The Paternal Time Investment and Human Capital Inequality *

Jiaqi Li
University of Warwick †

Yaolang Zhong
University of Warwick ‡

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Abstract

This study explores the impact of parental time investment on children's human capital development, with a particular emphasis on distinguishing between maternal and paternal contributions. Using household data from the PSID time diary and Child Development Supplement, our empirical analysis reveals significant heterogeneity and endogeneity in paternal time investment. To quantify these effects, we employ a dynamic factor model framework to estimate the production function for children's human capital, focusing on cognitive and health outcomes. To address the endogeneity in parental time investment, we implement a control function approach, leveraging local labor demand shocks as instruments. Our counterfactual analysis demonstrates that equalizing paternal time investment across different stages of child development can significantly reduce inequalities in cognitive and health outcomes by adolescence. Specifically, equalizing paternal time during both early and middle childhood results in a 22 percent reduction in cognitive disparities and a 49 percent reduction in health disparities. These findings underscore the pivotal role of paternal time investment in shaping the inequality of children's human capital development.

Keywords: Intergenerational Mobility, Paternal Time Allocation, Child Development, Inequality, Dynamic Latent Factor Models, Human Capital Production Functions

JEL: D13, J13, J21, J22, J24, I14

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† Department of Economics, University of Warwick . Email: j.li.51@warwick.ac.uk

‡ Department of Economics, University of Warwick. Email: yaolang.zhong@warwick.ac.uk

1 Introduction

This paper contributes to the literature on child development and intergenerational mobility by investigating the impact of parental time investment on the development of children's human capital, with a particular focus on distinguishing between maternal and paternal contributions. By estimating the dynamic production function of children's human capital with both maternal and paternal inputs, this study provides insights into how these investments influence intergenerational mobility.

1.1 Background

Intergenerational mobility is a central issue in economics, as it reflects the extent to which individuals can improve their economic standing compared to their parents. The persistence of earnings across generations has been linked to various parental characteristics, including income, education, socio-economic status, and wealth, as well as the genetic transmission of traits. Additionally, such persistence is often seen as an indicator of a lack of equal opportunities in society [Chetty et al., 2014, Blanden, 2019].

This study aims to explore the extent to which intergenerational persistence is driven by parental actions rather than inherent characteristics. Specifically, it examines the heterogeneity in the effects of parental time investments on children's cognitive and health outcomes, with a particular emphasis on the differences between maternal and paternal investments. This is a critical area of research, as the role of fathers - especially in terms of time investment rather than financial contributions - has often been overlooked in the literature.

Parental investment is vital for child development. Extensive research has explored the relationship between maternal parenting time, female labor supply, and child development, showing that mothers often face a trade-off between spending time with their children and participating in the workforce [Baum II, 2003, Ruhm, 2004, Bernal, 2008, Agostinelli, 2021]. Although working longer hours can increase household income and provide more resources for children, it often comes at the cost of time spent with them.

However, there is surprisingly little research on the paternal role in child development. Does the trade-off between work and parenting time apply equally to fathers? Is there significant heterogeneity in fathers' time investment? If so, does this heterogeneity contribute to inequality in child development?

Not only in academic research, but also in policy debates, the paternal role in child development has been largely overlooked. This may reflect societal perceptions and even the tacit acceptance of fathers' limited involvement in childrearing. For instance, Bernal [2008] excludes fathers from their model of child cognitive development, assuming they play no active role in the process. However, seminal work by Del Boca et al. [2014] shows that the time investments of both parents are equally important for their children's cognitive development. In contrast, the authors find that financial expenditures are much less effective in fostering child development.

1.2 Stylized Facts

This paper contributes to the literature by investigating the role of paternal time investment in the development of children's human capital. To achieve this, we first document stylized facts about fathers' time investment using the 1997 PSID time diary dataset. Figure 1 illustrates the relationship between family income and parental time investment, revealing substantial heterogeneity in the amount of time fathers spend with their children—both actively and passively—on weekdays and weekends, across various income levels. Notably, while maternal time shows a more nuanced relationship with family income, there is a significant positive correlation between family income and paternal time investment. This pattern persists regardless of the child's race, age, or gender[1], suggesting that paternal behavior may be highly endogenous. For instance, fathers with higher incomes may be more educated and thus more aware of the importance of time investment in their children's development.

