Selecting the wisdom of an expert

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Abstract

This paper considers uncertainty about bias and the endogenous information acquisition by an expert in strategic communication. We consider an expert who is privately informed about his bias as well as about whether he is informed, in addition can also engage in observable and costly information acquisition. In this setup, information acquisition simultaneously serves the purposes of getting informed and increasing credibility before communicating through cheap talk to a decision maker. We define the *signaling* and the *intrinsic value* of information and find the conditions under which separation in the information acquisition behavior can arise. Communication is most precise with an initially uninformed expert at an intermediary cost value, and the DM preferred level of expertise is non monotone in the cost. If the DM can choose the level of expertise, he does it in a way to achieve separation and there is no wasteful investment. The overall welfare increases as cost increases until enabling separation.

Keywords: information acquisition; cheap talk; communication; signaling; credibility.

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1 Introduction

In markets and organizations, decision makers often rely on the advice of experts. However, it is often difficult to assess the credibility of experts and the quality of their information. The literature on strategic information transmission pioneered by **?** mostly assumes that the expert's bias is known and that the expert has free access to information. These assumptions have been relaxed in different setups. Uncertainty about bias has been explored in the literature on the reputation of experts in repeated cheap talk initiated by **?** and **?**. Costly information acquisition before cheap talk was first introduced by **?** and recently **?** described this as an open question[1]. To study the interaction between bias and information acquisition, we consider an expert may or may not be biased and who may or may not be informed when the expert can observably acquire information, making it a signal of credibility. These ingredients make the problem one of costly signaling and cheap talk.

To understand the strategic consideration, contemplate a politician consulting a policy advisor about the effects of pollution on the occurrence of lung disease. The advisor may be getting kickbacks from a polluting industry and hence be biased. Maybe the advisor has no conflict of interest, but already has access to information due to his experience. Finally, it may be that the advisor, although unbiased, isn't informed on this issue but can pay a team of researchers to get information, which is costly. Complication arises because a biased advisor could also mimic this behavior and incur cost only to enhance the credibility of his advice. There are instances when experts incur costs in order to enhance their credibility while they are being backed by third parties. For instance, Andrew Wakefield, a former gastroenterologist and medical researcher, was found guilty of misconduct in his research paper that claimed the MMR vaccine was linked to autism and bowel disease, years after his research was published and had impact. In addition, it was claimed that he had been paid by lawyers who were trying to prove that the vaccine was unsafe.(**?**)

Similar issues arise when a division manager advising the CEO of his company may have a bias towards launching a product due to career concerns. The division manager may indeed know the potential of this product due to his expertise or may ask his team to carry out a market research, the cost being the time spent by the employees. Motivated by these examples where experts can overtly spend money, time or resources on information acquisition either to learn or to increase their credibility before communicating to a decision maker (DM), this paper studies how bias interacts with endogenous acquisition of information. When the decision maker cannot differentiate between biased and unbiased experts, what type of expert, in terms of how informed they are, is best for the decision maker? It turns out that a more informed expert is not necessarily better, and may indeed hurt the decision maker as this expert doesn't have enough incentive to separate himself from a biased expert. Indeed, there is a value to having an expert who is uninformed initially but is willing to acquire information and thus can be

[1] Some recent examples include **?**, **?** and **?**

identified as unbiased.

In the model the state of the world is $\omega \in \{0, 1\}$ and the DM's action space is $y \in [0, 1]$. The expert is biased with known probability β and with probability α he is already perfectly informed about the state of the world. The decision maker and the unbiased type of expert want the action to match the state of the world, while the biased type always wants the highest action regardless of the state. The incentives of the biased and unbiased expert informed of high state are identical as both want the highest action. At the outset, the expert has an option to make costly investment in information acquisition, which is observable even though its outcome is private to the expert. As there is a probability that the expert is already informed, an expert who does not invest in information does not reveal that he is uninformed, or that he is biased.

The equilibrium investment strategies of different expert types determine the DM's belief, and hence the credibility of communicating the high state. The expert cares about his credibility only insofar as it influences the DM's action: reputation is instrumental, not intrinsic. Investment in information serves to enhance credibility for the biased and informed unbiased experts who may wastefully invest in order to pool with uninformed unbiased types who genuinely want to get informed. Hence, even an unbiased expert who knows the state is high may end up investing in information to enhance his credibility.

We identify the *intrinsic value of information* for the unbiased uninformed expert and the *signaling value* for the biased and unbiased informed expert. If the signaling value is above the intrinsic value, then a biased expert always invests whenever an unbiased one does. Hence, we focus on the case when signaling value of information is less than its intrinsic value, in which case a separating equilibrium can arises for some cost values. The separating equilibrium is the most precise one, plus from an efficiency perspective, it is optimal if the only type that invests in information acquisition is the uninformed unbiased expert type, which is the case in the separating equilibrium. This is why an uninformed expert who gets informed later always communicates more precisely than an expert who is already informed that the state is high and cannot separate himself from the biased type who wants to pool with him.

In this equilibrium, the unbiased informed expert doesn't invest and pools with the biased type whenever the state is high. This highlights the value of an uninformed expert, as this type is the only one who can separate himself from the biased type in either state of the world, conditional on investment in information being optimal. In any other case, whether there is pooling in investment or no investment at all, an unbiased informed expert cannot separate from a biased expert when informed of $w = 1$. For the separating equilibrium to arise, there should be a sufficiently high probability that the expert is informed, α high enough, as this makes communication upon no investment more credible. In other words, the probability that the expert is already informed helps the biased type avoid too much prejudice by the DM when he sends high message without investment and decreases his incentive to pool by investing. In the separating equilibrium, the DM's welfare is decreasing in α, as the informed types pool with the biased type whenever the state is high. Hence, in an ex-ante sense, the DM's payoff is maximized when α is just high enough for separating equilibrium to realize but not higher. In an ex-post sense, the DM achieves a higher payoff if matched with an expert who is initially uninformed who separates himself from the biased type by investing in information.

The game also admits interesting mixed strategy equilibria in which the biased and unbiased informed (w=1) types are indifferent and play different probabilistic investment strategies. When the cost of information acquisition is less, proportionately more biased types than informed and unbiased types must invest in order to keep these types indifferent. In the separating region of cost, there is a unique mixed strategy equilibrium preferred both by the expert and DM. This equilibrium is dominated in terms of expert payoff by the separating equilibrium while the DM prefers the mixed strategy equilibrium. However, total welfare is higher in the separating equilibrium. Indeed, for a given cost of information acquisition, the DM's payoff is bounded by the separating equilibrium with the minimal portion of informed experts. This unique cutoff α^* gives the highest possible DM welfare among all pure and mixed strategy equilibria.

The DM's preferred expertise level, α^*,is non monotone in the cost of information acquisition. When cost is low enough that pooling arises even for α close to 1, expertise doesn't matter for the DM. When cost is higher, the DM prefers α to be just high enough to achieve separation. The intuition is as follows: conditional on the separating equilibrium arising, uninformed unbiased types who invest can perfectly communicate while the ex-ante informed unbiased types with high signa pool with biased types. Then, it is optimal to have just enough informed types to attain separation due to the loss in communication with these types in the high state. Indeed, at α^*, the separating equilibrium is also the best equilibrium (in terms of expert and DM payoff) over all mixed strategy equilibria. Hence, there is no issue of multiplicity and no wasteful investment when the DM can ex-ante pick a certain expert type. Interestingly, this optimal level of expertise decreases with the information acquisition cost. The reason is that there is a loss in communication from the ex-ante informed types pooling with biased types but some informed types are necessary to prevent biased types from pooling. As the cost of information rises, pooling becomes more costly so a lower credibility upon no investment (fewer informed types) suffices to prevent wasteful investment by these types.

When the cost is high enough that the separating equilibrium arises even if the expert is uninformed for sure, then $\alpha^* = 0$. When this is so, the biased types find it too costly to invest even if the DM perfectly identifies the expert who does not invest as biased. Indeed, if the separating equilibrium arises even with a perfectly uninformed expert, this type of expert maximizes DM welfare as all unbiased experts become informed and separate from biased types. Finally, when the cost is so high that no type will invest, an expert who is informed for sure is optimal. This follows because no one invests so that the DM cannot distinguish between a biased and an unbiased expert through investment behavior.

The key takeaway is that information acquisition is not just a way to become informed but also a way for expert types to signal credibility. When this is the case, a decision maker may not benefit from a more informed expert, and can even be hurt as this expert relying on his prior information lacks sufficient incentives to separate from a biased type. The paper emphasizes the value of an uninformed expert who is willing to invest in order to become informed and by doing so, signal that he is unbiased. Finally, if the decision maker can select an expert according to the level of expertise, he should do so in such a way that separation and there is no wasteful investment.

The model delivers interesting welfare results with respect to the cost of information. We show that less wasteful investment more than compensates for a higher cost of information acquisition when cost increase move the equilibrium from the pooling region to separating region, while the total welfare relating to decision making remains constant. Hence, total welfare is non-monotone in the cost of information: it decreases as cost rises in a given equilibrium, but if the increase enables separation, then total welfare increases.

Finally, we contrast outcomes in this overt information acquisition setting with the outcomes that emerge with covert information acquisition where the investment choice is not observable. Then there is no wasteful investment as informed types can pool with unbiased types without incurring any cost, but separation between biased and unbiased types is not possible whenever the state is high. Overall welfare is higher with overt information acquisition when costs are such that the separating equilibrium arises in overt information acquisition, due to more precise communication. Otherwise, when pooling in investment arises with overt information acquisition, covert information acquisition always leads to higher welfare because it results in less wasteful investment as there is no signaling.

