Strategic Inaccuracy in Bargaining*

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Abstract

This paper studies a buyer seller game in which the buyer communicates taste through cheap talk before the seller makes a take it or leave it offer. The amount of information revealed determines the fit of the offer, but also results in the seller asking for a higher price. Lack of commitment by the seller creates a hold-up problem and a trade off between efficiency and surplus sharing, resulting in coarse information. The buyer optimal equilibrium has the least informative partition that ensures trade: buyer types pool in intervals which are the largest possible subject to trade taking place. When extended to multiple periods of bargaining, it is seen that the buyer is better off sending informative messages only at the first period implying that gradual revelation of information is not profitable.

Keywords: information; cheap-talk; bargaining; buyer-seller relation
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1 Introduction

Sellers often use consumer information in order to personalize products and pricing. Although beneficial, customization comes at a cost for buyers: sellers can charge a higher price the more they know about the buyer's preferences. Hence, buyers face a tradeoff between revealing their private information in order to get a better fit versus being less transparent in order to ensure a lower price. This paper deals with the question of how much information buyers should reveal when facing an uncommitted seller.

Personalized offers have since long existed in settings where upon the information revealed by the potential buyer, the seller proposes a specific good at a personalized price, such as on bazaar-stalls where most of the time the price is not transparent. In business to business relations, negotiations lead to personalized products and pricing, especially when a service has many different features. In online markets, retailers acquire information about consumers through cookies tracking search and purchase history, and loyalty subscriptions asking for personal information such as age, job or even postcode. Users have control over how much information they want to provide. Based on these, sellers can predict what kind of products a particular user is interested in buying at what prices and send them targeted advertisements, promotion codes, etc. It is an important question to study how consumers' welfare is affected by the amount of information they reveal.

In order to study this problem, I introduce information transmission through cheap talk communication from a buyer to a seller in a simple framework. The buyer sends a message about his horizontal taste parameter before the seller comes up with an offer consisting of a good and a price. The seller is indifferent among the horizontal varieties of the good and lacks commitment. This leads to a hold-up problem and the buyer can keep information rents only in case his type is not perfectly revealed.

The main objective of the paper is the characterization of the buyer's optimal equilibrium among multiple equilibria. This multiplicity is due to the wide range of possibilities of communication, which is a common feature in the cheap talk literature, pioneered by Crawford and Sobel (1982) (CS hereafter). However, there is a crucial departure, which is the lack of a *bias* on the choice of the good which maximizes welfare and hence a conflict on this choice. Finding a better fit is always surplus enhancing, but there is a conflict on the sharing of the surplus in terms of the price chosen. In addition, there is a third stage in which the buyer decides whether to accept the offer made by the seller or not. I show that the perfect revelation equilibrium does exist, but is not prone to neologism proofness refinement[1]. The reason is that there are always some types of the buyer are better off in equilibria in which they pool in intervals. Although perfect revelation equilibrium maximizes both the seller and the social surplus, it leaves zero surplus to the buyer. I then move on to study equilibria in which information is coarse, in other words interval partition equilibria. Among these, I search for the one that is ex-ante surplus maximizing for the buyer, and call this buyer-optimal.

The one round communication and offer game provides the main result on the optimal amount of information revelation by the buyer. I show that the buyer-optimal equilibrium has the *coarsest* information structure (the minimum number of intervals) that *covers the market* (trade takes place for all types). In this equilibrium, the buyer's messages are the least informative such that trade is ensured: partition intervals are the largest subject to the constraint that the seller doesn't want to exclude any type. In other words, the buyer ensures his message is just precise enough to get an acceptable good, as his surplus is increasing in the length of the partition intervals as long as no type is excluded. If the intervals become even larger, then the buyer's surplus starts decreasing as some types start being excluded.

Next, the dynamics are explored in a two period game, in which a second round of communication and offer takes place in case the first period offer is rejected. I show that the buyer's ex-ante optimal equilibrium in this setting consists of sending informative messages in the first period and an uninformative message if period 2 is reached, subject to the condition that trade is still ensured. For the seller, the presence of a second period means that the interval inside which the buyer can be found shrinks after the offer is rejected in addition to the possibility of receiving another informative signal. When period 2 is uninformative (babbling), the seller no longer enjoys as much informational benefit which disciplines her while making the period 1 offer and reduces expected delay.

Compared to the one period game, the seller is worse off only if the discount factor is high enough that there is expected delay. When the discount factor is

[1]An equilibrium refinement introduced by Farrell (1993)

sufficiently low, the buyer-optimal equilibrium features no delay, as delay is sufficiently costly that the buyer is better off revealing more information in order to avoid it. This equilibrium results in higher profits for the seller as more information is revealed and trade takes place without delay, and results in higher welfare compared to the one period game. Hence, the threat by the seller of excluding more types leading to delay may result in a more informative equilibrium than in the one period game.

The buyer obtains his highest surplus in the one period game and is always worse off by the introduction of a second or more periods. As the number of periods increases, the buyer surplus in the equilibrium with delay decreases while the no delay equilibrium surplus is independent of the number of periods. Hence, when the number of periods increases, the no delay equilibrium is more likely to become the buyer optimal one.

The buyer doesn't benefit from gradual revelation of information as the seller has more incentives to choose a higher price and risk delaying trade if she anticipates more information in the future. Hence, the buyer is better off revealing information in the beginning and waiting for the seller to make offers. This result contrasts with some literature on selling information where gradual revelation of information is shown to be optimal. For example, Horner and Skrzypacz (2016) study a dynamic problem between a firm and an agent who has valuable and verifiable information, and show that gradual revelation of information is optimal. Although the setting of my paper is different, it provides an example in which gradual revelation of information is not profitable for the owner of information, which in our case is the buyer.

Farrell and Gibbons (1989) are the first to study the role of cheap talk before bargaining takes place, in a setting in which bargaining happens on a predetermined good and there is incomplete information about the buyer's valuation. In the current paper, cheap talk has a more significant role as it also determines the type of the good offered.

There has been some recent contributions on dynamic cheap talk. Golosov et al. (2014) study the dynamic version of the CS model in which the seller takes a decision in every period and construct a perfect revelation equilibrium.[2]

[2]They show that full revelation is possible by constructing a non-monotonic partition equilibrium, in which far away types pool initially and separate later on. In contrast, in this paper

This paper also relates to the literature on consumer information and privacy. For a discussion on consumer privacy see Varian (1997) and for a recent survey on the literature see Acquisti, Taylor, and Wagman (JEL 2016). Varian (2000) discusses versioning of information goods in order to extract surplus from different groups of consumers. The main departure from this literature is that the current paper focuses on the buyer's choice of information revelation. Bergemann and Bonatti (2015) consider the problem of a data provider who sells consumer information to advertisers who tailor their advertisements to the individual match value. There is also a recent literature on consumer information design. There are two papers on consumer information design which are Roesler and Szentes (2016) who consider a buyer who acquires a signal about her valuation for an exogenously fixed good, while the seller chooses a price after observing the distribution of signals but not the signal itself and Condorelli and Szentes (2016) consider a buyer who chooses a CDF which then determines her valuation. Lastly, a follow up paper by Vellodi (2016), is very closely related and considers the one period game in my setting by focusing on the set of neologism proof equilibria.

2 The Model

A buyer wants to buy one unit of good (or service) from a seller and is privately informed about his type θ while the seller knows it is uniformly distributed in $[0, 1]$. The utility of the buyer from a good located at y is $U(\theta, y) = k - f(\theta - y)$, where f, the cost of mismatch between the good and the buyer's type, is symmetric around 0, $\frac{\partial^2 f}{\partial y^2} > 0$ and $\frac{\partial f}{\partial y}|_{y=\theta} = 0$ (f is strictly convex and single peaked), and $\frac{\partial^2 f}{\partial \theta \partial y} < 0$ (the single crossing condition).

The maximum buyer valuation, k, is small enough that in the absence of information revelation the market will not be covered, in other words the seller's offer will be rejected by some buyer types. Outside options are zero. The seller can choose to offer any good $y \in [0, 1]$ and her valuation is normalized to zero, which implies that trade is always optimal. The seller would like to find the best fit for the buyer in order to charge the highest possible price. However, this is only possible if the buyer perfectly reveals his type.

there is a one time decision and it is costly to delay trade. Hence, perfect revelation is inferior for the buyer from an ex-ante point of view.

The game proceeds as follows. The buyer, after observing his type, sends a message $m \in M$ to the seller, to which the seller responds by an offer consisting of a good $y(m)$ and a transfer $\tau(m)$. If the buyer accepts the offer, the game ends yielding payoffs $U(\theta, y) - \tau$ and τ respectively for the buyer and the seller.

A Perfect Bayesian Equilibrium (PBE) consists of a message strategy $m : [0,1] \to M$ for the buyer and an offer strategy $\alpha(m) = (y(m), \tau(m))$ for the seller, to which the buyer responds by $\sigma_\theta(y, \tau) \in \{0, 1\}$ where 1 denotes the decision to accept and 0 the decision to reject the offer. Strategies that constitute a PBE satisfy:

- for any $\theta \in (0,1)$, $m(\theta)$ satisfies $\arg\max_m U(\theta, \alpha(m))$ where $U(\theta, \alpha(m)) = max\{0, U(\theta, y(m)) - \tau(m)\}$ as the buyer has the option to reject the offer. (the buyer's message maximizes his utility among feasible messages given the seller's best response.)
- for any m, $\alpha(m) \in \arg\max_\alpha \tau \int_0^1 \sigma_\theta(\alpha)\rho(\theta|m)d\theta$ where $\rho(\theta|m)$ is the belief that agent is of type θ given his message m, and is derived from Bayes' rule whenever possible. (the seller's offer maximizes her expected profit given her belief about the buyer's type.)

