Privacy, Personalization and Price Discrimination

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Abstract

We study a bilateral trade setting in which a buyer has private valuations over a multi-product seller's inventory. We introduce the notion of an *incentive-compatible market segmentation (IC-MS)* – a market segmentation compatible with the buyer's incentives to voluntarily reveal their preferences. Our main result is a characterization of the buyer-optimal IC-MS. It is partially revealing, comprised primarily of pooling segments wide enough to keep prices low but narrow enough to ensure trade over relevant products. We use our results to study a novel design problem in which a retail platform seeks to attract consumers by calibrating the coarseness of its search interface. Our analysis speaks directly to consumer privacy and the debate regarding *product steering* versus *price discrimination* in online retail.

JEL Classification: D82, L11, L12. **Keywords**: market segmentation, price discrimination, product differentiation, communication, search design.

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1 Introduction

We study the interaction between a buyer who is privately informed regarding her preferences over a multi-product seller's inventory. We ask: to what extent does the buyer have incentives to *voluntarily reveal* information to the seller prior to trade? The answer turns on a fundamental trade-off: precise information leads to trade over her preferred item, but allows the seller to extract trade surplus through higher personalized prices.

Such deliberations are central to the debate regarding consumer privacy in online retail. Consider a consumer searching via an online retail site. The consumer naturally benefits from supplying the site with detailed information, in order that product recommendations be of greater relevance – a practice we refer to as *steering*. However, detailed information allows the site to potentially engage in *price discrimination*, securing the gains from the supply of information for itself. A recent White House Report on personalized pricing describes the problem succinctly:[1]

"Much of what companies learn…is used to design products and services that deliver more value to the individual consumer. At the same time, if sellers can accurately predict what a customer is willing to pay, they may set prices so as to capture much of the value in a given transaction"

Price discrimination is of first-order concern in online settings; Hannak et al. (2014) find that 9 out the 10 e-commerce sites they studied engaged in some form of both steering and price discrimination based on consumer-specific information, with prices differing in some instances by hundreds of dollars.[2] As the White House report continues to argue, "*Whilst there are substantial concerns about differential pricing,…whether [privacy] helps or harms the average consumer depends on how and where it is used.*"

Much of the academic literature on consumer privacy has focussed on cases where firms use historical patterns of purchase and browsing behaviour to estimate consumers' preferences and

[1] See https://tinyurl.com/y2oeq7sw

[2] Consumers are themselves increasingly aware of these issues. According to a study commissioned by the European Commission, 62% were aware of how firms used their information for personalised offers, and 44% were aware of how personalised pricing worked. See European Commission (2016)

capture gains from trade.[3] However, a natural recommendation might be to cede control of information regarding their personal attributes directly to consumers, which has formed the basis for various existing policies. For instance, the General Data Protection Regulation (GDPR) passed by the European Union, requires the anonymization of consumer information and dissemination only with explicit consent, whilst the US Federal Trade Commission recommends giving consumers greater control over the collection and use of their personal data.[4] Such policies speak to the potential gains from *voluntary* communication on the consumer's part, as opposed to *involuntary* disclosure via the unconsented monitoring of their actions. Furthermore, some companies e.g. Amazon and AT&T offer to pay consumers for the right to track their behaviour.[5] By studying a model of costless communication, our analysis not only complements existing theoretical treatments of consumer privacy, but can also quantify the willingness-to-pay consumers might have for such transactions and more generally the extent to which consumers benefit from control over and dissemination of personal information.[6]

Aside from the applied contribution outlined above, our paper also contributes to the theoretical literature on market segmentation and third-degree price discrimination, recently re-invigorated by Bergemann, Brooks and Morris (2015) (henceforth, BBM). Their analysis traces the division of surplus between a buyer and a single-good monopolist, as the monopolist's information regarding the buyer's valuation varies. However, their setting does not permit incentives for voluntary communication.[7] By extending their analysis to a multi-product setting with horizontal differentiation, standard single-crossing intuition now provides types with incentives to separate and thus endogenously segment the market.

In our model, a buyer faces the seller of multiple, heterogeneous goods, over which the buyer

[3] See Section 3 for a detailed discussion.

[4] See https://tinyurl.com/y6euao98.

[5] See https://tinyurl.com/satubgq. Recent work by Dinerstein et al. (2018) provides extensive empirical evidence documenting precisely the trade-off faced by consumers between gains from better targeting against losses from higher prices in the context of voluntary consumer search via a large online retail platform.

[6] A closely related paper that also shares an interest in voluntary communication and its implications for privacy is Ali et al. (2019). While our applied motivations are similar, our approaches are complementary, as we discuss in further detail in Section 3.

[7] The authors themselves note this: see page 926, footnote 1.

has private valuations. The seller makes a take-it-or-leave-it price offer for one good from her stock. As in BBM, we are interested in outcomes as we vary the information set of the seller, and as such, we start our analysis by extending their notion of a *market segmentation* to our multi-product setting. Unlike BBM, we are interested in the buyer's incentives to voluntary reveal their preferences, and as such, we introduce the notion of an *incentive-compatible market segmentation* (IC-MS) – a market segmentation which is consistent with the incentives of different buyer types to truthfully report their preferences. In Section 2.5, we show how such segmentations arise as the set of interim beliefs formed in equilibria of a game in which the buyer sends a cheap-talk message to the seller, who then makes an offer that the buyer accepts or rejects.

We begin our analysis by examining each player's preferred IC-MS. For the seller, full segmentation is optimal, as it allows for perfect price discrimination — Lemma 1. As in BBM, this outcome is efficient and transfers the total surplus to the seller. However, a subtlety is at play here that is absent in BBM – efficiency hinges not only on guaranteed trade, but also on the buyer being offered and ultimately purchasing his preferred good.

Characterizing the buyer-optimal IC-MS — Theorem 1 — constitutes the main result of the paper. Whilst the construction is subtle, the segmentation itself admits a simple description; it is the *least informative segmentation that guarantees trade*. The intuition relies on the buyer's mixed incentives to pool. The buyer's gain from pooling comes through price discounts; if types pool, the seller's optimal price offer decreases, as she attempts to trade with a wider segment. If these discounts are sufficiently large, this gain outweighs the buyer's loss from being offered a sub-optimal good. However, if the seller's price offer is not discounted enough, the buyer's loss from being mis-matched outweighs the price gain, resulting in a breakdown in trade. In the language of online retail, the buyer seeks to balance the *gains from steering* against the *losses from price discrimination*. This simple characterization extends to richer menus of offers and more general preferences, as discussed in the Online Appendix. At first glance, this characterization bears a stiking resemblance to the results in both BBM and Roesler and Szentes (2017), which also find a buyer-optimal information structure that admits a similar description. However, their settings

lack the sorting dimension present here, and thus admit a qualitatively distinct economic rationale. In particular, unlike in those papers, our buyer-optimal IC-MS is *inefficient*, as almost all types do not purchase their preferred items. Beyond this main result, our model also delivers several comparative static results. For instance, as goods become less substitutable, the buyer's preferred IC-MS involves increased information revelation – Corollary 1 – as there is little point buying a worthless good, even at a discount.

To bring our results closer to describing online retail, in Section 2.6, we embed the analysis in a richer setting in which a platform intermediates trade between the seller and the buyer. The platform engages in *search design*, whereby it constructs coarse categories that the buyer self-selects into. Formally, the platform presents an IC-MS to the buyer, who selects a segment from its support. The seller sets prices using their equilibrium inference of both the platform's search design and buyer's selection rule. Buyer participation is costly, whilst the platform maximizes profits obtained through ad valorem transaction fees. We characterize optimal search design in this setting, and by so doing are able to draw out several testable implications of the theory: when platforms compete more fiercely for consumers, this leads to lower prices, coarser search categories and lower total surplus; in markets for more specialized goods, search categories are more precise but no more profitable for the platform than in markets for more substitutable goods. We view this section as being of significant interest in and of itself, as the use of market segmentation as modelling the design of a search interface is novel; many platforms offer different search interfaces that vary in the extent to which they elicit personal preferences from users.[8] Our paper provides a novel theoretical counterpart to (Dinerstein et al., 2018), who empirically study the use of platform search design in shaping targetted offerings and personalized pricing.

The paper is structured as follows. Section 2 introduces the benchmark model. Section 2.1 introduces the notion of an *incentive-compatible market segmentation*, the key building-block of

[8]For instance, beyond its proprietary search tool which requires specific search parameters, kayak.com offers KAYAK Explore, which does not require a direct search query, thus allowing users flexible on match quality to search over prices. Similar functionalities exist via Skyline, Google Flights and other airline sites. The motivation for these coarser search functionalities is typically to allow consumers to trade-off match quality against prices without providing details, which fits our setting.

our analysis. Section 2.3 provides the central intuitions that lead to the main results contained in Secion 2.4. Section 2.5 provides an extensive-form implementation of the set of IC-MS as cheap-talk equilibria of a bargaining game, and Section 2.6 embeds the analysis in a richer market setting wherein a profit-maximizing platform intermediates trade. Section 3 outlines the contribution of our paper to various literatures, whilst Section 4 concludes with some possible extensions. All proofs are relegated to the Appendix.

2 Model

A buyer and a seller interact. The seller has a commonly known stock of indivisible, heterogeneous goods, each indexed by $v \in V \triangleq [0,1]$. The buyer's privately known valuation for a good v is captured by some $\theta \in \Theta \triangleq [0,1]$. A buyer of type θ has willingness-to-pay for good v given by $u(v,\theta)$. The seller's belief over Θ is uniform and common knowledge. We employ a standard model of pure horizontal differentiation in the main part of the paper: $u(v,\theta) = \bar{u} - a(v-\theta)^2$, where $\bar{u}, a > 0$. Here, the buyer's type represents their preferred good, and suffers a quadratic loss from inferior products.

2.1 Incentive-compatible market segmentation

We now revise and extend various definitions from BBM. A *market* is a distribution over types, $x \in X \triangleq \Delta(\Theta)$, and let $\tilde{x}$ denote the aggregate market, i.e. $\tilde{x} = \mathcal{U}[0,1]$. An *offer* is a combination of a good and a price, $(v,p) \in V \times \mathbb{R}_+$. Let $\Theta(v,p) \triangleq \{\theta \in \Theta \mid u(v,\theta) \geqslant p\}$ be the set of types that would buy good v at price p. The offer (v,p) is *optimal* for market x if $\int_{\Theta(v,p)} px(d\theta) \geqslant \int_{\Theta(v',p')} p'x(d\theta)$ for all $(v',p') \in V \times \mathbb{R}_+$, that is it maximizes the seller's expected revenue given the market she faces. Let $X(v,p) \triangleq \{x \in X \mid (v,p) \text{ optimal for } x\}$ denote the set of markets for which the offer (v,p) is optimal.

A *segmentation* $\sigma \in \Sigma \triangleq \{\sigma \in \Delta(X) \mid \int x(Y)\sigma(dx) = \tilde{x}(Y),\ \forall Y \in \mathcal{B}(\Theta)\}$, where $x(Y)$ is the measure of the Borel set Y. In words, a segmentation splits the aggregate market into sub-markets.

Each $x \in \text{supp}\sigma$ is a *segment*. An *offer rule* for a segmentation σ specifies an offer for each segment, that is a function $\phi : \text{supp}\sigma \to \Delta(V \times \mathbb{R}_+)$, and ϕ is *optimal for* σ if each offer is optimal on its respective segment, that is for all $x \in X$, $(v, p) \in \text{supp}\phi(x) \Rightarrow x \in X(v, p)$. σ is a *full market segmentation* if each type forms its own segment, that is if $x \in \text{supp}\sigma \Rightarrow x = \delta(\theta)$, for some $\theta \in \Theta$ and where $\delta(\theta)$ is the Dirac-delta function. We abuse notation and denote by $\tilde{x}$ the segmentation that puts full mass on $\tilde{x}$, and by $[a, b] \subset \Theta$ the market that places uniform probability over $[a, b]$.

