Information Acquisition and Credibility in Cheap Talk

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Abstract

This paper explores the interaction between uncertainty in bias and endogeneous information acquisition in strategic communication. I consider an expert who is privately informed about his bias as well as about whether he is informed, in addition can also engage in costly information acquisition. In this setup, information acquisition simultaneously serves the purposes of getting informed and increasing credibility before communicating through cheap talk to a decision maker. I define the *signaling* and the *intrinsic value* of information and find the conditions under which separation in the information acquisition behavior can arise. I solve for equilibria as a function of cost of information acquisition and show that communication is most precise with an initially uninformed expert at an intermediary cost value. The overall welfare is non-monotone in cost, and it increases as cost increases to enable separation. When covert information acquisition is considered, there is a tradeoff between less wasteful investment versus less precise communication compared to the overt case in terms of welfare.

Keywords: information acquisition; cheap talk; communication; signaling; credibility.

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1 Introduction

Decision makers often rely on the advice of experts. However, it is difficult to assess the credibility of experts as well as their knowledge. The literature on strategic information transmission pioneered by the seminal paper Crawford and Sobel (1982) has made two most common assumptions: that the bias of the expert is known and that the expert has access to information for free. These assumptions have been relaxed separately in different setups. Uncertainty about bias has been explored in the literature on the reputation of experts in repeated cheap talk initiated by Sobel (1985) and Morris (2001). Costly information acquisition before cheap talk has been first introduced by Austen-Smith (1994) and recently Sobel (2013) described this area as an open question. In order to study the interaction between bias and information acquisition, in this paper both of these assumptions are relaxed: the expert may or may not be aligned, in addition he may or may not be ex-ante informed. Finally, costly acquisition of information is observable by the decision maker, which means it is a signal. These ingredients make the problem rich in that there is simultaneously costly signaling and cheap talk.

To understand the relevance of this question, consider a government consulting a policy advisor about the affects of pollution on the occurrence of lung disease. Assume the advisor can be of three types: one type who gets kickbacks from a polluting industry and hence is biased. The other type of advisor has no conflict of interest, but already has done some work and hence is informed on this issue. Finally, there is the third type of advisor who needs to incur cost in order to get informed. One way to get informed is to hire a researcher to carry out some studies, which is costly but will make later advice more credible. Then, this means even a biased adviser could mimic this behavior only to increase his credibility while making a suggestion.[1]. Similar issues arise when a CEO asks a team manager about whether to launch a new product. The manager may have a bias towards launching the product due to his career concerns. In addition, the manager may already know the potential of this product due to his experience or else he may ask his team to carry out a market research. Motivated by these settings, the question this paper asks is how does bias interact with endogenous acquisition of information which may take place before cheap talk communication.

This paper considers a setup in which the sender's information acquisition influences the decision maker's belief about his type which determines the credibility of his communication. Hence, there are 2 different motives for information acquisition for different types of experts: the first one is the *intrinsic value of information* and the second one is its *signaling value*. The expert cares about his credibility only insofar as it affects the decision maker's (DM) action: reputation is instrumental and not intrinsic, as in Sobel (1985) and Morris (2001) who consider

[1]There are many instances when experts incur costs in order to produce fake information to enhance their credibility. For instance, Andrew Wakefield, a former gastroenterologist and medical researcher was found guilty of misconduct in his research paper in order to support a claim that MMR vaccine was linked to autism and bowel disease. It was later discovered that he had been paid by lawyers who were trying to prove that the vaccine was unsafe. Deer (2006)https://www.thetimes.co.uk/article/mmr-doctor-given-legal-aid-thousands-00ftl80msbs

repeated cheap talk. By endogenizing the information acquisition process, in this paper this effect is captured without dynamics, as there is an initial stage in which the expert chooses whether to invest in information acquisition or not before communication takes place. The investment stage also serves the role of a reputation building or signaling stage. Wasteful investment in information arises in order to pool with aligned types. However, investment is not always wasteful as it leads to learning for the uninformed and unbiased type of expert.

An effect similar to *bad reputation*, as defined by Ely and Valimaki (2003), arises: even an unbiased informed expert ends up wastefully investing in information when the cost is low enough. When the *signaling value* of information is higher than its *intrinsic value*, there exists no equilibrium in which information acquisition is efficient. Surprisingly, the decision maker's payoff may be higher when matched with an expert who is uninformed, as this type in overall communicates more efficiently than one that is ex-ante informed.

A simple model is considered with binary state of the world, $\omega \in \{0, 1\}$ and perfectly revealing signals. The decision maker's action space is $y \in [0, 1]$. With a probability β the expert is biased, and with probability α he is already informed about the state of the world. The decision maker and the aligned type of expert share the same incentives and want to match the state of the world, while the biased type wants highest action regardless of the state. As the biased expert has state independent preferences, information has no intrinsic value for him. This assumption enables to disentangle the intrinsic value of information from its signaling value. It turns out that the incentives of the biased expert and the unbiased expert endowed with signal 1 are identical: both want highest action to be taken. The expert can incur cost and acquire a perfectly revealing signal, where investment in information is observable by the DM but not its outcome. As there is a probability that the expert is already informed, that an expert does not invest in information does not reveal that he is uninformed, neither that he is biased.

Assuming information acquisition is a binary decision with a fixed cost, I solve for the different types of equilibria that arise for different costs of information acquisition. As there is costly signaling, equilibrium refinements, mainly the Intuitive Criterion introduced by Cho and Kreps (1987), is useful to rule out unreasonable equilibria and obtain uniqueness for some regions. When the cost is high enough, in the unique equilibrium no investment takes place. Below the no investment region of cost, there is the separating equilibrium which is the most informative one among pure strategy equilibria. In this equilibrium, the only type that invests is the uninformed unbiased expert, hence there is no *wasteful* investment. This equilibrium exists for some cost values under certain parameter restrictions and fails to exist whenever the "signaling value" of information is higher than its "intrinsic value". It is less likely to arise when the probability of the expert being biased is very high as this implies that the biased type and informed (1) type have higher incentives to invest.

The signaling value of investment is higher the lower the proportion of initially informed unbiased expert. Indeed, the proportion of informed unbiased types determine the payoff for the biased types when not investing: the higher the proportion of these types, communication is more credible hence the DM's action will be higher. The DM's payoff is higher when just sufficiently many types are informed that the equilibrium is separating. However, once in the separating region, the DM's as well as the overall welfare is decreasing in the proportion of informed unbiased experts, as these types cannot separate themselves from the biased type whenever the state is 1. This is the intuition for the result that the decision maker is better off having an expert who is initially uninformed: the uninformed type can separate himself from the biased type and communicate perfectly in the separating equilibrium while the informed type is always pooled with the biased type whenever the state is 1 in any equilibrium.

When the cost of information acquisition is low enough, all three types except the informed (0) type pool by investing in information. Only the unbiased uninformed type invests for the value of information. However, due to the "contamination" by the biased types, the communication of the unbiased type is less credible in this equilibrium.

In the region in between the pooling and separating equilibrium, there exists no pure strategy equilibria but only mixed strategy equilibria with the feature that the unbiased uninformed type always strictly prefers investing, while the biased and the unbiased informed (1) types are indifferent. As the cost of information acquisition goes down, proportionately more biased types than informed unbiased types should be investing in order to keep the indifference condition of these types themselves. Indeed, in the region of pooling and separating equilibria as well, there exists mixed strategy equilibria. However, I show that in these regions, mixed strategy equilibria are Pareto efficient as they are dominated in terms of expert payoff by pure strategy equilibria.

After characterizing the equilibria for every cost region, I go on to make welfare comparisons. Unsurprisingly, the separating equilibrium maximizes the DM's surplus. Surprisingly, the ex-ante total welfare over all expert types plus the DM is higher when cost increases so that the equilibrium moves from pooling to the separating equilibrium. This means less wasteful investment more than compensates for a higher information acquisition cost when the cost moves from the pooling region to the separating one.

Then, I consider covert information acquisition in which investment is no longer observable. The only investment that can arise in this setup is efficient (uninformed unbiased types) and the biased and informed types can pool without incurring any cost with the unbiased type. The overall welfare is higher in the overt case whenever the separating equilibrium arises due to more precise communication. Otherwise, when pooling in investment arises, the covert case leads to higher welfare due to less wasteful investment and same communication precision.

As an extension, I generalize the information acquisition choice from binary to a continuum of effort levels, $e \in [0, 1]$, where e is the probability of getting information and $c(e)$ the convex cost function. I show that the result that a separating equilibrium fails to exists whenever the signaling value of information is more than the intrinsic value remains robust. In addition, in the least cost separating equilibrium there may be over investment by the unbiased type in order to separate himself from the biased type. Indeed, for some parameter values, the decision maker is better off having uncertainty about the type of the expert compared to having an unbiased expert with probability one. This is the case when over investment in information by the unbiased type more than compensates for the possibility that the expert is biased and communication is uninformative. There are also pooling equilibria at positive investment levels which survive the Intuitive Criterion. However, whenever a LCS equilibrium does exist, I argue that it will be the unique outcome due to the use of a babbling threat by the decision maker.

2 Literature

This paper relates closely to the literature on reputational concerns in cheap talk introduced by Sobel (1985) and Morris (2001) who consider repeated cheap talk with uncertainty about sender type. The concern about reputation in this paper is similar to Morris (2001) in that it is "instrumental" and not "intrinsic": it matters insofar as the decision maker's action can be influenced. Morris captures this in a two period setup in which the DM updates the prior about the agent's type as a function of his message and the outcome realized, whereas in my setting the decision is taken only once but the information acquisition serves as a credibility building stage. In his setting, the advice of the unbiased type of advisor is distorted in order to avoid being perceived as a biased type and to enhance his credibility in period 2. In my setup, the messages are never distorted as decision making takes place only once, but distortion takes place in form of wasteful investment in information acquisition. In Morris, as reputational concerns increase, the good advisor more incentive to be politically correct that when these concerns are sufficiently high, there is only a babbling equilibrium. In my setup, when the signaling incentive is high enough, there is always wasteful investment and no possibility for an unbiased type to separate himself from a biased one in both states of the world.

