Recruitment, Wage Inequality, and Shrinking Task Differences *

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Abstract

Although the college wage premium increased by 30% from 1980 to 2020, college workers today work at jobs requiring fewer cognitive abstract tasks compared to graduates in 1980. I argue this is due to improved recruitment technology benefiting college workers even when they are in less specialized roles. Using a general equilibrium search model with endogenous recruitment, I show that when more college-requiring jobs are created through either technical change or the expansion of colleges, recruitment efficiency for college workers improves, raising wages even as average abstract task content decreases. Quantitatively, accounting for endogenous recruitment is necessary to reconcile the observed shift in college employment toward less abstract-intensive jobs. Recruitment contributes 12.6% to the increase in the college wage premium from 1980 to 2020 through technical change and college expansion. While technical change and college expansion displace non-college workers, the model predicts that a search assistance program for displaced workers would inadvertently accelerate their displacement from the workforce.

Keywords: Recruitment; competitive search; college wage premium; employment distribution

JEL Classification: E24, J24, J31, J64

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1 Introduction

The rise in the U.S. college wage premium since the 1980s, measured as the ratio of average real wages of degree workers to non-degree workers, has been given great attention in economic studies as an important source of wage inequality.[1] Due to the concurrent rise in the labor supply of college workers, many have attributed the rising college wage premium to technology-induced changes in labor demand. For instance, capital-skill complementarity could enhance the productivity of college-educated workers as more capital is adopted (Krusell et al., 2000); and automation could displace workers in performing repetitive and programmable routine tasks and increase labor inputs in cognitive tasks (Autor et al., 2003). Following the technical change mechanism, we should expect college workers nowadays to perform more tasks that utilize their comparative advantage over less-educated workers, especially in more cognitive-demanding abstract tasks.[2]

Yet, this does not seem to be the case empirically. Instead, college workers today are working at jobs that are more similar in task contents to those of non-college workers than they were decades ago. An average college graduate job demanded abstract task input comparable to those of a secondary school teacher or a paralegal in 1980. By 2020, the average job content of college workers is more akin to that of a general office clerk. This gives rise to the following question: As college and non-college workers are doing increasingly similar jobs, why does the college wage premium continue to rise?

In this paper, I propose a potential explanation that college workers benefit from the endogenous evolution of recruitment technology, in addition to the gain from skill-biased technical change, allowing them to earn high wages even in less specialized roles. I develop a general equilibrium search model in which a search platform, such as a job board or a staffing agency, owns and operates the recruitment technologies to illustrate the recruitment channel. I show theoretically that the search platform enhances the recruitment efficiency for a worker group when

[1]See Katz and Murphy (1992); Goldin and Katz (2008); Acemoglu and Autor (2011); Autor (2019) for an overview.

[2]Following the definition of Autor and Dorn (2013), examples of abstract tasks include creative thinking, problem-solving, and coordination.

the share of vacancies for that group increases. Hence, as either technical change or the expansion of colleges displace non-college workers and allocate more jobs for college graduates, the evolution of recruitment disproportionately benefits college workers. Conceptualizing the search platform also enables evaluations of active labor market policies designed to assist displaced workers with different policy mechanisms.

I deliver several key quantitative results using the model. First, I show that the recruitment channel is quantitatively necessary to generate the observed shifts of college employment distribution toward less abstract-intensive jobs, given the rise in the college wage premium from 1980 to 2020. If endogenous recruitment were shut down, the model prediction would have college workers working at more abstract-intensive jobs to deliver the same rise in the college wage premium. Second, the recruitment channel accounts for 12.6 percent of the increase in the college wage premium from 1980 to 2020 through technical changes and college expansion. Ignoring the recruitment channel would overestimate technology's positive impact and college expansion's negative effect on the wage premium. Third, as more jobs are reallocated away from non-college workers, active labor market policies such as job search assistance and employment subsidies are commonly used to improve the employment and wages of the displaced worker groups. However, I find search assistance reduces overall job creation and allocates jobs away from targeted workers, inadvertently accelerating job displacement.

Overall, I develop my analysis in two stages. In the first part, I illustrate the recruitment channel mechanism by extending the task-based framework (Acemoglu and Restrepo, 2018) with competitive search (Moen, 1997) and endogenous recruitment efficiency. The model features heterogeneous workers and jobs, and an independent search platform deciding recruitment efficiencies for heterogeneous workers. Unlike in the standard search and matching model (Mortensen and Pissarides, 1999), where search intermediaries are abstracted away from, vacancies created are posted at some search intermediaries to advertise, such as in newspapers and online job boards (Kroft and Pope, 2014). These search intermediaries act as a third type of agent in the hiring process that endogenously responds to changes in the compositions of jobs and workers to maximize their profit. In the absence of endogenous recruitment, a standard search and matching model with free entry (Mortensen and Pissarides, 1999) would have the job creation of firms fully offset the changes in worker composition.[3] The incorporation of endogenous recruitment provides a microfoundation that enables changes in worker and vacancy composition to affect wages and job-matching probability.

In the model, the search platform is a profit-maximizing agent that owns and operates the matching technologies in the labor market. It is compensated by firms for the job-filling services and is subjected to a recruitment cost function. The model suggests that a certain worker group's recruitment efficiency improves as the share of vacancies that specifically target this group increases. This is because the enhanced efficiency of the matching technology can benefit multiple job openings. For example, enhancing job candidate screening algorithms or expanding shared candidate pools benefits all vacancies simultaneously. These non-rival elements allow search intermediaries to experience decreasing marginal costs as vacancies increase. Consequently, from an aggregate perspective, the matching function exhibits increasing returns to scale when considering the recruitment channel. The strength of this channel depends on the relative importance of non-rival components in recruitment activities, which is disciplined by data in the calibration.

In a frictional labor market, a worker's wage depends on (1) labor productivity of the pool of jobs that are assigned to them and (2) the job-matching probability. The recruitment channel can improve the job-matching probability and provide additional upward pressure on wages when the labor demand for a particular group of workers increases. When there is a biased technical change, the recruitment channel amplifies the overall impact on the relative wages. More importantly, when the relative supply of a worker group increases, firms would reallocate more jobs toward this group because of their enhanced availability in the labor market, the recruitment channel can also generate upward pressure on the relative wages of a worker group when their labor supply increases.

[3] In a standard model (Mortensen and Pissarides, 1999), firms choose the optimal market tightness — the ratio of vacancy to jobseekers — to solve their job creation problem. The equilibrium wage is thus a function of market tightness. With free entry, any change in the number of jobseekers induces a corresponding change in vacancy that does not affect the optimal choice of market tightness.

How can the recruitment channel provide upward pressure on college wages even when their labor supply increases? When the supply of college workers increases, firms are more likely to fill positions with degree holders, leading them to reassign some jobs from non-college to college workers. However, college workers have less comparative advantage in these newly reassigned jobs, reducing their average relative labor productivity. This productivity channel, performing the role of the 'Law of Demand' in a conventional demand-supply framework, puts downward pressure on relative wages as supply increases. Wages are also affected by the recruitment channel. As more posted jobs require a college degree, the search platform improves the recruitment efficiency for college workers. This enhanced matching efficiency leads to better matching probabilities and higher overall match surpluses between vacancies and college workers. Consequently, the recruitment channel puts upward pressure on college wages and counteracts the negative effect of the productivity channel. If the recruitment channel dominates, an increase in the relative supply of college workers can even lead to a higher college wage premium.

Several pieces of empirical evidence support the recruitment channel. I examine the geographical expansion patterns of Craigslist, a free online platform for classified advertisements, during its rapid growth in the 2000s. I find that Craigslist prioritized entry into areas with higher concentrations of college-educated residents in the earlier stage of its expansion. This trend persists even when accounting for correlations between local college shares and other socioeconomic factors and internet coverage. Second, I find the ratio of matching efficiency – the job-finding probability of workers conditional on the ratio of vacancy to job seekers – of college workers to non-college workers has trended up since the 1980s. Hence, the advancement in matching technology has disproportionately benefited college workers while the college-educated workforce has grown. Third, I show that college workers' matching efficiency positively correlates with their share among the unemployed in local markets. This indicates that a worker group's job-finding probability improves when that group comprises a larger portion of job seekers, aligning with the model's predictions about recruitment efficiency.

In the second stage of my analysis, I calibrate the model to the U.S. labor market and conduct several counterfactual exercises. First, I compare predictions for the 2020 employment distribution of college workers using two models: one with an active recruitment channel and another without endogenous recruitment, when the economy underwent technical change and the expansion of colleges from 1980. This analysis yields several insights. First, technology shocks alone push college workers towards more abstract-intensive jobs, which is counterfactual. Second, when accounting for increased college labor supply, only the model with an active recruitment channel can accurately predict the observed shift of college workers to less abstract-intensive jobs compared to their 1980 allocation. Without the recruitment channel providing additional upward pressure on college wages, college workers would be concentrated in high-paying, abstract-intensive jobs to generate the observed rise in the college wage premium.

Second, I quantify the contribution of the recruitment channel to the rise of the college wage premium from 1980 to 2020. I find endogenous recruitment would mitigate over 40 percent of the wage effect of the productivity channel when colleges expand. Accounting for both technical changes and college expansion, the rise in the college wage premium would have been reduced by 5.6 percentage points without the recruitment channel. This reduction is equivalent to 12.6 percent of the increase in relative wages of college workers from 1980 to 2020.

To mitigate the damage of job displacement due to changes in technology and worker composition, policymakers sometimes implement active labor market policies to support displaced workers (Card et al., 2018). In the third exercise, I evaluate and compare the effects of two active labor market policies targeting non-college workers suffering from job displacement: job search assistance and employment subsidies. Both policies reduce unemployment and increase wages for non-college workers, but their effects on job creation and allocation differ significantly. Employment subsidies directly benefit firms and encourage vacancy creation and job allocation to non-college workers. In contrast, since search assistance increases non-college workers' wages through the recruitment channel but does not directly benefit firms, firms cut back on the costly hiring of non-college workers. This reduces vacancy creation and job allocation to non-college workers.

Related literature. I make several contributions to the literature in this paper. First, I contribute to the growing literature on recruitment's impact on the labor market. Following the pioneering work by Abraham (1983), aggregate vacancies have become a critical measure in the analysis of the labor market. While earlier work focuses on the job creation aspect of the hiring process, Davis et al. (2013) document heterogeneity in vacancy filling rates beyond the total number of vacancies posted. This has led to new advancements in both theoretical models and empirical analyses regarding employers' recruitment methods.[4]

Kaas and Kircher (2015), Gavazza et al. (2018), and Leduc and Liu (2020) present equilibrium models that examine how firms adjust their recruiting efforts in response to business cycle fluctuations. For instance, Gavazza et al. (2018) shows that aggregate recruitment efficiency is procyclical as it responds positively to labor market slackness and aggregate job creation variations. In this paper, I add to the theoretical framework and illustrate that recruitment also reacts to vacancy composition. Birinci et al. (2024) examine how increased applications cause firms to seek services from search intermediaries to mitigate information friction and improve their hiring quality. In this paper, I study the role of recruitment as a match facilitator in the labor market, another important function of search intermediaries. In addition, while previous literature focuses on recruitment's impact on labor market dynamics, I contribute by theoretically establishing a relationship between recruitment and wages.

Second, this paper contributes to the broad literature on the rising college wage premium by studying the role of evolving search technology. While the rise in the college wage premium has been well-studied in the literature over the past decades, most of the focus is on technology-induced changes in labor demand. For instance, in earlier work, studies focus on the enhanced factor-augmenting technology and capital-skill complementary (Katz and Murphy, 1992; Krusell et al., 2000; Card and Lemieux, 2001). The literature evolves and emphasizes the roles of task-based technical changes and, more recently, the impact of automation (Autor et al., 2003; Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018). For in-

[4]See Mueller et al. (2024); Mongey and Violante (2019); Lochner et al. (2021); Kuhn et al. (2021); Carrillo-Tudela et al. (2023) for other empirical evidence.

stance, Acemoglu and Restrepo (2022) document that the displacement of workers in routine-intensive jobs by automation can account for 50 to 70 percent of changes in the U.S. wage structure since 1980. Despite the extended findings from the labor demand side, we have a limited understanding of how the search technology can also appear skill-biased and affect wage inequality and worker allocations across different jobs. In this paper, I extend the task-based framework of Acemoglu and Restrepo (2018) with competitive search (Moen, 1997) and examine how the responses of search intermediaries affect relative wages and employment allocations across heterogeneous jobs. In particular, the recruitment channel provides a potential mechanism to increase the relative wages of college workers without having college workers allocated to more productive jobs.

In addition to the college wage premium, I also contribute by emphasizing the role of college expansion in understanding the changes in employment allocation within worker types. While the phenomenon of employment polarization in the aggregate labor market is well-documented in the literature (Goos and Manning, 2007; Goos et al., 2014; Autor and Dorn, 2013), we do not observe college workers today more concentratedly employed at high-paying, abstract-intensive jobs compared to their counterparts in the 1980s. In other words, the task difference between college workers and non-college workers has been shrinking since the 1980s. Since the technical change mechanism would predict greater differences in tasks performed by college and non-college workers, the observed shrinkage in task differences is caused by college expansion. Moreover, I show that endogenous recruitment is quantitatively essential to deliver the observed shifts of college workers toward less abstract-intensive jobs.

Finally, this paper proposes another possibility that an increasing relative supply can lead to a rise in relative wages. Acemoglu (2002) presents an equilibrium framework of directed technical change to show that factor-biased technological innovation can be induced by the increased supply of that factor. This is because of the increasing return of scale nature of the innovation process as new technology can be shared among many workers. When the elasticity of substitution between factors is sufficiently large, biased innovation can generate a long-run relative demand curve that is upward-sloping.

The model in this paper shares the spirit of directed technical change but applies this to the innovation of search technology. A key difference between the recruitment channel and the directed technical change is that the recruitment channel does not directly affect the labor productivity of jobs. Hence, the recruitment channel enables the possibility of relative wages going up without increasing the average labor productivity of the worker type. Shephard and Sidibe (2019) presents an equilibrium model to show that an increased supply of educated workers can shift the vacancy distribution toward more productive jobs and increase wage inequality. The recruitment channel can complement this mechanism by providing an additional force to inequality without allocating college workers to more productive jobs and non-college workers to worse jobs.

This paper proceeds as follows. In Section 2, I present some motivating evidence of the shrinking task difference between college and non-college workers. Section 3 presents the model of endogenous recruitment and some stylized empirical support of the recruitment channel. Section 4 presents the calibration strategy and the quantitative exercises. Section 5 concludes.

