Selecting the wisdom of an expert

Sinem Hidir*†

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Abstract

This paper explores the interaction between uncertainty about bias and the endogenous information acquisition by an expert before strategic communication. We consider an expert who is privately informed about his bias as well as about whether he is informed, in addition can also engage in observable and costly information acquisition. In this setup, information acquisition simultaneously serves the purposes of getting informed and increasing credibility before communicating through cheap talk to a decision maker. We define the *signaling* and the *intrinsic value* of information and find the conditions under which separation in the information acquisition behavior can arise. Communication is most precise with an initially uninformed expert at an intermediary cost value, and that the optimal level of expertise for the DM is non monotone in the cost of information acquisition. The overall welfare is also non-monotone in cost, and specifically it increases as cost increases to enable more separation. When covert information acquisition is considered, there is less wasteful investment but also less precise communication compared to the overt case.

Keywords: information acquisition; cheap talk; communication; signaling; credibility.

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*University of Warwick. email: s.hidir@warwick.ac.uk.

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1 Introduction

In markets and organizations, decision makers often rely on the advice of experts. However, it is difficult to assess the credibility of experts as well as how informed they are. The literature on strategic information transmission pioneered by the seminal paper Crawford and Sobel (1982) has made two most common assumptions: that the bias of the expert is known and that the expert has access to information for free. These assumptions have been relaxed in different setups. Uncertainty about bias has been explored in the literature on the reputation of experts in repeated cheap talk initiated by Sobel (1985) and Morris (2001). Costly information acquisition before cheap talk has been first introduced by Austen-Smith (1994) and recently Sobel (2013) described this area as an open question[1]. In order to study the interaction between bias and information acquisition, in this paper both of these assumptions are relaxed: the expert may or may not be biased, and he may or may not be ex-ante informed. In addition, costly acquisition of information is observable by the decision maker which makes it a signal. These ingredients make the problem rich in that there is costly signaling and cheap talk.

To understand the relevance of this question, consider a politician consulting a policy advisor about the affects of pollution on the occurrence of lung disease. The advisor may be getting kickbacks from a polluting industry and hence biased. Maybe the advisor has no conflict of interest, but already has access to information due to his experience. Finally, it is possible that the advisor, although unbiased, isn't informed on this specific issue and may pay a team of researchers, which is costly. This implies even a biased advisor could mimic this behavior only to increase the credibility of his advice. There are instances when experts incur costs in order to enhance their credibility while they are being backed by third parties. For instance, Andrew Wakefield, a former gastroenterologist and medical researcher, was found guilty of misconduct in his research paper in order to support a claim that the MMR vaccine was linked to autism and bowel disease. It was later discovered that he had been paid by lawyers who were trying to prove that the vaccine was unsafe.(Deer (2006))

Similar issue arises in the case of a division manager reporting to the CEO of his company suggesting to launch a new product. The manager may have a bias towards launching the product due to his career concerns. In addition, he may also know about the potential of this product due to his expertise or he may ask his team to carry out a market research, the cost being the time spent by them. Motivated by these examples where experts can overtly spend money, time or resources on information acquisition, the question this paper asks is how does bias interact with endogenous acquisition of information when advisers communicate to a decision maker? Given that the decision maker cannot differentiate between aligned and biased experts, what type of expert is optimal for the decision maker in terms of how informed the expert is?

In this setup, which types of experts invest in information acquisition determines the decision maker's equilibrium belief hence the credibility of his communication. There are two separate

[1] Some recent examples include Pei (2015), Argenziano et al. (2016) and Deimen and Szalay (2016)

motives for information acquisition depending on the type of expert: the *intrinsic value of information* and its *signaling value*. I focus on the case where the *signaling value* of information is lower than its *intrinsic value*, so that separation in information acquisition behavior is possible. [2] The expert cares about his credibility only insofar as it affects the decision maker's (DM) action: reputation is instrumental and not intrinsic, as in Sobel (1985) and Morris (2001) who consider repeated cheap talk. By endogenizing the information acquisition process, a similar effect arises without dynamics, when the expert chooses whether to invest in information acquisition or not before communication takes place. This serves as a credibility building stage for the biased or informed unbiased types who may invest wastefully in order to pool with the unbiased types who genuinely wants to get informed. Somewhat reminiscent of *bad reputation*, as defined by Ely and Valimaki (2003), even an unbiased informed expert ends up investing in information in order to increase his credibility. Surprisingly, the ex-ante uninformed expert who later becomes informed always communicates more precisely than the expert ex-ante informed who cannot separate himself from the biased type when the state of the world is high.

In the model considered there is a state of the world $\omega \in \{0, 1\}$, while the DM's action space is $y \in [0, 1]$. There is a known probability β with which the expert is biased, and a known probability α that he is already perfectly informed about the state of the world. The decision maker and the unbiased type of expert want the action to match the state of the world, while the biased type wants the highest action regardless of the state. That the biased expert has state independent preferences enables us to perfectly distinguish the intrinsic value of information from its signaling value. The incentives of the biased expert and the unbiased expert informed with signal 1 are identical as both want the highest action to be taken. This is key to many of our interesting results. The expert has a binary decision of investing in information which is observable but its outcome is private. As there is a probability that the expert is already informed, that an expert does not invest in information does not reveal that he is uninformed, neither that he is biased.

We solve for equilibria that arise as a function of the cost of information acquisition, focusing on the most informative ones. Separating equilibrium is one in which the only type who invests is the uninformed unbiased type. This equilibrium exists for some cost values and only under certain parameter restrictions, specifically it always exists for some cost values when $\rho > 0.5$ and fails to exist if the *signaling value* of information is higher than its *intrinsic value*. When the cost of information acquisition is low enough, biased types and informed unbiased types who have high signal also invest and due to the "contamination" by the biased types, communication by the unbiased type is less credible in this equilibrium.

In order for the separating equilibrium to realise, it must be that there is a sufficiently high probability that the expert is already informed, as the proportion of informed types make the communication upon no investment more credible. In other words, they help the biased type avoid too much **prejudice** by the DM when he send high message without investing. The

[2] This is the case which gives the richest set of equilibria. The other case is also studied in the appendix.

DM's payoff is highest when just sufficiently many types are informed so that separation in investment behavior arises. However, conditional on the separating equilibrium arising, the DM's welfare decreases in the proportion of informed unbiased experts, as these types cannot separate themselves from the biased type whenever the state is 1. This is the intuition for why the decision maker achieves a higher payoff if matched with an expert who is initially uninformed: the uninformed type can separate himself from the biased type and communicate perfectly in the separating equilibrium while the informed type is always pooled with the biased type whenever the state is 1.

The game also admits mixed strategy equilibria in which the biased and the unbiased informed (1) types are indifferent and play different probabilistic investment strategies. As the cost of information acquisition goes down, proportionately more biased types than informed unbiased types should be investing in order to keep the indifference condition of these types. There is a unique mixed strategy equilibrium preferred by both expert and the DM. This equilibrium is dominated in terms of expert payoff by pure strategy equilibria, while the decision maker's payoff is highest in this mixed strategy equilibrium. However total welfare is higher in the separating equilibrium.

After characterizing the equilibria for different cost regions, we go on to make welfare comparisons. We show that less wasteful investment and more informative communication more than compensate for a higher information acquisition cost when the cost moves from the pooling region to the separating one. Hence, the total welfare is non monotone in cost: it decreases as cost increases in one equilibrium, but if it increases from the pooling region until enabling separation, then the total welfare increases.

An important implication is that if the decision maker could pick the expert type α, it would be non monotone in cost. When the cost is so low that pooling will arise, then any α is going to give the same payoff to the DM, hence expertise doesn't matter. When cost is higher, the decision maker prefers α to be just high enough that the separating equilibrium realises but not higher. The intuition is as follows: as long as the separating equilibrium is played, the uninformed types who invest will perfectly communicate while the initially informed types with high signal are pooled with biased types. But then, it is optimal for there to be no more than just sufficient proportion of these informed types as there is loss in communication whenever the state is high. When the cost increases further, the separating equilibrium arises even when there is no possibility that the expert is informed, at which point it is optimal to have an expert who is definitely uninformed. Finally, when the cost is so high that no type will invest, a perfectly informed expert is the optimal one.

An interesting takeaway is that the decision maker doesn't necessarily benefit from a more informed expert, and may even be hurt as this type doesn't have enough incentives to separate himself from a biased type.

We then extend the setting to covert information acquisition where investment choice is not observable. There is no wasteful investment as informed types can now pool with the unbiased types without incurring any cost, but also separation between biased and unbiased types is not possible when the state is high. The overall welfare is higher in the overt case in case the separating equilibrium arises, due to more precise communication. Otherwise, when pooling in investment arises in the overt case, the covert case leads to higher welfare due to less wasteful investment and identical precision of communication.

