Selecting the wisdom of an expert

Sinem Hidir*†

October 23, 2018

Abstract

We study the signaling role of information acquisition when there is uncertainty about an expert's bias as well as about his information. Observable investment in information simultaneously serves as a way to get informed and build credibility before communicating to a decision maker. We define the *signaling* and the *intrinsic value* of information and find the conditions under which separation in information acquisition behavior arises. We find that a more informed expert may often result in worse decision making for the DM. Indeed, most informative communication happens with an initially uninformed expert at an intermediary information cost, and the DM's preferred level of expertise is non monotone in this cost. This optimal expertise level ensures that a separating equilibrium arises and there is no wasteful investment. Surprisingly, the overall welfare increases when the cost of information increases to enable separation.

Keywords: information acquisition; cheap talk; communication; signaling; credibility.

JEL Codes: D82, D83.

*University of Warwick. email: s.hidir@warwick.ac.uk.

†I thank Dan Bernhardt, Costas Cavounidis, Wioletta Dizuida, Daniel Habermacher, Luciana Nicollier, Francesco Squintani, Johannes Hörner, Bruno Jullien and Dezsö Szalay for helpful comments as well as participants at SAET 2017, Lancaster Game Theory Conference 2017 and seminar participants at Birkbeck University.

1 Introduction

In markets and organizations, decision makers often rely on the advice of experts. However, it is difficult to assess the credibility of experts and the quality of their information. The literature on strategic information transmission pioneered by Crawford and Sobel (1982) mostly assumes that the expert's bias is common knowledge and that the expert has free access to information. These assumptions have been relaxed in different setups. Uncertainty about bias has been explored in the literature on the reputation of experts in repeated cheap talk initiated by Sobel (1985) and Morris (2001). Costly information acquisition before cheap talk was first considered by Austen-Smith (1994) and recently Sobel (2013) described this area as an open question[1]. In order to study the signaling role of information acquisition, we consider an expert who may or may not be biased and may or may not be informed plus can make an observable investment in information. These ingredients make the problem one of costly signaling and cheap talk.

To understand the strategic consideration, contemplate a politician consulting a policy advisor about the effects of pollution on the occurrence of lung disease. The advisor may be getting kickbacks from a polluting industry and hence be biased. Maybe the advisor has no conflict of interest, but already has access to information due to his experience. Finally, it may be that the advisor, although unbiased, isn't informed on the issue and may hire a team of researchers to provide information. Complication arises because a biased advisor could mimic this behavior only to make his advice more credible. There are instances when experts incur costs in order to enhance their credibility while in fact they are biased due to being backed by third parties. For example, Andrew Wakefield, a former gastroenterologist and medical researcher, was found guilty of misconduct in his research paper that claimed the MMR vaccine was linked to autism and bowel disease, years after his research was published and had impact. It was later found that he had been paid by lawyers who were trying to prove that the vaccine was unsafe.[2]

Similar issues arise when a division manager advising the CEO of his company may be biased towards launching a product due to career concerns. The division manager may claim to know the potential of this product due to his expertise or may ask his team to carry out a market research, the cost being the time spent by the employees. A lawyer may know whether it is optimal settle a lawsuit or not, but may also have a bias to fight in court to increase the fees. Although the lawyer may make a recommendation right away by relying on his experience, it is natural to imagine that a lawyer that spends some time to study the case in detail before making a suggestion appears more credible. Motivated by these examples where experts can overtly spend money, time or resources in order to get informed as well as to increase their credibility before communicating to a decision maker (DM), this paper studies the question of how bias affects incentives of endogenous information acquisition. Furthermore, given that the DM cannot identify the bias of the expert, what type of expert, in terms of how informed they are at the outset, provides the most accurate information?

[1]Some recent examples include Pei (2015), Argenziano et al. (2016) and Deimen and Szalay (2016)
[2]"MMR doctor given legal aid thousands" *The Sunday Times*, 31 December 2006.

We find that an expert who is more informed often does not provide more information. The reason is that this type, relying on his prior information, doesn't have enough incentives to separate himself from a biased expert by incurring costly investment. On the other hand, an uninformed expert faces sufficient uncertainty that he is willing to acquire information and whilst doing so may end up separating himself from a biased type. Hence, acquired information often turns out easier to transmit than ex-ante information in case costly investment in information also allows the expert to reveal himself as unbiased. If the decision maker can pick an expert according to his level of expertise, it is optimal to do so in a way that separation in investment behavior arises and there is no wasteful investment. This optimal level of expertise is non-monotone in the cost of information acquisition. Specifically, higher cost of information may lead to a less informed expert being optimal. Finally, we find that higher cost of information acquisition leads to higher overall welfare when it leads to separation.

We consider a model in which the state of the world is $\omega \in \{0, 1\}$ and the DM's action space is $y \in [0, 1]$. The expert is biased with a known probability β and with probability α he is already perfectly informed about the state of the world. The decision maker and the unbiased type of expert want the action to match the state of the world, while the biased type always wants the highest action regardless of the state. Hence, whether the biased expert is informed or not is irrelevant. We focus on equilibria in which an unbiased expert initially informed of low state truthfully communicates to the DM without investment. On the other hand, when an unbiased expert is informed of high state, his incentives are the same as a biased expert as both want the highest action. At the outset, the expert has an option to make a costly investment in information acquisition, which is observable although its outcome is private to the expert. As there is a probability that the expert is already informed, that an expert does not invest in information does not reveal that he is uninformed, nor that he is biased.

The equilibrium investment strategies of different expert types determine the DM's posterior, and hence the credibility of communicating the high state. The expert cares about his credibility only insofar as it influences the DM's action: reputation is instrumental, not intrinsic. Hence, a biased expert, as well as an unbiased expert who is already informed of high state may wastefully invest in order to pool with uninformed unbiased types who want to get informed in order to transmit the information. We identify the *intrinsic value of information* for the unbiased uninformed expert, which is the value of getting informed when his communication is taken at face value, and the *signaling value* for the biased and unbiased informed expert which is the value of deviating to invest and pool with the unbiased uninformed type. We focus on the case in which the signaling value of information is less than its intrinsic value, which allows a separating equilibrium to arise for some cost values. In a separating equilibrium, the only type that invests in information acquisition is the uninformed unbiased expert, while a biased expert pools with unbiased type informed of high state. Hence, as well as being the most informative pure strategy equilibrium, from an efficiency perspective, the separating equilibrium is the one in which information acquisition is efficient.

That the unbiased informed expert pools with the biased type whenever the state is high means that an expert can separate himself from the biased one in both states of the world only if he is initially uninformed. This type communicates more precisely than an expert who is already informed, as by investing he gets informed plus identifies himself as unbiased, conditional on separating equilibrium arising. At the same time, for the separating equilibrium to arise, there should be a sufficiently high probability that the expert is already informed, i.e. α is high enough, as this makes communication without investment more credible. In other words, the probability that the expert is already informed allows the biased type to avoid too much prejudice when sending a high message without investment hence prevents him from deviating to invest. Conditional on the separating equilibrium arising, the DM's welfare is decreasing in α, due to loss in communication with the ex-ante informed types who pool with the biased type. Hence, in an ex-ante sense, the DM's payoff is maximized when α is just high enough for separating equilibrium to arise but not more. In an ex-post sense, the DM's highest welfare arises if she was indeed matched with an unbiased expert initially uninformed but ends up investing.

A pooling equilibrium exists for lower costs of information acquisition, in which all types except the unbiased one with signal 0 invests in information. In this equilibrium, the expert is informed for sure but the DM cannot differentiate between a biased and an unbiased type whenever the high state realizes. Hence, the DM is strictly worse off in this equilibrium compared to the separating one in which all unbiased types are informed as well and in addition, he can identify an expert who invests in information as unbiased. The game also admits interesting mixed strategy equilibria in which the biased types and unbiased informed types with high signal are indifferent but play different probabilistic investment strategies. As the cost of information acquisition decreases, proportionately more biased types than unbiased informed types invests so that the indifference condition of these types is satisfied. We show that for any given cost, the DM's payoff among all mixed strategy equilibria is given by the separating equilibrium with the minimum portion of informed experts. We define this unique α^* which gives the highest possible DM welfare among any equilibria.

After characterising equilibria we define the DM's optimal expertise level, α^*, which is non monotone in the cost of information acquisition. If this cost is low enough that pooling arises even for α almost equal to 1, expertise doesn't matter for the DM: it is so cheap to mimic information acquisition behavior that pooling cannot be avoided. When cost is higher, the DM prefers α just high enough that a separating equilibrium arises. The intuition is as follows: it is optimal to have just enough informed types to achieve separation to prevent the biased types from pooling and not more, due to the loss in communication with these types who pool with biased types in high state. Moreover, this optimal level of expertise decreases in the cost of information acquisition over a certain interval. As the cost of information rises, a lower credibility upon no investment (fewer informed types) is sufficient to prevent wasteful investment by these types. When the cost of information acquisition is high enough that the separating equilibrium arises even when the expert is uninformed for sure, then $\alpha^* = 0$. In this case, the biased types find it too costly to invest even if the DM identifies an expert that does not invest as a biased one. Hence, the unbiased expert can perfectly transmit acquired information and it is optimal that he is uninformed at the outset. Finally, when the cost is so high that no type will invest, a perfectly informed expert is optimal: when the DM cannot distinguish between a biased and an unbiased expert as no investment will take place, he prefers the expert to be informed.

A surprising result relating to welfare is that less wasteful investment more than compensates for a higher cost of information acquisition when the cost increases to move the equilibrium from the pooling to the separating region, while the total welfare relating to communication and decision making remains constant. Hence, total welfare is non-monotone in the cost of information: it decreases as cost increases in a given equilibrium, but if the increase shifts the equilibrium to a separating one, then total welfare increases.

Finally, we contrast outcomes in this overt information acquisition setting with the outcomes that emerge with covert information acquisition when the investment choice of the agent is not observable. In this case, there is no wasteful investment as informed types can pool with unbiased types without incurring any cost, but separation between biased and unbiased types is not possible whenever the state is high. Overall welfare is higher with overt information acquisition when costs are such that separating equilibrium arises in overt information acquisition, due to more precise communication. In the overt case, some unbiased types can separate from biased types through the observability of investment which is not the case in covert information acquisition. On the other hand, when pooling in investment arises with overt information acquisition, covert information acquisition always leads to higher welfare because it results in less wasteful investment as there is no signaling. The DM is better off in overt information acquisition in any case, as he doesn't incur the cost of investment plus he has the chance to identify an unbiased expert only in the overt case.