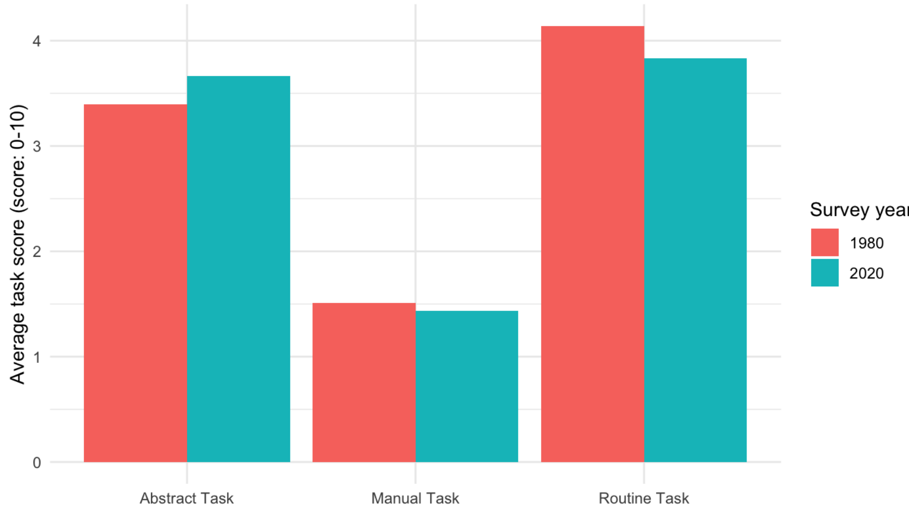
2 Motivating Facts

In this section, I present the motivating facts that the task contents of college and non-college workers have shrunk since 1980. I show that the "hollowing out" phenomenon in employment allocation — the increases in employment shares in high- and low-income occupations while shares in middle-income occupations fell — was mainly a result of composition changes due to the increased supply of college-educated workers. An average college worker today works at jobs that require less abstract task inputs, which college-educated workers should have comparative advantages in performing.

It is well-documented that the employment allocations across occupations of workers have undergone substantial changes since 1980. Autor et al. (2003) and Autor and Dorn (2013) present evidence on the phenomenon of "Hollowing out" or "employment polarization" in the US economy.[5] This refers to the fact that ag-

[5]See Goos and Manning (2007); Goos et al. (2009, 2014) for empirical studies on employment

Figure 1: Aggregate changes in the average task content between 1980 and 2020.



Note: This figure compares the average task scores of U.S. workers in 1976-1980 and 2016-2020 by education levels using the data from the CPS ASEC. The task score of each occupation is provided by Autor and Dorn (2013) and is measured along three dimensions: abstract, routine, and manual. The left panel shows the average task contents of workers without a college degree. The right panel shows the task contents of college workers. The red columns indicate the average task score in 1976-1980, while the blue columns present the score in 2016-2020.

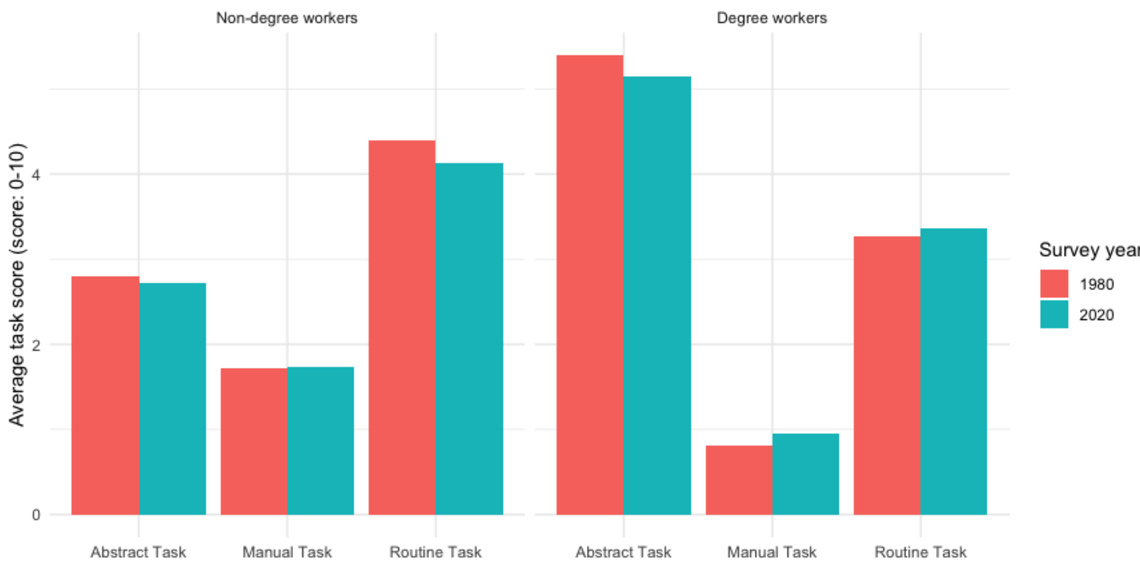
gregate employment shares in middle-income occupations shrank relative to their levels in 1980, whereas employment shares of higher- and lower-income occupations were expanding. To put this phenomenon in a task-based framework, one can observe an increase in employment shares in occupations that focus on performing abstract tasks and a falling share in occupations that heavily involve the performance of routine tasks.

Autor and Dorn (2013) provides the task content measures of each occupation by categorizing tasks into *abstract*, *routine*, and *manual* tasks. Based on the share of tasks in each category, the occupation is given a score ranging from 0 to 10, with a higher score indicating that the job has a larger share of task components.[6] Adopting this task measure, I compute the average task contents of workers in 1980 and compared it to that in 2020 using the Current Population Survey (CPS) Annual Social and Economic Supplements (ASEC) from the U.S. Bureau of Labor

polarization in other countries.

[6] Autor and Dorn (2013) computes the task scores based on the *Dictionary of Occupational Titles* by the U.S. Department of Labor in 1977.

Figure 2: Changes in task content between 1980 and 2020 by education levels.



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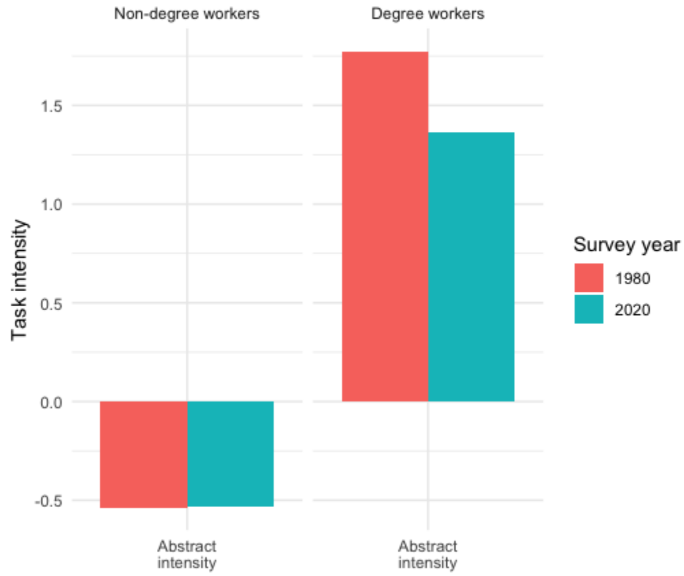
Statistics (BLS). The result is displayed in figure 1.

At the aggregate level, we observe an increase in the performance of abstract tasks and a fall in routine task contents for today's workers than in 1980, as documented in Autor and Dorn (2013). However, if we look at the changes in task contents separately for degree and non-degree workers across decades, as shown in figure 2, the increase in abstract task contents is absent for both types of workers. Hence, we do not observe apparent employment shifts toward jobs that contain higher abstract task contents within worker groups. In other words, the observed increase in abstract task content in the aggregate is mainly due to a compositional increase in the share of college workers in the labor market.

Of course, figure 2 accounts for each task separately but in reality tasks are bundled in each occupation. To better compare the task contents of the worker today relative to their 1980 counterparts, I compute the abstract task intensity (ATI) of each occupation in the spirit of the routine task intensity in Autor and

Figure 3: Decline in abstract task intensity from 1980 and 2020 for college workers.



Note: This figure compares the average task intensity (ATI) of U.S. workers in 1976-1980 and 2016-2020 by education levels using the data from the CPS ASEC. Following Autor and Dorn (2013), the abstract task intensity of occupation is computed using the task scores with the expression: $ATI = \ln(Abstract) - \ln(Routine) - \ln(Manual)$. The red columns indicate the ATI in 1976-1980, while the blue columns present the average in 2016-2020.

Dorn (2013). Specifically, the abstract task intensity of an occupation is given by

$$ATI = \ln(Abstract) - \ln(Routine) - \ln(Manual).$$

Comparing the average ATI level of degree workers in 1980 and 2020 in figure 3, we can see that college-educated workers today are working at jobs that are less abstract intensive. Meanwhile, since non-degree workers in recent years are working at jobs that are at a similar level of ATI as in 1980, the relative gap in ATI between these two types of workers shrank over time. Hence, as college-educated workers on average have a comparative advantage in productivity over non-college workers in performing abstract tasks, average college workers have less comparative advantage at jobs over non-college workers compared to 1980.

The fact of shrinking task differences between college and non-college workers gives rise to the following question: how do we reconcile the facts that college workers today are performing more similar tasks as non-college workers, but the college wage premium has been rising? I propose a new channel in the following section that the college workers benefit from endogenous recruitment activities, on top and over technical changes, which allows them to work at less specialized jobs but still earn a higher wage.

3 Model

In this section, I present a labor market model with endogenous recruitment efficiency. The model features heterogeneous jobs and workers, and a third agent in the form of a search platform to endogenously decide the recruitment efficiency in the labor market. The model serves three main purposes: (1) to provide a mechanism to reconcile the facts of the rising college wage premium and shrinking task differences; (2) to demonstrate how the endogenous response of recruitment efficiency reacts to labor demand and supply changes; (3) to provide a quantitative framework for counterfactual exercises.

3.1 Environment

Time is continuous and agents discount the future with rate r. There are three groups of risk-neutral agents: 1) two types of workers $i \in \{H, L\}$, degree worker H and non-degree worker L, who decide what vacancy j to apply for; 2) firms, which create heterogeneous vacancies $j \in [0, 1]$ from an exogenous continuous distribution Φ and decide which worker type i to target and what wages to offer given the vacancy type j created; 3) search platform, which decides the level of recruitment efficiency for each worker type μ_i and fill vacancies for firms. Wages are posted and search is directed. Hence, a submarket i-j is defined as worker type i looking for vacancy type j. Denote n_i the total measure of worker type i, u_{ij} the share of worker type i searching for vacancy j and v_j the measure of vacancy j created. Search technology follows a constant-return-to-scale matching function, with total number of matches M_{ij} between worker type i and vacancy type j given by

$$M_{ij} = \mu_i (n_i u_{ij})^{\sigma} v_j^{1-\sigma} \tag{1}$$

where σ is the matching elasticity with value between zero and one. The matching function states that total matches within submarket *i*-*j* is proportional to the total number of worker i ($n_i u_{ij}$) searching for job j and the total number of vacancy j created (v_j).

Workers of different type i have heterogeneous match-specific productivity y_{ij} across different job types j, where j is standardized to be between zero and one. Match-specific productivity y_{ij} for both worker type i are increasing in job type j. Specifcally, we have the following assumption on relative productivity.

Assumption 1 *Match-specific productivity y_{ij} is log-supermodular. That is for all j, $j' \in [0, 1]$ and $j < j'$,*

$$y_{Hj'} \cdot y_{Lj} \geq y_{Hj} \cdot y_{Lj'}.$$

In other words, we have $d \ln y_{Hj}/dj \geq d \ln y_{Lj}/dj$ for all j and thus the relative productivity y_{Hj}/y_{Lj} is increasing in j. This assumption on labor productivity ensures we have positive assortative matching in the equilibrium (Eeckhout and Kircher, 2010). Vacancy creation and the sorting of workers will be discussed in detail later when we discuss the behavior of firms.

3.2 Worker

There is a unit measure of workers in the economy. They are classified into two types. n_H of them are degree worker H; and $n_L = 1 - n_H$ of them are non-degree workers L. Here, the share of college workers is exogenous determined. Although whether to receive higher education is clearly an endogenous choice by individuals, the question on how career choice might be affected changes in recruitment efficiency is beyond the scope of this paper. The main focus of the model is on the behavior of job creation and recruitment activities. From the perspectives of firms and search intermediaries, it is a reasonable starting point to model labor supply choice as exogenous.

Workers can be employed or unemployed. Denote U_{ij} the expected value of an unemployed worker i looking for job type j; and E_{ij} the expected value of a worker i employed at job j. The availability of heterogeneous job j will be discussed in the later part While workers are unemployed, they receive an exogenous flow unemployment benefit b. Workers know their own type and decide which vacancy type j to search for. A worker type i searching for job j would then match with a vacancy at the Poisson rate $p(\theta_{ij}; \mu_i, n_i) = \frac{M_{ij}}{n_i u_{ij}} = \mu_i(\theta_{ij}/n_i)^{1-\sigma}$, where $\theta_{ij} = v_j/u_{ij}$ is the market tightness in submarket i-j. Since workers are atomic in the model and n_H is exogenously determined, they take the recruitment efficiency μ_i as given. The Bellman equation of an unemployed worker i searching for job j is given by

$$rU_{ij} = b + p(\theta_{ij}; \mu_i, n_i)(E_{ij} - U_{ij}).$$

When a worker i is employed at job j, the person receives wages w_{ij} and the match might be destroyed exogenously at a Poisson rate δ. Hence, the Bellman equation of a worker i is employed at job j is given by

$$rE_{ij} = w_{ij} - \delta(E_{ij} - U_{ij}).$$

Unemployed workers maximize their expected value by choosing the optimal job type j to look for. Since workers can observe all posted wages and decide which vacancy type j to search for, worker's decision of which submarket to look for job follows a reservation utility rule. Specifically, suppose a submarket i-j delivers an expected value of unemployment U_{ij}. An unemployed worker of type i will search for an alternative job j' if and only if the expected value of searching for job j $U_{ij'}$ is at least as high as expected level U_{ij}.

