest as they can pool with the unbiased uninformed expert without incurring any cost and the only type that may invest is the unbiased uninformed type. There are 2 types of pure strategy equilibria as a function of the cost which is summarized below:

1. Investment takes place by the uninformed unbiased type (group 2). Upon $m = 0$, the DM chooses $y = 0$ and upon $m = 1$, the DM takes action $y = \tilde{\rho}$. This equilibrium is similar to the pooling equilibrium except that the biased and informed (1) types do not actually invest. The payoffs of the biased and informed (1) types are higher in this equilibrium compared to the pooling equilibrium in our main model, as the same action is induced

without having to incur an investment cost.

The condition for the unbiased uninformed type to invest is:

$$c \leq (\rho - \rho^2) - \rho(1 - \tilde{\rho})^2$$ [17]

This means the unbiased type acquires information for a smaller range of cost values in the covert information acquisition than in the overt case, as this cutoff is above the pooling cutoff cost but below the separating equilibrium cutoff cost of the overt case. [18]

2. No investment takes place. Upon $m = 1$, the DM chooses $\hat{\rho}$ inferring that this message is sent by a group (1) type of expert. The uninformed expert sends $m = \emptyset$ and the DM chooses $y = \rho$. This equilibrium is equivalent to the no investment equilibrium in the overt information acquisition case. This equilibrium arises for the following cost values:

$$c > (\rho - \rho^2) - \rho(1 - \tilde{\rho})^2$$

3. In the region $(\rho - \rho^2) - \rho(1 - \tilde{\rho})^2 < c < (\rho - \rho^2) - \rho(1 - \tilde{\rho})^2$, mixed strategy equilibria exist in which the unbiased uninformed type invests with some probability.

The types that benefit from information acquisition being covert as opposed to overt are the group 1 types and only in case the cost is low enough that the uninformed unbiased type acquires information. Even though the cost does not affect these types directly as they don't invest in information, the investment of the unbiased uninformed type makes their message more credible. In this equilibrium, the payoffs of the uninformed unbiased expert and the decision maker are the same as in the pooling equilibrium in the overt case. When cost increases, we move to the no investment equilibrium in which payoffs are identical to the overt case without investment.

[18] Realize that in this region, there are as well mixed strategy equilibria in which the unbiased uninformed expert invests with a probability. This probability should increase as c decreases to keep the indifference condition satisfied.

The unbiased type invests in information less often and is worse off in covert case, as he can never separate himself from the biased type. This result shows that, even though the signaling value of information undermines its intrinsic value, in case it leads to separating equilibrium, overt information acquisition does strictly better than the covert one due to more precise communication of information.

However, when the cost of information acquisition is low enough that in the overt case pooling in investment arises, covert information acquisition does better in terms of the overall welfare as there is no wasteful investment, although the precision of communication is identical. The next proposition summarizes these results.

Proposition 5. *Whenever $c \geq (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$, overt information acquisition leads to higher overall welfare while whenever $c \leq (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$, covert information acquisition leads to higher welfare.*

The tradeoff is between more informative communication versus wasteful investment in information. Whenever no investment arises in the covert case, overt information acquisition does better as welfare is always higher when some types invest than when there is no investment. However, when cost is low enough that there is pooling in investment in the overt case, total welfare is higher in the covert case as although the communication precision is equivalent less cost is incurred. This is because in both cases, all the unbiased types are informed but cannot be separated from biased types, plus in the overt case there is more wasteful investment.

8 Conclusion

This paper considers situations when costly information acquisition may serve as a signalling device in addition to getting informed before communicating to a decision maker, in a simple model building on the communication and costly signaling literatures. We find that an unbiased expert, as well as a biased one, may wastefully invest in information, which leads to inefficiencies in decision making and lower overall welfare. Indeed, an uninformed expert who becomes informed later always communicates more precisely than an ex-ante informed expert. This is due the insufficient incentives of an unbiased and ex-ante informed expert to separate himself from a biased one. As such, our results highlight the value of an uninformed expert who faces sufficient uncertainty that he is willing to incur cost in order to become informed, and while doing so can also identify himself as unbiased. Acquired information turns out easier to transmit than ex-ante information.

In addition, we find that the optimal level of expertise which maximizes the decision maker'a welfare is non monotone in the cost of information acquisition, and specifically for higher information acquisition cost a less informed expert is optimal. This optimal expertise level trades off giving incentives for unbiased types to invest versus preventing biased types from costly signalling via investing. We also find that higher information acquisition cost increases overall welfare as the equilibrium moves from a pooling to a separating one.

The simplicity of the model allows for numerous extensions remaining for future research. An interesting feature of the model is that even though the ability to acquire information is uncorrelated with the type of the expert, the value of getting informed depends on the expert's type. Hence, it is possible to use information acquisition as a screening device by taxing experts for getting informed. Now, assume that the cost of information acquisition falls in the pooling region. We know that an expert who is uninformed and unbiased has the highest incentives to acquire information. Hence, this type will be willing to pay more than biased and informed experts so that wasteful investment by these types can be prevented.

9 Appendix A

The No Investment Equilibrium Cutoff:

When $c \geq \rho - \rho^2$ even the unbiased uninformed type doesn't want to invest. The right hand side is the cutoff below which the separating equilibrium exists, in which the only type that invests is the unbiased uninformed type. Even if the out of equilibrium belief upon investment places probability 0 to the expert being a biased type, the gain in utility from doing so for the uninformed unbiased type is $\rho - \rho^2$ and defines the cost value above which the unique equilibrium has no investment. Then, in the region $c \geq \rho - \rho^2$, the unique equilibrium has no investment by any type.

Welfare Calculations in Different Equilibria

It is easy to see that the DM's payoff is minimized in the equilibrium in which no type invests, which is the least informative equilibrium. Hence, presence of investment can only make communication more informative and increase the payoff of the DM. Furthermore, we will show that separating equilibrium is better than the pooling one for the DM.