2 Literature

This paper relates to several strands of the long-standing signaling and cheap talk literatures. Specifically, it relates to the literature on costly information acquisition in cheap talk. The first paper in this literature is Austen-Smith (1994) where costly information acquisition leads to perfect information and is not observable, plus there is uncertainty about the cost of information acquisition. As the expert can prove he is informed but can feign ignorance, low types pool with uninformed types to achieve a higher outcome, which improves communication for higher types. Since Austen-Smith (1994), only recently has there been more work on strategic communication with endogenous information acquisition, main examples being Pei (2015) and Argenziano et al. (2016). Pei (2015) considers the setup of Crawford and Sobel (1982) but endogenises information acquisition and finds that the expert truthfully transmits all the information he acquires, in other words the expert doesn't acquire information that he will not transmit.[3] and Argenziano et al. (2016) considers a similar setup and contrasts overt versus covert information acquisition. Deimen and Szalay (2016) also consider endogenous information where a biased expert can choose on which issues to gather information and show that communication dominates delegation. Suurmond et al. (2004) consider the effect of reputation in a delegation setup with information acquisition and an unbiased expert. They show that reputational concerns may help by incentivizing the expert to acquire information when he doesn't know his ability. In contrast if the expert privately knows his ability, he may take inefficient actions in order to mimic an efficient type. Dur and Swank (2005) study the tradeoff between the incentive to acquire information versus the precision of communication as a function of expert's bias. There is no uncertainty about bias and hence no signaling motive for the expert in these mentioned papers.

The paper also relates to the literature in cheap talk with uncertainty about expert's bias and reputational concerns. Morgan and Stocken (2003) consider strategic communication as in Crawford and Sobel (1982) with added uncertainty about the expert's type. They show that truthful communication cannot arise even with an unbiased analyst whenever the state of the world is sufficiently high. Sobel (1985) and Morris (2001) consider repeated cheap talk, where reputation is instrumental as in our setup, but communication and decision making is repeated. Outside of the communication literature, Ely and Valimaki (2003) consider a long run player facing short run players who takes a payoff relevant action and highlight the distortional consequences today of the incentives to avoid bad reputations in the future. In our setting the decision is taken only once but the observable information acquisition affects the credibility of the expert in the communicating stage. Hence, messages are never distorted as decision making takes place only once, but distortion takes the form of wasteful investment in information acquisition. Meng (2015) considers a two period setup as in Morris (2001) and endogenizes the precision of the expert's information, where investment is not a signal as it is not observable and finds that reputation building enhances the incentives to invest in information for both types in the first period to build credibility for the second period. There is also a literature (e.g. Ottaviani and Sorensen and Ottaviani and Sorensen (2006a)) showing that reputation concerns may lead to inefficient herding when experts bias their recommendation in order to appear more informed, where there is an intrinsic value of reputation. While our setup features neither dynamics nor an intrinsic value of reputation, similar effects to those found in the literature on reputation arises nonetheless due to the endogenous information acquisition and signaling incentives.

Finally, the wasteful investment in our setup which serves for signalling is reminiscent of *burning money* as identified by Austen-Smith and Banks (2000), when cheap talk is not the only way to communicate but senders may also incur a loss in utility in order to enhance their communication. However, contrary to pure money burning, the aligned type of experts in our

[3] Esö and Szalay (2010) consider a game in which the expert has no bias and endogenous information acquisition, and show that restricting the message space can induce the sender to acquire information more often.

setup do value information per se.

3 Model

There is a decision maker (DM, she) and an expert (he). There is a state of the world $\omega \in \{0,1\}$ and a commonly known prior $\Pr(\omega = 1) = \rho$. At the beginning of the game, the expert learns his two dimensional private type. With commonly known probability $(1-\beta)$ the expert is unbiased and shares the same payoff as the DM of $-(\omega - y)^2$ and with probability β he is biased and always wants the highest possible action, with payoff $-(1-y)^2$, where $y \in [0,1]$ is the decision maker's eventual decision.[4] The expert's type is denoted by $\theta \in \{u, b\}$ corresponding to unbiased or biased. Second, with commonly known probability α the expert is perfectly informed about the state of the world ex-ante; while an uninformed expert shares the same prior as the DM. We interpret α as an expertise parameter[5], which relates to the experience that the expert may have derived from having advised on similar issues. The decision maker does not observe whether the expert is biased or informed, but knows α and β. In addition, the expert can optionally invest in information by incurring cost c to get a private signal that perfectly reveals the state. The DM observes the investment decision but not its outcome.[6] Indeed, any type of expert could invest in information, including a biased or an informed one for whom investment is only meant for signaling. We will call such signaling wasteful investment. As the expert is already perfectly informed with probability α, the fact that he doesn't invest doesn't reveal that he is uninformed, nor that he is biased. Finally, communication happens through cheap talk following the investment decision. Below is a summary of the stages of the game:

1. the state of the world and the expert's type is realised.
2. the expert decides whether to acquire a perfect signal by incurring c, a decision denoted $x \in [0,1]$.
3. the expert sends a message $m \in M$ to the DM.
4. the DM takes an action, $y \in [0,1]$.

The decision maker interprets the expert's message as a function of her posterior about the expert's type, which depends on her prior and the equilibrium information acquisition behavior of the expert types. As the biased expert always wants the highest action regardless of the state, there is only 1 type of biased expert and whether he is informed or not has no relevance.[7] An equivalent assumption would be that the biased expert type is never informed. We can then

[4] This type of assumption about the expert type and utility is made by others (see e.g. in Morris (2001)).

[5] Bhattacharya et al. (2018) also make the assumption that the expert is either perfectly informed or uninformed and interpret this as an expertise parameter, when looking at optimal composition of expert panels without information acquisition.

[6] That the information acquisition process is **observable** to the decision maker but not its **outcome** is a common assumption in the literature, see e.g. Fischer and Stocken (2010), Argenziano et al. (2016), and Deimen and Szalay (2016) who also consider information acquisition before cheap talk in different setups with known bias of the sender.

[7] This would be different were the biased type's payoff not state independent, as in Morgan and Stocken (2003)

summarize the types of experts at the beginning of the game into four:

1. biased
2. unbiased and uninformed
3. unbiased and informed with signal 1
4. unbiased and informed with signal 0

Call $\Phi \in \{0, 1, \varnothing\}$ the information structure of the expert at the communication stage as a result of his initial information and choice of information acquisition. The expert's communication strategy denoted $m(\Phi, \theta)$ will be pure as each type of expert strictly prefers one of the three messages. The equilibrium concept is Perfect Bayesian Equilibrium (PBE). A strategy profile $\langle x, m, y\rangle$ along with the DM's posterior $\mu(x, m) = Pr(\omega = 1|x, m)$ forms a PBE if and only if:

- The DM's action maximizes her payoff given her posterior:

$$y^*(x, m) = \arg\max_y -\mu(x, m)(1-y)^2 - (1-\mu(x, m))y^2$$

  which is found as $y^*(x, m) = \mu(x, m)$.

- The DM's posterior $\mu(x, m)$ is consistent with the expert types' investment strategies and the prior ρ.

- The expert's strategy, (x, m) maximizes his payoff given the DM's belief updating and best response.

4 Equilibrium Analysis

As this is a game of signaling followed by cheap talk, there are multiple equilibria. There always exists an equilibrium in which no one acquires information and the DM interprets any message as a babbling one. Other than this, depending on the cost of information acquisition, there exist equilibria with information acquisition. Whenever there is investment, we focus on the most informative equilibrium. When there exists multiple informative equilibria we use the Intuitive Criterion (Cho and Kreps (1987)) to establish the most reasonable out-of-equilibrium belief. The next lemma summarizes the set of messages that are sent in any such equilibrium.

Lemma 1. *As the expert is either perfectly aligned or extremely biased, he has a unique optimal communication strategy following investment or no investment. The set of messages sent in equilibrium after investment are $m^i \in \{0, 1\}$ and after no investment, $m^n \in \{0, 1, \varnothing\}$.*

If there is investment, it is sure that the expert is informed and wants to induce either the highest or the lowest action. If there is no investment, there is a possibility that the expert is uninformed and if he is unbiased, he wants to send $\varnothing$. Given that the messages sent conditional on information set is understood, we now study the equilibrium investment behavior. First, we establish the behavior of the unbiased expert informed of low state.

Lemma 2. *The equilibria in which the unbiased type informed with signal 0 send message $m = 0$ without investing after which the DM chooses $y = 0$ are the only ones that survive the Intuitive Criterion.*

We restrict attention to equilibria in which communication by this type of expert is perfect. This is rather trivial: as the biased type would never want to mimic him, communicating low state without investing also reveals that he is unbiased and he has no incentive to invest. Hence, from now on, we only have to deal with the equilibrium behaviors of the remaining expert types. In addition, the unbiased type of expert always sends a truthful message: whenever uninformed, this type sends $m = \varnothing$ and whenever informed he sends $m = \omega$, as this expert never has incentives to lie about the state. The only uncertainty about this type of expert is whether he will invest or not. As the biased and unbiased types informed of high state share the same payoff function, they follow the same information acquisition strategies in strict Nash Equilibria and always send message $m = 1$ regardless of their investment strategy[8]. That these types have the same incentives underlines many interesting results, in particular that an uninformed expert who becomes informed later on can reveal himself as unbiased while an expert who is already informed ends up pooling with the biased type in the high state. This also leads to interesting mixed strategy equilibria. Even though these types have the same payoff, what they do is significant for the DM: the unbiased type is communicating truthfully while the biased type sends an uninformative message.

We define "*pooling*" and "*separating*" in this setup as a function of the investment decisions of the two groups of expert types:

Group 1: biased type and unbiased informed type with state 1.
Group 2: unbiased uninformed type.

We characterize all equilibria with information acquisition for different levels of cost:

1. **Separating equilibrium:** Only group 2 type invests hence the DM takes communication at face value. Group 1 types send $m = 1$ without investing upon which the DM chooses $y = \hat{\rho} > \rho$.

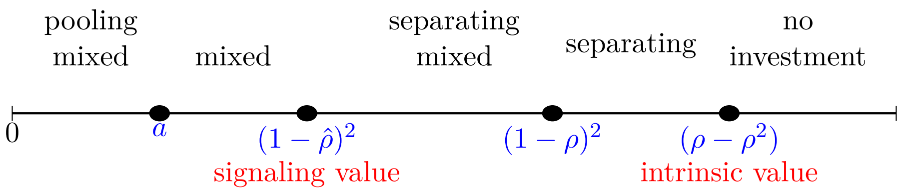
2. **Pooling equilibrium:** Both groups invest. The unbiased type sends $m = \omega$ while the biased type sends $m = 1$. Upon $m = 1$, the DM chooses $y = \tilde{\rho} > \hat{\rho}$.

3. **Mixed strategy (semi-pooling) equilibria:** Group 1 types play different probabilistic investment strategies that keep these types indifferent. As cost decreases, the portion of biased types who invest increases compared to unbiased types.

