Closing small open economy models with collateral constraints

Online Appendix

(*for the use of referees*)

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1 Appendix A. A model with an endogenous discount factor

1.1 The model

The small open economy is inhabited by a large number of identical and infinitely lived households. Their preferences are described by the following utility function, which features an endogenous rate of time preference as in SGU (Section 2):

$$E_0 \sum_{t=0}^{\infty} \theta_t u\left(c_t, h_t\right) \tag{A.1}$$

$$\theta_0 = 1 \tag{A.2}$$

$$\theta_{t+1} = \beta\left(\widetilde{c}_t, \widetilde{h}_t\right)\theta_t, t \geq 0 \tag{A.3}$$

where c_t denotes consumption, h_t denotes hours of work, $\tilde{c}_t$ and $\widetilde{h}_t$ denotes the aggregate levels of consumption and hours of work respectively, which the individual household take as given. As in SGU the discount factor is decreasing in consumption and increasing in hours $\left(\beta_{\widetilde{c}} < 0, \beta_{\widetilde{h}} > 0\right)$. Under this specification, in the steady state, the Euler equation of foreign assets will pin down the steady state level of consumption as a function solely of foreign interest rate and the parameters included in the function $\beta\left(\cdot\right)$.

For our numerical solutions, we use the following functional forms for preferences as in SGU:

$$u(c_t) = \frac{\left(c_t - \omega^{-1} h_t^{\omega}\right)^{1-\gamma} - 1}{1-\gamma}$$

$$\beta\left(\widetilde{c}_t, \widetilde{h}_t\right) = \left(1 + \widetilde{c}_t - \omega^{-1} \widetilde{h}_t^{\omega}\right)^{-\psi_1}$$

where γ and ψ_1 are constant parameters.

Households period budget constraint is:

$$d_t = (1 + r_t)\, d_{t-1} - y_t + c_t + i_t \tag{A.4}$$

where, y_t denotes output, c_t denotes consumption, i_t denotes investment and d_t denotes end-of-period one-period risk-free international bonds, which pay the world real interest rate r_t. In this model, the foreign interest rate faced by domestic households is assumed to be constant over time ($r_r = r$). Also, for simplicity, we assume away capital adjustment costs.

The world credit markets are imperfect. Specifically, households' borrowing ability is constrained by a fraction of their collateralized assets:

$$d_t \leq \phi k_t \tag{A.5}$$

This constraint resembles a margin-requirement; households have to finance the fraction $(1 - \phi)$, the margin, of their new capital investment with their own funding. Or, alternatively, could be seen as a contract with imperfect enforcement, where foreign lenders cannot collect more than a fraction of the value of debtor's assets. A similar type of collateral constraint can, also, be imposed due to default risk as Aguiar and Gopinath (2007).

The law of motion of capital is given by:

$$k_t = i_t + (1 - \delta)\, k_{t-1} \tag{A.6}$$

where $0 < \delta < 1$ is the depreciation rate.

Output y_t is produced by a linear production function:

$$y_t = A_t F(k_t, h_t) \tag{A.7}$$

where, $F(k_t, h_t) = k_t^{\alpha}, h_t^{1-\alpha}, 0 < \alpha < 1$.

We assume a simple AR (1) process for the productivity shock:

$$A_{t+1} = \rho A_t + \varepsilon_{t+1} \tag{A.8}$$

where, $0 < \rho < 1$ is the persistence parameter and $\varepsilon_t \sim NIID\left(0, \sigma_\varepsilon^2\right)$.

Each household chooses sequences of consumption, hours, investment, capital and one-period international bonds,$\{c_t, h_t, i_t, k_t, d_t\}_{t=0}^{\infty}$, so as to maximize their utility function (A.1) subject to the budget and the collateral constraint and the law of motion of capital, equations (A.4), (A.5) and (A.6) respectively. Letting λ_t and μ_t denote the non-negative Lagrange multipliers on (A.4) and (A.5), the optimality conditions of the households are:

$$\lambda_t = u_c\left(c_t, h_t\right) \tag{A.9}$$

$$-u_c\left(c_t, h_t\right) = \lambda_t A_t F_h\left(k_t, h_t\right) \tag{A.10}$$

$$\lambda_t - \phi\mu_t = \beta\left(\widetilde{c}_t, \widetilde{h}_t\right) E_t\lambda_{t+1}\left[A_{t+1}F_k\left(k_{t+1}, h_{t+1}\right) + 1 - \delta\right] \tag{A.11}$$

$$\lambda_t - \mu_t = \beta\left(\widetilde{c}_t, \widetilde{h}_t\right)\left(1 + r_t\right) E_t\lambda_{t+1} \tag{A.12}$$