2 Literature

This paper relates to several strands of the signaling and cheap talk literature. First, it relates to the literature on costly information acquisition in cheap talk. This was first considered by **?** where costly information acquisition that leads to perfect information is not observable and there is uncertainty about the cost of information acquisition. As the expert can prove he is informed but can feign ignorance, low types pool with uninformed types to achieve a higher outcome, which improves communication for higher types. Since **?**, only recently has there been work on strategic communication with endogenous information acquisition, mainly **?** and **?**. **?** considers the setup of **?** endogenising information acquisition and finds that the expert truthfully transmits all the information he acquires, in other words the expert doesn't acquire information that he will not transmit.[2] **?** also consider endogenous information where a biased expert can choose the issues on which to gather information and show that communication dominates delegation. **?** consider the effect of reputation in a delegation setup with information acquisition and an unbiased expert. They show that reputational concerns may help by incentivizing the expert to acquire information when he doesn't know his ability. In contrast if the expert privately

[2] **?** consider a game in which the expert has no bias and endogenous information acquisition, and show that restricting the message space can induce the sender to acquire information more often.

knows his ability, he may take inefficient actions in order to mimic an efficient type. **?** study the tradeoff between the incentive to acquire information versus the precision of communication as a function of expert's bias. There is no uncertainty about bias and hence no signaling motive for the expert in these models.

The paper also relates to the literature in cheap talk with uncertainty about expert's bias and reputational concerns. **?** and **?** consider repeated cheap talk. **?** consider strategic communication as in **?** with uncertainty about the expert's type. They show that truthful communication cannot arise even with an unbiased analyst whenever the state of the world is sufficiently high. Outside of the communication literature, **?** consider a long run player facing short run players who takes a payoff relevant action and highlight the distortional consequences today of the incentives to avoid bad reputations in the future. In contrast to our setting, these incentives arise either due to dynamics or to some intrinsic value of reputation. In this paper, the expert cares about the DM's belief about his type only insofar as his action can be influenced. However, in our setting the decision is taken only once but the observable information acquisition affects the credibility of the expert. Hence, messages are never distorted as decision making takes place only once, but distortion takes the form of wasteful investment in information acquisition. **?** considers a two period setup as in **?** and endogenizes the precision of the expert's information, where investment is not a signal as it is not observable. She finds that reputation building enhances the incentives to invest in information for both types in the first period. There is also a literature (e.g. **?** and **?**) showing that reputation concerns may lead to inefficient herding when experts bias their recommendation in order to appear more informed, where there is an intrinsic value of reputation. While our setup features neither dynamics nor an intrinsic value of reputation, similar effects to those found in the literature on reputation arises nonetheless due to the endogenous information acquisition and signaling incentives.

The wasteful investment in our setup is reminiscent of *burning money* identified by **?** where cheap talk is not the only way to communicate but senders may also incur loss in utility in order to enhance their communication. However, contrary to pure money burning, the aligned type of experts in our setup do value information per se.

3 Model

There is a decision maker (DM, she) and an expert (he). There is a state of the world $\omega \in \{0, 1\}$ and a commonly known prior $Pr(\omega = 1) = \rho$. At the beginning of the game, the expert learns his two dimensional private type. With commonly known probability $(1 - \beta)$ the expert shares the same payoff form as the DM, of $-(\omega - y)^2$ and with probability β he wants the highest possible action, with payoff $-(1 - y)^2$, where $y \in [0, 1]$ is the decision maker's action.[3] The expert's type is denoted by $\theta \in \{u, b\}$ corresponding to unbiased or biased. Second, with commonly known probability α the expert is perfectly informed about the state of the world ex-ante; while an

[3]This type of assumption about the expert type and utility is made by others (see e.g. in **?**).

uninformed expert shares the same prior as the DM. We interpret α as an expertise parameter[4], which relates to the experience that the expert may have had in the past derived from working on similar issues. The decision maker does not observe whether the expert is biased or informed, but knows α and β. In addition, the expert can optionally invest in information by incurring cost c to get a private signal that perfectly reveals the state. The DM observes the investment decision but not its outcome.[5] Indeed, any type of expert could invest in information, including a biased or an informed one for whom investment only serves the purpose of signaling. We will call such signaling wasteful investment. As the expert is already perfectly informed with probability α, the fact that he doesn't invest doesn't necessarily mean that he is uninformed, nor that he is biased. Finally, communication happens through cheap talk following the investment decision. Below is a summary of the stages of the game:

1. the state of the world, ω, and the expert's type is realised.
2. the expert decides whether to acquire a perfect signal by incurring c, a decision $x \in [0,1]$.
3. the expert sends a message $m \in M$ to the DM.
4. the DM takes an action, $y \in [0,1]$.

The decision maker interprets the expert's message as a function of her posterior about the expert's type, which depends on her prior and the information acquisition behavior of the expert. As the biased expert always wants the highest action regardless of the state, there is only 1 type of biased expert and whether he is informed or not has no relevance.[6] It could equivalently be assumed that the biased expert type is never informed. We can then summarize the types of experts at the beginning of the game into four:

1. biased
2. unbiased and uninformed
3. unbiased and informed with signal 1
4. unbiased and informed with signal 0

Call $\Phi \in \{0, 1, \varnothing\}$ the information structure of the expert at the communication stage as a result of his prior information and the information acquisition process. The expert's communication strategy denoted $m(\Phi, \theta)$ is pure as each type of expert always strictly prefers one of the three messages. The equilibrium concept is Perfect Bayesian Equilibrium (PBE). A strategy profile $\langle x, m, y \rangle$ along with the DM's posterior $\mu(x, m) = Pr(\omega = 1|x, m)$ forms a PBE if and only if:

- The DM's action maximizes her payoff given her posterior:

$$y^*(x, m) = \arg\max_y -\mu(x, m)(1 - y)^2 - (1 - \mu(x, m))y^2$$

[4]Bhattacharya et al. (2018) also make the assumption that the expert is either perfectly informed or uninformed, when looking at optimal composition of expert panels without information acquisition.

[5]That the information acquisition process is **observable** to the decision maker but not its **outcome** is a common assumption, (see e.g. Fischer and Stocken (2009), Argenziona, Severinov and Squintani (2016), Deimenn and Szalay (2017) who also consider information acquisition before cheap talk in different setups with known bias of the sender).

[6]This would be different were the biased type's payoff not state independent, as in **?**

- The expert's strategy, (x, m) maximizes his payoff given the DM's belief updating and decision making process.
- The DM's posterior $\mu(x, m)$ is consistent with the expert types' investment strategies and the prior ρ.
- The DM's optimal action is $y^*(x, m) = \mu(x, m)$.

4 Equilibrium Analysis

As this is a game of signaling followed by cheap talk, there are multiple equilibria. There always exists an equilibrium in which no one acquires information and the DM interprets any message as a babbling one. Other than this, depending on the cost, there exist equilibria with information acquisition. Whenever there is investment, we focus on the most informative equilibrium. When there exist multiple informative equilibria we use the Intuitive Criterion (**?**) to establish the most reasonable out-of-equilibrium belief. The next lemma summarizes the set of messages that are sent in equilibrium.

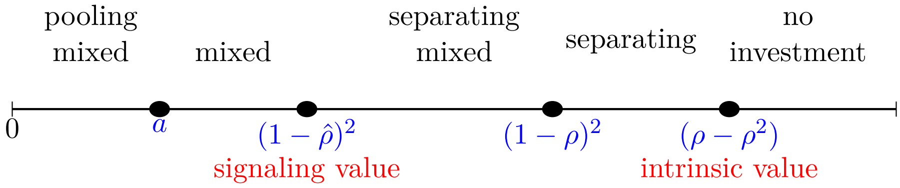
Lemma 1. *As the expert is either perfectly aligned or extremely biased, there is a unique optimal communication strategy conditional on investment or no investment. The set of messages sent in equilibrium after investment are $m^i \in \{0, 1\}$. Following no investment, the set of messages sent are $m^n \in \{0, 1, \varnothing\}$.*

If there is investment, it is sure that the expert is informed and either wants the highest or lowest action. If there is no investment, there is a possibility that the expert is uninformed and if he is unbiased, he wants to send $\varnothing$ indicating this. Given that the messages sent conditional on investment decision is clear, we now study the equilibrium investment behavior. First, we establish the behavior of the unbiased expert informed of low state.

Lemma 2. *The equilibria in which the unbiased types who are informed with signal 0 send message $m = 0$ without investing after which the DM chooses $y = 0$ are the only ones that survive the Intuitive Criterion.*

We restrict attention to equilibria in which communication by this expert type is perfect. This is rather trivial: as no other type wants to mimic this type he can credibly communicate without having to invest. Hence, from now on, we only have to deal with the equilibrium behaviors of the remaining expert types. In addition, we focus on equilibria in which the unbiased type of expert always sends a truthful message: whenever uninformed, this type sends $m = \varnothing$ and whenever informed he sends $m = \omega$, as this expert never has incentives to lie about the state. The only uncertainty about this type of expert is whether he will invest or not. As the biased and unbiased informed 1 types share the same payoff function, they follow the same information acquisition strategies in strict Nash Equilibria and always send message $m = 1$ regardless of

Figure 1: Equilibria as a function of cost, for given α and β



$\hat{\rho} = \mu(0, 1)$ in the separating equilibrium.
$\tilde{\rho} = \mu(1, 1)$ in the pooling equilibrium.

their investment strategy. That these types have the same incentives underlines many interesting results, in particular that an uninformed expert who becomes informed later communicates more precisely than an expert who is already informed and pools with the biased type in the high state. It also leads to interesting mixed strategy equilibria. Even though these types have the same payoff, what they do is important for the DM: the unbiased type is communicating truthfully while the biased type sends an uninformative message. The DM may be able to distinguish between these types in mixed strategy equilibria.