Figure 1 shows the regions for different equilibria as a function of the cost of information

[8] When these types are indifferent, they can play different strategies, which we will define as mixed strategy equilibria

Figure 1: Equilibria as a function of cost, for given α and β



$\hat{\rho} = \mu(0,1)$ in the separating equilibrium.
$\tilde{\rho} = \mu(1,1)$ in the pooling equilibrium.

acquisition, highlighting that multiple equilibria exist in some regions. When $c \geq \rho - \rho^2$, the unique equilibrium has no investment. As a result we focus on the region $c < \rho - \rho^2$ in order to study the range of equilibria. Our primary focus will be the separating equilibrium as it is the most informative one and we will identify the conditions under which this equilibrium can arise.

Separating Equilibrium: In a separating equilibrium, the only types who invest are the group 2 types of expert, while group 1 types send high message without investing. This equilibrium exists only when the **signaling value** is lower than the **intrinsic value** of information and the cost of information acquisition is intermediary. The **signaling value** of information is the gain in payoff for group 1 types of deviating to invest and sending message $m = 1$, given that the decision maker interprets the message as coming from an unbiased type. This gain is given by $-(1-\mu(1,1))^2 + (1-\mu(0,1))^2$, where $-(1-\mu(1,1))^2$ is the payoff upon investing and $-(1-\mu(0,1))^2$ is the payoff upon not investing in a separating equilibrium, while sending $m = 1$ in either case. In the separating equilibrium, $\mu(0,1)$ is given by:

$$\hat{\rho} = \frac{\alpha(1-\beta)\rho + \beta\rho}{\alpha(1-\beta)\rho + \beta}$$

as the DM infers that the high message either comes from a biased agent (with probability $\beta\rho$) or from an unbiased informed agent (with probability $\alpha(1-\beta)\rho$), and optimally chooses $y = \hat{\rho}$. The payoffs to both the biased and unbiased informed expert types are $-(1-\hat{\rho})^2$. Upon deviating to invest, they can induce the DM to choose $y = \mu(1,1) = 1$ and obtain a payoff of 0. Thus, $(1-\hat{\rho})^2$ is the signaling value of investment. The condition for this deviation not to be profitable for group 1 types is:

$$c \geq (1-\hat{\rho})^2 = \left[\frac{\beta(1-\rho)}{\alpha(1-\beta)\rho + \beta}\right]^2 \tag{1}$$

The **intrinsic value** of information is the payoff gain for the uninformed unbiased types from getting informed and obtaining the maximum payoff of 0 after investment, rather than deviating to not invest and sending $m = \varnothing$ upon which the DM would choose $y^* = \rho$ leading to a payoff of $-(\rho - \rho^2)$[9]. Then, group 2 type invests if and only if $c \leq \rho - \rho^2$. Hence, the cost values for

[9]To see this, realise that the expert's expected payoff when $y = \rho$ is given by: $-\rho(1-\rho)^2 - (1-\rho)\rho^2$ which leads to $-(\rho - \rho^2)$

which a separating equilibrium exists is:

$$(1-\hat{\rho})^2 \leq c \leq \rho - \rho^2 \tag{2}$$

A separating equilibrium exists when c falls in between the signaling and intrinsic value of information, in other words it exists when cost is neither too high nor too low. This region widens as ρ and α increase and tightens as β increases. High α and ρ lower the biased type's incentives to invest by making communication upon no investment more credible. On the other hand, the uninformed type is willing to invest whenever $c \leq \rho - \rho^2$, which is more likely to be satisfied when ρ is closer to 0.5, in other words when uncertainty is higher and the intrinsic value of information is higher. We will make the following assumption for the rest of the paper:

Assumption 4.1. $\rho_0 > 0.5$

The signaling value can at most be $(1-\rho)^2$ which arises for $\alpha = 0$.[10] Then, for $\rho > 0.5$, $(1-\rho)^2 < (\rho - \rho^2)$ and for any α there exists a region of cost values for which the separating equilibrium arises. This assumption gives us the richest set of equilibria, while our results do not depend on it.[11] If instead $\rho < 0.5$, a separating equilibrium region of cost exists only for α high enough and it never exists for $\alpha = 0$. If the signaling value of information is above the intrinsic value, then separation is never possible in pure strategies as a biased expert then has an incentive to invest whenever the uninformed unbiased one does.

Now, consider the region $c < (1-\hat{\rho})^2$. There exists no separating equilibrium as the biased type would have an incentive to deviate and invest if the DM attributes the investment to an unbiased type. However, just below this threshold it cannot be that the group 1 types invest with probability 1 either, as in equilibrium this would mean $\mu(1,1) < 1$ and the biased and informed (1) types would not find it profitable to invest. Then, only mixed strategy equilibria may exist in this region where group 1 types play probabilistic investment strategies.

Mixed strategy (semi-pooling) equilibria: In this type of equilibrium, biased types and unbiased types informed with high signal (group 1 types) play different and possibly probabilistic investment strategies, determining the posterior for the decision maker and hence the incentives of these types themselves, while the uninformed type still invests with probability 1. Specifically, in this equilibrium the strategies of group 1 players must be consistent with their indifference condition. We use σ and γ to denote respectively the probability that the biased sender and the unbiased informed type invest, while both send $m = 1$. When these types are indifferent, the unbiased uninformed type strictly prefers to invest. In this equilibrium, the DM's posterior upon investment is:

$$\mu(1,1) = \frac{(1-\beta)(\alpha + \gamma(1-\alpha))\rho + \beta\sigma\rho}{(1-\beta)(\alpha + \gamma(1-\alpha))\rho + \beta\sigma} > \hat{\rho}$$

[10]Without investment, the biased experts can at worse get the outcome $y = \rho$, when they are the only ones to not invest and the DM can identify them as biased.

[11][12]e consider the case with $(1-\rho)^2 > \rho - \rho^2$ in the appendix.

while upon no investment, it is:

$$\mu(0,1) = \frac{(1-\beta)\alpha(1-\gamma)\rho + \beta(1-\sigma)\rho}{(1-\beta)\alpha(1-\gamma)\rho + \beta(1-\sigma)}$$

. The indifference condition that should be satisfied in any mixed strategy equilibrium is:

$$c = (1-\mu(0,1))^2 - (1-\mu(1,1))^2$$

. From the equation above we can verify that σ and γ are complements and there are many pairs of $\{\sigma, \gamma\}$ that form a mixed strategy equilibrium for a given c.

When $c \geq (1-\rho)^2$, there exists no mixed strategy equilibria, as even when $\mu(1,1) = 1$, group 1 types prefer not to invest. Mixed strategy equilibria exist for $c \in (k, (1-\rho)^2)$[13]. In the region $c \in [(1-\hat{\rho})^2 - (1-\tilde{\rho})^2, (1-\hat{\rho})^2]$, mixed strategy equilibria are the unique equilibria while outside this region, they co-exist with pure strategy equilibria and survive the Intuitive Criterion.[14]

Let's consider the region $c \in [(1-\hat{\rho})^2, (1-\rho)^2]$ where mixed and separating equilibria coexist. Mixed strategy equilibria in this region feature $\mu(0,1) < \hat{\rho}$: as group 1 types strictly prefer not to invest when $\mu(1,1) = 1$ and $\mu(0,1) = \hat{\rho}$, and given that $\mu(1,1)$ cannot increase further, $\mu(0,1)$ must decrease to make these types indifferent between investing or not. In other words, the no investment payoff must be made worse to keep the indifference condition of the biased types. Then it should be that $\gamma > \sigma$: more unbiased informed types should invest in proportion to biased types.

Lemma 3. *For any cost value $c \in [(1-\hat{\rho})^2, (1-\rho^2)]$ where mixed strategy and separating equilibria co-exist, there exists a unique mixed strategy equilibrium which maximizes both the DM's and the expert's welfare and has $\gamma > 0$ and $\sigma = 0$.*

Hence, there is agreement between expert types and the DM on the best mixed strategy equilibria and from now on, when we talk about mixed strategy equilibrium in this region, we will refer to this specific one. When $c < (1-\hat{\rho})^2$, mixed strategy equilibria should have $\sigma > 0$, as in any other case $\mu(1,1) = 1$ and $\mu(0,1) \leq \hat{\rho}$ and the group 1 types will strictly prefer to invest. The best mixed strategy equilibrium in this region has $\gamma = 0$ and $\sigma > 0$. Plus, as c increases, $\frac{\mu(1,1)}{\mu(0,1)}$ should increase: communication after investment should become relatively more credible. Realize that group 2 types keep investing with probability 1 in any mixed strategy equilibria given that $c \leq (\rho - \rho^2)$.[15]

[13] $(1-\rho)^2$ is the highest cost level at which it is possible to have a mixed strategy equilibrium, as above this value it is impossible to have an equilibrium in which group 1 types invest, given that they prefer the prior which gives them $-(1-\rho)^2$ rather than incurring the cost and inducing the decision maker to choose the highest action 1. k is defined as the lowest value of cost for which there exists mixed strategy equilibria, and it is shown in the Appendix.

[14] To see why mixed strategy equilibria always survive the Intuitive Criterion, realise that in a mixed strategy equilibrium, both investment and no investment can take place, hence there is no out of equilibrium action in terms of investment choice. The only out of equilibrium belief can arise if a message $\varnothing$ is sent, however given that this would lead to a choice $y = \rho$, this deviation can never be optimal to any type.

[15] If $\rho > 0.5$, then $(1-\rho)^2 > \rho - \rho^2$ and hence this equilibrium couldn't arise for any value of c.

Proposition 1. *A separating equilibrium exists for cost values* $(1-\hat{\rho})^2 \leq c \leq (\rho-\rho^2)$ *and:*

- *Given β, ρ, and c, there exists a minimum α for which a separating equilibrium exists and satisfies $(1-\hat{\rho})^2 = c$ where $\hat{\rho}$ is an increasing function of α.*
- *In the region $(1-\hat{\rho})^2 \leq c \leq (1-\rho)^2$, the separating equilibrium coexists with mixed strategy equilibria. In this region, separating equilibrium is the expert optimal and welfare maximizing equilibrium while the mixed strategy equilibrium is the DM optimal one.*
- *In the region $(1-\rho)^2 < c < (\rho-\rho^2)$, separating equilibrium is the unique equilibrium.*

There exists an equilibrium in which all unbiased types invest, in which $\sigma = 0$ and $\gamma = 1$. This equilibrium exists exactly at the cost value $c = (1-\rho)^2$, which is the maximum cost at which a mixed strategy equilibrium can exist. In this equilibrium, we have $\mu(1,1) = 1$ and $\mu(0,1) = \rho$ and a group 1 type is indifferent between investing and not as $c = (1-\rho)^2$.[16] This is also the highest cost level for a mixed strategy equilibrium to arise. It is easy to see that this equilibrium is the ideal one for the decision maker, as he can perfectly identify biased types. However, as a very specific condition on either c or ρ needs to be satisfied for this type of equilibrium, it will not be our focus. **Pooling Equilibrium:** When the cost of information acquisition is low enough, a pooling equilibrium exists in which all types except the unbiased type informed with signal 0 invest. When cost is even lower, this is the unique equilibrium.