Separating equilibrium:

The DM's payoff

$$[(1-\beta)\alpha\rho + \beta][-(\hat{\rho} - \hat{\rho}^2)] = -\hat{\rho}\beta(1-\rho) \tag{3}$$

The only time when the DM gets a payoff less than 0 is if high message is communicated without investment, in which case his posterior is $\hat{\rho}$ and happens either because the expert is unbiased but informed with high signal ($(1-\beta)\alpha\rho$) or he is biased (β). This payoff is decreasing in α and β which both lead to higher $\hat{\rho}$. As α increases, more of the unbiased types will be pooled with biased types which leads to lower payoff. Also, the DM payoff also decreases in β, which makes a biased type more likely. The payoff is increasing in ρ whenever $\rho > 0.5$. This is intuitive: the DM's loss from biased communication is lower when the prior is already in favor of the action that the biased type of expert wants.

The expert's payoff

The payoff of the biased expert is $-(1-\hat{\rho})^2$, which is increasing in α and ρ and decreasing in β: the more unbiased types there are that are informed, the higher the DM's posterior upon no investment and high message. The ex-ante payoff of the unbiased expert is $-\alpha\rho(1-\hat{\rho})^2 - \alpha(1-\rho)0 - (1-\alpha)c = -\alpha\rho(1-\hat{\rho})^2 - (1-\alpha)c$, which is increasing in α, decreasing in ρ and decreasing in β.

Then, the expected payoff over all expert types in the separating equilibrium is:

$$-(1-\hat{\rho})^2[\beta + (1-\beta)\alpha\rho] - (1-\beta)(1-\alpha)c$$

Pooling equilibrium:

The DM's payoff

$$[(1-\beta)\rho + \beta][-(\tilde{\rho} - \tilde{\rho}^2)] = -\rho(1-\tilde{\rho}) \tag{4}$$

In the pooling equilibrium, the DM gets a payoff less than 0 if an investment happens and a high message is sent, which comes from either a biased type (β) or an unbiased type who has high signal ($(1-\beta)\rho$), upon which her posterior is $\tilde{\rho}$. This payoff is decreasing in β and when $\rho > 0.5$, it is increasing in ρ. When $\rho < 0.5$ and β is high enough it may be decreasing in ρ. The pooling payoff is independent of α, as all unbiased types invest in this case and the DM doesn't internalize the cost of investment. When we replace the values of $\hat{\rho}$ and $\tilde{\rho}$, we find that he separating equilibrium payoff dominates the pooling equilibrium payoff for the DM if:

$$\alpha(1-\rho)+\rho \leq 1$$

which is always satisfied. Hence, the DM's payoff is unambiguously higher in the separating equilibrium in which information is more precise. The DM benefits from having some probability of expert being initially informed and pool with the biased type, as this makes pooling less attractive for the biased type. If α is very low, there are more incentives to "pool" and the separating region shrinks. However, once in the separating region, the DM's payoff is decreasing in α. Hence, the DM's payoff is non-monotone in α: to maximize the DM's payoff it has to be just high enough for biased types not to wastefully invest.

The expert's payoff

The payoff of the biased expert in pooling equilibrium $-(1-\tilde{\rho})^2 - c$. This is increasing in ρ and decreasing in β. The ex-ante payoff of the unbiased expert is $-\rho(1-\tilde{\rho})^2 - (1-\alpha(1-\rho))c$, which is decreasing in ρ and in β. It is increasing in α as less cost investment will be incurred given the unbiased expert with a low signal doesn't invest.

It is trivial that the biased type of expert is unambiguously better off in the pooling equilibrium than in the separating one, as otherwise he would not invest and still get the same payoff as in the separating equilibrium. On the other hand, the unbiased expert's expected payoff increases when moving from the separating to the pooling equilibrium, when we compare the payoff at the minimum cost at which there is separation and maximum cost at which there is pooling.

The expected payoff over all expert types in the pooling equilibrium is:

$$-[(1-\beta)\rho+\beta](1-\tilde{\rho})^2 - [1-(1-\beta)(1-\rho)\alpha]c$$

Mixed strategy equilibria

The DM's payoff The DM's payoff given σ and γ is:

$$-(\beta(1-\sigma)+(1-\beta)\alpha\rho(1-\gamma))(\mu(0,1)-\mu(0,1)^2)-((1-\beta)\rho(\alpha\gamma+1-\alpha)+\beta\sigma)(\mu(1,1)-\mu(1,1)^2)$$

which simplifies to:

$$-\beta(1-\rho)[(1-\sigma)\mu(0,1)+\sigma\mu(1,1)] \tag{5}$$

Mixed strategy equilibria exist for $c < (1-\rho)^2$. In the region $(1-\hat{\rho})^2 < c < (1-\rho)^2$, mixed strategy equilibria are such that some of the unbiased experts informed of high state invest and none of the biased experts invest. Hence, $\mu(0,1) > \hat{\rho}$ and $\mu(1,1) = 1$.

In the region where $c < (1-\hat{\rho})^2$, mixed strategy equilibria have $\mu(0,1) > \hat{\rho}$, as it cannot be the case that $\mu(1,1) > 1$. Then, it must be that $\sigma > 0$ hence $\mu(1,1) < 1$. We can see that the DM's payoff is highest as ρ decreases, specifically when $\rho = 0$ it is the highest, as $\mu(0,1) = \hat{\rho}$ also takes the minimum value.

At a given cost c, the DM optimal equilibrium, which also coincides with expert optimal mixed strategy equilibrium is the one in which $\sigma = 0$ and γ takes the value which satisfies:

$$c = (1-\mu(0,1))^2$$

as $\mu(1,1) = 1$.

