The Race to the Base*

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Abstract

We study multi-district legislative elections between two office-seeking parties when the election pits a relatively strong party against a weaker party; when each party faces uncertainty about how voter preferences will evolve during the campaign; and, when each party cares not only about winning a majority, but also about its share of seats in the event that it holds majority or minority status. When the initial imbalance favoring one party is small, each party targets the median voter in the median district, in pursuit of a majority. When the imbalance is moderate, the advantaged party continues to hold the centre-ground, but the disadvantaged party retreats to target its core supporters; it does so to fortify its minority share of seats in the likely event that it fails to secure a majority. Finally, when the imbalance is large, the advantaged party advances toward its opponent, raiding its moderate supporters in pursuit of an outsized majority.

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1. Introduction

A near-axiomatic logic of two-party elections is that to win the contest, a party must carry the support of the median voter. To the extent that political parties care solely about winning the election, their platforms should therefore converge to the median voter's most-preferred policy (e.g., Hotelling 1929, Downs 1957). In legislative elections, however, winning is *not* everything. In fact, winning a majority of legislative seats may be neither necessary nor sufficient for a party to achieve its goals.

Two examples help illustrate this point. In 1992, John Major's Conservative party won a majority of seats in the House of Commons, and the largest number of votes of any party in British electoral history. Nonetheless, Major's parliamentary majority fell from 102 to a mere 21 seats. Despite its victory, Major's government was persistently hampered by its small majority, which contributed to its first legislative defeat just over one year later.

In 2017, by contrast, Jeremy Corbyn's Labour party failed to win a majority of seats. Nonetheless, the party advanced its minority seat share by 26 seats, and successfully denied the Conservative party its previously-held parliamentary majority. Since the Conservatives had enjoyed a 20-point lead in the polls at the moment Theresa May called the election, the press concluded that, despite its failure to achieve outright victory, Labour had triumphed—in particular, over expectations of an electoral rout. The outcome was summarized by one commentator as "the sweetest of defeats", while Labour MP and campaign strategist John Trickett boasted that "every lesson all these politics professors ever learned has been proved wrong".[1]

At the start of the 2017 election campaign, Theresa May enjoyed a 39 percentage point popularity advantage over Jeremy Corbyn.[2] She opted for "an aggressive strategy, influenced by her strong lead in the initial polls... parking her tank on Labour's lawn in heartlands such as the North East and the North West of England."[3] May devoted 61% of her campaign visits to Labour-held constituencies, and courted moderate Labour supporters with policy proposals that included a price cap on energy bills—a policy commitment that had featured in Labour's 2015 election manifesto.[4]

[1] "The Jeremy Corbyn factor", *BBC News*, 9 June 2017.

[2] YouGov, 18-19 April 2018.

[3] "What Theresa May's campaign stops tell us about her failed strategy", *The Telegraph*, 13 June 2017.

[4] In that election cycle, David Cameron ridiculed energy price caps as evidence of Ed Miliband's desire to live in a 'Marxist universe'. See "Tories accused of stealing Labour's energy price cap promise", *The Guardian*, 23 April, 2017.

By contrast, Labour's campaign opted for a defensive strategy, eschewing centrist voters in favor of its core supporters. Jeremy Corbyn devoted 42% of his campaign visits to constituencies that Labour had won in the previous 2015 election with a victory margin of more than 20 percentage points, and made only 5% of his visits to constituencies that Labour had won with less than a 15 percentage point lead.[5] The party opted for a radical manifesto that promised to nationalize public utilities, abolish university tuition fees, and levy new taxes on firms with highly-paid staff.[6] To observers who believed that a more moderate platform would maximize Labour's election performance, the party's strategy was "baffling".[7] Why, then, did it forego the centrist—or even right-leaning—route that led Tony Blair's party to a majority of 179 seats in 1997?

More broadly, we ask: under what circumstances does an office-seeking party in a legislative election want to choose its electoral platform to target to its traditional supporters, rather than centrist voters? If it targets its traditional supporters, should the opposing party try to maintain its hold on the centre-ground, cater to its own base, or instead try to raid its opponent's more moderate supporters? And, how do the answers to these questions depend on parties' expectations about their popularity, the extent of voters' partisan loyalties, and the relative marginal value that a party derives from winning additional seats below, at or above the majority threshold?

Our Approach. To address these questions, we develop a model of two-party competition between office-motivated parties in a multi-district legislative election. For example, the election could determine control of a legislative chamber such as the U.S. House of Representatives, or the British House of Commons. One of the parties holds an initial net valence advantage—for example, its leadership may perceived as more competent, its opponent may be dogged by scandal or simply worn out by a long period of incumbency. After the parties simultaneously choose platforms, an aggregate net valence payoff shock in favor of one party is realized, and every voter in every district subsequently casts his or her ballot for one of the two parties.

We assume throughout that each party's payoff depends solely on its share of districts, or seats in the legislature. However, this does not imply that parties care solely about *winning* the election. If a party wins more than half of the total districts (seats), it not only derives a large fixed payoff from majority status, i.e., from winning the election, it also receives a strictly increasing payoff from any additional seats that it wins beyond the majority threshold. The fixed office rent

[5] "Analysis shows Theresa May spent half of campaign targeting Labour seats", *The Guardian*, 8 June 2017.

[6] "For the Many, Not the Few", *Labour 2017 Election Manifesto*, https://goo.gl/GZaTbk.

[7] "The baffling world of Labour's election strategy", *The Spectator*, 27 April 2017.

reflects the value of majority status *per se*: in a parliamentary democracy, majority status confers the right to form the government regardless of the size of a party's majority. Even in presidential systems, majority status grants a party control over crucial aspects of the legislative process, including scheduling bills and staffing committees. However, additional seats beyond the majority threshold are also valuable: they further insulate the majority party from the threat of confidence votes in a parliamentary context, insure against defections of a few party members on key votes, and mitigate the obstructionist legislative tactics that a minority party can employ.

If, instead, a party holds minority status, i.e., if its share of seats falls below one half, its payoff nonetheless strictly increases in its share of seats. This reflects that a stronger minority receives more committee positions, and can more effectively derail the majority party's agenda by use of parliamentary procedures that privilege a more numerous minority. Our key substantive assumption is that, conditional on winning minority status, the minority party cares sufficiently about the number of seats it holds. For example, more seats may secure a larger share of committee assignments, and greater recourse to obstructionist tactics that require supermajorities to override. It may also reflect non-institutional factors: winning any sized majority is sufficient to secure a reprieve for an embattled leader (e.g., John Major), but the strength of a losing performance may be crucial for a party leader's short-term survival (e.g., Jeremy Corbyn). In fact, members of Corbyn's own party speculated that he was "trying to maximise the popular Labour vote to help bolster his argument for staying on in the event of a defeat."[8]

Results. We obtain a unique equilibrium, in pure strategies, for all levels of the initial popularity imbalance between the parties. The equilibrium characterization can be indexed according to whether the initial imbalance is small, moderate, or large.

If the advantage is *small*, both parties locate at the policy preferred by the median voter in the median district. The reason is that—even with an imbalance—both parties remain competitive for majority status, encouraging them to compete aggressively to win the election, outright. This reflects that while winning isn't everything, it certainly matters a lot.

If the advantage is *moderate*, the disadvantaged party assesses that its prospect of winning an outright majority is distant enough that it no longer finds it worthwhile to single-mindedly pursue outright victory. Instead, its strategy reverts to moving its policy platform away from the median voter in the median district, and in the direction of its core supporters. This choice may

[8] "General Election 2017: A tale of two campaigns." *BBC News*, 15 May, 2017.

seem paradoxical, since this shift in strategy renders the party's prospects of winning even more distant. Nonetheless, it also *increases* its anticipated share of seats in the relatively more likely event that the election consigns the party to minority status. The reason is that the party raises its attractiveness to its core supporters by differentiating itself ideologically from the advantaged party. With further increases in imbalance, the disadvantaged party further retreats to its base, as the prospect of losing the election rises.

By contrast, the advantaged party maintains its strategy of targeting its platform at the policy preferred by the median voter in the median district. While it could chase the disadvantaged party into its own ideological turf in order to push for an even larger share of districts, its advantage is only moderate, and alienating centrist voters would risk its prospects of majority status —a cost that is high relative to the prospective gains from bolstering its seat share in the event that it wins.

Finally, if the imbalance is *large*, the disadvantaged party continues its retreat by locating its platform even further from the centre and toward its core supporters. But now the advantaged party gives chase, moving its platform beyond the median voter in the median district and into the disadvantaged party's ideological territory. This is strategically appealing for three reasons. First, the party's very strong advantage makes it less concerned about the risk of losing the election—i.e., failing to win a majority of seats; instead, its focus shifts to generating a comfortable seat advantage *conditional on winning majority status*. Second, it reduces the policy wedge between the parties, which heightens the salience of the advantaged party's net valence advantage, raising its appeal amongst all voters. Third, it capitalizes on the opportunity created by the disadvantaged party's increasingly extreme lurch to raid its more moderate supporters.

While platforms fully converge when initial imbalances are small, we show how changes in political primitives in the context of either a moderate or large initial imbalance either exacerbate or mitigate the disadvantaged party's incentive to revert to its base, and platform polarization.

When the initial imbalance is moderate, the disadvantaged party increasingly retreats to its base whenever its initial disadvantage grows, whenever the marginal value of seats conditional on minority status rises, or whenever uncertainty about voter preferences increases. It also further retreats when there is a decline in the strength of partisan loyalty amongst its traditional supporters, since these voters are less easily taken for granted. Because the advantaged party maintains its position in the centre, these changes thus trigger increased platform polarization.

Once the imbalance is large enough, however, further increases in the popularity imbalance induce *both* parties to move toward the disadvantaged party's core supporters. And, in contrast with moderate imbalances, the distance between platforms *falls*, reducing platform polarization. Thus, our model predicts that platform divergence is maximized when the initial electoral imbalance is neither very small, nor very large, and especially where partisan loyalties are in flux.

Contribution. Our premise and results contrast starkly with existing models of party positioning in elections. In the framework developed by Calvert (1985) and Wittman (1983), policy-motivated parties face uncertainty about the preferences of the electorate—specifically, the median voter's most preferred platform. In equilibrium, if a party becomes *more* advantaged, i.e., the expected location of the median voter moves toward its most-preferred policy, *both* parties advance toward the advantaged party's ideal policy.

Our framework predicts the opposite; consistent with the campaigns of Tony Blair and Theresa May, when the advantaged party's net valence advantage is large enough, an increased electoral imbalance encourages both parties to move in the direction of *disadvantaged* party's base. The advantaged party invades its opponent's ideological turf in pursuit of a strong majority, while the disadvantaged party retreats to its base in an attempt to rally its core supporters. The first implication seems eminently suited to interpreting Tony Blair's electoral strategy in 1997 to transition his party to *New Labour*, at a time when the party enjoyed a clear preference advantage amongst British voters. This advantage was so strong that even *The Sun* newspaper, which had supported the Conservatives in every election in the previous twenty years, endorsed Labour, condemning the Conservatives as "tired, divided and rudderless".[9] Our prediction also more accurately characterizes Theresa May's efforts to win over moderate Labour supporters in 2017. The second implication more closely corresponds to Bogdanor's summary of the Conservative lurch to the right from 2001 to 2010, in which "three successive Conservative leaders... responded to defeat by seeking to mobilize the Tory 'core' vote".[10]

While our analysis focuses on legislative elections, our finding that an advantaged party advances on its weaker opponent—rather than catering to its own core voters—extends to the candidate-centered elections that are the focus of the Calvert-Wittman framework. Like both Bill Clinton and Tony Blair, Emanuel Macron—at one time a Socialist party minister—leveraged his large popularity advantage in his 2017 presidential campaign to adopt a '*Third Way*' manifesto that included reductions in corporate taxes and public spending, increased defense spending,

[9] See Butler and Kavanagh (1997).

[10] "The Conservative Party: From Thatcher to Cameron", *New Statesman*.

and allowing firms to negotiate additional working hours beyond the country's 35-hour work week.

Groseclose (2001) augments the Calvert-Wittman framework by introducing a deterministic valence advantage for one party. However, Groseclose does not establish existence or uniqueness of an equilibrium. Moreover, his main theoretical results are limited to a context with a small valence advantage (specifically, moving from no advantage to an arbitrarily small advantage), and his framework features a single (median) voter—precluding the question of whom to target that drives our framework. The predictions that he derives when an equilibrium exists differ substantially from our office-motivated context. For example, his framework predicts that an increase in the advantaged party's net valence advantage *always* raises platform differentiation; and, if the advantaged party's net valence advantage is very large, it always adopts more extreme policy positions in the direction of its ideal policy.