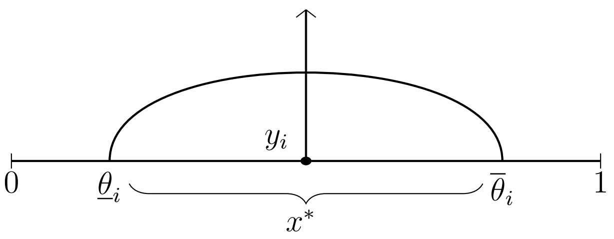
3 One Period Game

In this section the equilibria of the one period game are explored, among which the buyer-optimal equilibrium will be found. I start by considering the two extreme cases, which are babbling and perfect revelation equilibrium, before moving onto solving for the buyer optimal equilibrium.

First, let us consider the equilibrium in which no information is revealed, in other words the *babbling equilibrium*. In this type of equilibrium, the seller does not get any information about the buyer's type through his message. Then, the seller's weakly dominant strategy is to choose $y = \frac{1}{2}$. The price that the seller chooses trades off the probability of acceptance versus higher price paid upon acceptance. Given price τ, the types $\theta \in (\underline{\theta}, \overline{\theta})$ which accept the offer are given by the condition:

$$k - f(\overline{\theta} - \frac{1}{2}) = \tau$$

Figure 1: Babbling equilibrium



and by the uniform distribution of θ and symmetry of f around 0, $\frac{1}{2} - \underline{\theta} = \overline{\theta} - \frac{1}{2}$. Hence, the probability of acceptance is $(\overline{\theta} - \underline{\theta}) = x$. Then the seller's problem is to find the optimal length, x^*:

$$\max_{\{x\}} x(k - f(\frac{x}{2}))$$

x^* solves the FOC:

$$k - f(\frac{x^*}{2}) = \frac{x^*}{2} f'(\frac{x^*}{2}) \tag{1}$$

where the left hand side denotes the increase in the seller's profit due to the higher probability of acceptance and the right hand side denotes the loss in revenue due to the decrease in price multiplied by the probability that trade happens. x^* is the interval of types that the seller would serve in the babbling equilibrium, corresponding to the lowest price she could be induced to charge. Any interval of length $x \leq x^*$ will also be covered by the seller.

Assumption 1. $x^* < 1$*: without communication, trade may not take place.*

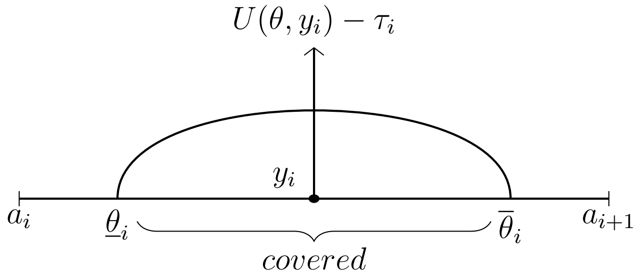
The above assumption

Second, let us consider the other extreme which is the perfect revelation equilibrium.

Proposition 1. *Perfect revelation equilibrium is one in which any buyer type sends a distinct message $m(\theta)$ revealing their type and inducing the offer (θ, k) leaving them 0 surplus. The expected buyer surplus is strictly positive in any other less informative equilibria.*

Perfect revelation equilibrium provides the highest social welfare as any type is getting the perfect fit and the highest seller profits while leaving zero surplus to the

Figure 2: interval i facing offer (y_i, τ_i)



buyer. However, this equilibrium does not survive the *neologism proofness* criteria introduced by Farrell (1993), where the presence of self signaling sets serves as a refinement criteria. In this case, if an out of equilibrium message is interpreted by the seller as coming from any type on the $[0, 1]$ line with equal probability, then her offer will be the one that is identical to the one in the babbling equilibrium, leaving positive surplus to some types.[3]

Now let us study an arbitrary interval partition in which there are n intervals. Types in interval i send a message m_i to which the seller responds by (y_i, τ_i). The buyer will accept this as long as $U(\theta, y_i) - \tau_i \geq 0$. By the single crossing condition, the types in an interval i for whom $U(\theta, y_i) - \tau_i \geq 0$ are connected. The threshold types $\underline{\theta}_i$ and $\overline{\theta}_i$ are those in interval i who obtain the lowest utility. The seller will not leave positive surplus to the threshold types. The boundary types coincide with the threshold types when the whole interval gets served. As the threshold types get 0 surplus, $\tau_i = k - f(\underline{\theta} - y_i) = k - f(\overline{\theta} - y_i)$ and by the symmetry around 0 of f, $y_i - \underline{\theta}_i = \overline{\theta}_i - y_i$, which leads to $y_i = \frac{\underline{\theta}_i + \overline{\theta}_i}{2}$. In case the whole interval is served, $y_i = \frac{a_i + a_{i+1}}{2}$. Figure 2 displays the response to a given offer (y_i, τ_i) by types in an interval i and their corresponding utilities.

In an interval in which there is no exclusion:

$$U(a_i, y(m_{i-1})) - \tau(m_{i-1}) = U(a_i, y(m_i)) - \tau(m_i)$$

The type a_i who is located at the boundary of intervals i and $i - 1$ is indifferent

[3]We consider messages that puts positive probability on types which would *weakly* benefit from a deviation. In a self signaling interval, some types will be indifferent and some will strictly benefit from this deviation. In this case, any type of buyer would be weakly better off in a babbling equilibrium, as some types would receive positive rents while others would still earn 0 surplus.

among the messages m_i and m_{i-1}. From now on, interval i will be used to denote the interval of types who send the message m_i.

Now we can classify the set of monotone partition equilibria into two: those in which there is no exclusion and those in which there are some intervals in which not every type gets served. The former is equivalent to saying that all the boundary types in each interval are accepting the seller's offer whereas the latter means there are some boundary types that are not accepting an offer.

A monotone partition equilibrium is one in which each message is sent by types that are found connected in intervals and no types in separate intervals send the same message. This is called uniform signaling as described by Crawford and Sobel (1982). Fewer number of intervals is equivalent to *coarser* communication: less information revelation. I will denote the message sent by types in an interval i by m_i and say that interval i sends this message.

I will now restrict attention to monotone partitions after ruling out non-monotone partitions. A non-monotone partition has types located in separate intervals that send the same message.[4].

Lemma 1. *If $m(\theta_1) = m(\theta_2) = m_i$, then $m(\theta_3) = m_i$ for all $\theta_3 \in (\theta_1, \theta_2)$: buyer's strategy is monotone in any equilibrium in which trade happens with certainty.*

There cannot exist any non-monotone partition equilibria in which all buyer types accept an offer.[5] As any equilibrium in which there is exclusion will be Pareto-dominated, the set of equilibria in which the buyer optimal equilibrium is found will have monotone partitions where the number of messages sent in equilibrium is equal to the number of intervals, n.

3.1 The buyer-optimal equilibrium

I will now focus on the equilibrium which is superior to any other one in terms of the *expected buyer surplus*, in other words from an *ex-ante perspective.*

Lemma 2. *In any interval $(\underline{\theta}_i, \overline{\theta}_i)$ where $\sigma_\theta(y_i, \tau_i) = 1$ for all θ, the expected buyer surplus is strictly positive: $\int_{\underline{\theta}_i}^{\overline{\theta}_i} [U(\theta, y_i) - \tau_i] d\theta > 0$.*

[4]this definition is used in Golosov et al. 2014

[5]In case some types are not accepting the offer they get in equilibrium, then these types should be indifferent among other equilibria in which they send different messages which will also leave them 0 surplus.

Given that perfect revelation always gives zero surplus to the buyer while pooling results in positive surplus, it follows that the buyer optimal equilibrium will not have any types perfectly reveal themselves. In any given interval, the buyer surplus is lower for types which are further away from y_i, with the type $\theta = y_i$ getting the highest surplus. Hence, the buyer types get informational rents which are increasing in their distance from the threshold types.

Lemma 3. *The expected surplus of the buyer is increasing and the seller's surplus is decreasing in the lengths of the partition intervals in equilibria in which trade happens with probability one.*

Lemma 3 displays the conflict between the buyer and the seller in terms of surplus sharing. While moving away from a fully revealing equilibrium to less informative ones, the buyer surplus increases while the seller profit decreases, until some types start being excluded from trade. When the seller starts excluding some buyer types, the surplus of both sides decreases.

Lemma 4. *Among the equilibria in which trade happens for all types, the buyer optimal one has the least number of intervals, where all but one interval is of size x^*.*

By making use of lemma 3 and 4, I conclude that x^* in definition 1 is the largest interval such that no type is excluded from trade. It remains to show that in the buyer-optimal equilibrium trade happens for all types. Next proposition summarizes the buyer optimal equilibrium of the 1 period game.

Proposition 2. *The buyer-optimal equilibrium of the one period game has the minimum amount of information revealed such that trade is ensured for all types with intervals of length x^* and possibly a final uneven interval.*[6]

Proof. First, in the babbling equilibrium, an interval of $x^* = \overline{\theta} - \underline{\theta}$ of buyer types are served by definition 1. The threshold types get utility 0: for $\overline{\theta}$ and $\underline{\theta}$, $U(\theta, y) - \tau = 0$.

[6] This partition equilibrium looks as follows:

- $\left\langle \frac{1}{x^*} \right\rangle$ intervals of size x^* (where $\left\langle \frac{1}{x^*} \right\rangle$ is the biggest integer smaller than $\frac{1}{x^*}$)
- if $(1 - \left\langle \frac{1}{x^*} \right\rangle x^*) > 0$, another interval of size $1 - \left\langle \frac{1}{x^*} \right\rangle x^*$

Now consider another equilibrium in which the types slightly below, $\theta \in (\underline{\theta} - \epsilon, \underline{\theta})$ send a different message, m', inducing the offer $(y(m'), \tau(m'))$ such that $\forall \theta \in (\underline{\theta} - \epsilon, \underline{\theta})$, $\sigma_\theta(y(m'), \tau(m')) = 1$ (as $\epsilon < x^*$, this interval is covered). As $\underline{\theta}$ is a threshold type, $U(\underline{\theta}, y(m')) - \tau(m') = 0$ which means for $\theta > \underline{\theta}$, $U(\theta, y(m')) - \tau(m') < 0$. Hence, the messages and surpluses of the types in the initial interval of length x^* do not change. For the new types who now get served, by lemma 2, $\int_{\underline{\theta}-\epsilon}^{\underline{\theta}} (U(\theta, y(m')) - \tau(m')) d\theta > 0$. Hence this new equilibrium dominates the initial equilibrium in terms of the buyer surplus. Repeating the same procedure, any equilibrium in which some types are excluded is dominated by a more informative equilibrium in which more types are served, until the equilibrium in which trade happens for all types is reached. The first part of the proof is done. The neologism proofness criteria applies here: in any equilibrium with exclusion, the excluded types can form a self signaling set by sending a separate message which leads to a weakly better payoff for any type. There will be some types will get positive surplus. Hence, any equilibrium with exclusion is not neologism proof.