Fix a segmentation and offer pair (σ, ϕ). Then for $x \in \text{supp}\sigma$, the (*interim*) seller and consumer surplus are

$$PS(\sigma, \phi; x) = \iint_{\Theta(v,p)} px(d\theta)\phi(x)(dv, dp)$$
$$CS(\sigma, \phi; x) = \iint_{\Theta(v,p)} (u(v, \theta) - p)x(d\theta)\phi(x)(dv, dp)$$

Total interim surplus $TS(\sigma, \phi; x) \triangleq CS(\sigma, \phi; x) + PS(\sigma, \phi; x)$. *Ex-ante* producer surplus is $PS(\sigma, \phi) = \int PS(\sigma, \phi; x)\sigma(dx)$ and similarly for consumer and total surplus. If σ has a unique optimal offer rule ϕ, then let $CS^*(\sigma), PS^*(\sigma)$ denote consumer and producer surplus under ϕ.

With these preliminaries in mind, we introduce the notion of an *incentive-compatible* market segmentation (IC-MS). Put simply, σ is an IC-MS if for some offer rule optimal for σ, each type θ cannot gain by selecting into a different segment of σ.

Definition 1. A segmentation σ is an **incentive-compatible market segmentation (IC-MS)** if there exists an offer rule ϕ optimal for σ such that:

$$\int \max\{u(v, \theta) - p, 0\}\phi(x)(dv, dp) \geqslant \int \max\{u(v, \theta) - p, 0\}\phi(x')(dv, dp), \tag{1}$$

for all $x, x' \in \text{supp}\sigma$ such that $\theta \in \text{supp}x$. Let $\Sigma_{IC} \triangleq \{\sigma \in \Sigma | \sigma \text{ is an IC-MS}\}$. An IC-MS is **partitional** if the supports of each segment partition Θ, and **n-partitional** if it is partitional and has n segments. Let Σ_P and Σ_P^n denote the set of partitional and n-partitional segmentations respectively.

Consider again BBM. Any market segmentation in which the optimal offer is not equal on each segment would clearly not be an IC-MS, as any type in a segment not being offered the lowest price would have an incentive to mis-report. In our multi-product setting, in which preferences satisfy standard single-crossing conditions, voluntary communication can play a non-trivial role in determining equilibrium allocations. With regards partitional IC-MS, the relation to partitional equilibria in cheap-talk games should be clear; the segmentation is formed through an endogenous communication strategy, with types that pool on the same message forming segments. We study this link further in Section 2.5.

2.2 Discussion of Model

We discuss the assumptions made above. The notion of an IC-MS assumes the buyer is privately informed of their type. This seems natural in most applications. We view our model as a reduced form description of consumer search via online platforms, in which communication is tantamount to performing a search query – see Section 2.6 for a full exploration of this interpretation. As such, it seems natural to assume the consumer knows his own taste before searching for products. Our notion of IC-MS makes ours a model of costless communication rather than costly signalling, e.g. sellers inferring buyers' valuations through past purchases. As discussed previously, our goal is to understand how information *alone* affects allocative efficiency.

The assumption that the seller can only offer one good from his range can be motivated on the grounds of limited attention on the part of the buyer or costly search through alternatives. This approach has both theoretical (Ravid (2017), Eliaz and Spiegler (2016)) and empirical support (Dinerstein et al. (2018)) and forms the basis for the *consideration set* approach to demand estimation (Honka et al. (2017), Goeree (2008)). Nevertheless, in the Online Appendix, we detail the results of allowing the seller to offer menus. In short, with all but the most complex menus available to the seller, the essential features of the main results still hold. The main takeaway is that more complex menus transfer surplus to the seller by forcing types to sort through their acceptance strategies.

This ties to another key assumption: the seller's inability to commit to prices. We thus belong to the large literature on third-degree price discrimination, begining with Pigou's seminal work and recently re-invigorated by BBM. We argue that the motivating debate surrounding consumer privacy and personalized pricing hinges on the seller being able to set prices *after* learning a consumer's preferences. Beyond the various sources outlined in the motivation, there is also evidence that seller's and platforms often renege on posted prices, thus seemingly contradicting any ability to commit to prices.[9]

To facilitate a clear exposition of the main idea, we assume a standard form of horizontal preferences. In the Online Appendix, we substantially generalize this preference structure, imposing but a few standard assumptions such as single-crossing, single-peakedness and strict concavity. The main results of the paper, namely that the seller prefers the most informative IC-MS whilst the buyer prefers the least informative IC-MS that guarantees trade, remain robustly in tact.

2.3 Efficiency and the Division of Surplus

How do different segmentations affect both overall welfare and the division of surplus between the buyer and seller? Recall the single-good setting of BBM. There, efficiency and trading probability are synonymous – any segmentation that guarantees trade is efficient. The width of a segment determines both efficiency – wider segments can lead to partial trade as the seller charges a monopoly price higher than the lowest valuations – and the distribution of surplus – conditional on guaranteed trade, wider segments transfer surplus from the seller to the buyer through standard information rents.

In our setting, the width of a segment not only determines these properties, but also the allocation of goods to types i.e. the profile of *sorting*. We formalize this discussion now.

Definition 2. A market x is **clearing** if $u(v,\theta) - p \geqslant 0$ for all $\theta \in \text{supp}x$ and all (v,p) such that $x \in X(v,p)$, i.e. any optimal offer rule results in all types accepting. A segment is **residual** if it

[9]Priceline notes that it often reduces posted prices ex-post based on a user's browsing behaviour. See https://tinyurl.com/tolkynx.

is not clearing. An IC-MS σ is **clearing** if each of its segments is clearing, and is residual if it is not clearing.

Proposition 1. *1. $x = [\theta, \theta']$ is clearing if and only if $|\theta' - \theta| \leqslant \Lambda(a, \bar{u})$, where $\Lambda(a, \bar{u}) = 2\sqrt{\frac{\bar{u}}{3a}}$*

2. If $\sigma \in \Sigma_{IC}$ is clearing, then σ has a unique optimal offer rule.

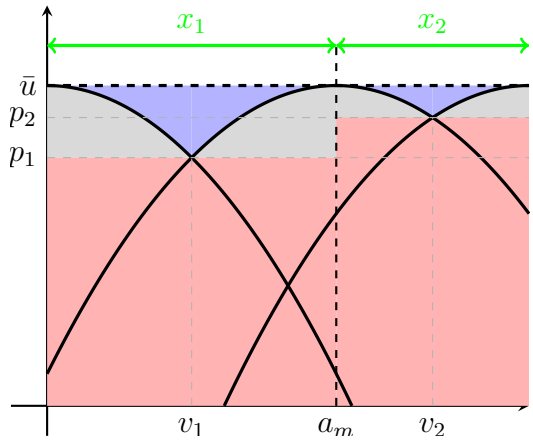
3. If $x = [\theta, \theta']$ is clearing and $x \in supp\sigma$ for some $\sigma \in \Sigma$, then for any ϕ optimal for σ, $CS(\sigma, \phi; x) = \frac{a}{6}(\theta' - \theta)^2, PS(\sigma, \phi; x) = \bar{u} - \frac{a}{4}(\theta' - \theta)^2$, and $TS(\sigma, \phi; x)$ is decreasing in $\theta' - \theta$.

4. Residual segmentations are ex-ante Pareto dominated. That is, if $x = [\theta, \theta']$ is residual and $x \in supp\sigma$ for some $\sigma \in \Sigma$, then there exists $\sigma' \in \Sigma$ such that $PS(\sigma', \phi') > PS(\sigma, \phi)$ and $CS(\sigma', \phi') > CS(\sigma, \phi)$, for all ϕ, ϕ' optimal for σ, σ' respectively.

The first part of Proposition 1 identifies a *critical width* $\Lambda(a, \bar{u})$ such that all partitional segmentations that comprise of segments smaller than this width are clearing, and all partitional segmentations with at least one interval wider than this width are residual. The economic interpretation of $\Lambda(a, \bar{u})$ mirrors the preceding discussion – a segment wider than $\Lambda(a, \bar{u})$ would result in the seller setting a monopoly price that forces some types to reject. The second part simply states that facing a clearing segment, the seller's optimal offer is unique. The third part states that conditional on being clearing, a wider segment benefits the consumer, hurts the seller, and destroys overall surplus through the sub-optimality of the buyer-product match. Intuitively, faced with a wider segment, the seller's optimal offer involves a lower price, as she attempts to ensure trade. These price discounts benefit the buyer at the seller's expense. Finally, the last part states that any residual segmentation is ex-ante dominated – the destruction of surplus through a lack of trade is entirely deadweight loss.

Figure 1 describes graphically a clearing IC-MS. In this example, the market splits into two segments, $x_1 \triangleq [0, a_m)$ and $x_2 \triangleq [a_m, 1]$. The seller's offer binds the IC constraints of the boundary types, in particular leaving type a_m indifferent between either segment. As segments widen, both the buyer's surplus and the inefficiency from product mismatch increase (the blue and grey areas

Figure 1: A 2-partitional clearing IC-MS σ and optimal offer rule ϕ.



supp$\sigma = \{x_1, x_2\}$, where $x_1 \triangleq [0, a_m), x_2 \triangleq [a_m, 1]$, $\phi(x_i) = (v_i, p_i)$ for $i = 1, 2$. Black thick lines are $u(v, \cdot)$ for $\theta = 0, a_m, 1$ respectively from left to right. Shaded areas: CS = blue, PS = red, deadweight loss = gray.

expand) at the expense of the seller (the red area contracts), who optimally offers an average product and lowers their price to ensure trade and bolster revenue.

2.4 Optimal Incentive-Compatible Market Segmentations

Proposition 1 provides rich guidance on how each player ranks various market segmentations. The buyer prefers wide segments that induce the seller to offer price discounts, but not too wide to undermine trade – segments with width $\Lambda(a, \bar{u})$ balance this trade-off perfectly. The seller prefers narrow segments, which increase total surplus through well-targeted offers but allow the seller to capture the gains through higher mark-ups. We build on this intuition by fully characterizing each player's optimal IC-MS. We call $\sigma \in \Sigma$ *seller-IC-optimal* if $PS(\sigma, \phi) \geqslant PS(\sigma', \phi')$ for all $\sigma' \in \Sigma_{IC}$ and ϕ, ϕ' optimal for σ, σ' respectively, i.e. if σ maximizes ex-ante producer surplus amongst all IC-MS, and similarly define a *buyer-IC-optimal* segmentation.

Lemma 1. *The unique seller-optimal $\sigma_S \in \Sigma$ is full market segmentation, with unique optimal offer $\phi\big(\delta(\theta)\big) = (\theta, \bar{u})$ for each $\theta \in \Theta$. Furthermore, $\sigma_S \in \Sigma_P$ and achieves the first-best allocation, i.e. $TS^*(\sigma_S) = \bar{u}$.*

The seller is perfectly informed regarding the type of buyer she faces, and offers them their ideal good at their maximal willingness-to-pay of $\bar{u}$. Clearly, each type has no incentive to deviate, as they would be paying the same price for an inferior good. The seller achieves her first-best payoff. Whilst full separation is not robust to more general preferences – see the online appendix for details –, the essential feature of this IC-MS remains in tact, namely that the seller's preferred IC-MS is as revealing as possible.

Characterizing the buyer's optimal IC-MS constitutes the main result of the paper. The market is segmented into as many intervals of width $\Lambda(a, \bar{u})$ as possible, with a final residual segment that covers the remainder of the market and guarantees trade. As such, we term a segmentation of this structure a **least separating clearing (LSC)** IC-MS and let Σ_{LSC} denote the set of all such segmentations.