We define "*pooling*" and "*separating*" in this setup as a function of the investment decisionS of the two groups of expert types:

Group 1: biased type and unbiased informed type with signal 1
Group 2: unbiased uninformed type.

We list all the resulting equilibria with information acquisition for different levels of cost:

1. **Separating equilibrium:** Only the unbiased uninformed type invests so the DM takes communication at face value. Group 1 types send $m = 1$ without investing and DM chooses $y = \hat{\rho} > \rho$.

2. **Pooling equilibrium:** Both groups invest. The unbiased type sends $m = \omega$ while the biased type sends $m = 1$. Upon $m = 1$, the DM chooses $y = \tilde{\rho} > \hat{\rho}$.

3. **Mixed strategy (semi-pooling) equilibria:** Group 1 types play different probabilistic investment strategies that keep these types indifferent. As cost decreases, the portion of biased types who invest increases compared to unbiased types.

Figure 1 shows the regions for different equilibria as a function of the cost of information acquisition, highlighting that multiple equilibria exist in some regions. When $c \geq \rho - \rho^2$, the unique equilibrium has no investment. As a result we focus on the region $c < \rho - \rho^2$. We will primarily focus on the separating equilibrium as it is the most informative one. We will identify the conditions under which this equilibrium can arise. In a **separating equilibrium** the only types who invest are uninformed and unbiased ones, making information acquisition is efficient while unbiased experts who know w=1 and biased types pool together. This equilibrium exists only when the costs are low enough and the **signaling value** is lower than the **intrinsic value** of information. The signaling value of information is the gain in payoff for group 1 types from deviating from a separating equilibrium to invest and send message $m = 1$, given that the decision maker interprets the message as coming from a group 1 type (either a biased or informed (1) type, according to their probabilities). This gain is given by $-(1-\mu(1,1))^2 + (1-\mu(0,1))^2$, where $-(1-\mu(1,1))^2$ is the payoff upon investing while $-(1-\mu(0,1))^2$ is the payoff upon not investing, while sending $m = 1$ in either case. In the separating equilibrium, $\mu(0,1)$ is given by:

$$\hat{\rho} = \frac{\alpha(1-\beta)\rho + \beta\rho}{\alpha(1-\beta)\rho + \beta}$$

as the DM infers that the high message either comes from a biased agent (with probability $\beta\rho$) or from an unbiased informed (1) agent (with probability $\alpha(1-\beta)\rho$), she chooses $y = \hat{\rho}$. The payoffs to both the biased and unbiased informed expert types are $-(1-\hat{\rho})^2$. Upon deviating to invest, these types would induce the DM to choose $y = \mu(1,1) = 1$ and obtain a payoff of 0. Thus, $(1-\hat{\rho})^2$ is the signaling value of investment. The condition for this deviation not to be profitable for group 1 types is:

$$c \geq (1-\hat{\rho})^2 = \left[\frac{\beta(1-\rho)}{\alpha(1-\beta)\rho + \beta}\right]^2 \tag{1}$$

The intrinsic value of information is the payoff gain for the uninformed unbiased (group 2) types from getting informed when their message is taken at face value and obtaining a payoff of 0 after investment, compared to not investing and sending $m = \varnothing$, upon which the DM optimally chooses $y^* = \rho$ leading to a payoff of $-(\rho - \rho^2)$[7]. Then, this type may invest if $c \leq \rho - \rho^2$. Hence, the cost values for which the separating equilibrium exists is:

$$(1-\hat{p})^2 \leq c \leq \rho - \rho^2 \tag{2}$$

A separating equilibrium exists if and only if $\rho[\alpha(1-\beta)\rho + \beta]^2 - \beta^2(1-\rho) > 0$, which is increasing in ρ and α and decreasing in β. Low β and high α lower the biased type's incentives to invest and when $c \leq \rho - \rho^2$, the uninformed type is willing to invest. This is easier to satisfy when ρ is closer to 0.5, in other words when uncertainty is higher so the intrinsic value of information is higher. We will make the following assumption in the paper.

Assumption 4.1. $\rho_0 > 0.5$

The signaling value is at most $(1-\rho)^2$ which arises when $\alpha = 0$.[8] Then, for $\rho > 0.5$, $(1-\rho)^2 < (\rho - \rho^2)$ and there always exists a region of cost values for which the separating

[7] To see this, realise that the expected payoff when $\mu(0, \varnothing)$ and $y = \rho$ is given by: $-\rho(1-\rho)^2 - (1-\rho)\rho^2$ which leads to $-(\rho - \rho^2)$

[8] Without investment, the biased and informed (1) experts can at worse get the outcome $y = \rho$, when no information is transmitted to the DM

equilibrium arises for any α. This assumption gives us the richest set of equilibria, but our results do not depend on it.[9] wHEN $\rho < 0.5$, a separating equilibrium region of cost existS only when α is high enough.

Now, consider $c < (1-\hat{\rho})^2$. There cannot be a separating equilibrium as the biased type has an incentive to deviate and invest if the DM attributes the investment to an unbiased type. However, just below this threshold it cannot be that the group 1 types invest with probability 1 either, as THEN $\mu(1,1) < 1$ and the biased and informed (1) types do not find it profitable to invest. Then, only mixed strategy equilibria exist. Although mixed strategy equilibria also exist when $c > (1-\hat{\rho})^2$, in this region they are unique.

Mixed strategy (semi-pooling) equilibria: In this type of equilibrium, types in group 1 who have the same payoff function play different and possibly probabilistic investment strategies, determining the posterior for the decision maker and hence the incentives of these types themselves, while the uninformed type still invests with probability 1. Specifically, in this type of equilibrium the strategies of group 1 players must satisfy their own indifference condition. We use σ and γ respectively to denote respectively the probability that the biased sender and the unbiased type informed of w=1 invest, while both send $m = 1$. When these types are indifferent, the unbiased uninformed type strictly prefers to invest. In this equilibrium, the DM's posterior upon investment is $\mu(1,1) = \frac{(1-\beta)(\alpha+\gamma(1-\alpha))\rho+\beta\sigma\rho}{(1-\beta)(\alpha+\gamma(1-\alpha))\rho+\beta\sigma} > \hat{\rho}$ while upon no investment it is $\mu(0,1) = \frac{(1-\beta)\alpha(1-\gamma)\rho+\beta(1-\sigma)\rho}{(1-\beta)\alpha(1-\gamma)\rho+\beta(1-\sigma)}$.

The indifference condition is $c = (1-\mu(0,1))^2 - (1-\mu(1,1))^2$ which is equivalent to:

$$c = \left[\frac{\beta(1-\sigma)(1-\rho)}{(1-\beta)\alpha(1-\gamma)\rho+\beta(1-\sigma)}\right]^2 - \left[\frac{\beta\sigma(1-\rho)}{(1-\beta)(\alpha+\gamma(1-\alpha))\rho+\beta\sigma}\right]^2 \tag{3}$$

The equation reveals that σ and γ are complements and for a given c, hence there are infinitely many pairs of $\{\sigma, \gamma\}$ that form a mixed strategy equilibrium for a given c.

Mixed strategy equilibria exist for $c \in (k, (1-\rho)^2$. [11] In the region $c \in [(1-\hat{\rho})^2 - (1-\tilde{\rho})^2, (1-\hat{\rho})^2]$, mixed strategy equilibria are the unique equilibria while outside this region, they co-exist with pure strategy equilibria and survive the Intuitive Criterion.

Our focus is on the separating equilibrium region $c \in [(1-\hat{\rho})^2, (1-\rho)^2]$. In this region, mixed strategy equilibria feature $\mu(0,1) < \hat{\rho}$. As group 1 types strictly prefer not to invest when $\mu(1,1) = 1$ and $\mu(0,1) = \hat{\rho}$, $\mu(0,1)$ must decrease in order to make these types indifferent between investing or not. In other words, the no investment payoff must be worsened for the biased types. Then it should be that $\gamma > \sigma$: more unbiased informed types should invest in proportion to biased types.

[9][10]e consider the case with $(1-\rho)^2 > \rho - \rho^2$ in the appendix.

[11]$(1-\rho)^2$ is the highest cost level at which it is possible to have a mixed strategy equilibrium, as above this value it is impossible to have an equilibrium in which biased and/or informed (1) types invest, given that they prefer the prior which gives them $-(1-\rho)^2$ rather than incurring the cost and inducing the decision maker to choose the highest action 1. k is defined as the lowest value of cost for which there exists mixed strategy equilibria.

Lemma 3. *For $(1-\hat{\rho})^2 < c < (1-\rho^2)$ where mixed strategy and separating equilibria co-exist, there exists a unique mixed strategy equilibrium which maximizes both the DM's and the expert's welfare and has $\gamma > 0$ and $\sigma = 0$.*

Hence, among the mixed strategy equilibria there is agreement between expert types and the DM on the best one and from now on, when we talk about the mixed strategy equilibrium in this region, we will refer to this specific one.

When $c \geq (1-\rho)^2$, there exists no mixed strategy equilibria, as even for $\mu(1,1) = 1$, group 1 types are not tempted to invest. In the region $c < (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$, mixed strategy equilibria should have $\gamma\alpha + (1-\alpha) < \sigma$. Indeed, as cost goes down, investment should become relatively less attractive to ensure that the indifference condition is still satisfied. Hence, as c increases, $\frac{\mu(1,1)}{\mu(0,1)}$ should increase: communication after investment becomes relatively more credible. Finally, as $c \leq (\rho - \rho^2)$, *unbiased uninformed* types are indeed investing in this region.[12]

Next proposition summarizes our findings until now.