If an unemployed worker i searches for both job types j and type j', it must be the case that both type j and j' offer the same expected value, i.e. $U_{ij} = U_{ij'}$. Following this logic, worker i searches for type j, i.e. $u_{ij} > 0$, if and only if the expected value of searching for job j deliver at least a market utility level U_i, i.e. $U_{ij} \geq U_i$. For all job type j' that $j' \neq j$, and both $u_{ij'} > 0$ and $u_{ij} > 0$, the expected value of searching for job type j' equals U_{ij} and market utility U_i, i.e. $U_{ij'} = U_{ij} = U_i$. Using the Bellman equations, we can obtain the **worker indifference condition** that governs the worker search decisions:

$$rU_i = \frac{(r+\delta)b + p(\theta_{ij}; \mu_i, n_i)w_{ij}}{r + \delta + p(\theta_{ij}; \mu_i, n_i)} \quad \Rightarrow \quad w_{ij} = \frac{r+\delta}{p(\theta_{ij}; \mu_i, n_i)}(rU_i - b) + rU_i. \tag{2}$$

The market utility value U_i of worker type i will then be endogenously decided in equilibrium.

3.3 Firm

Entry. Firms are responsible of creating vacancies, deciding which worker type i to recruit for the job, and determining wages that they would offer for the job. First, firms decide whether to enter and create a vacancy by paying a fixed cost $\kappa > 0$. One can also consider this as creation of a single-worker firm. Once a firm enters and pays κ, the job type j will be randomly drawn from the exogenous continuous distribution Φ with density ϕ_j. As mentioned above, each job type j has a heterogeneous match-specific productivity y_{ij} conditional on the worker type i that the job would eventually matched with.

Market opening. Second, after job j is created, the firm can decide on whether to post this vacancy j just created on the search platform and recruit a worker for it. Any posted vacancy will incur a maintenance flow cost $k > 0$. Not all job j are posted at the platform to be filled. Firms can also decide to not recruit a worker for the job. Suppose firms obtain an outside option V_0 when they decide not to post the job. This outside option can either be simply destroying the job and get a zero value ($V_0 = 0$), or an alternative to assign the job to a different factor like machines for some exogenous non-negative values V_M ($V_0 = V_M \geq 0$). Denote $V_j = \max\{V_{Hj}, V_{Lj}\}$ the expected value of job type j preformed by human labor, i.e. the maximum expected value of having vacancy j to be filled by worker type H or L. Job j will be posted to recruit a human labor if and only if $V_j \geq V_0$. Hence, there exists a **market opening threshold** j_0 such that $V_{j0} = V_0$.

Sorting and wage determination. Third, once the firms choose to post job j on the platform to recruit, they decide on the type of workers i they want for the job and the corresponding wage w_{ij} they wish to offer. Apart from the maintenance flow cost k, firms will have to pay the search platform a flat fee ρ when the vacancy is filled. Given the search technology, an open vacancy j that target worker type i is filled with Poisson rate $q(\theta_{ij}; \mu_i, n_i) = M_{ij}/v_j = \mu_i n_i^{\sigma}(\theta_{ij})^{-\sigma}$. Note that since a single vacancy of type j is atomic in the labor market, the optimization problem for a job type j takes the recruitment efficiency for both types and the exogenous share of worker n_H as given. To keep the notation concise, I omit μ_i and n_H in the arguments of the match probability from here onward as long as it does not generate confusions. The Bellman equation of a vacancy j to hire a worker i is given by

$$rV_{ij} = -k + q(\theta_{ij})(J_{ij} - V_{ij} - \rho),$$

and that of a filled vacancy is given by

$$rJ_{ij} = y_{ij} - w_{ij} - \delta J_{ij}.$$

Combining these equations, the value of vacancy j that targets worker i is

$$V_{ij} = \frac{q(\theta_{ij})}{r + q(\theta_{ij})} \left(\frac{y_{ij} - w_{ij}}{r + \delta} \right) - \frac{q(\theta_{ij}\rho + k}{r + q(\theta_{ij})}. \tag{3}$$

The first component of equation 3 refers to the expected match surplus that job j generates when it matches with worker type i. It increases with the probability of job being filled and reduces with the wages given the level of y_{ij}]. The second component consists of the total search cost, including the vacancy maintenance flow cost and the placement fee to the search platform once the position is filled.

Since wages are posted on the search platform, workers can observe all wages and choose which job to search for, as in Moen (1997). For a job j that targets worker i, firm maximizes the expected of vacancy V_{ij} subject to the worker i's indifference condition, while taking equilibrium object $\{U_H, U_L, \mu_H, \mu_L\}$ as given. Specifically, wage determination solves the following problem:

$$\max_{\{\theta_{ij}, w_{ij}\}} V(w_{ij}, \theta_{ij}) = \frac{q(\theta_{ij})}{r + q(\theta_{ij})} \left(\frac{y_{ij} - w_{ij}}{r + \delta} - \rho \right) - \frac{k}{r + q(\theta_{ij})}$$

$$\text{s.t. } w_{ij} = \frac{r + \delta}{p(\theta_{ij})}(rU_i - b) + rU_i \quad \text{(Worker Indifference Condition).}$$

By substituting out w_{ij} from the objective function, we obtain the first-order condition of the wage determination problem with respect to θ_{ij}.

$$\frac{r+(1-\sigma)q(\theta_{ij})}{p(\theta_{ij})}(rU_i - b) = \sigma\left[r\left(\frac{y_{ij}-rU_i}{r+\delta}-\rho\right)+k\right]. \tag{4}$$

This equation gives the relationship between y_{ij} and θ_{ij}. Given the market utility U_i and recruitment efficiency μ_i, a job with higher y_{ij} chooses a lower θ_{ij}, thus a higher $q(\theta_{ij})$ and w_{ij}. This follows a classic result in the directed search literature, that more productive jobs have a higher opportunity cost of staying vacant and were eager to be filled faster by offering higher wages. Therefore, while these jobs are better paid, they are also harder to match with from workers' perspective.

With this setup, a firm will recruit a college worker H for job j if and only if the expected value of recruiting type H for job j is greater or equal to the value of recruiting job L, i.e. $V_{Hj}(U_H, \mu_H) \geq V_{Lj}(U_L, \mu_L)$, given market utility U_i and recruitment efficiency μ_i. Moreover, equilibrium sorting will follow a threshold rule.

Proposition 1 *Given assumption 1, we have positive assortative matching. Specifically, for $j^* \in [0,1]$ such that $V_{Hj^*}(U_H, \mu_H) = V_{Lj^*}(U_L, \mu_L)$, all job $j \geq j^*$ would be assigned to college worker H, i.e. $V_{Hj}(U_H, \mu_H) \geq V_{Lj}(U_L, \mu_L)$ for all $j \geq j^*$. The value j^* is thus the* ***sorting threshold****.*

Proof — See appendix.

Overall, the total number of vacancy v created is pinned down by the **entry condition**:

$$\bar{V}(U_L, U_H, \mu_H, \mu_L) = \Phi(j_0)V_0 + \int_{j_0}^{1} \phi_j \max\{V_{Lj}, V_{Hj}\}\, dj = \kappa, \tag{5}$$

where $\Phi(\bar{x}) = \Pr(x < \bar{x})$ is the cdf of the vacancy distribution. Intuitively, a firm will create new vacancies until the *ex-ante* expected value of a vacancy equals the fixed cost κ.

3.4 Search Platform

In this model, firms do not directly recruit workers. Instead, all created job vacancies need to be posted on the search platform to be filled. The platform decides the recruitment efficiency μ_i on each worker type $i \in \{H, L\}$ and earns a flat and exogenous fee $\rho > 0$ from the firms for each vacancy successfully filled. The platform also faces a total recruitment cost function $c(v_H, \mu_H, v_L, \mu_L)$, where v_i is the total number of vacancies that target worker type i. Hence, the platform chooses the optimal recruitment efficiency μ_i for each worker type i by solving the following problem.

$$\max_{\mu_H, \mu_L} \pi_p(\mu_H, \mu_L) = \int_{j_0}^{j^*} v_j P_{Lj}(\mu_L) dj + \int_{j^*}^{1} v_j P_{Hj}(\mu_H) dj - c(v_H, \mu_H, v_L, \mu_L),$$

where P_{ij} is the value of a posted vacancy j that targets worker i and is derived from its Bellman equation

$$rP_{ij}(\mu_i) = q(\mu_i; \theta_{ij}, n_i)(\rho - P_{ij}(\mu_i)) \quad \Rightarrow \quad P_{ij}(\mu_i) = \frac{q(\mu_i; \theta_{ij}, n_i)\rho}{r + q(\mu_i; \theta_{ij}, n_i)},$$

taking the firm's choice of market tightness θ_{ij} as given.

The first order condition of the platform's optimization problem helps pin down the μ_i in equilibrium for each worker type i:

$$r\rho \underbrace{v}_{\substack{\text{Total}\\\text{Vacancies}}} \underbrace{\int_{j \in J_i} \phi_j}_{\text{Composition}} \underbrace{\frac{q_{\mu_i}(\theta_{ij})}{\left(r + q(\theta_{ij})\right)^2}}_{\substack{\text{Marginal value}\\\text{of job post}}} dj = c_{\mu_i}(v_H, \mu_H, v_L, \mu_L), \tag{6}$$

where J_i is the range of jobs assigned to type i, i.e. $J_L = [j_0, j^*]$ and $J_H = [j^*, 1]$.

Equation 6 gives the relationship where the marginal revenue of enhancing μ_i equals the marginal costs in equilibrium. The marginal benefit on the left-hand side can be decomposed into three components. First, the aggregate effect is that the marginal benefit increases with total vacancies created by the firm. This is the procyclical component that the matching efficiency improves when more vacancies are created in the aggregate, as discussed in Gavazza et al. (2018). Second, marginal benefit increases with the composition effect, that is when the share of vacancies allocated to a particular worker type increases. This creates a key asymmetric impact on different worker types as the sorting decision of the jobs changes. As more share of vacancies are allocated to a worker type, improving the recruitment efficiency for this particular worker type is profit-enhancing as there are more potential placement fees to be earned. Third, the recruitment efficiency also responds to changes in the marginal value of posted vacancy j, which is decreasing in the job-filling probability $q(\theta_{ij})$ and the recruitment efficiency μ_i. If the submarket i-j is slack, i.e. θ_{ij} was low, the job-filling probability conditional on the recruitment efficiency would be high. This enhanced filling probability reduces the marginal value of the posted vacancy. This would induce the search platform to reduce recruitment efficiency in finding the worker for these jobs since the job-filling probability is already adequately high due to market conditions. This third effect of the marginal value of job posts is like the slackness effect described in Gavazza et al. (2018). However, since each job i created is atomic and the wage determination problem is solved by taking μ_i as given, the optimal θ_{ij} does not vary much when there is a composition change to the labor market under free entry.

How recruitment efficiency μ_i changes in equilibrium also depends on how the marginal cost varies with v_i and μ_i. The total recruitment cost function $c(v_H, \mu_H, v_L, \mu_L)$ takes the following functional form:

$$c(v_H, \mu_H, v_L, \mu_L) = c^r_H v^\eta_H \mu_H + c^r_L v^\eta_L \mu_L, \tag{7}$$

where $\eta \geq 0$; $v_i = v \int_{J_i} \phi_j dj$ is the total number of vacancies assigned to worker type i; $c^r_i > 0$ are unit cost of μ_i given v_i. The total recruitment cost is increasing in both v_i and μ_i. Although μ_i is linear in the total cost function, as the marginal revenue is diminishing in μ_i, there exists a solution for the equilibrium μ_i. The scaling parameter η determine how the marginal costs of recruitment efficiency $c_{\mu_i}(v_H, \mu_H, v_L, \mu_L)$ changes with v_i. Given this functional form, the marginal recruitment cost with respect to μ_i is $c_{\mu_i}(v_H, \mu_H, v_L, \mu_L) = c^r_i v^\eta_i$.

The scaling parameter η is essential in affecting how recruitment efficiency responds to the composition of workers and sorting of jobs as it determines how the marginal cost changes with the additional vacancy created for worker type i. If $\eta = 1$, the recruitment cost is constant return to scale with respect to vacancies. The marginal cost of recruitment also increases linearly to v_i. Hence, the aggregate effect and the composition effect on the marginal revenue would be canceled out by the increases in marginal cost and thus μ_i in equilibrium might not be affected by a change in the allocation of vacancies. Intuitively, when η, any additional vacancy would incur the same total cost to be filled at the same recruitment efficiency μ_i as previous vacancies of the same type. Put this in the scenario as a recruiter, this implies the recruiter would start from scratch to fill any additional vacancy that requires the same workers. She would have to set up the whole hardware system and her previously accumulated candidate pool would not be available. On the contrary, if $\eta = 0$, the total recruitment cost is independent of the number of vacancies. This refers to a scenario where vacancies and workers were matched by an automated algorithm. Once the efficiency of this algorithm was enhanced, this upgrade allowed all vacancies to be filled faster. In this case, μ_i would be very responsive to the changes in the allocation of jobs across worker types. In reality, η should lie somewhere between 0 and 1 as job matches are neither formed fully automatically by an algorithm nor do recruiters need to start locating candidates from scratch again. When $\eta < 1$, the marginal cost per vacancy is decreasing in total vacancy and the search platform is operating an increasing-return-to-scale recruitment technology. In other words, the recruitment channel is active when $\eta < 1$. Due to the centrality of the recruitment cost function in the overall effect of the recruitment channel, we discuss how the cost function is disciplined by data in the calibration section.

