PO906: Quantitative Data Analysis and Interpretation

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Quantitative Data Analysis

Descriptive statistics: description of central variables by statistical measures such as median, mean, standard deviation and variance

Inferential statistics: test for the relationship between two variables (at least one independent variable and one dependent variable)

For the application of quantitative data analysis it is crucial that the selected method is appropriate for the data structure:

- DV:
  - Dimensionality: spatial and dynamic
  - continuous or discrete
  - Binary, ordinal categories, count
  - Distribution: normal, logistic, poison, negative binomial
- Critical points
  - Measurement level of the DV and IV
  - Expected and actual distribution of the variables
  - Number of observations and variance

Quantitative Methods I

Variables:

- A variable is any measured characteristic or attribute that differs for different subjects.
- OED: Something which is liable to vary or change; a changeable factor, feature, or element.
- *Math.* and *Phys.* A quantity or force which, throughout a mathematical calculation or investigation, is assumed to vary or be capable of varying in value.
- *Logic*. A symbol whose exact meaning or referend is unspecified, though the range of possible meanings usually is.
- Independent variables – explanatory variables – exogenous variables – explanans: variables that are causal for a specific outcome (necessary conditions)
- Intervening variables: factors that impact the influence of independent variables, variables that interact with explanatory variables and alter the outcome (sufficient conditions)
- Dependent variables – endogenous variables – explanandum: outcome variables, that we want to explain.

Measurement Level

The appropriate method largely depends on the measurement level, type, and distribution of the dependent variable!

Measurement levels of variables:
The level of measurement refers to the relationship among the values that are assigned to the attributes for a variable.

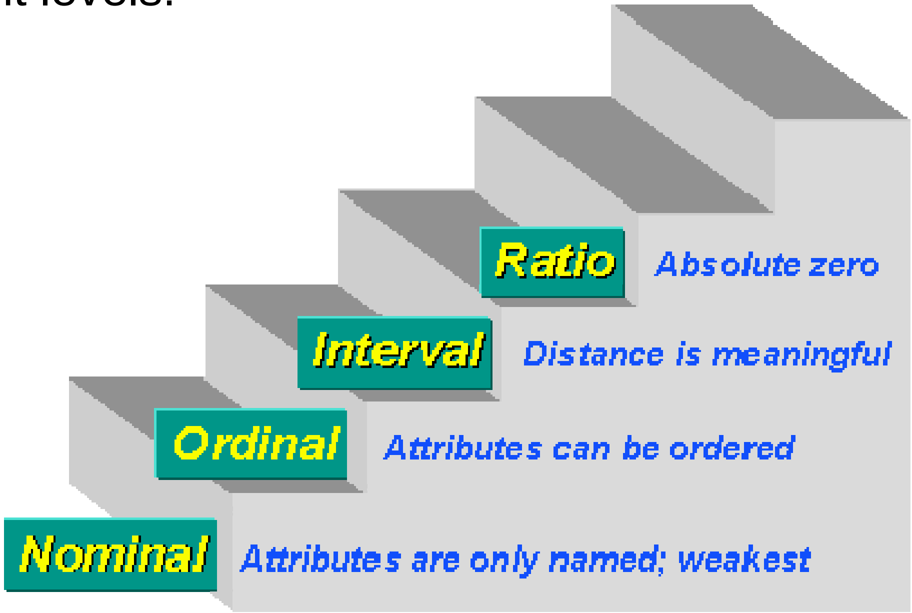
– Nominal: the numerical values just "name" the attribute uniquely. No ordering of the cases is implied. For example, party affiliation is measured nominally, e.g. republican=1, democrat=2, independent=3: 2 is not more than one and certainly not double. (qualitative variable)

- Ordinal: the attributes can be rank-ordered. distances between attributes do not have any meaning. For example, on a survey one might code Educational Attainment as 0=less than H.S.; 1=some H.S.; 2=H.S. degree; 3=some college; 4=college degree; 5=post college. In this measure, higher numbers mean *more* education. But is distance from 0 to 1 same as 3 to 4? The interval between values is not interpretable in an ordinal measure. Averaging data doesn‘t make sense.