Lemma 4. *A pooling equilibrium exists when:*

$$c \leq (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$$

and the DM's posteriors in this equilibrium are $\mu(1,1) = \tilde{\rho} = \frac{\rho}{\beta+(1-\beta)\rho}$ *and* $\mu(0,1) = \hat{\rho}$.

As β increases, so does the incentive of the biased type to invest making pooling more attractive. Now, let us compare overall welfare among equilibria. The DM gets the lowest surplus in the no investment equilibrium in which no type gets informed plus it is impossible to distinguish between biased and unbiased types. Among equilibria in which there is investment, the DM gets the lowest surplus in the pooling equilibrium in which all expert types are informed but she cannot differentiate between a biased and unbiased expert. It is not surprising that the DM's surplus increases when cost increases to move the equilibrium from pooling into the separating one as there is more separation between biased and unbiased types. In the separating equilibrium, all the unbiased types are informed as well plus the DM can identify the types who invest as unbiased. However, the same might not be true for the expert payoff as it becomes more costly to acquire information, although the equilibrium becomes more informative. Surprisingly, we find that as a result the overall welfare increases in cost as summarised in the next proposition.

[16] Where $-(1-\rho)^2$ is the payoff for the group 1 type from not investing, as $\mu(0,1) = \rho$ and payoff from investment is $-c$ as $\mu(1,1) = 1$.

Proposition 2. *Total welfare increases when cost of information increases from* $(1-\hat{\rho})^2-(1-\tilde{\rho})^2$ *to the separating region* $(1-\hat{\rho})^2$*. That is, welfare is non-monotone in the cost: in a given type of equilibrium it decreases in cost of information acquisition while it increases if the increase in costs moves the equilibrium from pooling to separating.*

To understand this result, let's separate total welfare into two parts: that related to information and decision making and that related to investment cost. When cost increases from the pooling region to the separating one, the welfare related to decision making does not change, and the only change concerns the total cost of information acquisition. The group 1 types lose while the uninformed unbiased type and DM gain in terms of value from the decision made which exactly cancel out. There is more investment in the pooling equilibrium at a lower cost, while there is less investment in the separating equilibrium at a higher cost which is overall lower than in pooling. This is an intuitive but non-trivial result. In both pooling and separating equilibria the uninformed unbiased expert is the only one to invest efficiently, so investment strategies of group 1 types do not affect the total amount of information present: the biased type doesn't use the information acquired while the unbiased type doesn't learn more than what he already knew before investment. However, when moving to the separating equilibrium, some decision related value is transferred from the group 1 types to the group 2 type and the DM. Hence, the only difference is indeed due to the total cost incurred in information acquisition, and this turns out lower when we move to the separating equilibrium.

5 DM's Welfare and Optimal Expertise Level

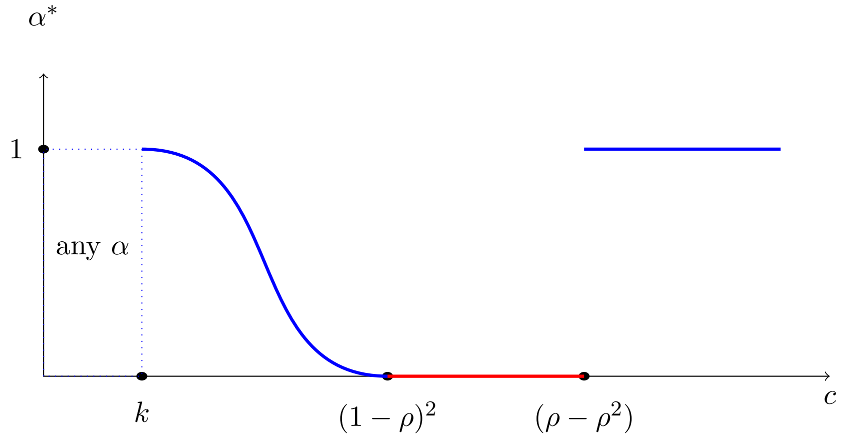
We now study the relation between the expert's type and the DM's welfare, and specifically what type of expert the DM should choose, given the option, as a function of the cost of information acquisition. The parameter we focus on is α, the **expertise** parameter, which denotes the probability an expert is informed on this specific issue as a function of his prior experience.

Proposition 3. *For any* $c \in [k,(1-\rho)^2]$[17]*, there is a unique* α^* *which satisfies* $c=(1-\hat{\rho})^2$ *and provides the DM her highest possible payoff among all equilibria. At this optimal level, the separating equilibrium and the best mixed strategy equilibrium coincide. This* α^* *is a decreasing function of cost meaning as information acquisition cost rises it is optimal to have a less informed expert.*

The intuition for this result is as follows: in the region $c<(1-\hat{\rho})^2$, the DM's welfare is increasing in α until the separating equilibrium is reached, given that $(1-\hat{\rho})^2$ is decreasing in α. However, conditional on separating equilibrium, the DM's welfare decreases in α once $c=(1-\hat{\rho})^2$. Hence, the optimal expertise level is non monotone in cost. That is, it is optimal to have the expert's probability of being informed just high enough to achieve separation but not

[17] Where $k=(1-\hat{\rho})^2$ where $\hat{\rho}$ is given by replacing $\alpha=1$ which is the cutoff below which pooling is the unique equilibrium for any α

Figure 2: α^* as a function of cost



higher. As the proposition says that at this unique α^*, the DM ensures the highest possible payoff independent of equilibrium selection issue, as the best mixed strategy equilibrium coincides with the separating equilibrium at this value. When $c < k$, even for α arbitrarily close to 1, a separating equilibrium cannot be achieved and pooling is the unique equilibrium when the cost of investment is so low.

We now summarize in the next proposition the optimal expert type for all possible cost levels.

Proposition 4. *The optimal expertise parameter α^* for the DM is non monotone in c:*

- *For $c > \rho - \rho^2$, $\alpha^* = 1$: no investment takes place => a perfectly informed expert is optimal.*
- *For $(1-\rho)^2 < c \leq \rho - \rho^2$, $\alpha^* = 0$: separating equilibrium is the unique equilibrium for any α => an uninformed expert is optimal.*
- *$k < c \leq \rho - \rho^2$, the unique optimal α^* solves $(1-\hat{\rho})^2 = c$ => α^* decreases in c.*
- *$c < k$, expertise doesn't matter as the unique equilibrium for any α is the pooling equilibrium.*

The DM's optimal type of expert is non-monotone in the cost of information acquisition. When $c > \rho - \rho^2$, in the unique equilibrium no expert invests. As it is impossible to separate between a biased and an unbiased expert through investment behavior, the DM is better off with a perfectly informed expert. Whenever $(1-\rho)^2 \leq c \leq \rho - \rho^2$, an unbiased uninformed expert strictly prefers to invest while informed (1) or biased types still strictly prefer not to invest when $\mu(1,1) = 1$, for any value of α. Then, it is optimal to have $\alpha = 0$ in order to avoid the loss in communication due to the pooling of informed type with biased type in the high state. This is because the investment behavior perfectly separates an unbiased type from a biased type when there are no ex-ante informed types. Indeed, this is the best situation for the DM, if it arises.

However, once the cost of information acquisition is lower, $c < (1-\rho)^2$, there exists no separating equilibrium for $\alpha = 0$, as whenever $\mu(1,1) = 1$ the biased type strictly prefers to invest. Now, to induce separation, there should be a sufficient portion of unbiased types who are informed, and the minimum α^* is given by the condition $c = (1-\hat{\rho})^2$. A lower α leads to a worse payoff for the DM as some biased types would then invest in any equilibrium. For any α higher than this critical level, the separating equilibrium payoff is lower for the DM while the mixed strategy equilibrium at most leads to the same payoff as the pure strategy separating equilibrium with α^*. This is because in the best mixed strategy equilibrium, some informed unbiased types reinvest. Hence, at α^* the best mixed strategy equilibrium coincides with the separating equilibrium and there is no issue of multiplicity. It follows that there is no benefit of having α higher than this critical level. This means, if it were the case that experts had a fee as a function of their expertise parameter, the DM shouldn't hire an expert more informed than this level.

Finally, k is defined as the threshold such that, when cost is so low, then it is inevitable that a pooling equilibrium arises, even for α arbitrarily close to 1. Then, in this region, expertise level has no importance for the DM's welfare.

Discussion: Our results show that when the bias of an expert is unobservable and information acquisition is endogeneous, a more informed expert often doesn't lead to better decision making. Indeed, in equilibrium, the only way a DM can distinguish an unbiased expert from a biased one in either state of the world is when there is separation in information acquisition behavior. In that case, the DM is lucky if the expert was an initially uninformed one who invest to get informed. However, having some probability that the expert is already informed is also necessary to discourage investment by biased types, by ensuring that an expert who communicates high state without investing is sufficiently credible. This is why, for a region of cost, the optimal expertise parameter decreases in the cost of information acquisition. Hence, **sunk** expertise in this setup is valuable only insofar as it discourages pooling by the biased types by giving them **credibility**, while **acquired** expertise is more valuable as it can be more easily transmitted. When the cost is high enough that separation arises even with a perfectly uninformed expert, then this type of expert is optimal. If the cost is so high that no type would invest, then given that investment behavior cannot separate a biased and unbiased expert, a perfectly informed expert is optimal.

6 Covert information acquisition

Now we consider covert information acquisition, in other words when the decision maker does not observe the investment made by the expert. In this setup, as investment has no signaling value, group 1 types never invest as they can pool with the unbiased uninformed expert without incurring any cost and the only type that may invest is the unbiased uninformed type. There are 2 types of pure strategy equilibria as a function of the cost which is summarized below:

1. Investment takes place by the uninformed unbiased type. Upon $m = 0$, the DM chooses

$y = 0$ and upon $m = 1$, the DM takes action $y = \tilde{\rho}$. This equilibrium looks like the pooling equilibrium except that the biased and informed (1) types do not actually invest. The payoffs of the biased and informed (1) types are higher in this equilibrium compared to the pooling equilibrium discussed, as they induce the same action without having to incur an investment cost.

The condition for the unbiased uninformed type to invest is:

$$c \leq (\rho - \rho^2) - \rho(1 - \tilde{\rho})^2$$

[18] This means the unbiased type acquires information for a smaller range of cost values in the covert information acquisition than in the overt case, as this cutoff is above the pooling cutoff cost but below the separating equilibrium cutoff cost of the overt case. [19]

2. No investment takes place. Upon $m = 1$, the DM chooses $\hat{\rho}$ inferring that this message is sent by a group (1) type of expert. The uninformed expert sends $m = \emptyset$ and the DM chooses $y = \rho$. This equilibrium is equivalent to the no investment equilibrium in the overt information acquisition case. This equilibrium arises for the following cost values:

$$c > (\rho - \rho^2) - \rho(1 - \tilde{\rho})^2$$

3. In the region $(\rho - \rho^2) - \rho(1 - \tilde{\rho})^2 < c < (\rho - \rho^2) - \rho(1 - \tilde{\rho})^2$, mixed strategy equilibria exist in which the unbiased uninformed type invests with some probability.