2 Literature

This paper relates to several strands of the signaling and cheap talk literature. First, it relates to the literature in cheap talk with uncertain bias and reputational concerns introduced by Sobel (1985) and Morris (2001) who consider repeated cheap talk. Outside of the communication literature, Ely and Valimaki (2003) consider a long run player facing short run players who takes a payoff relevant action and highlight the distortional consequences today of the incentives to avoid bad reputations in the future. Contrary to our setting, these arise when there are dynamics or when there is an intrinsic value of reputation of the experts. In this paper, as in Morris, the expert cares about the DM's belief about his type only insofar as his action can be influenced. However, while Morris considers a two period setup in which the DM updates the prior about the agent's type as a function of his message and the realized outcome, in our setting the decision is taken only once but the observable information acquisition affects the credibility of the expert. Hence, in my setup the messages are never distorted as decision making takes place only once, but distortion takes place in form of wasteful investment in information acquisition. Meng (2015) considers the setup of Morris (2001) and endogenizes the precision of the expert's information, where investment is not a signal as it is not observable, but it allows for better decision making. She finds that reputation building enhances the incentives to invest in information for both types in the first period.

Ottaviani and Sorensen and Ottaviani and Sorensen (2006a) show that reputation concerns may lead to inefficient herding when experts bias their recommendation in order to appear more informed. Morgan and Stocken (2003) consider strategic communication as in Crawford and Sobel (1982) with uncertainty about the expert's type and show that truthful communication cannot arise even for the unbiased type of analyst whenever the state of the world is sufficiently high. The distinctive feature of my setting compared to the literature is that there is neither dynamics involved which is present in the literature on reputation, nor an intrinsic value of reputation, yet similar effects arise due to the endogenous information acquisition and signaling incentives.

This paper also relates to the literature on costly information acquisition in cheap talk. The first example is Austen-Smith (1994) who considers the transmission of costly information where information acquisition leads to perfect information and is not observable. As the expert can prove being informed but can feign ignorance, low types can pool with uninformed types to achieve a higher outcome, which improves communication for higher types. Since Austen-Smith (1994), only recently there has been few work on strategic communication with endogenous information acquisition, such as Pei (2015) and Argenziano et al. (2016). One finding is that the expert truthfully transmits all the information he acquires, in other words the expert doesn't acquire information that he will not transmit. Esö and Szalay (2010) consider a game in which the expert has no bias and endogenous information acquisition, and show that restricting the message space can induce the sender to acquire information more often. Deimen and Szalay (2016) also consider endogenous information when a biased expert can choose on which issues to gather information and show that communication dominates delegation. Suurmond et al. (2004) consider the effect of reputation in a delegation setup with information acquisition and an unbiased agent and show that reputational concerns may be good as they incentivize the expert to acquire information when he doesn't know his ability. While when the expert has private information about his ability, he may take inefficient actions in order to mimic an efficient type. Dur and Swank (2005) also study endogenous information acquisition and study the tradeoff between incentive to acquire information versus precision of communication as a function of expert's bias. There is no uncertainty about the bias hence no signaling motive for the expert in these setups.

The wasteful investment in my setup is reminiscent of *burning money* first identified by aus where cheap talk is not the only way to communicate but senders may also incur loss in utility in order to enhance their communication. However, contrary to pure money burning, the aligned type of experts in our setup do value information per se. [3]

3 Model

There is a decision maker (DM, she) and an expert. There is a state of the world $\omega \in \{0, 1\}$ and a commonly known prior $Pr(\omega = 1) = \rho$. At the beginning of the game, the expert learns his two dimensional private type. First, he is either aligned with commonly known probability $(1 - \beta)$ and shares the same utility function as the DM, which is $-(\omega - y)^2$ or he is biased with probability β and always wants the highest possible action, with utility function $-(1 - y)^2$ [4]. The type of expert is denoted $\theta \in \{u, b\}$ which respectively mean unbiased or biased. Second, with commonly known probability α the expert is perfectly informed about the state of the world ex-ante, otherwise he has the same prior as the DM. We refer to this parameter as the **quality** of the expert. [5] This relates to the experience that the expert has in the past over similar issues. The decision maker does not observe whether the expert is biased or not and

[3] An important assumption I make is that information acquisition is observable. Argenziano et al. (2016) and Deimen and Szalay (2016) also assume that investment in information is observable. Plus, in my setup the expert has no incentive to hide his information acquisition, on the contrary he wants to show that he acquires information.

[4] This type of assumption about the expert type and utility function is also present in Morris (2001)

[5] Bhattacharya et al. (2018) also make this assumption on the possibility of expert being perfectly informed or not, when looking at optimal composition of expert panels without information acquisition.

neither whether the expert is informed or not, but knows α and β. In addition, the agent can optionally make an investment in information by incurring a cost c and get a private signal that perfectly reveals the state. The investment is observable but its outcome is not.[6] Indeed, any type of expert could invest in information, including a biased or an informed unbiased one in which case investment only serves the purpose of signaling, which we will call **wasteful** investment. As the expert may already be perfectly informed with probability α, that he doesn't invest doesn't mean that he is uninformed, neither that he is biased. Finally, communication happens through cheap talk following the investment decision. Below are the stages of the game:

0. the state of the world, ω, and the expert's type is realised.
1. the expert decides whether to acquire a perfectly revealing signal by incurring c, a decision $x \in [0, 1]$.
2. the expert sends a message $m \in M$ to the DM.
3. the DM takes an action, $y \in [0, 1]$.

The decision maker interprets the expert's message as a function of her posterior about the expert's type, which depends on her prior and the equilibrium information acquisition strategies of different types of experts. As the biased type of expert always wants the highest action irrespective of the state, we only have 1 type of biased expert and whether he is informed or not has no relevance.[7] It could equivalently be assumed that the biased type of expert is never informed. We can then summarize the types of experts at the beginning of the game into four:

1. biased
2. unbiased and uninformed
3. unbiased and informed with signal (1)
4. unbiased and informed with signal (0)

Call $\Phi \in \{0, 1, \varnothing\}$ the information structure of the expert at the communication stage as a result of his initial information and the information acquisition process. The communication strategy of the expert denoted by $m(\Phi, \theta)$ is pure as each type of expert will always strictly prefer one of the three messages. The equilibrium concept is Perfect Bayesian Equilibrium (PBE). A strategy profile $\langle x, m, y\rangle$ along with the DM's posterior $\mu(x, m) = Pr(\omega = 1|x, m)$ forms a PBE if and only if:

- The DM's action maximizes her payoff given her posterior:

$$y^*(x, m) = \arg\max_{y} -\mu(x, m)(1 - y)^2 - (1 - \mu(x, m))y^2$$

[6]That the information acquisition process is **observable** to the decision maker but not its **outcome** is also assumed in Fischer and Stocken (2009), Argenziola et al. (2016), Deimenn and Szalay (2017) who also consider information acquisition before cheap talk in different setups with known bias of the sender.

[7]This would be different if the biased type's payoff was not state independent, such as in Morgan and Stocken (2003)

- The expert's strategy, (x, m) maximizes his payoff given the DM's belief updating and decision process.
- The DM's posterior $\mu(x, m)$ is consistent with the expert types' equilibrium strategies and the prior ρ.

The DM's optimal action is $y^*(x, m) = \mu(x, m)$.

Communication strategies:

In the communication stage following investment, it is certain that the expert is informed. We focus on equilibria in which after investment there are at most two messages in equilibrium, $m^i \in \{0, 1\}$. To see this, realize that the unbiased expert either knows $\omega = 0$ and wants the lowest possible action or knows $\omega = 1$ and wants the highest possible action. The biased expert wants the highest possible action regardless of the state, hence strictly prefers $m = 1$.

In the communication stage following no investment, there are four possible types of experts. First, the informed expert with signal 0 who would strictly prefer to send the message that induces the lowest action hence will send $m = 0$. Second, the biased and unbiased informed (1) expert both want to send the message that induces the highest action hence will send $m = 1$. Finally, the unbiased uninformed expert who shares the DM's preferences. We will restrict attention to equilibria in which an empty message is available, $m = \varnothing$, which means "I am not informed" and will induce $y = \rho$.[8]. Then, following $x = 0$, the set of messages is $m^n \in \{0, 1, \varnothing\}$.

4 Equilibrium Analysis

There exist 3 types of pure strategy equilibria and a continuum of mixed strategy equilibria depending on the cost of information acquisition. I define "*pooling*" and "*separating*" in this setup in terms of the investment decision of the two groups of expert types defined as:

1. biased type and unbiased informed (1) type
2. unbiased uninformed type.

We will focus on parameter values such that separation in the investment decision of the two groups exist for some cost values.

Let us now define the **signaling value** and the **intrinsic value** of information. The signaling value of information is the gain in payoff for group 1 types from investing and sending message $m = 1$, compared to the case when they don't invest and the decision maker infers that the message is coming from a group 1 type. This is given by $-(1 - \mu(1, 1))^2 + (1 - \mu(0, 1))^2$ which is the incremental value due to investment, given these types send message 1 whether they invest or not. The intrinsic value of information is the gain in payoff for the uninformed unbiased

[8]As it becomes clear later, the biased expert can always induce a higher posterior by sending $m = 1$ as opposed to any other message

(group 2) types from getting informed when their message is taken at face value, compared to not having information and sending $m = \varnothing$. The value of information for the DM and the unbiased uninformed type of expert are identical and given by $(\rho - \rho^2)$: without any meaningful communication, the DM optimally chooses $y^* = \rho$ which provides a payoff of $-(\rho - \rho^2)$ for him as well as for the unbiased agent [9]. This is the maximum possible gain in payoff for the unbiased type from getting informed, when no biased type is willing to invest. The value of information for the unbiased type and the DM is maximized when $\rho = 0.5$, in other words when uncertainty is highest.