This means, as α increases until the separating equilibrium is reached, the payoff of DM increases. Then, once the separating equilibrium is attained, the payoff of DM decreases in α.

The expert's payoff The expected payoff over expert types in a mixed strategy equilibrium is:

$$-(\beta+(1-\beta)\alpha\rho)(1-\mu(0,1))^2-(1-\beta)(1-\alpha)\rho(1-\mu(1,1))^2-(1-\beta)(1-\alpha)c$$

The above is found by using the fact that the biased and informed (1) types of experts are indifferent between investing or not, and taking their payoffs as the payoff when they don't invest. This simplifies, if we replace $c=(1-\mu(0,1))^2-(1-\mu(1,1))^2$, to:

$$-[(1-\beta)(\alpha\rho+1-\alpha)+\beta](1-\mu(0,1))^2+(1-\beta)(1-\alpha)(1-\mu(1,1))^2$$

In the region where mixed strategy and separating equilibrium coexist, $(1-\hat{\rho})^2<c<(1-\rho)^2$, for the group 1 types to be indifferent between investing or not, in any mixed strategy equilibrium we should have $\mu(0,1)<\hat{\rho}$ as it cannot be the case that $\mu(1,1)>1$. We find in that case that the expert preferred mixed strategy equilibrium has $\sigma=0$, and $\gamma>0$. Hence, in the region where mixed equilibria coexist with pure strategy separating equilibrium, there exists a unique mixed strategy equilibrium preferred by both the DM and expert. However, the mixed strategy equilibrium is swill worse for the experts than the separating equilibrium.

No investment equilibrium:

The DM's utility in the equilibrium in which no one invests is:

$$\frac{-(1-\rho)\rho(\beta+(1-\alpha)\alpha(1-\beta)^2\rho)}{\beta+\alpha(1-\beta)\rho} \tag{6}$$

which is found by simplifying $-[\beta+(1-\beta)\alpha\rho](\hat{p}-\hat{p}^2)-(1-\beta)(1-\alpha)(\rho-\rho^2)$. We see that pooling in investment always leads to higher payoff for the DM than no investment. Although she cannot differentiate between biased and unbiased types in either case, in the pooling one at least all experts are informed. Hence, the no investment equilibrium provides the minimum possible payoff to the DM.

The overall expert utility:

$$(\beta + (1-\beta)\rho\alpha)(1-\hat{\rho})^2 - (1-\beta)(1-\alpha)(\rho-\rho^2)$$

Extension: the case with $\rho < 0.5$, hence $(1-\rho)^2 > \rho - \rho^2$

In this case, there is no region of cost for which separating equilibrium arises regardless of α, as was the case under the assumption made in the paper. In this case, separating equilibrium exists if α is high enough such that $(1-\hat{\rho})^2 < \rho - \rho^2$, and for the region of cost $(1-\hat{\rho})^2 < c < \rho - \rho^2$. Plus, whenever $c > (1-\rho)^2$, no type is going to invest. Indeed, even though the cost is below $(1-\rho)^2$, as long as the uninformed unbiased types are not willing to, the group 1 types will not invest either.

Under this assumption, the main difference to the case considered in the paper is that there is not a region in which $\alpha^* = 0$, as whenever the uninformed type is willing to invest, the group 1 types are strictly willing to invest.

Whenever $c > \rho - \rho^2$, $\alpha^* = 1$, as in this region the uninformed types will never acquire information.

When $k \leq c \leq \rho - \rho^2$, it is optimal to set $c = (1-\hat{\rho})^2$. In this region, α is decreasing in cost.

As in the case with $\rho > 0.5$, for $c < k$, only pooling equilibrium can arise for any value of α as the cost of investment is low. Hence, the main results do not change if $\rho < 0.5$ however there is less variety of equilibria that exist.

10 Appendix B

Proof of lemma 1:

Proof. In the communication stage following investment, it is certain that the expert is informed. We can then focus on equilibria in which after investment there are at most two messages, $m^i \in \{0, 1\}$. To see this, realize that the unbiased expert either knows $\omega = 0$ and wants to induce the lowest possible action or knows $\omega = 1$ and wants to induce the highest possible action. The biased expert wants to induce highest possible action regardless of the state. Hence, in equilibrium, after investment there need to be two types of messages only.

In the communication stage following no investment, there are four possible types of experts. First, the informed expert with signal 0 who strictly prefers the message which induces the lowest action hence will send $m = 0$. Second, the biased and unbiased informed (1) expert both want to send the message that induces the highest action hence send $m = 1$. Finally, the unbiased uninformed expert who shares the DM's preferences. It is then without loss of generality to restrict attention to equilibria in which an empty message is available, $m = \varnothing$, which means "I am not informed" and will induce $y = \rho$.[19]. Then, following $x = 0$, the set of messages is $m^n \in \{0, 1, \varnothing\}$.

□

Proof of lemma 2

Proof. The unbiased informed (0) type's payoff is maximized when $y = 0$ is chosen by the DM and there is no other type that could benefit from sending this message, given the set of other possible messages. For the biased types informed of high state, this message leads to the lowest possible payoff of -1 and they are better off sending any other message. For the unbiased uninformed type who has not invested, sending $m = \varnothing$ upon which the DM chooses $y = \rho$ can only lead to a better payoff than sending this message. Given that, it is consistent that the DM, upon hearing $m = 0$, interprets that this message is coming from an unbiased informed type. Then, $\mu(0, 0) = 0$ and the DM will indeed choose $y = 0$. The unbiased expert informed of signal 0 achieves maximal payoff under this strategy. By contradiction, assume there were any other equilibrium in which this type sent another message, or in which this type invested.