The types that gain from information acquisition being covert as opposed to overt are the group 1 types and only in case the cost is low enough that the uninformed unbiased type acquires information. Even though the cost does not affect these types directly as they don't invest in information, the investment of the unbiased uninformed type makes their message more credible. In this equilibrium, the payoffs of the uninformed unbiased expert and the decision maker are the same as in the pooling equilibrium in the overt case. When cost increases, we move to the no investment equilibrium in which payoffs are identical to the overt case without investment.

The unbiased type invests in information less often and is worse off in covert case, as he can never separate himself from the biased type. This result shows that, even though the signaling value of information undermines its intrinsic value, in case it leads to separating equilibrium, overt information acquisition does strictly better than the covert one due to more precise communication of information.

However, when the cost of information acquisition is low enough that in the overt case pooling

[18] This cutoff is found by considering: upon no investment the payoff of unbiased type is $-(\rho - \rho^2)$ and upon investment, it is 0 if the state is 0 but it is $-\rho(1 - \tilde{\rho})^2$ if the state is 1, as $\mu(1, 1) = \tilde{\rho}$ which is identical to the pooling equilibrium.

[19] Realize that in this region, there are as well mixed strategy equilibria in which the unbiased uninformed expert invests with a probability. This probability should increase as c decreases to keep the indifference condition satisfied.

in investment arises, then covert information acquisition does better in terms of the overall welfare as there is no wasteful investment, although the precision of communication is identical. The next proposition summarizes these results.

Proposition 5. *Whenever $c \geq (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$, overt information acquisition leads to higher overall welfare while whenever $c \leq (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$, covert information acquisition leads to higher welfare.*

The tradeoff is between more informative communication versus wasteful investment in information. Whenever no investment arises in the covert case, overt information acquisition does better as welfare is always higher when some types invest than when there is no investment. However, when cost is low enough that there is pooling in investment in the overt case, total welfare is higher in the covert case as although the communication precision is equivalent less cost is incurred. This is because in both cases, all the unbiased types are informed but cannot be separated from biased types, plus in the overt case there is more wasteful investment.

7 Conclusion

This paper considers a setup where costly information acquisition serves as a signalling device as well as for getting informed before communicating to a decision maker, using a simple model building on the communication and costly signaling literatures. We find that an unbiased expert, as well as a biased one, may wastefully invest in information, which leads to inefficiencies in decision making and lower overall welfare. Indeed, an uninformed expert who becomes informed later always communicates more precisely than an ex-ante informed expert. This is due the insufficient incentives of an unbiased and ex-ante informed expert to separate himself from a biased one. As such, our results highlight the value of an uninformed expert who has sufficient uncertainty that he is willing to incur cost in order to become informed, and while doing so can also identify himself as unbiased. In addition, the optimal level of expertise for the decision maker is non monotone in the cost of information acquisition, and specifically for higher information acquisition cost a less informed expert is optimal. Surprisingly, we show that higher information acquisition cost increases overall welfare as the equilibrium moves from a pooling to a separating one.

The simplicity of the model allows for numerous extensions remaining for future research. An interesting feature of the model is that even though the ability to acquire information is uncorrelated with the type of the expert, the value of getting informed depends on the expert's type. Hence, it is possible to use information acquisition as a screening device by taxing experts for getting informed. Now, assume that the cost of information acquisition falls in the pooling region. We know that the expert who is uninformed and unbiased has the highest incentives to acquire information. Hence, this type will be willing to pay more than biased and informed experts so that wasteful investment by these types can be prevented.

8 Appendix A

The No Investment Equilibrium Cutoff:

When $c \geq \rho - \rho^2$ even the unbiased uninformed type doesn't want to invest. The right hand side is the cutoff below which the separating equilibrium exists, in which the only type that invests is the unbiased uninformed type. We know that in the region $(1-\hat{\rho})^2 \leq c \leq \rho - \rho^2$, the biased type doesn't find it profitable to invest even if his message were taken at face value. Then, in the region $c \geq \rho - \rho^2$, following intuitive criterion, the out of equilibrium belief upon investment should place probability 0 to the expert being a biased type, as this type doesn't gain from deviating to invest even if he were perceived to be an unbiased type. Hence, the message upon a deviation to invest should be taken at face value by the DM. The gain in utility from doing so for the uninformed unbiased type is $\rho - \rho^2$ and defines the cost value above which the unique equilibrium has no investment.

Welfare Calculations in Different Equilibria

It is easy to see that the DM's payoff is minimized in the equilibrium in which no type invests, which is the least informative equilibrium. Hence, presence of investment can only make communication more informative. Plus, we will show that separating equilibrium is better than the pooling one for the DM.

Separating equilibrium:

The DM's payoff

$$[(1-\beta)\alpha\rho + \beta][-(\hat{\rho} - \hat{\rho}^2)] = -\hat{\rho}\beta(1-\rho) \tag{3}$$

The only time when the DM gets a payoff less than 0 is if high message is communicated without investment, in which case his posterior is $\hat{\rho}$ and happens either because the expert is unbiased but informed with high signal ($(1-\beta)\alpha\rho$) or he is biased (β). This payoff is decreasing in α and β which both lead to higher $\hat{\rho}$. As α increases, more of the unbiased types will be pooled with biased types which leads to lower payoff as these types get pooled with biased types. Also, the payoff decreases in β, which makes a biased type more likely. The payoff is also increasing in ρ whenever $\rho > 0.5$. This is intuitive: the DM's loss from biased communication is less when the prior is already in favor of the action that the biased type of expert wants.

The expert's payoff

The payoff of the biased expert is $-(1-\hat{\rho})^2$, which is increasing in α and ρ and decreasing in β: the more unbiased types there are that are informed, the higher the DM's posterior upon no investment in this type of equilibrium. The ex-ante payoff of the unbiased expert is $-\alpha\rho(1-\hat{\rho})^2 - \alpha(1-\rho)0 - (1-\alpha)c = -\alpha\rho(1-\hat{\rho})^2 - (1-\alpha)c$, which is increasing in α, decreasing in ρ and decreasing in β.

Then, the expected payoff over all expert types in the separating equilibrium is:

$$-(1-\hat{\rho})^2[\beta+(1-\beta)\alpha\rho]-(1-\beta)(1-\alpha)c$$

Pooling equilibrium:

The DM's payoff

$$[(1-\beta)\rho+\beta][-(\tilde{\rho}-\tilde{\rho}^2)]=-\rho(1-\tilde{\rho}) \tag{4}$$

In the pooling equilibrium, the DM gets a payoff less than 0 if an investment happens and a high message is sent, which is either a biased type (β) or an unbiased type who has high signal ($(1-\beta)\rho$), upon which his posterior is $\tilde{\rho}$. This payoff is decreasing in β and when $\rho > 0.5$, it is increasing in ρ. When $\rho < 0.5$ and β is high enough it may be decreasing in ρ. The pooling payoff is independent of α, as all unbiased types invest in this case and the DM doesn't internalize the cost of investment. When we replace the values of $\hat{\rho}$ and $\tilde{\rho}$, we find that he separating equilibrium payoff dominates the pooling equilibrium payoff for the DM if:

$$\alpha(1-\rho)+\rho\leq 1$$

which is always satisfied. Hence, the DM's payoff is unambiguously higher in the separating equilibrium in which information is more precise. The DM benefits from having some probability of expert being initially informed and pool with the biased type, as this makes pooling less attractive for the biased type. If α is very low, there are more incentives to "pool" and the separating region shrinks. However, once in the separating region, the DM's payoff is decreasing in α. Hence, the DM's payoff is non-monotone in α: it has to be just high enough for biased types not to invest.

The expert's payoff

The payoff of the biased expert is $-(1-\tilde{\rho})^2-c$. This is increasing in ρ and decreasing in β. The ex-ante payoff of the unbiased expert is $-\rho(1-\tilde{\rho})^2-(1-\alpha(1-\rho))c$, which is decreasing in ρ and in β. It is increasing in α as less cost investment will be incurred given the unbiased expert with a low signal doesn't invest.

It is trivial that the biased type of expert is unambiguously better off in the pooling equilibrium than in the separating one, as otherwise he would not invest and still get the same payoff as in the separating equilibrium. On the other hand, the unbiased expert's expected payoff increases when moving from the separating to the pooling equilibrium, when we compare the payoff at the minimum cost at which there is separation and maximum cost at which there is pooling.

The expected payoff over all expert types in the pooling equilibrium is:

$$-[(1-\beta)\rho+\beta](1-\tilde{\rho})^2-[1-(1-\beta)(1-\rho)\alpha]c$$

Mixed strategy equilibria

The DM's payoff The DM's payoff given σ and γ is:

$$-(\beta(1-\sigma)+(1-\beta)\alpha\rho(1-\gamma))(\mu(0,1)-\mu(0,1)^2)-((1-\beta)(\alpha\gamma+1-\alpha)+\beta\sigma)(\mu(1,1)-\mu(1,1)^2)$$

which simplifies to:

$$-\beta(1-\rho)[(1-\sigma)\mu(0,1)+\sigma\mu(1,1)] \tag{5}$$

A mixed strategy equilibrium exists for $c<(1-\rho)^2$. In the region $(1-\hat{\rho})^2<c<(1-\rho)^2$, mixed strategy equilibria are such that some of the unbiased experts informed of high signal re-invest, and none of the biased experts invest. Hence, $\mu(0,1)>\hat{\rho}$ and $\mu(1,1)=1$.

In the region where $c<(1-\hat{\rho})^2$, now the mixed strategy equilibria should have $\mu(0,1)>\hat{\rho}$, as it cannot be the case that $\mu(1,1)>1$. Then, it must be that $\sigma>0$ and $\mu(1,1)<1$. Then, we can see that the DM's payoff is highest as ρ decreases, specifically when $\rho=0$ it is the highest, as $\mu(0,1)=\hat{\rho}$ also takes the minimum value.

At a given cost c, the DM optimal equilibrium, which also coincides with expert optimal mixed strategy equilibrium is the one in which $\sigma=0$ and γ takes the value which satisfies:

$$c=(1-\mu(0,1))^2$$

as $\mu(1,1)=1$.

This means, as α increases from the mixed strategy region until the separating equilibrium is reached, the payoff of DM increases. Then, once the separating equilibrium is attained, the payoff of DM now decreases in α.