Theorem 1 (**Least Separating Clearing IC-MS**). *Let $N^* = \lceil \frac{1}{\Lambda(a,\bar{u})} \rceil$, where $\lceil y \rceil$ maps $y \in \mathbb{R}$ to the smallest integer greater than y. Then σ_{LSC} is buyer-IC-optimal if and only if $\sigma_{LSC} \in \Sigma_P^{N^*}$ consists of $N^* - 1$ segments with width $\Lambda(a, \bar{u})$ and one segment with width $1 - \Lambda(a, \bar{u})(N^* - 1)$.*

Note that for $a \leqslant \frac{4\bar{u}}{3}$, $N^* = 1$, and hence any LSC IC-MS is simply $\tilde{x}$. In other words, if goods are sufficiently substitutable, the buyer is willing to fully concede on the choice of good in favor of a price discount. Conversely, as the buyer becomes less willing to be mis-matched, his preferred segmentation becomes increasingly informative. After all, if the buyer wants only one particular item, he can't help but reveal this. We formalize this logic with the following comparative static result, the proof of which is direct from the expression for N^* given in Theorem 1.

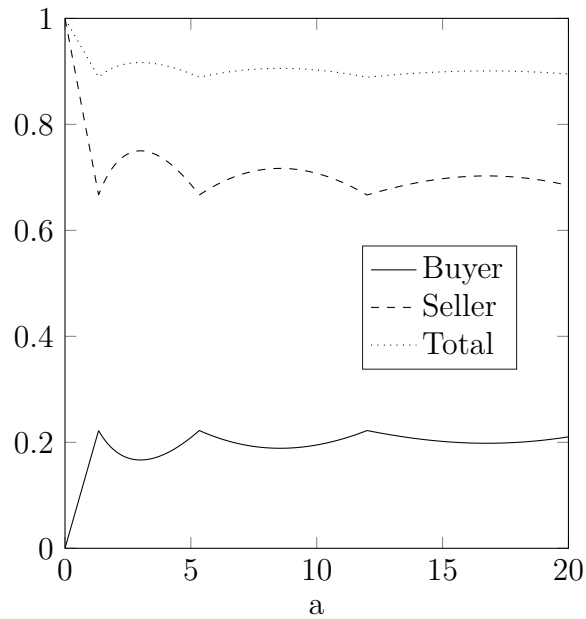
Corollary 1. *N^* is (weakly) increasing in a.*

We conclude this section by performing a second comparative static exercise; how does *ex ante* buyer welfare in an LSC IC-MS vary as goods become less substitutable? The answer is, perhaps surprisingly, non-monotonically.

Corollary 2.

$$CS^*(\sigma_{LSC}) = \frac{a}{6}\big[(N^* - 1)\Lambda(a, \bar{u})^3 + \big(1 - (N^* - 1)\Lambda(a, \bar{u})\big)^3\big]$$

Figure 2: *Ex ante* welfare in an LSC IC-MS ($\bar{u} = 1$)



In particular, $CS^*(\sigma_{LSC})$ is continuous and non-monotone when considered as a function of the parameter a.

One might expect that as the buyer becomes more discerning, he is forced to reveal his preferences more precisely whilst facing higher prices. This intuition is only partially correct. In an LSC IC-MS, the buyer reveals precisely enough information to force the seller to charge the *same* (monopoly) price as often as possible. Thus, whilst the buyer's loss from product mismatch increases, increased information provision offsets this loss. This is best understood through Figure 2.

2.5 Cheap-talk Implementation

Consider the following extensive form game $\mathcal{G}$. First, the buyer sends a message $m \in \mathcal{M}$ to the seller, where $\mathcal{M}$ is some set large enough to reveal θ, e.g. $\mathcal{M} = [0, 1]$. The seller then makes an offer of good v to the buyer at price p, which the buyer either accepts or rejects. In a *perfect Bayesian equilibrium (PBE)* of this game, the seller's beliefs form a family of conditional distributions, $\pi(\cdot|m),\ m \in \mathcal{M}$. It is immediate then that $\{\pi(\cdot|m)\}_{m\in\mathcal{M}}$ forms an IC-MS – observe that the

inequalities (1) are precisely the no-deviation constraints on the buyer's optimal communication strategy in $\mathcal{G}$. The converse also holds – any $\sigma \in \Sigma_P$ forms the set of conditional beliefs of some PBE of $\mathcal{G}$, in which all types within a given segment pool on the same message. We formalize this discussion in the Appendix.

This cheap-talk implementation brings our analysis closer to the setting of online retail. In particular, we may view the buyer's message as a search query submitted to an online, multi-product seller who can tailor both the good and price offered to the buyer based on the query. The seller forms a belief regarding the types of buyer she faces based on the particular search query she receives.

Beyond this, by appealing to well-known equilibrium *refinement concepts* within the cheap-talk literature, this implementation allows us to study several alternative focal outcomes beyond the buyer- and seller-IC-optimal segmentations. We analyse these at length in the Online Appendix. For instance, we examine outcomes when the buyer finds lying costly, and particular attention is directed toward the set of *neologism-proof* equilibria, which in this case admits a compelling structure.

2.6 Platform-mediated Search

We close the gap to online consumer search by embedding the analysis into a richer market setting. Various salient features of online search are now incorporated into the model. Firstly, we introduce a *profit-maximizing* platform, which designs the search process by choosing the coarseness with which the buyer may convey his preferences to the seller. Using the theory we have so far developed, this takes the form of a market segmentation. For instance, using the example in Figure 1, the platform provides two categories from which the buyer chooses whether his preferred good is to the left or right of a_m. Second, we introduce a costly, ex-ante participation choice on the buyer's part, interpreted as either direct or opportunity costs, allowing us to recast the division of surplus as a classic hold-up problem – the platform must provide the buyer sufficient downstream value in order to induce him to participate in search.

Formally, consider the following extension to the game above. The buyer now faces a participation cost $K \sim F(.)$, where the differentiable distribution F is commonly known, has support $[0, \bar{K}]$ and satisfies the standard monotone hazard-rate assumption $\frac{f(K)}{1-F(K)}$ is non-decreasing, where the realization K is the buyer's private information. A platform commits to a *search design* σ, which is simply a market segmentation. The timing of the game, along with strategies, is as follows:

1. Buyer privately draws $K \sim F(.)$.
2. Platform chooses $\sigma \in \Sigma$.
3. Buyer chooses participation.
4. Buyer draws $\theta \sim \mathcal{U}[0, 1]$.
5. Buyer selects $x \in \text{supp}\sigma$.
6. Seller chooses an offer $\phi(x)$. Allocations realized as per $\phi(x)$.

We should note that the timing could be altered to allow the seller to select an offer rule ϕ *before* the buyer selects their chosen segment x. This would best fit situations in which third-party sellers set prices anticipating the equilibrium search behaviour of consumers e.g. Amazon Marketplace, whereas the above timing fits a platform selling its own products and personalizing prices after search queries are submitted, e.g. Amazon retail.[10] The key assumption is that the platform's choice of search design is fixed before prices are set. Payoffs must now be amended to account for the presence of the platform. We focus on the case of *ad valorem fees* – the platform extracts a fixed commission α on the seller's trading price. Thus, for a given search design σ, offer rule ϕ and assuming buyer participation, the platform's expected profit is $\mathbb{E}_{\sigma,\phi}[\alpha p] \triangleq \alpha \cdot PS$, the seller's expected profit is $\mathbb{E}_{\sigma,\phi}[(1-\alpha)p]$ and the buyer's expected payoff is $\mathbb{E}_{\sigma,\phi}[u(v,\theta) - p] - K$. Were it not for the buyer's participation constraint, the platform would fully segment the market as it is aligned with the seller's preferences.

[10] Of course, the case of $\alpha \in (0, 1)$ seems inconsistent with the Amazon retail interpretation. We endogenize the choice of α in the Online Appendix to address this issue.

We focus on PBE of the game. The problem of the platform can be succinctly described as follows. Firstly, given the timing of the game, it is immediate that any optimal search design must be incentive compatible, and hence $\sigma \in \Sigma_{IC}$ at a PBE. Next, since residual segmentations incur deadweight loss, we may immediately rule these out. Suppose the platform aims to guarantee the buyer precisely K in expected surplus. Which search design would maximize the seller's expected profit whilst satisfying this constraint? Amongst clearing segmentations, the buyer's gain is the seller's loss, and so *any* clearing segmentation that provides K to the buyer is optimal. Next, what value of K should the platform optimally target? Higher K leads to lower prices but higher participation rates.

Proposition 2. *Let K^* solve $K^* = \frac{2\bar{u}}{3} - \frac{F(K^*)}{f(K^*)}$.*

1. *If $K^* \leqslant CS^*(\sigma_{LSC})$, then σ is an equilibrium segmentation if and only if $\sigma \in \Sigma_{IC}$ is clearing and $CS^*(\sigma) = K^*$.*

2. *If $K^* \geqslant CS^*(\sigma_{LSC})$, then σ is an equilibrium segmentation if and only if $\sigma \in \Sigma_{LSC}$.*

When combined with the various findings in the preceding analysis, these simple results can be leveraged to yield several testable implications. First, a rightward shift of the support of the distribution F – interpreted as an increase in the buyer's outside options – shifts the downward-sloping curve $\frac{2\bar{u}}{3} - \frac{F(K^*)}{f(K^*)}$ to the right, and thus K^* increases. The following implication is then immediate from Proposition 2:

Prediction 1. *An increase in the value of the buyer's outside options leads to: 1) lower prices and 2) coarser search design.*

Prediction 1 is a *within-market* implication. Following Corollary 1 by using a as a measure of how specialized the market is, and noting that the expression for K^* is invariant to a, we can draw out *between-market* implications:

Prediction 2. *More specialized markets exhibit: 1) finer search design but 2) similar profit margins for platforms.*

Finally, a basic result the recurs throughout the paper is that the more precise the seller's estimate of the buyer's type, the higher her price offers will be:

Prediction 3. *Finer search categories lead to higher prices.*

This last prediction enjoys empirical support. Dinerstein et al. (2018) find that greater horizontal differentiation between sellers – synonymous in our setting with finer segmentations – leads to higher prices and mark-ups for sellers. Recently, Hillenbrand and Hippel (2019) study experimentally the question of buyer-optimal information disclosure. The buyer chooses how to filter search results and the seller chooses prices accordingly. They find evidence that sellers set higher prices in response to finer search queries and that buyers strategically obfuscate search.

To lend further credence to this predictions, we show in the Online Appendix that the results of this section remain true in the presence of seller participation constraints as well as allowing the platform to set the commission rate α endogenously. In summary, higher α extracts grater surplus for the platform from each transaction, but reduces the seller's incentives to participate, whilst the proof of Proposition 2 shows that the platform's optimal segmentation is independent of α and thus its structure is unaffected by seller participation constraints.

We conclude this section with a few points of discussion. As per Section 2.5, this game has an equivalent implementation, in which the platform commits to transmitting a garbling of the buyer's reported type to the seller, who then makes an offer which the buyer accepts or rejects. Whilst analytically identical, these two extensive forms have distinct applied interpretations. In the game above, the platform never learns the true type of the buyer, whereas in its cheap-talk analogue it does. This distinction is of first-order importance regarding the privacy of consumer data and the sharing of this data between firms, an important issue that goes beyond the scope of the current paper. Finally, whilst we focus on the ad valorem platform setting as an empirically leading example, the model is tractable enough to consider a range of objectives. For instance, were the platform targeting buyer subscription fees, the platform would choose the segmentation that maximizes CS, i.e. an LSC IC-MS.