As a complement, Figure 2 illustrates the relationship between labor hours and parental time investment, again both active and passive, on weekdays and weekends. Maternal time investment exhibits a clear negative correlation with labor hours, consistent with the tradeoff between labor supply and maternal time that we discussed in the background section and that is well-documented in the literature. In contrast, paternal time investment initially shows a positive correlation with labor hours, but this relationship reverses to a negative correlation after reaching a peak. This pattern further supports the idea that paternal time investment decisions are endogenous.

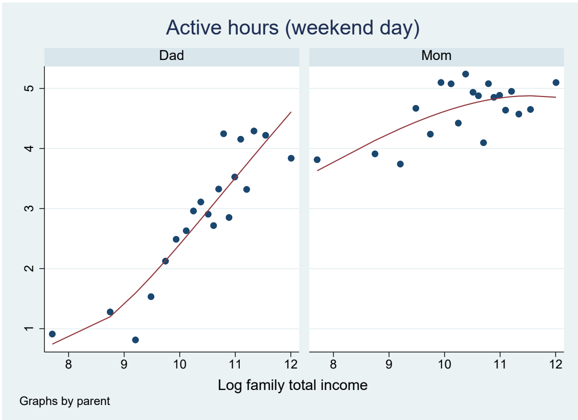
1.3 Methodologies

Given these empirical findings, this paper aims to quantify the extent to which the heterogeneity in paternal time investment explains the inequality in child development. We begin by estimating a dynamic latent factor model of the human capital production function, following a series of earlier works [Cunha and Heckman, 2007, 2008, Cunha et al., 2010, Attanasio, 2015, Attanasio et al., 2020]. Specifically, we examine children's human capital in terms of both cognitive and health levels, where their development depends on the current periods of cognition and health levels as well as parental time investments, with a distinction between maternal and paternal inputs. The analysis is conducted using the PSID dataset,

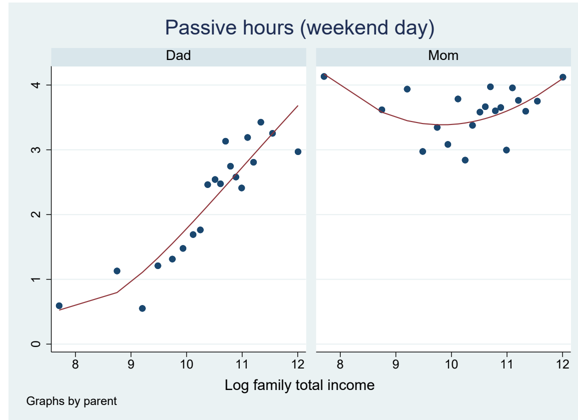
[1] Additional figures illustrating these relationships will be provided in the appendix in future versions of this paper.

Figure 1: Family income and parenting hours

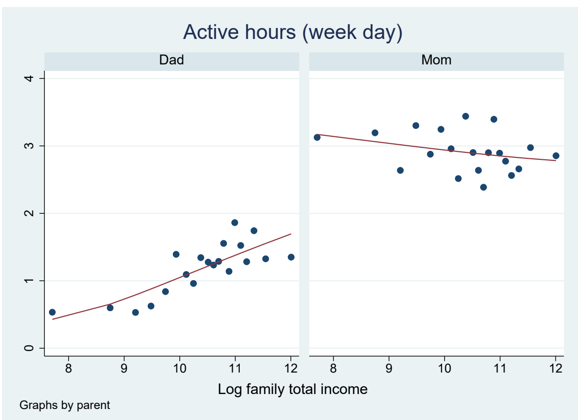
(a) Active parenting (Weekend)



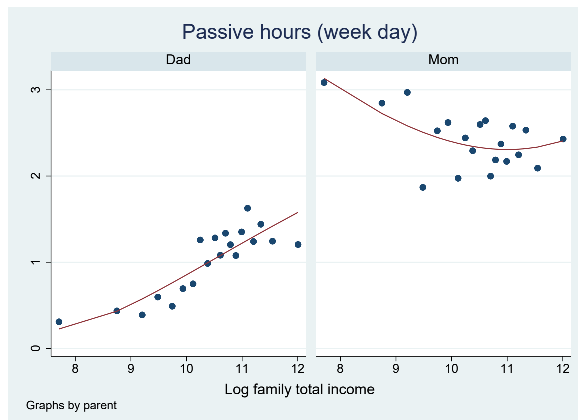
(b) Passive parenting (Weekend)



(c) Active parenting (Weekday)



(d) Passive parenting (Weekday)



Note: This figure shows the relationship between family income and both active and passive parenting hours on weekdays and weekends.