The second part is to show that moving to more informative equilibria leads to lower buyer surplus. Lemma 4 showed that any equilibrium partition which has more than 1 interval finer than x^* is inferior in terms of buyer surplus. The interval length x^* induces the seller to charge the minimum price. It then concludes that the partition with interval length x^* does better than any other more or less informative equilibria in terms of the buyer surplus. □

The equilibrium which is buyer-optimal has the least number of intervals with no exclusion. In other words, as larger intervals imply less informative (coarser) equilibria, messages are just informative enough for trade to take place with certainty. The buyer does not benefit from being more informative, as he guarantees the lowest price with no risk of exclusion. In addition, he does not benefit from being less informative either, because some types would then be excluded from trade.

3.2 Comparative statics

As the maximum willingness to pay k increases, x^* increases, in other words less information is required to ensure that the market is covered. When preferences are less important relative to the intrinsic value of the good, the buyer can be less precise and still obtain a good. In addition, as f' increases, meaning as f becomes more convex, x^* decreases: when the cost of mismatch is more important, more informative messages are required to ensure trade. While moving from a fully informative equilibrium to less informative ones, the seller surplus is decreasing and the buyer surplus is increasing, while total welfare is decreasing. This is true as long as trade is ensured. If the intervals get wide enough that some buyer types start being excluded, then both buyer and seller surpluses decrease.

3.3 A numerical example

Let us consider an example, where $f(x) = \frac{x^2}{2}$. If we call the optimal partition length x^*, the price charged by the seller is $k - f(\frac{x^*}{2})$. Then x^* is given by the following:

$$x^* = \arg\max_x \frac{1}{x} 2 \int_0^{\frac{x}{2}} (f(\frac{x^*}{2}) - f(\theta)) d\theta \tag{2}$$

This leads to the optimal partition length $x^* = 2\sqrt{\frac{2k}{3}}$, which is increasing in k. For $k \geq 0.375$, $x^* = 1$ and hence the buyer optimal equilibrium is babbling. For example, $k = 0.06$ leads to $x^* = 0.4$. This corresponds to a partition with 3 intervals, 2 of which are of size 0.4 and a final one of size 0.2. There are 3 different partitions that satisfy this, which differ in the location of the uneven interval. One such partition would be the one with boundary points $a_1 = 0.4, a_2 = 0.8$ and $a_3 = 1$.

4 Dynamics of the game: two periods

In this section, a second round of communication and offer is allowed for in case the first period offer is rejected, the buyer still wants to buy only 1 unit and his type is constant over the periods. There is a discount factor δ which makes delay costly. The seller's offer at period 2 is now a function of the period 1 and period 2 messages.

Let us start studying the game from the beginning of period 2. The buyer's strategy in period 2 is to accept the offer if and only if $U(y_2, \theta) - \tau_2 \geq 0$. Hence, the seller's period 2 offer will leave 0 surplus to the threshold types who accept the offer. The seller's strategy in this period is identical to the one period game. Then, when faced with the offer at period 1, the buyer already estimates the seller's posterior in case period 2 is reached as well as the offer she will make.

Remark 1. *At period 2, the largest interval of types that the seller's offer can cover is x^*, which is the optimal interval length in the one period game.*

Now, let us consider period 1, after a message is sent by an interval x_i. In this section, the notation for intervals also denotes the length of that interval. The seller's best response is to make an offer (y_i, t_i) such that at least some types inside this interval accept. I call $z(x_i)$ the interval of types that accept the offer inside an interval x_i at period 1, and $\{\underline{\theta}_i, \overline{\theta_i}\}$ are the threshold types. There may be an interval of types, called $b_i = x_i \setminus z(x_i)$ who reject the offer. After a rejection, the seller knows that the buyer is located in interval i outside of $[\underline{\theta}_i, \overline{\theta}_i]$. If $\underline{\theta}_i = a_i$ or $\overline{\theta}_i = a_{i+1}$, there is a single interval inside which the buyer can be found, which is called b_i. Otherwise, there are 2 separate and disconnected intervals, $b_{i1} = (a_i, \underline{\theta})$ and $b_{i2} = (\overline{\theta}, a_{i+1})$ where the buyer could be found.

The presence of a second period gives the seller the opportunity to better identify the buyer's taste and differentiate among the types over the two periods. For identical period 1 partition intervals, the seller charges a higher price in period 1 of the 2 period game compared to what she charges in the one period game, as she can make a new offer in period 2 in case the first one is rejected. The seller updates her belief about the buyer's type after a rejection. To sum up, the presence of period 2 provides two types of informational benefits to the seller: the shrinking of the interval inside which the buyer can be found, and the possibility of receiving another informative message in period 2.

Lemma 5. *A threshold type at period* 1 *will also be the boundary type in his interval at period* 2 *in the case of rejecting the period* 1 *offer, and hence will get* 0 *surplus in either period.*

Lemma 5 says that the dynamic incentive compatibility of the threshold types is satisfied. Once this holds, the incentive compatibility condition of all the types inside $(\underline{\theta}_i, \overline{\theta}_i)$ is also satisfied, due to the single crossing condition. Hence, when the threshold types are indifferent to accepting an offer in a given period, the types inside the interval are strictly better off doing so. This means the types inside an interval are not affected by the messages of the types in the consecutive intervals. It follows that in every period 1 interval, the types that accept an offer are found to be connected.

Definition 1. *A subdivision happens when a connected interval of types that reject the period* 1 *offer divide into at least* 2 *intervals by sending different messages in period 2.*

The possibility of subdivision implies that the set of possible period 2 partitions is huge. First, we focus on the least informative period 2 partition that ensures trade. For an interval b_i of types that reject the period 1 offer, the least informative partition equilibrium at period 2 which ensures trade is equivalent to the same partition rule as in the one period game. In the last period, the seller is willing to cover at most an interval of x^*. This means if $b_i \geq x^*$, this interval should divide into n intervals of measure x^* and a final interval of measure $b_i - nx^*$, these types would send separate messages. In case $b_i \leq x^*$, the types pool in their messages. Hence, x^* is the least informative partition rule that ensures trade in period 2.

Lemma 6. *If reached, period 2 intervals should be the largest possible ones which ensure trade:*

- *if $b_i \leq x^*$, the types in b_i pool in their period two messages.*
- *if $b_i > x^*$, the types in b_i divide into intervals of length x^* and a final interval of different length by sending separate messages.*

If period 2 is informative, then the types who reject the period 1 offer should send a period 2 message just precise enough to ensure that trade happens with certainty. On the other hand, if period 2 messages are babbling, then the interval of excluded types should be fine enough to ensure trade over the 2 periods. This leads us to conclude that in the buyer-optimal equilibrium of the two period game, trade happens for any type of the buyer either at period 1 or at period 2, due to lemma 6. Hence, there should be no exclusion at period 2.

Lemma 7. *Any equilibrium in which informative messages are sent at period* 1 *and a subdivision happens at period* 2 *is Pareto dominated by another equilibrium with no subdivision and a more informative period* 1 *partition.*

If there are any buyer types who are dividing at period 2 into several intervals, they would have been better off if they had instead sent separate messages at period 1. In the buyer-optimal equilibrium, if some information is provided at period 1, it should be sufficient to ensure that trade will take place over the 2 periods without a subdivision. Hence, equilibria with subdivision can be discarded in the search for the buyer-optimal equilibrium. After the next section, it will be clear that this type of equilibrium is dominated. Together with lemma 6, no subdivision leads to the condition $b_i \leq x^*$ for all i to trade at period 2 without a subdivision.

I will now classify the set of equilibria into two, named equilibria with delay and the no-delay equilibrium. Equilibria in which period 2 is never reached are called *no-delay equilibria*. Among the equilibria with delay, I restrict attention to those in which trade takes place at the end of the two periods and solve for the one that is buyer optimal in that class.[7] Finally, I will select the buyer-optimal equilibrium among all these equilibria.

4.1 Seller's offer given buyer's revelation strategy

Let us now study the different kinds of equilibria that can arise by finding the offer strategy of the seller given the message strategies of the buyer. We restrict attention to equilibria with no subdivision and trade certainty.

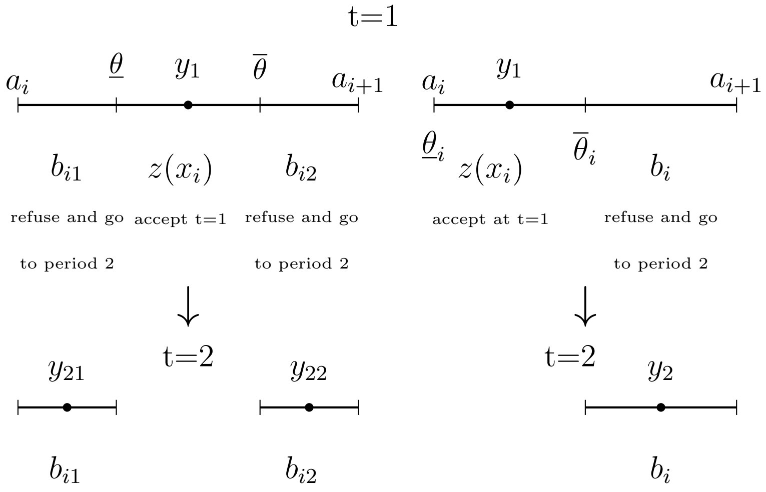
4.1.1 When period 2 is informative

Period 2 is said to be *informative* if the buyer types in a period 1 interval that refuse the offer send at least 2 distinct messages in period 2.