[19]As it becomes clear later, the biased expert can always induce a higher posterior by sending $m = 1$ as opposed to any other message

Then, it must be that his equilibrium payoff is lower than 0. However, in any such equilibrium, it would have an incentive to deviate to not invest and send $m = 0$. The DM should attribute this message to the informed 0 type: as any other type has a better message to send, which is either $m = 1$ or $m = \varnothing$. Hence, this type would indeed have an incentive to deviate and no other type has an incentive to do so. □

Proof of lemma 3

Proof. The expected payoff over all expert types in a mixed strategy equilibrium is:

$$-(\beta + (1-\beta)\alpha\rho)(1-\mu(0,1))^2 - (1-\beta)(1-\alpha)\rho(1-\mu(1,1))^2 - (1-\beta)(1-\alpha)c$$

We can see that the expert preferred mixed strategy equilibrium has $\sigma = 0$, and $\gamma > 0$ and indeed this is the best for all expert types. This is because for the uninformed expert who is getting informed, his payoff is maximized when $mu(1,1) = 1$. For the group 1 types, their payoff is also maximized when $\mu(1,1)$ takes the maximum value as they are indifferent between investing or not.

The DM's payoff given σ and γ is:

$$-(\beta(1-\sigma)+(1-\beta)\alpha\rho(1-\gamma))(\mu(0,1)-\mu(0,1)^2)-((1-\beta)(\alpha\rho\gamma+1-\alpha)+\beta\sigma)(\mu(1,1)-\mu(1,1)^2)$$

which simplifies to:

$$-\beta(1-\rho)[(1-\sigma)\mu(0,1) + \sigma\mu(1,1)]$$

Which is also maximized when $\sigma = 0$. Finally, for a given $(1-\hat{\rho})^2 < c < (1-\rho)^2$, the DM as well as expert optimal mixed strategy equilibrium is the one in which $\sigma = 0$ and $\gamma > 0$ such that:

$$c = (1 - \mu(0,1))^2$$

Then, at any c, there is a unique welfare maximizing mixed strategy equilibrium with $\sigma = 0$ and $\gamma > 0$. □

Proof of proposition 1

Proof. We show the separating equilibrium is the unique pure strategy equilibrium in $(1-\rho)^2 < c < (\rho - \rho^2)$. First, there cannot be any equilibrium in this region in which unbiased types informed of high state and biased types invest with positive probability, as even if $\mu(1,1) = 1$ these types do not find it profitable to invest given the cost is too high. Then, the only pure strategy equilibrium that could arise is the no investment equilibrium in which even the uninformed unbiased expert doesn't invest. For some out of equilibrium beliefs, this equilibrium can arise, as discussed below.

Assume that the DM believes any type except the informed (0) unbiased one is equally likely to invest, then her belief and optimal choice will be $\tilde{\rho}$ which is:

$$\tilde{\rho} = \frac{\rho(1-\beta) + \beta\rho}{\rho(1-\beta) + \beta} = \frac{\rho}{\rho(1-\beta) + \beta}$$

Let us show the biased type doesn't have the incentive to incur the cost c. Now, upon the message $m = 1$, the DM chooses $y^* = \hat{\rho}$ as there is only biased and unbiased informed (1) types who choose $m = 1$. For the biased and unbiased informed (1), the payoff from investing should be less than that from not investing:

$$(1 - \tilde{\rho})^2 + c \geq (1 - \hat{\rho})^2$$

As without investment and $m = 1$, the DM's belief is $\hat{\rho}$ as in case 1. This is equivalent to:

$$\left[\frac{\beta(1-\rho)}{\rho(1-\beta)+\beta}\right]^2 + c \geq \left[\frac{\beta(1-\rho)}{\alpha(1-\beta)\rho+\beta}\right]^2$$

For the uninformed agent, the payoff from not investing is $-(\rho-\rho^2)$ and from investing it will be:

$$-(1-\rho)0 - \rho(1-\tilde{\rho})^2 - c$$

Then the condition that should be satisfied is:

$$c \geq \rho - \rho^2 - \rho(1-\tilde{\rho})^2$$

Finally, the equilibrium in which no type wants to invest, for the specified out of equilibrium beliefs, exists for the following cost values:

$$c \geq \max\{\rho - \rho^2 - \rho(1-\tilde{\rho})^2, (1-\hat{\rho})^2 - (1-\tilde{\rho})^2\}$$

Now consider that when there is a separating equilibrium, the condition $\rho - \rho^2 > (1-\hat{\rho})^2$ is satisfied. Then, it is the case that $\rho - \rho^2 - \rho(1-\tilde{\rho})^2 > (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$. This means, the condition above becomes $c \geq \rho - \rho^2 - \rho(1-\tilde{\rho})^2$, which is less than $\rho - \rho^2$. Then, there is also a no investment equilibrium in this region. However, we are able to rule out this type of equilibrium. This is because, the uninformed type, when his message is taken at face value, is willing to deviate to invest while the group 1 types do not find it profitable to invest even if $\mu(1,1) = 1$. Hence, as there is a best response from the DM to this deviation that makes unbiased uninformed types better off and the group 1 types worse off and is consistent given her beliefs about the deviator, this type of equilibrium can be ruled out by the Intuitive Criterion.

Finally, we can also rule out an equilibrium in which only group 1 types invest, as there is no belief of the DM for which these types find it profitable to invest in this region.

In the region $c \in [\rho - \rho^2 - \rho(1-\tilde{p})^2, \rho - \rho^2]$, the separating equilibrium is the unique equilibrium that survives the Intuitive Criterion.

Whenever $c \geq (1-\hat{\rho})^2$, even for the highest belief $\mu(1,1) = 1$, the biased and uninformed (1) agent do not benefit from deviating to invest. Hence, in this region, the out of equilibrium belief should assign probability 1 to the expert being unbiased and uninformed. Then, whenever $c \geq (\rho - \rho^2)$, even the unbiased uninformed type doesn't want to invest, which provides the upper boundary for the no investment equilibrium.