Meng (2015) considers a similar setup to Morris (2001) with two periods and endogenizes the precision of expert's information, where investment is not a signal as it is not observable, but it allows for better decision making. Reputation building enhances the incentives to invest in information for both types in period 1, and aligned experts acquire more information. The expert's objective is similar to Morris that he cares about his future communication being credible.

There is a huge literature that considers bad reputation, where distortions caused by reputational concerns lead to loss of valuable information. These usually arise when there are dynamics, or when there is an intrinsic value of reputation for the experts. Outside of the communication literature, Ely and Valimaki (2003) consider a long run player who takes a payoff relevant action facing short run players and highlight the distortional consequences today of the incentives to avoid bad reputations in the future. In terms of intrinsic value of reputation, Ottaviani and Sorensen and Ottaviani and Sorensen (2006a) show that reputation doesn't always give the right incentives and may lead to herding. In their setup, experts have a utility from being perceived as good and bias their recommendation in order to appear more informed. Suurmond et al.

(2004) consider the effect of reputation in a delegation setup with information acquisition and an unbiased agent. In this setup, reputational concerns may be good as they incentivize the agent to acquire information when the agent doesn't know his ability. When the agent has private information about his ability, the expert may take inefficient actions in order to mimic an efficient type. The welfare effect of reputational concerns is ambiguous.

In the disclosure literature, Boujarde and Jullien (2011) consider an expert of known bias plus an intrinsic reputational concern about ability. The probability that the expert is informed depends on his ability hence disclosure enhances the expert's reputation. They study the interaction between the reputational concerns and incentives to disclose information and compare this to transparency about the expert type.

The distinctive feature of my setting compared to the literature is that there is neither dynamics involved which is present in the literature on reputation, nor an intrinsic value of reputation, yet a similar effect arises due to the endogenous information acquisition and signaling motive.

This paper also relates to the literature on costly information acquisition in cheap talk. The first example is Austen-Smith (1994) who considers the transmission of costly information where information acquisition leads to perfect information and is not observable. As the expert can prove being informed but can feign ignorance, the low types can now pool with uninformed types to achieve a higher outcome, which improves communication for higher types. In recent years there has been some work on strategic communication with endogenous information acquisition, such as Pei (2015) and Argenziano et al. (2016). A common finding is that the expert truthfully transmits all the information he acquires, in other words the expert doesn't acquire information that he will not transmit. Argenziano et al. (2016) show that the sender over-invests in information compared to what the decision maker would have incurred himself, due to the use of a babbling threat by the decision maker. In my setup, in the extension considered, there is an over-investment result when there is uncertainty about the expert's bias compared to having an unbiased expert for sure. The difference is that it is the expert who over invests compared to what he would have done if his type were known. Esö and Szalay (2010) consider a game in which the expert has no bias and endogenous information acquisition, and show that restricting the message set can induce the sender to acquire information more often. Deimen and Szalay (2016) also consider endogenous information when a biased expert can choose on which issues to gather information and show that communication dominates delegation. Frug (2017) considers dynamic information acquisition in cheap talk. Dur and Swank (2005) also study endogenous information acquisition and show that an unbiased advisor puts higher effort into acquiring information as the value of information is higher for them. Hence, when a decision maker is biased, it may be better for him to hire an advisor who is less biased than himself. There is no uncertainty about the bias hence no signaling motive for the expert in these papers.

In a disclosure setup, Che and Kartik (2009) show that the DM benefits having an advisor with some difference of opinion as this gives more incentives to invest in information, due to the incentive to manipulate the decision and also in order to avoid prejudice. The tradeoff is between the advisor acquiring more information versus hiding more strategically. Kartik et al. (2017) study disclosure with two experts having opposite biases, and show that information acquisition decisions of the experts are strategic substitutes.

There is also some literature in which the bias of the expert is unknown. Morgan and Stocken (2003) consider strategic communication as in Crawford and Sobel (1982) introducing uncertainty about the expert's type. They show that truthful communication cannot arise even for the unbiased type whenever the state of the world is high enough. In a disclosure setup, Wolinsky (2003) considers a setting in which the expert has uncertain bias and shows that the expert conceals more information when his bias is unknown. There are no information acquisition or signaling incentives in these settings.

In Austen-Smith (2000) cheap talk is not the only way to communicate but senders may incur some loss in utility in form of *burning money*. The wasteful investment in my setup is reminiscent of burning money, in that for some types there is no informational value but only a signaling one. However, contrary to burning money, some expert types do value information per se.

3 Model

There is a decision maker (DM) who wants to take an action, $y \in [0, 1]$. The state of the world is binary, $\omega \in \{0, 1\}$ and there is a commonly known prior probability $Pr(\omega = 1) = p_0$. The DM does not know the state of the world but has access to an expert advisor. The expert has a two dimensional private type. First, he is either unbiased with probability $(1 - \beta)$ and shares the same objective function as the DM, which is $u(\omega, y) = -(\omega - y)^2$ or he is biased and always wants the highest possible action with utility function, with utility function $u^b(\omega, y) = -(1-y)^2$[2]. Second, with probability α the expert is perfectly informed about the state of the world ex-ante, otherwise he has the same belief as the DM.[3] The decision maker does not observe whether the expert is biased or not and neither whether the expert is informed or not, but knows β and α. In addition, the agent can invest in information by incurring a cost c leading to a perfectly revealing signal. The investment in information acquisition is observable while its outcome is not. Indeed, any type of expert could invest in information acquisition, including a biased or an informed one, which is crucial in the setup. Finally, communication happens through cheap talk after the investment decision. Below are the stages of the game:

- the expert decides whether to acquire a signal by incurring c and become perfectly informed, binary decision $x \in \{0, 1\}$.

[2]This type of assumption about the expert type is also present in Morris (2001) and Morgan and Stocken (2003) who consider uncertainty about bias

[3]Indeed, it could also be thought that getting informed is costless for some experts.

- the expert sends a message $m \in M$ to the DM.
- the DM takes an action, $y \in [0,1]$.

The set of available messages is $M = \{\varnothing, 0, 1\}$, where $\varnothing$ means the sender doesn't have information. The decision maker interprets the expert's message as a function of her belief about the expert's type, which is updated depending on the equilibrium information acquisition strategies of different type of experts.

Let us now summarize the types of experts at the beginning of the game. As the biased type of expert always wants the highest action irrespective of the state, we only have 1 type of biased expert and whether he is informed or not has no relevance.[4] It could be assumed that the biased type of agent is never informed. We can then summarize the types of experts at the beginning of the game by separating them into four:

1. biased agent
2. unbiased agent who is uninformed
3. unbiased agent who is informed with signal (1)
4. unbiased agent who is informed with signal (0)

Equilibrium Concept: I look for Perfect Bayesian Equilibrium. After observing x and m, the DM's updated belief is $\mu(\omega|x,m)$, as a function of the equilibrium strategies of each expert type. I will denote the DM's updated belief by $\mu(x,m)$ hereafter, and the DM's problem is:

$$y^* = \arg\max_y -\mu(x,m)(1-y)^2 - (1-\mu(x,m))y^2 \tag{1}$$

this simplifies to $-y^2 - \mu(x,m) + 2\mu(x,m)y$ which is maximized for $y = \mu(x,m)$. Hence, $y^* = \mu(x,m)$. Indeed, the updating of the DM's belief takes place in two stages. The first updating happens upon observing the decision to invest, while the second updating happens upon receiving the message m. The PBE requires that the DM's belief $\mu(x,m)$ is consistent with the experts' equilibrium strategies while the sequential rationality of expert requires that, the expert's strategy, $\{x,m\}$ maximize the expert's payoff given $\mu(x,m)$.

Value of information: For the biased and informed (1) agents, information has no intrinsic value as these types strictly prefer sending $m = 1$. The only situation in which they may find it valuable to invest in information acquisition is when $\mu(1,1) - \mu(0,1)$ is large enough, as in that case the credibility of communicating $m = 1$ is higher after investing.

Without any meaningful communication, the DM would choose $y^* = p_0$ which provides her a payoff $-(p_0 - {p_0}^2)$, as well as for the unbiased agent. Then, the value of perfect information for the DM and the unbiased expert are identical and given by $(p_0 - {p_0}^2)$. This is the gain in payoff

[4] This would be different if the biased type's payoff was not state independent, such as in Morgan and Stocken (2003).

for the unbiased type from getting informed, conditional on his communication being perfect. The value of information for the unbiased type and the DM is maximized when $p_0 = 0.5$, in other words when uncertainty is highest. We will define the value of signaling shortly as we discuss equilibria.

4 Equilibrium Analysis

As this is a game of signaling followed by cheap talk, there is multiplicity of equilibria for some cost values in which case the Intuitive Criterion (due to Cho and Kreps (1987)) is used.

Lemma 1. *In any equilibrium in which the message space is $\{0, 1, \varnothing\}$, the unbiased informed (0) type sends $m = 0$ without additional investment, upon which the DM chooses $y^* = 0$.*

Proof. The informed (0) type's payoff is maximized when $y = 0$ and there is no other type that would benefit from sending this message, when the 3 separate messages are allowed for. For the biased and informed (1) types, this message would lead to the lowest possible payoff of -1. For the unbiased uninformed type who has not invested, sending a babbling message upon which DM chooses $y = p_0$ leads to a better payoff, $-(p_0 - {p_0}^2)^2$ than sending this message which leads to $-p_0$. Given that, the DM infers that this message comes from an unbiased informed type, $\mu(0, 0) = 0$ and she will choose $y = 0$. Finally, this type can only be worse off by sending the same message as any other type. □

By this lemma, we can restrict attention to equilibria in which at least communication happens perfectly by these types. Then, from now on we will focus on the equilibrium behaviors of the 3 other types.