- Interval: distance between attributes *does* have meaning. E.g. temperature (in Fahrenheit), the distance from 30-40 is same as distance from 70-80. The interval between values is interpretable. It makes sense to compute an average of an interval variable. But: in interval measurement ratios don't make any sense - 80 degrees is not twice as hot as 40 degrees.

- Ratio: there is always an absolute zero that is meaningful. This means that one can construct a meaningful fraction (or ratio). Weight is a ratio variable. In applied social research most "count" variables are ratio: number of wars. But also other continuous variables like gdp or government consumption.

Measurement levels:



- It's important to recognize that there is a hierarchy implied in the level of measurement idea. At lower levels of measurement, assumptions tend to be less restrictive and data analyses tend to be less sensitive. At each level up the hierarchy, the current level includes all of the qualities of the one below it and adds something new. In general, it is desirable to have a higher level of measurement (e.g., interval or ratio) rather than a lower one (nominal or ordinal).
- Knowing the level of measurement helps you decide how to interpret the data from a variable and what statistical analysis is appropriate on the values that were assigned.

Variable types:

- Discrete vs. Continuous variables: A discrete variable is one that cannot take on all values within the limits of the variable. For example, responses to a five-point rating scale can only take on the values 1, 2, 3, 4, and 5. The variable cannot have the value 1.7. A variable such as a person's height can take on any value. Variables that can take on any value and therefore are not discrete are called continuous. – for statistical analysis it is important whether the dependent variable is discrete or continuous.
- Count variables: discrete – specific distribution, positive values, number of wars/ terrorist attacks, numbers of acqui communautaire chapters closed
- Binary variables: discrete, either 1 or 0, yes/no, Gender, parliamentary/presidential,
- Truncated variables: only observations are used that are larger or smaller than a certain value: analysis of the determinants of poverty – only poor people are analyzed
- Censored variables: values above or below a certain threshold cannot be observed: income categories
- Categorical variables: answering categories in surveys
- Nominal variables with more than 2 categories: party affiliation

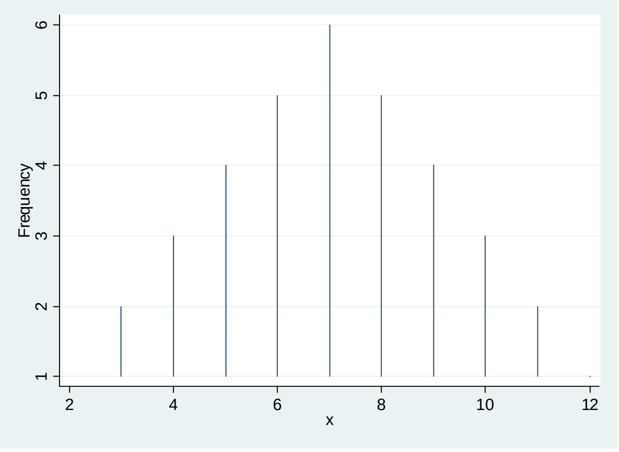
The appropriate statistical model heavily depends on the type of the dependent variable: probit/logit models for binary variables, poisson/negative binomial models for count variables etc.

Discrete Random Variables

- Basis of most statistical estimators
- Example: experiment with two (fair) dices – 36 possible experimental outcomes - sum of the values of the 2 dices:

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	2	3	4	5	6	5	4	3	2	1
probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

- Observations are independent – IMPORTANT!
- Adding all probabilities gives 1, since it is certain that one has to get one of the values in each experiment
- The set of all possible values of a random variables is the population from which it is drawn
- If we graphically depict the possible values and their frequencies we get the frequency distribution of the random variable, which is a symmetric distribution with mean 7



	1	2	3	4	5	6
1	2	3	4	5	6	**7**
2	3	4	5	6	**7**	8
3	4	5	6	**7**	8	9
4	5	6	**7**	8	9	10
5	6	**7**	8	9	10	11
6	**7**	8	9	10	11	12