Proposition 1.

- *Given α, a separating equilibrium exists for cost values $(1-\hat{\rho})^2 \leq c \leq (\rho - \rho^2)$. As $\hat{\rho}$ is an increasing function of α, higher α leads to a wider separating region of costs.*
- *In the region $(1-\hat{\rho})^2 \leq c \leq (1-\rho)^2$, separating equilibrium coexists along with mixed strategy equilibria. In this region, separating equilibrium is the expert preferred and welfare maximizing while the mixed strategy is the DM optimal equilibrium.*
- *In $(1-\rho)^2 < c < (\rho - \rho^2)$, separating equilibrium is the unique equilibrium.*

It is useful to note that there exists an equilibrium in which all unbiased types invest, where $\sigma = 0$ and $\gamma = 1$. This equilibrium exists exactly at the cost value $c = (1-\rho)^2$, which is the maximum cost for a mixed strategy equilibrium to exist. In this equilibrium, we have $\mu(1,1) = 1$ and $\mu(0,1) = \rho$ and a group 1 type is indifferent between investing and not as $c = (1-\rho)^2$.[13] This is also the highest cost level for a mixed strategy equilibrium to arise. It is easy to see that this equilibrium is the ideal one for the decision maker, as he can perfectly identify biased types. However, a very specific condition on either c or ρ needs to be satisfied so this type of equilibrium will not be our focus.

Pooling Equilibrium: When the cost of information acquisition is low enough, a pooling equilibrium exists in which all types except the informed 0 type invest. When cost is even lower, the pooling equilibrium is the unique equilibrium.

Lemma 4. *Let $\mu(1,1) = \tilde{\rho}$ and $\mu(0,1) = \hat{\rho}$, then a pooling equilibrium exists when:*

[12] If $\rho > 0.5$, then $(1-\rho)^2 > \rho - \rho^2$ and hence this equilibrium couldn't arise for any value of c.

[13] Where $-(1-\rho)^2$ is the payoff from the outside option for the group 1 type of not investing, as $\mu(0,1) = \rho$ and payoff from investment is $-c$ as $\mu(1,1) = 1$.

$$c \leq (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$$

Because $\frac{\partial(1-\hat{\rho})}{\partial\beta} - \frac{\partial(1-\tilde{\rho})}{\partial\beta} > 0$, when β increases, so does the incentive of the biased type to invest making pooling more attractive.

Now, compare welfare among equilibria. It is easy to see that the DM gets his lowest surplus in the pooling equilibrium when she cannot differentiate between a biased and unbiased expert. It is not surprising that the DM's surplus increases past pooling into the separating region,. More surprisingly, total welfare also increases as summarised in the next proposition.

Proposition 2. *Total welfare increases when cost of information increases past* $(1-\hat{\rho})^2-(1-\tilde{\rho})^2$ *to the separating region* $(1-\hat{\rho})^2$*. That is, welfare is non-monotone in the cost: in a given type of equilibrium it decreases in cost while it increases in cost when the equilibrium moves from pooling to separating.*

We can separate total welfare into two parts: the parts related to decision making and those related to investment cost. When cost increases from the pooling region to the separating one, the welfare related to decision related value does not change, and the only change concerns the total cost of information acquisition. The group 1 types lose while the uninformed type and DM gain in terms of decision related value which exactly cancel out. There is more investment in the pooling equilibrium at a lower cost, while there is less investment in the separating equilibrium at a higher cost which more than compensates for the higher cost. This is an intuitive but non-trivial result. In both pooling and separating equilibria the uninformed unbiased expert is the only one to invest efficiently, so investment strategies of group 1 types do not affect the total amount of decision related value: the biased type doesn't use the information acquired while the unbiased type doesn't learn more than what he already knew. However, when moving to the separating equilibrium, some decision related value is transferred from the biased and unbiased informed (1) types to the unbiased uninformed type and the DM.

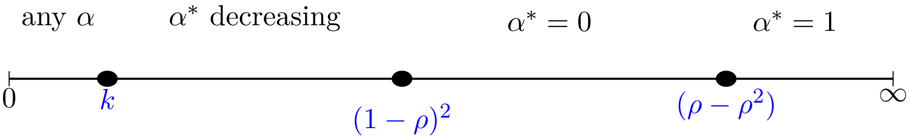
5 The DM Optimal Expertise Level

The model has implications for which expert type is best for the DM when she cannot distinguish between biased and an unbiased experts. The parameter we focus on is α, **expertise** parameter, which describes how likely an expert is to be informed. This parameter can be viewed as a measure of expertise based on an expert's prior experience.

Proposition 3. *For any* $c \in [k, (1-\rho)^2]$[14]*, there is a unique* α^* *which satisfies* $c = (1-\hat{\rho})^2$ *and provides the DM her highest possible payoff. At this point, the separating equilibrium is also the best mixed strategy equilibrium. This* α^* *decreases in cost implying as information costs rise it is optimal to have a less informed expert.*

[14] Where $k = (1-\hat{\rho})^2$ with $\alpha = 1$ which is the cutoff below which only pooling equilibrium exists for any α

Figure 2: α^* as a function of cost



The intuition is as follows: when $c < (1-\hat{\rho})^2$, the DM's welfare increases in α until the separating equilibrium is attained (as $\hat{\rho}$ increases in α) while conditional on the separating equilibrium arising, the DM's welfare decreases in α (when $c > (1-\hat{\rho})^2$. Hence, the DM's welfare is non monotone in α. That is, it is optimal that the expert's probability of being informed is just high enough to achieve separation, but not more. The proposition says that at the unique α^*, the DM can ensure the highest payoff regardless of equilibrium selection, as at this point the best mixed strategy equilibrium coincides with the pure strategy separating equilibrium. Hence, there is no issue of multiplicity or disagreement over equilibrium selection.

We next summarize the optimal expert type for all possible cost levels.

Proposition 4. *The optimal expertise parameter α for the DM is non monotone in c:*

- *For $c > \rho - \rho^2$, $\alpha^* = 1$: in this region, no expert type is willing to invest. Thus a perfectly informed expert is optimal.*
- *For $(1-\rho)^2 < c \leq \rho - \rho^2$, $\alpha^* = 0$: the separating equilibrium is the unique equilibrium for any α in this region, it is optimal to have an uninformed expert.*
- *$k < c \leq \rho - \rho^2$, the optimal α^* solves $(1-\hat{\rho})^2 = c$ where $\hat{\rho}$ is an increasing function of α. As c increases, α^* decreases.*
- *$c < k$, expertise doesn't matter as the sole equilibrium for any α is the pooling equilibrium.*

The DM's optimal α is non-monotone in the cost of information acquisition. The intuition is as follows: whenever $(1-\rho)^2 \leq c \leq \rho - \rho^2$, an unbiased uninformed expert strictly prefers to invest while they are the only ones to do so while informed (1) or biased experts strictly prefer not to invest even if $\mu(1,1) = 1$. Then, it is optimal to have $\alpha = 0$, as having informed experts leads to a loss in communication due to pooling with biased type in the high state. As the DM can perfectly identify the type of the expert when there are no informed types, in this region it is strictly optimal to have a perfectly uninformed expert.

However, once cost is lower, so that $c < (1-\rho)^2$, it is no longer possible to have separation with $\alpha = 0$, as biased expert would then have an incentive to invest, which would lead to $\mu(1,1) < 1$. Now, to induce separation, there should be sufficient portion of unbiased types who are informed, which gives the minimum α subject to separation, given by the condition $c = (1-\hat{\rho})^2$. A lower α leads to worse payoff for the DM as some biased types would invest. Moreover, for any α that exceeds this critical level, the separating equilibrium leads to a lower payoff for the DM while the best mixed strategy equilibrium always leads to the same payoff as the pure strategy separating equilibrium with α^*, as in the best mixed strategy equilibrium, some informed types reinvest. At α^* there is no issue of multiplicity. It follows that there is no point in raising the probability that the expert is informed pas this critical level. If, in addition, we had a setting in which experts demanded a fee as a function of their expertise parameter, this would reinforce why DM shouldn't pay for an expert more informed than this level.

The model predicts that when the bias of an expert is unobservable and information acquisition is endogeneous, a more informed expert isn't necessarily better for the DM. Indeed, in equilibrium the only type of unbiased expert who can communicate perfectly is the one who gets informed later by investing. However, having some probability that the expert is already informed is also necessary to keep biased types from investing, by avoiding too much prejudice against an expert who communicates high state without investing. This is why, over a region of cost, the optimal expertise parameter decreases in the cost of information acquisition. Expertise in this setup is valuable only as far as it discourages pooling by the biased types. When the cost is high enough that separation is achievable with a perfectly uninformed expert, then this is the optimal type of expert. Unsurprisingly, if the cost is so high that no type would invest, then a perfectly informed expert is optimal.

6 Covert information acquisition

Now we consider covert information acquisition, in other words when the decision maker does not observe the investment made by the expert. In this setup, as investment has no signaling value, group 1 types never invest as they can pool with the unbiased uninformed expert without incurring any cost. The only type that may invest is the unbiased uninformed type. There are 2 types of pure strategy equilibria as a function of the cost which as summarized below:

1. Investment takes place by the uninformed unbiased type. Upon $m = 0$, the DM chooses $y = 0$ and upon $m = 1$, the DM takes action $y = \tilde{\rho}$. This equilibrium looks like the pooling equilibrium except that the biased and informed (1) types do not actually invest. The payoffs of the biased and informed (1) types are higher in this equilibrium compared to the pooling equilibrium discussed, as they achieve the same outcome without having to incur an investment cost.