Lemma 8. *When the period 2 strategy of the buyer is informative, then in period 1, $y_i = \frac{a_i + a_{i+1}}{2}$ is a weakly dominant offer for the seller in response to a message m_i, so that the types who refuse the offer will be found in 2 separate and equal intervals.*

[7]It follows from the one period game that any equilibrium in which some types are excluded is Pareto dominated. To see how this applies to 2 period game: if some types are excluded at the end of 2 periods, then it means in period 2 they were pooling an interval which is larger than x^* and x^* is the interval length the seller will cover in that case. Then, these types could have at period 2 separated by sending another message and get an acceptable offer at period 2, without affecting the offer of the other types either in period 1 or 2.

Figure 3: Dynamics of the game



When informative messages are expected at period 2, the seller is weakly better off having the types excluded at period 1 in two separate and equal intervals. The seller's problem is then one of choosing the measure of types to be served at period 1 which I call z, hence at the same time b, the measure of each of the two identical separate intervals excluded in period 1. As we restrict attention to equilibria with no subdivision, b is small enough to be covered at period 2, meaning $b \leq x^*$. The principal's problem can be written as:

$$(z^*, b^*) = \arg\max_{z,b} z(k - f(\frac{z}{2})) + 2\delta b(k - f(\frac{b}{2}))$$

$$\text{s.t. } z + 2b \leq x$$

with $x \leq 3x^*$, the condition for trade to be ensured. The FOC leads to: $k - f(\frac{z}{2}) - zf'(\frac{z}{2}) = \delta(k - f(\frac{b}{2}) - bf'(\frac{b}{2}))$ if $\{z^*, b^*\} > 0$. As $\delta < 1$ and f' is non-decreasing, $b \leq z$ and $z(x) \geq \frac{x}{3}$. The largest period 1 interval such that trade is ensured at the end of period 2 is $x = 3x^*$, which leads to $z(x) = b = x^*$. This implies that one third of the types get the offer at period 1, in other words with probability $\frac{1}{3}$ trade happens at period 1, and with probability $\frac{2}{3}$ it happens at period 2.

Figure 3 provides a demonstration of the case in which period 2 is informative (left) and babbling (right).

4.1.2 When period 2 is babbling

Now I consider the equilibrium in which only the first period messages are informative and second period is babbling. This means all the buyer types who reject the period 1 offer send the same period 2 messages, in other words no information is revealed at period 2.

Lemma 9. *If the buyer's period 2 message is anticipated to be babbling, the seller weakly prefers to make an offer (y_i, τ_i) such that either $\underline{\theta}_i = a_i$ or $\overline{\theta}_i = a_{i+1}$ so that the types who reject the period 1 offer are found connected. This maximizes the seller's profits in case period 2 is reached.*

When the excluded types are found in a single interval, at period 2 the seller knows that the buyer's type is found in b_i. In case the types that refuse the offer are found in 2 separated intervals which I will call b_{i1} and b_{i2}, the seller knows $\theta \in b_{i1} \cup b_{i2}$, so she may find it optimal to cover totally or partially one of the intervals, meaning trade may not occur for a positive measure of types. In case she wants to sell to some types in both intervals, for any y she has to ask for a lower price than she would ask for if they were found connected.[8]

The seller's problem is then:

$$(z^*, b^*) = \arg\max_{z,b} z(k - f(\frac{z}{2})) + \delta b(k - f(\frac{b}{2}))$$

$$\text{s.t. } z + b \leq x$$

for $x \leq 2x^*$, the condition for trade to be ensured. The FOC leads to: $k - f(\frac{z}{2}) - zf'(\frac{z}{2}) = \delta(k - f(\frac{b}{2}) - zf'(\frac{b}{2}))$ when both z^* and b^* are strictly positive, which implies $b \leq z$ and $z(x) \geq \frac{x}{2}$. The maximum value the pooling interval x can take so that there is no exclusion is $2x^*$, which leads to $z(x) = b = x^*$: half of the buyer

[8]The reason she is weakly better off is that there can be some cases in which the seller will be indifferent between having the excluded types in a single interval or in two separated intervals. For example if the measure of buyer types excluded is $b \leq x^*$, then the seller is strictly better off when these types are found in a single interval, as she will serve all of them. In case they are separated, then either he will serve only one interval or has to charge a lower price in order to serve all the types. However, in case $b > x^*$, as the seller will sell only to x^* measure of types at period 2, she only has to make sure that one of the intervals that reject the offer measures at least x^*.

types accept the period 1 offer and the other half accept the period 2 offer. While x increases in the region with delay, the increase in b is higher than the increase in z, until both reach the value x^*. By comparing this with the FOC of the informative equilibrium, it is seen that $x = 2x^*$ in the babbling equilibrium is equivalent to $x = 3x^*$ in the informative equilibrium in terms of the price proposed in both periods, but more types get the offer at period 1 in the informative equilibrium. This provides the intuition for our result on the buyer-optimal equilibrium in which there is expected delay, which will be provided later in this section.

4.1.3 No-delay equilibrium

If the period 1 partition interval sending the message is fine enough, the seller does not find it profitable to exclude any type. This threshold length will be denoted by x^{nd}. One immediate result is $x^{nd} < x^*$, where x^* is the interval covered in the presence of only one period.

Take an equilibrium which is babbling at period two[9], where z denotes the interval of types who get served at period 1 and $(x - z)$ those who move to period 2, the seller's profit is:

$$(k - f(\frac{z}{2}))z + \delta(x - z)(k - f(\frac{x - z}{2}))$$

Taking the derivative with respect to z and replacing $z = x$:

$$k(1 - \delta) - f(\frac{x}{2}) - \frac{x}{2}f'(\frac{x}{2}) \tag{3}$$

As long as this expression is weakly positive, no type is excluded at period 1 and there is no expected delay (the condition is identical when period 2 is informative). As $f(\frac{x}{2}) + \frac{x}{2}f'(\frac{x}{2})$ is increasing in x, the maximum interval length such that there is no delay, x^{nd}, is found by setting the equation (3) equal to zero which leads to:

$$f(\frac{x^{nd}}{2}) + \frac{x^{nd}}{2}f'(\frac{x^{nd}}{2}) = k(1 - \delta)$$

For all $x \leq x^{nd}$, the game has no delay and any type gets served in the first round. Whenever $x_i \leq x^{nd}$ for all i, trade happens without delay.

[9] The result is the same when period 2 is informative. Indeed, the no delay threshold doesn't depend on the period 2 partition structure

The last type of equilibrium is the one which is babbling at period 1 and informative at period 2. This equilibrium is Pareto dominated for the buyer types: if period 2 is informative, then buyer optimality implies the partition intervals at period 2 are of length x^*. However, this means if the period 1 there were intervals of size $2x^*$, then half of the buyer types would get the offer at period 1 at the same price.

Proposition 3. *In any equilibrium of the dynamic game, the buyer's expected surplus is strictly lower than his surplus in the buyer-optimal equilibrium of the one period game.*

In case the one period optimal partition is played in the 2 period game, some types will be excluded at period 1 as the seller would charge a higher price. In order to induce the seller to charge the same price as in the one period game, the buyer's message has to be sufficiently uninformative that there is expected delay, in which case the surplus will again be lower than in the one period game.

4.2 The buyer-optimal equilibrium

Now that we have solved the seller's strategy for different strategies of the buyer, the next step is to solve for the buyer optimal equilibrium. This is the best equilibrium for the buyer from an ex-ante point of view (before he learns his type) as it maximizes the surplus over buyer types, as well as the best equilibrium once period two is reached as it maximizes the expected surplus of the types that may reach period 2[10]. Hence, this is the equilibrium the buyers would choose to coordinate on before learning their type.

By a similar argument as in the single offer game, I will conclude that the presence of a single asymmetric interval does not affect the optimality of the partition rule. The next proposition summarizes the buyer-optimal equilibrium:

Proposition 4. *In the buyer-optimal equilibrium, informative messages are sent only at period 1 and there is a threshold discount factor $\hat{\delta}$ such that:*

[10]The reason we verify that this is also the buyer optimal partition rule in period 2 conditional on being reached is in order to ensure neologism proofness at period 2, in other words that there is not a self signalling set in the beginning of period 2.

1. *for $\delta \geq \hat{\delta}$, partition intervals at period* 1 *are of length* $2x^*$, *in other words the maximum interval that ensures trade.*

    - *half of the types in each interval are served at period* 1 *and the other half at period* 2,
    - *in the case that it is reached, period 2 messages are babbling.*

2. *for $\delta < \hat{\delta}$, the equilibrium has no delay with period 1 intervals of length x^{nd}:*

    - *trade takes place at period 1 with certainty.*

The proof proceeds as follows: by lemma 6 and 7, we know that in equilibrium trade happens for any type of the buyer without a subdivision. The candidate equilibria either have informative messages in both periods, or informative messages in period 1 and babbling ones in period 2. Then, we can see that any informative equilibrium is Pareto dominated by a babbling equilibrium which is more informative at period 1 and has the same prices in both periods with less expected delay. This is because more types are being served at period 1 in the babbling equilibrium compared to informative one. I then conclude that the buyer optimal equilibrium has babbling messages at period 2 in the case that it is reached.

When the discount factor is low enough, the buyer is better off pooling in intervals just fine enough to avoid delay. In this region, the buyer surplus is decreasing in δ, as the partition intervals become finer in order to satisfy the no-delay condition. The seller's surplus in this equilibrium is even higher than in the static game, due to the finer partition intervals with no delay. In this region, as the match between the good and the buyer improves compared to the one period game and there is no delay, efficiency increases.