3 Literature and Contribution

Our main contributions are two-fold. Our first main contribution is to the theoretical literature on market segmentation, recently pioneered by BBM. In a standard setting with a single-good monopolist who can engage in third-degree price discrimination a buyer with unknown valuation, they trace out the consequences of varying the information the seller has regarding the buyer's valuation – market segmentations – for both efficiency and the division of surplus. Along the way, they show how the buyer-optimal market segmentation is partially uninformative, securing the buyer information rents whilst guaranteeing trade occurs. Roesler and Szentes (2017) find a similar characterization in a similar setting but where the buyer is initially uniformed and commits to an information structure. Condorelli, Szentes (2019) extend this analysis to a costly acquisition problem, which has some similarity to the hold-up problem embodied in our platform extension. We contribute to this literature in two novel ways. First, we extend the notion of a market segmentation to a multi-product setting, which allows us to speak to the issue of product steering. Second, we introduce the notion of an IC-MS, and thus incorporate the study of voluntary information revelation into this research agenda. In recent work, Hagnapah and Siegel (2019) study multi-product market segmentation in a more general setting than ours whilst abstracting from incentives for disclosure. Ali et al. (2019) also study disclosure incentives as applied to consumer privacy, focusing on the disclosure of hard evidence. Our work is complementary in that we study the communication of soft information to a monopolist when there are multiple potential products that could be sold. Their analysis abstracts from match quality by studying a single-product setting, and instead highlights the benefits that emerge purely from personalized pricing when a consumer can disclose hard rather than soft information.[11]

Our second main contribution is to the ongoing debate regarding consumer privacy in online retail. Acquisti, Taylor and Wagman (2016) and Varian (1996) provide extensive surveys on the privacy literature, Fudenberg and Villas-Boas (2006) provide a survey on models with behavioral

[11] In addition, they study how disclosure amplifies competition in a duopoly model with horizontal differentiation.

price discrimination and Bergemann and Bonatti (2019) survey papers on markets for information. Bonatti and Cisternas (2019) consider a model where a consumer's historical behaviour informs a *score* that is then used by firms to set prices. [12] Taylor (2004) considers the sale of consumer information and finds that when consumers are strategic, firms cannot benefit from targeted pricing. Calzolari and Pavan (2006) consider the sharing of consumer information between two principals who sequentially contract with the same agent and give conditions under which commitment to privacy is optimal.[13] Acquisti and Varian (2005) and Conitzer, Taylor and Wagman (2012) consider a monopolist who sells a good over two periods where customers are strategic and may choose to maintain anonymity at a cost. In contrast to these settings, we study information revelation by the buyer regarding his preferences, rather than through costly equilibrium purchase choices. This choice affords several advantages. First, we speak directly to the question of whether ceding control over their information benefits consumers and to what extent. Second, by abstracting from the incentives to make inefficient purchase decisions, we can focus on the *direct* role that data sharing has for both efficiency and surplus division. Finally, our paper is amongst the first to explicitly consider the interaction between privacy concerns and participation incentives. Liang and Madsen (2020) explore such a linkage in a setting that also exhibits moral hazard and cross-agent externalities.

That the good offered is chosen conditional on buyer information connects our analysis to the literature on targeted advertising. Shen and Villas-Boas (2018) consider behavior-based advertising by a monopolist where purchase decisions determine future advertising decisions. Bergemann and Bonatti (2015) consider the problem of a data provider who sells consumer information to firms which then tailor their advertisements to the individual match value. De Corniere and De Nijs (2016) consider an online platform that auctions an advertising slot to competing firms.[14] In their setup, consumer information is taken as freely available to the platform and improves the

[12] Villas-Boas (1999) and Fudenberg and Tirole (2000) consider behavior- based price discrimination in duopoly setting.

[13] Some other examples of behavior based price discrimination include Gehrig and Stenbacka (2007), Chen and Zhang (2009), Jing (2011) , Shy and Stenbacka (2016).

[14] See also Rhodes and Wilson (2018) for a model of strategi information transmission in an advertising context.

match between the advertiser and consumer while also increasing the prices. They abstract from the consumer's strategic incentives to reveal information, and allow the seller to commit to prices. The same is true of Gomes and Pavan (2019), who compare uniform to personalized pricing in a broader matching market setting.

In concurrent work, Ichihashi (2019) studies information disclosure by consumers to a multi-product firm, focussing on welfare comparisons between discriminatory and non-discriminatory pricing. Some key distinctions exist between our papers. First, by studying a persuasion model, he abstracts from the incentive-compatibility property central to our analysis. Second, valuations are independently distributed over products in his setting, rather than perfectly correlated as in ours. Our preference structure not only allows for a complete characterization of buyer-optimal outcomes but also novel comparative static results such as Corollaries 1 and 2 and Predictions 2 and 3. These all relate to our key parameter $\Lambda(a, \bar{u})$, which has no counterpart in his analysis.

The tension driving incentives to reveal information bears some resemblance to the so-called *ratchet effect* identified in the early literature on dynamic adverse selection (see Hart and Tirole (1988), Laffont and Tirole (1988)). Ickes and Samuelson (1987) show how switching between skill-independent jobs can break the link between current and future incentives, thereby reducing the ratchet effect. Coles, Kushnir and Niederle (2013) model signalling in matching markets in which preferences have both horizontal and vertical differentiation.

4 Conclusion

This paper presented a model of bilateral trade between a privately informed buyer and a multi-product seller. We introduced the notion of an *incentive-compatible market segmentation (IC-MS)*, i.e. a splitting of the market that is consistent with the buyer's *voluntary* incentives to reveal their type. Our main result is a characterization of the buyer-optimal IC-MS - the buyer reveals as little information as possible regarding his tastes to ensure low prices, whilst providing enough information to ensure the seller offers relevant products. These conflicting objectives map directly

into the ongoing debate regarding consumer privacy, in the form of two well-known practices - *steering* and *price discrimination*. We concluded by embedding the analysis into a broader applied setting in which a profit-maximizing platform intermediates trade and design the search interface used by the buyer.

Whilst the simplicity of the model allows a transparent investigation of the key economic forces, it paves the way for a number of extensions. For instance, incorporating multiple rounds of trade into the game might produce a more realistic extensive form. We posit that introducing multiple trading rounds would transfer surplus to the seller, whilst reducing the information revealed by the buyer via his cheap-talk messaging.[15] We leave a formal analysis of this and other extensions for future work.

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A Proofs

A.1 Proof of Proposition 1

1. We begin with a lemma:

   Lemma 2. *If the offer (v, p) optimal for $[\theta, \theta']$ then $u(v, \theta) - p \leqslant 0, u(v, \theta') - p \leqslant 0$.*

   Proof. Suppose not, i.e. $p < u(v, \theta)$. By the single-crossing property, there exists a type $\theta_m \in (\theta, \theta')$ such that $p = u(v, \theta_m)$ and $p > u(v, y)$ for all $y \in (\theta_m, \theta']$. Hence, the seller's profits with offer (v, p) are $\Pi = p(\theta_m - \theta)$. Now consider the profile $(v + \epsilon, p)$. By continuity, we can find a sufficiently small ϵ such that $p < u(v + \epsilon, \theta)$ and $p < u(v + \epsilon, \theta_m)$. Hence by continuity again, there exists a type $\theta_n > \theta_m$ such that $p = u(v + \epsilon, \theta_n)$. Profits are now given by $\Pi^{**} = p(\theta_n - \theta) > p(\theta_m - \theta) = \Pi^*$, a contradiction. Furthermore, without loss, we can take $v_m = \frac{\theta + \theta'}{2}$, since this mid-point good offers (weakly) greater surplus to the seller. The seller's profit from offer (v_m, p) is $\Pi(p) \triangleq 2p\left(\frac{\bar{u} - p}{a}\right)^{\frac{1}{2}}$. □

   Now, from the definition of clearing segments, $[\theta, \theta']$ is clearing if and only if the seller makes greater profit from selling to all types than charging the monopoly price that solves $\Pi'(p) = 0$. Thus, by Lemma 2, $[\theta, \theta']$ is clearing if and only if $\bar{u} - a\left(\theta' - \theta\right)^2 \leqslant \frac{2\bar{u}}{3}$, i.e. $\theta' - \theta \leqslant 2\sqrt{\frac{\bar{u}}{3a}}$.

2. By symmetry and Lemma 2, the only optimal offer that can clear a segment $x = [\theta, \theta']$ is $(v, p) = \big(v_m, u(v_m, \theta)\big)$, and thus uniqueness is immediate.

3. Combining these arguments, the expressions are simple algebra:

$$
\begin{aligned}
CS^*(\sigma, \phi; x) &= \frac{1}{\theta' - \theta} \int_{\theta}^{\theta'} \bar{u} - a\Big(\frac{\theta + \theta'}{2} - y\Big)^2 - \bar{u} + a\Big(\frac{\theta' - \theta}{2}\Big)^2 dy \\
&= \frac{a}{6}(\theta' - \theta)^2 \\
PS^*(\sigma, \phi; x) &= u(v_m, \theta) \\
&= \bar{u} - \frac{a}{4}(\theta' - \theta)^2
\end{aligned}
$$

4. Since x is residual, there exists a Borel set $Y \subset [\theta, \theta']$ with strictly positive measure such that if $\theta_m \in Y$ then θ_m rejects any optimal offer. Let $\sigma' \in \Sigma$ agree with σ except at x, where we split x into $[\theta, \theta'] \setminus Y$ and Y. Note that there exists $Y' \subset Y$ of strictly positive measure such that Y is clearing; else, the seller would trade with measure zero and hence make zero profit.

A.2 Proof of Lemma 1

To see that full market segmentation is an IC-MS is straightforward. Each type θ receives 0 under $\{\sigma_S, \phi\}$, and $\bar{u} - a(v - \theta')^2 - \bar{u} < 0$ under any other segment. That this segmentation achieves first-best $\bar{u}$ is immediate, as the loss function $-(\theta - v)^2$ is clearly minimized by setting $v = \theta$.

A.3 Proof of Theorem 1

We begin with a claim:

Claim 1. *If $\sigma \in \Sigma_{IC}$ is buyer-optimal, then $\sigma \in \Sigma_P^n$.*

Proof. We first claim that any $\sigma \in \Sigma_{IC} \setminus \Sigma_P$ is residual, and thus cannot be buyer-optimal. To see this, note that by the arguments in the proof of Proposition 1, $\sigma \in \Sigma_{IC}$ implies that the IC constraint of a boundary type must bind. If $\sigma \in \Sigma_{IC} \setminus \Sigma_P$, then there exist $x, x' \in \text{supp}\sigma$ and $\tilde{\theta} \in \text{supp}x, \text{supp}x'$ that is a boundary type x and in the strict interior of x'. The fact that $\sigma \in \Sigma_{IC}$ implies that $\tilde{\sigma}$ must receive 0 in either segment. But then by single crossing, there must be a positive measure set $Y \subset \text{supp}x'$ such that all $\theta \in Y$ that also receive 0, and hence σ is residual. Finally, suppose that $\sigma \in \Sigma_P \setminus (\cup_{n\in\mathbb{N}}\Sigma_P^n)$. Then there exists $[a, b] \in \text{supp}\sigma$ such that σ fully segments $[a, b]$. Consider $\sigma' \in \Sigma_P$ that is identical to σ everywhere except on $[a, b]$, on which σ' pools $[a', b'] \subset [a, b]$ and where $b' - a' < \Lambda(a, \bar{u})$. Then clearly $CS(\sigma', \phi') > CS(\sigma, \phi)$ for any ϕ, ϕ' optimal for σ, σ' respectively. □

Note that each clearing $\sigma \in \Sigma_P^n$ is uniquely defined by boundary types $0 = a_0, \ldots, a_n = 1$, where between a_i and a_{i+1}, the segmentation is either pooling or fully segmented, since $|\Phi(\sigma)| = 1$. Let $M(\sigma) \triangleq (M_1, \ldots, M_n) = ([a_0, a_1], \ldots, [a_{n-1}, a_n])$. We will abuse notation and denote the widths of these intervals by M_i as well.