As we know that $\mu(0,1) > \rho$, the signaling value can at most take the value $(1-\rho)^2$ which arises when $\alpha = 0$[10]. The equilibrium which is the most informative one among the pure strategy equilibria is the **separating equilibrium**, in which the only types who invest are the uninformed unbiased types. This equilibrium can exist only for some cost values and under certain parameter conditions: when the signaling value is lower than the intrinsic value of information. Then, for $\rho > 0.5$, $(1-\rho)^2 < (\rho - \rho^2)$ so there always exists a region of cost values for which the separating equilibrium arises and we will assume this is the case. Indeed, under this condition, even for $\alpha = 0$, there is a region in which separating equilibrium arises. [11]

As this is a game of signaling followed by cheap talk, there is multiplicity of equilibria. There always exists an equilibrium in which no one acquires information and the DM interprets any message as a babbling one. Other than this, depending on the cost, there exist equilibria with information acquisition. Whenever there is investment, we will focus on the most informative type of equilibrium which dominates those equilibria without information acquisition. When there exists multiple informative equilibria we make use of the Intuitive Criterion (due to Cho and Kreps (1987)) in order to choose the reasonable out of equilibrium belief.

Lemma 1. *The equilibria in which the unbiased informed (0) type sends $m = 0$ without investing and the DM chooses $y^* = 0$ are the Pareto Optimal ones.*

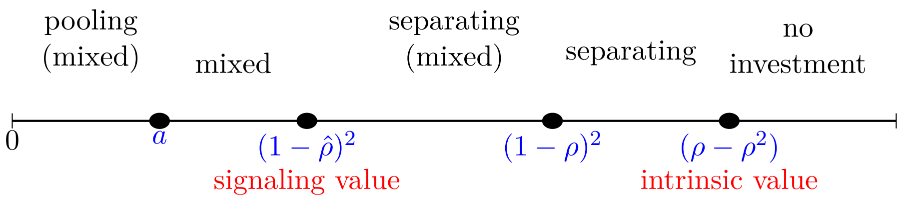
Proof. The unbiased informed (0) type's payoff is maximized when $y = 0$ is chosen by the DM and there is no other type that could benefit from sending this message, given the other possible messages. For the biased and informed (1) types, this message leads to the lowest possible payoff of -1 and they are better off sending any other message. For the unbiased uninformed type who has not invested, sending $m = \varnothing$ upon which the DM chooses $y = \rho$ leads to a better payoff than sending this message. Given that, the DM infers that $m = 0$ can only come from an unbiased informed type, $\mu(0,0) = 0$ and will indeed choose $y = 0$. Finally, the informed 0 expert achieves maximal payoff under this strategy and can only be worse off by sending the same message as another type or by incurring the cost of investment. □

[9] To see this, realise that the expected payoff when $\mu(0,\varnothing)$ and $y = \rho$ is given by: $-\rho(1-\rho)^2 - (1-\rho)\rho^2$ which leads to $-(\rho - \rho^2)$

[10] This is because without investment, the biased and informed (1) experts can at worse get the outcome $y = \rho$

[11] The main results do not depend on this assumption and we also consider the other case, with $(1-\rho)^2 > \rho - \rho^2$ in the appendix.

Figure 1: Equilibria as a function of cost, for given α and β



$\hat{\rho} = \mu(0,1)$ in the separating equilibrium.
$\tilde{\rho} = \mu(1,1)$ in the pooling equilibrium.

We will restrict attention to equilibria in which communication by the unbiased informed 0 type is perfectly informative. Hence, from now on, we will only deal with the equilibrium behaviors of the remaining types of experts. In all the equilibria we focus on the unbiased type of expert always sends a truthful message: whenever uninformed, this type sends $m = \varnothing$ and whenever informed he sends $m = \omega$, as this expert never has incentives to lie about the state. The only uncertainty about this type of expert is about whether he will invest or not.

As the biased and unbiased informed 1 (group 1) types share the same payoff function, they have the same information acquisition strategies in strict Nash Equilibria and always send the message $m = 1$ regardless of their investment strategy. That these types have the same incentives is key to many interesting results, mainly that an uninformed expert who acquires information later communicates more precisely than an expert who is already informed. It also leads to interesting mixed strategy equilibria, as although these types have the same payoff, their message has different meaning for the decision maker: the unbiased type is communicating truthfully while the biased type sends an uninformative message. Hence, the DM would be better off if she could distinguish between these types which can only arise in certain mixed strategy equilibria. Let's list the set of possible equilibria with information acquisition that may arise:

1. **Separating equilibrium:** Only the unbiased uninformed type invests upon which the DM takes communication at face value. The group 1 types do not invest and send $m = 1$ resulting in $y = \hat{p} > \rho$.

2. **Pooling equilibrium:** Both groups invest. The unbiased type sends $m = \omega$ while the biased type sends $m = 1$. Upon $m = 1$, the DM chooses $y = \tilde{p} > \hat{p}$.

3. **Mixed strategy (semi-pooling) equilibria:** The group one types play different and probabilistic investment strategies in a way that keeps these types indifferent. As cost decreases, the portion of biased types who invest increases compared to unbiased types.

The figure shows the region for different equilibria under this assumption as a function of the cost of information acquisition.

When $c \geq \rho - \rho^2$, the unique equilibrium has no investment in information. Hence, we focus on the more interesting region $c < \rho - \rho^2$ for the next part. Specifically, most focus will be on the separating equilibrium which is the most informative one for the DM.

Separating Equilibrium

The separating equilibrium is the one in which the only type that acquires information is the uninformed unbiased type who truthfully communicates, hence information acquisition is efficient. We use efficient investment to mean the investment made only by the types who will use that information. Upon communication of $m = 1$ without investment, the DM's belief $\mu(0, 1)$ is given by:

$$\hat{p} = \frac{\alpha(1-\beta)\rho + \beta\rho}{\alpha(1-\beta)\rho + \beta}$$

as the DM infers that it either comes from a biased agent (with probability β) or from an unbiased informed (1) agent (with probability $\alpha(1-\beta)\rho$), she chooses $y = \hat{p}$. The payoffs to both the biased and unbiased informed expert types are $-(1-\hat{p})^2$. Upon deviating to invest, these types would induce the DM to choose $y = 1$ and obtain a payoff of 0, meaning $(1-\hat{p})^2$ is the signaling value of investment. Hence, the condition for this deviation not to be profitable for these types is:

$$c \geq (1-\hat{p})^2 = \left[\frac{\beta(1-\rho)}{\alpha(1-\beta)\rho + \beta}\right]^2 \tag{1}$$

For the unbiased uninformed expert that invests, the payoff in case of deviating to not invest is $-(\rho - \rho^2) < 0$ as in that case he would send $m = \varnothing$ and the DM would choose $y = \rho$ while upon investment he obtains a payoff of 0 as the DM will treat his message as truthful in either state. Then, this expert type prefers to invest if and only if $\rho - \rho^2 \geq c$. Hence, the cost values for which the separating equilibrium exists is:

$$(1-\hat{p})^2 \leq c \leq \rho - \rho^2 \tag{2}$$

The best equilibrium payoff for this type is that the unbiased informed 1 type doesn't invest either, so that $\hat{p}$ takes a higher value. The separating region exists if and only if:

$$\rho[\alpha(1-\beta)\rho + \beta]^2 - \beta^2(1-\rho) > 0$$

which can be satisfied for ρ and α large enough and specifically, it is always satisfied when $\rho \geq 0.5$ even for $\alpha = 0$ which is the case we focus on. As we have $\frac{\partial(1-\hat{p})}{\partial\beta} > 0$ and $\frac{\partial(1-\hat{p})}{\partial\alpha} < 0$, this equilibrium is more likely to exist for low β and high α. When β is high and α is low, the types that do not invest are more likely to be biased ones so $\hat{p}$ is low, making the payoff of the biased types from not investing lower, hence they will have more incentives to invest. For the uninformed type to be willing to invest, the condition $c \leq \rho - \rho^2$ is easier to satisfy the closer ρ is to 0.5, in other words when the intrinsic value of information is sufficiently high. Hence, ρ should be high enough to increase the non investment value for the biased types, but not too much so that investment is sufficiently profitable for the unbiased uninformed types.

Now, consider the equilibria in the region $c < (1-\hat{p})^2$. We know there cannot be a separating equilibrium as the biased type would have an incentive to deviate and invest if the DM anticipated the investment to come from an unbiased type. However, it cannot be the case that just below this threshold, the group 1 types invest with probability 1 either, as in that equilibrium $\mu(1,1) < 1$ and the biased and informed (1) types would not find it profitable to invest. Then, in this region there can only exist mixed strategy equilibria. Although mixed strategy equilibria are not restricted to this region, they are the unique equilibria only in this region.