Our framework predicts the opposite: the advantaged candidate responds to a large advantage not by adopting more extreme positions favored by its own core supporters, but instead by targeting its opponent's moderate supporters. Our analysis reconciles campaigning by the Australian Labor Party (ALP) during the first of several election victories, in 1983. The election came at a time of high unemployment, high inflation, industrial unrest and a prime minister (Malcolm Fraser) who had only recently survived an internal leadership challenge. The incumbent government was so unpopular that former ALP leader Bill Hayden quipped that "*a drover's dog could lead the Labor Party to victory, the way the country is and the way the opinion polls are showing up...*".[11] During the election and in government, the party—whose constitution still declares it to have "the objective of the democratic socialization of industry, production, distribution and exchange"—promoted tariff reductions, tax reforms, limits on union activity, transitioning from centralized bargaining to enterprise bargaining, the privatization of government enterprises, and banking deregulation.

Also in contrast with Groseclose (2001), we find that policy divergence is maximized by an *intermediate* electoral imbalance in favor of one party. If the imbalance is very small, both parties compete for the support of the median voter in the median district, resulting in complete policy convergence; and if the imbalance is large, the advantaged party chases the disadvantaged party into its own turf, reducing policy divergence. When the imbalance is intermediate, the disadvan-

[11] "Statements from Hayden Bowen, Hawke". *The Canberra Times*, 4 February 1983.

taged party retreats to its base, but the advantaged party maintains the centre-ground. Finally, we prove existence and uniqueness of an equilibrium for all levels of the valence advantage.

Aragones and Palfrey (2002) and Hummel (2010) characterize equilibria in a Downsian setup with purely office-motivated candidates and a deterministic net valence advantage. As in our setting, the advantaged candidate benefits by raising the salience of this valence advantage. This encourages the advantaged candidate to mimic the disadvantaged candidate, and the disadvantaged candidate to try to differentiate itself from the advantaged party. Both papers are limited to characterizing a particular mixed strategy equilibrium, under the restriction either of a small (Aragones and Palfrey, 2002) or large (Hummel, 2010) initial valence advantage.

Our framework offers an explanation for why parties may instrumentally choose relatively extreme policies. In Eguia and Giovannoni (2017), a party that is sufficiently disadvantaged today may give up on a mainstream policy, and instead invest in an extreme policy; it does so not to increase its office-motivated payoffs today, but instead to gamble on a shock to voters' preferences in a subsequent election. Our explanation emphasizes that the instrumental adoption of extreme policies in the face of a likely election defeat arises not only via dynamic office-holding incentives, but also via static office-holding incentives that emphasize the value of a strong minority position.

Our multi-district framework is closest to Callander (2005), in which two parties simultaneously choose national platforms, facing entry by local candidates, generating equilibrium platforms that differ greatly from ours. Other authors—for example, Austen-Smith (1984), Kittsteiner and Eyster (2007), and Krasa and Polborn (2018)—study multi-district competition in which party platforms are an aggregate of decentralized choices by local candidates. Our framework, like Callander's, instead reflects a context in which voters predominantly assess their view of the party on the basis of its national platform.

2. Model

Preliminaries. Two parties, L and R, simultaneously choose campaign platforms, x_L and x_R, prior to an election. The policy space is the one-dimensional continuum, $\mathbb{R}$. Competition involves multiple districts, with the winner of each individual district determined by a plurality rule. Each district features a continuum of voters; each voter i is indexed by his or her preferred policy, x_i. There are a continuum of districts: in a district with median ideology m, voters' preferred policies are uniformly distributed on the interval $[m - Z, m + Z]$; and district medians are uniformly distributed on the interval $[-1, 1]$. We assume $Z > 1$ to capture the idea that there is more preference heterogeneity within districts than across different district medians.

Voter Payoffs. If party L implements platform x_L, a voter i with preferred policy x_i derives payoff

$$u(i, x_L) = -\gamma|x_L - x_i| - \theta x_i/2. \tag{1}$$

If, instead, party R implements its platform x_R, the voter derives the payoff

$$u(i, x_R) = -\gamma|x_R - x_i| + \theta x_i/2 + \rho_0 + \rho_1. \tag{2}$$

Here, ρ_0 is an initial valence advantage in favor of party R, commonly known by all agents, and ρ_1 is a preference shock, uniformly distributed on the interval $[-\psi, \psi]$.[12] The valence advantage ρ_0 reflects voters' relative assessment of the parties at the outset of a campaign—for example, evaluations of its leadership that are inherited from a party's previous spell in government. The valence shock ρ_1, by contrast, summarizes unanticipated developments that unfold over the course of an election campaign— right up to election day—including performances by party leaders in public debates or town hall meetings, or scandal revelations. If the legislative election coincides with a presidential election, ρ_1 could also capture evaluations of a party arising from its presidential candidate's campaign. Without loss of generality, we assume $\rho_0 \geq 0$.

The policy-related part of voters' preferences has two distinct components. The first component is a linear policy loss that increases with the distance between the party's policy platform and the voter's preferred policy. The parameter γ can be interpreted as the salience of the policy dimension on which parties choose platforms, or alternatively it could reflect the perceived credibility of the candidates' pre-electoral policy commitments.

The second component, which implies that voter i derives an additional net value $-\theta x_i$ from party L, has multiple interpretations. For example, it could reflect a fixed policy position on another dimension of policy conflict, e.g., social issues such as abortion, gay marriage, or on trade and immigration, and voter preferences on this issue are correlated with their preferences on taxation—over which parties are proposing policies—where the extent to which voters care about this second issue is proportional to θ.

Our running interpretation is that it reflects *partisanship*, i.e., a voter's "early-socialized, en-

[12] One could alternatively interpret ρ_0 as the mean of the preference shock ρ_1, i.e., $\rho_1 \sim U[\rho_0 - \psi, \rho_0 + \psi]$.

during, affective... identification with a specific political party" (Dalton, 2016, 1) that transcends short-term policy platforms that parties adopt from one election to the next. For example, while the British Labour Party has vacillated between centrism and more left-wing policies many times in the twentieth century, its loyalty amongst its core voters in the north of England has remained firm. In the United States, southern support for the Republican party is robust to changes in the party's platform across elections.

In sum, a voter with preferred policy x_i prefers party L if and only if:

$$\Delta(x_i) \equiv \underbrace{\gamma|x_R - x_i| - \gamma|x_L - x_i|}_{\text{Platform gap}} - \underbrace{\theta x_i}_{\text{Partisan gap}} - \underbrace{(\rho_0 + \rho_1)}_{\text{Valence gap}} \geq 0.$$

Party Payoffs. Let $d_P \in [0,1]$ denote the share of districts won by party $P \in \{L, R\}$, and let $M_P = \mathbb{I}[d_P > 1/2]$ denote the event that party P wins a majority of districts. Party P's payoff is

$$u_P(d_P) = M_P[r + \beta(d_P - 1/2)] + (1 - M_P)\alpha d_P. \tag{3}$$

A party receives a fixed payoff of $r > 0$ if it wins the election, i.e., if $d_P > 1/2$. Higher values of r reflect the majoritarian organization of a legislature: winning a majority gives a party agenda-setting authority, and control over committee assignments and leadership. And, in a parliamentary democracy, winning a majority yields formal control over the executive branch.

Parties also value winning additional seats both below and above the majoritarian threshold. Even if a party fails to achieve a majority, i.e., $d_P < 1/2$, it still gains from winning more seats. And, if a party achieves a majority, it values increasing its share of seats above the majority threshold. To capture this idea in the simplest possible way, we let $\alpha > 0$ denote the marginal value of winning districts that nonetheless keep a party's total share of districts less than a majority; similarly, $\beta > 0$ denotes the marginal value of winning districts above and beyond the majority threshold of one half. This piece-wise linear formulation facilitates tractable solutions, and may be viewed as an approximation of more sophisticated payoff schedules.

We impose two assumptions; the first assumption focuses on party preferences, while the second assumption focuses on voter preferences.

Assumption 1: $r > \frac{1}{2}\left(\alpha + \frac{\psi}{\theta}(\alpha - \beta)\right)$, and $\alpha \geq \beta$.

The first restriction states that parties sufficiently value winning majority status. Notice that for majority status to convey a benefit, it *must* be that $r > \alpha/2$.[13]

The second preference restriction, $\alpha \geq \beta$ states that the marginal value of additional seats to a minority party exceeds the marginal value of additional seats to a majority party, above and beyond its gains from majority status that are captured by r. We later describe properties of equilibrium policy platforms under the alternative assumption that $\beta > \alpha$; nonetheless, we view the restriction in Assumption 1 as inherently more plausible. For example, one can view $\alpha > \beta$ as a reduced-form preference assumption that captures the value of extra seats to the majority party when it obtains additional leeway to move policy outcomes closer to its most-preferred goal. Then, a larger majority allows the majority party to shift policy in its preferred direction. If party leaders have concave utility over policy, these incremental policy movements harm the minority party by more than they benefit the majority party, implying that $\alpha > \beta$.[14]

Assumption 2: $\theta > 2\gamma$.

This assumption eases analysis by ensuring that for any pair of platforms, there is some ideal policy x^* such that a voter prefers party L if and only if her ideal policy lies to the left of x^*.[15] This implies that in any district, the median voter is decisive for the outcome of that district's election. It further implies that the median voter in the median district determines which party wins the national election.[16]

Timing. The interaction proceeds as follows.

1. The parties simultaneously select platforms x_L and x_R.

2. The preference shock ρ_1 is realized and observed by all agents.

3. Each voter chooses to vote for one of the two parties.

[13] The restriction on r is not needed for our qualitative results. We make it to streamline exposition; moreover, we view it as a reasonable description of real-world contexts, in which the value of achieving majority status per se is large relative both to the value of minority status and relative to any incremental gains from an ever-larger share of seats beyond the majority threshold.

[14] We thank Pablo Montagnes for this observation. Alternatively, the reduced-form preference assumption could reflect a party leader's calculation about how the election outcome will affect her risk of being replaced. While an embattled leader who wins an election may secure a reprieve from the threat of replacement, regardless of her margin of victory, her survival if she loses an election may depend very sensitively on just how *badly* she loses.

[15] In particular, Assumption 2 implies that this is true even if the parties adopt platforms satisfying $x_R < x_L$.

[16] For tractability, we also assume that ψ is large enough that each party wins with positive probability in all districts, for any platform pair $(x_L, x_R) \in [-1, 1]^2$.

4. The party that wins a majority of districts implements its promised platform, and payoffs are realized.

Discussion. In our framework, parties know voters' policy preferences, but they face uncertainty about whether a popularity advantage at the start of the campaign (ρ_0) will subsequently increase, narrow, or even reverse during the election (via ρ_1). Both Aragones and Palfrey (2002) and Groseclose (2001) adopt the opposite perspective that at the time parties choose platforms, they perfectly forecast their relative popularity on election day, but face uncertainty about voters' policy preferences—specifically, the median voter's preferred policy.

Our approach reflects the view that an individual's perceptions of a party or party leader's competence, honesty and charisma—arising from campaign rallies, public debates and town halls, and (social) media coverage—fluctuate much more over the course of a single election cycle than his or her views on policy issues such as taxation, health care or gay marriage. They therefore constitute the first-order source of uncertainty facing parties in a given election. For example, while Theresa May started the 2017 election with a 39 percentage point popularity advantage, her popularity fluctuated throughout the campaign, and by polling day her margin had diminished to 10 percentage points.[17] In addition to its substantive motivation, our modeling framework yields a unique equilibrium in pure strategies, facilitating our goal of describing strategic behavior in real-world election campaigns.

3. Results

Preliminary Results. We begin by identifying the share of districts won by each party for any platform pair (x_L, x_R) and net valence advantage $\rho_0 + \rho_1$—and thus each party's probability of winning the election. Under Assumption 2, preferences are single-peaked, so there is a unique voter that is indifferent between the candidates: there is some ideal policy $x_i^*(x_L, x_R, \rho_0 + \rho_1)$ such that a voter prefers party L if and only if her ideal policy lies to the left of x_i^*. Because voters' ideal policies in a district with median m are uniformly distributed, and district medians are uniformly distributed on $[-1, 1]$, party L's share of districts is

$$\frac{1 + x_i^*(x_L, x_R, \rho_0 + \rho_1)}{2}.$$

[17] "Opinion Polling for the United Kingdom General Election, 2017", https://goo.gl/7mTYQW.