Observation 1. *The biased type and the unbiased informed (1) type share the same incentives as their payoff functions are given by $-(1 - y)^2$. These types have the same information acquisition strategies in strict Nash Equilibria and they always send the message $m = 1$ regardless of their investment strategy.*

As the payoff functions of these two types are identical, whenever they are playing a strictly dominant strategy, they should follow the same investment strategy. In case they are playing a mixed strategy, then their investment strategies can differ. This will be important, as even though both types always prefer sending $m = 1$, they have different meaning for the decision maker: the unbiased informed (1) type is communicating truthfully while the biased type's message is "babbling" as he always announces state 1. Hence, if she could, the DM would be willing to distinguish between these two types. This can only be possible in mixed strategy equilibria in which investment decisions of these types differ.

Observation 2. *The unbiased type of expert always prefers sending a truthful message. In particular, whenever uninformed, this type sends $m = \emptyset$.*

The aligned type never has incentives to lie about the state. Whenever uninformed, he strictly prefers sending $m = \varnothing$ which is always a credible message leading to $y^* = p_0$ and whenever informed, he prefers sending $m = \omega$.

There are 3 different equilibria in pure strategies and a continuum of semi-pooling equilibria in mixed strategies. Which equilibrium arises depends on the cost of information acquisition. After having specified the strategy of the unbiased informed (0) type, I define "pooling" and "separating" in this setup in terms of the investment decision of the two groups of expert types defined as:

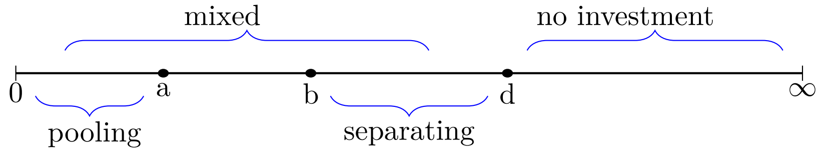
1. biased type and unbiased informed (1) type,
2. unbiased uninformed type.

We focus on parameter values such that separation between the two groups does exist for some cost values. Then, following is the list of all possible equilibria that arise:

1. **Separating equilibrium:** Only the unbiased uninformed type invests upon which the DM takes communication at face value. The group 1 types do not invest and send $m = 1$ leading to $y = \hat{p} > p_0$.
2. **Pooling equilibrium:** Both groups of types invest. The unbiased type sends $m = \omega$ while the biased type sends $m = 1$. Upon $m = 1$, the DM chooses $y = \tilde{p} > \hat{p}$.
3. **No investment equilibrium:** No type invests. Upon $m = 1$ the DM chooses $y = \hat{p}$.
4. **Semi-pooling (mixed strategy) equilibria:** For each cost value, the strategies of biased and unbiased types lead to an updated belief of the DM which satisfy the indifference condition of these types. As cost decreases, the portion of biased types who invest increases compared to unbiased types.

The equilibrium maximizing the decision maker's and overall surplus among the pure strategy equilibria is the separating one which is also the most informative equilibrium in pure strategies. This equilibrium only exists for some cost values under certain parameter conditions: when the **signaling (reputational) value of information** is lower than the **intrinsic value of information**. The signaling value of information is the gain in payoff for the group 1 types from being perceived as unbiased, in other words if their message is taken at face value, compared to the case when the decision maker infers that the message is coming from group 1. The intrinsic value of information is the gain in payoff for the uninformed unbiased (group 2) type from getting informed and being perceived as an unbiased type, i.e. when his message is taken at face value, compared to not having information and sending $m = \emptyset$. When this condition is violated, there exists no separating equilibrium, in other words there is no equilibrium in which investment in information is efficient. In that case, there is always some biased types that invest hence the communication of unbiased uninformed type is never perfect.

Figure 1: Equilibria as a function of cost



$$a = (1-\hat{p})^2 - (1-\tilde{p})^2$$
$$b = (1-\hat{p})^2$$
$$d = p_0 - {p_0}^2$$

The value of information for the unbiased uninformed type is higher the closer p_0 is to 0.5 while the signaling value of information decreases in p_0. For any $p_0 \geq 0.5$, there exists a region of cost values for which the separating equilibrium arises. When it does exist, the separating equilibrium arises for *intermediate* values of the investment cost, as for low enough cost values there is pooling and for high enough cost values even the unbiased uninformed type doesn't find it profitable to invest. For separation to exist, we also need α to be high enough: sufficiently many of the unbiased types should be initially informed so that the biased types find it attractive to not invest and pool with these types. The figure shows the region for different equilibria under our assumption.

Separating Equilibrium

The separating equilibrium is the one in which the only type that acquires information is the uninformed unbiased type. Hence, information acquisition is efficient. Upon communication of $m = 1$ without investment, the DM's belief $\mu(0, 1)$ is given by:

$$\hat{p} = \frac{\alpha(1-\beta)p_0 + \beta p_0}{\alpha(1-\beta)p_0 + \beta}$$

When the DM receives $m = 1$ without investment and knows that it either comes from the biased agent (with probability β) or from the unbiased informed (1) agent (with probability $\alpha(1-\beta)$), she chooses $y = \hat{p}$. The payoffs to both the biased and unbiased informed expert types are $-(1-\hat{p})^2$. If instead they deviate to invest, they induce the DM to choose $y = 1$ by sending $m = 1$ and obtain a payoff of 0. Hence, the condition for this deviation to not be profitable for these types is:

$$c \geq (1-\hat{p})^2 = \left[\frac{\beta(1-p_0)}{\alpha(1-\beta)p_0 + \beta}\right]^2 \tag{2}$$

For the unbiased uninformed expert who invests, payoff in case of deviating to not invest is $-(p_0 - {p_0}^2) < 0$ as in that case she would send $m = \varnothing$ and the DM would choose $y = p_0$. On the other hand, upon investing he obtains a payoff of 0 as the DM will treat his message as truthful in either state. Then, this expert prefers to invest if and only if:

$$p_0 - {p_0}^2 \geq c$$

This means, together with equation (2), that the cost values for which the separating equilibrium exists is:

$$(1-\hat{p})^2 \leq c \leq p_0 - {p_0}^2 \tag{3}$$

Then, for the separating equilibrium to exist at least for some cost values, we should have:

$$(1-\hat{p})^2 \leq p_0 - {p_0}^2$$

If the above condition is violated, then there exists no cost values for which the separating equilibrium arises. To see this, realize that whenever the biased type is not investing, the best case for this type is that the unbiased informed 1 type is also not investing, so that $\hat{p}$ takes its maximum value. If even in this case, the payoff for these types to be perceived as unbiased ($(1-\hat{p})^2$) is higher than the payoff to get informed for the unbiased uninformed types ($p_0 - {p_0}^2$), then there is no cost value for which the equilibrium in which no biased type invests can be sustained.

We find that $p_0 - {p_0}^2 \geq (1-\hat{p})^2$ if and only if:

$$p_0[\alpha(1-\beta)p_0 + \beta]^2 - \beta^2(1-p_0) > 0$$

which can be satisfied for p_0 and α large enough and specifically, it is always satisfied when $p_0 \geq 0.5$. As we have $\frac{\partial(1-\hat{p})}{\partial\beta} > 0$ and $\frac{\partial(1-\hat{p})}{\partial\alpha} < 0$, this equilibrium is more likely to exist for low β and high α. When β is high, the incentive to pool is high. When α and $\hat{p}$ are low enough, the outside option of the biased types of not investing to pool only with the unbiased informed types is lower, hence they will be more tempted to invest. For the uninformed type to be willing to invest, the condition $c \leq p_0 - {p_0}^2$ is easier to satisfy the closer p_0 is to 0.5, in other words when the intrinsic value of information is sufficiently high. Hence, p_0 should be high enough to increase the non investment value for the biased types, but not too much, for investment to be profitable for the unbiased uninformed types.

No Investment Equilibrium:

It is intuitive that when c is high enough, no type wants to invest. This is the case when:

$$c \geq p_0 - {p_0}^2$$

The right hand side is the upper boundary of the separating equilibrium, in which the only type that invests is the unbiased uninformed type. We know that in the region $(1-\hat{p})^2 \leq c \leq p_0 - {p_0}^2$, the biased type doesn't find it profitable to invest, even when his message is credible. Then, in the region $c \geq p_0 - {p_0}^2$, the out of equilibrium belief upon investment should put probability 0 to the expert being a biased type, as this type can never gain from deviating to invest even if he would be believed to be an unbiased type. Then, the message upon a deviation to invest should be taken at face value. The gain in utility from doing so for the uninformed unbiased type defines the boundary of the no investment equilibrium. Then, above this cost level, the unique equilibrium has no investment. Next proposition summarizes the results until now.

Proposition 1.

- *In the region $(1-\hat{p})^2 \leq c \leq p_0 - {p_0}^2$, the separating equilibrium, in which the only investment is made by the uninformed unbiased type, is the unique pure strategy equilibrium that survives the Intuitive Criterion.*
- *In the region $c > p_0 - {p_0}^2$, the unique equilibrium is the one in which there is no investment.*

Now, consider the equilibria in the region $c < (1-\hat{p})^2$. We know that there is no separating equilibrium as the biased types will have an incentive to deviate and invest if the DM expects the investment to come from an unbiased type. However, it cannot be the case that just below this threshold, the biased and informed (1) types invest with probability 1, as then $\mu(1,1) < 1$, and the biased and informed (1) types would not find it profitable to invest anymore at this cost. Then, in this region there can only be mixed strategy equilibria with semi-pooling in investment. However, although mixed strategy equilibria are not restricted to this region, they are only unique in this region. First, we will study the pooling equilibrium which arises for lower cost values, before studying the mixed strategy equilibria in the cost region in between.