3.5 Equilibrium

The model is solved at the steady state distribution. Before I define the equilibrium, I discuss the steady state distribution of vacancies, employment and unemployment. First, since the vacancy distribution is determined exogenously, the total measure of vacancy type j equals the total vacancies created times the density ϕ_j, specifically $v_j = v\phi_j$. Second, the steady state employment distribution e_{ij} of worker i at job j can be solved by equating the outflow of inflow into job j

$$\delta e_{ij} = v_j q(\theta_{ij}) \quad \Rightarrow \quad e_{ij} = \frac{v_j q(\theta_{ij})}{\delta}. \tag{8}$$

Likewise, the steady state unemployment distribution is solved from its law of motion. Denote u_{ij} the share of unemployed worker i looking for job type j and it is given by

$$u_{ij}p(\theta_{ij}) = (1 - u_i)\delta \frac{e_{ij}}{\int_{j \in J_i} e_{ij}\, dj}. \tag{9}$$

The total unemployment rate of worker type i (u_i) is the sum of u_{ij} across all $j \in J_i$, i.e. $u_i = \int_{j \in J_i} u_{ij} dj$, where J_i is the range of job j that assigned to worker type i. Finally, by substituting the steady state distribution of v_j, e_{ij} into the expression of u_{ij}, we can derive the implied total vacancy created v from the steady state distribution of worker type i:

$$v = \frac{n_i(1 - u_i)\delta}{\int_{j \in J_i} \phi_j q(\theta_{ij})\, dj}. \tag{10}$$

Market clearing then requires that the implied total vacancy derived from the distributions of both types of worker to consistent. Hence the equilibrium of the model is defined below

Definition 1 *Equilibrium of the model is a set of* $\{w_{ij}, \theta_{ij}\}_{i \in \{H,L\},j}$ *and* $\{U_H, U_L, j^*, j_0, \mu_H, \mu_L\}$ *that satisfy the following conditions:*

1. θ_{ij} *solves the* ***firm's problem*** *and maximize profit in submarket ij, given* U_i *and* μ_i*;*

2. w_{ij} *satisfies the* ***worker indifference condition*** *given* U_i*,* μ_i *and* θ_{ij}*;*

3. ***Optimal recruitment intensity*** μ_H, μ_L *satisfy* ***FOCs*** *of search platform;*

4. ***Market opening condition****:* $V_{Lj_0}(U_L, \mu_L) = V_0$ *;*

5. ***Sorting condition****:* $V_{Lj^*}(U_L, \mu_L) = V_{Hj^*}(U_H, \mu_H)$*;*

6. ***Entry condition****:* $\Phi(j_0)V_0 + \int_{j_0}^{j^*} \phi_j V_{Lj}(U_L)\, dj + \int_{j^*}^{1} \phi_j V_{Hj}(U_H)\, dj = \kappa$*;*

7. ***Market clearing condition****:*

$$\frac{n_H(1 - u_H)\delta}{\int_{j^*}^{1} \phi_j q(\theta_{Hj})\, dj} = \frac{(1 - n_H)(1 - u_L)\delta}{\int_{j_0}^{j^*} \phi_j q(\theta_{Lj})\, dj}.$$

3.6 Mechanism of the recruitment channel

The model introduces a third agent in the form of a search platform in the labor market. This enables the recruitment responsibility to be split from firms, which are responsible for the creation of vacancies and wage determinations. The independence of the search platform allows it to endogenously react to changes in the allocation of jobs and composition of workers. Consequently, its decision on recruitment efficiency affects various equilibrium outcomes, including wages and unemployment.

To see how the recruitment channel works, let's consider the case of college expansion as the college share in the labor force increases. When college workers become more available, two things occur. First, job-filling probability with college workers is enhanced and the expected value of vacancies V_{Hj} targeting college workers rises. More jobs are assigned to college workers as firms want to take advantage of the enhanced availability of college workers. Hence, the sorting threshold j^* falls. However, since these newly reallocated jobs to college workers are of relatively lower match-specific productivity, it puts downward pressure on the average wages of college workers.

Meanwhile, the opposite occurs for non-college workers. Some jobs targeting these workers find it too costly to recruit non-college workers as they become scarce and exit the market. Hence, the market opening threshold j_0 rises. The remaining jobs for college workers are of relatively higher productivity and thus put upward pressure on their average wages. These combined effects form the productivity channel, which delivers how the relative price reacts to an increase in relative quantity in a conventional model.

Second, the reallocation of jobs toward college workers affects the optimal recruitment efficiency for each worker type. In the case of $\eta < 1$, the positive composition effect generates a net positive value on the marginal revenue of filling jobs with college workers. Hence, the search platform finds it more profitable to enhance μ_H, and job matches are more efficiently formed. This enhances the match surplus for filling jobs with college worker and puts upward pressure on their wages. Since the opposite occurs for non-college workers and the jobs assigned to them, the recruitment channel exerts a positive effect on the relative wages of college workers.

The overall effect on relative wage depends on the contributions of the productivity channel and the recruitment channel. Yet, even if the productivity channel dominates, an active recruitment channel will mitigate the effect of the productivity channel. The reduction in relative wages would be smaller than in a model when the recruitment is inactive. Moreover, if the recruitment channel dominates, an increase in the relative supply of college workers can increase relative wages.

3.7 Empirical evidence of the recruitment channel

I present several stylized evidence that the evolution of search technology is associated with the composition of labor supply. First, using the expansion of Craigslist as a case study, I document that online job boards expanded their coverage to locations where the college share was higher before expanding elsewhere. Second, I show that the relative matching efficiency of college workers to non-college workers, measured as the ratio of the job-finding probability after controlling the variation in market tightness, was trending upwards in past decades. This aligns with the increasing supply of college workers and the evolution of staffing agencies and online job boards. Third, I show that the matching efficiency of college workers is positively associated with the share of college workers in the local labor market.

3.7.1 Fact #1: Craigslist expansion sequences

How do college shares relate to the entry decision of search intermediaries? If the evolution of search technology is directed, search platforms should react to college expansion by increasing recruitment efficiency for college workers. In real life, while recruitment efficiency is not observed, we can look at the realized entry decisions of search intermediaries as their revealed preferences.

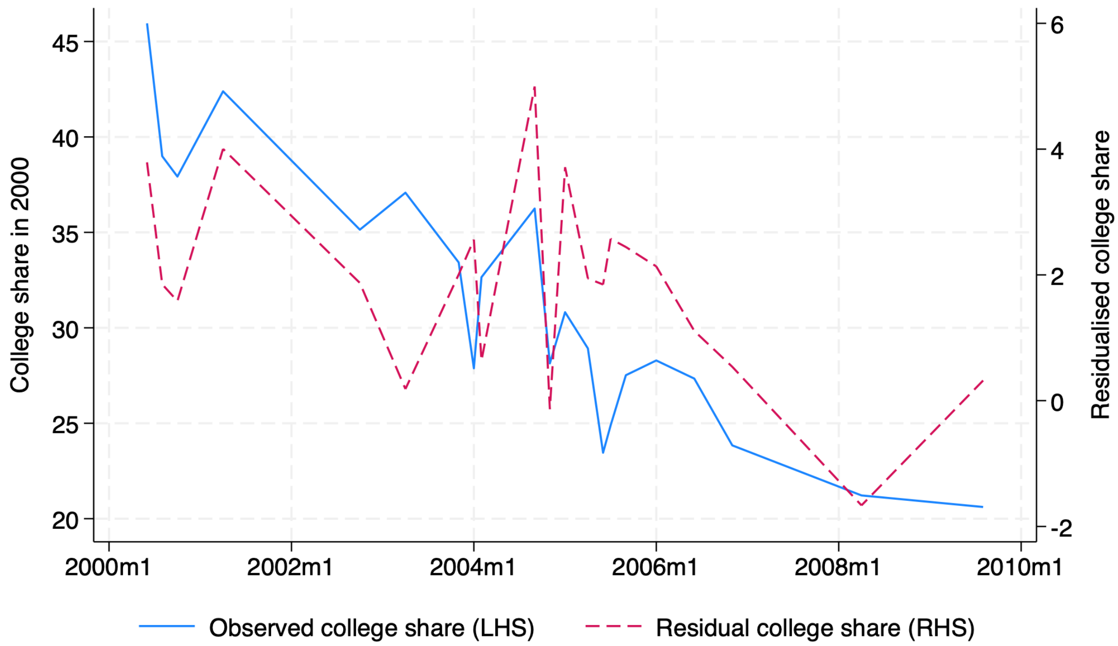
The subject in this exercise is Craigslist, an advertisement website to post for goods, services, jobs, and housing in their local areas. The website started in 1995 serving only the Bay Area in the United States. It expanded its coverage in the 2000s to numerous locations in the U.S. and other countries. By 2005, Craigslist covered 115 locations, and up to 416 locations in 2024. The subject in this exercise is Craigslist, an advertisement website to post for goods, services, jobs, and housing in their local areas. The website started in 1995 serving only the Bay Area in the United States. It expanded its coverage in the 2000s to numerous locations in the U.S. and other countries. By 2005, Craigslist covered 115 locations, and up to 416 locations in 2024. According to the website's founder Craig Newmark in an interview in 2004, Craigslist's expansion decision depends on "how many people are asking us to do so" and the team's "perception of a city's demographics" [7]. The expansion of Craigslist in the 2000s is a common subject to study for the impacts of online platforms on local markets. For instance, Kroft and Pope (2014) studies the impact of the introduction of Craigslist on traditional job search methods and labor market efficiency. Balgova (2024) investigates the impact of online job boards on labor market geography, finding that early access to online recruitment in U.S. cities increased migration flows and wages. Djourelova et al. (2024) examines how the introduction of Craigslist affected local newspapers' coverage of political news and voting preference.

In this exercise, I study the timing of the expansion between 2000 and 2010 and the college shares of the location in 2000 where Craigslist was expanding. I use the data on Craigslist's expansion at the county level collected by Djourelova et al. (2024). The local characteristics, including college shares, are collected from Census 2000. The results are presented in figure 4. It shows clearly Craigslist was expanding into locations where the college share was higher before entering elsewhere. For instance, earlier expansions were in counties where college share was above 40 percent in 2000. The average local college shares of counties Craigslist entered later were about 20 to 30 percent. For those areas where the website did not enter by 2010, the average college share was 19 percent.

Since the local college share is also strongly associated with other favorable factors for entry, I present a series where the college share is residualized with other local characteristics. These control variables include the number of internet service providers in the area, income, age, local population, the share of white,

[7] See the full interview here: https://www.sfgate.com/business/ontherecord/article/CRAIGSLIST-On-the-record-Craig-Newmark-2733312.php

Figure 4: Craigslist expanded first to regions with higher college share



Note: The geographical expansion sequence of Craigslist is from the data made available by Djourelova et al. (2024). College share (solid line) is measured as the share of the population with college education from the 2000 Census. The time stamp on the x-axis indicates the month Craigslist expanded its coverage into a county; the value on the y-axis indicates the average college share of the county Craigslist was entering. The residual college share (dashed line) is obtained from the regression of the county college share on other local characteristics, including income per capita, average age, total population, shares of white, black, and Hispanic ethnic groups, local population density, urban population share from the 2000 Census, and the number of Internet service providers (ISPs) collected by Djourelova et al. (2024).

black, and Hispanic ethnic groups, local population density, and urban population share. The result is presented in the dashed line in figure 4. While the residualized result isn't as prominent as the observed college shares, the general downward trend remains.

3.7.2 Fact #2: Changes in relative matching efficiency

The directed response of recruitment technology predicts that search intermediaries would respond to an increased supply of college workers by enhancing their recruitment efficiency in the labor market. Therefore, we should observe a persistent rise in the relative matching efficiency of college workers to non-college workers alongside the increased supply of college workers in recent decades. Moreover, if the evolution of search intermediaries, including the rise of recruitment agencies and online job boards, was an endogenous outcome of the increased supply of college workers, we should see the timing of the surge in relative matching efficiency coincide with the growing popularity of these search intermediaries.

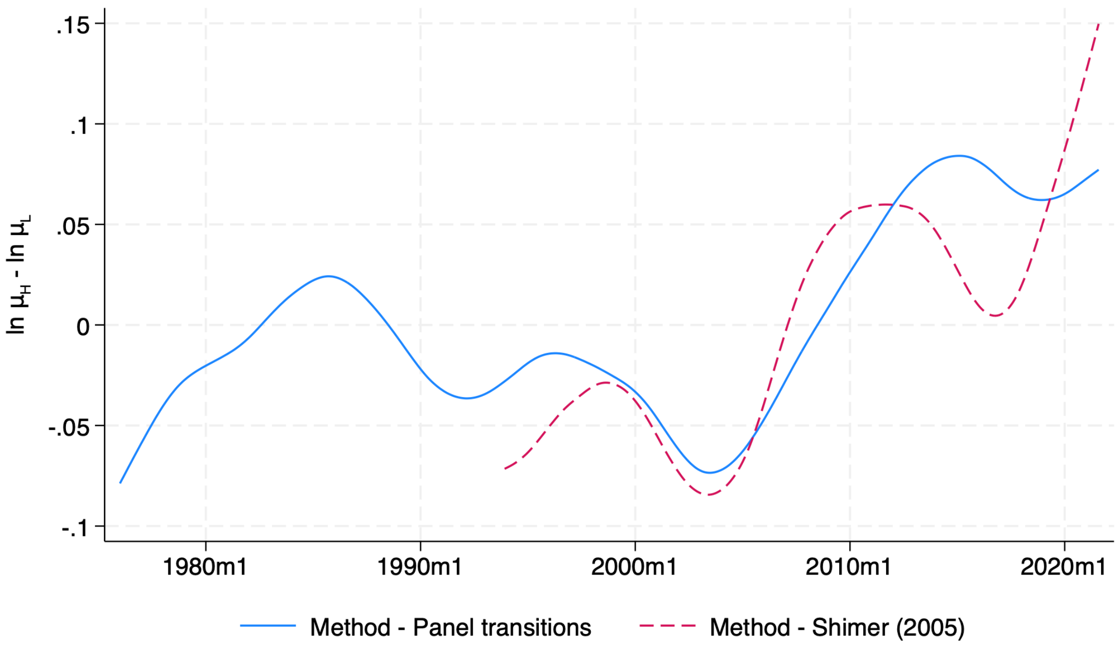
To measure the aggregate matching efficiency for college and non-college workers, I follow the conventional method in the literature by taking the residuals of an aggregate matching function that generates job matches given the number of job seekers and vacancies. I take the constant-return-to-scale Cobb-Douglas matching function of equation 1 defined in the model above. Applying a log transformation to job-finding probability $p(\theta_{ij}$, and we get

$$\ln p_{it} = (1-\sigma)\ln\theta_{it} + \ln\mu_{it} \qquad\qquad \text{for } i \in \{H, L\}$$

where H refers to college workers and L for non-college workers; and market tightness θ_{it} is the ratio of vacancies over job seekers.