Party L therefore wins the election if and only if $x_i^* \geq 0$, i.e., if and only if it is preferred by the median voter in the median district, with ideal policy zero. We have:

$$x_i^* \geq 0 \iff \gamma|x_R| - \gamma|x_L| - \rho_0 \geq \rho_1 \equiv \rho_1^*(x_L, x_R, \rho_0).$$

Henceforth, we call the median voter in the median district the *median voter*. Substituting into the party payoff function in equation (3) yields party L's expected payoff:

$$\pi_L(x_L, x_R) = \int_{-\psi}^{\rho_1^*(x_L,x_R,\rho_0)} \left(r + \beta\left(\frac{1 + x_i^*(x_L, x_R, \rho_0 + \rho_1)}{2} - \frac{1}{2}\right)\right) f(\rho_1)d\rho_1 + \int_{\rho_1^*(x_L,x_R,\rho_0)}^{\psi} \alpha \frac{1 + x_i^*(x_L, x_R, \rho_0 + \rho_1)}{2} f(\rho_1)d\rho_1. \tag{4}$$

Party R's corresponding expected payoff is:

$$\pi_R(x_L, x_R) = \int_{\rho_1^*(x_L,x_R,\rho_0)}^{\psi} \left(r + \beta\left(\frac{1 - x_i^*(x_L, x_R, \rho_0 + \rho_1)}{2} - \frac{1}{2}\right)\right) f(\rho_1)d\rho_1 + \int_{-\psi}^{\rho_1^*(x_L,x_R,\rho_0)} \alpha \frac{1 - x_i^*(x_L, x_R, \rho_0 + \rho_1)}{2} f(\rho_1)d\rho_1. \tag{5}$$

Main Results. We now characterize equilibrium platforms choices and highlight how they depend on R's initial advantage (ρ_0), uncertainty about how preferences will evolve over the course of the election (i.e., uncertainty about ρ_1), the relative value of seats to the minority party (α) versus the majority (β), and the value of winning a legislative majority (r). We first establish that our framework produces a unique equilibrium, in pure strategies.

Theorem 1. *Under Assumptions 1 and 2, there exists a unique pure strategy equilibrium.*

To understand why—and to preview the incentives that govern equilibrium platform choices—we consider a platform pair (x_L, x_R) satisfying $x_L < x_R = 0$ and study the local incentives of party L. Recall that there exists a unique voter that is indifferent between the two parties, whose preferred platform x^* satisfies:

$$\Delta(x^*) = \gamma|x_R - x^*| - \gamma|x_L - x^*| - \theta x^* - \rho_0 - \rho_1 = 0,$$

such that a voter prefers party L if and only if her ideal policy lies to the left of x^*. We refer to this indifferent agent as the *marginal voter*. Notice that there are three possible intervals from which

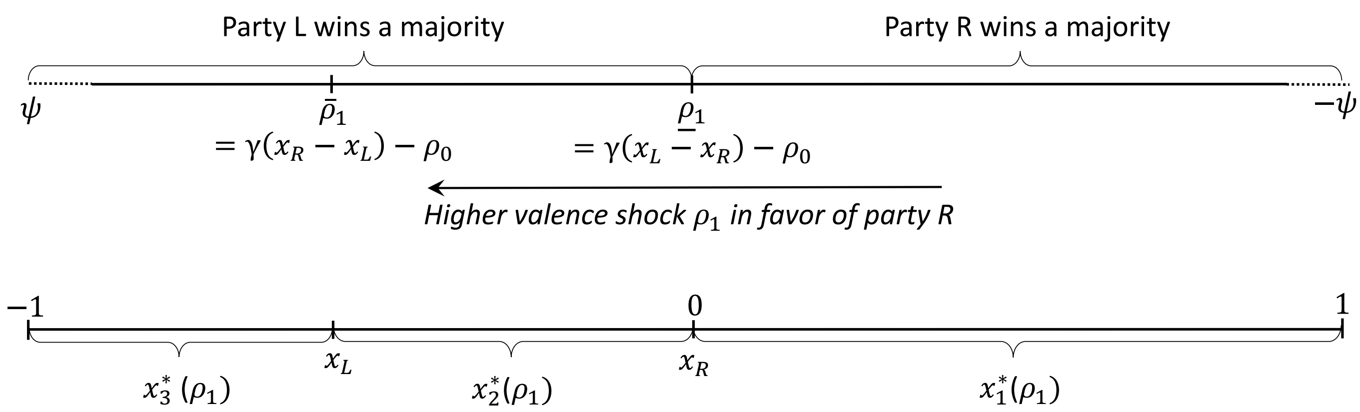

Figure 1 – Possible locations for the marginal voter (bottom row), depending on the valence shock ρ_1 (higher row).

the marginal voter's preferred policy could be realized, depending on the sign and magnitude of the valence shock ρ_1. These correspond to the three intervals highlighted in Figure 1, in which we identify thresholds $\underline{\rho}_1$ and $\overline{\rho}_1$ such that:

1. if $\rho_1 < \underline{\rho}_1$, the marginal voter belongs to one of advantaged party R's core districts:

$$x_1^* = \frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{\theta} \geq x_R. \tag{6}$$

2. If $\overline{\rho}_1 \leq \rho_1 \leq \underline{\rho}_1$, the marginal voter belongs to a moderate district:

$$x_2^* = \frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2\gamma + \theta} \in [x_L, x_R]. \tag{7}$$

3. If $\rho_1 > \underline{\rho}_1$, the marginal voter belongs to one of disadvantaged party L's relatively extreme core districts:

$$x_3^* = \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta} \leq x_L. \tag{8}$$

Notice that the marginal voter's ideal policy lies to the right of zero if and only if $\rho_1 \leq \underline{\rho}_1$, in which case party L wins the election.[18]

[18] If $\theta = 0$, the marginal voter's preferred policy always lies on the interior $[x_L, x_R]$—for example, this carries the implausible implication that either *every* voter with ideal policy to the left of x_L supports party L, or *every* such voter supports party R. When parties care about their seat share above ($\beta > 0$) or below ($\alpha > 0$) a majority, $\theta = 0$ generates payoff discontinuities which result in mixed strategies.

We may therefore re-write equation (4), party L's expected payoff from a platform x_L when party R chooses $x_R = 0$:[19]

$$\pi_L(x_L, x_R) = r\int_{\rho_1<\underline{\rho}_1} f(\rho_1)d\rho_1 + \frac{\beta}{2}\int_{\rho_1<\underline{\rho}_1} x_1^*(\rho_1)f(\rho_1)\,d\rho_1 + \frac{\alpha}{2}\int_{\underline{\rho}_1<\rho_1<\overline{\rho}_1} x_2^*(\rho_1)f(\rho_1)\,d\rho_1 + \frac{\alpha}{2}\int_{\rho_1<\overline{\rho}_1} x_3^*(\rho_1)f(\rho_1)\,d\rho_1 + \frac{\alpha}{2}\int_{\rho_1<\overline{\rho}_1} f(\rho_1)d\rho_1. \quad (9)$$

This first term reflects that if the valence shock is sufficiently favorable to party L, i.e., if $\rho_1 < \underline{\rho}_1$, then it wins a majority of votes, and thus enjoys the majority status payoff, r. The second term captures the additional return that party L receives from winning any additional seats beyond the majority threshold, which it values at a rate $\beta \geq 0$. Finally, the bottom line reflects party L's value $\alpha \geq 0$ from winning seats when it holds minority status.

If party L moves to the left, away from party R, the change in its payoffs is:

$$\frac{\partial \pi_L(x_L, 0)}{\partial(-x_L)} \propto r\underbrace{\frac{\partial \underline{\rho}_1}{\partial(-x_L)}}_{<0} + \frac{\beta}{2}\int_{\rho_1<\underline{\rho}_1}\underbrace{\frac{\partial x_1^*(\rho_1)}{\partial(-x_L)}}_{<0}d\rho_1 + \frac{\alpha}{2}\int_{\underline{\rho}_1\leq\rho_1\leq\overline{\rho}_1}\underbrace{\frac{\partial x_2^*(\rho_1)}{\partial(-x_L)}}_{<0}d\rho_1 + \frac{\alpha}{2}\int_{\rho_1>\overline{\rho}_1}\underbrace{\frac{\partial x_3^*(\rho_1)}{\partial(-x_L)}}_{>0}d\rho_1. \quad (10)$$

This expression highlights the two critical agents that determine L's electoral trade-offs: the *median* voter, and the *marginal* voter.

Median voter. Party L faces an uncertain prospect of winning, because of the stochastic valence shock ρ_1. However, it does *not* face uncertainty about the policy that maximizes this prospect. Moving away from the policy preferred by the median voter, with ideal policy zero, differentiates party L from its stronger opponent. This increases L's attractiveness to voters that *already* prefer L's policy to its moderate opponent's policy. Nonetheless, it harms L's standing with the voter that is decisive for the election, who is now better served by party R. This harms L in proportion to r, the value of majority status, reflected in the first term on the RHS of equation (10).

Marginal voter. While parties can forecast the policy preferences of the *median* voter, they nonetheless face uncertainty about the identity of the *marginal* voter. When L differentiates its platform from R's by moving to the left, the consequences depend critically on the whether the stochastic

[19] Notice that if we had conjectured $x_R < 0$, we would need to account for a fourth case, in which the marginal voter's preferred policy lies on the interval $[x_R, 0]$. This additional consideration does not alter the intuitions we develop in this section.

valence shock ρ_1 exacerbates, diminishes or even reverses party R's initial valence advantage.

1. If the valence shock ρ_1 strongly favors L, i.e., if $\rho_1 < \underline{\rho}_1$, then the marginal voter prefers R's policy to L's. Equation (6) highlights that L's move toward its core supporters shifts the marginal voter's ideal policy x_1^* to the left. This *lowers* L's incremental majority seat share, which it values at a rate β. This loss is reflected in the second term on the RHS of equation (10).

2. If the valence shock weakly favors party R, i.e., if $\rho_1 \in [\underline{\rho}_1, \overline{\rho}_1]$, then the marginal voter is a relatively moderate agent with ideal policy $i_2 \in (x_L, x_R)$. Equation (7) reveals that L's move toward its core supporters shifts the marginal voter's ideal policy x_2^* to the left. This *lowers* L's incremental minority seat share, which it values at a rate α. This harms L in proportion to α, which is reflected in the third term on the RHS of equation (10).

3. If the valence shock strongly favors R, i.e., if $\rho_1 < \underline{\rho}_1$, then the marginal voter prefers L's policy to R's. Equation (8) reveals that L's move toward its core supporters shifts the marginal voter's ideal policy x_1^* to the right. This *raises* L's incremental minority seat share, which it values at a rate α. This gain is reflected in the fourth term on the RHS of equation (10).

The relative strength of these incentives depends on the value of majority status (r), and the relative value of additional minority (α) and majority (β) seats. It also depends on the magnitude of the initial valence advantage that favors party R, ρ_0: for any fixed pair of platforms, a higher ρ_0 *lowers* the prospect of a valence shock $\rho_1 < \underline{\rho}_1$ that carries party L to victory and *raises* the prospect of a valence shock $\rho_1 > \overline{\rho}_1$, such that the marginal voter is drawn from one of L's more extreme core districts. In this latter case, L's core vote becomes its swing vote!

If $\rho_1 > \underline{\rho}_1$, then the valence shock sufficiently favors advantaged party R that the marginal voter's ideal policy lies to the left of its platform, i.e., zero. The party therefore wins a majority. Moving its policy toward its weaker opponent ("chasing" party L) can help it to secure a larger share of seats in this event. However, if $\rho_1 < \underline{\rho}_1$, then the valence shock favors its opponent so strongly that the marginal voter's ideal policy lies to the right of party R's platform. Party R is therefore consigned to minority status. Moving its policy toward its weaker opponent only exacerbates its losses in this event, because the party further alienates its core supporters—who have become swing voters due to the large valence shock that favors party L.

In sum: each party's trade-offs depend on their beliefs about the likely location of the marginal voter, and thus where the front lines of the electoral battle will be fought. This location depends *both* on primitives *and* platform choices. How these trade-offs resolve, and thus the characterization of the unique equilibrium, can be indexed according to whether the advantaged party's initial imbalance is small, intermediate, or large.