Mixed strategy (semi-pooling) equilibria

A mixed strategy equilibrium in this setting is defined as one in which the two types in group 1 play different, possibly probabilistic investment strategies. There exists mixed strategy equilibria for $c \in (k, (1-\rho)^2]$. [12] Specifically, for $c \in [(1-\hat{p})^2 - (1-\tilde{p})^2, (1-\hat{p})^2]$, mixed strategy equilibria are the only equilibria. Outside of this region, mixed strategy equilibria co-exist with pure strategy (separating or pooling in investment) equilibria and do survive the Intuitive Criterion.

In any mixed strategy equilibrium, the indifference condition of group 1 types is satisfied. The biased sender invests with probability σ and the unbiased informed (1) type invests with probability γ, while both these types always send $m = 1$. We know that when these types are indifferent, the unbiased uninformed type strictly prefers to invest. The DM's posteriors are: $\mu(1,1) = \frac{(1-\beta)(\alpha+\gamma(1-\alpha))\rho+\beta\sigma\rho}{(1-\beta)(\alpha+\gamma(1-\alpha))\rho+\beta\sigma} > \hat{p}$ and $\mu(0,1) = \frac{(1-\beta)\alpha(1-\gamma)\rho+\beta(1-\sigma)\rho}{(1-\beta)\alpha(1-\gamma)\rho+\beta(1-\sigma)}$.

The indifference condition of group 1 types implies $c = (1-\mu(0,1))^2 - (1-\mu(1,1))^2$, which leads to:

$$c = \left[\frac{\beta(1-\sigma)(1-\rho)}{(1-\beta)\alpha(1-\gamma)\rho+\beta(1-\sigma)}\right]^2 - \left[\frac{\beta\sigma(1-\rho)}{(1-\beta)(\alpha+\gamma(1-\alpha))\rho+\beta\sigma}\right]^2 \quad (3)$$

The first expression on the right hand side of 3 is decreasing and the second one is is increasing in σ for a given γ. We see that σ and γ are complements and for a given c, and multiple pairs of $\{\gamma, \sigma\}$ satisfy the condition.

Finally, it is easy to show that the unbiased uninformed type strictly prefers to continue investing.

In the region $[(1-\hat{p})^2 - (1-\tilde{p})^2, (1-\hat{p})^2]$, mixed strategy equilibria are the only equilibria

[12] $(1-\rho)^2$ is the highest cost level at which it is possible to have a mixed strategy equilibrium, as above this value it is impossible for biased (and informed (1)) types to invest, given that they prefer the prior which gives them $-(1-\rho)^2$ to incurring the cost and inducing the decision maker to choose the highest action 1. k is defined as the lowest value of cost for which there exists mixed strategy equilibria.

while outside this region, mixed strategy equilibria co-exist with pure strategy equilibria.

Specifically, in the separating region $\geq (1-\hat{p})^2 \leq c \leq (1-\rho)^2$, there exists mixed strategy equilibria in which $\mu(0,1) < \hat{p}$. As group 1 types strictly prefer not to invest when $\mu(1,1) = 1$, then $\mu(0,1)$ must decrease in order to make these types indifferent between investing or not. To have $\mu(0,1) < \hat{p}$ we need $\gamma > \sigma$: more unbiased informed types should invest in proportion than biased types. Once $c \geq (1-\rho)^2$, there exists no mixed strategy equilibria, as even if $\mu(1,1) = 1$, the group 1 types do not find it profitable to invest. Second, in the pooling region $c < (1-\hat{p})^2 - (1-\tilde{p})^2$, mixed strategy equilibria should have $\gamma\alpha + (1-\alpha) < \sigma$. Indeed, the composition of types that invest and those that do not invest should be such that as cost goes up, investment becomes relatively less attractive so that indifference between investing or not is satisfied at higher costs. Hence, as c increases, $\frac{\mu(1,1)}{\mu(0,1)}$ should be increasing: communication after investment should be more credible. The interesting feature of this type of equilibria is that although the types inside group 1 share the same payoff function, their strategies lead to different belief for the decision maker hence determine the incentives of these types themselves.

Lemma 2. *In the region $(1-\hat{p})^2 < c < (1-\rho^2)$ where mixed and separating equilibria co-exist, there exists a unique mixed strategy equilibrium at any c which maximizes both the DM and expert's welfare and has $\gamma > 0$ and $\sigma = 0$.*

Proof. The expected payoff over expert types in a mixed strategy equilibrium is:

$$-(\beta + (1-\beta)\alpha\rho)(1-\mu(0,1))^2 - (1-\beta)(1-\alpha)\rho(1-\mu(1,1))^2 - (1-\beta)(1-\alpha)c$$

The above is found by using the fact that the biased and informed (1) types of experts are indifferent between investing or not, and taking their payoffs as they didn't invest. This simplifies, if we replace $c = (1-\mu(0,1))^2 - (1-\mu(1,1))^2$, to:

$$-[(1-\beta)(\alpha\rho + 1 - \alpha) + \beta](1-\mu(0,1))^2 + (1-\beta)(1-\alpha)(1-\mu(1,1))^2$$

From the above, we can see that the expert preferred mixed strategy equilibrium has $\sigma = 0$, and $\gamma > 0$. The DM's payoff given σ and γ is:

$$-(\beta(1-\sigma) + (1-\beta)\alpha\rho(1-\gamma))(\mu(0,1) - \mu(0,1)^2) - ((1-\beta)(\alpha\gamma + 1 - \alpha) + \beta\sigma)(\mu(1,1) - \mu(1,1)^2)$$

which simplifies to:

$$-\beta(1-\rho)[(1-\sigma)\mu(0,1) + \sigma\mu(1,1)]$$

At a given cost c, the DM optimal mixed strategy equilibrium is the one in which $\sigma = 0$ and

$\gamma > 0$ such that:

$$c = (1 - \mu(0,1))^2 \tag{4}$$

Then, at any c, there is a unique welfare maximizing mixed strategy equilibrium with $\sigma = 0$ and $\gamma > 0$. □

Perfect communication by the unbiased types: There exists an equilibrium in the set of mixed strategy equilibria in which all of the unbiased types invest. In this equilibrium, the informed (1) unbiased type invests with probability one while the biased type does not invest with probability one. We call this a mixed strategy equilibrium as the group 1 types play different investment strategies, although pure. This equilibrium coexists with the separating equilibrium exactly at the cost value $c = (1-\rho)^2$. In this equilibrium, we have $\mu(1,1) = 1$ and $\mu(0,1) = \rho$. Hence, a group 1 type is indifferent between investing or not if and only if: $(1-\rho)^2 = c$.[13] Finally, as $c \leq (\rho - \rho^2)$, *unbiased uninformed* types are indeed investing.[14] Indeed, this is also the highest cost level for any mixed strategy equilibrium to exist, with $\sigma = 0$ and $\gamma = 1$. It is easy to see that this equilibrium is the preferred one for the decision maker, as he can perfectly identify the biased types plus the unbiased types are all investing. However, a very specific condition on either c or ρ needs to be satisfied for this equilibrium to realize.

Proposition 1.

- *In the region $(1-\hat{p})^2 \leq c \leq (1-\rho)^2$, mixed strategy and separating equilibrium coexist.*
- *In $(1-\rho)^2 < c < (\rho - \rho^2)$ the separating equilibrium is the unique equilibrium.*

Proof. I find the separating equilibrium to be the unique pure strategy equilibrium in this region. First, there cannot be any equilibrium in this region in which unbiased informed (1) and biased types invest with probability one, as even when $\mu(1,1) = 1$, in this region the group 1 types not find it profitable to invest as the cost is too high. Then, the only pure strategy equilibrium that could arise is the no investment equilibrium in which even the uninformed unbiased expert doesn't invest. For some out of equilibrium beliefs, this equilibrium can arise, as discussed below.

Assume that the DM believes any type except the informed (0) unbiased one is equally likely to invest, then his belief and optimal choice will be $\tilde{p}$ which is:

$$\tilde{p} = \frac{\rho(1-\beta) + \beta\rho}{\rho(1-\beta) + \beta} = \frac{\rho}{\rho(1-\beta) + \beta} \tag{5}$$

Let us show the biased type doesn't have the incentive to incur the cost c. Now, upon the message $m = 1$, the DM chooses $y^* = \hat{p}$ as there is only biased and unbiased informed(1) types

[13] Where $-(1-\rho)^2$ is the payoff from the outside option for the group 1 type of not investing, as $\mu(0,1) = \rho$ and payoff from investment is $-c$ as $\mu(1,1) = 1$.