   The condition for the unbiased uninformed type to invest is:

   $$c \leq \rho - \rho^2 - \rho(1 - \tilde{\rho})^2$$

   This means the unbiased type acquires information for a smaller range of cost values in the covert information acquisition than in the overt case, as this cutoff is above the pooling cutoff cost but below the separating equilibrium cutoff cost of the overt case. [15]

[15]Realize that in this region, there are as well mixed strategy equilibria in which the unbiased uninformed expert invests with a probability. This probability should increase as c decreases to keep the indifference condition

2. No investment takes place. Upon $m = 1$, the DM chooses $\hat{\rho}$ inferring that this message is sent by a group (1) type of expert. The uninformed expert sends $m = \emptyset$ and the DM chooses $y = \rho$. This equilibrium is equivalent to the no investment equilibrium in the overt information acquisition case. This equilibrium arises for the following cost values:

$$c > \rho - \rho^2 - \rho(1 - \tilde{\rho})^2$$

The types that gain from information acquisition being covert as opposed to overt are the group 1 types and only in case the cost is low enough that the uninformed unbiased type acquires information. Even though the cost does not affect these types directly as they don't acquire information, the investment of the unbiased uninformed type makes their message freely more credible. In this equilibrium, the payoffs of the uninformed unbiased expert and the decision maker are the same as in the pooling equilibrium in the overt case while the payoff of the biased and informed (1) expert are higher only because they don't invest. When cost increases, we move to the no investment equilibrium in which payoffs are identical to the overt case without investment.

The unbiased type invests in information less often and is worse off in covert case, as he can never perfectly separate himself from the biased type. This result shows that, even though the signaling value of information undermines its intrinsic value, under certain parameters overt information acquisition still does strictly better than the covert one, specifically the separating equilibrium does exists in the overt information acquisition.

However, when the cost of information acquisition is low enough that in the overt case pooling in investment arises, then covert information acquisition does better in terms of the overall welfare as there is no wasteful investment, although the precision of communication is identical. Then the next corollary follows.

Proposition 5. *Whenever $c \geq (1 - \hat{\rho})^2 - (1 - \tilde{\rho})^2$, overt information acquisition leads to higher overall welfare while whenever $c \leq (1 - \hat{\rho})^2 - (1 - \tilde{\rho})^2$, covert information acquisition leads to higher welfare.*

The tradeoff is between more informative communication versus wasteful investment in information. Whenever there is no investment in the covert case, overt information acquisition does better as we have seen that welfare is always higher when some types invest than when there is no investment. However, when cost is low enough that there is pooling in investment in the overt case, total welfare is higher in the covert case as although the communication precision is equivalent less cost is incurred. This is because in both cases, all the unbiased types are informed but cannot be separated from biased types, plus in the overt case there is more wasteful investment.

satisfied.

7 Conclusion

This paper studies the interaction of credibility concerns and costly information acquisition and its implications for decision making, by using a simple model building on the communication and signaling literatures. It is found that an unbiased expert, as well as a biased one, may wastefully invest in information acquisition, which leads to inefficiencies in decision making and lowers overall welfare. Indeed an uninformed expert who later becomes informed always communicates more precisely than an ex-ante informed expert. This is due the inability of the unbiased type to separate himself from a biased one whenever he knows that the state of the world is high whereas the uninformed type can perfectly separate by being the only one to invest in information. As such, our results highlight the value for an expert to be uninformed but willing to incur cost in order to become informed. When considering what type of expert is optimal for the decision maker assuming he can pick one, we showed that the optimal level of expertise is non monotone in the cost. Another interesting result was that higher information acquisition cost increases the overall welfare when the equilibrium moves from the pooling into the separating region.

The simplicity of the model allows for numerous extensions remaining for future research. An interesting feature of the model is that even though the ability to acquire information is not correlated with the type of the expert, the value of getting informed depends on the expert's type. Hence, it is possible to use information acquisition as a screening device by taxing the experts who get informed. Assume the cost falls in the pooling region. We know that the expert who is uninformed and unbiased has the highest incentives to acquire information. Hence, this type will be willing to pay more than biased and informed (1) experts so that wasteful investment by these types can be prevented by taxing experts for getting informed.

8 Appendix A

The No Investment Equilibrium cutoff:

When $c \geq \rho - \rho^2$ even the unbiased uninformed type doesn't want to invest. The right hand side is the boundary at which the separating equilibrium starts, in which the only type that invests is the unbiased uninformed type. We know that in the region $(1-\hat{p})^2 \leq c \leq \rho - \rho^2$, the biased type doesn't find it profitable to invest, even if his message were taken at face value. Then, in the region $c \geq \rho - \rho^2$, by the intuitive criterion, the out of equilibrium belief upon investment should place probability 0 to the expert being a biased type, as this type can never gain from deviating to invest even if he were to be perceived to be an unbiased type. Hence, the message upon a deviation to invest should be taken at face value by the DM. The gain in utility from doing so for the uninformed unbiased type is $\rho - \rho^2$ and defines the cost value above which the unique equilibrium has no investment.

Welfare Comparisons

The DM's payoff

It is easy to see that the DM's payoff is minimized in the equilibrium in which no type invests, which is the least informative equilibrium. Hence, presence of investment can only make communication more informative. Plus, it can be easily seen that separating equilibria are better than pooling ones for the DM.

Separating equilibrium:

$$[(1-\beta)\alpha\rho + \beta][-(\hat{p} - \hat{p}^2)] = -\hat{p}\beta(1-\rho) \tag{4}$$

This is decreasing in α and β which both lead to higher $\hat{p}$. As α increases, more of the informed unbiased types will be pooled with biased types, and as β increases, facing a biased type becomes more likely. The payoff is also increasing in ρ whenever $\rho > 0.5$. This is intuitive: the DM's loss from biased communication is less when the prior already favors the action that the biased type of expert wants.

Pooling equilibrium:

$$[(1-\beta)\rho + \beta][-(\tilde{p} - \tilde{p}^2)] = -\rho(1-\tilde{p}) \tag{5}$$

This is decreasing in β and when $\rho > 0.5$, it is increasing in ρ. When $\rho < 0.5$, and β high enough it may be decreasing in ρ. The pooling payoff is independent of α, as all unbiased types do invest in this case and the DM doesn't internalize the cost of investment. The separating equilibrium payoff dominates the pooling equilibrium payoff for the DM iff:

$$\alpha(1-\rho)+\rho \leq 1$$

which is always satisfied. Hence, the DM's payoff is unambiguously higher in the separating equilibrium in which information is more precise. The DM benefits from having some probability of expert being initially informed and pool with the biased type, as this makes pooling less attractive for the biased type. If α is very low, there are more incentives to "pool" and the separating region shrinks. However, inside the separating region, the DM's payoff is decreasing in α. Hence, the DM's payoff is non-monotone in α: it has to be just high enough for biased types not to invest.

No investment equilibrium:

The DM's utility in the equilibrium in which no one invests is:

$$\frac{-(1-\rho)\rho(\beta+(1-\alpha)\alpha(1-\beta)^2\rho)}{\beta+\alpha(1-\beta)\rho} \tag{6}$$

which is found by simplifying $-[\beta+(1-\beta)\alpha\rho](\hat{p}-\hat{p}^2)-(1-\beta)(1-\alpha)(\rho-\rho^2)$. We see that pooling in investment always leads to higher payoff for the DM than no investment. Hence, the no investment equilibrium provides the minimum possible payoff to the DM.

Mixed strategy equilibria

The DM's payoff given σ and γ is:

$$-(\beta(1-\sigma)+(1-\beta)\alpha\rho(1-\gamma))(\mu(0,1)-\mu(0,1)^2)-((1-\beta)(\alpha\gamma+1-\alpha)+\beta\sigma)(\mu(1,1)-\mu(1,1)^2)$$

which simplifies to:

$$-\beta(1-\rho)[(1-\sigma)\mu(0,1)+\sigma\mu(1,1)]$$

At a given cost c, the DM optimal equilibrium, which also coincides with expert optimal mixed strategy equilibrium is the one in which $\sigma=0$ and γ takes the value which satisfies:

$$c=(1-\mu(0,1))^2$$

The expert's payoff

Separating equilibrium:

The payoff of the biased expert is $-(1-\hat{p})^2$, which is increasing in α and ρ and decreasing in β. The ex-ante payoff of the unbiased expert is $-\alpha\rho(1-\hat{p})^2 - \alpha(1-\rho)0 - (1-\alpha)c = -\alpha\rho(1-\hat{p})^2 - (1-\alpha)c$, which is increasing in α, decreasing in ρ and decreasing in β.

Then, the expected payoff over expert types in the separating equilibrium is:

$$-(1-\hat{p})^2[\beta + (1-\beta)\alpha\rho] - (1-\beta)(1-\alpha)c$$

Pooling equilibrium:

The payoff of the biased expert is $-(1-\tilde{p})^2 - c$. This is increasing in ρ and decreasing in β. The ex-ante payoff of the unbiased expert is $-\rho(1-\tilde{p})^2 - (1-\alpha(1-\rho))c$. This is decreasing in ρ and in β. It is increasing in α as less cost will have to be incurred.