Distribution of variables

- Frequency distribution/ density: measures the frequency with which a certain value occurs in a sample
- Probability distribution/ density: measures the probability with which a certain value occurs in a population, the sum of probabilities equals 1
- Distributions are uniquely characterized by the determining parameters and their moments
- Moments are: mean, variance, skewness, kurtosis etc.
- We always distinguish between the “true value” and the sampling value of a moment
- 1st moment: central tendency of a distribution, most common is the *mean* (in a sample, also called expected value of a variable):

$$\mu = E(x) = \overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

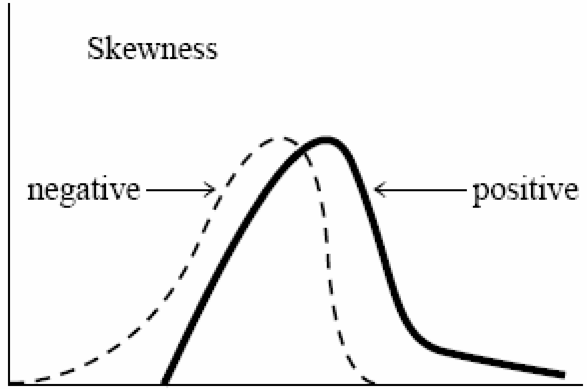
- 2nd moment: “width” or “variability” around the central value, most common is the *variance* or its square root the standard deviation:

$$\sigma^2 = Var(x_1 \ldots x_n) = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2;\ \sigma = \sqrt{Var(x_1 \ldots x_n)}$$

Higher moments are almost always less robust than lower moments!

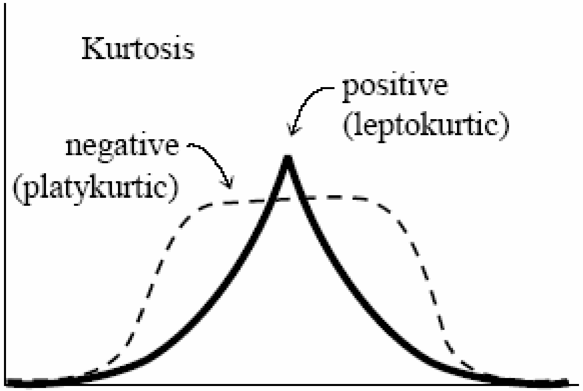
- 3rd moment: skewness – characterizes the degree of asymmetry of a distribution around its mean: A positive value of skewness signifies a distribution with an asymmetric tail extending out towards more positive *x*; a negative value signifies a distribution whose tail extends out towards more negative *x*

$$Skew(x_1 \ldots x_n) = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{x_i - \overline{x}}{\sigma}\right]^3$$



- 4th moment: kurtosis - measures the relative peakedness or flatness of a distribution, relative to a normal distribution, a distribution with positive kurtosis is termed *leptokurtic*; a distribution with negative kurtosis is termed *platykurtic*; an in-between distribution is termed *mesokurtic*.

$$Kurt(x_1 \ldots x_n) = \left\{\frac{1}{N}\sum_{i=1}^{N}\left[\frac{x_i - \overline{x}}{\sigma}\right]^4\right\} - 3$$



True Values and Sampling Values of Moments

- True value refers to the underlying population and its distribution:
  - Expected value and population variance: the probability that a certain value occurs is known (see the 2 dice experiment), or
  - Draw a sample from the same population and infinite number of times and calculate the mean, there will be some variation – the result is a distribution with a mean the equals the true value

- The sampling value refers to the single draw, the measured variable
  - Mean and sampling variance

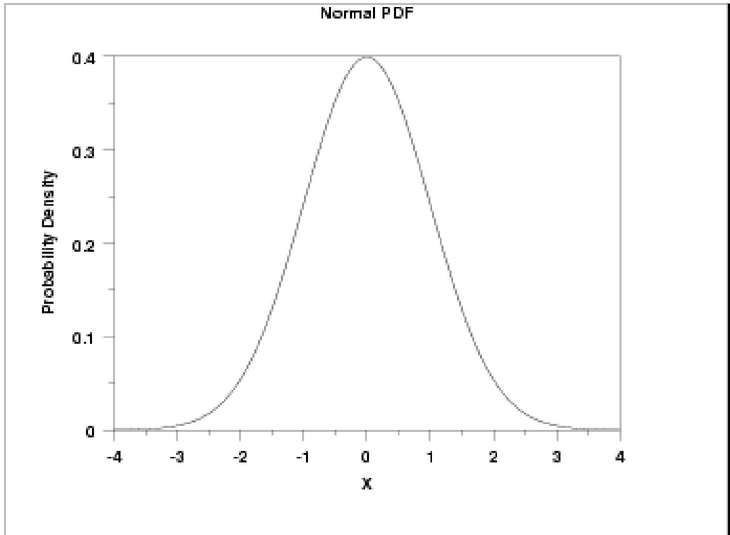
PDF vs. CDF: Probability Density Function vs. Cumulative Distribution Function of a variable:

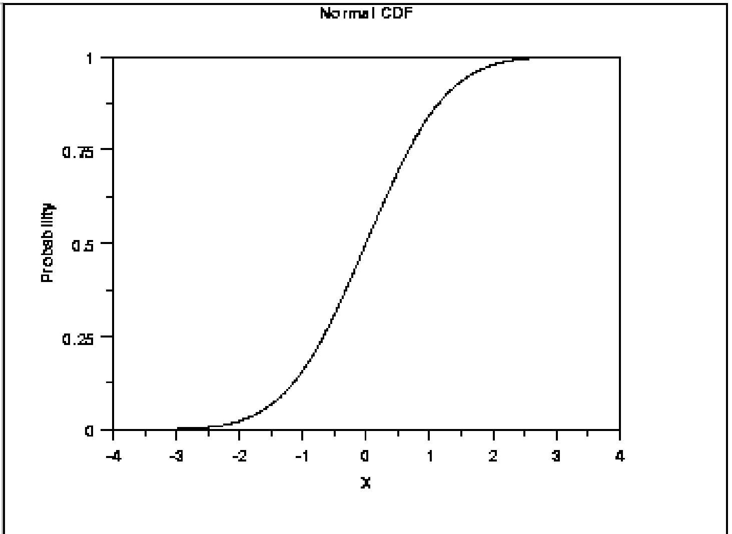
- PDF: For a continuous variable, the probability density function (pdf) is the probability that the variate has the value x.
- CDF: The cumulative distribution function (cdf) is the probability that the variable takes a value less than or equal to x.
- The CDF is the antiderivative or integral of the PDF and the PDF is the derivative of the CDF.
- Example: normal distribution: PDF:

$$f(x) = \varphi = \frac{e^{-(x-\mu)^2/\left(2\sigma^2\right)}}{\sigma\sqrt{2\pi}} = F'\left(x\right)$$

CDF:

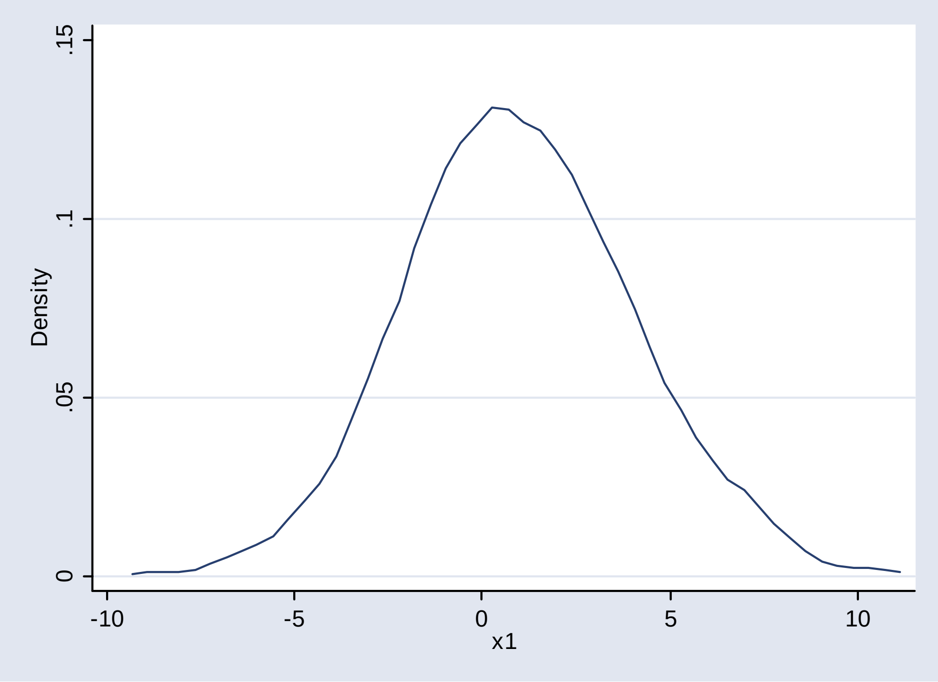
$$F(x) = \Phi = \int_{-\infty}^{x} f(x)dx$$