To obtain the measure of the matching efficiency, we need the job-finding probability and market tightness for both types of workers. I compute the job-finding probability for each worker type from 1993 to 2020 from the monthly CPS of the U.S. Bureau of Labor Statistics using the method introduced in Shimer (2005). I obtain monthly vacancy data from the Job Openings and Labor Turnover Survey (JOLTS) of the Bureau of Labor Statistics and the Composite Help-wanted Index (HWI) in Barnichon (2010). Since vacancy data is collected in aggregate and does not directly distinguish whether the vacancy requires a college degree or not, I follow Barnichon and Figura (2015) and split the job openings of a given month between college and non-college workers according to their employment share of that month. As a robustness check, I repeat the exercise by splitting the vacancies by the employment share of occupations that require a bachelor's degree or above, based on the Typical entry-level educational requirement in the Occupational Employment and Wage Statistics of the Bureau of Labor Statistics. The market tightness is then computed as the ratio of vacancies to the number of unemployed workers from the CPS. The variations in the log of the aggregate matching

Figure 5: Trend of relative matching efficiency of college workers



Note: The relative matching efficiency is the log difference in matching efficiencies between college and non-college workers. The corresponding matching efficiencies are obtained from the residuals by regressing the log job-finding probability $\ln p_{it}$ of unemployed workers on the log market tightness $\ln \theta_{it}$. The solid line shows the relative matching efficiency obtained from the Panel transition in the monthly CPS data. The dashed line shows the relative matching efficiency obtained using the method of Shimer (2005) to account for time aggregation bias. The lines displayed in this figure are the trend components after applying the HP filter.

function of each worker type $\hat{\mu}_{it}$ can then be obtained by regressing the log of their job-finding probability to the corresponding market tightness. Finally, the relative matching efficiency between the two types of workers is the log difference between college matching efficiency and that of the non-college workers each month.

Figure 5 shows the evolution of the relative matching efficiency from 1976 to 2020, smoothed using the HP filter with a smoothing parameter of 14,400 for monthly data. First, the series shows a clear upward trend. This implies that the matching efficiency of college workers is increasing over time relative to non-college workers. Since the relative supply of college workers has been growing over the same period, this is consistent with the prediction of the recruitment channel.

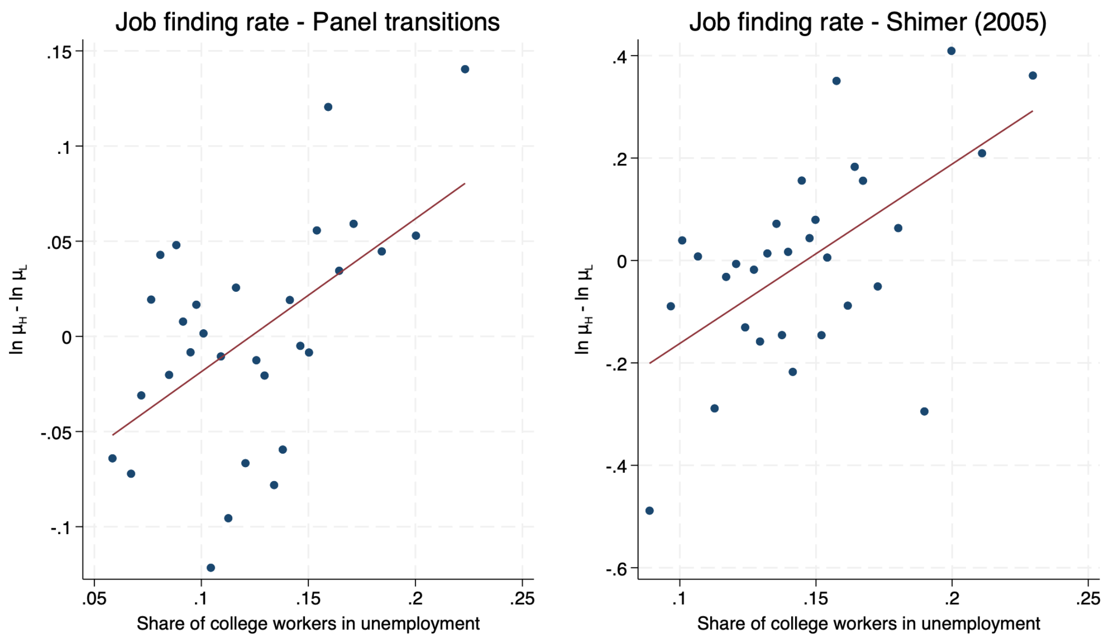
Second, the rise of relative matching efficiency also coincides with the time when search technology was advancing. There were three periods where the relative matching efficiency rose the most rapidly. The first period was in the 1970s and early 80s. Apart from technological advancements such as the adoption of commercial fax machines, this was also the period when courts in U.S. states rapidly adopted exceptions to the common law doctrine of employment-at-will. This limited employers' discretion to terminate workers and subsequently expanded businesses for staffing agencies (Autor, 2003). The second period was from 1995 to 1999. This was when personal computers and the World Wide Web were more generally adopted in households. Meanwhile, this was also when staffing agencies became more popular as the employment of temporary help services grew by 140 percent between 1990 and 2000, according to statistics from the Quarterly Census of Employment and Wages data from the BLS. The third period started in 2005. This was when online job boards and professional social networks gained popularity. For instance, Craigslist was expanding the fastest in 2005; and LinkedIn was launched in 2003 and reached 10 million users in 2007. In addition, De Leon et al. (2024) documents an 85 percent growth in the share of revenue that manufacturing establishments spent on staffing agencies from 2006 to 2017.

3.7.3 Fact #3: Matching efficiency and unemployment compositions

Having analyzed the long-term trends, I will now focus on the short-term response of relative matching efficiency to the availability of various worker types among job seekers. Figure 6 shows how the log difference of aggregate matching efficiency between college and non-college workers is associated with the share of college workers in the unemployed in the aggregate data. The matching efficiency is taken from the estimates above and the college share of unemployed is computed from the monthly CPS. The figure shows a statistically significant positive relationship. This indicates that the job-finding probability of the college workers, conditional on the market conditions, is higher when the college workers are more abundant among the pool of unemployed job seekers.

To check the robustness of this relationship, I extend the analysis to the state level for a panel setup, utilizing monthly job opening data from JOLTS for each state from 2000 to 2020. Utilizing the Shimer (2005) method illustrated above, I

Figure 6: Relative matching efficiency and college share of unemployment



Note: The left panel shows the matching efficiency obtained from the job-finding probability estimated using the panel transition in the monthly CPS from 1976 to 2020. The slope coefficient in the left panel is 0.803, with the standard error being 0.189. The right panel shows the matching efficiency obtained from the job-finding probability estimated using the Shimer (2005) method using monthly CPS from 1994 to 2020. The slope coefficient in the right panel is 3.503, with the standard error being 0.827.

estimate the matching efficiency of college and non-college workers in each state. I examine the relationship between relative matching efficiency and local college share of unemployment. Table 1 reports the estimated coefficients of different model specifications. Although the positive coefficient of the pooled regression in column 1 is statistically indifferent from zero, the positive estimate in column 2 becomes statistically significant when state and year fixed effects are added. Therefore, the within-variation in the matching efficiency is positively associated with the college share of unemployment. Column 3 reports estimates that control for each state's college share of the labor force. These findings indicate that even when accounting for labor force composition, a higher college share of unemployment correlates with higher relative matching efficiency.

Table 1: College share of unemployment on relative matching efficiency

Relative Matching Efficiency	(1)	(2)	(3)
Log College Share of Unemployment	0.0142	0.136**	0.336***
	(0.0481)	(0.0584)	(0.0417)
Log College Share of Labor Force			-1.223***
			(0.164)
Observations	5,820	5,820	5,820
Number of states		51	51
State FE		YES	YES
Year FE		YES	YES

Note: *** p<0.01, ** p<0.05, * p<0.1. Standard errors in parentheses clustered at the state level. The first column reports the estimated coefficient of pooled regression. The second column reports the coefficient estimated using the within estimator with year fixed effects. The third column reports the coefficient estimated using the within estimator with year fixed effects and added college share in the labor force as control.

4 Quantification

In this section, I quantify the importance of the recruitment channel in affecting wages and job allocations when college expands. I first discuss the parameterization of the model and the calibration strategy. The model is calibrated to replicate key features of the U.S. labor market in 1980. Then I present the results of the quantitative exercises.

4.1 Calibration

4.1.1 Parameterization

I first discuss the data measure for job types j in the model. Since the quantitative exercise aims to estimate how the recruitment channel has contributed to the changes in relative wages and allocation of tasks across worker types, job types are defined by the relative task score of abstract tasks and routine tasks in an occupation. Specifically, I define job type j as the relative abstract-routine intensity (ARI) of occupations as the difference between abstract and routine task content:

$$j = \ln(Abstract) - \ln(Routine) \tag{11}$$

Table 2: Externally Set Parameter Values

Parameter		Value	Target
Discount rate (monthly)	r	0.004	Annual rate 0.05
Matching Elasticity wrt job seekers	σ	0.5	Pissarides (2009)
Home production	b_H, b_L	0.614, 0.455	40% of wages (Shimer 2005)
Separation rate	δ_H, δ_L	0.005, 0.020	CPS
Entry cost	κ	2	Normalization
Placement fee	ρ	2.93	20% of annual wage.
L-type production shifter	A_L	1	Normalisation

Each occupation's task scores are taken from Autor and Dorn (2013). To allow the exogenous vacancy distribution to be flexible, I adopt a beta distribution $\mathcal{B}(\alpha, \beta)$ and calibrate the distribution parameters (α, β) using the data. The ARI measure is further standardized to range from zero to one to fit the support of the beta distribution. Hence, jobs with ARI j close to zero are the most routine-intensive jobs with little problem-solving components.

Second, I specify the functional form of the match-specific production function

$$y_{ij} = A_i \exp(\psi_i \cdot j),$$

for $i \in \{H, L\}$, where A_i is the production shifting parameter and ψ_i measures how a type i worker's productivity grow across job type j. Intuitively, A_i measures the productivity of worker i at $j = 0$. The difference in ψ_H and ψ_L gives how the relative productivity changes over j. For y_{ij} to be log-supermodular, we need $\psi_H - \psi_L > 0$.

4.1.2 Externally Calibrated

Discount rate r is set to 0.004, equivalent to an annual rate of 5 percent. In line with the practice in the literature, the elasticity of the matches with respect to the number of job seekers is set to 0.5, as in Pissarides (2009). The flow value of home production is set to 40 percent of the average wages of college and non-college workers, respectively. The placement fee ρ is set to be 20 percent of the first year's salary, which is the average level that recruitment firms charge for each placement in U.S.[8] The levels of average wages of each worker type are targeted in the internal calibration. The entry fixed cost is set to 2 as a normalization.

Exogenous separation rates are set to 2 percent for non-college workers and 0.54 percent for college workers. These are the average monthly separation rates in the 1990s estimated using the CPS following the method of Shimer (2005). Since individuals who were unemployed for less than 4 weeks cannot be identified in the monthly CPS before 1993, I use the earliest available separation rates for college and non-college workers for the calibration. The average separation rate, weighted by the employment rate in 1980, equals 1.6 percent, which is close to the aggregate employment-to-unemployment flow rate of 1.7 percent in 1980 as reported by Shimer (2005).

Finally, the productivity shifter of non-college worker A_L is normalized to 1. Hence, the productivity of non-college workers matched with the most routine-intensive job, i.e. $j = 0$, is equal to one. Table 2 summarizes the externally calibrated parameters.

4.1.3 Internally Calibrated

There are 10 internally calibrated parameters. They include the firm's outside option V_0, two gradient parameters ψ_i for each worker type's match-specific productivity, and the productivity shifter of college worker A_H, two parameters (α, β) for the vacancy distribution, vacancy maintenance flow cost k, and three parameters (c^r_H, c^r_L, η) associated with the recruitment cost function of the search platform.

Match-specific productivity and firm's outside option. The match-specific production function across different job types j and the vacancy distribution jointly determine the wage and employment distribution. The production gradient parameters ψ_H and ψ_L are pinned down by the mean value of the residualized wages

[8] The number of 20 percent is reported by multiple sources online. See https://gohire.io/blog/how-much-does-a-recruitment-agency-charge-by-country and https://eddy.com/hr-encyclopedia/recruitment-fees/. I also reconciled the number with current consultants in the recruitment industry.

in the CPS ASEC between 1976 and 1980. [9] I let the production shifting parameters A_H target the average ARI of college workers between 1976 and 1980 in the CPS ASEC. Since the firm's outside option V_0 determines the minimum expected return of hiring a non-college worker, V_0 is identified from the average ARI of non-college workers.

Vacancy distribution. The vacancy distribution is assumed to follow a Beta distribution, which contains two shape parameters (α, β). Since the vacancy distribution shapes the employment distribution, I allow the vacancy distribution to match the median and the 60th percentile of non-college workers in CPS ASEC between 1976 and 1980. I only use information on the employment distribution of non-college workers to identify the vacancy distribution because the employment distribution of college workers is left out as an untargeted moment in the counterfactual exercise.

Firm's hiring cost. There are two cost components when a vacancy is created. First, a vacant job incurs a maintenance flow cost k, which can be seen as the opportunity cost of leaving the job vacant. When the job is filled, the firm pays a one-off placement cost ρ to the search platform. Hence, the firms' expected total hiring cost equals $k + q(\theta, \mu)\rho$. To pin down the flow vacancy maintenance cost, I follow Pissarides (2009) by targeting the average aggregate market tightness in 1979, computed using the unemployment level reported by BLS and the vacancy level from the composite Help-Wanted Index by Barnichon (2010). The placement fee is then identified by the ratio of the total hiring cost to the average wage, which is reported to be 0.93 in Gavazza et al. (2018).

Search platform's recruitment cost. The recruitment cost function of the search platform contains three parameters. They are the efficient unit cost of recruitment for each worker type i (c^r_H, c^r_L) and the cost elasticity of vacancy η. In equation 6, we know that the marginal revenue to the search platform is diminishing in

[9] Wages are residualised by running a regression of individual log wages on demographic characteristics, including gender, age, and race, using the CPS ASEC data.

the recruitment efficiency. Since a higher c_i^r will be associated with a lower level of μ_i and subsequently the job-finding probability and the unemployment rate of worker type i, I let c_i^r to replicate the corresponding unemployment rate of college and non-college workers in 1980 obtained from the CPS.