When $\delta > \hat{\delta}$, the amount of information the buyer has to provide in order to avoid delay is too much, hence he is better off in the equilibrium with delay. In this equilibrium, the buyer and seller surpluses are both lower than the one period game. The seller surplus is lower compared to the one with no delay and also lower than in the static game, as the buyer's period 1 message is less informative resulting in delay.

The buyer is better off when information revelation is restricted to period 1, as this disciplines the seller and increases the possibility that trade takes place earlier. The seller is willing to sell to more buyer types at period 1 when period 2 messages are babbling. When the period 1 messages are the only information the seller will acquire, she is less likely to risk delaying trade by asking for a higher price, compared to the case in which she expects more information to arrive at period 2. The buyer is always worse off from the introduction of more periods, as he could always replicate the two period outcome in one period without delay.

The presence of 2 periods does not always benefit the seller, as the buyer's strategy in return is modified. The seller is worse off in the two period game if there is expected delay: information transmission does not improve compared to the single period game, and in addition there is expected delay. However, in the no-delay equilibrium, information transmission and overall welfare are both higher. Hence, the presence of a second round increases the gains from trade if and only if delay is costly enough that the buyer prefers revealing more information in order to avoid it.

5 Extension to T Periods and Infinite Horizon

This section discusses extending the game first to T periods, and then to an infinite horizon in which rounds of communication and offer continue until trade takes place. A result that carries on from the two period game is the no-delay equilibrium interval length x^{nd}, which is independent of the number of periods that are introduced. The intuition is simple: the marginal type in the no delay equilibrium would have been served at period 2 in case excluded at period 1. In other words, the presence of a third, fourth or more periods does not modify the features of the no delay equilibrium.

Now let us consider a T periods game. At any period t that may be reached, after the buyer's message m_t, the seller chooses an offer (y_t, τ_t). If trade takes place at period t, the buyer's utility is $\delta^{t-1}(U(\theta, y) - \tau)$ and the seller's utility is $\delta^{t-1}\tau$. The seller's information at period t and her offer are a function of the history, which consists of the messages sent up to and including period t, in other words the public history $h_t = \{m_s\}_{s=0}^{t}$.

An interval at period t will be denoted by x_t and z_t will denote the interval of types that accept and b_t the types that reject the offer in period t (which may consist of a single or two separate intervals depending on the seller's offer).

Two kinds of equilibria which ensure trade without subdivision are identified and explored. The first type is the **informative equilibrium** in which the messages are informative in each period. As in the two period game, if future periods are informative, the seller's weakly optimal strategy is to offer the good located in the middle of the interval, $y_i = \frac{a_i+a_{i+1}}{2}$. Continuing in this fashion, from a period 1 point of view, an initial interval will have divided into 2^{t-1} possible intervals z_t at any period t in the informative equilibrium.

The second type of equilibrium is the **babbling** one in which the only informative messages are sent at period 1 and the types which reject the offer pool together at every future period. In this equilibrium, by lemma 9, a single interval z_t is served at each period and the types which are excluded are found connected. Hence, from a period 1 point of view, there is a single interval z_t served and a single message sent at each period corresponding to a period 1 interval inside which the buyer type is found.

The intervals evolve by getting finer over the periods by $x_{t+1} = x_t - z_t$ in the babbling equilibrium and $x_{t+1} = \frac{x_t - z_t}{2}$ in the informative one. Given the first period partition and the future revelation strategy of the buyer, the seller determines at period 1 the future offers $(y_t, \tau_t)_{t=0}^{T}$.

Lemma 10. *In a game which can last at most T periods, $z_1 \geq z_2 \geq .... \geq z_T$ and $\tau_1 \leq \tau_2 \leq \tau_3 .... \leq \tau_T$.*

Proof. I look at the seller's strategy in a T periods game in which trade is ensured. Upon receiving a message from an interval of length x at period 1 and given the expected revelation strategy, the seller's problem is to maximize profits by determining z_t that will be covered at each t, and hence $\hat{T} \leq T$ which is the last period in which trade can take place. The discounted expected surplus of the seller when future periods are informative is:

$$\max_{z_t} \Pi = \sum_{t=1}^{T} 2^{t-1} \delta^{t-1} z_t (k - f(\frac{z_t}{2}))$$

subject to:

$$\sum_{t=1}^{T} 2^{t-1} z_t \leq x \tag{4}$$

which will hold as an equality when trade is ensured. The marginal gain from a threshold type in any interval z_t is:

$$\delta^{t-1}2^{t-1}[k - f(\frac{z_t}{2}) - \frac{z_t}{2}f'(\frac{z_t}{2})]$$

where the expression 2^{t-1} is normalized as it also appears in the budget constraint in equation (4). Then, as $\delta < 1$, this condition implies that $z_1 \geq z_2 \geq z_3 ... \geq z_{\hat{T}}$, with $\hat{T}$ denoting the latest period which can be reached. Period $\hat{T}$ is the first period in which $x_{\hat{T}} = z_{\hat{T}} \leq x^{nd}$, in which case trade will take place with certainty at period T.

Second, in case all future periods $t > 1$ are babbling, the expected seller surplus given a first period pooling interval x is:

$$\max_{z_t} \Pi = \sum_{t=1}^{T} \delta^{t-1} z_t(k - f(\frac{z_t}{2}))$$

subject to:

$$\sum_{t=1}^{T} z_t \leq x$$

Again the marginal profit from a threshold type in any interval z_t is:

$$\delta^{t-1}[k - f(\frac{z_t}{2}) - \frac{z_t}{2}f'(\frac{z_t}{2})]$$

which leads to $z_1 \geq z_2 ... \geq z_{\hat{T}}$ where $\hat{T}$ denotes the latest period which can be reached.

Now, realize that the derivatives normalized by the coefficients in the constraints are identical for the informative and babbling cases, however more delay is expected in the informative equilibrium for identical prices. This result is similar to the two period game.

Given that the terminal period T is the one in which the condition $f(\frac{z_T}{2}) + \frac{z_T}{2}f'(\frac{z_T}{2}) = k(1-\delta)$ is satisfied, in previous periods t, we should have: $f(\frac{z_t}{2}) + \frac{z_t}{2}f'(\frac{z_t}{2}) = k(1 - \delta^{T-t+1})$. □

The seller's problem is solved for an interval of buyer types at period 1 and their revelation strategy in the future. An important observation is that in any equilibrium with delay the price proposed by the seller weakly increases over time. The seller's optimality requires the benefit of serving a marginal buyer type now or in the next period to be equivalent. However, due to the discount factor this is only possible if the price is weakly increasing. Another way to interpret this is that over time, the seller acquires more precise information about the buyer and proposes a higher price.

In any period in which all the partition intervals which may be present satisfy $x \leq x^{nd}$, trade will take place without delay. As x^{nd} is the largest possible interval such that trade happens with no delay, then for any pooling interval of length $x > x^{nd}$, the seller will serve at least x^{nd} measure of types, meaning $z_t \geq x^{nd}$, but there may be delay if the initial interval is large enough. Hence, in any game, whenever the pooling interval is larger than x^{nd}, a positive measure of types will be excluded: $b_t > 0$.

Proposition 5. *Any equilibrium strategy which has informative messages in any period* $t > 1$ *is weakly dominated for any buyer type and strictly dominated for the seller by another equilibrium which is informative only at period* 1 *and babbling in future periods.*

If we take any informative equilibrium which can last up to T periods, there exists a corresponding babbling one which also lasts at most T periods with identical interval lengths z_{it} for all $t \in \{1, T\}$ and a finer period 1 partition rule (meaning at period 1, $x^b < x^i$ where b and i respectively denote babbling and informative equilibria) which leads to higher buyer surplus due to less delay at identical prices. In other words, any informative equilibrium can be transformed into one in which all information is revealed at period 1 rather than over several periods and provides a weakly higher surplus to any type of buyer. This equilibrium is strictly preferred by the seller as it leads to lower expected delay. This means the buyer optimal equilibrium is the one in which the buyer types send informative messages only at period 1 and further information is revealed only through the rejection of offers, hence the shrinking of the partition intervals. This equilibrium has less delay but prices in each period identical to the informative one. Finally, in the buyer optimal equilibrium, no informative messages are sent in periods $t > 1$.

After solving the seller's problem for a given period 1 interval, I conclude that for any T, the buyer optimal equilibrium either has no-delay or has delay with the minimum amount of information revelation that ensures trade and no information revelation for $t > 1$. This is analogous to the result in the 2 periods game.

Proposition 6. *The buyer optimal equilibrium in the T period and infinite horizon game is either the one with no delay or the least informative one that ensures trade. As the number of periods increases, the equilibrium with delay becomes less profitable whereas the no delay equilibrium surplus remains constant for the buyer.*

It is optimal to send information at once and babbling messages afterwards. The benefit to the buyer of sending informative messages is due to the higher probability that trade can take place earlier. Hence, the coarser the current information structure (larger the pooling intervals), the more incentives there are for the buyer types to separate by sending informative messages. This means, as the intervals become finer each period, sending babbling messages in one period and informative ones in the future cannot be optimal.

6 Discussion: Uncertainty on the Value of k

An interesting question to ask is how the information revelation strategy and outcome would be modified if uncertainty about k (common value of the good) is introduced, especially whether truthful revelation by the high value buyer could be achieved. Consider a one period game with two types. The type of the buyer now consists of two variables: (k, θ) where k can take a high ($\overline{k}$) or a low ($\underline{k}$) value. In this case, some high valuation types will always have an incentive to pool with the low valuation ones on the vertical dimension as long as the low types are not excluded by the seller. The reason is the high valuation types located at the boundaries of any interval would always prefer pooling with the low types in order to secure strictly positive rents.

To sum up, in case of two possible values of k, either the low types are excluded by the seller's offer, or there is pooling and the high types are better off pooling with the low types. There is no equilibrium in which $\overline{k}$ types separate themselves from $\underline{k}$ types, which leads to conclude that no information can be provided on the vertical attribute.