The expert's payoff The expected payoff over expert types in a mixed strategy equilibrium is:

$$-(\beta+(1-\beta)\alpha\rho)(1-\mu(0,1))^2-(1-\beta)(1-\alpha)\rho(1-\mu(1,1))^2-(1-\beta)(1-\alpha)c$$

The above is found by using the fact that the biased and informed (1) types of experts are indifferent between investing or not, and taking their payoffs as the payoff when they don't invest. This simplifies, if we replace $c = (1-\mu(0,1))^2 - (1-\mu(1,1))^2$, to:

$$-[(1-\beta)(\alpha\rho + 1 - \alpha) + \beta](1-\mu(0,1))^2 + (1-\beta)(1-\alpha)(1-\mu(1,1))^2$$

In the region where mixed strategy and separating equilibrium coexist, $(1-\hat{\rho})^2 < c < (1-\rho)^2$, in order to make the group 1 types indifferent to invest or not, in any mixed strategy equilibrium we should have $\mu(0,1) < \hat{\rho}$ as it cannot be the case that $\mu(1,1) > 1$. We find in that case that the expert preferred mixed strategy equilibrium has $\sigma = 0$, and $\gamma > 0$. Hence, in the region where mixed equilibria coexist with pure strategy separating equilibrium, there exists a unique mixed strategy equilibrium preferred by both the DM and expert.

No investment equilibrium:

The DM's utility in the equilibrium in which no one invests is:

$$\frac{-(1-\rho)\rho(\beta + (1-\alpha)\alpha(1-\beta)^2\rho)}{\beta + \alpha(1-\beta)\rho} \tag{6}$$

which is found by simplifying $-[\beta + (1-\beta)\alpha\rho](\hat{p} - \hat{p}^2) - (1-\beta)(1-\alpha)(\rho - \rho^2)$. We see that pooling in investment always leads to higher payoff for the DM than no investment. Hence, the no investment equilibrium provides the minimum possible payoff to the DM.

The expert's utility:

$$(\beta + (1-\beta)\rho\alpha)$$

The case with $\rho < 0.5$, hence $(1-\rho)^2 > \rho - \rho^2$

In this case, there is no region of cost for which separating equilibrium arises regardless of α, as was the case under the assumption made in the paper. In this case, separating equilibrium exists if α is high enough such that $(1-\hat{\rho})^2 < \rho - \rho^2$, and for the region $(1-\hat{\rho})^2 < c < \rho - \rho^2$. Plus, whenever $c > (1-\rho)^2$, no type is going to invest. Indeed, even though the cost is below $(1-\rho)^2$, as long as the uninformed unbiased types are not willing to, the group 1 types will not invest either.

Under this assumption, the main difference to the case in the paper is that there is not a region in which $\alpha^* = 0$, as whenever the uninformed type is willing to invest, the group 1 types are strictly willing to invest.

Whenever $c > \rho - \rho^2$, $\alpha^* = 1$, as in this region the uninformed types will never acquire information.

When $k \leq c \leq \rho - \rho^2$, it is optimal to set $c = (1 - \hat{\rho})^2$. In this region, α is decreasing in cost.

As in the case with $\rho > 0.5$, for $c < k$, only pooling equilibrium can arise for any value of α as the cost of investment is low. Hence, the main results do not change if $\rho < 0.5$ however there is less variety of equilibria that exist.

9 Appendix B

Proof of lemma (1):

Proof. In the communication stage following investment, it is certain that the expert is informed. We can then focus on equilibria in which after investment there are at most two messages, $m^i \in \{0, 1\}$. To see this, realize that the unbiased expert either knows $\omega = 0$ and wants to induce the lowest possible action or knows $\omega = 1$ and wants to induce the highest possible action. The biased expert wants to induce highest possible action regardless of the state. Hence, in equilibrium, after investment there need to be two types of messages only.

In the communication stage following no investment, there are four possible types of experts. First, the informed expert with signal 0 who strictly prefers the message which induces the lowest action hence will send $m = 0$. Second, the biased and unbiased informed (1) expert both want to send the message that induces the highest action hence send $m = 1$. Finally, the unbiased uninformed expert who shares the DM's preferences. It is then without loss of generality to restrict attention to equilibria in which an empty message is available, $m = \varnothing$, which means "I am not informed" and will induce $y = \rho$.[20]. Then, following $x = 0$, the set of messages is $m^n \in \{0, 1, \varnothing\}$.

□

Proof of lemma 2

Proof. The unbiased informed (0) type's payoff is maximized when $y = 0$ is chosen by the DM and there is no other type that could benefit from sending this message, given the set of other possible messages. For the biased types informed of high state, this message leads to the lowest possible payoff of -1 and they are better off sending any other message. For the unbiased uninformed type who has not invested, sending $m = \varnothing$ upon which the DM chooses $y = \rho$ can only lead to a better payoff than sending this message. Given that, it is consistent that the DM, upon hearing $m = 0$, interprets that this message is coming from an unbiased informed type. Then, $\mu(0, 0) = 0$ and the DM will indeed choose $y = 0$. The unbiased expert informed of signal 0 achieves maximal payoff under this strategy. For contradiction, assume there were any other

[20] As it becomes clear later, the biased expert can always induce a higher posterior by sending $m = 1$ as opposed to any other message

equilibrium in which this type sent another message, or it invested. Then, it must be that his equilibrium payoff is lower than 0. However, in any such equilibrium, it would have an incentive to deviate to not invest and send $m = \varnothing$. By the intuitive criterion, the DM should attribute this message to the informed 0 type. Then, he would indeed have an incentive to deviate and no other type has an incentive to do so. $\square$

Proof of lemma 3

Proof. The expected payoff over all expert types in a mixed strategy equilibrium is:

$$-(\beta + (1-\beta)\alpha\rho)(1-\mu(0,1))^2 - (1-\beta)(1-\alpha)\rho(1-\mu(1,1))^2 - (1-\beta)(1-\alpha)c$$

We can see that the expert preferred mixed strategy equilibrium has $\sigma = 0$, and $\gamma > 0$. This is because for the uninformed expert who is getting informed, his payoff is maximized when $mu(1,1) = 0$. For the group 1 types, their payoff is also maximized when $\mu(1,1)$ takes the maximum value as they are indifferent between investing or not.

The DM's payoff given σ and γ is:

$$-(\beta(1-\sigma)+(1-\beta)\alpha\rho(1-\gamma))(\mu(0,1)-\mu(0,1)^2)-((1-\beta)(\alpha\gamma+1-\alpha)+\beta\sigma)(\mu(1,1)-\mu(1,1)^2)$$

which simplifies to:

$$-\beta(1-\rho)[(1-\sigma)\mu(0,1)+\sigma\mu(1,1)]$$

Which is also maximized when $\sigma = 0$. Finally, for a given $(1-\hat{\rho})^2 < c < (1-\rho)^2$, the DM optimal mixed strategy equilibrium is the one in which $\sigma = 0$ and $\gamma > 0$ such that:

$$c = (1-\mu(0,1))^2$$

Then, at any c, there is a unique welfare maximizing mixed strategy equilibrium with $\sigma = 0$ and $\gamma > 0$. $\square$

Proof of proposition 1

Proof. We find the separating equilibrium to be the unique pure strategy equilibrium in $(1-\rho)^2 < c < (\rho-\rho^2)$. First, there cannot be any equilibrium in this region in which unbiased informed (1) and biased types invest with positive probability, as even if $\mu(1,1) = 1$, these types do not find it profitable to invest as the cost is too high. Then, the only pure strategy equilibrium that could arise is the no investment equilibrium in which even the uninformed unbiased expert doesn't invest. For some out of equilibrium beliefs, this equilibrium can arise, as discussed below.

Assume that the DM believes any type except the informed (0) unbiased one is equally likely to invest, then his belief and optimal choice will be $\tilde{\rho}$ which is:

$$\tilde{\rho} = \frac{\rho(1-\beta)+\beta\rho}{\rho(1-\beta)+\beta} = \frac{\rho}{\rho(1-\beta)+\beta}$$

Let us show the biased type doesn't have the incentive to incur the cost c. Now, upon the message $m = 1$, the DM chooses $y^* = \hat{\rho}$ as there is only biased and unbiased informed(1) types who choose $m = 1$. For the biased and unbiased informed (1), the payoff from investing should be less than that from not investing:

$$(1-\tilde{\rho})^2 + c \geq (1-\hat{\rho})^2$$

As without investment and $m = 1$, the DM's belief is $\hat{\rho}$ as in case 1. This is equivalent to:

$$\left[\frac{\beta(1-\rho)}{\rho(1-\beta)+\beta}\right]^2 + c \geq \left[\frac{\beta(1-\rho)}{\alpha(1-\beta)\rho+\beta}\right]^2$$

For the uninformed agent, the payoff from not investing is $-(\rho-\rho^2)$ and from investing it will be:

$$-(1-\rho)0 - \rho(1-\tilde{\rho})^2 - c$$

Then the condition that should be satisfied is:

$$c \geq \rho - \rho^2 - \rho(1-\tilde{\rho})^2$$

Finally, the equilibrium in which no type wants to invest, for the specified out of equilibrium beliefs, exists for the following cost values:

$$c \geq \max\{\rho - \rho^2 - \rho(1-\tilde{\rho})^2, (1-\hat{\rho})^2 - (1-\tilde{\rho})^2\}$$

Now consider that when there is a separating equilibrium, the condition $\rho - \rho^2 > (1-\hat{\rho})^2$ is satisfied. Then, it is the case that $\rho - \rho^2 - \rho(1-\tilde{\rho})^2 > (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$. This means, the condition above becomes $c \geq \rho - \rho^2 - \rho(1-\tilde{\rho})^2$, which is less than $\rho - \rho^2$. Then, there is also a no investment equilibrium in this region. However, we are able to rule out this type of equilibrium. This is because, the uninformed type, when his message is taken at face value, is willing to deviate to invest while the group 1 types do not find it profitable to invest even if $\mu(1,1) = 1$. Hence, as there is a best response from the DM to this deviation that makes unbiased uninformed types better off and the group 1 types worse off and is consistent given his beliefs about the deviator, this type of equilibrium can be ruled out by the Intuitive Criterion.

Finally, we can also rule out an equilibrium in which only group 1 types invest, as there is no belief of the DM these types find it profitable to invest in this region.

In the region $c \in [\rho - \rho^2 - \rho(1-\tilde{p})^2, \rho - \rho^2]$, the separating equilibrium is the unique equilibrium that survives the Intuitive Criterion.

Whenever $c \geq (1 - \hat{\rho})^2$, even for the highest belief $\mu(1,1) = 1$, the biased and uninformed (1) agent do not benefit from deviating to invest. Hence, in this region, the out of equilibrium belief should assign probability 1 to the expert being unbiased and uninformed. Then, whenever $c \geq (\rho - \rho^2)$, even the unbiased uninformed type doesn't want to invest, which provides the boundary of the no investment equilibrium.