To identify the cost elasticity of vacancy η, I start with equation 6, the first order condition of the search platform that gives the equilibrium level of μ_i. Multiplying both sides of equation 6 with μ_i and substituting the job-filling probability in sub-market i-j with the observed job-filling probability $\bar{q}_i$ of a worker type i, we obtain the following equation

$$r\rho v_i \frac{\bar{q}_i}{(r+\bar{q}_i)^2} = c_i^r v_i^\eta \mu_i \qquad \text{for } i \in \{H, L\}. \tag{12}$$

Dividing equation 12 of college worker H with that of non-college worker L, we have the following relationship

$$\begin{aligned} \left(\frac{Q_H}{Q_L}\right) &= \left(\frac{c_H^r}{c_L^r}\right)\left(\frac{v_H}{v_L}\right)^{\eta-1}\left(\frac{\mu_H}{\mu_L}\right) \\ \Rightarrow \ln\left(\frac{Q_H}{Q_L}\right) &= \ln\left(\frac{c_H^r}{c_L^r}\right) + (\eta-1)\ln\left(\frac{v_H}{v_L}\right) + \ln\left(\frac{\mu_H}{\mu_L}\right), \end{aligned} \tag{13}$$

where $Q_i = \frac{\bar{q}_i}{(r+\bar{q}_i)^2}$ for $i \in \{H, L\}$. Equation 13 is obtained after applying a log transformation to the first line, and we have an empirical relationship between the ratio of the job-filling probability, the vacancy ratio, and the relative recruitment efficiency. Hence, the cost elasticity η can be identified by targeting $\ln(Q_H/Q_L)$ in the data, given other model outcomes and parameters. Using the state-level JOLTS data from 2000-2005, I estimate the average daily job-filling probability for college and non-college workers and compute the average $\ln(Q_H/Q_L)$ to be 0.611.[10]

The expression of equation 13 seems to suggest the ratio of c_H^r/c_L^r and η can be identified through regression analysis using observed job-filling and vacancy

[10]Job-filling probability is computed as the number of hires over the number of open job vacancies in a given month, further divided by the number of working days (26 days). Number of job vacancies are allocated across college and non-college workers by their share of employment at the beginning of the month. Total hires of college workers are measured as the total hires of the state times the share of hires implied by the job-finding probability of the college workers.

Table 3: Parameter Values Estimated Internally

Parameter		Value	Target	Model	Data
Firm's outside option	V_0	0.542	Mean standardized ARI L - $\bar{J}_L$	0.562	0.569
H-type production shifter	A_H	0.947	Mean standardized ARI H - $\bar{J}_H$	0.778	0.779
L-type production gradient	ψ_L	0.518	Mean residualized wages L	1.139	1.138
H-type production gradient	ψ_H	0.790	Mean residualized wages H	1.532	1.531
Vacancy distribution	α	2.543	p60 of ARI L - J_L^{p60}	0.576	0.563
Vacancy distribution	β	4.081	p50 of ARI L - J_L^{p50}	0.552	0.545
Flow vacancy cost	k	0.466	Hiring cost/wage	0.938	0.928
H-type recruitment cost	c_H^r	0.015	Unemployment rate H	2.4%	2.2%
L-type recruitment cost	c_L^r	0.007	Unemployment rate L	5.8%	6.4%
Recruitment cost elasticity w.r.t. vacancy	η	0.141	ln ratio of Q_H and Q_L	0.571	0.611

Note: This table provides a list of internally calibrated parameters. H and L refers to college and non-college workers respectively. $Q_i = \bar{q}_i/(r+\bar{q}_i)^2$, where $\bar{q}$ is the daily job-filling probability of worker type $i \in \{H, L\}$ computed using the state-level data (2000-2004) from JOLTS. Abstract-routine intensity (ARI) is calculated from the task score given by Autor and Dorn (2013). ARI is further standardized to range from 0 to 1 to fit the beta distribution. Mean market tightness is measured as the mean vacancy number over the mean unemployment level in 1979. Vacancy is given by Barnichon (2010) and unemployment level is taken from BLS. The hiring cost to wage ratio is reported in Gavazza et al. (2018). Finally, the unemployment rates are taken from the CPS.

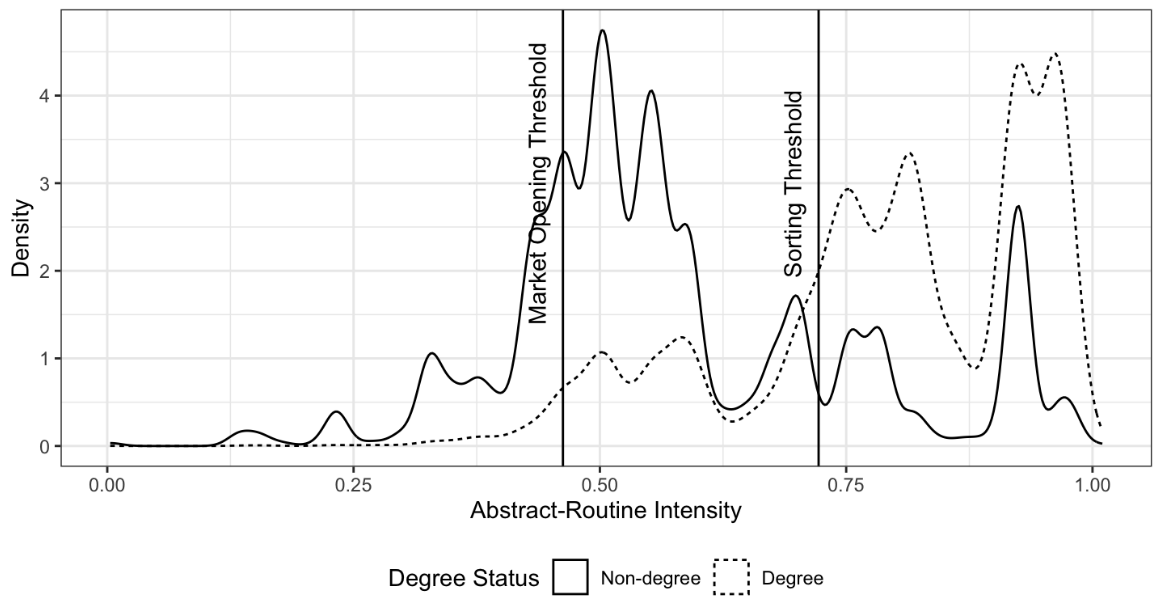
variations. However, it is important to note that equation 13 is derived from the first-order condition of the recruitment given the optimal value on market tightness θ decided by the firms. Since the variation in vacancies also affects θ and average job-filling rate $\bar{q}$, a linear regression of $\ln(Q_H/Q_L)$ on v_H/v_L and μ_H/μ_L cannot identify η and c_H^r/c_L^r. Instead, these parameter values are calibrated internally based on equilibrium outcomes.

Table 3 shows the internally calibrated parameters. While I only target several points of the ARI distribution for college and non-college workers, figure 7 shows that the market opening threshold j_0 and the sorting threshold j^* of the calibrated model can match the region where most of the college and non-college workers were employed along the ARI distribution in 1976-1980.

4.2 Recruitment's contribution to task difference shrinkage

In this counterfactual exercise, I examine the role of endogenous recruitment on the shifts in college workers' allocation toward less abstract-intensive jobs. I start with the model calibrated to the 1980 U.S. labor market and recalibrate some of the technology parameters to allow the model to match the productivity-related moments in the 2020 U.S. labor market. The targeted 2020 moments include the

Figure 7: Model output and employment distribution in 1976-1980 across ARI.



Note: This figure shows the employment distribution of non-college (solid) and college (dashed) workers across the standardized abstract-routine intensity (ARI) computed using equation 11 and the corresponding thresholds predicted by the calibrated model. The market opening threshold j_0, given by the equilibrium of the calibrated model, indicates the cutoff along the ARI below which no job is assigned to human labor. The sorting threshold j^*, also given by the equilibrium of the calibrated model, indicates the cutoff above which all jobs are assigned to college workers.

mean wage levels in 2020 for college and non-college workers, and information related to the employment distribution of non-college workers (mean ARI, median ARI, and 60th percentile of ARI of non-college workers). These moments help identify the productivity gradients (ψ_H and ψ_L), the firm's outside option V_0, and the new vacancy distribution (α, β). By assuming the college worker to have the same productivity in performing the most routine-intensive job in the market in 2020, I keep the productivity shifter of college workers A_H at the same value as in the base calibration of the 1980 U.S. labor market. The rest of the parameters, including the vacancy maintenance cost k and the search platform's recruitment cost function (c^r_H, c^r_L, η), target the same labor market moments in the 1980 period as described in table 3.

In the recalibrations, I do not target any moment associated with the employment distribution of college workers in 2020. Hence, I can examine how different

Table 4: Parameter calibrated to match the 2020 labor market

	1980	2020			
	Base	w/o Recruit	with Recruit	w/o Recruit	with Recruit
n_H	0.200	0.200	0.200	0.400	0.400
V_0	0.542	1.100	1.257	1.009	0.920
ψ_L	0.518	0.450	0.461	0.421	0.387
ψ_H	0.790	0.962	0.960	1.043	1.082
α	2.543	0.401	0.364	0.316	1.489
β	4.081	1.697	1.749	1.660	3.191
k	0.466	0.469	0.483	0.605	0.597
c^r_L	0.015	–	0.016	–	0.009
c^r_H	0.007	–	0.008	–	0.010
η	0.141	–	0.137	–	0.127

Note: This table summarizes the parameters calibrated to the wage level and non-college employment distribution in the 2020 U.S. labor market. The first column reported the parameter values of the baseline calibration to the 1980 economy for reference. The columns underneath 2020 reports parameter values from four cases: college share of labor supply n_H kept at the 1980 level of 20 percent with or without endogenous recruitment; and n_H raised to the 2020 level of 40 percent with or without endogenous recruitment. The targeted moments and the model fits are reported in table 10 in the appendix.

model scenarios predict the employment allocation of college workers. I study four distinct cases: (1) a model without an active recruitment channel and college labor supply n_H at 0.2; (2) a model with the recruitment channel and n_H at 0.2; (3) a model without recruitment channel and n_H equals the 2020 level at 0.4; and (4) a model with an active recruitment channel and n_H at 0.4.

The recalibrated parameters for each case are reported in table 4. The changes in parameter values in all cases present three common technology-related trends. First, the rise in V_0 reflects the enhanced return of automation, as documented in Acemoglu and Restrepo (2022). Second, the increase in the difference between ψ_H and ψ_L represents the biased technical changes that enhance the productivity of college workers at more abstract-intensive jobs. Finally, the calibrated vacancy distribution in 2020 is flatter than the one calibrated to the 1980 labor market. Specifically, the 2020 distribution has greater densities in areas closer to $j = 1$ and to $j = 0$ relative to the 1980 counterpart, while the densities in the middle of the distribution declined. The change in the vacancy distribution is consistent with the documented phenomenon of job polarization (Autor and Dorn, 2013).

Table 5 shows the predicted mean ARI of college workers required for each model scenario to generate the college wage premium and the mean ARI of non-college workers in 2020. The model fits to targeted moments are reported in table 10 in the appendix. In the case of only technology shocks and college supply at 0.2, the mean ARI of college workers increased from 0.78 to 0.88, indicating that college workers are working at more abstract-intensive jobs. Hence, the increase in college supply is necessary to generate a down-skilling shift for college workers.

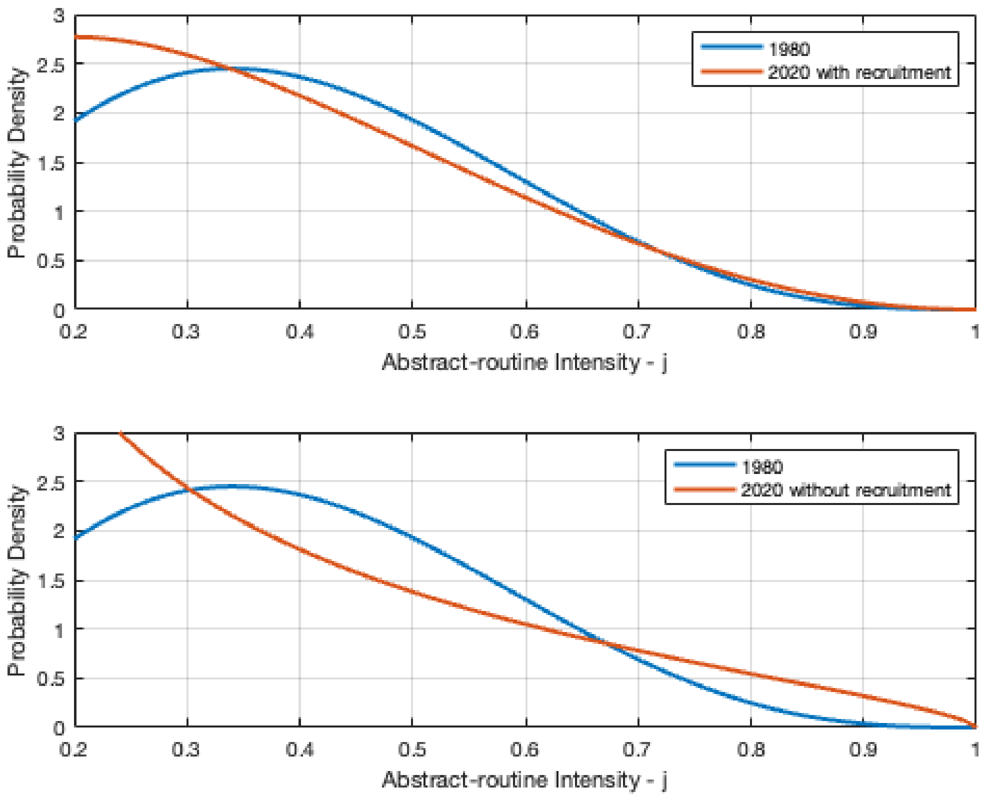
Table 5: Counterfactual college wage premium and employment allocation.

	1980		2020				
	(1) Data	(2) Base	(3) w/o Recruit	(4) Recruit	(5) w/o Recruit	(6) Recruit	(7) Data
n_H	0.2	0.2	0.2	0.2	0.4	0.4	0.4
w_H/w_L	1.345	1.345	1.757	1.757	1.754	1.756	1.756
$\bar{J}_L$	0.569	0.562	0.603	0.603	0.601	0.603	0.613
$\bar{J}_H$	0.779	0.778	0.882	0.883	0.834	0.774	0.772

Note: This table reports the model predictions on the college wage premium w_H/w_L and employment distribution ($\bar{J}_L$, $\bar{J}_H$) in different cases. The "Data" columns in (1) and (7) reported the observed levels in 1980 and 2020 correspondingly. In the calibrations to the 2020 U.S. labor market, the models target moments associated with w_H/w_L and non-college employment distribution $\bar{J}_L$. The model predictions for the 2020 $\bar{J}_H$ are untargeted. Columns (3) to (6) report model predictions in different scenarios: Columns (3) and (4) report results with college labor share at 0.2 (1980 level), without and with endogenous recruitment; likewise, column (5) and (6) report results when college share is at 0.4 (2020 level).