Proposition 3. *If $\sigma = (a_1, \ldots, a_{n-1})$ is a clearing equilibrium with associated intervals $(M_1, \ldots, M_n)$, then*

$$CS^*(\sigma) = \frac{a}{6}\sum_{i=1}^{n} M_i^3$$

Furthermore, CS is convex in $(M_1, \ldots, M_n)$.

Proof.

$$\begin{aligned}
CS^*(\sigma) &= \sum_{i=1}^{n}\int_{a_{i-1}}^{a_i} \bar{u} - a\Big(\frac{a_i + a_{i-1}}{2} - \theta\Big)^2 - \bar{u} + a\Big(\frac{a_i - a_{i-1}}{2}\Big)^2 d\theta \\
&= \sum_{i=1}^{n} a\int_{a_{i-1}}^{a_i} \Big(\frac{a_i - a_{i-1}}{2}\Big)^2 - \Big(\frac{a_i + a_{i-1}}{2} - \theta\Big)^2 d\theta \\
&= a\sum_{i=1}^{n}\Big[\Big(\frac{a_i - a_{i-1}}{2}\Big)^2\theta - \frac{1}{3}\Big(\frac{a_i + a_{i-1}}{2} - \theta\Big)^3\Big]_{a_{i-1}}^{a_i} \\
&= \frac{a}{6}\sum_{i=1}^{n}(a_i - a_{i-1})^3 \\
&= \frac{a}{6}\sum_{i=1}^{n} M_i^3
\end{aligned}$$

$CS^*(\sigma)$ is clearly then invariant over $\sigma \in \Sigma_P^n$ with identical intervals, regardless of their order. Since $(M_1, \ldots, M_n)$ are such that $M_i \in [0, 1]$ for all i, and $\sum_{i=1}^{n} M_i = 1$, we may consider the consumer surplus

of $\sigma \in \Sigma_P^n$ as a function $CS : \Delta_n \to \mathbb{R}_+$, where Δ_n is the n-dimensional simplex. Since Δ_n is convex, a sufficient condition for CS to be a convex function is for its Hessian

$$\mathbf{D}^2 CS = \frac{a}{6} \begin{pmatrix} 6M_1 & 0 & \dots & 0 \\ 0 & 6M_2 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 6M_n \end{pmatrix}$$

to be positive semi-definite. But $|\mathbf{D}^2 CS - x\mathbf{I}| = \frac{a}{6} \prod_{i=1}^n (6M_i - x)$, hence $\mathbf{D}^2 CS$ has all positive eigenvalues. □

Finally, note that any $\sigma \in \Sigma_P^n$ that is not LSC is such that there exist at least two intervals $M_i, M_j \in M(\sigma)$ such that $M_i \leqslant M_j < \Lambda(a, \bar{u})$, where the first inequality is without loss. Construct $\sigma' \in \Sigma_P^n$ such that $M(\sigma') = (M_1, \dots, M_i - \epsilon, \dots, M_j + \epsilon, \dots, M_n')$. For sufficiently small ϵ, σ' is a clearing. Since CS is convex in $(M_1, \dots, M_n)$, it must be that $CS^*(\sigma') > CS^*(\sigma)$.

A.4 Proof of Corollary 2

In an LSC IC-MS, $N^* - 1$ segments have width $\Lambda(a, \bar{u})$, with a residual segment having width $1 - (N^* - 1)\Lambda(a, \bar{u})$. Hence

$$\begin{aligned} CS^*(\sigma_{LSC}) &= \frac{a}{6} \sum_{i=1}^n M_i^3 \\ &= \frac{a}{6} \Big[\sum_{i=1}^{N^*-1} \Lambda(a, \bar{u})^3 + \big(1 - (N^* - 1)\Lambda(a, \bar{u})\big)^3 \Big] \\ &= \frac{a}{6} \big[(N^* - 1)\Lambda(a, \bar{u})^3 + \big(1 - (N^* - 1)\Lambda(a, \bar{u})\big)^3 \big] \end{aligned}$$

To prove non-monotonicity, it is sufficient to prove that if a is such that $1/\Lambda(a, \bar{u}) \in \mathbb{N}$, then $CS_{LSC}(a) = \frac{2\bar{u}}{9}$; the result then follows immediately as clearly $CS^*(\sigma_{LSC})$ is non-constant and continuous over regions where N^* is constant. At such values of a, all segments have width $\Lambda(a, \bar{u})$, and the seller charges a price $\frac{2\bar{u}}{3}$. Let $1/\Lambda(a, \bar{u}) = N$. Then $\frac{1}{2}\sqrt{\frac{3a}{\bar{u}}} = N$ and so $a = \frac{4N^2\bar{u}}{3}$. Hence

$$\begin{aligned} CS^*(\sigma_{LSC}) &= \sum_{i=1}^N \Big[\int_0^{\frac{1}{N}} \bar{u} - \frac{4N^2\bar{u}}{3} \Big(\frac{1}{2N} - \theta \Big)^2 d\theta \Big] - \frac{2\bar{u}}{3} \\ &= N \Big[\frac{\bar{u}}{N} - \frac{2a}{3} \cdot \frac{1}{8N^3} \Big] - \frac{2\bar{u}}{3} \\ &= \frac{2\bar{u}}{9} \end{aligned}$$

A.5 Cheap-talk Implementation

Let $m : \theta \to \mathcal{M}$ be the profile of messages sent by the buyer. Let $S = V \times \mathbb{R}_+$ be the space of offers - a good and a price, and let $s : \mathcal{M} \to S$ the seller's choice of offer, conditional on having received message m. Finally, let $t : [0,1] \times S \to \{0,1\}$ be the acceptance strategies for each buyer type (where 0 indicates a rejection), given an offer (v,p). A *strategy profile* $\sigma \triangleq (m,s,t)$ is a profile of message strategies m, offer strategies s and acceptance strategies t.

Type θ's expected utility given a strategy profile $\zeta = (m,s,t)$ is $\mathbb{E}_\zeta(u(v,\theta) - p)$, whilst the seller's expected profit is $\mathbb{E}_\zeta(p)$. In the event of a rejection, payoffs to both agents are zero.

Definition 3. A strategy profile ζ is an equilibrium if for each $\theta \in [0,1]$, $m(\theta) \in \arg\max_{m\in\mathcal{M}} \mathbb{E}_\sigma\big(u(v,\theta) - p\big)$; given m, $s(m) \in \arg\max_{v,p} \mathbb{E}_\sigma(p)$; and $t(s) = 1$ if and only if $u(v,\theta) - p \geqslant 0$ for all $(v,p) \in \text{supp}s\,(m(\theta))$.

Let $\mathcal{G}$ denote this game and $\mathcal{E}$ denote its equilibrium set. As usual, with each $\zeta \in \mathcal{E}$ can be associated a family of conditional distributions $\mu(\cdot|m)$ which in turn can be viewed as an element $\mu^\zeta \in \Sigma$. The following remark follows immediately from noting that for any $\zeta \triangleq \{m,s,t\} \in \mathcal{E}$ and for each $\theta \in [0,1]$, we have that $m(\theta) \in \arg\max_{m\in\mathcal{M}} \mathbb{E}_\zeta\big(u(v,\theta) - p\big)$ if and only if for all $\theta' \in [0,1]$

$$\iint \max\{u(v,\theta) - p, 0\} s\big(m(\theta)\big)(dv,dp) \geqslant \iint \max\{u(v,\theta) - p, 0\} s\big(m(\theta')\big)(dv,dp),$$

Remark 1. *Suppose $\sigma \in \Sigma_P$. Then there exists $\zeta \triangleq \{m,s,t\} \in \mathcal{E}$ such that $\mu^\zeta = \sigma$. Conversely, if $\zeta \in \mathcal{E}$, then $\mu^\zeta \in \Sigma_P$.*

A.6 Proof of Proposition 2

First, note that by Proposition 1 part 4, it is without loss to focus on clearing segmentations $\sigma \in \Sigma_{IC}$. Type θ selects $x \in \text{supp}\sigma$ such that $\theta \in \text{supp}x$, and the seller's optimal offer is unique. Let $\Pi(\sigma, K)$ denote the platform's profit from choosing a clearing $\sigma \in \Sigma_{IC}$ such that $CS^*(\sigma) = K$. Recall from the proof of Theorem 1 that each such segmentation is comprised of intervals pooling intervals $M_1, \ldots, M_n$ with the remaining market fully segmented. Then

$$\begin{aligned}\Pi(\sigma, K) &= \alpha F(K)\Big[\sum_i M_i \bar{u} - \frac{a}{4} M_i^3 + \big(1 - \sum_i M_i\big)\bar{u}\Big] \\ &= \alpha F(K)\Big[\bar{u} - \frac{3K}{2}\Big]\end{aligned}$$

where the second equality holds since $K \triangleq CS^*(\sigma) = \frac{a}{6}\sum_i M_i^3$. Thus, $\Pi(\sigma, K)$ is clearly invariant to the structure of σ. Finally, to derive the expression for K^*, note that $\Pi'(K^*) = 0 \Rightarrow K^* = \frac{2\bar{u}}{3} - \frac{F(K^*)}{f(K^*)}$ has a unique solution by the hazard-rate assumption imposed on F.

Online Appendix

"Personalization, Discrimination and Information Disclosure"

Sinem Hidir, Nikhil Vellodi

B Menus

Suppose the seller can offer a *complete* menu of offers, so that an offer rule $\phi : \text{supp}\sigma \to \mathbb{R}_+^{[0,1]}$. Then, for any $\sigma \in \Sigma$, an optimal offer for the seller is simply $\phi(x)(v) = \bar{u}$ for all $v \in [0,1]$, i.e. offer the maximal price $\bar{u}$ for each good. All types accept the offer pertaining to their ideal good, and receive 0, much as under full market segmentation. Indeed, σ itself becomes completely irrelevant. Such a situation would best model competitive retailers, e.g. supermarkets, who clearly have the ability to commit to a menu of prices on all goods in their stock. Buyers rarely hesitate when telling a supermarket assistant what they are looking for.

A more moderate approach would be to ask what happens if the seller can offer bundles with *finitely* many goods, i.e. that there are more buyer types than there are different products on offer. This mirrors the *consideration set* approach discussed in Section 2.2. As with the complete menu case, offering multiple goods greatly helps the seller, allowing him to set higher prices with the knowledge that types will be forced to sort themselves more selectively through their acceptance rules. Hence, richer menus can be viewed as playing an analogous role to more finely segmented markets. The following result summarizes this intuition formally.

Proposition 4. *Suppose the seller can offer menus of k goods. Then*

- *The LSC IC-MS is clearing and N_k^*-partitional, with $N_k^* = \lceil \frac{1}{2k}\sqrt{\frac{3a}{\bar{u}}} \rceil$*
- *The LSC IC-MS is buyer-optimal.*

Proof. To prove the first part of the proposition, note that as before, the seller is unwilling to charge a price of lower than $\frac{2\bar{u}}{3}$, and hence we can derive the critical width as $\Lambda_k(a, \bar{u}) = 2k\sqrt{\frac{\bar{u}}{3a}}$.

The remainder of the construction is identical to that in the proof of Proposition 1.