Pooling Equilibrium:

We call an equilibrium pooling equilibrium if all types except for the informed (0) type invest. The out of equilibrium belief of the DM upon no investment and $m = 1$ is $\mu(0,1) = \hat{p}$. The out of equilibrium belief puts probability 0 on the unbiased uninformed expert as this type always prefers to send $m = \varnothing$ if he were to deviate to no investment. On the other hand, the group 1 types always prefer sending $m = 1$ over any other message. Then, by the Intuitive Criterion, $\mu(0,1) = \hat{p}$ as this deviation can only come from a biased or informed (1) type.

The DM's updated belief $\mu(1,1)$ is:

$$\tilde{p} = \frac{p_0(1-\beta) + \beta p_0}{p_0(1-\beta) + \beta} = \frac{p_0}{p_0(1-\beta) + \beta}$$

First, consider the investment choice of the group 1 types. If the biased or unbiased agent were to deviate to not invest and send $m = 1$, then the DM would choose $y = \hat{p}$. Then, the following should hold for pooling equilibrium to arise:

$$-(1-\tilde{p})^2 - c \leq -(1-\hat{p})^2$$

Second, for the uninformed unbiased type, the payoff from not investing is $-(p_0 - {p_0}^2)$ as before, in which case they would send $m = \emptyset$. Then, consider the payoff of this type from investing. If the signal turns out to be 0, then the DM takes the message at face value and chooses $y = 0$ whereas if the signal is 1, this type will be pooled with the group 1 types and the decision maker will choose $\tilde{p}$. Then this type prefers investing to not if and only if:

$$-p_0(1-\tilde{p})^2 - c \geq -(p_0 - {p_0}^2)$$

These conditions together result in:

$$c \leq min\{(1-\hat{p})^2 - (1-\tilde{p})^2, p_0 - {p_0}^2 - p_0(1-\tilde{p})^2\}$$

When $(1-\hat{p})^2 < (p_0 - {p_0}^2)$, which is the case we consider, we have $min\{(1-\hat{p})^2 - (1-\tilde{p})^2, p_0 - {p_0}^2 - p_0(1-\tilde{p})^2\} = (1-\hat{p})^2 - (1-\tilde{p})^2$. To see this, realize that the value of getting informed is higher for unbiased uninformed types than for the group 1 types: $(1-\hat{p})^2 - (1-\tilde{p})^2 < p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$. This is because unbiased uninformed types get their bliss point in case the state of the world is 0, while communication is distorted when the message is 1. However, from the point of view of the group 1 types, communication is always distorted as their bliss point is 1 and $\mu(1,1) = \tilde{p} < 1$. Then, the condition for the pooling equilibrium is given by the condition for the biased and informed (1) types to be willing to invest which is $c \leq (1-\hat{p})^2 - (1-\tilde{p})^2$.

As we have $\frac{\partial(1-\hat{p})}{\partial\beta} - \frac{\partial(1-\tilde{p})}{\partial\beta} > 0$, when β increases the incentive of the biased type to invest increases and pooling equilibrium becomes more likely.

Now consider the expression $c \leq p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$ which is the condition for the unbiased uninformed types to invest in the pooling equilibrium. As $p_0 - {p_0}^2 > p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$, where the left hand side is the cutoff for investment in the separating equilibrium, the condition for unbiased uninformed types to invest is easier to satisfy in the separating equilibrium. The difference between these two is due to the biased types' "crowding out" the unbiased uninformed types: investment of the biased types make information acquisition by the unbiased types less profitable, hence cost has to be lower in order to satisfy their participation.

Mixed strategy equilibria

There exists mixed strategy equilibria for $c \in (0, (1-p_0)^2]$ if $p_0 \geq 0.5$ and otherwise for $c \in (0, (p_0 - {p_0}^2)]$. Specifically, for $c \in [(1-\hat{p})^2 - (1-\tilde{p})^2, (1-\hat{p})^2]$, mixed strategy equilibria are the unique equilibria. Outside of this region, mixed strategy equilibria exist along with pure strategy (separating or pooling in investment) equilibria. Indeed, these equilibria survive the Intuitive Criterion. However, as will be shown in lemma 2, in these regions the mixed strategy equilibria are Pareto dominated for the expert compared by pure strategy equilibria. Hence, mixed strategy equilibria are not Pareto efficient when they coexist with pure strategy equilibria. I use this as a selection criterion in the regions where mixed and pure strategy equilibria coexist.

In mixed strategy equilibria, the biased and informed (1) types no longer play the same strategy and for any c, their strategies should be such that the indifference condition for these types is satisfied. In this type of equilibrium, the biased sender mixes between investing and not with probability of investment being σ. The unbiased informed (1) type is also indifferent and invests with probability γ. Both these types always send $m = 1$. The mixing strategies are such that $\mu(1, 1)$ equals the value which makes these group 1 types indifferent between investing and not. More specifically, as c increases, $\frac{1-\sigma}{1-\gamma}$ should be increasing in order for $\mu(0, 1)$ to decrease and to keep the indifference condition satisfied, hence a larger portion of unbiased types should be investing in order for $\mu(1, 1)$ to increase. We have that when $\gamma = \sigma = 1$, the indifference condition is satisfied at $c = (1 - \hat{p})^2 - (1 - \tilde{p})^2$ which is the cutoff below which the pooling equilibrium arises. For the indifference condition to be satisfied at higher c, we need $\mu(0, 1) < \hat{p}$. Then, we know that when these types are indifferent, the unbiased uninformed type strictly prefers to invest. We also know that $\sigma < 1$, as when $\sigma = 1$, $\mu(1, 1) \leq \hat{p}$ for any $\gamma \leq 1$.

Given σ and γ, $\mu(1, 1)$ is:

$$p' = \frac{(1-\beta)(\alpha + \gamma(1-\alpha))p_0 + \beta\sigma p_0}{(1-\beta)(\alpha + \gamma(1-\alpha))p_0 + \beta\sigma} > \hat{p}$$

Then, $\mu(0, 1)$ is:

$$p^* = \frac{(1-\beta)\alpha(1-\gamma)p_0 + \beta(1-\sigma)p_0}{(1-\beta)\alpha(1-\gamma)p_0 + \beta(1-\sigma)}$$

For the indifference condition of the biased types (and informed (1) unbiased types) to be satisfied, it should be that $c = (1 - p^*)^2 - (1 - p')^2$, which leads to:

$$c = \left[\frac{\beta(1-\sigma)(1-p_0)}{(1-\beta)\alpha(1-\gamma)p_0 + \beta(1-\sigma)}\right]^2 - \left[\frac{\beta\sigma(1-p_0)}{(1-\beta)(\alpha + \gamma(1-\alpha))p_0 + \beta\sigma}\right]^2 \tag{4}$$

The left hand side of equation 4 is decreasing and the right hand side is increasing in σ for a given γ. Hence, for every c and γ, there is a σ for which this equality holds. But for a given c, there are multiple pairs of $\{\gamma, \sigma\}$ that satisfy the condition.

Finally, we will check that the unbiased uninformed type strictly prefers to continue investing. To see this, consider the condition for this type to invest:

$$-p_0(1 - p')^2 - c \geq -(p_0 - {p_0}^2)$$

leading to:

$$c \leq (p_0 - {p_0}^2) - p_0(1 - p')^2$$

when the indifference condition of the biased type is satisfied, $c(e) = (1 - p^*)^2 - (1 - p')^2$, the incentive compatibility of the unbiased agent also is satisfied, as $p^* > \hat{p} \to (1 - p^*) < 1 - \hat{p} \to (1 - p^*)^2 < (1 - \hat{p})^2 \to (1 - p^*)^2 < (1 - \hat{p})^2 < p_0 - {p_0}^2$ plus $(1 - p')^2 > p_0(1 - p')^2$:

$$c = (1 - p^*)^2 - (1 - p')^2 < (p_0 - {p_0}^2) - p_0(1 - p')$$

The mixed strategy equilibria outside the region $[(1-\hat{p})^2-(1-\tilde{p})^2,(1-\hat{p})^2]$ exist simultaneously with other pure strategy equilibria. While $c \geq (1-\hat{p})^2$, there exists mixed strategy equilibria in which $\mu(0,1) < \hat{p}$, in other words the outside option of the biased and informed (1) types is worse than in the separating equilibrium and for this we need $\gamma > \sigma$: more unbiased informed types investing than biased types. Finally, in the region $c < (1-\hat{p})^2 - (1-\tilde{p})^2$, there exists mixed strategy equilibria in which $\mu(1,1) < \tilde{p}$, so that the investment option is less attractive to satisfy the indifference condition. This requires $\gamma\alpha + (1-\alpha) > \sigma$ which is a stronger condition than $\gamma > \sigma$.

Indeed, the fraction of the investment and non-investment types should be such that as cost goes up, the outside option of not investing becomes *better*, while the option of investment becomes *worse* for the group 1 types, who would like the highest action to be taken by the decision maker. In overall, the mixed strategy equilibria have the property that as c increases, $\mu(1,1)$ is increasing and/or $\mu(0,1)$ should be decreasing: investing should be more attractive compared to the outside option of not investing, to keep the indifference condition of group 1 types. Then, the strategies should be determined so that the beliefs will satisfy the indifference condition at each cost value. The interesting feature of this type of equilibria is that although the types inside group 1 share the same payoff function, their strategies lead to differences in the decision maker's updated beliefs hence determine the incentives of these types themselves. This is a feature distinct to my model compared to other signaling setups.