1.2 Macroeconomic equilibrium

We solve for a symmetric equilibrium in which all households are alike ex post. Thus, in equilibrium the individual and aggregate variables are identical:

$$c_t = \widetilde{c}_t \tag{A.13}$$

$$h_t = \widetilde{h}_t \tag{A.14}$$

The equilibrium system can be summarized by the following 8 equations in 8 endogenous variables $\{c_t, h_t, i_t, k_t, d_t, \lambda_t, \mu_t, y_t\}_{t=0}^{\infty}$, given the exogenous variable $\{A_t\}$ and the initial stocks k_0, d_0.

$$\lambda_t = u_c\left(c_t, h_t\right) \tag{A.15}$$

$$-u_c\left(c_t, h_t\right) = \lambda_t A_{t+1} F_h\left(k_t, h_t\right) \tag{A.16}$$

$$\lambda_t - \phi\mu_t = \beta\left(\widetilde{c}_t, \widetilde{h}_t\right) E_t\lambda_{t+1}\left[A_t F_k\left(k_{t+1}, h_{t+1}\right) + 1 - \delta\right] \tag{A.17}$$

$$\lambda_t - \mu_t = \beta\left(\widetilde{c}_t, \widetilde{h}_t\right)\left(1 + r_t\right) E_t\lambda_{t+1} \tag{A.18}$$

$$d_t = \left(1 + r_t\right) d_{t-1} - y_t + c_t + i_t \tag{A.19}$$

$$d_t = \phi k_t \tag{A.20}$$

$$k_t = i_t + (1 - \delta)k_{t-1} \tag{A.21}$$

$$y_t = A_t F\left(k_t, h_t\right) \tag{A.22}$$

2 Appendix B. A model with a debt-elastic interest rate

2.1 The model

In this subsection we consider again the small open economy model with collateral constraints of the subsection (1.1), with the only difference that stationarity is induced by assuming a debt-elastic foreign interest rate, while the discount factor is constant. Specifically, we follow Section 3 of SGU and assume that the real interest rate at which households borrow from abroad, r_t, is time and state contingent. In fact, the foreign interest rate is an increasing function of the aggregate level of foreign debt, $\widetilde{d}_t$, which households take as given (i.e. agents fail to internalize the effect of their debt decisions on the borrowing rate). Following SGU (2003), we employ for the functional form:

$$r_t = r + \psi_2\left(e^{\left(\widetilde{d}_t - \widetilde{d}\right)} - 1\right) \tag{B.1}$$

where r is the exogenously given world interest rate and $\widetilde{d}$, ψ_2 are constant parameters. When the aggregate per capita foreign debt ratio rises above $\widetilde{d}$, domestic agents face a country-specific interest rate premium, the elasticity of which to aforementioned deviation, is measured by the parameter ψ_2. The reason why this specification induces stationarity in the model is that in the steady state, the Euler equation of foreign assets will pin down the steady state level of net foreign asset position (instead of consumption in the previous model) as a function solely of world interest rate and the parameters $\widetilde{d}$,ψ_2, included in the premium function.

In more details, households' preferences are given by equation (A.1). However, unlike to the EDF model, we now assume that the discount factor is constant:

$$\theta_{t+1} = \beta^t \tag{B.2}$$

where, $0 < \beta < 1$ is a parameter.

Each household chooses again sequences of consumption, hours, investment, capital and one-period international bonds,$\{c_t, h_t, i_t, k_t, d_t\}_{t=0}^{\infty}$, so as to maximize their utility function (A.1) subject to the budget constraint, the law of motion of capital and the collateral constraint, equations (A.4), (A.6) and (A.5) respectively. The optimality conditions are now:

$$\lambda_t = u_c\left(c_t, h_t\right) \tag{B.3}$$

$$-u_c\left(c_t, h_t\right) = \lambda_t A_t F_h\left(k_t, h_t\right) \tag{B.4}$$

$$\lambda_t - \phi\mu_t = \beta E_t \lambda_{t+1}\left[A_{t+1} F_k\left(k_{t+1}, h_{t+1}\right) + 1 - \delta\right] \tag{B.5}$$

$$\lambda_t - \mu_t = \beta\left(1 + r_t\right) E_t \lambda_{t+1} \tag{B.6}$$

where λ_t is the multiplier associated with the budget constraint and μ_t is the multiplier associated with the collateral constraint.