For the second part, it is again without loss to focus on clearing equilibria. Let $0 = a_0 < \cdots < a_n = 1$ denote the boundary types on pooling intervals as before. Let $(v_{i,j}, p_{i,j}), i = 1, \ldots, n, j = 1, \ldots, k$ denote the seller's offer and $b_{i,j}, i = 1, \ldots, n, j = 1, \ldots, k$ be types indifferent between $(v_{i,j}, p_{i,j})$ and $(v_{i,j+1}, p_{i,j+1})$, so that $b_{i,k} = b_{i+1,1} = a_{i+1}$. Further, let $M_{i,j} = b_{i+1,j} - b_{i,j}$ be the widths of intervals accepting the same option within a menu, and $M_i = a_{i+1} - a_i$ the widths of pooling types. As before, it must be that each $b_{i,j}$ receives 0 surplus, and hence the seller's expected profit from a given menu can be computed as

$$\begin{aligned}\Pi(v_{i,j}, p_{i,j}) &= \sum_j M_{i,j}(\bar{u} - a(\frac{M_{i,j}}{2})^2) \\ &= \bar{u}M_i - \frac{a}{4}\sum_j M_{i,j}^3\end{aligned}$$

Refer to a menu $(v_{i,j}, p_{i,j})$ as *balanced* if $M_{i,j} = M_{i,j+1}$ for all $j = 1, \ldots, k$. That is, a menu is balanced if the seller's offer partitions each pooling interval into equal groups. We now prove that the seller's optimal offer is balanced. Suppose not, i.e there exists an i and r, s such that $M_{i,r} \neq M_{i,s}$. Without loss, and since the segmentation is clearing, we take $M_{i,r} < M_{i,s} \leqslant \Lambda_k(a, \bar{u})$. Consider a new menu $(v'_{i,j}, p'_{i,j})$ with $M'_{i,r} = M_{i,r} + \epsilon$, $M'_{i,s} = M_{i,s} - \epsilon$ for small $\epsilon > 0$, and all remaining $M_{i,j}$ unchanged. For small enough ϵ, such a menu is still clearing and hence is incentive compatible, whilst the above formula implies that the new menu yields greater profit to the seller.

Finally, using this result, the buyer's ex-ante welfare for a given clearing segmentation σ can be computed as

$$\begin{aligned}CS(\sigma) &= \frac{a}{6}\sum_{i=1}^{n} k\Big(\frac{M_i}{k}\Big)^3 \\ &= \frac{a}{6k^2}\sum_{i=1}^{n} M_i^3\end{aligned}$$

Notice that this expression is identical to that in the proof of Theorem 1 scaled by $\frac{1}{k^2}$, and

hence the result follows by the same argument. □

The key economic forces from the single-good benchmark remain in tact in this setting, since N_k^* remains finite and, for $a \geqslant \frac{4k^2\bar{u}}{3}$, greater than 1, and so the LSC IC-MS remains partially informative. Furthermore, N_k^* is decreasing in k, showing how as menus become richer, the buyer reveals less information.

C Alternative Refinements

In this section, we briefly discuss alternative refinements purpose-built for cheap talk games. As such, we use the language of the cheap-talk implementation for the remainder of this document, rather than that of market segmentations.

C.1 Neologism-Proofness

Section 2.4 showed how the buyer's preferred equilibrium carefully balances the gains from pooling that come through price discounts with the losses from pooling that come through product mis-match and the subsequent reduction in trade. In this section, we will show how a well-known refinement concept introduced specifically for cheap-talk games – the "neologism-proofness"concept of Farrell (1993) – operates through very similar forces. Although being one of the earliest and best-known refinement concepts for cheap-talk games, neologism-proofness typically suffers from non-existence as well as stability issues.[16] In the current setting, not only do these equilibria always exist and demonstrate a form of stability, but they admit a simple characterization (Proposition 5). In particular, they are all clearing and involve significant pooling, as was true for the LSC equilibrium.

Neologism-proofness starts with a putative equilibrium, and asks,"what would the seller do in response to facing some set of types G, and would said types prefer this response to the equilibrium?"If the answer leaves types in G better of, and if types not in G do not stand to gain

[16]For example, the leading example of Crawford and Sobel (1982) has no neologism-proof equilibria.

by joining G, then G is referred to as a self-signalling set. An equilibrium σ is neologism-proof if no such set exists.

Formally, for $G \subset \Theta$, let

$$\mathrm{br}(G) = \arg\max_{v,p} \int_G p \mathbb{I}_{u(v,\theta) \geqslant p} d\theta$$

That is, $\mathrm{br}(G)$ is the seller's best-response to the belief that he faces types G, given that they behave optimally at the accept/reject stage that follows and under the presumption that they indeed are in G.

Definition 4. Let $G \subset \Theta$. For all $\theta \in G$, we define

$$\underline{U}(\theta|G) \triangleq \min_{(v,p)\in \mathrm{br}(G)} \mathbb{E}_{\mathrm{br}(G)}(U(v,p,\theta))$$

$$\bar{U}(\theta|G) \triangleq \max_{(v,p)\in \mathrm{br}(G)} \mathbb{E}_{\mathrm{br}(G)}(U(v,p,\theta))$$

Given an equilibrium σ, a set $G \subset \Theta$ is **self-signalling** if

$$\underline{U}(\theta|G) \overset{a.s.}{>} \mathbb{E}_\sigma(U(v,p,\theta)) \;\; \forall \theta \in G$$

$$\bar{U}(\theta|G) \leqslant \mathbb{E}_\sigma(U(v,p,\theta)) \;\; \forall \theta \in \Theta \setminus G$$

σ is **neologism-proof** if no self-signalling set exists relative to it.

A few technical remarks are needed at this point. Firstly, if the seller's best response to G is not unique, we use the convention introduced by Matthews et al. (1990) - a deviating type assesses his worst-case deviation against the putative equilibrium, whereas a non-deviating type assesses his best-case deviation. I abuse terminology and maintain the moniker "neologism-proofness". This is not to undermine the contribution of Matthews et al. (1990), but simply because my adapted definition seems closer in essence to Farrell (1993). Secondly, I slightly adapt Farrell's original definition for finite type spaces to the current setting. I impose that at most a measure 0 of types in G are indifferent between the deviation and the equilibrium payoffs. Whilst this convention seems the most natural, how one takes a stand on this point turns out to be important for the

existence of self-signalling sets. Were I to impose that the deviation be strict *for all* types in G, then all equilibria would be neologism-proof.

Towards a complete description, note that in any equilibrium which admits some zone of separation, all types in that zone receive a zero payoff. Any interval subset G of that zone can serve as a profitable, self-signalling set so long as the seller serves that set entirely with their subsequent pricing decision. Proposition 1 guarantees that such a G exists, as long as its width is below the critical width $\Lambda(a, \bar{u})$. Thus, if an equilibrium is either semi-separating, fully-separating or residual, then it is not neologism-proof.

Proposition 5 provides the key features of NP equilibria. Firstly, it gives bounds on interval lengths. The first bound comes directly from Proposition 1, whereas the second comes from the fact that almost all types in adjacent intervals that could pool together and still form a clearing segment would strictly prefer such a situation, proven within the proof of Theorem 1. Hence, viewed as a deviating set G, this would constitute a self-signalling set. Combining these two bounds yields a tight bound on the number of intervals.

Proposition 5. *An equilibrium is neologism-proof if and only if* $|a_{i+1} - a_i| \leqslant \Lambda(a, \bar{u})$ *for all* $i = 0, \ldots, n-1$ *and* $|a_{i+2} - a_i| > \Lambda(a, \bar{u})$ *for all* $i = 0, \ldots, n-2$. *In particular, if an* n*-partitional equilibrium is neologism-proof, then* $n \in \{N^*, \ldots, 2N^* - 1\}$. *The LSC equilibrium is neologism-proof.*

Proof. We prove the result with a sequence of propositions. The main idea is to find necessary and sufficient conditions for a set G to be self-signalling.

Proposition 6. *Take a clearing* n*-equilibrium* σ *with boundary types* $0 = a_0 < \cdots < a_n = 1$. *If* $G \subset \Theta$ *is self-signalling wrt* σ*, then* $G = [a_j, a_{k+1}]$*, for some* $j \in \{1, \ldots, n\}$*,* $k > j$.

Proof. Clearly $G = [a_i, a_{i+1}]$ cannot be self-signalling, since σ is an equilibrium. By single-crossing, we may then restrict our attention to interval sets $G = [b, c], b < c$. Suppose b is not a boundary

type, i.e. $b \in (a_i, a_{i+1})$ for some i. For $\epsilon > 0$ and define

$$\mathcal{O}_\epsilon(b) = \{\theta \in [a_i, a_{i+1}] | |\theta - b| < \epsilon\}$$

$$U(\mathcal{O}_\epsilon^-(b); \sigma) = \inf_{\theta \in \mathcal{O}_\epsilon(b)} \mathbb{E}_\sigma(U(v, p, \theta))$$

$$U(\mathcal{O}_\epsilon^+(b); \sigma) = \sup_{\theta \in \mathcal{O}_\epsilon(b)} \mathbb{E}_\sigma(U(v, p, \theta))$$

For sufficiently small ϵ, we have that $\mathcal{O}_\epsilon(b) \subset (a_i, a_{i+1})$ and $U(\mathcal{O}_\epsilon^-(b); \sigma) = \delta(\epsilon) > 0$. Fix such an ϵ. Then we have that $U(b; br(G)) = 0$, and hence by continuity of U, there exists $\epsilon^* > 0$ such that $U(\mathcal{O}_{\epsilon^*}^+(b); br(G)) < \delta(\epsilon)$. That is, the positive measure of types $\mathcal{O}_{\epsilon^*}(b)$ are strictly worse off from the deviation. An identical argument shows that c must also be a boundary type. □

Proposition 7. *σ is NP if and only if σ is clearing and for all $G = [a_i, a_{i+2}]$, G is residual. Hence, if σ is babbling, then σ is NP if and only if it is clearing.*

Proof. For the first part, note that the $\Rightarrow$ implication follows directly from the proof of Theorem 1. For the $\Leftarrow$ implication, note first that if $[a_i, a_{i+2}]$ is residual, then by Lemma 1 so too is $[a_i, a_{i+k}]$ for all $k > 2$. Now take any residual G. We claim this cannot be a self-signalling set. To this end, fix an offer $(v, p) \in br(G)$. Since G is residual, there exists an open set $\mathcal{O}(v, p) = \{\theta \in G | E_{br(G)}(U(v, p, \theta)) = 0\}$, and hence $\mathcal{O}(v, p)$ cannot strictly gain from the deviation. Since $(v, p) \in br(G)$ was arbitrary, G cannot be self-signalling. The implication then follows from Proposition 6. □

Proposition 8. *An equilibrium is NP if and only if it is clearing and $[a_i, a_{i+2}] > \Lambda(a, \bar{u})$ for all consecutive boundary types a_i, a_{i+1}, a_{i+2}. In particular, if an n-partitional equilibrium is NP, then $n \in \{N^*, \ldots, 2N^* - 1\}$.*

Proof. The bounds are now immediate from Proposition 7. For the last part, a simple counting argument shows that the upper bound on the number of intervals is given by $\lfloor \frac{1-\Lambda(a,\bar{u})}{\Lambda(a,\bar{u})/2} \rfloor + 2$.

Applying Hermite's Identity (see Graham and Knuth (1994)), we have that

$$\begin{aligned}
\lfloor \frac{1-\Lambda(a,\bar{u})}{\Lambda(a,\bar{u})/2} \rfloor + 2 &= \lfloor \frac{2}{\Lambda(a,\bar{u})} \rfloor \\
&= \lfloor \frac{1}{\Lambda(a,\bar{u})} \rfloor + \lfloor \frac{2}{\Lambda(a,\bar{u})} + \frac{1}{2} \rfloor \\
&= N^* - 1 + N^* \\
&= 2N^* - 1
\end{aligned}$$

□

This completes the proof of Proposition 8, and thus Proposition 5 follows immediately. □

Finally, the concept of neologism-proofness is known to lack certain stability properties. Here, we employ the notion of *communication-proofness* as in Blume and Sobel (1995) - a concept invented specifically to study stability in the context of cheap-talk games - to determine whether in this context, NP equilibria are stable. In the bechmark model, an equilibrium is communication-proof if and only if it is clearing. Hence, all NP equilibria are communication-proof. In the language of Blume and Sobel (1995), the clearing equilibria are "good", and hence cannot be de-stabilized by other clearing equilibria. The NP set is thus a subset of the stable set with respect to this concept. Intuitively, any residual equilibrium is de-stabilized by one in which the set of types not served are served in intervals. Clearing equilibria cannot be ex-post Pareto-ranked; simply consider two clearing equilibria with different boundary types.