Perfect communication equilibrium: I would like to highlight one interesting equilibrium among the mixed strategy equilibria in which perfect communication arises for any type that invests. This is the equilibrium in which the informed (1) unbiased type invests with probability 1 as well as the uninformed type while the biased type chooses not to invest with probability one. We call this a mixed strategy equilibrium as the group 1 types play pure but differing strategies. This equilibrium exists in case $\rho_0 > \frac{1}{2}$ and exactly at the cost value $c = (1-p_0)^2$. In this equilibrium, we have $\mu(1,1) = 1$ while $\mu(0,1) = p_0$. Hence, a group 1 type is indifferent between investment or not if and only if:

$$(1-p_0)^2 = c$$

Where the left hand side provides the outside option for the group 1 type of not investing, where $\mu(0,1) = p_0$ and upon investment $\mu(1,1) = 1$. Then, exactly at this cost value and beliefs, this type of equilibrium exists. Finally, in order to ensure the unbiased uninformed types are investing, we need the following condition:

$$(1-p_0)^2 \leq (p_0 - {p_0}^2)$$

which is satisfied if $p_0 \geq \frac{1}{2}$. Indeed, this is also the highest cost level for the mixed strategy equilibria. For $c > (1-p_0)^2$ there exists no mixed strategy equilibria.

5 Welfare analysis

Now we will make welfare comparison among theequilibria defined. Let us summarize some results first:

- The DM is better off if facing an uninformed expert whenever there is investment. If the expert is unbiased and informed, he cannot distinguish himself from the biased type when the state is 1 (except for some mixed strategy equilibria). On the other hand, it is possible for the uninformed unbiased type to perfectly communicate if the separating equilibrium arises.
- DM's payoff (and welfare) is non-monotone in α: it should be high enough for separation as the amount of informed unbiased types serves as an outside option for the biased types and decreases incentives to invest. However, above the level that leads to separation, the DM's payoff and total welfare decrease in α as there are more and more unbiased informed types pooling with the biased type leading to loss of precision in communication.
- Total welfare is non-monotone in the cost of information acquisition: it is increasing as cost increases while moving from pooling to separating equilibria, however once in the separating equilibrium, total welfare decreases in cost.

5.1 The DM's payoff

It is easy to see that the DM's payoff is minimized in the equilibrium in which no type invests, which is the least informative equilibrium. Then, the comparison will be between the separating and pooling equilibria.

Separating equilibrium:

$$[(1-\beta)\alpha p_0 + \beta][-(\hat{p} - \hat{p}^2)] = -\hat{p}\beta(1-p_0) \tag{5}$$

This is decreasing in α and β. As α increases, more of the informed unbiased types will be pooled with biased types, and as β increases, the DM's welfare decreases as there are more biased types.

Pooling equilibrium:

$$[(1-\beta)p_0 + \beta][-(\tilde{p} - \tilde{p}^2)] = -p_0(1-\tilde{p}) \tag{6}$$

This is decreasing in β. If $p_0 > 0.5$, this is increasing in p_0. When $p_0 < 0.5$, and β high enough it may be decreasing in p_0. It is independent of α, as in the end all types do get informed. The condition for the separating equilibrium payoff to dominate the pooling equilibrium payoff for the DM is:

$$\alpha(1 - p_0) + p_0 \leq 1$$

which is always satisfied. Hence, the DM's payoff is unambiguously higher in the separating equilibrium, in which information is more precise.

The DM benefits from having probability of some initially informed experts who pool with the biased type, as otherwise the separating equilibrium cannot arise. If α is very low, there are more incentives to "pool" and the separating region shrinks. However, inside the separating region, the DM's payoff is decreasing in α: she is better off when more types are uninformed as the informed types endowed with signal 1 are not able to separate themselves from the biased types. Hence, the DM's payoff is non-monotone in α: it has to be high enough for biased types not to invest, but once in the separating region the DM's payoff decreases in α. Also, the DM's payoff is unambiguously decreasing in β.

No investment equilibrium:

The DM's utility in the equilibrium in which no one invests:

$$\frac{-(1 - p_0)p_0(\beta + (1 - \alpha)\alpha(1 - \beta)^2 p_0)}{\beta + \alpha(1 - \beta)p_0} \tag{7}$$

which is found by simplifying $-[\beta + (1 - \beta)\alpha p_0](\hat{p} - \hat{p}^2) - (1 - \beta)(1 - \alpha)(p_0 - {p_0}^2)$. When we compare the no investment equilibrium with the pooling in investment equilibrium, we find that pooling in investment always leads to higher payoff for the DM than no investment. Hence, no investment equilibrium provides the minimum payoff to the DM.

Mixed strategy equilibria

The principal's payoff given σ and γ is:

$$-(\beta(1-\sigma)+(1-\beta)\alpha p_0(1-\gamma))(\mu(0,1)-\mu(0,1)^2)-(p_0(1-\beta)(\alpha\gamma+1-\alpha)+\beta\sigma)(\mu(1,1)-\mu(1,1)^2)$$

which simplifies to:

$$-\beta(1 - p_0)[(1 - \sigma)\mu(0,1) - \gamma\mu(1,1)]$$

5.2 The expert's payoff

Separating equilibrium:

The payoff of the biased expert is $-(1-\hat{p})^2$. This is increasing in α and p_0 and decreasing in β.

The ex-ante payoff of the unbiased expert is $-\alpha p_0(1-\hat{p})^2 - \alpha(1-p_0)0 - (1-\alpha)c = -\alpha p_0(1-\hat{p})^2 - (1-\alpha)c$. This is increasing in α, decreasing in p_0 and decreasing in β.

Pooling equilibrium:

The payoff of the biased expert is $-(1-\tilde{p})^2 - c$. This is increasing in p_0 and decreasing in β.

The ex-ante payoff of the unbiased expert is $-p_0(1-\tilde{p})^2 - (1-\alpha(1-p_0))c$. This is decreasing in p_0 as this type can perfectly reveal information when $\omega = 0$, and it is also decreasing in β due to the contamination of the biased types. It is increasing in α as less cost will have to be incurred.

It is trivial that the biased type of agent is unambiguously better off in pooling equilibrium than in separating equilibrium, as otherwise, he would not invest and get the same payoff as in the separating equilibrium.

On the other hand, the unbiased agent's payoff increases when moving from the separating to the pooling equilibrium, when we compare the payoff at the minimum cost at which there is separation and maximum cost at which there is pooling.

Mixed strategy equilibria

The expert's payoff in mixed strategy equilibria is:

$$-(\beta + (1-\beta)(p_0 + 1 - \alpha)(1-\mu(0,1))^2 + (1-\beta)(1-\alpha)(1-\mu(1,1))^2$$

which simplifies to:

$$-(\beta + (1-\beta)p_0)(1-\mu(0,1))^2 - (1-\beta)(1-\alpha)c \tag{8}$$

Hence, for a given c, among the mixed strategy equilibria, the one that maximizes the expert's payoff is the maximum $\mu(0,1)$.

Lemma 2. *In the region $c \geq (1-\hat{p})^2$ and $c \leq (1-\hat{p})^2 - (1-\tilde{p})^2$ which are the regions where mixed and pure strategy equilibria co-exist, the pure strategy equilibria payoff dominate mixed strategy equilibria for all the expert types.*

Proof. First, consider the region $c \geq (1-\hat{p})^2$, where mixed strategy and the separating equilibria coexist, we can see that the separating equilibrium provides higher payoff for any expert type for any c. To see this: we know that in the separating equilibrium, the uninformed unbiased type's communication is taken at face value and for the group 1 types' payoff is $-(1-\hat{p})^2$, independent of cost. In the mixed strategy equilibria, for any $c > (1-\hat{p})^2$, $\mu(0,1) < \hat{p}$ hence the payoff of group 1 types are $-(1-\mu(0,1))^2 < -(1-\hat{p})^2$ given their indifference condition, and for the uninformed type's payoff will also be lower as $\mu(1,1) < 1$. Then, all expert types are strictly worse off in the mixed strategy equilibria compared to pure strategy in this region.

The same is true for the region $c \leq (1-\hat{p})^2 - (1-\tilde{p})^2$ where pooling in investment and mixed strategy equilibria coexist. In the pooling equilibrium, the payoff of group 1 types is higher than $-(1-\hat{p})^2$ which is their payoff in case of not investing and sending $m=1$. In the mixed strategy equilibria, we have that for any c, $\mu(1,1) < \tilde{p}$. Then, as by the indifference condition, the group 1 types have payoff $-(1-\mu(1,1))^2 - c$ in the mixed strategy equilibria, their payoff is again lower than in the pooling in investment equilibria which gives them payoff $-(1-\tilde{p})^2 - c$. For the uninformed type, the payoff is also lower in mixed strategy as $\mu(1,1) < \tilde{(p)}$. Hence, all expert types are again strictly worse off in the mixed strategy equilibria in this region than in the pooling in investment equilibria. Finally, we can conclude that mixed strategy equilibria are payoff dominated for the expert types by the pure strategy equilibria. $\square$

Hence, sender optimality rules out mixed strategy equilibria in the regions where a pure strategy equilibrium exists. In addition, the receiver could not enforce this kind of equilibria by a babbling equilibrium threat as the behavior of sender cannot be detected in the pure strategy equilibria compared to the mixed one: the mixed strategy equilibria has both investment and no investment strategy and both messages are sent with or without investment.

5.3 Total Welfare Comparison

Given the argument for Pareto dominance of mixed strategy equilibria by pure strategy equilibria, we make the total welfare comparison ruling out mixed equilibria whenever pure strategy equilibrium exists.