Proposition 1. *If party R's advantage is **small** in the sense that*

$$0 \leq \rho_0 \leq \frac{\theta(2r-\alpha)-(\alpha-\beta)\psi}{\alpha+\beta} \equiv \underline{\rho}_0,$$

then both parties locate at the ideal policy of the median voter in the median district:

$$x_L^*(\rho_0)=0, \quad x_R^*(\rho_0)=0.$$

A party wins a majority of districts if and only if it is most-preferred by the median voter in the median district, i.e., with most-preferred policy zero. When the parties are initially balanced, i.e., when ρ_0 is zero, each party is competitive for a majority. Because winning a majority r, each party aggressively pursues an outright victory.

Starting from a position of initial symmetry, i.e., starting from $\rho_0 = 0$, increases in ρ_0 reduce L's chances of winning, but do *not* alter the policy platform that maximizes this probability. Thus—and to an extent that is proportional to its value from majority status, r—L's electoral strategy continues to target a legislative majority by way of a centrist policy platform even as its prospects of winning deteriorate. Notice that as $(\alpha-\beta)\psi$ increases—implying a greater relative concern for incremental minority versus majority seat shares, $\alpha-\beta$, combined with the greater electoral risk encapsulated in ψ—the upper bound of imbalance for which the disadvantaged party wants to compete directly with the advantaged party ($\underline{\rho}_0$) *falls*.

When the imbalance between the parties is large enough, however, L no longer prefers unmitigated competition with R for outright victory.

Proposition 2. *If party R's advantage is **intermediate** in the sense that*

$$\underline{\rho}_0 \leq \rho_0 < \underline{\rho}_0 + \frac{\psi(\alpha-\beta)(2\theta\alpha+(\alpha-\beta)\gamma)}{(\alpha+\beta)(\alpha\theta+(\alpha-\beta)\gamma)} \equiv \overline{\rho}_0,$$

then party L retreats to its base,

$$x_L^*(\rho_0) = \frac{\theta(2r-\alpha) - \alpha(\rho_0+\psi) + \beta(\psi-\rho_0)}{\gamma(\alpha-\beta)+2\alpha\theta} < 0,$$

but R still locates at the ideal policy of the median voter in the median district, choosing $x_R^*(\rho_0) = 0$.

When the electoral imbalance in favor of party R surpasses an initial threshold $\underline{\rho}_0 > 0$, two features of the disadvantaged party L's competitive environment shift enough to merit a change in electoral strategy. First, a sufficiently high ρ_0 implies that the prospect of winning a majority—even when targeting the *median* voter, directly—becomes a distant prospect. Second, when $x_L = x_R = 0$, equation (8) reveals that a large ρ_0 implies that in the event L fails to win a majority, the *marginal* voter with ideology $x_3^* = -\frac{\rho_0+\rho_1}{\theta}$ is decisive in one of L's core districts. This further implies that (a) when L loses the election on a strategy designed to win outright, it loses *very badly*, and (b) it can make the best of its likely opposition status only by buttressing its support amongst these core districts.

As a result, L's best electoral strategy reverts to galvanizing its base, i.e., by selecting a platform $x_L(\rho_0) < 0$. By distancing itself from party R, it creates a meaningful ideological alternative to R's centrist platform: policy differentiation partly mitigates L's valence disadvantage amongst voters that value more left-wing policies. While retreating from the political centre further lowers L's prospect of winning a majority of districts, $\rho_0 > \underline{\rho}_0$ implies that party L no longer finds it worthwhile to target an outright victory. That is, acknowledging that is very likely to hold minority status, its priority smoothly reverts from solely pursuing a majority to instead balancing this objective with the need to secure the most advantageous minority share of seats possible.

By contrast, the same primitives encourage party R to maintain its hold on the ideological centre-ground. Its prospect of winning the election is maximized by selecting the policy preferred by the median voter in the median district. Party R *could* chase L into its own ideological turf, in order to increase its seat advantage conditional on holding a majority. However, its initial electoral advantage is still small enough ($\rho_0 < \overline{\rho}_0$) that it does not want to risk its prospect of winning. Chasing disadvantaged party L makes advantaged party R more palatable to moderate left-wing districts, but harms R's standing with both the median voter and R's own core voters. And, in the event that R fails to win a majority, the marginal voter—with ideology $x_1^* > 0$—will indeed be one of R's core supporters. To the extent that R values insuring itself against an adverse popularity shock, it prefers not to give chase.

To see this point, more clearly, notice that the size of the interval $[\underline{\rho}_0, \overline{\rho}_0]$ is proportional to $\psi(\alpha - \beta)$, and the interval is empty when $\alpha = \beta$. This reflects the advantaged party's incentive to hold back versus give chase. As it advances on its retreating opponent by shifting its platform to the left:

1. it raises its appeal amongst its opponent's core supporters and therefore—conditional on winning—raises its share of districts by shifting the marginal voter's ideal policy $x_3^* = \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta}$ to the *left* (recall equation (8)). It values these districts at rate β; but,

2. it lowers its appeal amongst its own core supporters, and therefore—-conditional on losing—lowers its share of districts by shifting the marginal voter's ideal policy $x_1^* = \frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{\theta}$ to the *right* (recall equation (6)). It values these districts at rate $\alpha \geq \beta$.

As the wedge $\alpha - \beta$ increases—amplified by the magnitude of the election risk ψ—the advantaged party increasingly prefers to 'play it safe', holding back even as its initial advantage increases.

These channels generate natural effects of primitives on party L's platform, and thus the degree of policy divergence between the parties.

Corollary 1. *Party L increasingly retreats to its base—and thus platform divergence* increases*—whenever*

1. *its initial disadvantage ρ_0* increases*,*

2. *the marginal value of minority seats α* increases*, or*

3. *uncertainty about voter preferences ψ* rises*.*

Conversely, L increasingly targets the median voter when

1. *the value of majority status r* increases*,*

2. *party loyalty θ* increases*, or*

3. *policy responsiveness γ* rises*.*

If party loyalty θ amongst more ideologically/polarized voters rises, party L becomes less worried about losing support amongst its core districts—the rate at which higher valence shocks ρ_1 shift the identity of the marginal voter further into its core districts slows. This encourages the party to target more centrist districts whose support is crucial for the party to win. Conversely, when voters are relatively more responsive to platform choices via γ, the weaker party must make greater concessions to its base in order to win their support.

Finally, suppose that parties anticipate a more volatile electorate via higher ψ. Then, for any pair of platforms, there is a heightened prospect of a large post-election imbalance between the majority and minority party via more extreme realizations of $\rho_1 \sim U[-\psi, \psi]$, because the marginal voter's identity becomes more volatile. If the disadvantaged party competes more aggressively by moving its platform toward its opponent, it could win more seats in the event of a strong majority (i.e., $\rho_1 < \underline{\rho}_1$), but it may lose more seats in the event of an unfavorable $\rho_1 > \overline{\rho}_1$ realization that consigns the party minority status. Here, with $\alpha > \beta$, risk-aversion encourages the weaker party to hasten its retreat. Thus, our framework predicts that platform polarization is greater when party loyalty is weaker (θ smaller) and voters' preferences are more volatile.

Finally, when the imbalance between the parties is very large, party R becomes so emboldened by its initial advantage over L that it abandons the mere pursuit of victory, and instead chases its weaker opponent in an effort to plunder its moderate supporters.

Proposition 3. *If party R's advantage is* ***large****, i.e., $\rho_0 > \overline{\rho}_0$, then party L retreats by more to its base:*

$$x_L^*(\rho_0) = \frac{((\alpha-\beta)\gamma+\beta\theta)(\theta(2r-\alpha)-(\alpha+\beta)\rho_0)-\beta\theta\psi(\alpha-\beta)}{\theta((\alpha^2-\beta^2)\gamma+2\alpha\beta\theta)}, \tag{11}$$

and party R advances toward party L's base:

$$x_R^*(\rho_0) = x_L^*(\rho_0) + (\alpha-\beta)\frac{(\alpha+\beta)(\psi-\rho_0)+\theta(2r-\alpha)}{(\alpha^2-\beta^2)\gamma+2\alpha\beta\theta}. \tag{12}$$

When the electoral imbalance in favor of party R is very large, party L overwhelmingly focuses on consolidating support amongst its base—the most likely location of the marginal voter, and thus the most likely front-line of the political battle. In turn, party R also advances into L's ideological territory to win over centre-left districts that are increasingly ill-served by the more extreme L party. It does so for two reasons. First, a sufficiently large advantage ($\rho_0 > \overline{\rho}_0$) makes party R less concerned about its chances of achieving majority status and instead more focused on generating the largest possible legislative majority in the event that it wins. Second, by reducing the policy differentiation between the parties, R intensifies its comparative valence advantage in the eyes of the likely marginal voter, further increasing its support. Notice that when $\alpha - \beta = 0$, the platforms converge, i.e., $x_L^*(\rho_0) = x_R^*(\rho_0)$, reflecting the chase-and-evade logic of Aragones and

Palfrey (2002).

Corollary 2 summarizes the effect of primitives on the parties' platforms, and their consequences for platform divergence.

Corollary 2. *As R's initial advantage $\rho_0 \geq \overline{\rho}_0$ increases, both party L and party R move toward L's base, and platform divergence* decreases*.*

As party L becomes further disadvantaged, it faces even greater incentives to target its base; by differentiating itself further from the advantaged party, it increases its appeal to its core supporters, consolidating its minority position. However, party R is also further emboldened to advance into its opponent's home turf. Their incentives are two-fold; a higher ρ_0 strengthens R's incentives to chase the increasingly weakened L *and*—independently—it wants to use its platform to turn centrist districts that L has abandoned, in pursuit of an outsized majority. The net effect is that platforms further converge, with the speed of convergence increasing in β, the marginal value of seats conditional on majority status.

Corollary 2 highlights that party R's platform moves to the left faster than party L's, so that the net effect is to reduce policy differentiation between the parties. Conversely, if ρ_0 *decreases*, both parties move their platforms toward the median voter in the median district, but party L moves more slowly than party R, *increasing* the degree of platform divergence.

Other changes in primitives may lead to different effects for the ex-ante advantaged versus disadvantaged party, and may hinge on other features of the political environment.

Corollary 3. *When the marginal value of minority seats, α, increases, party L increasingly retreats to its base by an amount that increases in preference volatility, ψ. By contrast, when α increases, there exists $\hat{\rho}_0 \geq \overline{\rho}_0$ such that party R moves toward L's base if and only its initial advantage ρ_0 exceeds $\hat{\rho}_0$.*

As α rises, party L grows more concerned about not losing the election too badly, so it increasingly targets its core supporters. Party R, however, faces two conflicting incentives. First, as α increases, it too has a stronger incentive to consolidate its core support by reverting to the right, i.e., in the direction of its base. This incentive increases with preference volatility, ψ, since more volatility implies a greater risk of a bad election result that consigns the party to minority status. However, as party L increasingly moves toward its base, party R also faces a stronger incentive to advance toward party L's platform in order to reduce the policy differentiation between parties, thereby heightening its comparative valence advantage.

The net effect on party R's equilibrium platform depends on the size of its initial advantage. If this initial advantage is low, party R's unwillingness to abandon its core supporters is the dominant force, encouraging it to move its platform back toward the median voter. If, instead, its initial advantage is large enough, party R chases party L even more aggressively, in order to reduce platform differentiation and further press its heightened advantage.

Corollary 4. *As the value of majority status r increases, both party L and party R revert toward the ideal policy of the median voter in the median district, but platform divergence* increases*.*

A party wins a majority if and only if it is preferred by the median voter. A higher value r of majority status encourages both parties to target this voter. Corollary 4 highlights that party R's platform moves faster than party L's. To see why, recall that party L remains at a competitive disadvantage; moving toward the centre raises its attractiveness to moderate voters, but dampens its relative appeal amongst its base. This represents a trade-off for party L. For party R, however, moving back toward the centre both raises its appeal to centrists *and* its core supporters.

Since both trade-offs are complementary to party R, but opposing for party L, the net effect is to increase platform divergence: L reluctantly abandons its base, while R's increased desire to win implies that its platform choice is governed less by the incentive to chase L, and more by the incentive to maximize its appeal to the decisive voter in a bid for outright victory.