[14] If $\rho > 0.5$, then $(1-\rho)^2 > \rho - \rho^2$ and hence this equilibrium couldn't arise for any value of c.

who choose $m = 1$. For the biased and unbiased informed(1), the payoff from investing should be less than that from not investing:

$$(1 - \tilde{p})^2 + c \geq (1 - \hat{p})^2 \tag{6}$$

As without investment and $m = 1$, the DM's belief is $\hat{p}$ as in case 1. This is equivalent to:

$$\left[\frac{\beta(1-\rho)}{\rho(1-\beta)+\beta}\right]^2 + c \geq \left[\frac{\beta(1-\rho)}{\alpha(1-\beta)\rho+\beta}\right]^2 \tag{7}$$

For the uninformed agent, the payoff from not investing is $-(\rho - \rho^2)$ and from investing it will be:

$$-(1-\rho)0 - \rho(1-\tilde{p})^2 - c \tag{8}$$

Then the condition that should be satisfied is:

$$c \geq \rho - \rho^2 - \rho(1-\tilde{p})^2 \tag{9}$$

Finally, the equilibrium in which no type wants to invest, for the specified out of equilibrium beliefs, is:

$$c \geq \max\{\rho - \rho^2 - \rho(1-\tilde{p})^2, (1-\hat{p})^2 - (1-\tilde{p})^2\} \tag{10}$$

Now consider that when there is a separating equilibrium, the condition $\rho - \rho^2 > (1 - \hat{p})^2$ is satisfied. Then, it is the case that $\rho - \rho^2 - \rho(1 - \tilde{p})^2 > (1 - \hat{p})^2 - (1 - \tilde{p})^2$. This means, the condition above becomes $c \geq \rho - \rho^2 - \rho(1 - \tilde{p})^2$, which is less than $\rho - \rho^2$. Then, there is also a no investment equilibrium in this region. However, we are able to rule out this type of equilibrium. This is because, the uninformed type, when his message is taken at face value, is willing to deviate to invest while the group 1 types do not find it profitable to invest. Hence, as there is a best response of the DM that makes unbiased uninformed types better off and the group 1 types worse off and is optimal given his beliefs about the deviator, this type of equilibrium can be ruled out by the Intuitive Criterion.

Finally, we can also rule out an equilibrium in which only group 1 types invest, as for no belief of the DM these types find it profitable to invest in this region.

In the region $c \in [\rho - \rho^2 - \rho(1-\tilde{p})^2, \rho - \rho^2]$, the separating equilibrium is the unique equilibrium that survives the Intuitive Criterion.

Whenever $c \geq (1-\hat{p})^2$, even for the highest belief $\mu(1,1) = 1$, the biased and uninformed (1) agent do not benefit from deviating to invest. Hence, in this region, the out of equilibrium belief should assign probability 1 to the expert being unbiased and uninformed. Then, whenever $c \geq -(\rho - \rho^2)$, even the unbiased uninformed type doesn't want to invest, which provides the boundary of the no investment equilibrium.

□

Pooling Equilibrium:

We call an equilibrium pooling if all types except for the informed (0) type invest. By the intuitive criterion (Cho and Kreps 1987) the out of equilibrium belief upon no investment and $m = 1$ should put probability 0 on the unbiased uninformed expert as this type always prefers to send $m = \varnothing$ if he were to deviate to no investment while the group 1 types always prefer sending $m = 1$ than any other message, which leads to $\mu(0,1) = \hat{p}$.

Lemma 3. *The pooling equilibrium exists for the cost region:*

$$c \leq (1-\hat{p})^2 - (1-\tilde{p})^2$$

and the out of equilibrium belief of the DM upon no investment and $m = 1$ will be $\mu(0,1) = \hat{p}$.

while $\mu(1,1) = \tilde{p} = \frac{\rho}{\rho(1-\beta)+\beta}$.

As we have $\frac{\partial(1-\hat{p})}{\partial\beta} - \frac{\partial(1-\tilde{p})}{\partial\beta} > 0$, when β increases the incentive of the biased type to invest increases and makes the pooling equilibrium more likely to arise.

5 Welfare analysis

Now we will make welfare comparisons among equilibria.

Lemma 4.
- *In the region $(1-\hat{p})^2 \leq c \leq (1-\rho)^2$, where separating equilibrium coexists along with mixed strategy equilibria, the separating equilibrium is the welfare maximizing one, whenever it exists, as well as being the expert preferred one.*
- *However, the DM prefers the mixed strategy equilibrium.*

Proof. First, consider the region $(1-\hat{\rho})^2 \leq c \leq (1-\rho)^2$ where mixed strategy and separating equilibria coexist. We can see that the separating equilibrium provides higher payoff for any expert type for any c. To see this: we know that in the separating equilibrium, the uninformed unbiased type's communication is taken at face value and the group 1 types' payoff is $-(1-\hat{p})^2$, independent of cost. In the mixed strategy equilibria in this region, for any $c > (1-\hat{\rho})^2$, we need to have $\mu(0,1) < \hat{p}$ hence the payoff of group 1 types will be $-(1-\mu(0,1))^2 < -(1-\hat{\rho})^2$ given their indifference between investment and not, and the uninformed type's payoff cannot be higher as $\mu(1,1) = 1$ in the separating equilibrium. Then, all expert types are weakly (and some strictly) worse off in mixed strategy equilibria compared to pure strategy in this region.

For the DM, when we compare the mixed strategy payoff in 8 to the pure strategy equilibrium payoff in equation **??**, we see that the mixed strategy equilibrium payoff is higher than the separating equilibrium. To see this, consider the mixed strategy equilibria of the type $\sigma = 0$ and $\gamma > 0$ which is the optimal one. Then, realizing that $\mu(0,1) < \hat{p}$ provides the result. The equilibrium with $\sigma = 0$ is also the one which maximizes the DM's payoff. In this type of equilibrium, as c increases, γ increases while keeping $\sigma = 0$. Hence, we can conclude that the DM's payoff is highest in mixed strategy equilibria in this region and especially it is highest in the equilibrium in which $\gamma = 1$ and $\sigma = 0$ which can arise at $c = (1-\rho)^2$.

The total payoff in separating equilibrium is $-\beta(1-\rho)-(1-\beta)(1-\alpha)c$ while in mixed strategy equilibrium it is $-\beta(1-\rho)\mu(0,1)-(\beta+(1-\beta)\alpha\rho)(1-\mu(0,1))^2-(1-\beta)(1-\alpha)c$. Now, consider $-(\beta+(1-\beta)\alpha\rho)(1-\mu(0,1))^2 = X$, then we have $X < -(1-\rho)\beta(1-\mu(0,1))$. As the payoff in mixed strategy would be equal to separating equilibrium if and only if $X \geq -(1-\rho)\beta(1-\mu(0,1))$, we can conclude that the separating equilibrium welfare is higher than mixed strategy. □

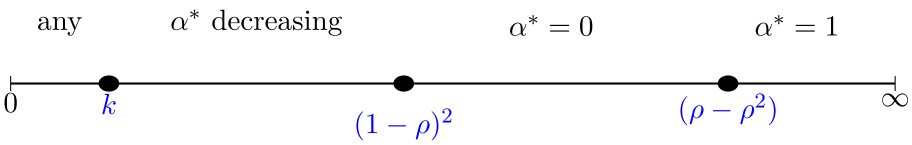
The pure strategy equilibrium payoffs dominate that of mixed strategy equilibria for all expert types while in the mixed strategy equilibria the DM's payoff is maximized whenever it exists.

Total welfare is non-monotone in the cost of information acquisition: In a given type of equilibrium, unsurprisingly total welfare decreases in cost. Surprisingly, total welfare increases as cost increases to shift the equilibrium from the pooling to the separating region.

Proposition 2. *Total welfare increases when cost increases from the pooling to the separating equilibrium region. Hence, total welfare is non-monotone in the cost level: in a given type of equilibrium it decreases in cost while it increases in cost if this increase shifts the equilibrium from pooling to separation.*

Indeed, welfare related to information values cancels out, and the only terms that remain are related to costs. This suggests that biased types lose and unbiased types as well as the DM gain when moving from pooling to separating equilibria in terms of informativeness. These types gain due to more precise communication upon investment in equilibrium while the biased and unbiased informed types lose as they find it too costly to invest in order to pool with the uninformed unbiased types. There is more investment in the pooling equilibrium at a lower cost, while there is less investment in the separating equilibrium at a higher cost which more than overcomes the higher cost. This is not a trivial but intuitive result. As in both pooling

Figure 2: α^* as a function of cost



and separating equilibria the uninformed unbiased expert who is the only one to learn from information, invests, the investment strategies of group 1 types do not affect the total amount of information available to the expert: the biased type doesn't use the information while the unbiased type doesn't learn more from his investment than what he already knew. However, in the separating equilibrium, the welfare is transferred from the biased and unbiased informed type to the unbiased uninformed type and the DM.

Our results have interesting implications. It turns out that the expert is already informed isn't necessarily good for the DM, and indeed in equilibrium the DM has a higher payoff when matched with an uninformed expert who becomes informed later, in front of her eyes. If the expert is unbiased but already informed, he cannot distinguish himself from the biased type whenever the high state is realized (in pure strategy equilibria). On the other hand, the uninformed unbiased type communicates perfectly in the separating equilibrium as well as in most mixed strategy equilibria. DM's (and total) welfare is non-monotone in α: a high enough α allows separation as the amount of informed unbiased types should be high enough to make the no investment payoff for the biased types high enough that they don't invest. However, once in the separating equilibrium, welfare decreases in α.[15]

5.1 What type of expert is best for the DM?

The model provides interesting implications as to what type of expert is the best one for the decision maker for different values of information acquisition, when she cannot distinguish a biased and aligned one. The parameter we focus on is α, the quality parameter, which is how likely an expert is to be informed. This parameter can be thought of as the reputation of the expert linked to their previous experience on the subject. We find the optimal α for the DM for different costs of effort.