More importantly, the model with an active recruitment channel is the only one that predicts a decline in the mean ARI of college workers $\bar{J}_H$ relative to the 1980 level. This is because, with the active recruitment channel, the model can deliver the college wage premium and employment distribution of non-college workers at the 2020 level with a smaller degree of job polarization, as displayed in figure 8 where the predicted 2020 vacancy distributions is compared with the 1980 distribution. As discussed in the previous section, the recruitment channel generates upward pressure on college wages when the college labor supply increases. Since college workers benefit from the endogenous response of recruitment, they can earn more even when they are placed at jobs with lower ARIs. As a result, the recruitment model does not require a greater degree of polarization to match the college relative wages and the employment distribution of non-college workers.

Figure 8: Predicted vacancy distribution with and without recruitment channel.



Note: This figure compares the probability density distributions of predicted vacancy distributions for 1980 and 2020 calibrations. The blue line represents the 1980 baseline calibration, while the orange line shows the 2020 calibrations. The upper panel compares the 1980 distribution to the 2020 prediction using a model with endogenous recruitment. The lower panel contrasts the 1980 baseline with the 2020 prediction using a model without endogenous recruitment.

4.3 Recruitment's contribution to changes in college wage premium

4.3.1 Recruitment response to college expansion

I now analyze how an increase in the relative supply of college workers can induce responses to recruitment activities and affect labor market outcomes. I do this by only changing the n_H from 0.20 to 0.40 to match the college share in the male labor force in 2016-2020. To see the contribution of the active recruitment channel, I compare the equilibrium outcomes with the model prediction without endogenous recruitment. To do that, I keep the recruitment efficiencies at the original level where n_H is 0.2.

Table 6 shows the equilibrium outcomes when n_H increases from 0.2 to 0.4, while keeping all other parameters unchanged. Before we examine why recruitment efficiency changes and its implications, we need to understand how the allocation of vacancies responds to the rise in n_H. First, an increase in n_H reduces the sorting threshold j^*. This is because an increased supply of college workers allows jobs to be more easily filled by college workers, increasing V_H. Hence, some jobs with lower ARI are reassigned to college workers and this puts downward pressure on the wages of college workers.

Second, the market opening threshold j_0 increases as n_H rises, meaning some routine-intensive jobs with low ARI are not open to human labor. Intuitively, this is because when n_H increases, the share of non-college workers also declines. This reduces the probability of filling a job with non-college workers and thus causes V_L to fall. For jobs with low ARI and close to j_0, they find it no longer profitable to post a vacancy as V_L falls and thus j_0 rises. As some routine-intensive jobs exit the market, this puts upward pressure on the wages of non-college workers. Changes on j^* and j_0 directly reduce the relative wage through the productivity channel.

Given the change in the vacancy allocations, I now discuss the equilibrium response of recruitment efficiency. An increase in n_H raises the relative recruitment efficiency of college workers through the composition effect. As j^* and j_0 change, the share of jobs assigned to college workers increases. The search platform finds it profitable to enhance recruitment efficiency for college workers and reduce the level of recruitment efficiency for non-college workers. As the probability of forming a match improves for college workers through the composition effect of the recruitment channel, it puts upward pressure on the reservation utility of college workers U_H despite being assigned less abstract-intensive jobs. This generates a positive force that counteracts the negative impact of the productivity channel on the relative wage.

In addition, while the composition effect positively affects U_H, it hampers

Table 6: Equilibrium outcomes to college expansion

	w_H/w_L	j^*	j_0	μ_H	μ_L	U_H	U_L
$n_H = 0.2$	1.345	0.722	0.463	0.173	0.281	386.61	253.48
$n_H = 0.4$	1.310	0.684	0.485	0.213	0.258	385.99	253.28
W/O recruit	1.284	0.672	0.496			381.37	256.25

Note: This table compares the equilibrium outcomes when the college share of the labor force n_H increases from 0.2 to 0.4, while all other parameters remain unchanged. The first row shows the equilibrium outcomes of the calibrated model. The second row shows the equilibrium outcomes when n_H increases to 0.4 with an active recruitment channel. The third row shows the equilibrium outcomes when n_H is raised to 0.4 but the recruitment channel is shut down and the recruitment efficiencies are kept at the same values as in the first row.

the reservation utility of non-college workers U_L. Despite some less productive routine-intensive jobs exiting the market, a lower U_L implies firms can hire non-college workers by offering them less utility. This allows some routine-intensive jobs to remain profitable enough to stay open by paying non-college workers less. Hence, the recruitment channel also indirectly contributes to the rise of relative wage by slowing down worker displacement at lower-paying jobs. Without recruitment, the reservation utility is directly determined by the worst job assigned to the corresponding worker type. In that case, a fall in j^* would surely imply a decline in U_H. Likewise, a rise in j_0 would imply an escalation of U_H.

Given the base calibration, the impact of the productivity channel dominates the effect of the recruitment channel. Comparing the changes in relative wages with the model prediction without endogenous recruitment, the recruitment channel mitigates about 43 percent of the fall in relative wages due to the productivity channel as n_H increases.

4.3.2 Recruitment responses to technical changes

Technology shocks consist of three components, each corresponding to a set of technology parameters listed in table 4. First, the rise in the firm's outside option V_0 reflects the change in return when firms assign a job to machines instead of human labor. Second, the greater difference in the productivity gradients ψ_H and ψ_L in 2020 captures the increased comparative advantage of college workers working at more abstract-intensive jobs (higher js). Third, the change in the vacancy dis-

Table 7: Equilibrium outcomes to technology shocks

Indicator	Base (1980)	Technology	Labor Supply
Relative wage w_H/w_L	1.345	1.862	1.794
Sorting threshold j^*	0.717	0.733	0.706
Market Opening j_0	0.458	0.459	0.508
College recruitment efficiency μ_H	0.173	0.194	0.239
Non-college recruitment efficiency μ_L	0.280	0.268	0.248
Vacancy created	0.209	0.287	0.345
Share of vacancy to college workers v_H/v	0.113	0.123	0.216

Note: This table summarizes the changes in equilibrium outcomes when the economy is subsequently hit by shocks to the technological parameters. Column 1 reports from the base calibration in 1980. Column 2 reports the equilibrium outcomes when technology parameters changes to the 2020 values with recruitment in table 4. Column 3 reports the equilibrium outcomes when n_H increases from 0.2 to 0.4 after technology parameters changes to the 2020 levels.

tribution captures the changes in the firm's overall labor demand across different occupations.

Table 7 summarizes the changes in equilibrium outcomes when technology and labor market shocks are turned on one after another. Column 1 displays the equilibrium outcomes when the technology shocks hit. In that case, both the productivity and recruitment channel contribute positively to the rise in relative wages. Looking within the recruitment channel, the composition effect raises the relative recruitment efficiency of college workers, whereas the aggregate effect of recruitment raises the recruitment efficiency further for both types of workers as total vacancies increase.

For job allocations, positive productivity shocks cause displacements of human labor in routine-intensive jobs (j_0 increases) and reallocations of college workers to more abstract-intensive jobs (j^* increases). This is because the technology shock causes more entry of new jobs. Additional entry of vacancies dilutes the job-filling probability and leads to more exits of jobs with lower ARI. Subsequently, the reduced job-filling probability around j^* also causes some jobs with lower abstract intensity to switch back to target non-college workers. Hence, technology shock alone cannot lead to college workers working at less abstract-intensive jobs. The shrinkage in task differences is thus a result of increased college labor supply, as displayed in column 3.

Table 8: Contribution of recruitment channel to changes in relative wages

	1980	2020		
	Base	Technology	College supply	Recruitment cost
College labor supply	0.200	0.200	0.400	0.400
Relative wages				
With recruitment	1.345	1.862	1.794	1.756
Without recruitment	–	1.837	1.738	1.740
Acc. change in % points				
With recruitment	–	0.517	0.449	0.412
Without recruitment	–	0.492	0.393	0.395
Recruitment Contribution				
Contribution by shock	–	0.025	0.031	-0.040
Acc. Contribution	–	0.025	0.056	0.017
% of total change	–	4.90%	12.57%	4.08%
Decomposition	–	44.88%	55.12%	-70.26%
j^* (sorting threshold)				
With recruitment	0.717	0.733	0.706	0.697
Without recruitment	–	0.726	0.694	0.694
j_0 (market opening)				
With recruitment	0.458	0.459	0.508	0.530
Without recruitment	–	0.468	0.537	0.539

Note: This table presents the contribution of the recruitment channel to the changes in relative wages. Each column represents the college wage premium when the technology, college labor supply, and recruitment costs parameters are changed subsequently. The accumulated change in % point is the difference between the base level of the college relative wage and the level after each changes realized. The recruitment contribution is computed by comparing the difference in changes in college relative wage with and without an active recruitment channel. The proportional contribution to the changes in relative wage is the accumulated contribution over the accumulated changes in the relative wage. The decomposition of recruitment contribution is computed as the contribution by shock divided by the value of accumulated contribution in column 2 when only technology and labor shocks are realized.

4.3.3 Contribution of recruitment to changes in relative wages

To see the contribution of endogenous recruitment on changes in relative wages, I compare the changes in college wage premium with and without the recruitment channel. Table 8 presents the results on the contribution of endogenous recruitment to changes in relative wages.

The recruitment channel affects relative wages in two ways. One directly affects the wages through the worker's market utility (U_H and U_L), and another indirectly affects the job allocations. First, the recruitment channel directly increases college wage premium by improving the recruitment efficiency μ_H and subsequently the reservation market utility of college workers U_H when the shares of vacancies assigned to college workers increase. Second, the recruitment channel indirectly increases relative wages by slowing down the displacement of non-college workers at less productive routine-intensive jobs and speeding up reallocation to more abstract-intensive jobs. This is because the fall in μ_L puts downward pressure on non-college market utility U_L, jobs at lower ARI can remain open as they can hire non-college workers at lower wages. Likewise, the rise in U_H makes hiring college workers more costly, putting upward pressure on j^*.

In column 2, I apply the same technology parameter values obtained from the 2020 recalibration with recruitment in table 4 to both models. The recruitment channel exaggerates the increase in the college wage premium by 2.5 percentage points. This is mainly attributed to the recruitment's indirect effects on job allocations. Specifically, in the presence of the recruitment channel, j^* moves from 0.717 to 0.733 rather than 0.726, and j_0 only moves to 0.459 rather than 0.468 in the case without endogenous recruitment.

In column 3, I report results when the college share is increased to 40 percent. When the relative supply of college workers increases, the recruitment channel generates a strong composition effect. The effect on recruitment efficiency contributes 3.1 percentage points to the rise in college relative wages. Accounting for technology and labor supply shocks only, the recruitment channel causes a 5.6 percentage point increase in relative wages as the economy underwent technological growth and college expansion. This is equivalent to about 12.6 percent of the total increase in relative wages.

Yet, the recruitment cost structure that the search platform faces also changes and causes a reduction in the overall contribution of the recruitment channel. In column 4, the changes in the recruitment cost structure cause a reduction of 4 percentage points in the contribution of the recruitment channel, leaving the final contribution to be only 1.7 percentage points.

Decomposing the contribution of the recruitment channel across demand and supply shocks, 45 percent of the total impact is from technical changes. The rest of 55 percent is attributed to the increase in n_H. The change in recruitment cost structure cuts back to contribution by 70 percent.

4.4 Implications on active labor market policies

Policymakers implement a variety of active labor market policies to assist workers affected by the changing labor demand. By conceptualizing firms and search platforms separately, the model enables us to conduct policy evaluations on some active labor market policies. Card et al. (2018) provide a meta-analysis on the effectiveness of different active labor market policies in improving employment. In addition to unemployment, I also analyze how other equilibrium outcomes, such as wages and job allocations, are impacted by these policies in this exercise. I concentrate on two types of active labor market policies: (1) job search assistance and (2) employment subsidies, targeting non-college workers. In the current exercise, my interest is in the impact of steady-state outcomes of these policies. The subsequent effects of fiscal burden on firms and workers are abstracted from this exercise.

To simulate a job search assistance program in the model, I introduce a permanent shock to decrease the value of the recruitment cost shifters c_L^r for the unemployed by 20 percent. This could be interpreted as government intervention in providing financial support to the search platform. For instance, the government might create a pool of disadvantaged candidates for the search platform or establish public job centers specifically for less-educated workers.

Another active labor market policy to examine is employment subsidy, which refers to the government providing financial incentives for firms when they hire a specific type of worker. The policy of subsidized employment is introduced in the model by having the government cover the placement fee for the firms when they hire non-college workers. Specifically, the firm pays zero placement fee *rho* when the job is filled by a non-college worker. However, the search platform still receives the fee when the job is filled as the payment is covered by the government.

Table 9: Policy implication of employment subsidy and job search assistance

	u_H	u_L	w_H	w_L	j^*	j_0	v	v_H/v
Base	2.37%	5.84%	1.531	1.138	0.717	0.458	0.070	0.113
Employment subsidy	2.37%	5.38%	1.532	1.207	0.719	0.457	0.071	0.111
Search assistance	2.38%	5.35%	1.527	1.143	0.713	0.460	0.065	0.120

Note: This table compares the equilibrium outcomes of two active labor market policies: (1) employment subsidy and (2) job search assistance. The first row shows the equilibrium outcomes of the baseline calibration. Employment subsidies are implemented by having the government cover the placement fee for firms when a job is filled by non-college workers and its equilibrium outcomes are presented in the second row. Job search assistance is implemented by an exogenous decline in the cost parameter c_L^R for recruiting non-college workers. The equilibrium outcomes with job assistance are presented in the third row.

I compare the equilibrium outcome by introducing the policies separately to the calibrated model in 1980. The results are presented in table 9. Consistent with the literature, both job search assistance and employment subsidies are effective in reducing the unemployment rate of the target group by inducing an increase in the recruitment efficiency for non-college workers.[11] Yet, the mechanism by which each policy causes the rise in recruitment efficiency is different. For employment subsidies, a job filled with non-college workers delivers a higher match surplus as it no longer incurs a placement fee. This increase in match surplus induces more jobs created and more vacancies allocated to non-college workers, and thus causes the recruitment efficiency for non-college workers to rise. The job search assistance program reduces the unemployment rate with a different mechanism. Lowering the platform's cost of recruitment for non-college workers directly increases recruitment efficiency.