Proposition 2. *The total welfare increases when moving from the pooling in investment equilibrium to the separating equilibrium at the minimum cost, where it takes the highest value among pure strategy equilibria. Hence, welfare is maximized at the minimum cost level at which there is no wasteful investment and it takes the minimum value in the no investment equilibrium for high enough cost values.*

6 Covert information acquisition

Now we consider the case when information acquisition process is covert, in other words when the decision maker does not observe the investment made by the expert. In this case, as there is no signaling value of investment, the biased and informed 1 types will never wastefully invest, as they can now pool with the unbiased uninformed expert without having to incur any information acquisition cost. The only type that may invest is the unbiased uninformed type. There are 2 types of pure strategy equilibria as a function of the cost, as summarized below:

1. There is investment only by the uninformed unbiased type. Upon $m = 0$, the DM chooses $y = 0$ and upon $m = 1$, the DM takes action $y = \tilde{p}$. This equilibrium looks like the pooling equilibrium except that the biased and informed (1) types do not actually invest. The payoffs of the biased and informed (1) types are higher in this equilibrium compared to the pooling equilibrium discussed, as they achieve the same outcome without having to pay the investment cost.

   The condition for the unbiased uninformed type to invest is:

   $$c \leq p_0 - {p_0}^2 - p_0(1 - \tilde{p})^2$$

   This means the unbiased type acquires information for smaller range of cost values in the covert information acquisition than in the overt case. This cutoff is above the pooling cutoff cost but below the separating equilibrium cutoff cost in the overt case studied earlier.

2. No investment takes place. Upon $m = 1$, the DM chooses $\hat{p}$ inferring that this message is sent either by the biased or the unbiased informed (1) type of expert. The uninformed expert sends $m = \emptyset$ and the DM chooses $y = p_0$. This equilibrium is equivalent to the no investment equilibrium in the overt information acquisition case. This equilibrium arises for the following cost values:

   $$c > p_0 - {p_0}^2 - p_0(1 - \tilde{p})^2$$

The types that gain from information acquisition being covert rather than overt are the biased and unbiased informed (1) types and only in case the cost is low enough that the uninformed unbiased type acquires information. Even though the cost does not affect these types directly as they never acquire information, the fact that the unbiased uninformed type does makes their message more credible as they can pretend to have acquired information. In this equilibrium, the payoffs of the uninformed unbiased expert and decision maker are the same as in the pooling equilibrium in the overt case while the payoff of the biased and informed (1) expert are higher. When cost is higher, we move to the no investment equilibrium in which payoffs are identical as the overt case with no investment.

The unbiased type invests in information less often and is worse off in covert case, as he can never perfectly separate himself from the biased type. This result shows that, even though the signaling value of information undermines its intrinsic value, under certain parameters overt information acquisition does strictly better than the covert one. This is true for the parameters under which the separating equilibrium does exist in the overt information acquisition.

However, when the cost of information acquisition is low enough that in the overt case there will be pooling in investment, then covert information acquisition does better than overt in terms of the overall welfare as there is no wasteful investment, although the precision of communication is equivalent. Then the next corollary follows.

Proposition 3. *Whenever* $c \in [p_0 - {p_0}^2 - p_0(1-\tilde{p})^2, p_0 - {p_0}^2]$*, overt information acquisition leads to higher welfare while whenever* $c \leq (1-\hat{p})^2 - (1-\tilde{p})^2$*, covert information acquisition leads to higher welfare.*

The tradeoff is between more informative communication versus wasteful investment in information. Whenever there is no investment in the covert case, then overt information does better as we have seen that welfare is always higher when some types invest than no investment. However, whenever there is pooling in investment in the overt case, as there is also investment by the unbiased types in the covert case and communication precision is the same, then welfare is higher in the covert case due to less wasteful investment.

7 General Information Acquisition Technology

In this part, a more general information acquisition technology is considered. I show that the presence of biased types can lead to over-investment by the unbiased types. The level of cost incurred in information acquisition is $e \in [0,1]$, where e is the probability with which the expert obtains perfect information, $c(e)$ is increasing and convex in e, $c'(e) > 0$, $c''(e) > 0$, $c(0) = 0$ and $c'(e) \to \infty$ as $e \to 1$.[5] The decision maker now observes the amount of effort exerted by the expert before communication takes place but does not know whether information did arrive, nor its realization.

Lemma 3. *Call the equilibrium effort level chosen by the unbiased uninformed expert as* $\hat{e}$*. The equilibrium effort level chosen by the biased and informed (1) types takes one of two values,* $e \in \{0, \hat{e}\}$*, in pure strategies.*

Proof. Assume effort took any other value. Then, given the effort level of the unbiased uninformed type is $\hat{e}$, upon observing a different effort level, the DM infers the expert to be a biased or an

[5] This is a standard assumption in the literature. For example, Che and Kartik (2009) use a similar information acquisition process (followed by strategic disclosure), where $c(p)$ is convex and p is the probability that information is acquired (investigation is successful), even though the information that is acquired is still noisy but has mean equal to the state.

unbiased uninformed type. But, this would also be the case when $e = 0$. Then, given that the biased and unbiased informed (1) types do not value information, they would choose either of the two effort levels: $\{0, \hat{e}\}$. Indeed, for some out of equilibrium beliefs, (such as $\mu(0,1) = p_0$ which puts probability 1 to biased expert upon no investment) it is possible to have a positive but still separating effort level for these types, but this type of equilibria are worse in terms of payoff for the experts, and not more informative for the DM either. So, we will ignore the possibility of this kind of out of equilibrium beliefs and the lemma follows. □

Lemma 4. *The unbiased uninformed type always exerts a positive effort level:* $\hat{e} > 0$*.*

Proof. Assume that the unbiased uninformed type chose effort level 0. Then it should be that the other types do not exert any effort either, by lemma 3. We have $c'(0) = 0$. Whenever $m = 0$, the DM takes it at face value: the possibility that this message comes from a biased type is 0. Hence, upon hearing $m = 0$ after a positive effort level, the DM will infer that it is an unbiased type and take action $y = 0$. Then, even only from the probability of receiving signal 0, the expert will increase his payoff by $e(1 - p_0)(p_0 - {p_0}^2)$. Then, the minimum effort he would exert when all other types choose $e = 0$ is given by the condition $c'(e) = (1 - p_0)(p_0 - {p_0}^2)$. This is the worst case, under the assumption that $m = 1$ be treated as a babbling message by the DM. □

The distinctive feature of this setup is that there exist pooling equilibria at positive effort level that cannot be ruled out by the Intuitive Criterion. As it is not necessarily the unbiased uninformed type who benefits more from deviating to invest more when believed to be an unbiased type, the intuitive criterion fails to rule out some pooling equilibria.

7.1 Separating Equilibrium

The separating equilibrium is one in which only the unbiased uninformed agent incurs a positive effort level, called $\hat{e}$. The payoff of the unbiased uninformed type in an equilibrium in which he is the only type to exert effort is:

$$-(1 - e)(p_0 - {p_0}^2) - c(e)$$

whereas without investment his payoff is $-(p_0 - {p_0}^2)$. The first best level of investment for the unbiased type is then:

$$(p_0 - {p_0}^2) = c'(e^*) \tag{9}$$

The biased type and informed (1) type get a payoff of $-(1 - \hat{p})^2$ when they do not invest and send $m = 1$. If they instead deviate to invest and pool with the unbiased type, they will send $m = 1$ which the DM will take at face value and get payoff 0. Then, the condition for these types not to deviate to invest is:

$$c(e) \geq (1 - \hat{p})^2$$

If $c(e^*) > ((1-\hat{p})^2$, then at the first best level of investment of the unbiased type, there are no incentives to mimic. Then, in that case there is a unique separating equilibrium in which the uninformed unbiased type chooses its first best investment level.

If $c(e^*) \leq (1-\hat{p})^2$, then at the first best investment level of the unbiased type, the biased and informed (1) type are willing to deviate to invest. If this is the case, the least cost separating equilibrium level of effort is given by $c(e) = (1-\hat{p})^2$. Then, we can say that whenever $c(e^*) < (1-\hat{p})^2$, there is over-investment by the unbiased type in any separating equilibrium, $e > e^*$. The separating effort level should also satisfy the participation constraint of the unbiased type, who could instead deviate to not invest and send $m = \emptyset$. The unbiased type is willing to incur any level of e which satisfies:

$$-(1-e)(p_0 - {p_0}^2) - c(e) \geq -(p_0 - {p_0}^2)$$

where the right hand side is the payoff from not investing. The above simplifies to:

$$e(p_0 - {p_0}^2) \geq c(e)$$

If there is incentive to deviate for the biased type at the best level of investment of the unbiased uninformed type, then $c(\tilde{e}) = (1-\hat{p})^2$ is the least cost separating (LCS) effort level and as long as:

$$\tilde{e}(p_0 - {p_0}^2) - (1-\hat{p})^2 \geq 0 \tag{10}$$

the LCS equilibrium will arise. Indeed, whenever the LCS equilibrium satisfies equation (10), it will be the unique equilibrium. This is possible as the DM could use a babbling threat: whenever $e < \tilde{e}$, the DM treats the message as babbling (coming from a biased type) and chooses $y = p_0$. In that case, the LCS equilibrium at $\tilde{e}$ will be the unique equilibrium. Then, the following proposition follows:

Proposition 4. *Given e^* the first best effort level for the unbiased uninformed type, following describes the equilibria that arise:*

- *When $c(e^*) > ((1-\hat{p})^2$, the separating equilibrium arises in which the first best effort level is chosen by the unbiased uninformed type, which is the level of effort he would choose if $\beta = 0$.*
- *When $c(e^*) \leq ((1-\hat{p})^2$, the least cost separating equilibrium effort level, $\tilde{e} > e^*$, is given by $c(\tilde{e}) = (1-\hat{p})^2$. This will be the unique equilibrium if $-\tilde{e}(p_0 - {p_0}^2) - (1-\hat{p})^2 \geq 0$.*
- *There exists no separating equilibrium if:*

$$(p_0 - {p_0}^2) \leq (1-\hat{p})^2$$

The last part is the condition for even when the unbiased type exerts the maximum effort level 1, the biased type to still find it profitable to mimic him by deviating to invest, in which case there exists no separating equilibrium. This is likely to be the case if β is high, p_0 is low, or α is low. When this condition is violated, there exists some $e \in [0, 1]$ for which separation is possible but even then it is not guaranteed: at the LCS effort level, it may be that the payoff for unbiased uninformed type is lower than the payoff without investment. In that case, a separating equilibrium will not exist either.