Next, we make the welfare comparisons. First, consider the region $(1-\hat{\rho})^2 \leq c \leq (1-\rho)^2$ where mixed strategy and separating equilibria coexist. We can see that the separating equilibrium provides higher payoff for any expert type for any c. To see this: we know that in the separating equilibrium, the uninformed unbiased type's communication is taken at face value and the group 1 types' payoff is $-(1-\hat{\rho})^2$, independent of cost. In the mixed strategy equilibria in this region, for any $c > (1-\hat{\rho})^2$, we need to have $\mu(0,1) < \hat{\rho}$ hence the payoff of group 1 types will be $-(1-\mu(0,1))^2 < -(1-\hat{\rho})^2$ given their indifference between investment and not, and the uninformed type's payoff cannot be higher as $\mu(1,1) = 1$ in the separating equilibrium. Then, all expert types are weakly (and some strictly) worse off in mixed strategy equilibria compared to pure strategy in this region.

For the DM, when we compare the pure strategy equilibrium payoff in equation 3 to the mixed strategy payoff in 5, we see that the mixed strategy equilibrium payoff is higher than the separating equilibrium. To see this, consider the mixed strategy equilibria of the type $\sigma = 0$ and $\gamma > 0$ which is the optimal one among the mixed strategy equilibria. Then, realizing that $\mu(0,1) < \hat{\rho}$ provides the result. The equilibrium with $\sigma = 0$ is also the one which maximizes the DM's payoff. In this type of equilibrium, as c increases, γ increases while keeping $\sigma = 0$. Hence, we can conclude that the DM's payoff is highest in mixed strategy equilibria in this region and especially it reaches the highest level in the equilibrium in which $\gamma = 1$ and $\sigma = 0$ which can arise at $c = (1-\rho)^2$.

The total payoff in separating equilibrium is $-\beta(1-\rho) - (1-\beta)(1-\alpha)c$ while in mixed strategy equilibrium it is $-\beta(1-\rho)\mu(0,1) - (\beta + (1-\beta)\alpha\rho)(1-\mu(0,1))^2 - (1-\beta)(1-\alpha)c$. Now, consider $-(\beta + (1-\beta)\alpha\rho)(1-\mu(0,1))^2 = X$, then we have $X < -(1-\rho)\beta(1-\mu(0,1))$. As the payoff in mixed strategy would be equal to separating equilibrium if and only if $X \geq -(1-\rho)\beta(1-\mu(0,1))$, we can conclude that the separating equilibrium welfare is higher than mixed strategy. □

Proof of lemma 4:

Proof. In this equilibrium, the DM's updated belief $\mu(1,1)$, given that both group 1 and 2 types are investing, is:

$$\tilde{\rho} = \frac{\rho(1-\beta) + \beta\rho}{\rho(1-\beta) + \beta} = \frac{\rho}{\rho(1-\beta) + \beta}$$

First, consider the investment choice of the group 1 types. If the biased or unbiased agent were to deviate to not invest and send $m = 1$, then the DM would choose $y = \hat{\rho}$ as the DM infers this out of equilibrium action can only come from a biased or unbiased expert informed with signal 1. Making use of the intuitive criterion (Cho and Kreps 1987) we define the out of equilibrium belief upon no investment and $m = 1$ to assign probability 0 to the unbiased uninformed expert as this type always prefer to send $m = \varnothing$ if he were to deviate to no investment while the group 1 types always prefer sending $m = 1$ to any other message, which leads to $\mu(0,1) = \hat{\rho}$.

Then, the following is the condition for a pooling equilibrium to arise:

$$-(1-\tilde{\rho})^2 - c \leq -(1-\hat{\rho})^2$$

Second, for the uninformed unbiased type, the payoff from not investing is $-(\rho - \rho^2)$, as in that case they would send $m = \varnothing$ which would be attributed to this type by the DM2. Then, consider the payoff of this type from investing. If the signal turns out to be 0, the DM takes the message at face value and chooses $y = 0$ whereas if the signal is 1, the decision maker will choose $\hat{\rho} < \tilde{\rho} < 1$ as the DM infers it can come from a group 1 or 2 type. Then, this type prefers investing to not if and only if:

$$-\rho(1-\tilde{\rho})^2 - c \geq -(\rho - \rho^2)$$

These conditions together lead to:

$$c \leq min\{(1-\hat{\rho})^2 - (1-\tilde{p})^2, \rho - \rho^2 - \rho(1-\tilde{\rho})^2\} = (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$$

When $(1-\hat{\rho})^2 < (\rho - \rho^2)$, which is the case we consider, we have $min\{(1-\hat{\rho})^2 - (1-\tilde{\rho})^2, \rho - \rho^2 - \rho(1-\tilde{\rho})^2\} = (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$. To see this, realize that the value of getting informed is higher for unbiased uninformed types than for the group 1 types: $(1-\hat{\rho})^2 - (1-\tilde{\rho})^2 < \rho - \rho^2 - \rho(1-\tilde{\rho})^2$. This is because unbiased uninformed types get maximum payoff of 0 when the state of the world is 0, while their communication is distorted when the message is 1. However, from the point of view of the group 1 types, communication is always distorted as their bliss point is 1 and $\mu(1,1) = \tilde{\rho} < 1$. Then, the condition for the pooling equilibrium is given by the condition for the biased and informed (1) types to be willing to invest which is $c \leq (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$.

Now consider the expression $c \leq \rho - \rho^2 - \rho(1-\tilde{\rho})^2$, the condition for the unbiased uninformed types to actually invest given a pooling equilibrium. As $\rho - \rho^2 > \rho - \rho^2 - \rho(1-\tilde{\rho})^2$, where the left hand side is the cutoff for investment in the separating equilibrium, the condition for unbiased uninformed types to invest is easier to satisfy in the separating region. The difference between these two is due to the biased types' "crowding out" the unbiased uninformed types: investment of the biased types makes information acquisition by the unbiased types less profitable, hence cost has to be lower in order to satisfy their participation.