Distribution of variables

- Mainly depends on the variable type
- Continuous variables (interval and ratio) are mainly normally distributed – at least the universe of cases is and so should be a random sample:



- Symmetric
- Median = mean = modus
- Standard normal: mean = 0, standard deviation = 1
- A normal distribution is uniquely defined by only two parameters: mean and variance, since it is uni-modal and symmetric

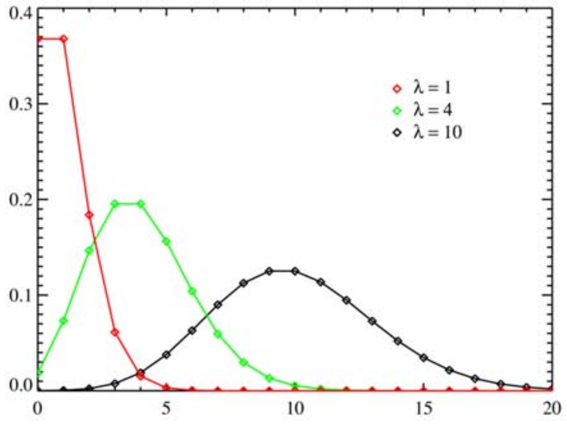
- Count data: poisson or negative binomial distributed: discrete variables, with positive integers, normally lower values occur with a higher probability, e.g. chapters closed, number of terrorist attacks in a year, number of wars, number of clients, sold cars…

  Poisson PDF:

$$f\left(k;\lambda\right)=\frac{e^{-\lambda}\lambda^{k}}{k!}$$

  lambda: expected number of occurrences

  k: number of occurrences



- Binary data: binomial distribution: the discrete probability distribution of the number of successes in a sequence of *n* independent yes/no experiments, each of which yields success with probability *p*. Such a success/failure experiment is also called a Bernoulli experiment or Bernoulli trial (n=1 – Bernoulli distribution):

  PDF (probability of getting exactly k successes):

$$f\left(k;n,p\right)=\binom{n}{k}p^{k}\left(1-p\right)^{n-k}$$

Samples and Random Samples

- Sample: a specific subset of a population (the universe of cases)
- Samples can be random or non-random=selected
- For most simple statistical models random samples are a crucial prerequisite
- Random sample: drawn from the population in a way that every item in the population has the same (according to occurrence in population) opportunity of being drawn – each draw is independent of another draw, the observations of the random sample are thus independent of each other.
  - e.g. if there are 100 balls in a bowl of which 10 are green, 30 are red and 60 are blue, then you should draw a green ball with a probability of 10/100=10%, a red ball with a probability of 30% and a blue ball with a probability of 60%. A perfect random sample of 10 observations would contain 1 green, 3 red and 6 blue balls. However, this should only hold true on average, say if you draw 50 random samples of 10 observations. If you draw just one sample there is some “sampling error”

- Sampling error: one sample will usually not be completely representative of the population from which it was drawn – this random variation in the results is known as sampling error.

- For random samples, mathematical theory is available to assess the sampling error, estimates obtained from random samples can be combined with measures of the uncertainty associated with the estimate, e.g. standard error, confidence intervals, central limit theorem (for large samples)

Characteristics of Random Samples:

- Observations are independent of each other
- The random sample mimics the distribution and all characteristics of the underlying population
- Sampling error is white noise, a random component with no structure and can therefore be assessed by mathematical and statistical tools
- Often: not observing a random sample renders statistical results biased and unreliable

Selected Samples:

- Sample selected on the basis of a specific criterion connected with the dependent variable (e.g. assessing the accession process for those countries that have applied to the EU, just looking at poor households, analyzing the voting behaviour of green party members)
- Sample selection often precludes inference beyond the sample and renders estimation results biased
- One has to be aware of possible sample selection and account for the possible bias especially of test statistics