Next, we make the welfare comparisons. First, consider the region $(1-\hat{\rho})^2 \leq c \leq (1-\rho)^2$ where mixed strategy and separating equilibria coexist. We can see that the separating equilibrium provides higher payoff for any expert type for any c. To see this: we know that in the separating equilibrium, the uninformed unbiased type's communication is taken at face value and the group 1 types' payoff is $-(1-\hat{\rho})^2$, independent of cost. In the mixed strategy equilibria in this region, for any $c > (1-\hat{\rho})^2$, we need to have $\mu(0,1) < \hat{\rho}$ hence the payoff of group 1 types will be $-(1-\mu(0,1))^2 < -(1-\hat{\rho})^2$ given their indifference between investment and not, and the uninformed type's payoff cannot be higher as $\mu(1,1) = 1$ in the separating equilibrium. Then, all expert types are weakly (and some strictly) worse off in mixed strategy equilibria compared to pure strategy in this region.

For the DM, when we compare the pure strategy equilibrium payoff in equation 3 to the mixed strategy payoff in 5, we see that the mixed strategy equilibrium payoff is higher than the separating equilibrium. To see this, consider the mixed strategy equilibria of the type $\sigma = 0$ and $\gamma > 0$ which is the optimal one. Then, realizing that $\mu(0,1) < \hat{\rho}$ provides the result. The equilibrium with $\sigma = 0$ is also the one which maximizes the DM's payoff. In this type of equilibrium, as c increases, γ increases while keeping $\sigma = 0$. Hence, we can conclude that the DM's payoff is highest in mixed strategy equilibria in this region and especially it is highest in the equilibrium in which $\gamma = 1$ and $\sigma = 0$ which can arise at $c = (1-\rho)^2$.

The total payoff in separating equilibrium is $-\beta(1-\rho) - (1-\beta)(1-\alpha)c$ while in mixed strategy equilibrium it is $-\beta(1-\rho)\mu(0,1) - (\beta + (1-\beta)\alpha\rho)(1-\mu(0,1))^2 - (1-\beta)(1-\alpha)c$. Now, consider $-(\beta + (1-\beta)\alpha\rho)(1-\mu(0,1))^2 = X$, then we have $X < -(1-\rho)\beta(1-\mu(0,1))$. As the payoff in mixed strategy would be equal to separating equilibrium if and only if $X \geq -(1-\rho)\beta(1-\mu(0,1))$, we can conclude that the separating equilibrium welfare is higher than mixed strategy. □

Proof of lemma 4:

Proof. In this equilibrium, the DM's updated belief $\mu(1,1)$, given that both group 1 and 2 types are investing, is:

$$\tilde{\rho} = \frac{\rho(1-\beta)+\beta\rho}{\rho(1-\beta)+\beta} = \frac{\rho}{\rho(1-\beta)+\beta}$$

First, consider the investment choice of the group 1 types. If the biased or unbiased agent were to deviate to not invest and send $m = 1$, then the DM would choose $y = \hat{\rho}$ as the DM infers this out of equilibrium action can only come from a biased or unbiased expert informed with signal 1. Making use of the intuitive criterion (Cho and Kreps 1987) we define the out of equilibrium belief upon no investment and $m = 1$ to assign probability 0 to the unbiased uninformed expert as this type always prefer to send $m = \varnothing$ if he were to deviate to no investment while the group 1 types always prefer sending $m = 1$ to any other message, which leads to $\mu(0,1) = \hat{\rho}$.

Then, the following is the condition for a pooling equilibrium to arise:

$$-(1-\tilde{\rho})^2 - c \leq -(1-\hat{\rho})^2$$

Second, for the uninformed unbiased type, the payoff from not investing is $-(\rho - \rho^2)$, as in that case they would send $m = \varnothing$. Then, consider the payoff of this type from investing. If the signal turns out to be 0, the DM takes the message at face value and chooses $y = 0$ whereas if the signal is 1, the decision maker will choose $\hat{\rho} < \tilde{\rho} < 1$ as the DM infers it can come from a group 1 or 2 type. Then, this type prefers investing to not if and only if:

$$-\rho(1-\tilde{\rho})^2 - c \geq -(\rho - \rho^2)$$

These conditions together lead to:

$$c \leq min\{(1-\hat{\rho})^2 - (1-\tilde{p})^2, \rho - \rho^2 - \rho(1-\tilde{\rho})^2\} = (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$$

When $(1-\hat{\rho})^2 < (\rho - \rho^2)$, which is the case we consider, we have $min\{(1-\hat{\rho})^2 - (1-\tilde{\rho})^2, \rho - \rho^2 - \rho(1-\tilde{\rho})^2\} = (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$. To see this, realize that the value of getting informed is higher for unbiased uninformed types than for the group 1 types: $(1-\hat{\rho})^2 - (1-\tilde{\rho})^2 < \rho - \rho^2 - \rho(1-\tilde{\rho})^2$. This is because unbiased uninformed types get maximum payoff of 0 when the state of the world is 0, while their communication is distorted when the message is 1. However, from the point of view of the group 1 types, communication is always distorted as their bliss point is 1 and $\mu(1,1) = \tilde{\rho} < 1$. Then, the condition for the pooling equilibrium is given by the condition for the biased and informed (1) types to be willing to invest which is $c \leq (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$.

Now consider the expression $c \leq \rho - \rho^2 - \rho(1-\tilde{\rho})^2$, the condition for the unbiased uninformed types to actually invest given a pooling equilibrium. As $\rho - \rho^2 > \rho - \rho^2 - \rho(1-\tilde{\rho})^2$, where the left hand side is the cutoff for investment in the separating equilibrium, the condition for unbiased uninformed types to invest is easier to satisfy in the separating region. The difference between these two is due to the biased types' "crowding out" the unbiased uninformed types: investment of the biased types makes information acquisition by the unbiased types less profitable, hence cost has to be lower in order to satisfy their participation.

□

Proof of proposition 2:

Proof. We compare the payoff in the pooling equilibrium to that in the separating equilibrium, before considering the equilibria with no investment. For this, we compare the payoff for cost values at which there is pooling equilibrium and separating equilibrium to find how the payoff changes when c increases from the pooling region to separation.

The DM's welfare:

$$[(1-\beta)\rho+\beta][-(\tilde{\rho}-\tilde{\rho}^2)] = -\rho(1-\tilde{\rho}) \tag{7}$$

The payoff of the biased expert is $-(1-\tilde{\rho})^2-c$. The ex-ante payoff of the unbiased expert is $-\rho(1-\tilde{\rho})^2-(1-\alpha(1-\rho))c$. This is decreasing in ρ and in β.

Then the total expert welfare found by summing these two is:

$$-[(1-\beta)\rho+\beta](1-\tilde{\rho})^2-[1-(1-\beta)(1-\rho)\alpha]c$$

When we sum the DM's and expert's welfare we get:

$$-\rho(1-\tilde{\rho})-\beta[(1-\tilde{\rho})^2+c]-(1-\beta)[\rho(1-\tilde{\rho})^2+(1-\alpha(1-\rho))c] \tag{8}$$

which simplifies to:

$$-(1-\tilde{\rho})^2[\beta+\rho(1-\beta)]-\rho(1-\tilde{\rho})-c[1-\alpha(1-\rho)] = -\beta(1-\rho)-c[1-\alpha(1-\rho)(1-\beta)] \tag{9}$$

In the **separating** equilibrium, the DM's welfare is:

$$[(1-\beta)\alpha\rho+\beta][-(\hat{\rho}-\hat{\rho}^2)] = -\hat{\rho}\beta(1-\rho) \tag{10}$$

The payoff of the biased expert is $-(1-\hat{\rho})^2$, while the ex-ante payoff of the unbiased expert is $-\alpha\rho(1-\hat{\rho})^2-\alpha(1-\rho)0-(1-\alpha)c = -\alpha\rho(1-\hat{\rho})^2-(1-\alpha)c$.

Then, the expected payoff over expert types in the separating equilibrium is:

$$-(1-\hat{\rho})^2[\beta + (1-\beta)\alpha\rho] - (1-\beta)(1-\alpha)c$$

Finally, the total welfare in the separating equilibrium is:

$$- \hat{\rho}\beta(1-\rho) - \beta(1-\hat{\rho})^2 - (1-\beta)[\alpha\rho(1-\hat{\rho})^2 + (1-\alpha)c] \tag{11}$$

which simplifies to:

$$- \beta(1-\rho) - c(1-\alpha)(1-\beta) \tag{12}$$

The difference in the total welfare in equation (12 − 9) is:

$$c_p[1-\alpha(1-\rho)(1-\beta)] - c_s(1-\beta)(1-\alpha) \tag{13}$$

The welfare in terms of decision values cancel out and the terms that remain are those related to the cost incurred for information acquisition. In the pooling equilibrium there is more investment at a lower price where some of this investment is wasteful, while in the separating equilibrium there is less investment at a higher price. When we plug in the maximum cost at which the pooling equilibrium exists and the minimum cost at which the separating equilibrium exists, it is seen that the welfare in separating equilibrium is higher than in the pooling one although the cost of information acquisition is higher.

At $c_p = (1-\hat{\rho})^2 - (1-\tilde{\rho})^2$ and $c_s = (1-\hat{\rho})^2$, equation (13) becomes:

$$(1-\hat{\rho})^2(\beta(1-\alpha\rho) + \alpha\rho) - (1-\tilde{\rho})^2(1-\alpha(1-\rho)(1-\beta)) > 0 \tag{14}$$

When we replace these values, finally we are left with the condition:

$$\beta(1-2\rho) + \rho(1-\beta)(\alpha - (1+\alpha)\rho) \leq 0 \tag{15}$$

which is satisfied whenever $\rho > \frac{1}{2}$ which is the initial assumption we made. The first term is negative. The second term is negative when $\rho > \frac{\alpha}{1+\alpha}$ which is always the case when $\rho > \frac{1}{2}$ and $\alpha < 1$.

Then, although the cost of information rises, welfare increases due to the lack of wasteful investment in information. In order to demonstrate this result, we considered the cost values at the boundaries. As expected, when we keep increasing the cost in the separating equilibrium region, the welfare will decrease and at some point, it will be lower than in the pooling equilibrium.