If the separating equilibrium exists and it is the LCS one, then the DM is sometimes better off due to the presence of the biased type compared to having an unbiased expert with probability 1, due to the higher investment by the unbiased uninformed type in order to separate himself from the biased type. This is the case if the gain to the DM from the over investment of the unbiased expert outweighs the negative effect of having a biased expert with some probability who sends uninformative advice.

Let us compare this to some results from the literature before showing in a numerical example. Argenziola et al. (2016) show that the expert overinvests compared to what the DM would have done by using an argument of babbling threat by the decision maker. There is only 1 type of expert in their setup whereas the over investment result in my setup arises because the expert wants to avoid being mimicked by the biased type. In addition, in my setup there is over investment compared to what the unbiased expert would have done without the presence of a biased expert. This result is also reminiscent of Che and Kartik (2009) who show that the DM prefers to have an advisor with different prior than herself compared to an aligned one, but in a setting in which there is no uncertainty about bias and communication happens through disclosure. The similarity is that their result is also due to the higher effort exerted by a biased expert given that no disclosure is perceived as withholding information.

A numerical example

Let us now demonstrate numerically the result that the DM's welfare may increase when introducing some uncertainty about the expert's type. Assume the cost of effort is $c(e) = e^2$, $\beta = 0.1$,$\alpha = 0.4$, $p_0 = 0.5$. For these parameters, we have $e^* = \frac{1}{8}$ as the first best investment level and $(1 - \hat{p}) = \frac{5}{28}$. Then, the DM's payoff when the expert is known to be an unbiased type (the advisor chooses $e^* = \frac{1}{8}$:

$$-(1-\alpha)\frac{7}{8}(p_0 - {p_0}^2) = -0.6 \times \frac{7}{8} \times 0.25 = -0.13125$$

When there is a probability of the advisor being biased ($\beta = 0.1$), then the LCS equilibrium has $e = \frac{5}{28}$. Then, the DM's payoff is:

$$-(0.1 + 0.9 \times 0.4 \times 0.5)\frac{5}{28}^2 - \frac{23}{28} \times 0.9 \times 0.6 \times 0.25 = -0.11982142856$$

As we see, the DM's payoff is higher in the least cost separating equilibrium when there is a possibility that the agent is biased compared to when the advisor is unbiased with certainty. This is a hold-up problem in the sense that the advisor invests in information and the DM does not pay for it. As the expert only takes into account his own benefit but the decision is valued both by him and the decision maker, he under invests in the case when there is no uncertainty about his type. However, when uncertainty about the expert type is introduced, then it can be the case that the adviser over-invests in information. Even though there is possibility that the expert may be a biased one giving wrong advice, the over investment by the expert may overcome this effect and as a result the DM may be better off.

7.2 Pooling Equilibrium

As we established that a separating equilibrium is not always guaranteed, we now study pooling equilibria in which all types except informed (0) unbiased invest. Upon a pooling level of investment e and message $m = 1$, the DM's updated belief is:

$$\tilde{p} = \frac{p_0\beta + (1-\beta)p_0\alpha + (1-\beta)(1-\alpha)ep_0}{\beta + (1-\beta)p_0\alpha + (1-\beta)(1-\alpha)ep_0}$$

which simplifies to:

$$\frac{p_0\beta + (1-\beta)(p_0\alpha + (1-\alpha)ep_0)}{\beta + (1-\beta)(p_0\alpha + (1-\alpha)ep_0)}$$

and $1-\tilde{p}$ becomes:

$$\frac{\beta(1-p_0)}{\beta + (1-\beta)(p_0\alpha + (1-\alpha)ep_0)}$$

If the unbiased type's investment doesn't result in a signal, he sends $m = \emptyset$, whereas the biased type always sends $m = 1$. The belief $\mu(0, 1) = \hat{p}$ is as in the separating equilibrium:

$$\hat{p} = \frac{\alpha(1-\beta)p_0 + \beta p_0}{\alpha(1-\beta)p_0 + \beta}$$

The gain to the biased agent from investing the pooling effort level e compared to no investment is: $(1-\hat{p})^2 - (1-\tilde{p})^2$, where e affects $\tilde{p}$ and $c(e)$. As $\hat{p}$ is decreasing in e, $(1-\tilde{p})^2$ is increasing and convex in e. Then, the question remaining is to find the pooling equilibrium level of investment. The condition for the unbiased type to exert the pooling equilibrium level of effort, e, is:

$$-ep_0(1-\tilde{p})^2 - (1-e)(p_0 - {p_0}^2) - c(e) \geq -(p_0 - {p_0}^2)$$

which leads to:

$$c(e) \leq -ep_0(1-\tilde{p})^2 + ep_0(1-p_0)$$

The net value of investment for the unbiased type is convex in e. The biased type is willing to invest if and only if:

$$(1-\hat{p})^2 - c(e) \geq (1-\tilde{p})^2$$

The hand right side is decreasing in e, as $\tilde{p}$ is increasing in e. The ex-ante payoff of the unbiased type in the pooling equilibrium with effort level e is:

$$-p_0(\alpha + (1-\alpha)e)(1-\tilde{p})^2 - c(e)(1-\alpha(1-p_0))$$

Payoff of the biased type in the pooling equilibrium with effort level e:

$$-(1-\tilde{p})^2 - c(e)$$

The first best investment level for the unbiased uninformed type in the pooling equilibrium is:

$$c'(e^*) = (p_0 - {p_0}^2) - p_0(1-\tilde{p})^2 + 2ep_0(1-\tilde{p})\frac{\partial \tilde{p}}{\partial e} = p_0[1 - p_0 - (1-\tilde{p})^2 + 2e(1-\tilde{p})\frac{\partial \tilde{p}}{\partial e}]$$

Finally, there is a continuum of pooling equilibria which survive the intuitive criterion.

7.3 Equilibrium Selection

Now, we argue that in case there exists a separating equilibrium, it will be the unique equilibrium. First, if separation is possible at e^*, then this is the unique equilibrium and we are done. Otherwise, the LCS equilibrium has over investment with effort level $\tilde{e} > e^*$. This will be the unique equilibrium if the payoff of the unbiased uninformed type at the LCS effort level is as good as his no investment payoff to sending $m = \emptyset$. To rule out lower investment levels in that case, the DM uses babbling as a threat: upon observing an effort level $e < \tilde{e}$, the DM treats the message as coming from a biased agent, hence a "babbling" message.

If there exists no separating equilibrium, this is because the minimum effort level the unbiased type should exert in order to achieve separation leads to a payoff less than $-(p_0 - {p_0}^2)$ for this type or because the biased type is willing to mimic the unbiased type for any possible effort level. Then, the only type of equilibria that arise are pooling equilibria at positive investment level, and there are multiple pooling effort levels can arise as an equilibrium. To refine the types of pooling equilibria in that case requires further work.

8 Conclusion

The paper studied a simple and novel question building on communication and signaling literature. It explored the interaction of credibility concerns with costly information acquisition and the welfare implications of this. The ex-ante informed expert who is unbiased is unable to separate himself from a biased expert whenever the state of the world is high. Hence, the decision maker can be better off when matched with an uninformed expert who will later on get informed and will be able to communicate more efficiently. I find that an unbiased expert, as well as a biased one, may wastefully invest in information acquisition. This leads to inefficiency in decision making and lower overall welfare. Higher information acquisition cost increases the overall ex-ante welfare when the equilibrium moves from pooling to separating region.

The simplicity of the model allows for numerous extensions left for future research. An interesting feature of the model is that even though the ability to acquire information is not correlated with the type of the expert, the value of getting informed is correlated to his type. This means it is possible to use information acquisition as a screening device by taxing the experts for getting informed. While in the pooling region, the expert who is uninformed and unbiased has the highest incentives to acquire information. Hence, this type will be willing to pay more than the biased and informed (1) agents implying wasteful investment by the biased and informed experts can be avoided. Another extension could be to consider commitment by the decision maker on action as a function of investment and communication.

9 Appendix

Proof of proposition (1):

I find the separating equilibrium to be the unique equilibrium in this region by using the intuitive criterion. First, there cannot be any equilibrium in this region in which unbiased informed (1) and biased types invest with probability one, as even when $\mu(1,1) = 1$, in this region, the group 1 types not find it profitable to invest as the cost is too high. Then, the only pure strategy equilibrium that could arise is the no investment equilibrium in which even the uninformed unbiased expert doesn't invest. For some out of equilibrium beliefs, this equilibrium can arise, as discussed below.