□

Proof of proposition 2:

Proof. We compare the total welfare in the pooling equilibrium to that in the separating equilibrium, finally also consider the equilibrium with no investment. For this, we compare the payoff for the region of cost values at which there is pooling equilibrium and separating equilibrium to find how the payoff changes when c increases from the pooling region to separating.

First, in the **pooling equilibrium**, the DM's welfare (all as found in appendix A):

$$[(1-\beta)\rho+\beta][-(\tilde{\rho}-\tilde{\rho}^2)] = -\rho(1-\tilde{\rho}) \tag{7}$$

The payoff of the biased expert is $-(1-\tilde{\rho})^2 - c$. The ex-ante payoff of the unbiased expert is $-\rho(1-\tilde{\rho})^2 - (1-\alpha(1-\rho))c$. This is decreasing in ρ and in β.

Then the total expert welfare found by summing these two is:

$$-[(1-\beta)\rho+\beta](1-\tilde{\rho})^2 - [1-(1-\beta)(1-\rho)\alpha]c$$

When we sum the DM's and expert's welfare we get:

$$-\rho(1-\tilde{\rho}) - \beta[(1-\tilde{\rho})^2 + c] - (1-\beta)[\rho(1-\tilde{\rho})^2 + (1-\alpha(1-\rho))c] \tag{8}$$

which simplifies to:

$$-(1-\tilde{\rho})^2[\beta+\rho(1-\beta)] - \rho(1-\tilde{\rho}) - c[1-\alpha(1-\rho)] = -\beta(1-\rho) - c[1-\alpha(1-\rho)(1-\beta)] \tag{9}$$

In the **separating** equilibrium, the DM's welfare is:

$$[(1-\beta)\alpha\rho + \beta][-(\hat{\rho} - \hat{\rho}^2)] = -\hat{\rho}\beta(1-\rho) \tag{10}$$

The payoff of the biased expert is $-(1-\hat{\rho})^2$, while the ex-ante payoff of the unbiased expert is $-\alpha\rho(1-\hat{\rho})^2 - \alpha(1-\rho)0 - (1-\alpha)c = -\alpha\rho(1-\hat{\rho})^2 - (1-\alpha)c$.

Then, the expected payoff over expert types in the separating equilibrium is:

$$-(1-\hat{\rho})^2[\beta + (1-\beta)\alpha\rho] - (1-\beta)(1-\alpha)c$$

Finally, the total welfare in the separating equilibrium is:

$$-\hat{\rho}\beta(1-\rho) - \beta(1-\hat{\rho})^2 - (1-\beta)[\alpha\rho(1-\hat{\rho})^2 + (1-\alpha)c] \tag{11}$$

which simplifies to:

$$-\beta(1-\rho) - c(1-\alpha)(1-\beta) \tag{12}$$

The difference in the total welfare in equation ($12 - 9$) is:

$$c_p[1-\alpha(1-\rho)(1-\beta)] - c_s(1-\beta)(1-\alpha) \tag{13}$$

The welfare in terms of decision values cancel out and the terms that remain are those related to the cost incurred for information acquisition, where c_p and c_s denote respectively the cost incurred in pooling and separating equilibria. In the pooling equilibrium there is more investment at a lower price where some of this investment is wasteful (signalling value), while in the separating equilibrium there is less investment at a higher price. When we replace the maximum cost at which the pooling equilibrium exists and the minimum cost at which the separating equilibrium exists, it is seen that the welfare in separating equilibrium is higher than in the pooling one although the cost of information acquisition is higher.

At $c_p = (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$ and $c_s = (1-\hat{\rho})^2$, equation (13) becomes:

$$(1-\hat{\rho})^2(\beta(1-\alpha\rho)+\alpha\rho) - (1-\tilde{\rho})^2(1-\alpha(1-\rho)(1-\beta)) > 0 \tag{14}$$

When we replace these values, finally we are left with the condition:

$$\beta(1-2\rho) + \rho(1-\beta)(\alpha-(1+\alpha)\rho) \leq 0 \tag{15}$$

which is satisfied whenever $\rho > \frac{1}{2}$ which is the initial assumption we made. The first term is negative. The second term is negative when $\rho > \frac{\alpha}{1+\alpha}$ which is always the case when $\rho > \frac{1}{2}$ and $\alpha < 1$.

Then, although the cost of information rises, welfare increases due to the lack of wasteful investment in information. In order to demonstrate this result, we considered the cost values at the boundaries. As expected, when we keep increasing the cost in the separating equilibrium region, the welfare will decrease and at some point, it will be lower than in the pooling equilibrium.

Finally, the total welfare in the **no investment** equilibrium is:

$$-\beta(1-\hat{\rho})^2 - (1-\beta)[\alpha\rho(1-\hat{\rho})^2 + 2(1-\alpha)(\rho-\rho^2) - (\hat{\rho}-\hat{\rho}^2)(\beta+(1-\beta)\alpha\rho)] \tag{16}$$

As no investment equilibrium surplus is unambiguously worse than the separating equilibrium, we compare it to the pooling in investment equilibrium and find that the payoff in the no investment equilibrium is also lower than the pooling in investment equilibrium. This is intuitive: first, the DM's payoff is unambiguously higher in the pooling in investment equilibrium compared to the no investment equilibrium, as more information is revealed. The welfare of the biased type also higher in the pooling in investment equilibrium as their outside option of not investing and getting $-(1-\hat{\rho})^2$ is still available. Hence, if this type does find it profitable to invest, then it must be getting a higher payoff. The same is true for the unbiased informed (1) type who would get $-(1-\hat{\rho})^2$ if deviating to not invest. Finally, for the unbiased uninformed type, it is true as well: if this type didn't invest they would get the payoff $-(\rho-\rho^2)$ which is still available if they deviate in the pooling equilibrium to send $m = \varnothing$. □

Proof of Proposition 3

Proof. First, we know that conditional on the separating equilibrium arising, the DM's payoff is decreasing in α. In addition, we know that the pooling equilibrium gives a strictly worse payoff for the DM. Then, the best payoff for the DM in pure strategies happens when $c = (1-\hat{\rho})^2$ which is satisfied for $\alpha = \alpha^*$: cost is at the boundary of the separating equilibrium.