Corollary 5. *As electoral volatility ψ increases, both party L and party R revert toward their respective core supporters, and platform divergence* increases*.*

When there is a large initial wedge in the parties' initial strength, more uncertainty *always* raises platform divergence. This reflects that both parties grow more concerned with insuring themselves against an adverse popularity shock by consolidating their core supporters. Greater volatility raises the prospect that the election will result in a larger imbalance in favor of one of the two parties. Because $\alpha - \beta > 0$, each risk-averse party resolves in favor of buttressing its seat share in the event that it is consigned to minority status.

We highlight our framework's predictions about the political contexts in which platform divergence is maximized. Maximal platform divergence occurs when $\rho_0 = \overline{\rho}_0$, i.e., when the initial imbalance between the parties is large, but not overwhelming. Platform divergence also rises when parties care substantially about bolstering the minority position (α large), when traditional party loyalties are in flux (θ small) and when there is significant uncertainty about the mood of the electorate, as reflected in uncertainty about ρ_1 (i.e., ψ is large).

What about $\beta > \alpha$? Our analysis focuses on settings where $\alpha \geq \beta$, i.e., where the marginal value of additional seats to the minority exceeds the marginal value of additional seats to the majority (β), above and beyond its *per se* benefit from majority status, r. In the less plausible context in which $\beta > \alpha$, the parties fully converge on the ideal policy of the median voter in the median district when R's advantage is not too large—as in our benchmark setting. As we detail in a Supplemental Appendix, however, with a very large initial advantage, the shape of preferences may induce parties to engage in risk-taking behavior, generating platform separation in which party R gambles on a left-wing platform, leaving the centre-ground to its weaker opponent. Our benchmark presentation, by contrast, reflects the more empirically relevant scenario in which parties may court their opponent's core supporters (as detailed in Proposition 3), but never to the extent that their own core voters are better served by their opponent.

4. Conclusion

We analyze two-party competition in multi-district legislative elections. We ask: how do initial electoral imbalances encourage an office-seeking party to target its traditional supporters, rather than the centrist voters that are crucial for outright victory? If it targets traditional supporters, when should the opposing party maintain its focus on courting centrist voters, and when instead should it chase its opponent, targeting voters that are more ideologically disposed toward its opponent? And, how do the answers to these questions depend on parties' expectations of how voters attitudes might change over the course of the campaign, the strength of pre-existing party loyalty, and the relative marginal value that a party derives from winning additional seats below, at, or even above the majority threshold?

A small initial imbalance does not deter a disadvantaged party from the sole pursuit of outright victory by way of a centrist policy agenda. However, a sufficiently large imbalance induces it to revert in favor of a strategy that consolidates its core supporters, in order to avoid a catastrophic defeat. Similarly, an advantaged party initially prefers to maintain uncontested control of the political centre to further fortify its prospects of a post-election majority. But, if the imbalance is large enough, it chases its opponent to plunder its increasingly ill-served moderate supporters; the advantaged party's goal evolves from seeking to win, to winning with a larger post-election majority.

Our framework generates novel predictions about the consequences of initial electoral imbalances for platform choices and polarization. In particular, we predict that a very advantaged party uses its strength as an opportunity to expand the frontier of its political support beyond the median voter; as illustrated by the campaigns of Tony Blair and Theresa May. This contrasts with Groseclose (2001), who predicts that a very advantaged party instead uses its advantage to revert toward its own base. This logic also implies that polarization between parties is maximized not when the imbalance between parties is very large, but instead when it is intermediate—small enough that the stronger party maintains rather than attempting to expand its support, but large enough that the weak party reverts toward a more defensive strategy. We also find that polarization is greatest *not* when party loyalties are strong, but rather when they are *weak* so that core supporters cannot be taken for granted.

In ongoing work, we use our framework to study the dynamics of political campaigns in contexts where some voters cast ballots early, or make up their minds before a campaign concludes. To wit, we assume that some voters cast their ballots after an initial valence shock that favors one of the parties, but before the parties have communicated their policy commitments, and prior to any other developments—such as leader debates, town hall meetings, or personal revelations—that occur over the course of a campaign. We interpret these voters as 'early deciders', who are relatively insensitive or inattentive to the twists and turns of election campaigns.

If the initial valence shock favoring one of the parties is small, the parties converge on a platform that—rather than targeting the median voter in the median district, as in Proposition 1—moves toward the advantaged party's core districts, by an increment that grows with both the magnitude of the initial valence shock and the fraction of early deciders. To see why, notice that the advantaged party enjoys a larger share of support amongst early deciders, and thus gains a starting lead in the polls. In order to win the election, the disadvantaged party therefore needs to offset its disadvantage by carrying strictly more than a majority of supporters amongst the remaining voters. This leads it to move beyond the ideological centre-ground, targeting voters that are ideologically disposed toward its advantaged opponent. Thus, the disadvantaged party designs its policy to appeal to its rival's voters even though ideology is *not* the source of its disadvantage.

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5. Appendix: Proofs of Results

Let $x_i^*(\rho_0 + \rho_1, x_L, x_R)$ denote the preferred policy of the ("marginal") voter who is indifferent between parties L and R given the realized net valence advantage to party R of $\rho_0 + \rho_1$ and the platform choices x_L and x_R. Given $\theta > 2\gamma$, for any pair (x_L, x_R), a voter j with ideal policy $x_j > x_i^*$ strictly prefers party R, and a voter j with ideal policy $x_j < x_i^*$ strictly prefers party L. Party L's total vote share in a district with median $m \in [-1, 1]$ is therefore

$$\frac{1}{2} + \frac{x_i^*(\rho_0 + \rho_1, x_L, x_R) - m}{2Z} \tag{13}$$

Party L therefore wins a district with median m if and only if Equation 13 exceeds one half, i.e., if $x_i^*(\rho_0 + \rho_1, x_L, x_R) \geq m$. Aggregating over districts, party L's seat share is:

$$\frac{1 + x_i^*}{2}, \tag{14}$$

and party L therefore wins the election if and only if $x_i^*(\rho_0 + \rho_1, x_L, x_R) \geq 0$.

Proof of Propositions 1, 2, 3 . We first rule out the existence of an equilibrium in which the platform profile (x_L, x_R) fails to satisfy $x_L \leq x_R \leq 0$. For this, we will rule out pure strategy equilibria in which (1) $x_L \leq 0 < x_R$, (2) $0 \leq x_L \leq x_R$ with at least one strict inequality, (3) $x_R \leq 0 \leq x_L$ with at least one strict inequality, (4) $x_R < x_L \leq 0$, and (5) $0 \leq x_R < x_L$.

Profile 1: $x_L \leq 0 < x_R$. There are 3 possible locations for the marginal voter:

1. *Location 1*: $x_i^* \geq x_R$, i.e. $\gamma(x_R - x_L) - \rho_0 - \rho_1 - \theta x_L \leq 0$, i.e. $\rho_1 \leq \gamma(x_L - x_R) - \rho_0 - \theta x_R$:

$$x^* = \frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{\theta} \equiv x_1^*.$$

2. *Location 2*: $x_L \leq x_i^* \leq x_R$, i.e. $\gamma(x_L - x_R) - \rho_0 - \theta x_R \leq \rho_1 \leq \gamma(x_R - x_L) - \rho_0 - \theta x_L$:

$$x^* = \frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2\gamma + \theta} \equiv x_2^*.$$

3. *Location 3*: $x_i^* \leq x_L$, i.e. $\rho_1 \geq \gamma(x_R - x_L) - \rho_0 - \theta x_L$:

$$x^* = \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta} \equiv x_3^*.$$

Party R wins if and only if ${x_2}^* \leq 0$, i.e.,

$$\rho_1 > \gamma(x_L + x_R) - \rho_0. \tag{15}$$

Party R's expected payoff is therefore:

$$\begin{aligned}
\pi_R(x_L, x_R) \; &= \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R)-\rho_0-\theta x_R} \left(\frac{\alpha}{2} - \alpha\frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{2\theta}\right) d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R)-\rho_0-\theta x_R}^{\gamma(x_L + x_R)-\rho_0} \left(\frac{\alpha}{2} - \alpha\frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(2\gamma + \theta)}\right) d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L + x_R)-\rho_0}^{\gamma(x_R - x_L)-\rho_0-\theta x_L} \left(r - \beta\frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(2\gamma + \theta)}\right) d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_R - x_L)-\rho_0-\theta x_L}^{\psi} \left(r - \beta\frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{2\theta}\right) d\rho_1.
\end{aligned}$$

Under Assumption 1, this expected payoff is strictly concave in x_R. Solving the first-order condition yields:

$$\hat{x}_R^{\text{int}}(x_L, \rho_0) = \frac{-\alpha(\theta - \rho_0 + \psi) + \beta(\rho_0 + \psi) + 2\theta r + \gamma x_L(\beta - \alpha)}{\beta\gamma - \alpha(\gamma + 2\theta)}, \tag{16}$$

which increases in x_L. Setting $x_L = 0$, we find that

$$\hat{x}_R^{\text{int}}(x_L) = \frac{-\alpha(\theta - \rho_0 + \psi) + \beta(\rho_0 + \psi) + 2\theta r}{\beta\gamma - \alpha(\gamma + 2\theta)},$$

which is strictly negative for all $\rho_0 \geq 0$, under Assumption 1. This contradicts the supposition that $x_R > 0$ is a best response.

Profile 2: $0 \leq x_L \leq x_R$, with at least one strict inequality. Under this profile, party R wins if and only if ${x_3}^* \leq 0$, which happens if and only if $\rho_1 \geq \gamma(x_R - x_L) - \rho_0$ where the marginal voter is given as in Profile 1. Party R's expected payoff is therefore:

$$\begin{aligned}
\pi_R(x_L, x_R) \; &= \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R)-\rho_0-\theta x_R} \left(\frac{\alpha}{2} - \alpha\frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{2\theta}\right) d\rho_1 \\
&+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R)-\rho_0-\theta x_R}^{\gamma(x_R - x_L)-\rho_0-\theta x_L} \left(\frac{\alpha}{2} - \alpha\frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(2\gamma + \theta)}\right) d\rho_1
\end{aligned}$$

$$+ \frac{1}{2\psi} \int_{\gamma(x_R - x_L) - \rho_0 - \theta x_L}^{\gamma(x_R - x_L) - \rho_0} \left(\frac{\alpha}{2} - \alpha \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{2\theta} \right) d\rho_1$$
$$+ \frac{1}{2\psi} \int_{\gamma(x_R - x_L) - \rho_0}^{\psi} \left(r - \beta \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{2\theta} \right) d\rho_1. \tag{17}$$

Under Assumption 1, this expected payoff is strictly concave in x_R. This objective admits a (unique) interior solution, $\hat{x}_R(x_L; \rho_0)$, satisfying:

$$\hat{x}_R(x_L; \rho_0) = \frac{(\alpha + \beta)\rho_0 - (\alpha - \beta)(\psi + \gamma x_L) + \theta(2r - \alpha)}{\beta\gamma - \alpha(\gamma + 2\theta)}. \tag{18}$$

This implies that $\hat{x}_R(x_L; \rho_0) - x_L$ strictly decreases in x_L and in ρ_0. It is easy to verify that $\hat{x}_R(0; 0) - x_L < 0$, contradicting the supposition $x_R > x_L$. The supposition $0 \leq x_L \leq x_R$ with at least one strict inequality then implies $0 < x_L = x_R$. Consider, therefore, party R's expected payoff from a choice of $x_R \in [0, x_L]$:

$$\pi_R(x_L, x_R) = \frac{1}{2\psi} \int_{-\psi}^{\gamma(x_L - x_R) - \rho_0 - \theta x_L} \left(\frac{\alpha}{2} - \alpha \left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta} \right) \right) d\rho_1$$
$$+ \frac{1}{2\psi} \int_{\gamma(x_L - x_R) - \rho_0 - \theta x_L}^{\gamma(x_R - x_L) - \rho_0 - \theta x_R} \left(\frac{\alpha}{2} - \alpha \left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)} \right) \right) d\rho_1$$
$$+ \frac{1}{2\psi} \int_{\gamma(x_R - x_L) - \rho_0 - \theta x_R}^{\gamma(x_R - x_L) - \rho_0} \left(\frac{\alpha}{2} - \alpha \left(\frac{(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta} \right) \right) d\rho_1$$
$$+ \frac{1}{2\psi} \int_{\gamma(x_R - x_L) - \rho_0}^{\psi} \left(r - \beta \left(\frac{(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta} \right) \right) d\rho_1. \tag{19}$$

Under Assumption 1, this expected payoff is strictly concave. Solving the first-order condition yields:

$$\hat{x}_R^{\text{int}} = \frac{-\alpha(\theta - \rho_0 + \psi) + \beta(\rho_0 + \psi) + 2\theta r + \gamma x_L(\beta - \alpha)}{\beta\gamma - \alpha(\gamma + 2\theta)},$$

which satisfies $\hat{x}_R^{\text{int}} < x_L$, contradicting the supposition that $0 < x_R = x_L$ is an equilibrium profile.