Assume that the DM believes any type except the informed (0) unbiased one is equally likely to invest, then his belief and optimal choice will be $\tilde{p}$ which is:

$$\tilde{p} = \frac{p_0(1-\beta) + \beta p_0}{p_0(1-\beta) + \beta} = \frac{p_0}{p_0(1-\beta) + \beta} \tag{11}$$

Let us show the biased type doesn't have the incentive to incur the cost c. Now, upon the message $m = 1$, the DM chooses $y^* = \hat{p}$ as there is only biased and unbiased informed(1) types who choose $m = 1$. For the biased and unbiased informed(1), the payoff from investing should be less than that from not investing:

$$(1-\tilde{p})^2 + c \geq (1-\hat{p})^2 \tag{12}$$

As without investment and $m = 1$, the DM's belief is $\hat{p}$ as in case 1. This is equivalent to:

$$\left[\frac{\beta(1-p_0)}{p_0(1-\beta) + \beta}\right]^2 + c \geq \left[\frac{\beta(1-p_0)}{\alpha(1-\beta)p_0 + \beta}\right]^2 \tag{13}$$

For the uninformed agent, the payoff from not investing is $-(p_0 - {p_0}^2)$ and from investing it will be:

$$-(1-p_0)0 - p_0(1-\tilde{p})^2 - c \tag{14}$$

Then the condition that should be satisfied is:

$$c \geq p_0 - {p_0}^2 - p_0(1-\tilde{p})^2 \tag{15}$$

Finally, the equilibrium in which no type wants to invest, for the specified out of equilibrium beliefs, is:

$$c \geq \max\{p_0 - {p_0}^2 - p_0(1-\tilde{p})^2, (1-\hat{p})^2 - (1-\tilde{p})^2\} \tag{16}$$

Now consider that when there is a separating equilibrium, the condition $p_0 - {p_0}^2 > (1-\hat{p})^2$ is satisfied. Then, it is the case that $p_0 - {p_0}^2 - p_0(1-\tilde{p})^2 > (1-\hat{p})^2 - (1-\tilde{p})^2$. This means, the condition above becomes $c \geq p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$, which is less than $p_0 - {p_0}^2$. Then, there is also a no investment equilibrium in this region. However, we are able to rule out this type of equilibrium. This is because, the uninformed type, if his message is taken at face value, is willing to deviate to invest while even if the message is taken truthfully, the group 1 types do not find it profitable to invest. Hence, as there is a best response of DM that makes only unbiased uninformed types better off, this type of equilibrium can be ruled out by the intuitive criterion.

Finally, we can also rule out an equilibrium in which only group 1 types invest, as for no belief of the DM these types find it profitable to invest in this region.

In the region $c \in [p_0 - {p_0}^2 - p_0(1-\tilde{p})^2, p_0 - {p_0}^2]$, the separating equilibrium is the unique equilibrium that satisfies the Intuitive Criterion. Then, in the region $c \in [p_0 - {p_0}^2 - p_0(1-\tilde{p})^2, p_0 - {p_0}^2]$, the separating equilibrium is the unique equilibrium that satisfies the Intuitive Criterion.

Whenever $c \geq (1-\hat{p})^2$, even for the highest belief $\mu(1,1) = 1$, the biased and uninformed (1) agent do not benefit from deviating to invest. Hence, in this region, the out of equilibrium belief should assign probability 1 to the expert being unbiased and uninformed, which provides the boundary of the no investment equilibrium.

Proof of proposition (2):

Proof. I compare the payoff in the pooling equilibrium versus separating equilibrium, before comparing to no investment equilibria. For this, I compare the payoff for cost values at which there is pooling equilibrium and separating equilibrium to find how the payoff changes when c increases from the pooling region to separation.

The total welfare in the **pooling** equilibrium is given by:

$$-p_0(1-\tilde{p})-\beta[(1-\tilde{p})^2+c]-(1-\beta)[p_0(1-\tilde{p})^2+(1-\alpha(1-p_0))c] \tag{17}$$

which simplifies to:

$$-(1-\tilde{p})^2[\beta+p_0(1-\beta)]-p_0(1-\tilde{p})-c[1-\alpha(1-p_0)]=-\beta(1-p_0)-c[1-\alpha(1-p_0)] \tag{18}$$

Total welfare in the **separating** equilibrium is:

$$-\hat{p}\beta(1-p_0)-\beta(1-\hat{p})^2-(1-\beta)[\alpha p_0(1-\hat{p})^2+(1-\alpha)c] \tag{19}$$

which simplifies to:

$$-\beta(1-p_0)-c(1-\alpha)(1-\beta) \tag{20}$$

The difference in the total welfare in equation ($18-20$) is:

$$-c_p[1-\alpha(1-p_0)]+c_s(1-\alpha-\beta(1-\alpha)) \tag{21}$$

The welfare in terms of information cancels out and what remain are the terms related to the cost incurred in information acquisition. In pooling equilibrium there is more investment in information at a lower price, while in the separating equilibrium there is less investment but at a higher price. When we plug in the maximum cost at which the pooling equilibrium exists and minimum cost at which the separating equilibrium exists, it is seen that welfare in separating equilibrium is higher than in the pooling one although the cost of information acquisition is higher.

At $c_p=(1-\hat{p})^2-(1-\tilde{p})^2$ and $c_s=(1-\hat{p})^2$, equation (21) becomes:

$$-(1-\hat{p})\beta(1-p_0)+(1-\tilde{p})^2[1-\alpha(1-p_0)]<0 \tag{22}$$

Then, although the cost of information rises, welfare increases due to no wasteful investment in information. In order to demonstrate this result, we considered the boundary cost values. Then, we have shown that the proof holds.

As expected, when we keep increasing the cost in the separating equilibrium region, the result may be reversed. When we plug in the maximum cost at which there is separation, $c = (p_0 - {p_0}^2)$, we get:

$$(-1 + p_0)(\beta + (1 + \alpha(-1 + p_0))p_0) \tag{23}$$

Then, when we plug this in the difference in the total welfare equation (21) becomes:

$$(-1 + p_0)p_0(-1 + 3\beta + \alpha(1 - 2\beta + p_0)) \tag{24}$$

This whole expression is negative if and only if:

$$3\beta + \alpha(1 - 2\beta + p_0) > 1 \tag{25}$$

This is satisfied for certain parameters, mainly more likely to hold for low p_0.

Total welfare in the **no investment** equilibrium is:

$$-\beta(1-\hat{p})^2 - (1-\beta)[\alpha p_0(1-\hat{p})^2 + 2(1-\alpha)(p_0 - {p_0}^2) - (\hat{p} - \hat{p}^2)(\beta + (1-\beta)\alpha p_0)] \tag{26}$$

When we compare this to pooling in investment, it is seen that it is unambiguously inferior. As no investment equilibrium surplus is unambiguously worse than the separating equilibrium, we compare it to the pooling in investment equilibrium and we find that the payoff in no investment equilibrium is also lower than the pooling in investment equilibrium. This is intuitive: first, the DM's payoff is unambiguously higher in the pooling in investment equilibrium compared to the no investment equilibrium, as more information is revealed. The welfare of the biased type also higher in the pooling in investment equilibrium as their outside option of not investing and getting $-(1-\hat{p})^2$ is still available. Hence, if this type does find it profitable to invest, then it must be getting a higher payoff. The same is true for the unbiased informed (1) type who would get $-(1-\hat{p})^2$ if deviating to not invest. Finally, for the unbiased uninformed type, it is true as well: if this type didn't invest they would get the payoff $-(p_0 - {p_0}^2)$ which is still available if they were to deviate in the pooling equilibrium and send $m = \emptyset$. □

Proof of proposition (??)

Proof. When $c > p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$, in the covert information acquisition case no type is getting informed, hence the payoff is the same as no investment equilibrium in the overt case. We know that in the overt information acquisition that welfare is always higher when there is some information acquisition compared to no investment, and in the region $[p_0 - {p_0}^2 - p_0(1-\tilde{p})^2, p_0 - {p_0}^2]$ there is investment in the overt case. Hence, the payoff in overt case is always higher under this condition.

When $c < (1-\hat{p})^2 - (1-\tilde{p})^2$, in the overt case, there is pooling in investment while in the covert case, the unbiased uninformed type invests only (realize that $(1-\hat{p})^2 - (1-\tilde{p})^2 < p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$ hence there is investment in covert case). In the end, the amount of information transmitted is the same. Hence, in the overt case there is more wasteful investment for the same precision of communication. Then, we can conclude that welfare is higher in the covert case under this assumption. □

Proof of proposition (4)

Proof. The first two items were shown earlier.

The last condition says that even if the uninformed unbiased expert exerts effort $e = 1$ in which case his gain in payoff will be $(p_0 - {p_0}^2)$, this is still lower than the gain of the biased type and informed (1) type from investing and pooling with him which is $(1-\hat{p})^2$. The outcome of investment is not relevant for these types and they will always claim to have received signal 1. Given that this is a deviation from an equilibrium in which the only investment is made by unbiased types, when they send $m = 1$ the DM will choose $y = 1$. Hence, their payoff from investment is 0, whereas without investment their payoff was $-(1-\hat{p})^2$, which means their gain is $(1-\hat{p})^2$.

On the other hand, the gain in profit from investment for the unbiased uninformed expert is $e(p_0 - {p_0}^2)$. To see this, realize that with probability e, a signal will arrive and if it does, then the expert's payoff will be 0 as the DM will take the message at face value. If no message arrives, then the expert will send $m = \emptyset$ and get a payoff of $-(p_0 - {p_0}^2)$. Hence, the gain in payoff for the uninformed type is $e(p_0 - {p_0}^2)$.

Then, the gain in payoff from deviating to pool is higher for the biased type than the gain in information for the unbiased uninformed type even when $e = 1$ if $(p_0 - {p_0}^2) \leq (1-\hat{p})^2$. Then, for any level of e, the uninformed unbiased type could not separate himself from the biased type of expert. Hence, no separating equilibrium exists under this condition.

□

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