Now, we will show that any other α gives a lower payoff to the DM. First, when α decreases, we will have $c < (1-\hat{\rho})^2$, given that $\hat{\rho}$ decreases. In this region, in order to keep the indifference condition of group 1 types, we need $\sigma > 0$ (some biased types investing), as otherwise we will still have $\mu(1,1) = 1$ and given that $\mu(0,1) \leq \hat{\rho}$, group 1 types would strictly want to invest. Hence, any equilibrium is mixed in between the separating and pooling regions should have $\mu(1,1) < 1$ or $\mu(0,1) > \hat{\rho}$. If we look at the DM's payoff, $-\beta(1-\rho)[(1-\sigma)\mu(0,1) + \sigma\mu(1,1)]$, we can see that whenever $\sigma > 0$ or $\mu(0,1) > \hat{\rho}$, it will be strictly lower than when $\sigma = 0$ and $\mu(0,1) = \hat{\rho}$, given that $\mu(0,1) < \mu(1,1)$. As α continues to decrease further, eventually it may lead to the pooling equilibrium. However, we know that pooling equilibrium provides lower payoff to the DM than the separating equilibrium. If the mixed strategy equilibrium is realized, we already showed that the DM's payoff increases in α until reaching the separating region.

Second, if we increase α above α^* so that $(1-\hat{\rho})^2 < c$, now c falls strictly inside the separating region. At this point, there is indeed multiplicity of equilibria: both mixed and separating equilibria exist. We will show that this multiplicity doesn't affect the optimal α^*. First for the separating equilibrium, the DM's payoff decreases in α. Second, there exists mixed strategy equilibria at this point, among which the one that maximizes the DM's payoff has $\gamma > 0$ and $\sigma = 0$. As $\mu(1,1) = 1$, γ should satisfy $c = (1-\mu(0,1))^2 = (1 - \frac{(1-\beta)\alpha(1-\gamma)\rho+\beta\rho}{(1-\beta)\alpha(1-\gamma)\rho+\beta})^2$. Now, if we modify α to $\hat{\alpha} = (1-\gamma)\alpha$, $\hat{\alpha}$ will now satisfy $c = (1-\hat{\rho})^2$ hence, the mixed and pure strategy equilibria coincide, and at this new separating equilibrium the DM gets the same payoff as the best mixed strategy equilibrium with α that we found. This means for any α such that $(1-\hat{\rho})^2 < c$, where there is both mixed and separating equilibria, there is a $\hat{\alpha} = \alpha^*$. To understand the intuition, realize that $\mu(0,1)$ is the same in both equilibria (the DM optimal mixed in α, and the separating equilibrium with $\hat{\alpha}$, plus $\mu(1,1) = 1$ in the best mixed strategy equilibrium for any $c > (1-\hat{\rho})^2$. In both equilibria, an equal portion of unbiased experts invest in overall, and while informed ones invest wastefully in the mixed strategy equilibria, in the separating equilibrium with $\hat{\alpha}$ all unbiased experts who invest are uninformed. The composition of types that do not invest are also constant in both equilibria. Whenever $\alpha > \alpha^*$ so that $(1-\hat{\rho})^2 < c$, the highest DM equilibrium payoff is in mixed strategies (while the expert preferred one is always the separating equilibrium). However, for any c and α, it is shown that this mixed strategy equilibrium payoff is bounded by the separating equilibrium payoff for $\alpha = \alpha^*$. In addition, at α^* the separating and best mixed strategy equilibria coincide, hence there is no disagreement over the equilibrium play between the expert and the DM.

This shows that there is a unique α which maximizes the DM's payoff regardless of the equilibrium selection. At α^*, the best mixed strategy equilibrium is indeed the separating equilibrium, and there is no conflict over the equilibrium selection.

Finally, we will identify the threshold k. This is the cost value below which, even for α arbitrarily close to 1, all types are still willing to invest. Then, we replace $\alpha = 1$ to find $\hat{\rho} = \frac{\rho}{(1-\beta)\rho+\beta}$, and then the threshold cost is given by $k = (1-\hat{\rho})^2 = \left(\frac{\beta(1-\rho)}{\beta(1-\rho)+\rho}\right)^2$. Whenever the cost is below this level, no matter what α is, all types are willing to invest regardless of whether they are informed or not. This means, as cost decreases, the DM's optimal level of expert is going to be increasingly informed, and once it becomes arbitrarily close to being perfectly informed, below that level any α will lead to the same result. □

Proof of proposition 4

Proof. Fix c. We have 4 different regions. First, for $c < k$, the unique equilibrium is the pooling one for any $\alpha > 0$ (assuming this is interior, so that $k > 0$) where $k = (1 - \hat{\rho})$ and $\hat{\rho}$ is found by setting $\alpha = 1$ (this means $(1 - r\hat{h}o) = \frac{\beta(1-\rho)}{\beta(1-\rho)+\rho}$). Realise also that when cost is so low, we are in the region where even if the probability that the expert is informed were very close to 1, there is still pooling equilibrium. Then below this level, expertise parameter doesn't matter for the DM's payoff.