It is trivial that the biased type of expert is unambiguously better off in the pooling equilibrium than in the separating one, as otherwise he would not invest and still get the same payoff as in the separating equilibrium. On the other hand, the unbiased expert's expected payoff increases when moving from the separating to the pooling equilibrium, when we compare the payoff at the minimum cost at which there is separation and maximum cost at which there is pooling.

The expected payoff over all expert types in the pooling equilibrium is:

$$-[(1-\beta)\rho + \beta](1-\tilde{p})^2 - [1-(1-\beta)(1-\rho)\alpha]c$$

Mixed strategy equilibria

The expected payoff over expert types in a mixed strategy equilibrium is:

$$-(\beta + (1-\beta)\alpha\rho)(1-\mu(0,1))^2 - (1-\beta)(1-\alpha)\rho(1-\mu(1,1))^2 - (1-\beta)(1-\alpha)c$$

The above is found by using the fact that the biased and informed (1) types of experts are indifferent between investing or not, and taking their payoffs as they didn't invest. This simplifies, if we replace $c = (1-\mu(0,1))^2 - (1-\mu(1,1))^2$, to:

$$-[(1-\beta)(\alpha\rho + 1 - \alpha) + \beta](1-\mu(0,1))^2 + (1-\beta)(1-\alpha)(1-\mu(1,1))^2$$

From the above, we can see that the expert preferred mixed strategy equilibrium has $\sigma = 0$, and $\gamma > 0$.

The case with $\rho < 0.5$, hence $(1-\rho)^2 > \rho - \rho^2$

In this case, there is no region of cost for which separating equilibrium arises regardless of α, as was the case in the main assumption of the paper. However, again if α is high enough such that $(1-\hat{\rho})^2 < \rho - \rho^2$, then for $(1-\hat{\rho})^2 < c < \rho - \rho^2$ the pure strategy separating equilibrium will realize. Plus, whenever $c > (1-\rho)^2$, no type is going to invest. Indeed, even though the cost is below $(1-\rho)^2$, as long as the uninformed unbiased types are not willing to, the group 1 types will not invest.

Under this assumption, the main distinction compared to the case in the paper is that there is not a region in which $\alpha^* = 0$, as whenever the uninformed type is willing to invest, the group 1 types are strictly willing to invest.

Whenever $c > \rho - \rho^2$, $\alpha^* = 1$, as in this region the uninformed types will never acquire information.

When $c \leq \rho - \rho^2$, it is optimal to set $c = (1-\hat{\rho})^2$. In this region, α is decreasing in cost.

As in the case with $\rho > 0.5$, again until k, which is the cost value less than which only pooling equilibrium can arise, for any value of α. Hence, the main results do not change if $\rho < 0.5$ however there is less diversity in terms of equilibria.

9 Appendix B

Proof of lemma (1):

Proof. In the communication stage following investment, it is certain that the expert is informed. We can then focus on equilibria in which after investment there are at most two messages, $m^i \in \{0,1\}$. To see this, realize that the unbiased expert either knows $\omega = 0$ and wants the lowest possible action or knows $\omega = 1$ and wants the highest possible action. The biased expert wants the highest possible action regardless of the state, hence strictly prefers $m = 1$.

In the communication stage following no investment, there are four possible types of experts. First, the informed expert with signal 0 who strictly prefers the message which induces the lowest action hence will send $m = 0$. Second, the biased and unbiased informed (1) expert both want to send the message that induces the highest action hence send $m = 1$. Finally, the unbiased uninformed expert who shares the DM's preferences. It is then without loss of generality to restrict attention to equilibria in which an empty message is available, $m = \varnothing$, which means "I am not informed" and will induce $y = \rho$.[16]. Then, following $x = 0$, the set of messages is $m^n \in \{0, 1, \varnothing\}$.

□

Proof of lemma (2)

[16] As it becomes clear later, the biased expert can always induce a higher posterior by sending $m = 1$ as opposed to any other message

Proof. The unbiased informed (0) type's payoff is maximized when $y = 0$ is chosen by the DM and there is no other type that could benefit from sending this message, given the set of other possible messages. For the biased and informed (1) types, this message leads to the lowest possible payoff of -1 and they are better off sending any other message. For the unbiased uninformed type who has not invested, sending $m = \varnothing$ upon which the DM chooses $y = \rho$ can only lead to a better payoff than sending this message. Given that, it is consistent that the DM infers $m = 0$ as coming from an unbiased informed type, $\mu(0,0) = 0$ and will indeed choose $y = 0$. Finally, the informed 0 expert achieves maximal payoff under this strategy and cannot be better off by sending the same message as another type or by incurring the cost of investment. For contradiction, assume there were any other equilibrium in which this type sent another message, or it invested. Then, it must be that his equilibrium payoff is lower than 0. However, in any such equilibrium, it would have an incentive to deviate to not invest and send $m = \varnothing$. By the intuitive criterion, DM attributes this message to the informed 0 type, then he does have an incentive to deviate and no other type has an incentive to do so. □

Proof of lemma *mixed*

Proof. The expected payoff over all expert types in a mixed strategy equilibrium is:

$$-(\beta + (1-\beta)\alpha\rho)(1-\mu(0,1))^2 - (1-\beta)(1-\alpha)\rho(1-\mu(1,1))^2 - (1-\beta)(1-\alpha)c$$

We can see that the expert preferred mixed strategy equilibrium has $\sigma = 0$, and $\gamma > 0$. This is because for the uninformed expert who is getting informed, his payoff is maximized when $mu(1,1) = 0$. For the group 1 types, their payoff is also maximized when $\mu(1,1)$ takes the maximum value as they are indifferent between investing or not.

The DM's payoff given σ and γ is:

$$-(\beta(1-\sigma)+(1-\beta)\alpha\rho(1-\gamma))(\mu(0,1)-\mu(0,1)^2)-((1-\beta)(\alpha\gamma+1-\alpha)+\beta\sigma)(\mu(1,1)-\mu(1,1)^2)$$

which simplifies to:

$$-\beta(1-\rho)[(1-\sigma)\mu(0,1) + \sigma\mu(1,1)]$$

Which is also maximized when $\sigma = 0$. Finally, for a given $c < (1-\rho)^2$, the DM optimal mixed strategy equilibrium is the one in which $\sigma = 0$ and $\gamma > 0$ such that:

$$c = (1-\mu(0,1))^2 \tag{7}$$

Then, at any c, there is a unique welfare maximizing mixed strategy equilibrium with $\sigma = 0$ and $\gamma > 0$. □

Proof of proposition (1)

Proof. We find the separating equilibrium to be the unique pure strategy equilibrium in $(1-\rho)^2 < c < (\rho - \rho^2)$. First, there cannot be any equilibrium in this region in which unbiased informed (1) and biased types invest with positive probability, as even if $\mu(1,1) = 1$, the group 1 types not find it profitable to invest as the cost is too high. Then, the only pure strategy equilibrium that could arise is the no investment equilibrium in which even the uninformed unbiased expert doesn't invest. For some out of equilibrium beliefs, this equilibrium can arise, as discussed below.

Assume that the DM believes any type except the informed (0) unbiased one is equally likely to invest, then his belief and optimal choice will be $\tilde{p}$ which is:

$$\tilde{p} = \frac{\rho(1-\beta) + \beta\rho}{\rho(1-\beta) + \beta} = \frac{\rho}{\rho(1-\beta) + \beta} \tag{8}$$

Let us show the biased type doesn't have the incentive to incur the cost c. Now, upon the message $m = 1$, the DM chooses $y^* = \hat{p}$ as there is only biased and unbiased informed(1) types who choose $m = 1$. For the biased and unbiased informed(1), the payoff from investing should be less than that from not investing:

$$(1-\tilde{p})^2 + c \geq (1-\hat{p})^2 \tag{9}$$

As without investment and $m = 1$, the DM's belief is $\hat{p}$ as in case 1. This is equivalent to:

$$\left[\frac{\beta(1-\rho)}{\rho(1-\beta) + \beta}\right]^2 + c \geq \left[\frac{\beta(1-\rho)}{\alpha(1-\beta)\rho + \beta}\right]^2 \tag{10}$$

For the uninformed agent, the payoff from not investing is $-(\rho - \rho^2)$ and from investing it will be:

$$-(1-\rho)0 - \rho(1-\tilde{p})^2 - c \tag{11}$$

Then the condition that should be satisfied is:

$$c \geq \rho - \rho^2 - \rho(1-\tilde{p})^2 \tag{12}$$

Finally, the equilibrium in which no type wants to invest, for the specified out of equilibrium beliefs, is:

$$c \geq \max\{\rho - \rho^2 - \rho(1-\tilde{p})^2, (1-\hat{p})^2 - (1-\tilde{p})^2\} \tag{13}$$

Now consider that when there is a separating equilibrium, the condition $\rho - \rho^2 > (1-\hat{p})^2$ is satisfied. Then, it is the case that $\rho - \rho^2 - \rho(1-\tilde{p})^2 > (1-\hat{p})^2 - (1-\tilde{p})^2$. This means, the condition above becomes $c \geq \rho - \rho^2 - \rho(1-\tilde{p})^2$, which is less than $\rho - \rho^2$. Then, there is also a no investment equilibrium in this region. However, we are able to rule out this type of equilibrium. This is because, the uninformed type, when his message is taken at face value, is willing to deviate to invest while the group 1 types do not find it profitable to invest even if $\mu(1,1) = 1$. Hence, as there is a best response from the DM to this deviation that makes unbiased uninformed types better off and the group 1 types worse off and is consistent given his beliefs about the deviator, this type of equilibrium can be ruled out by the Intuitive Criterion.