Profile 3: $x_R \leq 0 < x_L$. There are three possible locations for the marginal voter:

Location 1: ${x_i}^* \leq x_R$, i.e., $\gamma(x_R - x_L) - \gamma(x_R - x_R) - \rho_0 - \rho_1 - \theta x_R \leq 0$, i.e., $\rho_1 \geq \gamma(x_R - x_L) - \rho_0 - \theta x_R$:

$$\gamma(x^* - x_L) - \gamma(x^* - x_R) - \rho_0 - \rho_1 - \theta x^* = 0 \iff x^* = \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta} \equiv {x_1}^*. \tag{20}$$

Location 2: ${x_i}^* \in (x_R, x_L)$, i.e., $\gamma(x_R - x_L) - \gamma(x_R - x_R) - \rho_0 - \rho_1 - \theta x_R > 0$ and $\gamma(x_L - x_L) - \gamma(x_R -$

$x_L) - \rho_0 - \rho_1 - \theta x_L < 0$, i.e., $\gamma(x_R - x_L) - \rho_0 - \theta x_R > \rho_1 > \gamma(x_L - x_R) - \rho_0 - \theta x_L$. This implies:

$$\gamma(x^* - x_L) - \gamma(x_R - x^*) - \theta x^* - \rho_0 - \rho_1 = 0 \iff x^* = \frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{\theta - 2\gamma} \equiv {x_2}^*. \tag{21}$$

<u>*Location 3*</u>: ${x_i}^* \geq x_L$, i.e., $\gamma(x_L - x_L) - \gamma(x_R - x_L) - \rho_0 - \rho_1 - \theta x_L > 0$, i.e., $\rho_1 < \gamma(x_L - x_R) - \rho_0 - \theta x_L$. This implies:

$$\gamma(x_L - x^*) - \gamma(x_R - x^*) - \theta x^* - \rho_0 - \rho_1 = 0 \iff x^* = \frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{\theta} \equiv {x_3}^*. \tag{22}$$

Party R wins if and only if ${x_2}^* \leq 0$, i.e.,

$$\rho_1 \geq -\gamma(x_L + x_R) - \rho_0.$$

Party R's expected payoff from $x_R \leq 0$ is therefore:

$$\begin{aligned}
\pi_R(x_L, x_R) = \; & \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R) - \rho_0 - \theta x_L} \left(\frac{\alpha}{2} - \alpha\left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \rho_0 - \theta x_L}^{-\gamma(x_L + x_R) - \rho_0} \left(\frac{\alpha}{2} - \alpha\left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{-\gamma(x_L + x_R) - \rho_0}^{\gamma(x_R - x_L) - \rho_0 - \theta x_R} \left(r - \beta\left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_R - x_L) - \rho_0 - \theta x_R}^{\psi} \left(r - \beta\left(\frac{(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1.
\end{aligned} \tag{23}$$

Under Assumption 1, this expected payoff is strictly concave in x_R. Solving the first-order condition yields:

$$\hat{x}_R^{\text{int}}(x_L) = \frac{-\alpha(\theta + \rho_0 - \psi) - \beta(\rho_0 + \psi) + 2\theta r + \gamma x_L(\alpha - \beta)}{\alpha\gamma - \beta\gamma + 2\beta\theta}.$$

Similarly, party L's expected payoff from $x_L \geq 0$ is:

$$\begin{aligned}
\pi_L(x_L, x_R) = \; & \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R) - \rho_0 - \theta x_L} \left(r + \beta\left(\frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{2\theta}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \rho_0 - \theta x_L}^{-\gamma(x_L + x_R) - \rho_0} \left(r + \beta\left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right) d\rho_1
\end{aligned}$$

$$+\frac{1}{2\psi}\int_{-\gamma(x_L+x_R)-\rho_0}^{\gamma(x_R-x_L)-\rho_0-\theta x_R}\left(\frac{\alpha}{2}+\alpha\left(\frac{-\gamma(x_L+x_R)-\rho_0-\rho_1}{2(\theta-2\gamma)}\right)\right)d\rho_1$$
$$+\frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_R}^{\psi}\left(\frac{\alpha}{2}+\alpha\left(\frac{(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1. \tag{24}$$

Under Assumption 1, this expected payoff is strictly concave in x_L. Solving the first-order condition yields:

$$\hat{x}_L^{\text{int}}(x_R)=\frac{\alpha(\theta-\rho_0-\psi)+\beta(\psi-\rho_0)-2\theta r+\gamma x_R(\alpha-\beta)}{\alpha\gamma-\beta\gamma+2\beta\theta}.$$

Solving these interior best responses, we obtain a pair (x_L^*, x_R^*) satisfying, under Assumption 1:

$$x_R^*-x_L^*=\frac{-\alpha\theta+\alpha\psi-\beta\psi+2\theta r}{\alpha\gamma-\beta\gamma+\beta\theta}>0.$$

Thus, there does not exist an equilibrium in which $x_R < 0 < x_L$.

Suppose, instead, $x_R = 0 < x_L$. Since $\hat{x}_L^{\text{int}}(x_R)$ strictly decreases in ρ_0 for all x_R, we may set $\rho_0 = 0$ and obtain:

$$\hat{x}_L^{\text{int}}(0)=\frac{\alpha(\theta-\psi)+\beta\psi-2\theta r}{\alpha\gamma-\beta\gamma+2\beta\theta}<0,$$

which contradicts $x_L > 0$.

Profile 4: $x_R < x_L \leq 0$. The three possible locations of the marginal voter are given in expressions (20) through (22). In contrast with Profile 3, however, Party R wins under this profile if and only if $x_3{}^* \leq 0$, i.e.,

$$\rho_1 \geq \gamma(x_L-x_R)-\rho_0.$$

Party R's expected payoff is:

$$\pi_L(x_L,x_R)=\frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\rho_0}\left(\frac{\alpha}{2}-\alpha\left(\frac{(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1$$
$$+\frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0}^{\gamma(x_L-x_R)-\rho_0-\theta x_L}\left(r-\beta\left(\frac{(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1$$
$$+\frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_L}^{\gamma(x_R-x_L)-\rho_0-\theta x_R}\left(r-\beta\left(\frac{-\gamma(x_L+x_R)-\rho_0-\rho_1}{2(\theta-2\gamma)}\right)\right)d\rho_1$$
$$+\frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_R}^{\psi}\left(r-\beta\left(\frac{(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1. \tag{25}$$

Under Assumption 1, this expected payoff is strictly concave in x_R. Solving the first-order condition yields:

$$\hat{x}_R^{\text{int}}(x_L) = \frac{-\alpha(\theta + \rho_0 - \psi) - \beta(\rho_0 + \psi) + 2\theta r + \gamma x_L(\alpha - \beta)}{\alpha\gamma - \beta\gamma + 2\beta\theta}.$$

Similarly, party L's expected payoff is:

$$\begin{aligned}\pi_L(x_L, x_R) = & \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R)-\rho_0} \left(r + \beta\left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1 \\ & + \frac{1}{2\psi}\int_{\gamma(x_L - x_R)-\rho_0}^{\gamma(x_L - x_R)-\rho_0-\theta x_L} \left(\frac{\alpha}{2} + \alpha\left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1 \\ & + \frac{1}{2\psi}\int_{\gamma(x_L - x_R)-\rho_0-\theta x_L}^{\gamma(x_R - x_L)-\rho_0-\theta x_R} \left(\frac{\alpha}{2} + \alpha\left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right) d\rho_1 \\ & + \frac{1}{2\psi}\int_{\gamma(x_R - x_L)-\rho_0-\theta x_R}^{\psi} \left(\frac{\alpha}{2} + \alpha\left(\frac{(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1. \end{aligned} \tag{26}$$

Under Assumption 1, this expected payoff is strictly concave in x_L. Solving the first-order condition yields:

$$\hat{x}_L^{\text{int}}(x_R) = \frac{\alpha(\theta + \rho_0 + \psi) + \beta(\rho_0 - \psi) - 2\theta r + \gamma x_R(\beta - \alpha)}{\beta\gamma - \alpha(\gamma + 2\theta)}.$$

Suppose, first, that $x_R < x_L < 0$. Solving the interior best responses simultaneously yields a pair (x_L^*, x_R^*) satisfying:

$$x_L^* - x_R^* = \frac{(\alpha - \beta)(\alpha(\theta + \rho_0 - \psi) + \beta(\rho_0 - \psi) - 2\theta r)}{\alpha^2\gamma + 2\alpha\beta\theta - \beta^2\gamma},$$

which strictly increases in ρ_0. Straightforward algebra establishes:

$$x_L^* - x_R^* \geq 0 \iff \rho_0 \geq \psi + \frac{\theta(2r - \alpha)}{\alpha + \beta}, \tag{27}$$

which violates our assumption that $\rho_0 \in [-\psi, \psi]$. Suppose, instead, that $x_R < x_L = 0$. We find that $x_R^{\text{int}}(0)$ strictly decreases in ρ_0, and moreover that:

$$x_R^{\text{int}}(0) \leq 0 \iff \rho_0 \geq \frac{-\alpha\theta + \alpha\psi - \beta\psi + 2\theta r}{\alpha + \beta} \equiv \hat{\rho}_0. \tag{28}$$

Straightforward algebra reveals that

$$\alpha \geq \beta \Rightarrow \left.\frac{\partial \pi_L(x_L, x_R)}{\partial x_L}\right|_{x_L=0, x_R=\hat{x}_R(0), \rho_0=\hat{\rho}_0} < 0, \tag{29}$$

which implies that a deviation by party L to a platform $x'_L < 0$ is profitable.

Profile 5: $0 < x_R < x_L$. The three possible locations of the marginal voter x^* are given in expressions (20) through (22). In contrast with Profiles 3 and 4, however, Party R wins under this profile if and only if ${x_1}^* \leq 0$, i.e.,

$$\rho_1 \geq \gamma(x_R - x_L) - \rho_0.$$

Party R's expected payoff is:

$$\begin{aligned}
\pi_R(x_L, x_R) = & \frac{1}{2\psi} \int_{-\psi}^{\gamma(x_L - x_R) - \rho_0 - \theta x_L} \left(\frac{\alpha}{2} - \alpha \left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta} \right) \right) d\rho_1 \\
& + \frac{1}{2\psi} \int_{\gamma(x_L - x_R) - \rho_0 - \theta x_L}^{\gamma(x_R - x_L) - \rho_0 - \theta x_R} \left(\frac{\alpha}{2} - \alpha \left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)} \right) \right) d\rho_1 \\
& + \frac{1}{2\psi} \int_{\gamma(x_R - x_L) - \rho_0 - \theta x_R}^{\gamma(x_R - x_L) - \rho_0} \left(\frac{\alpha}{2} - \alpha \left(\frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{2\theta} \right) \right) d\rho_1 \\
& + \frac{1}{2\psi} \int_{\gamma(x_R - x_L) - \rho_0}^{\psi} \left(r - \beta \left(\frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{2\theta} \right) \right) d\rho_1.
\end{aligned} \tag{30}$$

Under Assumption 1, this expected payoff is strictly concave in x_R. Solving the first-order condition yields:

$$\hat{x}_R^{\text{int}}(x_L) = \frac{(2r - \alpha)\theta - x_L(\alpha - \beta)\gamma + \rho_0(\alpha + \beta) - (\alpha - \beta)\psi}{-((\alpha - \beta)\gamma + 2\alpha\theta)}.$$

Similarly, party L's expected payoff is:

$$\begin{aligned}
\pi_L(x_L, x_R) = & \frac{1}{2\psi} \int_{-\psi}^{\gamma(x_L - x_R) - \rho_0 - \theta x_L} \left(r + \beta \left(\frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{2\theta} \right) \right) d\rho_1 \\
& + \frac{1}{2\psi} \int_{\gamma(x_L - x_R) - \rho_0 - \theta x_L}^{\gamma(x_R - x_L) - \rho_0 - \theta x_R} \left(r + \beta \left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)} \right) \right) d\rho_1 \\
& + \frac{1}{2\psi} \int_{\gamma(x_R - x_L) - \rho_0 - \theta x_R}^{\gamma(x_R - x_L) - \rho_0} \left(r + \beta \left(\frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{2\theta} \right) \right) d\rho_1 \\
& + \frac{1}{2\psi} \int_{\gamma(x_R - x_L) - \rho_0}^{\psi} \left(\frac{\alpha}{2} + \alpha \left(\frac{(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta} \right) \right) d\rho_1.
\end{aligned} \tag{31}$$

Under Assumption 1, this expected payoff is strictly concave in x_R. Solving the first-order condition yields:

$$\hat{x}_L^{\text{int}}(x_R) = \frac{(2r-\alpha)\theta - x_R(\alpha-\beta)\gamma + \rho_0(\alpha+\beta) - (\alpha-\beta)\psi}{-((\alpha-\beta)\gamma + 2\beta\theta)}.$$

Solving the interior best responses, simultaneously, yields a pair (x_L^*, x_R^*) which, under Assumption 1, satisfy:

$$x_L^* - x_R^* = -\frac{(\alpha-\beta)(\alpha(\rho_0+\psi) + \beta(\rho_0+\psi) + \theta(2r-\alpha))}{\alpha^2\gamma + 2\alpha\beta\theta - \beta^2\gamma} < 0,$$

which contradicts $x_R < x_L$. □

Existence of equilibrium. We now verify that there exists an equilibrium in which $x_L \leq x_R \leq 0$, and moreover that it is unique under Assumptions 1, 2 and 3.