Proposition 3. *The optimal α for the DM, the expertise parameter, is non monotone in c:*

- *For $c > \rho - \rho^2$, $\alpha^* = 1$:in this region, no type is willing to invest hence better to get a perfectly informed expert.*
- *$c \leq \rho - \rho^2$, α^* such that $(1 - \hat{p})^2 = c$ where $\hat{p}$ is an increasing function of α. This also means that as c increases, α^* for the DM decreases.*

[15] This is because as α increases, there are more and more unbiased informed types pooling with the biased type leading to loss of precision in communication.

- *For* $(1-\rho)^2 \leq c \leq \rho - \rho^2$, $\alpha^* = 0$*: in this region, separating equilibrium arises for any* α*.*

Proof. First, fix c. We have 3 different regions. Initially consider the region $c < (1-\rho)^2$. We make use of the fact that $(1-\hat{p})^2$ is a decreasing function of α as $\hat{p}$ is increasing in α. Among the equilibria that arise in this region, we know that the DM's welfare is maximized with α high enough to ensure $(1-\hat{p})^2 = c$ as opposed to lower α, which leads to $\alpha^* = \frac{\beta(1-\rho-\sqrt{c})}{(1-\beta)\rho\sqrt{c}}$. Hence, increasing α up to this point definitely leads to a better equilibrium payoff for the DM. Plus, we know that whenever we are in the separating region, as α increases further so that $(1-\hat{p})^2 > c$, the separating equilibrium payoff decreases for the DM. At α^*, the DM optimal mixed strategy equilibrium and the separating equilibria coincide and both provide the highest payoff for the DM.

Indeed, consider α such that the separating equilibrium arises, as $c > (1-\hat{p})^2$. Then, there exist both pure strategy and mixed strategy equilibria, with the DM preferred being the mixed strategy equilibrium with $\sigma = 0$, while the separating equilibrium payoff is strictly lower than the separating equilibrium with α at which $(1-\hat{p})^2 = c$. Indeed, this mixed strategy equilibrium payoff is equivalent for the DM (leads to the same posteriors) to the one in which $\hat{\alpha} = \alpha(1-\gamma)$, and which would lead to $c = (1-\hat{p})^2$, meaning the minimum α such that separating equilibrium realizes. Then we can say that, higher α can never increase the DM's payoff plus it leads to multiplicity of equilibria. Indeed, even when there is multiplicity, the welfare maximizing equilibrium is the separating one, for which the welfare is lower than the separating one. Let us interpret this result: the DM's payoff doesn't increase in α once the separating region: there is an α^* that is decreasing in cost which maximizes the DM's welfare without regards to the equilibrium selection. This implies, there is no point to hire a more informed expert than necessary. Indeed, depending on the equilibrium selection, it can even hurt the DM. Among the separating and mixed strategy equilibria in this region, for any α, we find that the separating equilibrium is the welfare maximizing one. Given that the separating equilibrium leads to higher welfare, but lower DM surplus, the more reason not to have a higher α.

We know that when $(1-\rho)^2 \leq c \leq \rho - \rho^2$, the unique equilibrium is the one in which only the unbiased uninformed types invest and the DM takes communication at face value. Then, it is optimal for DM's payoff when $\alpha = 0$ so that no unbiased type has to pool with a biased type, making communication most precise.

Finally, when $c > \rho - \rho^2$, there is no possibility of any investment by any type. In that case, it is optimal that the expert is informed with probability 1. When there is no way for the DM to separate between biased and unbiased types, still it is optimal that the unbiased types are informed.

□

Hence, the optimal level of α for the DM is non-monotone in how costly information is. The intuition is as follows: whenever $(1-\rho)^2 \leq c \leq \rho - \rho^2$, all the unbiased experts strictly prefer to invest if they can communicate perfectly while informed (1) or biased experts strictly prefer not to invest. Then, in order to decrease the loss of communication by the ex-ante informed aligned experts, it is optimal to choose $\alpha = 0$, so that any unbiased expert invests in information and any biased expert does not. Hence the DM can perfectly identify the type of the expert. Then, in this region it is strictly optimal to choose an uninformed expert. This is due to the unwillingness of an already informed expert to incur the cost in order to make their message more credible, while an uninformed expert does face more uncertainty that he wants to invest.

On the other hand, once $c < (1-\rho)^2$, it is no longer possible to satisfy the above when $\alpha = 0$, as the biased experts would have an incentive to deviate to invest. In order to have separation, there should be just sufficient portion of unbiased types who are informed, which gives the minimum α such that $c = (1-\hat{p})^2$. Plus, for any α that is higher than this critical level, the separating equilibrium leads to a lower payoff for the DM while the best mixed strategy equilibrium always leads to the same payoff which is identical to the pure strategy separating equilibrium with α^*. This means, there is no point having an expert more informed than that critical minimum level. If the experts required a fee which depended on their α parameter, then there is more reason that the DM should not pay for an expert more informed than this level.

The model predicts that when the bias of an expert is unobservable, a more informed expert isn't necessarily good for the DM. Indeed, in equilibrium the only type of unbiased expert who can communicate perfectly is the one who invests in order to get informed. However, having some probability that the expert is already informed is also good to keep the biased types from investing, by avoiding too much prejudice against an expert who communicates without investing. This is why over a region, the optimal expertise parameter is decreasing in cost.

6 Covert information acquisition

Now we consider the case when information acquisition is covert, in other words when the decision maker does not observe the investment made by the expert. In this case, as investment has no signaling value, the group 1 types will never invest, as they can pool with the unbiased uninformed expert without incurring any cost. The only type that may invest is the unbiased uninformed type. There are 2 types of pure strategy equilibria as a function of the cost which are summarized below:

1. Investment takes place by the uninformed unbiased type. Upon $m = 0$, the DM chooses $y = 0$ and upon $m = 1$, the DM takes action $y = \tilde{p}$. This equilibrium looks like the pooling equilibrium except that the biased and informed (1) types do not actually invest. The payoffs of the biased and informed (1) types are higher in this equilibrium compared to the pooling equilibrium discussed, as they achieve the same outcome without having to incur the investment cost.

The condition for the unbiased uninformed type to invest is:

$$c \leq \rho - \rho^2 - \rho(1-\tilde{p})^2$$

This means the unbiased type acquires information for smaller range of cost values in the covert information acquisition than in the overt case. This cutoff is above the pooling cutoff cost but below the separating equilibrium cutoff cost of the overt case. [16]

2. No investment takes place. Upon $m = 1$, the DM chooses $\hat{p}$ inferring that this message is sent either by the group (1) type of expert. The uninformed expert sends $m = \emptyset$ and the DM chooses $y = \rho$. This equilibrium is equivalent to the no investment equilibrium in the overt information acquisition case. This equilibrium arises for the following cost values:

$$c > \rho - \rho^2 - \rho(1-\tilde{p})^2$$

The types that gain from information acquisition being covert as opposed to overt are the group 1 types and only in case the cost is low enough that the uninformed unbiased type acquires information. Even though the cost does not affect these types directly as they don't acquire information, the investment of the unbiased uninformed type makes their message freely more credible. In this equilibrium, the payoffs of the uninformed unbiased expert and the decision maker are the same as in the pooling equilibrium in the overt case while the payoff of the biased and informed (1) expert are higher only because they don't invest. When cost increases, we move to the no investment equilibrium in which payoffs are identical to the overt case without investment.

The unbiased type invests in information less often and is worse off in covert case, as he can never perfectly separate himself from the biased type. This result shows that, even though the signaling value of information undermines its intrinsic value, under certain parameters overt information acquisition still does strictly better than the covert one, specifically the separating equilibrium does exists in the overt information acquisition.

However, when the cost of information acquisition is low enough that in the overt case pooling in investment arises, then covert information acquisition does better in terms of the overall welfare as there is no wasteful investment, although the precision of communication is identical. Then the next corollary follows.

Proposition 4. *Whenever* $c \geq (1-\hat{p})^2 - (1-\tilde{p})^2$, *overt information acquisition leads to higher overall welfare while whenever* $c \leq (1-\hat{p})^2 - (1-\tilde{p})^2$, *covert information acquisition leads to higher welfare.*

The tradeoff is between more informative communication versus wasteful investment in

[16] Realize that in this region, there are as well mixed strategy equilibria in which the unbiased uninformed expert invests with a probability. This probability should increase as c decreases to keep the indifference condition satisfied.

information. Whenever there is no investment in the covert case, overt information acquisition does better as we have seen that welfare is always higher when some types invest than when there is no investment. However, when cost is low enough that there is pooling in investment in the overt case, total welfare is higher in the covert case as although the communication precision is equivalent less cost is incurred.

7 Conclusion

This paper studies a novel question in a simple model building on the communication and signaling literatures. It explores the interaction of credibility concerns with costly information acquisition and its implications for welfare. We have shown that an unbiased expert, as well as a biased one, may wastefully invest in information acquisition. This leads to inefficiency in decision making and lowers welfare, and as a result the decision maker may be better off if matched with an uninformed expert. This is due the inability of the informed unbiased type to separate himself from a biased type whenever the high state of the world is realized. There are interesting comparative statics among which one is the non monotonicity of welfare in the cost of information acquisition. Higher information acquisition cost increases the overall welfare when the equilibrium moves from the pooling to separating region. Finally, we considered what type of expert would be optimal for the decision maker and we found that there is also non monotonicity in the optimal level of **informedness** of the expert.