Finally, we can also rule out an equilibrium in which only group 1 types invest, as there is no belief of the DM these types find it profitable to invest in this region.

In the region $c \in [\rho - \rho^2 - \rho(1-\tilde{p})^2, \rho - \rho^2]$, the separating equilibrium is the unique equilibrium that survives the Intuitive Criterion.

Whenever $c \geq (1-\hat{p})^2$, even for the highest belief $\mu(1,1) = 1$, the biased and uninformed (1) agent do not benefit from deviating to invest. Hence, in this region, the out of equilibrium belief should assign probability 1 to the expert being unbiased and uninformed. Then, whenever $c \geq (\rho - \rho^2)$, even the unbiased uninformed type doesn't want to invest, which provides the boundary of the no investment equilibrium.

Next, we make the welfare comparisons. First, consider the region $(1-\hat{\rho})^2 \leq c \leq (1-\rho)^2$ where mixed strategy and separating equilibria coexist. We can see that the separating equilibrium provides higher payoff for any expert type for any c. To see this: we know that in the separating equilibrium, the uninformed unbiased type's communication is taken at face value and the group 1 types' payoff is $-(1-\hat{p})^2$, independent of cost. In the mixed strategy equilibria in this region, for any $c > (1-\hat{\rho})^2$, we need to have $\mu(0,1) < \hat{p}$ hence the payoff of group 1 types will be $-(1-\mu(0,1))^2 < -(1-\hat{\rho})^2$ given their indifference between investment and not, and the uninformed type's payoff cannot be higher as $\mu(1,1) = 1$ in the separating equilibrium. Then, all expert types are weakly (and some strictly) worse off in mixed strategy equilibria compared to pure strategy in this region.

For the DM, when we compare the mixed strategy payoff in 9 to the pure strategy equilibrium payoff in equation **??**, we see that the mixed strategy equilibrium payoff is higher than the separating equilibrium. To see this, consider the mixed strategy equilibria of the type $\sigma = 0$ and $\gamma > 0$ which is the optimal one. Then, realizing that $\mu(0,1) < \hat{p}$ provides the result. The equilibrium with $\sigma = 0$ is also the one which maximizes the DM's payoff. In this type of equilibrium, as c increases, γ increases while keeping $\sigma = 0$. Hence, we can conclude that the DM's payoff is highest in mixed strategy equilibria in this region and especially it is highest in the equilibrium in which $\gamma = 1$ and $\sigma = 0$ which can arise at $c = (1-\rho)^2$.

The total payoff in separating equilibrium is $-\beta(1-\rho)-(1-\beta)(1-\alpha)c$ while in mixed strategy equilibrium it is $-\beta(1-\rho)\mu(0,1)-(\beta+(1-\beta)\alpha\rho)(1-\mu(0,1))^2-(1-\beta)(1-\alpha)c$. Now, consider $-(\beta+(1-\beta)\alpha\rho)(1-\mu(0,1))^2 = X$, then we have $X < -(1-\rho)\beta(1-\mu(0,1))$. As the payoff in mixed strategy would be equal to separating equilibrium if and only if $X \geq -(1-\rho)\beta(1-\mu(0,1))$, we can conclude that the separating equilibrium welfare is higher than mixed strategy. □

Proof of lemma (4):

Proof. In this equilibrium, the DM's updated belief $\mu(1,1)$, given that all three types are investing, is:

$$\tilde{p} = \frac{\rho(1-\beta)+\beta\rho}{\rho(1-\beta)+\beta} = \frac{\rho}{\rho(1-\beta)+\beta}$$

First, consider the investment choice of the group 1 types. If the biased or unbiased agent were to deviate to not invest and send $m = 1$, then the DM would choose $y = \hat{p}$ as the DM infers this out of equilibrium action can only come from a biased or unbiased but informed 1 expert. Making use of the intuitive criterion (Cho and Kreps 1987) we define the out of equilibrium belief upon no investment and $m = 1$ to assign probability 0 to the unbiased uninformed expert as this type always prefers to send $m = \varnothing$ if he were to deviate to no investment while the group 1 types always prefer sending $m = 1$ rather than sending any other message, which leads to $\mu(0,1) = \hat{p}$.

Then, the following should hold for the pooling equilibrium to arise:

$$-(1-\tilde{p})^2 - c \leq -(1-\hat{p})^2$$

Second, for the uninformed unbiased type, the payoff from not investing is $-(\rho-\rho^2)$ as before, as in that case they would send $m = \varnothing$. Then, consider the payoff of this type from investing. If the signal turns out to be 0, the DM takes the message at face value and chooses $y = 0$ whereas if the signal is 1, the decision maker will choose $\hat{p} < \tilde{p} < 1$ as the DM infers it can come from a group 1 or 2 type. Then, this type prefers investing to not if and only if:

$$-\rho(1-\tilde{p})^2 - c \geq -(\rho-\rho^2)$$

These conditions together lead to:

$$c \leq min\{(1-\hat{p})^2-(1-\tilde{p})^2, \rho-\rho^2-\rho(1-\tilde{p})^2\} = (1-\hat{p})^2-(1-\tilde{p})^2$$

When $(1-\hat{p})^2 < (\rho-\rho^2)$, which is the case we consider, we have $min\{(1-\hat{p})^2-(1-\tilde{p})^2, \rho-\rho^2-\rho(1-\tilde{p})^2\} = (1-\hat{p})^2-(1-\tilde{p})^2$. To see this, realize that the value of getting informed is higher for unbiased uninformed types than for the group 1 types: $(1-\hat{p})^2-(1-\tilde{p})^2 < \rho-\rho^2-\rho(1-\tilde{p})^2$. This is because unbiased uninformed types get maximum payoff of 0 when the state of the world is 0, while their communication is distorted when the message is 1. However, from the point of view of the group 1 types, communication is always distorted as their bliss point is 1 and $\mu(1,1) = \tilde{p} < 1$. Then, the condition for the pooling equilibrium is given by the condition for the biased and informed (1) types to be willing to invest which is $c \leq (1-\hat{p})^2-(1-\tilde{p})^2$.

Now consider the expression $c \leq \rho-\rho^2-\rho(1-\tilde{p})^2$, the condition for the unbiased uninformed types to actually invest given a pooling equilibrium. As $\rho-\rho^2 > \rho-\rho^2-\rho(1-\tilde{p})^2$, where the left hand side is the cutoff for investment in the separating equilibrium, the condition for unbiased uninformed types to invest is easier to satisfy in the separating region. The difference between these two is due to the biased types' "crowding out" the unbiased uninformed types: investment of the biased types makes information acquisition by the unbiased types less profitable, hence cost has to be lower in order to satisfy their participation.

□

Proof of proposition (2):

Proof. We compare the payoff in the pooling equilibrium to that in the separating equilibrium, before considering the equilibria with no investment. For this, we compare the payoff for cost values at which there is pooling equilibrium and separating equilibrium to find how the payoff changes when c increases from the pooling region to separation.

The total welfare in the **pooling** equilibrium is given by:

The DM's welfare:

$$[(1-\beta)\rho+\beta][-(\tilde{p}-\tilde{p}^2)] = -\rho(1-\tilde{p}) \tag{14}$$

The expert's welfare: The payoff of the biased expert is $-(1-\tilde{p})^2 - c$. The ex-ante payoff of the unbiased expert is $-\rho(1-\tilde{p})^2-(1-\alpha(1-\rho))c$. This is decreasing in ρ and in β.

Then the total expert welfare is found by summing these:

$$-[(1-\beta)\rho+\beta](1-\tilde{p})^2 - [1-(1-\beta)(1-\rho)\alpha]c$$

When we sum the DM and expert welfare we get:

$$-\rho(1-\tilde{p}) - \beta[(1-\tilde{p})^2+c] - (1-\beta)[\rho(1-\tilde{p})^2+(1-\alpha(1-\rho))c] \tag{15}$$

which simplifies to:

$$-(1-\tilde{p})^2[\beta+\rho(1-\beta)]-\rho(1-\tilde{p})-c[1-\alpha(1-\rho)]=-\beta(1-\rho)-c[1-\alpha(1-\rho)(1-\beta)] \tag{16}$$

In the **separating** equilibrium, the DM's welfare is:

$$[(1-\beta)\alpha\rho+\beta][-(\hat{p}-\hat{p}^2)]=-\hat{p}\beta(1-\rho) \tag{17}$$

The payoff of the biased expert is $-(1-\hat{p})^2$, while the ex-ante payoff of the unbiased expert is $-\alpha\rho(1-\hat{p})^2-\alpha(1-\rho)0-(1-\alpha)c=-\alpha\rho(1-\hat{p})^2-(1-\alpha)c$.

Then, the expected payoff over expert types in the separating equilibrium is:

$$-(1-\hat{p})^2[\beta+(1-\beta)\alpha\rho]-(1-\beta)(1-\alpha)c$$

Finally, the total welfare in the separating equilibrium is:

$$-\hat{p}\beta(1-\rho)-\beta(1-\hat{p})^2-(1-\beta)[\alpha\rho(1-\hat{p})^2+(1-\alpha)c] \tag{18}$$

which simplifies to:

$$-\beta(1-\rho)-c(1-\alpha)(1-\beta) \tag{19}$$

The difference in the total welfare in equation (19 − 16) is:

$$c_p[1-\alpha(1-\rho)(1-\beta)]-c_s(1-\beta)(1-\alpha) \tag{20}$$

The welfare in terms of decision values cancel out and the terms that remain are those related to the cost incurred for information acquisition. In the pooling equilibrium there is more investment at a lower price where some of this investment is wasteful, while in the separating equilibrium there is less investment at a higher price. When we plug in the maximum cost at which the pooling equilibrium exists and the minimum cost at which the separating equilibrium exists, it is seen that the welfare in separating equilibrium is higher than in the pooling one although the cost of information acquisition is higher.