Finally, the total welfare in the **no investment** equilibrium is:

$$-\beta(1-\hat{\rho})^2 - (1-\beta)[\alpha\rho(1-\hat{\rho})^2 + 2(1-\alpha)(\rho-\rho^2) - (\hat{\rho}-\hat{\rho}^2)(\beta+(1-\beta)\alpha\rho)] \tag{16}$$

As no investment equilibrium surplus is unambiguously worse than the separating equilibrium, we compare it to the pooling in investment equilibrium and find that the payoff in the no investment equilibrium is also lower than the pooling in investment equilibrium. This is intuitive: first, the DM's payoff is unambiguously higher in the pooling in investment equilibrium compared to the no investment equilibrium, as more information is revealed. The welfare of the biased type also higher in the pooling in investment equilibrium as their outside option of not investing and getting $-(1-\hat{\rho})^2$ is still available. Hence, if this type does find it profitable to invest, then it must be getting a higher payoff. The same is true for the unbiased informed (1) type who would get $-(1-\hat{\rho})^2$ if deviating to not invest. Finally, for the unbiased uninformed type, it is true as well: if this type didn't invest they would get the payoff $-(\rho-\rho^2)$ which is still available if they deviate in the pooling equilibrium to send $m = \varnothing$. □

Proof of Proposition 3

Proof. First, we know that conditional on the separating equilibrium arising, the DM's payoff is decreasing in α. In addition, we know that the pooling equilibrium gives a strictly worse payoff for the DM. Then, the best payoff for the DM in pure strategies happens when $c = (1-\hat{\rho})^2$: at the boundary of the separating equilibrium.

Now, we will show that any other α gives a lower payoff to the DM. First, lower α leads to $c < (1-\hat{\rho})^2$. In this region, in order to keep the indifference condition of group 1 types, we need $\sigma > 0$, as otherwise we will still have $\mu(1,1) = 1$ and $\mu(0,1) \leq \hat{\rho}$ and the group 1 types would strictly want to invest. Hence, in any mixed strategy equilibrium in between the separating and pooling regions, we should have $\mu(1,1) < 1$ or $\mu(0,1) > \hat{\rho}$. If we look at the DM's payoff, $-\beta(1-\rho)[(1-\sigma)\mu(0,1) + \sigma\mu(1,1)]$, we can see that whenever $\sigma > 0$ or $\mu(0,1) > \hat{\rho}$, it will be strictly lower than when $\sigma = 0$ and $\mu(0,1) = \hat{\rho}$, given that $\mu(0,1) < \mu(1,1)$. As α continues to decrease further eventually it may lead to pooling equilibrium.

We know that pooling equilibrium provides lower payoff to the DM than the separating equilibrium. If the mixed strategy equilibrium is realized, we know that the payoff increases in α in that region.

Second, if we increase α above α^* so that $(1-\hat{\rho})^2 < c$, now c falls strictly inside the separating region. At this point, there is indeed multiplicity of equilibria. We will show that this multiplicity doesn't affect α^*. First there is the separating equilibrium, whose payoff decreases in α. Second, there exists mixed strategy equilibria at this point, among which the one that maximizes the DM's payoff has $\gamma > 0$ and $\sigma = 0$. As $\mu(1,1) = 1$, γ should satisfy $c = (1-\mu(0,1))^2 = (1-\frac{(1-\beta)\alpha(1-\gamma)\rho+\beta\rho}{(1-\beta)\alpha(1-\gamma)\rho+\beta})^2$. Now, if we replace α by $\hat{\alpha} = (1-\gamma)\alpha$, $\hat{\alpha}$ will now satisfy $c = (1-\hat{\rho})^2$ hence the mixed and pure strategy equilibria coincide, and at this new separating equilibrium the DM gets the same payoff. This means for any α such that $(1-\hat{\rho})^2 < c$, there is a $\hat{\alpha} = \alpha^*$. To understand the intuition, realize that $\mu(0,1)$ is the same in both equilibria, plus $\mu(1,1) = 1$ in the best mixed strategy equilibrium for any $c > (1-\hat{\rho})^2$. In both equilibria, an equal portion of unbiased experts invest in overall, and while informed ones invest wastefully in the mixed strategy equilibria, in the separating equilibrium with $\hat{\alpha}$ all unbiased experts who will invest are uninformed. The composition of types that do not invest are constant in both equilibria. This means, whenever $\alpha > \alpha^*$ so that $(1-\hat{\rho})^2 < c$, the highest DM equilibrium payoff is in mixed strategies. However, for any c and α, it is shown that this mixed strategy equilibrium payoff is bounded by the separating equilibrium payoff for $\alpha = \alpha^*$.

This shows that there is a unique α which maximizes the DM's payoff regardless of the equilibrium selection. At α^*, the best mixed strategy equilibrium is the separating equilibrium, and there is no conflict over equilibrium selection.

Finally, we will identify the threshold k. This is the cost value below which, even for α arbitrarily close to 1, all types are still willing to invest. Then, we replace $\alpha = 1$ to find $\hat{\rho} = \frac{\rho}{(1-\beta)\rho+\beta}$, and then the threshold cost is given by $k = (1-\hat{\rho})^2 = \left(\frac{\beta(1-\rho)}{\beta(1-\rho)+\rho}\right)^2$. Whenever the cost is below this level, no matter what α is, all types are willing to invest regardless of whether they are informed or not. This means, as cost increases, the DM's optimal level of expert is going to be increasingly informed, and once it becomes arbitrarily close to being perfectly informed, below that level any α will lead to the same result. □

Proof of proposition 4

Proof. Fix c. We have 4 different regions. First, for $c < k$, the unique equilibrium is the pooling one for any $\alpha > 0$. Realise also that when cost is so low, assuming this is interior, so that $c < k$ where $k = (1-\hat{\rho})$ with $\alpha = 1$ (this means $(1-\rho) = \frac{\beta(1-\rho)}{\beta(1-\rho)+\rho}$, we are in the region where even if the probability that the expert is informed is very close to 1, there is still pooling equilibrium. Then below this level, expertise parameter doesn't matter for the DM. Hence in this region the DM's payoff doesn't depend on the expertise parameter.

Second, consider the region $k < c \leq (1-\rho)^2$. We know that $(1-\hat{\rho})^2$ is a decreasing function of α as $\hat{\rho}$ is increasing in α. Among the equilibria that arise in this region, we know that the DM's welfare is maximized with α high enough to ensure $(1-\hat{\rho})^2 = c$, which leads to $\alpha^* = \frac{\beta(1-\rho-\sqrt{c})}{(1-\beta)\rho\sqrt{c}}$. When α is slightly lower, mixed strategy equilibrium arises in which the equilibrium payoff for the DM is lower. Plus, we know that conditional on the separating equilibrium realizing, if α increases further so that $(1-\hat{\rho})^2 > c$, the separating equilibrium payoff is decreasing for the DM due to the loss in communication for informed unbiased types. At α^*, the DM optimal mixed strategy equilibrium is also the separating equilibria and provides the highest possible payoff for the DM.

When cost increases further so that $(1-\rho)^2 \leq c \leq \rho - \rho^2$, the unique equilibrium is the one in which only the group 2 types invest, as even though $\mu(1,1) = 1$ the group 1 types find it too costly to invest now. Then, it is DM optimal that $\alpha = 0$ given that the unbiased uninformed types are willing to invest and separate themselves from a biased type, making communication most precise. If the biased types can be prevented from pooling when $\mu(0,1) = \rho$, in other words when they are identified as biased, then it is optimal to have no informed experts at all. This is the ideal scenario for the DM and provides him the highest possible payoff.

Finally, when $c > \rho - \rho^2$, there is no possibility of investment by any type. In that case, it is optimal that the expert is informed with probability 1. When there is no way for the DM to separate between biased and unbiased types through investment behavior, it is optimal that the expert perfectly informed, in the least.

□

Proof of proposition 5

Proof. When $c > \rho - \rho^2 - \rho(1-\tilde{p})^2$, in the covert information acquisition case no type is getting informed, hence the payoff is the same as in the no investment equilibrium in the overt case. We know that in the overt information acquisition case, welfare is always higher when there is some information acquisition compared to none, and in the region $[\rho - \rho^2 - \rho(1-\tilde{p})^2, \rho - \rho^2]$ there is investment in the overt case. Hence, the payoff in overt case is always higher under this condition.

When $c < (1-\hat{p})^2 - (1-\tilde{p})^2$, in the overt case, there is pooling in investment while in the covert case, the unbiased uninformed type invests only (realize that $(1-\hat{p})^2 - (1-\tilde{p})^2 < \rho - \rho^2 - \rho(1-\tilde{p})^2$ hence there is investment in covert case). In the end, the amount of information transmitted is the same. Hence, in the overt case there is more wasteful investment for the same precision of communication. Then, we can conclude that welfare is higher in the covert case under this assumption. □

References

Rosella Argenziano, Sergei Severinov, and Francesco Squintani. Strategic information acquisition and transmission. *American Economic Journal: Microeconomics*, 8(3):119–155, 2016.

David Austen-Smith. Strategic transmission of costly information. *Econometrica*, 62:955–963, 1994.

David Austen-Smith and Jeffrey S. Banks. Cheap talk and burned money. *Journal of Economic Theory*, 91:1–16, 2000.

Sourav Bhattacharya, Maria Goltsman, and Arijit Mukherjee. On the optimality of diverse expert panels in persuasion. *Games and Economic Behavior*, 107:345–363, 2018.

In-Koo Cho and David M. Kreps. Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102:179–221, 1987.

Vincent Crawford and Joel Sobel. Strategic information transmission. *Econometrica*, 50: 1431–1451, 1982.

Inga Deimen and Dezs̋o Szalay. Delegated expertise, authority, and communication. *working paper*, 2016.

Robert Dur and Otto H. Swank. Producing and manipulating information. *Economic Journal*, 115, 2005.

Jeffrey Ely and Juuso Valimaki. Bad reputation. *Quarterly Journal of Economics*, pages 785–814, 2003.

Péter Esö and Dezsö Szalay. Incomplete language as an incentive device. *working paper*, 2010.

Paul E. Fischer and Phillip C. Stocken. Analyst information acquisition and communication. *The Accounting Review*, 2010.

Xiaojing Meng. Analyst reputation, communication, and information acquisition. *Journal of Accounting Research*, pages 119–173, 2015.

John Morgan and Philipp C. Stocken. An analysis of stock recommendations. *RAND journal of economics*, 34(1):183–203, 2003.

Stephen Morris. Political correctness. *Journal of Political Economy*, 109(2), 2001.

Marco Ottaviani and Peter N. Sorensen. Reputational cheap talk. *RAND Journal of Economics*, 37.

Marco Ottaviani and Peter N. Sorensen. Professional advice: The theory of reputational cheap talk. *Journal of Economic Theory*, 126:120–142, 2006a.

Harry Di Pei. Communication with endogenous information acquisition. *Journal of Economic*

Theory, 160:132–149, 2015.

Joel Sobel. A theory of credibility. *The Review of Economic Studies*, 1985.

Joel Sobel. Giving and receiving advice. *Advances in Economics and Econometrics*, 2013.

Guido Suurmond, Otto H. Swank, and Bauke Visser. On the bad reputation of reputational concerns. *Journal of Public Economics*, 88:2817–2838, 2004.