2.2 Macroeconomic equilibrium

We solve for a symmetric equilibrium in which all households are alike ex post. Thus, in equilibrium the individual and aggregate variables are identical:

$$d_t = \widetilde{d}_t \tag{B.7}$$

The equilibrium system can be summarized by the following 8 equations in 8 endogenous variables $\{c_t, h_t, i_t, k_t, d_t, \lambda_t, \mu_t, y_t\}_{t=0}^{\infty}$, given the exogenous variable $\{A_t\}$ and the initial stocks k_0, d_0.

$$\lambda_t = u_c\left(c_t, h_t\right) \tag{B.8}$$

$$-u_c\left(c_t, h_t\right) = \lambda_t A_t F_h\left(k_t, h_t\right) \tag{B.9}$$

$$\lambda_t - \phi\mu_t = \beta E_t \lambda_{t+1}\left[A_{t+1} F_k\left(k_{t+1}, h_{t+1}\right) + 1 - \delta\right] \tag{B.10}$$

$$\lambda_t - \mu_t = \beta\left(1 + r_t\right) E_t \lambda_{t+1} \tag{B.11}$$

$$d_t = \left(1 + r_t\right) d_{t-1} - y_t + c_t + i_t \tag{B.12}$$

$$d_t = \phi k_t \tag{B.13}$$

$$k_t = i_t + (1 - \delta) k_{t-1} \tag{B.14}$$

$$y_t = A_t F\left(k_t, h_t\right) \tag{B.15}$$

3 Appendix C. Parameter values

Table C.1 presents the parameter values, which are as in Schmitt-Grohé and Uribe (2003) (Table 1). Regarding the parameter in the collateral constraint, we set ϕ at 0.2, as in Korinek and Mendoza (2014).

Table C.1: Parameter values

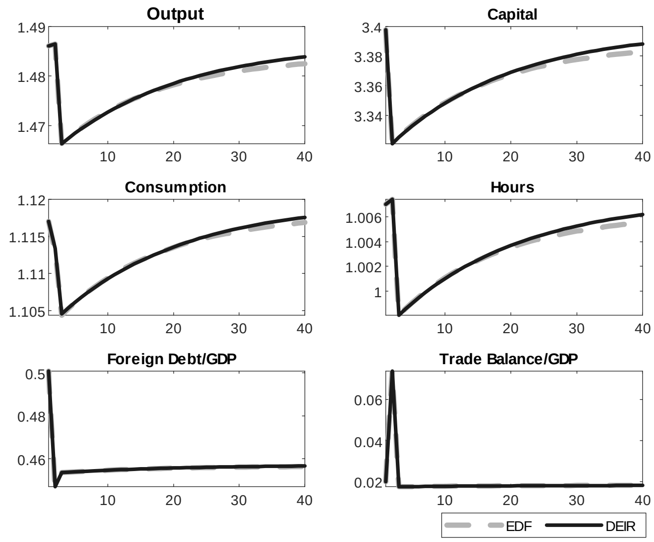
parameter	value	description
α	0.32	share of capital
δ	0.1	rate of capital depreciation
γ	2	elasticity of intertemopral substitution
ω	1.455	elasticity of labour
β	0.96	discount factor in DEIR
ψ_1	0.11	elasticity of discount factor in EDF
ψ_2	0.000742	risk premium parameter in DEIR
$\widetilde{d}$	0.7442	foreign debt threshold in DEIR
r	0.04	risk-free interest rate on foreign assets
ϕ	0.2	collateral constraint parameter
ρ	0.42	persistence parameter in TFP process
σ_ε	0.0129	standard deviation of the TFP shock

4 Appendix D. Transition dynamics

In this section we report what happens when we depart from the unconstrained and travel towards the constrained steady state equilibrium for both model specifications. To compute the transition paths, we assume that the economies are in the unconstrained regime at the time the collateral constraint binds permanently. In that way we generate a sudden stop event. For each model we check its saddle-path stability when we use as initial conditions for the state variables their steady state values of the unconstrained economy.

Figure D.1 presents the dynamic paths of output, capital, consumption, hours and debt and trade balance as a share of output, towards the long-run equilibrium of the financially constrained economy.

Figure D.1: From the unconstrained to the constrained economy



The main message from Figure D.1 is that the transition paths in both models are very close and indicate the impact costs of binding collateral constraints, namely the output, capitals and hours contraction, the debt deleveraging and the sharp but short-lived improvement of the trade balance. When the collateral constraint binds agents sell capital to meet the constraint which results in a decrease in investment, output and consumption. Even in the DEIR model, where households end up better-off with respect to output, capital and consumption in the constrained regime, they initially decrease their foreign bond and capital holdings, as well as consumption, in response to the positive marginal cost of relaxing the collateral constraint ($\mu_t > 0$).