Second, consider the region $k < c \leq (1 - \rho)^2$. We know that $(1 - \hat{\rho})^2$ is a decreasing function of α as $\hat{\rho}$ is increasing in α. Among the equilibria that arise in this region, we know that the DM's welfare is maximized with α high enough to ensure $(1 - \hat{\rho})^2 = c$, which leads to $\alpha^* = \frac{\beta(1-\rho-\sqrt{c})}{(1-\beta)\rho\sqrt{c}}$. When α is slightly lower, mixed strategy equilibrium arises in which the equilibrium payoff for the DM is lower. Plus, we know that conditional on the separating equilibrium realizing, if α increases further so that $(1 - \hat{\rho})^2 > c$, the separating equilibrium payoff is decreasing for the DM due to the loss in communication for informed unbiased types. At α^*, the DM optimal mixed strategy equilibrium is also the separating equilibria and provides the highest possible payoff for the DM.

When cost increases further so that $(1 - \rho)^2 \leq c \leq \rho - \rho^2$, for any value of α the unique equilibrium is the separating one in which only the group 2 types invest, as even though $\mu(1, 1) = 1$ the group 1 types find it too costly to invest now. Then, it is DM optimal that $\alpha = 0$ given that the unbiased uninformed types are willing to invest and separate themselves from a biased type, making communication most precise. If the biased types can be prevented from pooling when $\mu(0, 1) = \rho$, in other words when they are identified as biased, then it is optimal to have no informed experts at all. This is the ideal scenario for the DM and provides him the highest possible payoff.

Finally, when $c > \rho - \rho^2$, there is no possibility of investment by any type. In that case, it is optimal that the expert is informed with probability 1. When there is no way for the DM to separate between biased and unbiased types through investment behavior, it is optimal that the expert perfectly informed, in the least.

□

Proof of proposition 5

Proof. When $c > \rho - \rho^2 - \rho(1-\tilde{p})^2$, in the covert information acquisition case no type is getting informed, hence the payoff is the same as in the no investment equilibrium in the overt case. We know that in the overt information acquisition case, welfare is always higher when there is some information acquisition compared to none, and in the region $[\rho - \rho^2 - \rho(1-\tilde{p})^2, \rho - \rho^2]$ there is investment in the overt case. Hence, the payoff in overt case is always higher under this condition.

When $c < (1-\hat{p})^2 - (1-\tilde{p})^2$, in the overt case, there is pooling in investment while in the covert case, the unbiased uninformed type invests only (realize that $(1-\hat{p})^2 - (1-\tilde{p})^2 < \rho - \rho^2 - \rho(1-\tilde{p})^2$ hence there is investment in covert case). In the end, the amount of information transmitted is the same. Hence, in the overt case there is more wasteful investment for the same precision of communication. Then, we can conclude that welfare is higher in the covert case under this assumption. □

References

Rosella Argenziano, Sergei Severinov, and Francesco Squintani. Strategic information acquisition and transmission. *American Economic Journal: Microeconomics*, 8(3):119–155, 2016.

David Austen-Smith. Strategic transmission of costly information. *Econometrica*, 62:955–963, 1994.

David Austen-Smith and Jeffrey S. Banks. Cheap talk and burned money. *Journal of Economic Theory*, 91:1–16, 2000.

Sourav Bhattacharya, Maria Goltsman, and Arijit Mukherjee. On the optimality of diverse expert panels in persuasion. *Games and Economic Behavior*, 107:345–363, 2018.

Yeon-Koo Che and Navin Kartik. Opinions as incentives. *Journal of Political Economy*, 117(5): 815–860, 2009.

In-Koo Cho and David M. Kreps. Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102:179–221, 1987.

Vincent Crawford and Joel Sobel. Strategic information transmission. *Econometrica*, 50: 1431–1451, 1982.

Inga Deimen and Dezs´o Szalay. Delegated expertise, authority, and communication. *American Economic Review*, 109(4):1349–1374, 2016.

Robert Dur and Otto H. Swank. Producing and manipulating information. *Economic Journal*, 115, 2005.

Jeffrey Ely and Juuso Valimaki. Bad reputation. *Quarterly Journal of Economics*, pages 785–814, 2003.

Péter Esö and Dezsö Szalay. Incomplete language as an incentive device. *working paper*, 2010.

Paul E. Fischer and Phillip C. Stocken. Analyst information acquisition and communication. *The Accounting Review*, 2010.

Navin Kartik, Frances Xu Lee, and Wing Suen. Investment in concealable information by biased experts. *RAND Journal of Economics*, 48(1):24–43, 2017.

Steven Matthews and Andrew Postlewaite. Quality testing and disclosure. *The RAND Journal of Economics*, 16(3):328–340, 1985.

Xiaojing Meng. Analyst reputation, communication, and information acquisition. *Journal of Accounting Research*, pages 119–173, 2015.

John Morgan and Philipp C. Stocken. An analysis of stock recommendations. *RAND journal of economics*, 34(1):183–203, 2003.

Stephen Morris. Political correctness. *Journal of Political Economy*, 109(2), 2001.

Marco Ottaviani and Peter N. Sorensen. Reputational cheap talk. *RAND Journal of Economics*, 37.

Marco Ottaviani and Peter N. Sorensen. Professional advice: The theory of reputational cheap talk. *Journal of Economic Theory*, 126:120–142, 2006b.

Harry Di Pei. Communication with endogenous information acquisition. *Journal of Economic Theory*, 160:132–149, 2015.

Steven Shavell. Acquisition and disclosure of information prior to sale. *RAND Journal of Economics*, 25(1):20–36.

Joel Sobel. A theory of credibility. *The Review of Economic Studies*, 1985.

Joel Sobel. Giving and receiving advice. *Advances in Economics and Econometrics*, 2013.

Guido Suurmond, Otto H. Swank, and Bauke Visser. On the bad reputation of reputational concerns. *Journal of Public Economics*, 88:2817–2838, 2004.