The simplicity of the model allows for numerous extensions remaining for future research. An interesting feature of the model is that even though the ability to acquire information is not correlated with the type of the expert, the value of getting informed depends on the expert's type. Hence, it is possible to use information acquisition as a screening device by taxing the experts for getting informed. While in the pooling region, the expert who is uninformed and unbiased has the highest incentives to acquire information. Hence, this type will be willing to pay more than biased and informed (1) experts so that wasteful investment by these types can be prevented.

8 Appendix

No Investment Equilibrium:

When $c \geq \rho - \rho^2$ even the unbiased uninformed type doesn't want to invest. The right hand side is the boundary at which the separating equilibrium starts, in which the only type that invests is the unbiased uninformed type. We know that in the region $(1-\hat{p})^2 \leq c \leq \rho - \rho^2$, the biased type doesn't find it profitable to invest, even if his message were taken at face value. Then, in the region $c \geq \rho - \rho^2$, by the intuitive criterion, the out of equilibrium belief upon investment should place probability 0 to the expert being a biased type, as this type can never gain from deviating to invest even if he were to be perceived to be an unbiased type. Hence, the message upon a deviation to invest should be taken at face value by the DM. The gain in utility from doing so for the uninformed unbiased type defines the cost value above which the unique equilibrium has no investment.

Proof of proposition (3):

Proof. In this equilibrium, the DM's updated belief $\mu(1,1)$, given that all three types are investing, is:

$$\tilde{p} = \frac{\rho(1-\beta)+\beta\rho}{\rho(1-\beta)+\beta} = \frac{\rho}{\rho(1-\beta)+\beta}$$

First, consider the investment choice of the group 1 types. If the biased or unbiased agent were to deviate to not invest and send $m = 1$, then the DM would choose $y = \hat{p}$ as the DM infers this out of equilibrium action can only come from a biased or unbiased but informed 1 expert. Then, the following should hold for the pooling equilibrium to arise:

$$-(1-\tilde{p})^2 - c \leq -(1-\hat{p})^2$$

Second, for the uninformed unbiased type, the payoff from not investing is $-(\rho-\rho^2)$ as before, as in that case they would send $m = \varnothing$. Then, consider the payoff of this type from investing. If the signal turns out to be 0, the DM takes the message at face value and chooses $y = 0$ whereas if the signal is 1, the decision maker will choose $\hat{p} < \tilde{p} < 1$ as the DM infers it can come from a group 1 or 2 type. Then, this type prefers investing to not if and only if:

$$-\rho(1-\tilde{p})^2 - c \geq -(\rho-\rho^2)$$

These conditions together lead to:

$$c \leq min\{(1-\hat{p})^2 - (1-\tilde{p})^2, \rho - \rho^2 - \rho(1-\tilde{p})^2\} = (1-\hat{p})^2 - (1-\tilde{p})^2$$

When $(1-\hat{p})^2 < (\rho - \rho^2)$, which is the case we consider, we have $min\{(1-\hat{p})^2 - (1-\tilde{p})^2, \rho - \rho^2 - \rho(1-\tilde{p})^2\} = (1-\hat{p})^2 - (1-\tilde{p})^2$. To see this, realize that the value of getting informed is higher for unbiased uninformed types than for the group 1 types: $(1-\hat{p})^2 - (1-\tilde{p})^2 < \rho - \rho^2 - \rho(1-\tilde{p})^2$. This is because unbiased uninformed types get maximum payoff of 0 when the state of the world is 0, while their communication is distorted when the message is 1. However, from the point of view of the group 1 types, communication is always distorted as their bliss point is 1 and $\mu(1,1) = \tilde{p} < 1$. Then, the condition for the pooling equilibrium is given by the condition for the biased and informed (1) types to be willing to invest which is $c \leq (1-\hat{p})^2 - (1-\tilde{p})^2$.

Now consider the expression $c \leq \rho - \rho^2 - \rho(1-\tilde{p})^2$, the condition for the unbiased uninformed types to actually invest given a pooling equilibrium. As $\rho - \rho^2 > \rho - \rho^2 - \rho(1-\tilde{p})^2$, where the left hand side is the cutoff for investment in the separating equilibrium, the condition for unbiased uninformed types to invest is easier to satisfy in the separating region. The difference between these two is due to the biased types' "crowding out" the unbiased uninformed types: investment of the biased types makes information acquisition by the unbiased types less profitable, hence cost has to be lower in order to satisfy their participation.

□

Proof of proposition (2):

Proof. I compare the payoff in the pooling equilibrium to that in the separating equilibrium, before considering the equilibria with investment. For this, I compare the payoff for cost values at which there is pooling equilibrium and separating equilibrium to find how the payoff changes when c increases from the pooling region to separation.

The total welfare in the **pooling** equilibrium is given by:

the DM's welfare:

$$[(1-\beta)\rho + \beta][-(\tilde{p} - \tilde{p}^2)] = -\rho(1-\tilde{p}) \tag{11}$$

The expert's welfare: The payoff of the biased expert is $-(1-\tilde{p})^2 - c$. The ex-ante payoff of the unbiased expert is $-\rho(1-\tilde{p})^2 - (1-\alpha(1-\rho))c$. This is decreasing in ρ and in β.

Then the total expert welfare is:

$$-[(1-\beta)\rho + \beta](1-\tilde{p})^2 - [1-(1-\beta)(1-\rho)\alpha]c$$

When we sum the DM and expert welfare:

$$-\rho(1-\tilde{p}) - \beta[(1-\tilde{p})^2 + c] - (1-\beta)[\rho(1-\tilde{p})^2 + (1-\alpha(1-\rho))c] \tag{12}$$

which simplifies to:

$$(13)\quad -(1-\tilde{p})^2[\beta+\rho(1-\beta)]-\rho(1-\tilde{p})-c[1-\alpha(1-\rho)]=-\beta(1-\rho)-c[1-\alpha(1-\rho)(1-\beta)]$$

In the **separating** equilibrium, the DM's welfare is:

$$(14)\qquad [(1-\beta)\alpha\rho+\beta][-(\hat{p}-\hat{p}^2)]=-\hat{p}\beta(1-\rho)$$

The payoff of the biased expert is $-(1-\hat{p})^2$, while the ex-ante payoff of the unbiased expert is $-\alpha\rho(1-\hat{p})^2-\alpha(1-\rho)0-(1-\alpha)c=-\alpha\rho(1-\hat{p})^2-(1-\alpha)c$.

Then, the expected payoff over expert types in the separating equilibrium is:

$$-(1-\hat{p})^2[\beta+(1-\beta)\alpha\rho]-(1-\beta)(1-\alpha)c$$

Finally, the total welfare in the **separating** equilibrium is:

$$(15)\qquad -\hat{p}\beta(1-\rho)-\beta(1-\hat{p})^2-(1-\beta)[\alpha\rho(1-\hat{p})^2+(1-\alpha)c]$$

which simplifies to:

$$(16)\qquad -\beta(1-\rho)-c(1-\alpha)(1-\beta)$$

The difference in the total welfare in equation (16 − 13) is:

$$(17)\qquad c_p[1-\alpha(1-\rho)(1-\beta)]-c_s(1-\beta)(1-\alpha)$$

The welfare in terms of information cancels out and the terms that remain are those related to the cost incurred for information acquisition. In the pooling equilibrium there is more investment in information at a lower price, while in the separating equilibrium there is less investment at a higher price. When we plug in the maximum cost at which the pooling equilibrium exists and the minimum cost at which the separating equilibrium exists, it is seen that the welfare in separating equilibrium is higher than in the pooling one although the cost of information acquisition is higher.

At $c_p = (1-\hat{p})^2 - (1-\tilde{p})^2$ and $c_s = (1-\hat{p})^2$, equation (17) becomes:

$$(1-\hat{p})^2(\beta(1-\alpha\rho)+\alpha\rho) - (1-\tilde{p})^2(1-\alpha(1-\rho)(1-\beta)) > 0 \tag{18}$$

When we replace these values, finally we are left with the condition:

$$\beta(1-2\rho) + \rho(1-\beta)(\alpha-(1+\alpha)\rho) \leq 0 \tag{19}$$

which is satisfied whenever $\rho > \frac{1}{2}$ which is the initial assumption we made. The first term is negative. The second term is negative when $\rho > \frac{\alpha}{1+\alpha}$ which is always the case when $\rho > \frac{1}{2}$ and $\alpha < 1$.

Then, although the cost of information rises, welfare increases due to the lack of wasteful investment in information. In order to demonstrate this result, we considered the cost values at the boundaries. As expected, when we keep increasing the cost in the separating equilibrium region, the welfare will decrease and at some point, it will be lower than in the pooling equilibrium.