Descriptive Statistics

- Variables can be describe by some useful descriptive measures
- Mean: most common measure of central tendency of a distribution

$$\mu = E(x) = \overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

- Median: depicts the border between two halves: ordered sample – value that divides sample in halves: half or the observations are smaller and half are larger than the median: median is resistant towards outliers and is therefore better suited as measure for central tendency than the mean in case the distribution is not normal (or symmetric)

$$\tilde{x} = \begin{cases} x_{(n+1)/2} & , n \ uneven \\ 1/2\left(x_{(n/2)} + x_{(n/2+1)}\right), & n \ even \end{cases}$$

- Modus/ Mode: the value with the highest frequency in a distribution, the value that has the highest probability of occurrence; only nominal level needed (ordinal for median, interval for mean)
- Minimal value, Maximal value
- Range: max value – min value
- Standard Deviation, Variance:

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}\left(x_i - \overline{x}\right)^2 \qquad \sigma = \sqrt{\sigma^2}$$

Example:

Variable: 0, 1, 10, 5, 0, 3, 7, 2, 1, 5, 6, 7, 4, 7, 1, 7, 10, 4, 9, 8,7

Mean: 4.952

Median: 0,0,1,1,1,2,3,4,4,5,**5**,6,7,7,7,7,7,8,9,10,10

Mode: 7

Measures of association between two variables

- Co-variance: association between two variables, e.g. years of schooling (x) and earnings (y): positive value for proportional relationship (if schooling increases earnings do as well) and negative value for inverse proportional relationship (if schooling increases, earnings decrease):

$$\mathrm{cov}_{xy} = \frac{1}{N}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)$$

- Correlation: measures essentially the same as co-variance, but values range between -1 and 1; better measure than co-variance since it is a standardized measure and does not depend on the units the variables are measured in, e.g. if hourly earnings are measured in pence instead of pounds the co-variance will be affected but the correlation coefficient won't change.

$$\rho_{xy} = \frac{\sum_{i=1}^{N}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sum_{i=1}^{N}\sqrt{\left(x_i - \bar{x}\right)^2}\sum_{i=1}^{N}\sqrt{\left(y_i - \bar{y}\right)^2}} = \frac{\mathrm{cov}_{xy}}{\sigma_x \sigma_y}$$

- Both measures of association are flawed since they cannot take into account other variables that might influence the relationship. No notion of causality between x and y involved;

Datasets:

- Datasets contain dependent, independent and intervening variables for answering a specific research question/ testing specific theoretical propositions
- All variables in the dataset have the same dimensionality (observations for the same cases, units and time points)
- Variables in a dataset can have different measurement levels, types and distributions:
- Example: study of tax competition in OECD countries: dataset contains the DV: tax rates in OECD countries over time, IV: economic variables: government debt, gdp, unemployment, FDI, capital formation etc.; political variables: colour of the government party, election dates, capital restrictions, corporatism etc.

Data-types:

Dimensionality of the data

- Cross-sectional data: observations for N units at one point in time
- Time series data: observations for 1 unit at different points in time
- Panel data: observations for N units at T points in time: N is significantly larger than T – mostly used for micro data – units are individuals
- Time series cross section (TSCS) data: = panel data, but mostly used for macro data – aggregated (country) data
- Cross section time series (CSTS) data: observations for N units at T points in time: T > N

The names Panel, TSCS and CSTS are used interchangeably, however the distinction is useful since asymptotic characteristics of estimators are derived always for the larger dimension.

Micro Data: Individual Data

- Survey data: Eurobarometer, National Election Study (US), British Election Study, socio-economic panel (Germany and other countries)
- Individual Income (LIS), firm data

→ Research questions from the fields of Political Behaviour, British Politics, Public Opinion and Polling

Macro Data: Aggregated Data on different levels

- Economic indicators: Inflation, Unemployment, GDP, growth, population (density) and demographic data, government spending, public debt, tax rates, government revenue, interest rates, exchange rates, income distribution, FDI, foreign aid, trade (exports/ imports), no of employees in different sectors etc.
- Political indicators: electoral system (majority, proportional), political system (parliamentary, presidential, federal), political institutions: CBI, exchange rate system (fixed, floating), federal courts, capital controls, number of veto players, regime type (democracy, autocracy), union density, labour market regulations, wage negotiation system (corporatism), human and civil rights, economic and financial openness, political particularism etc.