7 Conclusion

This paper deals with the question of how a buyer could optimally communicate his information when that information determines both the seller's offer and price. This is an important problem which has not previously been explored.

To address this problem, first I studied a one round communication and offer game. It is shown that the buyer surplus is maximized when information revelation is just sufficient to ensure that the seller does not want to exclude any buyer type from trade. Hence, the buyer preferred equilibrium is the least informative one in which trade is ensured. This result shows that the incentive to keep information rents is an explanation to why buyers may want to be less transparent about their preferences.

Next, it is shown in a two period game that the buyer is better off restricting information revelation to period 1. This decreases the seller's incentive to ask for a higher price as well as the expected delay. When the discount factor is sufficiently low, the buyer is willing to reveal sufficient information in order to avoid delay, and efficiency increases. When the discount factor is sufficiently high, avoiding delay requires too much information revelation. In that case, there is expected delay and efficiency is lower compared to the one period game. This result shows that a longer bargaining horizon may or may not improve the efficiency of trade, but the buyer is always hurt from the introduction of a second period. Finally, I show that the main results persist when extended to any game length, including infinite horizon.

Appendix A

Proof of Lemma 1:

Proof. Suppose there exists a non-monotone partition such that there are θ_1 and θ_3 who send the same message m, and θ_2 sending a different message, m' and all types accepting the offer they receive. If $(y(m), \tau(m))$ is the offer induced by message m, then the following is true for θ_1 and θ_3:

$$U(\theta_i, y(m)) - \tau(m) \geq U(\theta_i, y(m')) - \tau(m') \tag{5}$$

and $U(\theta_2, y(m')) - \tau(m') \geq U(\theta_2, y(m)) - \tau(m)$. Assume WLOG that $y(m) \in (\theta_2, \theta_3)$. Then we have $U(\theta_2, y(m)) > U(\theta_1, y(m))$ by the single crossing condition. Then $U(\theta_2, y(m')) - \tau(m') \geq U(\theta_2, y(m)) - \tau(m) > U(\theta_1, y(m)) - \tau(m)$.

First, assume $y(m') > y(m)$. Then, $U(\theta_3, y(m')) > U(\theta_3, y(m))$. As $U(\theta_2, y(m')) - \tau(m') \geq U(\theta_2, y(m)) - \tau(m)$ and $U(\theta_2, y(m)) > U(\theta_2, y(m'))$, we conclude $\tau(m') < \tau(m)$. Then it should be the case that $U(\theta_3, y(m')) - \tau(m') > U(\theta_3, y(m)) - \tau(m)$, which means θ_3 would initially prefer sending the message m'.

Second, if $y(m') < y(m)$, then $U(\theta_1, y(m')) > U(\theta_1, y(m))$ and $U(\theta_2, y(m')) - \tau(m') > U(\theta_2, y(m)) - \tau(m)$ which results in $U(\theta_2, y(m')) - U(\theta_2, y(m)) > \tau(m') - \tau(m)$, and due to $U_{11} > 0$, $U(\theta_1, y(m')) - U(\theta_1, y(m)) > U(\theta_2, y(m')) - U(\theta_2, y(m)) > \tau(m') - \tau(m)$, which finally results in $U(\theta_1, y(m')) - \tau(m') > U(\theta_1, y(m)) - \tau(m)$. □

Proof of Proposition 1**:**

Proof. First, consider the fully revealing equilibrium in which each type sends message $m(\theta)$ and gets 0 surplus from the offer. Now consider another equilibrium in which a positive measure of buyer types, $\theta \in (\underline{\theta}, \overline{\theta})$ pool together where $0 < \overline{\theta} - \underline{\theta} < x^*$ and send the same message. By assumption 1 this interval is covered by the seller's offer, and the buyer surplus is strictly positive. It concludes that the buyer surplus is strictly positive in any equilibrium in which some buyer types pool. □

Proof of Lemma 2**:**

Proof. The threshold types in any interval i get 0 surplus: $k - f(\overline{\theta}_i - y_i) = k - f(y_i - \underline{\theta}_i) = \tau_i$ and by symmetry, $y_i = \frac{\overline{\theta}_i + \underline{\theta}_i}{2}$. As $U_{12} > 0$, $U(\theta, y_i) - \tau_i > 0$ for all $\theta \in (\underline{\theta}, \overline{\theta})$, which leads to $\int_{\underline{\theta}_i}^{\overline{\theta}_i} (U(\theta, y_i) - \tau_i) d\theta > 0$. □

Proof of Lemma 3**:**

Proof. Let us take an interval i inside which all buyer types are accepting an offer. As the boundary types get 0 surplus, $\tau_i = k - f(y_i - a_i) = k - f(a_{i+1} - y_i)$ so that $U(\theta, y_i) - \tau_i = f(y_i - a_i) - f(\theta - y_i)$ gives the surplus of type a_i. Then, the expected utility of the types that belong to an interval i is:

$$\int_{a_i}^{a_{i+1}} [f(y_i - a_i) - f(\theta - y_i)]d\theta \tag{6}$$

as $y_i = \frac{a_i + a_{i+1}}{2}$ and f is symmetric around 0, this simplifies to $2[(y_i - a_i)f(y_i - a_i) - \int_{a_i}^{y_i} f(y_i - \theta)d\theta]$. Then, replacing for simplicity $a_i = 0$ and $a_{i+1} - a_i = x$ we get $f(y_i - a_i) = f(\frac{x}{2})$, and equation (6) simplifies to:

$$2\int_0^{\frac{x}{2}} (f(\frac{x}{2}) - f(\theta))d\theta \tag{7}$$

If we take the expected buyer surplus, by dividing equation (7) by the mass of buyer types, x:

$$x^* = \arg\max_x f(\frac{x}{2}) - \frac{2}{x}\int_0^{\frac{x}{2}} f(\theta)d\theta$$

the derivative of this equation with respect to x is:

$$\frac{1}{2}f'(\frac{x}{2}) - \frac{f(\frac{x}{2})}{2x} + \frac{2}{x^2}\int_0^{\frac{x}{2}} f(\theta)d\theta \tag{8}$$

then, as $f'(\frac{x}{2}) \geq \frac{f(\frac{x}{2})}{x}$ (given f' nondecreasing) and $\int_0^{\frac{x}{2}} f(\theta)d\theta > 0$, equation (8) is always positive for any x, so the expected buyer surplus is increasing in the interval length as long as no type is being excluded. When the number of intervals decreases subject to the constraint that there is no exclusion, the total buyer surplus always increases.

It is seen that the expected buyer surplus increases as the partition intervals get wider until reaching the length x^*, after which the seller would start excluding some types. The price $\tau_i = k - f(\frac{x}{2})$ is decreasing as x increases, in other words as the covered interval gets wider, until reaching x^*. The seller surplus on the other hand is decreasing in the length of the partition intervals, as the seller is induced to ask for a lower price when the buyer reveals less information. Hence, the seller's profit increases as the number of intervals increases leading to a more informative equilibrium. □

Proof of Lemma (4)**:**

Proof. When there are 2 intervals which are both finer than x^*, the buyer surplus increases as one enlarges at the expense of the other one until reaching the threshold length x^*. This means, if x^* is the optimal partition interval length, then there can be only one interval which is finer while all others are of length x^*. Consider 2 adjacent intervals having lengths x' and x'' such that $x' + x'' \leq x^*$, sending respectively messages m' and m''. Then, by moving to another equilibrium in which these types pool to send message m', the expected buyer surplus increases due to lemma (3). Any equilibrium partition in which there are 2 intervals with lengths $x' < x^*$, $x'' < x^*$ and $x' + x'' = g > x^*$ can be improved into an equilibrium in which $x' = x^*$ and $x'' = g - x^*$. Consider 2 intervals, x' and x'', such that $x' + x'' = \overline{x}$. The expected utility over the two intervals is written as:

$$x' f(\frac{x'}{2}) - 2\int_{s=0}^{\frac{x'}{2}} f(s)ds + x'' f(\frac{x''}{2}) - 2\int_{s=0}^{\frac{x''}{2}} f(s)ds$$
$$\text{s.t. } x' + x'' = \overline{x}$$

The derivative with respect to each term is $\frac{x'}{2} f'(\frac{x'}{2})$ and $\frac{x''}{2} f'(\frac{x''}{2})$, which means once $x' > x''$, it is optimal to have it increase to x^* as the first derivatives are non decreasing. Hence, either of the two intervals should reach maximal length. Finally, after showing that in the buyer-optimal equilibrium there is no exclusion I can conclude that x^* is the optimal partition rule. □

Proof of Lemma 5

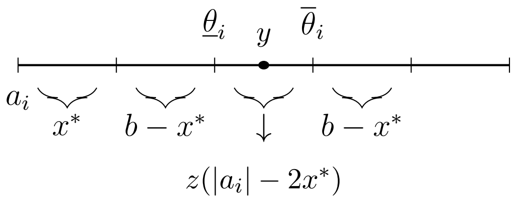
Proof. At period 1, interval i sends a message and types $\theta \in (\underline{\theta}_i, \overline{\theta}_i)$ accept the offer. In case the period 1 offer is rejected, at period 2, $\underline{\theta}_i$ is the higher boundary type in the interval $(a_i, \underline{\theta}_i)$, and $\overline{\theta}_i$ is the lower boundary type in the interval $(\overline{\theta}_i, a_{i+1})$. The seller's offer at period 2 will never leave any positive surplus to the boundary types. Then, as these types are indifferent to accepting the offer in either period, they are given 0 surplus at the period 1 offer as well. □

Proof of Lemma 6**:**

Proof. First, I will show that this strategy is optimal once $t = 2$ is reached. If period 2 is reached, the surplus maximizing partition rule for the types in b_i has intervals of length x^* as in the one period game. So, this strategy is optimal once period 2 is reached.