Finally, the total welfare in the **no investment** equilibrium is:

$$-\beta(1-\hat{p})^2 - (1-\beta)[\alpha\rho(1-\hat{p})^2 + 2(1-\alpha)(\rho-\rho^2) - (\hat{p}-\hat{p}^2)(\beta+(1-\beta)\alpha\rho)] \tag{20}$$

As no investment equilibrium surplus is unambiguously worse than the separating equilibrium, we compare it to the pooling in investment equilibrium and find that the payoff in the no investment equilibrium is also lower than the pooling in investment equilibrium. This is intuitive: first, the DM's payoff is unambiguously higher in the pooling in investment equilibrium compared to the no investment equilibrium, as more information is revealed. The welfare of the biased type also higher in the pooling in investment equilibrium as their outside option of not investing and getting $-(1-\hat{p})^2$ is still available. Hence, if this type does find it profitable to invest, then it must be getting a higher payoff. The same is true for the unbiased informed (1) type who would get $-(1-\hat{p})^2$ if deviating to not invest. Finally, for the unbiased uninformed type, it is true as well: if this type didn't invest they would get the payoff $-(\rho-\rho^2)$ which is still available if they deviate in the pooling equilibrium to send $m = \emptyset$. □

Proof of proposition (??)

Proof. The first two items were shown earlier.

The last condition says that even if the uninformed unbiased expert exerts effort $e = 1$ in which case his gain in payoff will be $(\rho-\rho^2)$, this is still lower than the gain of the biased type and informed (1) type from investing and pooling with him which is $(1-\hat{p})^2$. The outcome of investment is not relevant for these types and they will always claim to have received signal 1. Given that this is a deviation from an equilibrium in which the only investment is made by unbiased types, when they send $m = 1$ the DM will choose $y = 1$. Hence, their payoff from investment is 0, whereas without investment their payoff was $-(1-\hat{p})^2$, which means their gain is $(1-\hat{p})^2$.

On the other hand, the gain in profit from investment for the unbiased uninformed expert is $e(\rho - \rho^2)$. To see this, realize that with probability e, a signal will arrive and if it does, then the expert's payoff will be 0 as the DM will take the message at face value. If no message arrives, then the expert will send $m = \emptyset$ and get a payoff of $-(\rho - \rho^2)$. Hence, the gain in payoff for the uninformed type is $e(\rho - \rho^2)$.

Then, the gain in payoff from deviating to pool is higher for the biased type than the gain in information for the unbiased uninformed type even when $e = 1$ if $(\rho - \rho^2) \leq (1-\hat{p})^2$. Then, for any level of e, the uninformed unbiased type could not separate himself from the biased type of expert. Hence, no separating equilibrium exists under this condition.

□

DM's and Expert Welfare Comparisons

The DM's payoff

It is easy to see that the DM's payoff is minimized in the equilibrium in which no type invests, which is the least informative equilibrium. Hence, presence of investment definitely makes communication more informative. Plus, it can be easily seen that separating equilibria are better than pooling ones for the DM.

Proof. **Separating equilibrium:**

$$[(1-\beta)\alpha\rho + \beta][-(\hat{p} - \hat{p}^2)] = -\hat{p}\beta(1-\rho) \tag{21}$$

This is decreasing in α and β which both increase $\hat{p}$. As α increases, more of the informed unbiased types will be pooled with biased types, and as β increases, facing a biased type becomes more likely. The payoff is also increasing in ρ whenever $\rho > 0.5$. This is intuitive: the DM's loss from biased communication is less when the prior already favors the action that the biased type of expert wants.

Pooling equilibrium:

$$[(1-\beta)\rho + \beta][-(\tilde{p} - \tilde{p}^2)] = -\rho(1-\tilde{p}) \tag{22}$$

This is decreasing in β and when $\rho > 0.5$, it is increasing in ρ. When $\rho < 0.5$, and β high enough it may be decreasing in ρ. The pooling payoff is independent of α, as all unbiased types do invest in this case and the DM doesn't internalize the cost of investment. The separating equilibrium payoff dominates the pooling equilibrium payoff for the DM iff:

$$\alpha(1-\rho) + \rho \leq 1$$

which is always satisfied. Hence, the DM's payoff is unambiguously higher in the separating equilibrium in which information is more precise. The DM benefits from having some probability of expert being initially informed and pool with the biased type, as this makes the separating equilibrium attractive for the biased type. If α is very low, there are more incentives to "pool" and the separating region shrinks. However, inside the separating region, the DM's payoff is decreasing in α. Hence, the DM's payoff is non-monotone in α: it has to be just high enough for biased types not to invest.

No investment equilibrium:

The DM's utility in the equilibrium in which no one invests is:

$$\frac{-(1-\rho)\rho(\beta + (1-\alpha)\alpha(1-\beta)^2\rho)}{\beta + \alpha(1-\beta)\rho} \tag{23}$$

which is found by simplifying $-[\beta + (1-\beta)\alpha\rho](\hat{p} - \hat{p}^2) - (1-\beta)(1-\alpha)(\rho - \rho^2)$. We see that pooling in investment always leads to higher payoff for the DM than no investment. Hence, the no investment equilibrium provides the minimum possible payoff to the DM.

Mixed strategy equilibria

The DM's payoff given σ and γ is:

$$-(\beta(1-\sigma) + (1-\beta)\alpha\rho(1-\gamma))(\mu(0,1) - \mu(0,1)^2) - ((1-\beta)(\alpha\gamma + 1 - \alpha) + \beta\sigma)(\mu(1,1) - \mu(1,1)^2)$$

which simplifies to:

$$-\beta(1-\rho)[(1-\sigma)\mu(0,1) + \sigma\mu(1,1)]$$

At a given cost c, the DM optimal equilibrium, which also coincides with expert optimal mixed strategy equilibrium is the one in which $\sigma = 0$ and γ takes the value which satisfies:

$$c = (1 - \mu(0,1))^2$$

□

The expert's payoff

Separating equilibrium:

The payoff of the biased expert is $-(1-\hat{p})^2$, which is increasing in α and ρ and decreasing in β. The ex-ante payoff of the unbiased expert is $-\alpha\rho(1-\hat{p})^2 - \alpha(1-\rho)0 - (1-\alpha)c = -\alpha\rho(1-\hat{p})^2 - (1-\alpha)c$, which is increasing in α, decreasing in ρ and decreasing in β.

Then, the expected payoff over expert types in the separating equilibrium is:

$$-(1-\hat{p})^2[\beta + (1-\beta)\alpha\rho] - (1-\beta)(1-\alpha)c$$

Pooling equilibrium:

The payoff of the biased expert is $-(1-\tilde{p})^2 - c$. This is increasing in ρ and decreasing in β. The ex-ante payoff of the unbiased expert is $-\rho(1-\tilde{p})^2 - (1-\alpha(1-\rho))c$. This is decreasing in ρ and in β. It is increasing in α as less cost will have to be incurred.

It is trivial that the biased type of expert is unambiguously better off in the pooling equilibrium than in the separating one, as otherwise he would not invest and still get the same payoff as in the separating equilibrium. On the other hand, the unbiased expert's expected payoff increases when moving from the separating to the pooling equilibrium, when we compare the payoff at the minimum cost at which there is separation and maximum cost at which there is pooling.

The expected payoff over all expert types in the pooling equilibrium is:

$$-[(1-\beta)\rho + \beta](1-\tilde{p})^2 - [1-(1-\beta)(1-\rho)\alpha]c$$

Mixed strategy equilibria

The expected payoff over expert types in a mixed strategy equilibrium is:

$$-(\beta + (1-\beta)\alpha\rho)(1-\mu(0,1))^2 - (1-\beta)(1-\alpha)\rho(1-\mu(1,1))^2 - (1-\beta)(1-\alpha)c$$

The above is found by using the fact that the biased and informed (1) types of experts are indifferent between investing or not, and taking their payoffs as they didn't invest. This simplifies, if we replace $c = (1-\mu(0,1))^2 - (1-\mu(1,1))^2$, to:

$$-[(1-\beta)(\alpha\rho + 1 - \alpha) + \beta](1-\mu(0,1))^2 + (1-\beta)(1-\alpha)(1-\mu(1,1))^2$$

From the above, we can see that the expert preferred mixed strategy equilibrium has $\sigma = 0$, and $\gamma > 0$.

The case with $\rho < 0.5$, hence $(1-\rho)^2 > \rho - \rho^2$

Proof of proposition (??)

Proof. When $c > \rho - \rho^2 - \rho(1-\tilde{p})^2$, in the covert information acquisition case no type is getting informed, hence the payoff is the same as in the no investment equilibrium in the overt case. We know that in the overt information acquisition, welfare is always higher when there is some information acquisition compared to none, and in the region $[\rho - \rho^2 - \rho(1-\tilde{p})^2, \rho - \rho^2]$ there is investment in the overt case. Hence, the payoff in overt case is always higher under this condition.

When $c < (1-\hat{p})^2 - (1-\tilde{p})^2$, in the overt case, there is pooling in investment while in the covert case, the unbiased uninformed type invests only (realize that $(1-\hat{p})^2 - (1-\tilde{p})^2 < \rho - \rho^2 - \rho(1-\tilde{p})^2$ hence there is investment in covert case). In the end, the amount of information transmitted is the same. Hence, in the overt case there is more wasteful investment for the same precision of communication. Then, we can conclude that welfare is higher in the covert case under this assumption. □

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