C.2 No Incentive to Separate

First, consider the *no incentive to separate* concept, recently introduced by
Chen, Kartik and Sobel (2008). An equilibrium σ satisfies *no incentive to separate* (NITS) if $\mathbb{E}_\sigma(U(v,p,0)) \geqslant \mathbb{E}_{\mathrm{br}(\{0\})}(U(v,p,0))$. That is, the lowest buyer type weakly prefers σ to revealing his type (if he could), and the seller best-responding to this revelation. It is immediately obvious that, in the current setting, such a refinement has no power. For if type 0 reveals himself truthfully,

the seller offers good 0 at price $\bar{u}$, which is accepted. Since type 0 gets 0 in this deviation, it is clear that any incentive-compatible strategy profile satisfies NITS.

C.3 Costly Lying

Here we study the implications of costly lying. In particular, fix $M = [0, 1]$ so that reporting one's type has meaning, and modify each type's payoff to be

$$U(v, \theta, m) = \bar{u} - a(v - \theta)^2 - kC(m, \theta)$$

where C is strictly decreasing for $m < \theta$ and strictly increasing for $m > \theta$. Further assume that $C(\theta, \theta) = 0$ so that the function C is readily interpreted as a cost of lying. Such a pertubation was studied at length in Kartik (2009) and was also discussed in Chen, Kartik and Sobel (2008). The latter discuss how, in games with monotonicity properties that the current game fails to satisfy, costly lying and NITS are closely related. This turns out to be far from the case in the current setting, as demonstrated by the following result.[17]

Lemma 3. *For any $k > 0$, the unique pure-strategy PBE is fully separating.*

Proof. First, to see that the fully separating equilibrium in which each type reports truthfully is a PBE, note that this profile incurs no lying costs, and hence the logic is unchanged from the benchmark analysis.

Next, we argue that any strategy profile involving pooling types cannot be part of an equilibrium. The lying cost $C(m, \theta)$ preserves single-crossing, and hence it is again sufficient to consider interval pooling. Suppose types θ_1, θ_2 are such that $[\theta_1, \theta_2]$ pool by sending message m. Now the boundary types receive $-kC(m, \theta_1), -kC(m, \theta_2)$ respectively. Suppose that $m = \theta_1$. Then type θ_2 receives $kC(\theta_1, \theta_2) < 0$, and hence can profitably deviate by reporting θ_2 and subsequently rejecting the seller's offer, thus securing himself 0. A similar argument rules out $m = \theta_2$ and $m \notin \{\theta_1, \theta_2\}$. □

[17] Technically, the statement of Proposition 6 in Chen, Kartik and Sobel (2008) still obtains, since the fully separating equilibrium trivially satisfies NITS.

D Generalizing the Buyer's Preferences

Horizontal differentiation is one of the two canonical models of consumer preferences over multiple products. The other - *vertical differentiation* - posits that all consumers have the same ranking over the product line. Of course, in reality, a combination of the two seems the most appropriate description. Many brands offer premium and budget lines for their products, with all consumers ranking the former higher, whereas within a given line, consumers might have subjective preferences across brands, e.g. a preference for Apple over Samsung based on compatibility concerns.[18]

For a start, take the standard model of vertical preferences, given by $u(v, \theta) = v\theta$ (see for instance Mussa and Rosen (1978)). In this setting, the conclusion of the analysis changes dramatically; no communication occurs in equilibrium. To see this, note that for a given offer (v, p), types $\theta \geqslant \frac{p}{v}$ accept, and types $\theta < \frac{p}{v}$ reject. For a given belief μ, the seller's profit from an offer (v, p) is then $\Pi(v, p|\mu) = \int_{\frac{p}{v}}^{1} p d\mu(\theta)$. Clearly then, offering $v = 1$ weakly dominates. Suppose the equilibrium was 2-partitional. Then the seller offers v for both intervals, at prices p_0, p_1. If $p_0 < p_1$, then types in μ_1 would strictly benefit by deviating and pooling with μ_0, and vice versa if $p_1 < p_0$. A similar argument holds for any equilibrium with some separation. Plainly put, when all types agree on the ranking of goods, they want the lowest price possible, knowing that the seller will offer the best good, leaving no room for effective communication.

Remark 2. *Suppose $u(v, \theta) = v\theta$. Then $\sigma \in \Sigma_{IC}$ if and only if it $\sigma = \tilde{x}$.*

That monotonicity in preferences leads to no communication is a well-known result in games with cheap-talk; Remark 2 is simply an expression of this[19]. A more natural exercise would be to allow for a more general $u \in \mathcal{C}^2(V \times \Theta)$, satisfying minimal conditions that combine both horizontal and vertical differentiation. To this end, we impose the following restrictions:

Assumption 1. (A1) $\frac{\partial u}{\partial v}|_{v=\theta} = 0$ (A2) $\frac{\partial u}{\partial \theta}|_{v=\theta} > 0$ (A3) $\frac{\partial^2 u}{\partial v^2} < 0$ (A4) $\frac{\partial^2 u}{\partial \theta \partial v} \geqslant 0$ (A5) $u(\theta, \theta) > 0$

[18] See Coles, Kushnir and Niederle (2013) for preferences that combine vertical and horizontal differentiation.
[19] See Sobel (2009) for a detailed discussion.

Properties (A1) and (A3) ensure that u is single-peaked and strictly concave in v. (A2) captures the idea that, although it is not true that all types share the same ranking, it is the case that higher v products entail potentially greater joint surplus. (A4) is a standard sufficient condition for single-crossing, and (A5) ensures gains from trade in every good.

How then do such restrictions affect the results from Sections 2 and C.1? The following result demonstrates the fragility of the fully separating equilibrium to even the smallest vertical perturbation. By (A2) the gain for type θ to deviate marginally to the left is first-order, whereas by (A1) the loss is second-order. This fragility provides yet more support for the efficacy of neologism-proofness as a refining concept in this setting.

Specifically, in a fully separating equilibrium, it must be that $s(v, p|\theta) = \big(\theta, u(\theta, \theta)\big)$ - by (A1) and (A3), $v = \theta$ uniquely maximizes $u(v, \theta)$. Suppose type θ considers a deviation to $\theta - \epsilon$, for some small $\epsilon > 0$. His payoff from accepting is then $u(\theta - \epsilon, \theta) - u(\theta - \epsilon, \theta - \epsilon)$. But for small ϵ,

$$u(\theta - \epsilon, \theta) - u(\theta - \epsilon, \theta - \epsilon) \approx \epsilon u_\theta(\theta, \theta) > 0$$

by (A2).[20]

Proposition 9. *If $u(v, \theta)$ satisfies Assumption 1, then a fully separating equilibrium cannot exist. Furthermore, there exists an M^* such that all partitional equilibria have at most M^* intervals.*

Proof. The first part is immediate from the preceding paragraph. For the remainder, we proceed with some propositions.

First, for $\theta \in [0, 1]$, let $b(\theta) \in [0, 1] \setminus \{\theta\}$ solve $u(b, b) = u(b, \theta)$ (if no solution to the equation exists in $[0, 1]$, set $b(\theta) = 0$). Then the function $b : [0, 1] \to [0, 1]$ is well-defined, and furthermore $b(\theta) \in [0, \theta)$ by (A2).

Define $\lambda : [0, 1] \to \mathbb{R}_+$ by $\lambda(x) = x - b(x)$.

[20] It should be noted that assumption (A2) is not necessary for the non-existence of full separation. All that is required is that there exists *a single* type θ for whom (A2) holds.

Proposition 10. *Given a partitional equilibrium σ, if $a_{i+1} - a_i < \lambda(a_{i+1})$, then $U(v_i, p_i, a_{i+1}) > 0$ and $U(v_i, p_i, a_i) = 0$.*

Proof. The last equality follows from the usual arguments forcing the lowest type IC to bind. Towards a contradiction, suppose $U(v_i, p_i, a_{i+1}) = 0$, i.e. $p_i = u(v_i, a_{i+1})$. Then there exists $v_m \in [a_i, a_{i+1})$ such that $x(\theta, v_i, p_i) = 0 \forall \theta \in [a_i, v_m], x(\theta, v_i, p_i) = 1 \forall \theta \in [v_m, a_{i+1}]$, i.e. v_m forms a cut-off type. Then the seller's profit is given by

$$\Pi(v_i, p_i) = \frac{p_i(a_{i+1} - v_m)}{a_{i+1} - a_i} = \frac{u(v_i, a_{i+1})(a_{i+1} - v_m)}{a_{i+1} - a_i}$$

Consider instead the offer $(v', p') = (v_i + \epsilon, u(v_i + \epsilon, v_m)$, for some small $\epsilon > 0$. By definition, v_m solves $(u(v_i, a_{i+1}) = u(v_i, v_m)$. Hence, for sufficiently small ϵ, $v' < v_m$, thus the acceptance set is the same as the original offer (v_i, p_i). The seller's profit becomes

$$\begin{aligned}
\Pi((v', p') &= \frac{u(v_i + \epsilon, v_m)(a_{i+1} - v_m)}{a_{i+1} - a_i} \\
&\approx \frac{\big(u(v_i, v_m) + \epsilon u_v(v_i, v_m)\big)(a_{i+1} - v_m)}{a_{i+1} - a_i} \\
&= \frac{u(v_i, v_m)(a_{i+1} - v_m)}{a_{i+1} - a_i} + \frac{\epsilon u_v(v_i, v_m)(a_{i+1} - v_m)}{a_{i+1} - a_i} \\
&> \Pi(v_i, p_i)
\end{aligned}$$

by (A2). Hence, (v', p') constitutes a profitable deviation for the seller.

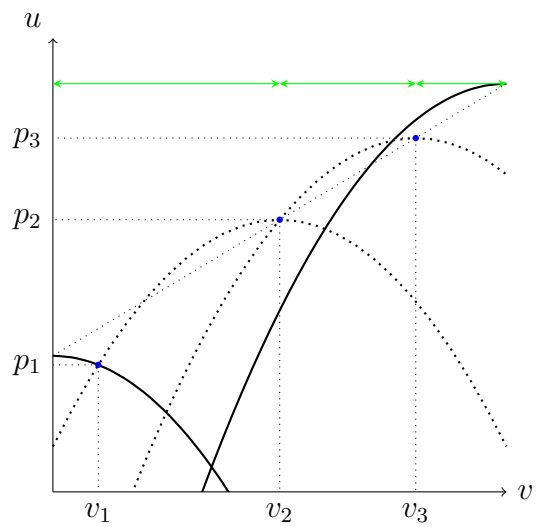
Set $\lambda(f) = \inf_{\theta \in \Theta} (\lambda(\theta)$. This is clearly well-defined, since $\lambda([0, 1]) \subset [0, 1]$.

□

Finally, to prove the proposition, suppose no such M^* exists. Then for large enough M, there must exist two consecutive intervals $[a_i, a_{i+1}], [a_{i+1}, a_{i+2}]$ such that $a_{i+j} - a_{i+j-1} < \lambda(f)\ \forall j = 1, 2$. By Proposition 10, type a_{i+1} receives positive surplus by pooling with $[a_i, a_{i+1}]$, but zero surplus by pooling with $[a_{i+1}, a_{i+2}]$, a contradiction. □

Much like in Crawford and Sobel (1982), M^* can be determined as the solution to a recursive

Figure 3: An M* equilibrium ($u(v,\theta) = 3 + \frac{3}{5}\theta - \frac{1}{5}(v-\theta)^2$)



difference equation. In this case, if a_{i+1} is a boundary type, then the solution to the difference equation

$$u(a_i, a_i) = u(a_i, a_{i+1}), \quad a_{i+1} \neq a_i$$

forms a lower bound for how close a_i can be to a_{i+1} in equilibrium. Figure 3 shows this construction graphically.