Next let us check whether this strategy is also optimal from an ex-ante point of view, by verifying whether the buyer could be better off under another period 2

Figure 4: interval i



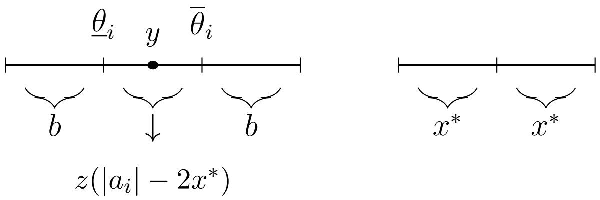
partition rule $x' < x^*$ from the viewpoint of period 1. When $b_i < x'$, this strategy is the same as the partition rule x^* as the intervals b_i will not separate in any case. In case $x' < b_i < 2x'$, this implies that b_i divides into 2 intervals of x' and $b_i - x' < x'$. As this increases the seller's period 2 profit, the only change (in case there is) in the seller's strategy would be charging higher and excluding more types at period 1. In addition, in case the game moves to period 2, a more informative partition leads to lower expected utility for the buyer given that no type is excluded in either case, by lemma 3. Hence, the expected surplus of buyer types that are served in period 1 and also at period 2 are both lower, and the possibility of reaching period 2 is higher leading to lower surplus due to delay. So, more informative partitions at period 2 can not improve buyer surplus.

Finally, let us verify that committing to a less informative period 2 strategy is not profitable either. A less informative strategy means that at $t = 2$, if $b_i > x^*$, the types pool in their messages. As the seller's offer covers at most an x^* measure of types for any given interval, there is $b_i - x^* > 0$ of types for whom trade will not happen. Assume WLOG that these types are located on the left end of the initial interval. Then, by separating and sending a different message at period 2, they wouldn't affect the allocation of the interval x^* of types, plus the expected surplus of these separating types would be positive. In case these types are found in the middle, the types located outside and excluded would have been better off if they sent 2 separate messages and received an offer, again not affecting the surplus of the other types. □

Proof of Lemma 7:

Proof. Figure 5 demonstrates 2 intervals i which subdivide at period 2. Look at the intervals of measure x^*. When these two intervals are pooling together with interval a_i, they are excluded at period 1 and get the period 2 offer, hence the

Figure 5: interval i without subdivision



expected surplus over these intervals is:

$$2\delta \int_0^{\frac{x}{2}} (f(\frac{x}{2}) - f(\theta))d\theta \tag{9}$$

Now, consider another partition in which these intervals were found consecutively on the right side of interval i and separate from the interval i and adopt a strategy of pooling together in period 1 and babbling at period 2. By the seller's problem, an interval of x^* would get the offer at period 1 and the other interval of x^* at period 2. Then, the surplus over the interval $2x^*$ will be:

$$(1+\delta)2 \int_0^{\frac{x}{2}} (f(\frac{x}{2}) - f(\theta))d\theta \tag{10}$$

where (10) > (9). From the seller's problem, we know that $z_i = z(x_i - 2x^*)$ which is not influenced by the change in the strategy of these two subintervals. In addition, all the types inside the intervals x^* are weakly better off as well: half of the types get the same offer now at period 1 instead of period 2. Hence, this new partition which has 2 intervals of lengths $x_i - 2x^*$ and $2x^*$ and dominates the previous partition equilibrium for the buyer types. To conclude, any equilibrium partition with subdivision is dominated by another one which is more informative at period 1 and has no subdivision at period 2. □

Proof of Lemma 8**:**

Proof. Let us look at a situation when the seller excludes at period 1 a total measure c of types in an interval i, which can be a single or 2 separated intervals. Assume that there are 2 separated intervals of lengths b_{i1} and b_{i2}, with $b_{i1} + b_{i2} = c$. Restrict attention to $b_{i1}, b_{i2} \leq x^*$ so all the types can be served with no subdivision. As we know that as long as $b_{i1}, b_{i2} \leq x^*$, the buyer optimal equilibrium which satisfies no exclusion has no subdivision. The seller's expected profit when t=2 is informative, so that the types in b_{i1} and b_{i2} send separate messages at $t = 2$ and both intervals are covered, will be:

$$b_{i1}(k - f(\frac{b_{i1}}{2})) + (c - b_{i1})(k - f(\frac{c - b_{i1}}{2}))$$

FOC wrt b_{i1}:

$$f(\frac{c - b_{i1}}{2}) - f(\frac{b_{i1}}{2}) + \frac{c - b_{i1}}{2} f'(\frac{c - b_{i1}}{2}) \frac{b_{i1}}{2} f'(\frac{b_{i1}}{2}) = 0$$

$c = 2b_{i1}$ satisfies the equality, so $b^*_{i1} = b^*_{i2} = \frac{c}{2}$. If the seller is going to exclude a measure of types c, it is weakly better that these types are found in two separated intervals of equal lengths when next period will be informative. □

Proof of Lemma 9:

Proof. I restrict attention to the case in which there are no informative messages at period 2. If $\underline{\theta}_i > a_i$ and $\overline{\theta}_i < a_{i+1}$, given that the game moves to period 2, the buyer may be found in either one of the 2 separated intervals: $\theta \in b_{i1} = (\overline{\theta}_i, a_{i+1})$ or $\theta \in b_{i2} = (a_i, \underline{\theta}_i)$ where $b_{i1} + b_{i2} = x_i - z_i$. In case $\underline{\theta}_i = a_i$ or $\overline{\theta}_i = a_{i+1}$, the buyer is known to be in a specific interval, $(\overline{\theta}, a_{i+1})$ or $(a_i, \underline{\theta}))$ where $\underline{\theta} - a_i = a_{i+1} - \overline{\theta} = x_i - z_i$. First possibility is that $x_i - z_i \leq x^*$. This means at period 2, the seller would serve the whole interval if they were found in a single interval by offering $(\frac{b_i}{2}, k - f(\frac{b_i}{2}))$, which gives profits $k - f(\frac{b_i}{2})$. In case of 2 separated intervals, the seller either has to charge lower if he wants to include all types or exclude some types in case she charges the same price. If she wants to sell to all types, her profit is $(k - f(\frac{a_i + a_{i+1}}{2})) * (x_i - z_i)$ or otherwise some types are excluded which gives profits lower than $k - f(\frac{b_i}{2})$. Second, in case $x_i - z_i > x^*$ and there is a single interval, the seller will charge $(k - f(\frac{x^*}{2}))$ and x^* measure of types will accept, giving her profits of $\frac{x^*}{x_i - z_i}(k - f(\frac{x^*}{2}))$ whereas when there are 2 separated intervals, she either chooses to sell to one of the intervals b_{i1} or b_{i2} which lower profits in case $max\{b_{i1}, b_{i2}\} < x^*$, or she has to offer a good located in $(\underline{\theta}_i, \overline{\theta}_i)$ and hence sell to $x < x^*$ measure of types. So she always gets weakly lower profits than when the types are found in a single interval. Hence, we conclude that the seller is weakly better off when the excluded types are found in a single interval. □

Proof of Proposition 3:

Proof. We know by proposition 2 that x^* is the length of an interval in the buyer optimal equilibrium of the one period game. In the 2 periods game, given z_i for all i the intervals getting the offer at $t = 1$ and b_i for all i the intervals which would get the offer at $t = 2$ inside each interval i, the expected surplus of the buyer types using equation 7 is:

$$\left[\sum_{i=1}^{n_z} 2\int_0^{f(\frac{z_i}{2})} (f(\frac{z_i}{2}) - f(\theta))d\theta + \sum_{i=1}^{n_b} 2\delta \int_0^{f(\frac{b_i}{2})} (f(\frac{b_i}{2}) - f(\theta))d\theta\right] \tag{11}$$

where $\sum_{i=1}^{n_z} z_i + \sum_{i=1}^{n_b} b_i \leq 1$. Hence, $\sum_{i=1}^{n_z} z_i \leq 1$. This surplus would be maximized if $z_i = x^*$ for all i and $\sum_{i=1}^{n_z} z_i = 1$: only if no type were excluded and the game ended at period 1 with certainty, the surplus would be equivalent to the surplus of the one period game. However, when x^* is a pooling interval at period 1, then $z(x) < x^*$ whether period 2 is informative or babbling, and $z(x) = x^*$ is achieved only when $x = 2x^*$ in the babbling equilibrium and when $x = 3x^*$ in the informative equilibrium, in which case a positive measure of types are excluded at period 1, implying there exists $b_i > 0$. Then, $\sum_{i=1}^{n_z} z_i < 1$, and due to the discount factor $\delta < 1$, the buyer surplus is always lower than in the static game.

Finally, if every type is served at period 1, then largest possible interval length is x^{nd}, which also gives lower surplus given $x^{nd} < x^*$. Then I conclude that the buyer surplus in the 2 period game is always lower than the buyer optimal equilibrium of the one period game. This generalizes to more than 2 periods, given that the seller has more incentives to exclude types when there are more periods left to play. □

Proof of proposition 4

Proof. This is proven by characterizing all the candidate equilibria. By using lemma 6 and 7, I restrict attention to equilibria where trade happens for all types without a subdivision. This leaves the following candidate equilibria:

1. period 1 is informative and:
   - period 2 is informative
   - period 2 is babbling

2. period 1 is babbling and period 2 is informative.

For $x = z + b$, we can write the buyer's expected surplus for the case with period 2 babbling as:

$$2\int_0^{\frac{z}{2}}(f(\frac{z}{2}) - f(\theta))d\theta + 2\int_0^{\frac{b}{2}}(f(\frac{b}{2}) - f(\theta))d\theta \qquad (12)$$

then, dividing by the mass of buyer types:

$$f(\frac{z}{2}) - \frac{2}{z}\int_0^{\frac{z}{2}} f(\theta)d\theta + \delta(f(\frac{b}{2}) - \frac{2}{b}\int_0^{\frac{b}{2}} f(\theta)d\theta)$$

We have shown in the one period game that the surplus in period 1 and 2 are respectively convex in z and b. However, the period 2 surplus is discounted. Hence this surplus function has 2 candidate maximum points, one when x takes its maximum value subject to no-delay $x = x^{nd}$ ($b = 0$) and the other one is when x takes its maximum value subject to ensuring trade over two periods. In the region $x > x^{nd}$, the buyer surplus is increasing in the interval length. Hence, the second candidate maximum is the least informative equilibrium which ensures trade. If it is optimal for some types to wait until period 2, then it is optimal that as many as possible types be served at period 2. Among this type of equilibria, the babbling one is shown to be the best one as it results in less delay for the same prices. When $\delta < \hat{\delta}$, the no-delay equilibrium dominates any equilibria with delay, and for $\delta \geq \hat{\delta}$ the babbling equilibrium is the dominating one.