Apart from unemployment, both policies can enhance the wages of non-college workers and reduce wage inequality, despite different degrees. Employment subsidy delivers a greater increase in wages for non-college workers because the improved match surplus from the subsidy is directly shared with workers. Job search assistance only affects wages by enhancing the job-finding probability for non-college workers through the recruitment channel. However, the two policies have opposite effects on job allocations and vacancy creation. First, while employment

[11] The size of the job search assistance is adjusted to match a similar reduction in the non-college unemployment rate induced by the employment subsidy.

subsidies increase the range of jobs allocated to non-college workers, job search assistance reduces it. Second, employment subsidies encourage more job creation, but job search assistance reduces the number of vacancies created.

The distinction in policy impacts on job allocation and job creation is due to the difference in the direct benefactor of the policies. Employment subsidy is a financial support for firms. It enhances the vacancy value of a job assigned to non-college workers. Consequently, this encourages more jobs to be created and allocated to non-college workers. However, the benefit of job search assistance is mainly enjoyed by the recruitment platform. From the firm's perspective, the enhanced recruitment efficiency improves the market utility of non-college workers and subsequently their wages through the recruitment channel. As firms' match surplus reduces with rising market utility, they cut back on job creation and allocate fewer jobs to non-college workers. This is reflected in the increase in opening threshold j_0 and a fall in sorting threshold j^*.

Although employment subsidies and job search assistance help reduce unemployment and increase wages for targeted workers, they have contrasting implications for job creation and allocations. These results have implications for policymakers in two matters. First, while employment subsidies encourage job creation and allocation to non-college workers, job search assistance does the opposite and reduces vacancy rates. Therefore, if a policy objective were to create more jobs and increase vacancy rates while lowering the unemployment rate of target worker groups, an employment subsidy program would be desired. Second, employment subsidies would slow down job displacement of non-college workers, but search assistance accelerates the process. If one of the main objectives of labor market policies is to slow down worker displacement in the economy and reduce unemployment, employment subsidies would be the preferred policy.

5 Conclusion

In this paper, I introduce a potential explanation for the paradoxical trend of the rising college wage premium coinciding with increasing task similarity between college and non-college workers in the U.S. I develop a model by extending the task-based framework with competitive search and endogenous recruitment. Using the model, I demonstrate how college workers benefit from improved matching efficiency, allowing for higher wages despite performing more similar tasks to non-college workers.

The model, featuring a profit-maximizing search platform, shows that recruitment efficiency for college workers improves with aggregate job openings and the share of college-requiring vacancies. I present empirical evidence, including Craigslist's expansion patterns and changes in matching efficiency, that supports the mechanism of the recruitment channel.

In the counterfactual analysis, I show that the recruitment channel is essential in predicting the changes in college employment distribution from 1980 to 2020. A model without an active recruitment channel would require an average college worker to work at a more abstract-intensive job to generate the same increment in college wage premium. Quantitatively, the recruitment channel contributes 12.6% to the increase in relative wages of college workers from 1980 to 2020 through technical changes and college expansion.

Furthermore, the model allows us to evaluate the implications of two different types of active labor market policies: employment subsidies and job search assistance for non-college workers. Both policies are predicted to deliver lower unemployment rates and higher wages for the targeted non-college workers. However, while employment subsidies directly benefit firms and encourage more creation of vacancies and more jobs assigned to non-college workers, the job search assistance program, which lowers the cost of the search platform, reduces job creation and displaces non-college workers from their jobs. This is because although the job assistance program benefits non-college workers by enhancing job-finding prospects, it makes them more costly for firms to hire. As a result, some jobs are reallocated away from these workers in the end.

The analysis so far provides several directions for future research. First, while the current paper explores the search platform's role as a general operator of matching technology, these technologies have undergone great evolution historically as they transformed from classified ads in newspapers in the 19th century to online job boards and large recruitment agencies today. I intend to extend the current framework to study the evolution of search technology on structural transformation and firm dynamics. Meanwhile, another key role of the search intermediaries is to mitigate the information cost of candidates. As discussed in Birinci et al. (2024), the increased number of job applicants and labor supply of workers with higher qualifications had made the screening of suitable candidates more costly. In future research, we can explore the rise in information friction as an additional mechanism that recruitment endogenously responds to. Third, the model can be extended to account for skill measures in multiple dimensions, this allows quantification of recruitment's contribution to skill mismatch (Şahin et al., 2014; Herz and Van Rens, 2020; Baley et al., 2022). For instance, as documented in Herz and Van Rens (2020), barriers to job mobility are the main source of mismatch unemployment. A potential source of barriers can be due to jobs requiring niche skill sets having better access to efficient recruitment technology, allowing the job requirements to be restrictive. I leave these questions for research in the future.

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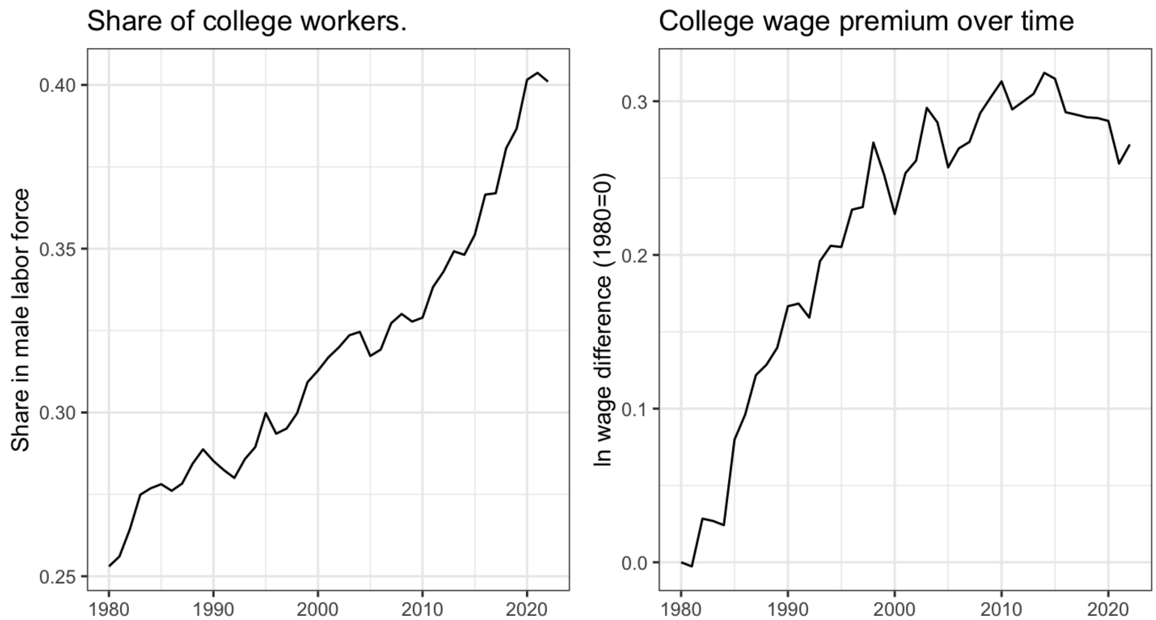
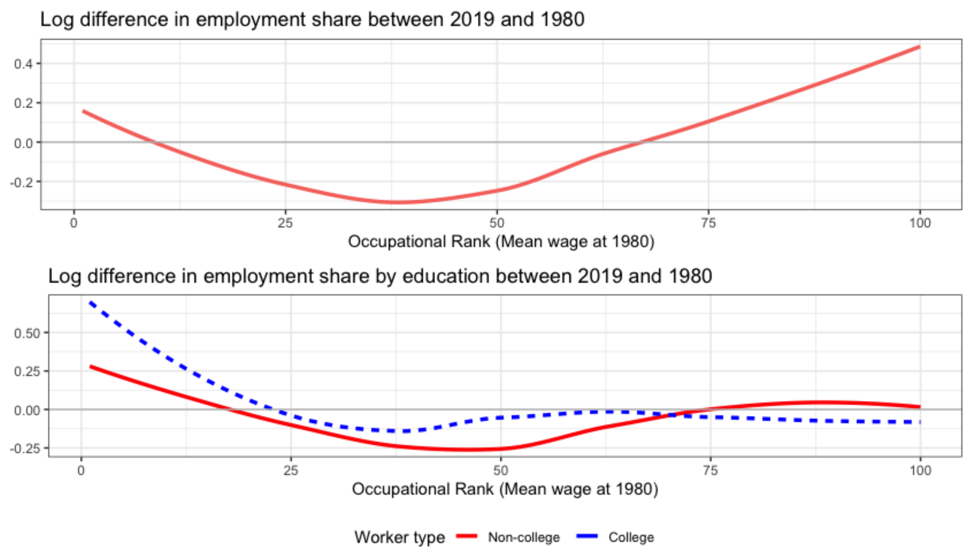
A Additional Tables and Figures

Figure 9: U.S. college wage premium and college labor supply from 1980-2020



Note: The figure uses March Current Population Survey Annual Social and Economic Supplement data (CPS ASEC). The left panel shows the share of college workers in the male labor force. The right panel shows the college wage premium, calculated as the log difference in predicted real hourly wages. The predicted wage is computed using a regression model of log wages on gender, age groups, race groups, and education levels. This approach of computing predicted wages is similar to figure 1 in Autor (2019).

Figure 10: Changes in employment share across jobs from 1980-2019



Note: The figure repeats the exercise in figure 1 of Autor and Dorn (2013) and is constructed using Census IPUMS and American community Survey data. It measures the change in employment share between 1980 and 2019. The occupations are ranked by the mean wages of workers in 1980. The upper panel shows the aggregate changes, consistent to the figure in Autor and Dorn (2013). The lower panel shows the same exercise by education level across the same occupation rank.

Table 10: Model fits for calibration to the 2020 labor market

	1980		2020				
	Data	Base	w/o Recruit	Recruit	w/o Recruit	Recruit	Data
n_H	0.200	0.200	0.200	0.200	0.400	0.400	0.400
u_H	2.2%	2.4%	2.1%	2.0%	2.3%	2.0%	–
u_L	6.4%	5.8%	6.1%	6.7%	6.9%	6.9%	–
w_H	1.531	1.531	1.906	1.906	1.905	1.905	1.906
w_L	1.139	1.138	1.085	1.085	1.086	1.085	1.085
w_H/w_L	1.345	1.345	1.757	1.757	1.754	1.756	1.756
$\bar{J}_L$	0.569	0.562	0.603	0.603	0.601	0.603	0.613
J_L^{p50}	0.545	0.552	0.590	0.590	0.594	0.598	0.581
J_L^{p60}	0.563	0.576	0.624	0.623	0.618	0.614	0.623
Hiring Cost/$\bar{w}$	0.928	0.938	0.928	0.914	0.926	0.927	–
$\ln(Q_H/Q_L)$	0.611	0.571	0.753	0.644	0.759	0.644	–

Note: This table provides a list of targeted moments and their corresponding model moments. Columns under 1980 report empirical data moments of the 1980 U.S. labor market and the baseline model moments. The models calibrated to the 2020 economy target wage levels (w_H and w_L) and information of the non-college employment distribution ($\bar{J}_L$, J_L^{p50}, and J_L^{p60}) in 2020. These 2020 moments are reported in the last "Data" column under 2020. For other labor market moments, including unemployment rates, hiring cost ratio, and $\ln(Q_H/Q_L)$, the 2020 models target the same values as the 1980 baseline model.

B Proof

B.1 Proof of proposition 1

Proof.— Denote θ^*_{ij} the solution and the maximizer of the wage determination problem for a job $j \in [0,1]$ targeting worker $i \in \{H, L\}$; and V^*_{ij} the expected value of the vacancy evaluated at the maximizer θ^*_{ij}.

To establish positive assortative matching, we need for some $\hat{j} \in [0,1]$ that $V^*_{H\hat{j}} = V^*_{L\hat{j}}$, we have $V^*_{Hj} \geq V^*_{Lj}$ for all $j \geq j^*$. Specifically, we need

$$\left.\frac{dV^*_{Hj}}{dj}\right|_{j=\hat{j}} \geq \left.\frac{dV^*_{Lj}}{dj}\right|_{j=\hat{j}}.$$

From the firm's wage determination problem, the expected value of the vacancy V^*_{ij} takes the following form for any $j \in [0,1]$ and :

$$V^*(\theta^*_{ij}) = \frac{q(\theta^*_{ij})}{r + q(\theta^*_{ij})}\left(\frac{y_{ij} - w_{ij}((\theta^*_{ij})}{r+\delta} - \rho\right) - \frac{k}{r + q(\theta^*_{ij})},$$

where w_{ij} is a function of θ^* given by the worker indifference equation. To save on notation and without loss of generality, I put parameters ρ and k to zero.

For a $j \in [0,1]$, the derivative of $V^*_{i\hat{j}}$ with respect to $\hat{j}$ is given by the envelope theorem

$$\begin{aligned}
\frac{dV^*_{ij}}{dj} &= \frac{q(\theta^*_{ij})}{r + q(\theta^*_{ij})}\frac{1}{r+\delta}\frac{dy_{ij}}{dj} \\
&= \frac{q(\theta^*_{ij})}{r + q(\theta^*_{ij})}\frac{y_{ij} - w_{ij}(\theta^*_{ij})}{r+\delta}\frac{d\ln y_{ij}}{dj} \\
&= V^*_{ij}\frac{d\ln y_{ij}}{dj}
\end{aligned}$$

Suppose for some $\hat{j} \in [0,1]$ that $V^*_{H\hat{j}} = V^*_{L\hat{j}}$. By assumption 1, we have $d\ln y_{Hj}/dj \geq$

$d \ln y_{Lj}/dj$ for all j. Hence, we establish the following

$$\left.\frac{dV^*_{Hj}}{dj}\right|_{j=\hat{j}} = V^*_{H\hat{j}} \frac{d \ln y_{ij}}{dj} \geq \left.\frac{dV^*_{Lj}}{dj}\right|_{j=\hat{j}} = V^*_{Hj} \frac{d \ln y_{ij}}{dj}.$$

Hence, for any $j \geq \hat{j}$, $V^*_{Hj} \geq V^*_{Lj}$ and positive assortative matching.

In addition, since the derivatives $\frac{dV^*_{ij}}{dj}$ is also increasing V^*_{ij}, the difference of derivative between type H and type L is also increasing in j, i.e. $\frac{dV^*_{Hj}}{dj} - \frac{dV^*_{Lj}}{dj} \geq 0$. Therefore, there exists a j^* such that $V_{Hj'} \geq V_{Lj'}$ for all $j' > j^*$. If j^* is above the market opening threshold j_0 or $j^* \neq 1$, then we have the **sorting condition** $V_{Hj^*} = V_{Lj^*}$. ■