At $c_p = (1-\hat{p})^2 - (1-\tilde{p})^2$ and $c_s = (1-\hat{p})^2$, equation (20) becomes:

$$(1-\hat{p})^2(\beta(1-\alpha\rho)+\alpha\rho) - (1-\tilde{p})^2(1-\alpha(1-\rho)(1-\beta)) > 0 \tag{21}$$

When we replace these values, finally we are left with the condition:

$$\beta(1-2\rho) + \rho(1-\beta)(\alpha-(1+\alpha)\rho) \leq 0 \tag{22}$$

which is satisfied whenever $\rho > \frac{1}{2}$ which is the initial assumption we made. The first term is negative. The second term is negative when $\rho > \frac{\alpha}{1+\alpha}$ which is always the case when $\rho > \frac{1}{2}$ and $\alpha < 1$.

Then, although the cost of information rises, welfare increases due to the lack of wasteful investment in information. In order to demonstrate this result, we considered the cost values at the boundaries. As expected, when we keep increasing the cost in the separating equilibrium region, the welfare will decrease and at some point, it will be lower than in the pooling equilibrium.

Finally, the total welfare in the **no investment** equilibrium is:

$$-\beta(1-\hat{p})^2 - (1-\beta)[\alpha\rho(1-\hat{p})^2 + 2(1-\alpha)(\rho-\rho^2) - (\hat{p}-\hat{p}^2)(\beta+(1-\beta)\alpha\rho)] \tag{23}$$

As no investment equilibrium surplus is unambiguously worse than the separating equilibrium, we compare it to the pooling in investment equilibrium and find that the payoff in the no investment equilibrium is also lower than the pooling in investment equilibrium. This is intuitive: first, the DM's payoff is unambiguously higher in the pooling in investment equilibrium compared to the no investment equilibrium, as more information is revealed. The welfare of the biased type also higher in the pooling in investment equilibrium as their outside option of not investing and getting $-(1-\hat{p})^2$ is still available. Hence, if this type does find it profitable to invest, then it must be getting a higher payoff. The same is true for the unbiased informed (1) type who would get $-(1-\hat{p})^2$ if deviating to not invest. Finally, for the unbiased uninformed type, it is true as well: if this type didn't invest they would get the payoff $-(\rho-\rho^2)$ which is still available if they deviate in the pooling equilibrium to send $m = \emptyset$. □

Proof of Proposition 3

Proof. First, we know that conditional on separating equilibrium, the DM's payoff is decreasing in α. In addition, the pooling equilibrium gives a worse payoff for the DM. Then, the best payoff for the DM in pure strategies happens when $c = (1-\hat{\rho})^2$ at the boundary of the separating equilibrium.

Now, we will show that at that c, any other α gives a lower payoff to the DM. First, lower α leads to mixed strategy first and if it decreases further it may lead to pooling equilibrium. We know that pooling equilibrium provides lower payoff to the DM than pure strategy equilibrium. If the mixed strategy equilibrium is realized, we know that the payoff increases in α in that region.

Second, if we increase α above α^* so that $(1-\hat{p})^2 < c < (1-\rho)^2$, now c falls strictly inside the separating region. At this point, there is multiplicity of equilibria: first there is the separating equilibrium which gives a lower payoff than α^*. Finally, there also exists mixed strategy equilibria at this point, among which the one that maximizes the DM's payoff has $\gamma > 0$ and $\sigma = 0$. However, this mixed strategy equilibrium is indeed equivalent to the separating equilibrium if initially we chose $\hat{\alpha} = (1-\gamma)\alpha$. To see this, realize that $\mu(0,1)$ is the same in both equilibria, plus $\mu(1,1) = 1$ in the best mixed strategy equilibrium when $c < (1-\hat{\rho})^2$. In both equilibria, an equal portion of unbiased experts invest in overall, and while informed ones invest wastefully in the mixed strategy equilibria, in the separating equilibrium with $\hat{\alpha}$ all unbiased experts who will invest are uninformed. The composition of types that do not invest are constant in both equilibria.

This shows that there is no other α which can make the DM as well (or better) off regardless of the equilibrium selection. At α^*, the best mixed strategy equilibrium is the separating equilibrium, and there is no conflict over the best equilibrium. Hence, I conclude that this is the unique optimal α that provides the highest DM payoff.

Finally, we can find the threshold k. This is the cost value below which, even for α arbitrarily close to 1, all types are still willing to invest. Then, we replace $\alpha = 1$ to find $\hat{\rho} = \frac{\rho}{(1-\beta)\rho+\beta}$, and then the threshold cost is given by $k = (1-\hat{\rho})^2 = \left(\frac{\beta(1-\rho)}{\beta(1-\rho)+\rho}\right)^2$. Whenever the cost is below this level, no matter what α is, all types are willing to invest regardless of whether they are informed or not. This means, as cost increases, the DM's optimal level of expert is going to be increasingly informed, and once it becomes arbitrarily close to being perfectly informed, below that level any α will lead to the same result. □

Proof of proposition 4

Proof. Fix c. We have 4 different regions. When $c < k$, for any $\alpha > 0$ the unique equilibrium is the pooling one. Hence in this region the DM's payoff doesn't depend on the expertise parameter. Second, consider the region $k < c \leq (1-\rho)^2$. We know that $(1-\hat{p})^2$ is a decreasing function of α as $\hat{p}$ is increasing in α. Among the equilibria that arise in this region, we know that the DM's welfare is maximized with α high enough to ensure $(1-\hat{p})^2 = c$, which leads to $\alpha^* = \frac{\beta(1-\rho-\sqrt{c})}{(1-\beta)\rho\sqrt{c}}$. When α is slightly lower, mixed strategy equilibrium arises and the equilibrium payoff for the DM is lower. Plus, we know that whenever the separating equilibrium realizes, if α increases further so that $(1-\hat{p})^2 > c$, the separating equilibrium payoff is also decreasing for the DM due to loss in communication for informed unbiased types. At α^*, the DM optimal mixed strategy equilibrium is also the separating equilibria and provides the highest payoff for the DM.

Consider α such that the separating equilibrium arises, $c > (1-\hat{p})^2$. Then, there exist both pure strategy and mixed strategy equilibria, with the DM preferred one being the mixed strategy equilibrium with $\sigma = 0$, while her separating equilibrium payoff is strictly lower than the separating equilibrium with α at which $(1-\hat{p})^2 = c$. Indeed, this mixed strategy equilibrium payoff is equivalent for the DM (leads to the same posteriors) to the one when $\hat{\alpha} = \alpha(1-\gamma)$. This new $\hat{\alpha}$ would satisfy $c = (1-\hat{p})^2$, meaning the minimum α such that separating equilibrium realizes. Then we can say that, higher α can never increase the DM's payoff plus it leads to multiplicity of equilibria. Even when there is multiplicity, the welfare maximizing equilibrium is the separating one, for which the welfare is lower than the separating one. Let us interpret this result: the DM's payoff doesn't increase in α once the separating region, as there is an α^* that is decreasing in cost which maximizes the DM's welfare independent of the equilibrium selection. This implies, there is no point in hiring a more informed expert than necessary. Indeed, depending on the equilibrium selection, it can even hurt the DM. Among the separating and mixed strategy equilibria in this region, for any α, we find that the separating equilibrium at the critical α is the one that maximizes the DM payoff, regardless of equilibrium selection. Realise also that when cost is so low, assuming this is interior, so that $c < k$ where $k = (1-\hat{\rho})$ with $\alpha = 1$ (this means $(1-\rho) = \frac{\beta(1-\rho)}{\beta(1-\rho)+\rho}$, we are in the region where even if the probability that the expert is informed is very close to 1, there is still pooling equilibrium. Then below this level, expertise parameter doesn't matter for the DM.

When cost increases further so that $(1-\rho)^2 \leq c \leq \rho - \rho^2$, the unique equilibrium is the one in which only the unbiased uninformed types invest and the DM takes communication at face value. Then, it is DM optimal that $\alpha = 0$ so that all the unbiased types can separate themselves from a biased type, making communication most precise.

Finally, when $c > \rho - \rho^2$, there is no possibility of any investment by any type. In that case, it is optimal that the expert is informed with probability 1. When there is no way for the DM to separate between biased and unbiased types, it is optimal that at least the expert perfectly informed.

□

Proof of proposition (??)

Proof. When $c > \rho - \rho^2 - \rho(1-\tilde{p})^2$, in the covert information acquisition case no type is getting informed, hence the payoff is the same as in the no investment equilibrium in the overt case. We know that in the overt information acquisition case, welfare is always higher when there is some information acquisition compared to none, and in the region $[\rho - \rho^2 - \rho(1-\tilde{p})^2, \rho - \rho^2]$ there is investment in the overt case. Hence, the payoff in overt case is always higher under this condition.

When $c < (1-\hat{p})^2 - (1-\tilde{p})^2$, in the overt case, there is pooling in investment while in the covert case, the unbiased uninformed type invests only (realize that $(1-\hat{p})^2 - (1-\tilde{p})^2 < \rho - \rho^2 - \rho(1-\tilde{p})^2$ hence there is investment in covert case). In the end, the amount of information transmitted is the same. Hence, in the overt case there is more wasteful investment for the same precision of communication. Then, we can conclude that welfare is higher in the covert case under this assumption. □

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