Period 1 informative

subcase 1: period 2 babbling

Restricted to the case where $a \leq 2x^*$, the condition for the market to be covered over the two periods given no subdivision at period 2. As x^* is the highest interval that the seller covers, then $2x^*$ is the upper bound on the period 1 partition. The partition rule that gives the highest surplus to the buyer when playing an informative strategy at period 1 and babbling at period 2 is:

The total surplus per an interval is written as:

$$z(f(\frac{z}{2}) - 2\int_0^{\frac{z}{2}} f(s)ds) + \delta z[f(\frac{z}{2}) - 2\int_0^{\frac{z}{2}} f(s)ds] \qquad (13)$$

The average buyer surplus is respectively $(f(\frac{z}{2}-2\int_0^{\frac{z}{2}} f(s)ds$ and $\delta[f(\frac{z}{2})-2\int_0^{\frac{z}{2}} f(s)ds]$ and from (3) we know that either expression is increasing in the length of interval. The second derivative with respect to z is:

$$\frac{1}{4}[f''(\frac{z}{2} - \frac{1}{z}f'(\frac{z}{2})] + \frac{1}{z^2}[f(\frac{z}{2}) - \frac{2}{z}\int_0^{\frac{z}{2}} f(s)ds] \tag{14}$$

The first expression is positive given that f' is non decreasing. The same is true for the second expression: given $\int_0^{\frac{z}{2}} f(s)ds \leq \frac{z}{2}f(\frac{z}{2})$, multiplying the two by $\frac{2}{z}$ we get that this term is also positive. The same holds for the derivative with respect to b. This implies that either $b = 0$ and z is the no-delay maximum length or both z and b take their maximum value (this is optimal in case δ is not very low). From the seller's problem, we know that for $x \leq x^{nd}$, $b = 0$ and for $x > x^{nd}$, b is increasing. Then, either δ is low enough hence it is optimal for the buyer to have $b = 0$ or in case $b > 0$, it is optimal to extend the pooling interval as much as possible subject to trade taking place.

- the partition rule $a^* = 2x^*$
- half of the types in each interval accepting the offer at period 1: $z = x^*$
- the other half refusing and moving on to period 2: $b = x^*$

when $\delta < \hat{\delta}$

- no-delay partition interval x^{nd}

subcase 2: period 2 informative

In order for trade to be ensured without any subdivision, the partition intervals should satisfy $a \leq 3x^*$ (in period 1 at most x^* types can be served, and at period 2, there will be 2 separate intervals of x^*.) If $a > 3x^*$, then as $z \leq x^*$, $b = \frac{a-z}{2} > x^*$, so some types would be excluded. The equilibrium surplus of the buyer types in a partition a when period 2 is informative is:

$$z(f(\frac{z}{2}) - 2\int_0^{\frac{z}{2}} f(s)ds) + 2\delta z[f(\frac{z}{2}) - 2\int_0^{\frac{z}{2}} f(s)ds] \tag{15}$$

Again, similar to the babbling equilibrium, either $a = x^{nd}$ or z and b both take maximum value. This happens when a take its maximum value which is $3x^*$, in which case $z(a) = x^*$ and $b = x^*$.

Then, the partition rule that gives the highest buyer surplus when both periods are informative is, for $\delta > \hat{\delta}$:

- $a^* = 3x^*$
- $z(a) = x^*$
- $b_1 = b_2 = x^*$

comparing case 1 and case 2:

In the buyer-optimal equilibrium with delay, it is seen that $z = b = x^*$ for both types of equilibria. However, in the informative one there are 2 intervals b for one z. Hence, more types get the offer at the same price in period 2 in the informative equilibrium compared to the babbling one. Then, due to the discount factor, it concludes that the buyer surplus is higher in the babbling equilibrium.

Babbling at period 1

There is a single interval of length 1 at period 1 and the seller offers y, τ such that x^* interval of types accept. Among the equilibria in which $t = 1$ is babbling, by using lemma 6 which ensures that trade should take place with certainty, the buyer optimal one is informative at t=2 and has the partition rule x^*. Let us show that this equilibrium is dominated. Assume that $y = \frac{1}{2}$ and that there are types $\theta \in (0, 2x^*)$ who are excluded at period 1 and in period 2 they separate into two intervals of x^*. Now consider another equilibrium in which the types in this interval send an informative message at period 1, and babble at period 2. This doesn't affect the surplus of other types. From the seller's problem, one of the intervals $(0, x^*)$ or $(x^*, 2x^*)$ will be served at $t = 1$ and the other one at $t = 2$. This means at the same price, half of the types now get the offer at period 1 instead of period 2. In fact, the surplus of some buyer types is identical as before, and for some it is strictly higher. Finally, babbling at period 1 cannot be the buyer-optimal strategy. Now we conclude that the surplus maximizing equilibrium strategy for the buyer over all candidate equilibria is the one in which period 1 is informative and period 2 is babbling in case it is reached. □

Proof of proposition 5

Proof. Take an informative equilibrium which lasts at most until period T. Then, consider a babbling equilibrium with the same z_t for all t which also lasts at most T periods. This means ${z_t}^b = {z_t}^i$ for all t where b denotes babbling, and i denotes informative equilibrium. The surplus from the informative equilibrium is:

$$\sum_{t=1}^{T} \delta^{t-1} 2^{t-1} z_t (k - f(\frac{z_t}{2}))$$

subject to:

$$\sum_{t=1}^{T} 2^{t-1} z_t = a^i$$

where a^i is the first period interval length in the informative case. The FOC with respect to z_t gives . The surplus from the babbling equilibrium is:

$$\sum_{t=1}^{T} \delta^{t-1} z_t (k - f(\frac{z_t}{2}))$$

subject to:

$$\sum_{t=1}^{T} z_t = a^b$$

The FOC with respect to z_t is also positive and decreasing in t. Hence the surplus would be higher when the intervals z_t which are served earlier are higher in number.

Next I will show that the period 1 intervals are finer in the babbling equilibrium: $\sum_{t=1}^{T} z_t < \sum_{t=1}^{T} 2^{t-1} z_t$, meaning $a^b < a^i$ and $n^b = \frac{1}{a^b} > \frac{1}{a^i} = n^i$: there are more intervals at period 1 in the babbling equilibrium than in the informative one. Now the surplus in the informative case can be written as:

$$n_i \sum_{t=1}^{T} \delta^{t-1} 2^{t-1} z_t (k - f(\frac{z_t}{2}))$$

subject to:

$$n_i \sum_{t=1}^{T} 2^{t-1} z_t = 1$$

The surplus for the babbling case:

$$n_b \sum_{t=1}^{T} \delta^{t-1} z_t (k - f(\frac{z_t}{2}))$$

subject to:

$$n_b \sum_{t=1}^{T} z_t = 1$$

where $n^i < n^b$. As z_t's fixed by the seller's problem are identical and decreasing in t, the surplus functions is higher when earlier z_t's have higher coefficients. If $\sum_{t=1}^{T} n^b z_t = \sum_{t=1}^{T} n^i 2^{t-1} z_t$, then there is t^* such that for $t < t^*$, $n^b > n^i 2^{t-1}$ and for $t > t^*$, $n^b < n^i 2^{t-1}$. As the surplus function is higher when the z_t's for lower t have higher coefficients, I conclude that the surplus in babbling equilibrium is higher. Hence, in the babbling equilibrium a higher proportion of types are getting earlier offers compared to the informative equilibrium at the same price, and there are more period 1 partition intervals implying a more informative equilibrium. The proportion of types accepting offers in each period in the informative equilibrium to the babbling one is increasing in t. So, any informative equilibrium is dominated by a babbling equilibrium in terms of the expected buyer surplus. Finally, in the buyer optimal equilibria, the only informative messages are sent at period 1. □

Proof of proposition 6

Proof. Following is the normalized buyer surplus for a period 1 partition x where $z + b = x$ in a game with $T = 2$:

$$\frac{2}{z}\int_0^{\frac{z}{2}} (f(\frac{z}{2}) - f(\theta))d\theta + \frac{2}{b}\int_0^{\frac{b}{2}} (f(\frac{b}{2}) - f(\theta))d\theta \tag{16}$$

We know that both terms are increasing respectively in z and b, however the marginal gain from increasing z is higher due to the lack of discounting. But z cannot increase above x^{nd}, as some types of the buyer will then be excluded which is costly due to the discount factor δ. But in the region when $b > 0$, the surplus is increasing in b and hence setting the initial interval x as large as possible is indeed optimal as this decreases the price the seller asks for at period 1 as well as the price she asks for at period 2. If it is optimal for the buyer surplus that some types get excluded, then the loss in surplus due to δ is less important than the increase in surplus due to lower prices. In order to extend the result, observe that the tradeoff between period 2 and 3 is also identical: once some types are excluded at $t = 2$, it is optimal to have the interval of excluded types as large as possible as this decreases the price in both periods 2 and 3. Hence, we get the result that for low values of δ, the no-delay equilibrium and for higher values of δ, the minimum informative equilibrium which ensures trade is the best one. The difference is that now as T increases, the surplus from the equilibrium with delay decreases while the surplus from the no-delay equilibrium remains constant. For any T, the best equilibrium with delay has the minimum amount of information revelation that ensures trade over T periods, in which the seller will serve an interval x^* of types each period. This means when $T > \frac{1}{x^*}$, the buyer-optimal equilibrium with delay will be the babbling one. $\square$

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