Whilst such a result reveals a technical connection to Crawford and Sobel (1982), the economic forces are quite different. In Crawford and Sobel (1982), whilst equilibria cannot be *ex-post* Pareto-ranked, it is the case that both sender and receiver *ex-ante* expected payoffs are increasing in the degree of equilibrium separation. Indeed, applications tend to use this as a criterion for selecting the most informative equilibrium, whilst many refinement concepts were proposed solely to select this outcome (see Chen, Kartik and Sobel (2008) for a detailed discussion). This is not the case here. Certainly, all residual equilibria are *ex-ante* dominated. However, within the class of clearing equilibria, as the degree of separation increases, the expected surplus transfers from the buyer to the seller.

Finally, does the "refining down"feature of Proposition 5 hold in this general setting? Consider

the case when $u(v,\theta) = a\theta - (v-\theta)^2$.[21] As $a \to \infty$, no solution to the equation $u(a_i, a_i) = u(a_i, a_{i+1})$ exists, preferences become fully vertical, and hence no communication occurs. It is vacuously false then that the most informative equilibria are refined away. This limit clearly embodies a convergence towards Remark 2. We can, however, recover the result if the vertical perturbation is sufficiently small. To formalize this, consider the function $f(\theta) = u(\theta, \theta)$ that describes the locus of local maxima of the function $u(v, .)$, and suppose $a > 0$ is such that[22]

$$a = \max_{\theta \in [0,1]} f'(\theta)$$

In the benchmark case, $f(\theta) = \bar{u}$ and $a = 0$. As a becomes small, the locus of maxima flattens, converging to the horizontal benchmark. As such, the first-order gain that prevents full separation starts to shrink, thus allowing finer partitions to emerge in equilibrium. Denoting this dependence as $M^*(a)$, we obtain the following generalizations of Proposition 5 and Theorem 1.

Proposition 11. *1. Suppose $u(v,\theta)$ satisfies Assumption 1, and $a > 0$ is defined as above. Then as $a \to 0$, $M^*(a) \to \infty$. Furthermore, there exists δ such that for all $a < \delta$, any $M^*(a)$-equilibrium is not NP.*

2. All LSC equilibria are NP. The ex-ante buyer optimal equilibrium is an LSC equilibrium. In particular, the NP set is non-empty.

Proof. For sufficiently small a, (A5) allows us to restrict attention to clearing equilibria. Using the definitions of the functions $b(\theta)$ and $\lambda(\theta)$ as in the proof of Proposition 9, we proceed in steps.

Proposition 12. *Take a boundary type a_i. As $a \to 0$, $b(a_i) \to a_i$.*

Proof. Let $a_{i-1} = b(a_i)$. Applying the Mean Value Theorem to $f(\theta)$ on the closed interval $[a_{i-1}, a_i]$,

[21] It is easily verified that this function satisfies Assumption 1.

[22] Note that since f' is continuous on a compact set, the max here is well-defined.

there exists $y \in (a_{i-1}, a_i)$ such that $f(a_i) - f(a_{i-1}) = f'(y)(a_i - a_{i-1})$. Hence

$$\begin{aligned}
|u(a_i, a_{i-1}) - u(a_{i-1}, a_{i-1})| &< |u(a_i, a_i) - u(a_{i-1}, a_{i-1})| \\
&= |f(a_i) - f(a_{i-1})| \\
&= |f'(y)||a_i - a_{i-1}| \\
&\leqslant \max_{\theta \in [0,1]} f'(\theta)|a_i - a_{i-1}| \\
&= a|a_i - a_{i-1}| \\
&\leqslant a \\
&\to 0
\end{aligned}$$

as $a \to 0$. The first inequality follows from (A1). The result follows from continuity of $u(., \theta)$. $\square$

In particular, the Mean value bound constructed in the proof of Proposition 12 says that the minimum interval width in an M^*-equilibrium scales with a, and hence $M^* \to \infty$ as $a \to 0$.

To prove the remainder of the proposition, take an M^*-equilibrium, and a sequence of boundary types a_i, a_{i+1}, a_{i+2}. We will show that the set $G = [a_i, a_{i+2}]$ forms a self-signalling set. Single-crossing (A4) ensures that types outside $[a_i, a_{i+2}]$ prefer the equilibrium, given it and G are clearing. By (A3), it is sufficient consider local IC constraints around the boundary types a_i, a_{i+2}.

First consider type $a_i + \epsilon$, for some small $\epsilon > 0$. In equilibrium, this type receives payoff $u(a_i, a_i + \epsilon) - u(a_i, a_i)$. Denote this $U_\sigma(a_i + \epsilon)$. Under the self-signalling deviation G, this type receives $u(v, a_i + \epsilon) - u(v, a_i)$, for some v; by (A4), we know that $a_i < v < a_{i+2}$, so let $\delta_1, \delta_2 > 0$

be such that $v = a_i + \delta_2 = a_{i+2} - \delta_2$. Then

$$\begin{aligned}
u(v, a_i + \epsilon) - u(v, a_i) &= u(a_i + \delta_1, a_i + \epsilon) - u(a_i + \delta_1, a_i) \\
&\approx u(a_i, a_i + \epsilon) + \delta_1 u_v(a_i, a_i + \epsilon) - \big[u(a_i, a_i) + \delta_1 u_v(a_i, a_i)\big] \\
&= U_\sigma(a_i + \epsilon) + \delta_1 \big[\underbrace{u_v(a_i, a_i + \epsilon)}_{>0} - \underbrace{u_v(a_i, a_i)}_{=0}\big] \\
&> U_\sigma(a_i + \epsilon)
\end{aligned}$$

Note that Proposition 12 validates the first-order approximation. Similarly, the payoff for type $a_{i+2} - \epsilon$ in equilibrium is $u(a_{i+1}, a_{i+2} - \epsilon) - u(a_{i+1}, a_{i+1}) = U_\sigma(a_{i+2} - \epsilon)$, whilst under G,

$$\begin{aligned}
u(v, a_{i+2} + \epsilon) - u(v, a_{i+2}) &= u(a_{i+2} - \delta_2, a_{i+2} - \epsilon) - u(a_{i+2} - \delta_2, a_{i+2}) \\
&\approx u(a_{i+2}, a_{i+2} - \epsilon) - \delta_2 u_v(a_{i+2}, a_{i+2} - \epsilon) - \big[u(a_{i+2}, a_{i+2}) - \delta_2 u_v(a_{i+2}, a_{i+2})\big] \\
&\geqslant U_\sigma(a_{i+2} - \epsilon) - \delta_2 \big[\underbrace{u_v(a_{i+2}, a_{i+2} - \epsilon)}_{<0} - \underbrace{u_v(a_{i+2}, a_{i+2})}_{=0}\big] \\
&> U_\sigma(a_{i+2} - \epsilon)
\end{aligned}$$

where the third inequality holds by (A4), since $u(a_{i+2}, a_{i+2} - \epsilon) - u(a_{i+2}, a_{i+2}) > u(a_{i+1}, a_{i+2} - \epsilon) - u(a_{i+1}, a_{i+1}) = U_\sigma(a_{i+2} - \epsilon)$. Hence only boundary types are left indifferent under the deviation G, and so G is self-signalling.

The proof of the second part of the proposition is immediate from the arguments in the preceding proof, which showed that the same necessary and sufficient condition for neologism-proofness – that no two intervals must be able to pool and face a clearing offer – still holds. □

Proposition 11 maintains the robust "refining down"feature of Proposition 5. Were we to impose further restrictions on the second derivative $\frac{\partial^2 u}{\partial v^2}$, the "refining up"feature might also be recovered. The second part demonstrates how the identity of the buyer's preferred outcome is also robust to these generalized preferences. Note that, in general, there might be many LSC equilibria.

E Seller Participation

We extend the results of Section 2.6 to allow for both endogenous seller participation as well as the ad-valorem fee α to be a choice variable for the platform. Specifically, suppose that the seller draws a participation cost $C \sim G(.)$, where G is defined over the support $[0, \bar{u}]$ and also satisfies the MLRP. Were the seller and buyer to make participation choices simultaneously, the model would admit multiple equilibria due to standard coordination frictions.[23] To avoid such issues, we adopt the standard approach of making the participation choice sequential – the seller first chooses to enter based on C, and observing this choice, the buyer chooses to enter based on K. Thus, the timing is now as follows:

1. Buyer privately draws $K \sim F(.)$, seller privately draws $C \sim G(.)$.
2. Platform chooses $\sigma \in \Sigma$.
3. Seller chooses participation.
4. Buyer chooses participation.
5. Buyer draws $\theta \sim \mathcal{U}[0, 1]$.
6. Buyer selects $x \in \text{supp}\sigma$.
7. Seller chooses an offer $\phi(x)$. Allocations realized as per $\phi(x)$.

We establish a generalization of Proposition 2. First, note that conditional on seller participation, the sub-game defined at the buyer's choice is identical to the previous analysis. Second, since the optimal choice of segmentation is independent of α, this choice is unaffected by endogenizing α. It remains to pin down α as a function of the distributions G and F, noting that the seller's choice to participate is of course reliant on his equilibrium conjecture of the buyer's choice.

Proposition 13. *Let K^* solve $K^* = \frac{2\bar{u}}{3} - \frac{F(K^*)}{f(K^*)}$. Let $C^*(K^*)$ solve $C^* = F(K^*)(\bar{u} - \frac{2}{3}K^*) - \frac{G(C^*)}{g(C^*)}$.*

[23] For instance, an equilibrium will always exist in which neither the buyer nor seller enter.

1. *If $K^* \leqslant CS^*(\sigma_{LSC})$, then σ is an equilibrium segmentation if and only if $\sigma \in \Sigma_{IC}$ is clearing and $CS^*(\sigma) = K^*$.*

2. *If $K^* \geqslant CS^*(\sigma_{LSC})$, then σ is an equilibrium segmentation if and only if $\sigma \in \Sigma_{LSC}$.*

3. *The optimal choice of $\alpha^* = \frac{G(C^*(K^*))}{g(C^*(K^*))F(K^*)(\bar{u}-\frac{2}{3}K^*)}$.*

Proof. Fix a buyer entry threshold K such that the buyer enters if they draw a cost lower than K. Then the seller's ex-ante profit is $F(K)(\bar{u}-\frac{2}{3}K)(1-\alpha)$ and so in equilibrium, $\alpha^* = 1-\frac{C^*(K^*)}{F(K^*)(\bar{u}-\frac{2}{3}K^*)}$. The platform's ex-ante profit for buyer and seller entry thresholds K, C is thus

$$F(K)G(C)\left(1-\frac{C}{F(K)(\bar{u}-\frac{2}{3}K)}\right)(\bar{u}-\frac{2}{3}K)$$

The first-order condition for K yields an equilibrium K^* as in Proposition 2 whereas the first-order condition for C yields an equilibrium $C^*(K^*)$ that satisfies

$$C^*(K^*) = F(K^*)(\bar{u}-\frac{2}{3}K^*) - \frac{G\left(C^*(K^*)\right)}{g(C^*\left(K^*\right))}$$

The expression for α^* follows immediately. Note that the second-order condition for $C^*(K^*)$ is satisfied as G has the MLRP. □

If F shifts to the right, we know that not only does K^* increase, but also that $F(K^*)(\bar{u}-\frac{2}{3}K^*)$ decreases. Thus, from the expression above for $C^*(K^*)$, we see that $C^*(K^*)$ increases. In particular, Prediction 1 still holds. Since α^* is independent of a, Prediction 2 also remains in tact. Prediction 3 is evidently independent of participation choices.

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