There are 3 possible locations for the marginal voter.

Location 1: $x_i{}^* \geq x_R$, i.e. $\rho_1 \leq \gamma(x_L - x_R) - \rho_0 - \theta x_R$:

$$x^* = \frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{\theta} = x_3{}^*. \tag{32}$$

Location 2: $x_L \leq x_i{}^* \leq x_R$, i.e. $\gamma(x_L - x_R) - \rho_0 - \theta x_R \leq \rho_1 \leq \gamma(x_R - x_L) - \rho_0 - \theta x_L$:

$$x^* = \frac{\gamma(x_L + x_R) - \rho_0 - \rho_1}{2\gamma + \theta} = x_2{}^*. \tag{33}$$

Location 3: $x_i{}^* \leq x_L$, i.e. $\rho_1 \geq \gamma(x_R - x_L) - \rho_0 - \theta x_L$:

$$x^* = \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta} \equiv x_1{}^*. \tag{34}$$

Party R wins if and only if $x_3{}^* \leq 0$, i.e.,

$$\rho_1 \geq \gamma(x_L - x_R) - \rho_0.$$

Party R's expected payoff from $x_R \in [x_L, 0]$ is therefore:

$$\pi_R(x_L, x_R) \ = \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R) - \rho_0} \left(\frac{\alpha}{2} - \alpha\left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1$$

$$
\begin{aligned}
&+\frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0}^{\gamma(x_L-x_R)-\rho_0-\theta x_R}\left(r\ -\beta\left(\frac{(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1\\
&+\frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_R}^{\gamma(x_R-x_L)-\rho_0-\theta x_L}\left(r\ -\beta\left(\frac{\gamma(x_L+x_R)-\rho_0-\rho_1}{2(2\gamma+\theta)}\right)\right)d\rho_1\\
&+\frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_L}^{\psi}\left(r\ -\beta\left(\frac{(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1. \qquad (35)
\end{aligned}
$$

Under Assumption 1, this expected payoff is strictly concave in x_R. Solving the first-order condition yields:

$$
\hat{x}_R^{\text{int}}(x_L) = \frac{-\alpha(\theta+\rho_0-\psi)-\beta(\rho_0+\psi)+2\theta r+\gamma x_L(\alpha-\beta)}{\alpha\gamma-\beta\gamma+2\beta\theta}. \qquad (36)
$$

Similarly, party L's expected payoff from $x_L \le x_R$ is:

$$
\begin{aligned}
\pi_L(x_L,x_R)\ &= \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L-x_R)-\rho_0}\left(r\ +\beta\left(\frac{(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1\\
&+\frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0}^{\gamma(x_L-x_R)-\rho_0-\theta x_R}\left(\frac{\alpha}{2}+\alpha\left(\frac{(\gamma(x_L-x_R)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1\\
&+\frac{1}{2\psi}\int_{\gamma(x_L-x_R)-\rho_0-\theta x_R}^{\gamma(x_R-x_L)-\rho_0-\theta x_L}\left(\frac{\alpha}{2}+\alpha\left(\frac{\gamma(x_L+x_R)-\rho_0-\rho_1}{2(2\gamma+\theta)}\right)\right)d\rho_1\\
&+\frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_L}^{\psi}\left(\frac{\alpha}{2}+\alpha\left(\frac{(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1. \qquad (37)
\end{aligned}
$$

Under Assumption 1, this expected payoff is strictly concave in x_L. Solving the first-order condition yields:

$$
\hat{x}_L^{\text{int}}(x_R) = \frac{\alpha(\theta+\rho_0+\psi)+\beta(\rho_0-\psi)-2\theta r+\gamma x_R(\beta-\alpha)}{\beta\gamma-\alpha(\gamma+2\theta)}. \qquad (38)
$$

Let (x_L^*, x_R^*) denote a pair that satisfies $x_L^* \le x_R^* \le 0$ and that solves the system of best responses (36) and (38).

First, we identify conditions for $x_L^* = x_R^* = 0$. We observe that $\hat{x}_L^{\text{int}}(0)$ strictly decreases in ρ_0, and also that $\hat{x}_R^{\text{int}}(0)$ strictly decreases in ρ_0. We find that:

$$
\hat{x}_L^{\text{int}}(0) \ge 0 \iff \rho_0 \le \frac{-\psi(\alpha-\beta)-\alpha\theta+2\theta r}{\alpha+\beta} \equiv \underline{\rho}_0 \qquad (39)
$$

and

$$
\hat{x}_R^{\text{int}}(0) \ge 0 \iff \rho_0 \le \frac{\psi(\alpha-\beta)-\alpha\theta+2\theta r}{\alpha+\beta} = \rho_0', \qquad (40)
$$

where Assumption 1 that $\alpha \geq \beta$ implies that $\rho_0' \geq \underline{\rho}_0$. Thus, $x_L^* = x_R^* = 0$ if $\rho_0 \leq \underline{\rho}_0$.

Second, we identify conditions for $x_L^* < x_R^* = 0$. In that case, we have

$$x_L^* = \hat{x}_L^{\text{int}}(0) = \frac{\alpha(\theta + \rho_0 + \psi) + \beta(\rho_0 - \psi) - 2\theta r}{\beta\gamma - \alpha(\gamma + 2\theta)}, \tag{41}$$

and further require that $\hat{x}_R^{\text{int}}(\hat{x}_L^{\text{int}}(0)) \geq 0$. Tedious but straightforward algebra establishes that

$$\hat{x}_L^{\text{int}}(0) < 0 \iff \rho_0 > \underline{\rho}_0, \tag{42}$$

and

$$\hat{x}_R^{\text{int}}(\hat{x}_L^{\text{int}}(0)) \geq 0 \iff \rho_0 \leq \frac{\theta\left(\alpha\left(\frac{\psi(\alpha-\beta)}{\alpha(\gamma+\theta)-\beta\gamma} - 1\right) + 2r\right)}{\alpha + \beta} \equiv \overline{\rho}_0. \tag{43}$$

Therefore, $x_L^* < x_R^* = 0$ if $\rho \in (\underline{\rho}_0, \overline{\rho}_0]$.

Third, we identify conditions for $x_L^* < x_R^* < 0$. In that case, we may solve the system of interior solutions, directly, to obtain:

$$\begin{aligned} x_L^* &= \frac{\beta\theta\psi(\beta - \alpha) - (\alpha\gamma + \beta(\theta - \gamma))(\alpha(\theta + \rho_0) + \beta\rho_0 - 2\theta r)}{\theta\left(\alpha^2\gamma + 2\alpha\beta\theta - \beta^2\gamma\right)}, \\ x_R^* &= x_L^* + \frac{(\alpha - \beta)(-\alpha(\theta + \rho_0 - \psi) + \beta(\psi - \rho_0) + 2\theta r)}{\alpha^2\gamma + 2\alpha\beta\theta - \beta^2\gamma}, \end{aligned} \tag{44}$$

where it is easily verified that indeed $x_L^* < x_R^* < 0$ for all $\rho_0 > \overline{\rho}_0$.

We now verify that for all $\rho_0 \geq 0$, the solution (x_L^*, x_R^*) is an equilibrium. To establish this, it is necessary and sufficient to verify that there are no profitable deviations for party L to an alternative platform $x_L > x_R^*$, and no profitable deviations for party R to an alternative platform $x_R < x_L^*$.

No profitable deviation by party L to $x_L' > x_R^*$. Consider a deviation by party L to a platform $x_L' \in (x_R^*, 0]$. This implies $\rho_0 > \overline{\rho}_0$. For any pair (x_L, x_R) satisfying $x_R < x_L$, there are three possible locations for the marginal voter.

<u>Location 1</u>: ${x_i}^* \leq x_R$, i.e., $\rho_1 \geq \gamma(x_R - x_L) - \rho_0 - \theta x_R$.

$$\gamma(x^* - x_L) - \gamma(x^* - x_R) - \rho_0 - \rho_1 - \theta x^* = 0 \iff x^* = \frac{\gamma(x_R - x_L) - \rho_0 - \rho_1}{\theta} \equiv {x_1}^*. \tag{45}$$

$\underline{Location\ 2}$: $x_i{}^* \in (x_R, x_L)$, i.e., $\gamma(x_R - x_L) - \rho_0 - \theta x_R > \rho_1 > \gamma(x_L - x_R) - \rho_0 - \theta x_L$. This implies:

$$\gamma(x^* - x_L) - \gamma(x_R - x^*) - \theta x^* - \rho_0 - \rho_1 = 0 \iff x^* = \frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{\theta - 2\gamma} \equiv x_2{}^*. \tag{46}$$

$\underline{Location\ 3}$: $x_i{}^* \geq x_L$, i.e., $\rho_1 < \gamma(x_L - x_R) - \rho_0 - \theta x_L$. This implies:

$$\gamma(x_L - x^*) - \gamma(x_R - x^*) - \theta x^* - \rho_0 - \rho_1 = 0 \iff x^* = \frac{\gamma(x_L - x_R) - \rho_0 - \rho_1}{\theta} \equiv x_3{}^*. \tag{47}$$

If party L locates at a platform $x_L \in (x_R^*, 0]$, it wins if and only if $x_3{}^* \geq 0$, which occurs if and only if $\rho_1 < \gamma(x_L - x_R^*) - \rho_0$. Party L's expected payoff from this deviation is then:

$$\begin{aligned}\pi_L(x_L, x_R) = &\frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R)-\rho_0} \left(r + \beta\left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1 \\ &+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R)-\rho_0}^{\gamma(x_L - x_R)-\rho_0-\theta x_L} \left(\frac{\alpha}{2} + \alpha\left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1 \\ &+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R)-\rho_0-\theta x_L}^{\gamma(x_R - x_L)-\rho_0-\theta x_R} \left(\frac{\alpha}{2} + \alpha\left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right)d\rho_1 \\ &+ \frac{1}{2\psi}\int_{\gamma(x_R - x_L)-\rho_0-\theta x_R}^{\psi} \left(\frac{\alpha}{2} + \alpha\left(\frac{(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1.\end{aligned} \tag{48}$$

Under Assumption 1, party L's expected payoff is strictly concave in x_L. This yields a first-order condition that is equivalent to the first-order condition identified in expression (38), which is therefore strictly negative evaluated at any $x_L > x_R^*$ when $\rho_0 > \overline{\rho}_0$.

Consider, instead, a deviation by party L to $x_L > 0$. The possible locations of the marginal voter are given in expressions (45) through (47). Moreover, party L wins if and only if $x_2{}^* \geq 0$. Party L's expected payoff from $x_L \geq 0$ is:

$$\begin{aligned}\pi_L(x_L, x_R^*) = &\frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R)-\rho_0-\theta x_L} \left(r\ + \beta\left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1 \\ &+ \frac{1}{2\psi}\int_{\gamma(x_L - x_R)-\rho_0-\theta x_L}^{-\gamma(x_L + x_R)-\rho_0} \left(r\ + \beta\left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right))d\rho_1 \\ &+ \frac{1}{2\psi}\int_{-\gamma(x_L + x_R)-\rho_0}^{\gamma(x_R - x_L)-\rho_0-\theta x_R} \left(\frac{\alpha}{2} + \alpha\left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right)d\rho_1 \\ &+ \frac{1}{2\psi}\int_{\gamma(x_R - x_L)-\rho_0-\theta x_R}^{\psi} \left(\frac{\alpha}{2} + \alpha\left(\frac{(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right)d\rho_1.\end{aligned} \tag{49}$$

Under Assumption 1, party L's expected payoff is strictly concave in x_L. Solving the first-order condition yields:

$$x_L'(x_R) = \frac{\alpha\theta - \alpha\rho_0 - \alpha\psi - \beta\rho_0 + \beta\psi - 2\theta r + \gamma x_R(\alpha - \beta)}{\alpha\gamma - \beta\gamma + 2\beta\theta}, \tag{50}$$

which strictly increases in x_R. Recalling that $x_R^* \leq 0$, straightforward algebra yields:

$$x_L'(0) = -\frac{\alpha(-\theta + \rho_0 + \psi) + \beta(\rho_0 - \psi) + 2\theta r}{\alpha\gamma - \beta\gamma + 2\beta\theta} < 0,$$

which establishes that a deviation by party L to $x_L' > 0$ is not profitable, for any $x_R^* \leq 0$.

No profitable deviation by party R to $x_R' > 0$ or $x_R' < x_L^*$. Consider a deviation by party R to a platform $x_R < x_L$. The locations of the marginal voter are given in expressions (45) through (47). In this case, party R wins if and only if ${x_3}^* \leq 0$. Party R's expected payoff is:

$$\begin{aligned}
\pi_R(x_L, x_R) = & \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R) - \rho_0} \left(\frac{\alpha}{2} - \alpha\left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \rho_0}^{\gamma(x_L - x_R) - \rho_0 - \theta x_L} \left(r - \beta\left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \rho_0 - \theta x_L}^{\gamma(x_R - x_L) - \rho_0 - \theta x_R} \left(r - \beta\left(\frac{-\gamma(x_L + x_R) - \rho_0 - \rho_1}{2(\theta - 2\gamma)}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_R - x_L) - \rho_0 - \theta x_R}^{\psi} \left(r - \beta\left(\frac{(\gamma(x_R - x_L) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1.
\end{aligned} \tag{51}$$

Under Assumption 1, party R's expected payoff is strictly concave in x_R. This yields a first-order condition that is equivalent to the first-order condition identified in expression (36), and which therefore implies that a deviation $x_R' < x_L^*$ cannot be profitable.

Consider, instead, a deviation by party R to a platform $x_R > 0$. The locations of the marginal voter are given in expressions (32) through (34). In this case, party R wins if and only if ${x_2}^* \leq 0$. Party R's expected payoff is:

$$\begin{aligned}
\pi_R(x_L, x_R) = & \frac{1}{2\psi}\int_{-\psi}^{\gamma(x_L - x_R) - \rho_0 - \theta x_R} \left(\frac{\alpha}{2} - \alpha\left(\frac{(\gamma(x_L - x_R) - \rho_0 - \rho_1)}{2\theta}\right)\right) d\rho_1 \\
& + \frac{1}{2\psi}\int_{\gamma(x_L - x_R) - \rho_0 - \theta x_R}^{\gamma(x_L + x_R) - \rho_0} \left(\frac{\alpha}{2} - \alpha\left(\frac{(\gamma(x_L + x_R) - \rho_0 - \rho_1)}{2(\theta + 2\gamma)}\right)\right) d\rho_1
\end{aligned}$$

$$+\frac{1}{2\psi}\int_{\gamma(x_L+x_R)-\rho_0}^{\gamma(x_R-x_L)-\rho_0-\theta x_L}\left(r-\beta\left(\frac{\gamma(x_L+x_R)-\rho_0-\rho_1}{2(2\gamma+\theta)}\right)\right)d\rho_1$$
$$+\frac{1}{2\psi}\int_{\gamma(x_R-x_L)-\rho_0-\theta x_L}^{\psi}\left(r-\beta\left(\frac{(\gamma(x_R-x_L)-\rho_0-\rho_1)}{2\theta}\right)\right)d\rho_1. \tag{52}$$

Under Assumption 1, party R's expected payoff is strictly concave in x_R. Solving the first-order condition yields:

$$\hat{x}'_R(x_L)=\frac{-\alpha(\theta-\rho_0+\psi)+\beta(\rho_0+\psi)+2\theta r+\gamma x_L(\beta-\alpha)}{\beta\gamma-\alpha(\gamma+2\theta)}, \tag{53}$$

which strictly increases in $x_L \leq 0$. Straightforward algebra establishes:

$$\hat{x}'_R(0)=\frac{-\alpha(\theta-\rho_0+\psi)+\beta(\rho_0+\psi)+2\theta r}{\beta\gamma-\alpha(\gamma+2\theta)}<0, \tag{54}$$

which establishes that a deviation to $x_R > 0$ is not profitable. □

Proof of Corollary 1. In this case, we have $x^*_R(\rho_0) = 0$, so that

$$x^*_L(\rho_0)=\frac{\alpha(\theta+\rho_0+\psi)+\beta(\rho_0-\psi)-2\theta r}{\beta\gamma-\alpha(\gamma+2\theta)}. \tag{55}$$

We obtain comparative statics for each of the primitives, in turn.

Higher ρ_0. We have $\frac{\partial x^*_L}{\rho_0}=\frac{\alpha+\beta}{\beta\gamma-\alpha(\gamma+2\theta)}<0$. Thus, x^*_L *decreases* in ρ_0.

Higher θ. We have

$$\frac{\partial x^*_L}{\partial\theta}=\frac{\alpha(-\alpha\gamma+2\alpha(\rho_0+\psi)+\beta(\gamma+2\rho_0-2\psi))+2\gamma r(\alpha-\beta)}{(\beta\gamma-\alpha(\gamma+2\theta))^2}. \tag{56}$$

The numerator of this expression strictly increases in ρ_0, and is therefore positive if and only if $\rho_0 \geq \frac{(\alpha-\beta)(\alpha(\gamma-2\psi)-2\gamma r)}{2\alpha(\alpha+\beta)}$. This threshold is strictly negative (and thus vacuously satisfied) if $r > \frac{\alpha}{2}-\frac{\alpha\psi}{\gamma}$, which holds. We conclude that x^*_L *increases* in θ.

Higher α. We have

$$\frac{\partial x^*_L}{\partial\alpha}=\frac{\beta\gamma(\theta+2\rho_0)+2\beta\theta(\rho_0-\psi)-2\theta r(\gamma+2\theta)}{(\beta\gamma-\alpha(\gamma+2\theta))^2}. \tag{57}$$

Calling $\nu(\rho_0)$ the numerator of this expression, we find that $\nu(\rho_0)$ strictly increases in ρ_0, and that $\nu(\overline{\rho}_0) < 0$. Thus, x^*_L strictly *decreases* in α.

Higher γ. We have

$$\frac{\partial x_L^*}{\partial \gamma} = \frac{(\alpha-\beta)(\alpha(\theta+\rho_0+\psi)+\beta(\rho_0-\psi)-2\theta r)}{(\beta\gamma-\alpha(\gamma+2\theta))^2}. \tag{58}$$

The numerator of this expression strictly increases in ρ_0, and is weakly positive evaluated at $\rho_0 = \underline{\rho}_0$. Therefore, x_L^* strictly *increases* in γ.

Higher ψ. $\frac{\partial x_L^*}{\partial \psi} = \frac{\beta-\alpha}{\alpha(\gamma+2\theta)-\beta\gamma} < 0$.

Higher r. We have $\frac{\partial x_L^*}{\partial r} = -\frac{2\theta}{\beta\gamma-\alpha(\gamma+2\theta)} > 0$. $\square$

Proof of Corollaries 2, 3 and 4 and 5.

$$\begin{aligned} x_L^*(\rho_0) = &\ \frac{\beta\theta\psi(\beta-\alpha)-(\alpha\gamma+\beta(\theta-\gamma))(\alpha(\theta+\rho_0)+\beta\rho_0-2\theta r)}{\theta\left(\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma\right)}. \\ x_R^*(\rho_0) = &\ x_L^*(\rho_0) + \frac{(\alpha-\beta)((\alpha+\beta)(\psi-\rho_0)+\theta(2r-\alpha))}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma}. \end{aligned} \tag{59}$$

We obtain comparative statics for each of the primitives, in turn.

Higher ρ_0. We find that $\frac{\partial x_L^*}{\partial \rho_0} = \frac{\beta(\alpha-\beta)}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma} - \frac{1}{\theta}$, which is strictly negative if and only if $\theta > \frac{\beta\gamma-\alpha\gamma}{\beta}$, which holds. Moreover, $\frac{\partial[x_L^*-x_R^*]}{\partial\rho_0} = \frac{\beta^2-\alpha^2}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma} < 0$, which implies that x_R^* also decreases in ρ_0, and faster than x_L^*. $\square$

Higher α. We start with the platform x_L^*. We find that $\frac{\partial x_L^*}{\partial \alpha}$ can be written as a quotient with a strictly positive denominator, and a numerator that we call $\nu(r,\psi)$, which strictly decreases in r. Assumption 1, $r > \frac{1}{2}\left(\alpha+\frac{\psi}{\theta}(\alpha-\beta)\right)$, yields:

$$\left.\frac{\partial\nu(r,\psi)}{\partial\psi}\right|_{r=\frac{1}{2}\left(\alpha+\frac{\psi}{\theta}(\alpha-\beta)\right)} = -2\alpha\beta^2\theta - \frac{\gamma^2(\alpha-\beta)^3}{\theta} - \beta\gamma(\alpha-\beta)^2 < 0. \quad \square \tag{60}$$

Since $\psi > \rho_0$, it is then sufficient to observe that:

$$\begin{aligned} \nu\left(\frac{1}{2}\left(\alpha+\frac{\psi}{\theta}(\alpha-\beta)\right),\rho_0\right) = &- \left(\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma\right)(\alpha\gamma+\beta(\theta-\gamma)) \\ &- \frac{\rho_0(\alpha-\beta)\left(\gamma^2(\alpha-\beta)^2+2\beta\gamma\theta(\alpha-\beta)+2\beta^2\theta^2\right)}{\theta}, \end{aligned} \tag{61}$$

which is strictly negative under Assumptions 1 and 2. Thus, $\frac{\partial x_L^*}{\partial\alpha} < 0$.

We next consider the platform x_R^*. We find that $\frac{\partial x_R^*}{\partial\alpha}$ can be written as a quotient with a strictly positive denominator, and a numerator that we call $\mu(\rho_0,\psi)$, which strictly decreases in ρ_0, and that there exists $\hat{\rho}_0$ such that $\mu(\rho_0, \psi) \geq 0$ if and only if $\rho_0 \leq \hat{\rho}_0$. Thus, $\rho > \hat{\rho}_0$ implies that x_R^* decreases in α, while $\rho > \hat{\rho}_0$ implies that x_R^* increases in α, and that $\hat{\rho}_0$ strictly *decreases* in α. It is straightforward to verify that there are parameter configurations for which $\hat{\rho}_0 > \overline{\rho}_0$. $\square$

Higher r. $\frac{\partial x_L^*}{\partial r} = \frac{2(\alpha\gamma+\beta(\theta-\gamma))}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma} > 0$, and $\frac{\partial x_R^*}{\partial r} = \frac{2\alpha(\gamma+\theta)-2\beta\gamma}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma} > 0$, and $\frac{\partial[x_L^*-x_R^*]}{\partial r} = \frac{2\theta(\alpha-\beta)}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma} > 0$. $\square$

Higher ψ. $\frac{\partial x_L^*}{\partial \psi} = \frac{\beta(\beta-\alpha)}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma} < 0$, and $\frac{\partial x_R^*}{\partial \psi} = \frac{\alpha(\alpha-\beta)}{\alpha^2\gamma+2\alpha\beta\theta-\beta^2\gamma} > 0$. $\square$