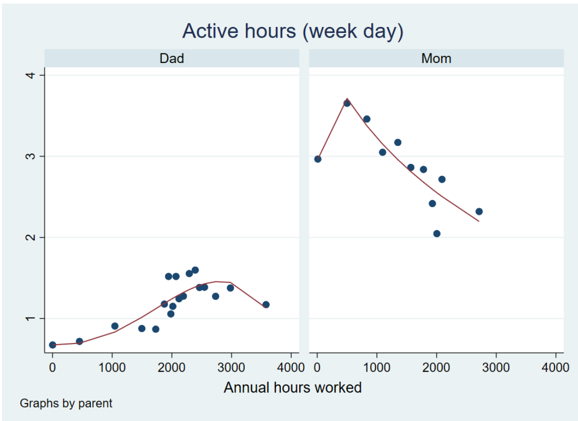
categorizing children into three distinct age groups: early childhood (ages 0 to 5), middle childhood (ages 6 to 11), and adolescence (ages 12 to 18).

Our estimation strategy consists of three steps. First, we assume a semi-log linear relationship between the observed data and the unobserved latent factors. Under the assumption that the joint distribution of the log of latent factors, as well as the measurement system, follows a mixture Gaussian distribution, we employ the EM algorithm to estimate the joint distribution of the measurement system. In the second step, we use minimum distance estimation to map the measurement system to the latent factors.

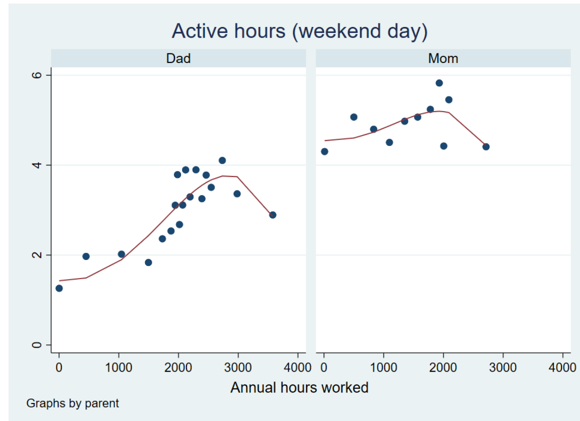
Finally, in the third step, we estimate the coefficients in the production function. To address the endogeneity in both maternal and paternal time investment decisions—one of the challenges in achieving consistent estimation of the production function coefficients—we employ a control function approach.This approach uses local gender-specific labor demand

Figure 2: Family income and parenting hours

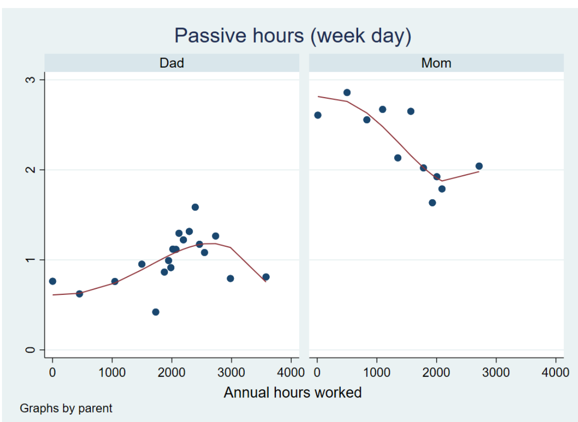
(a) Active parenting (Weekend)



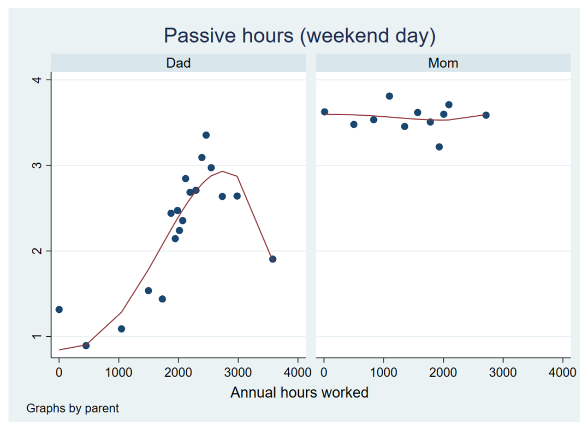
(b) Passive parenting (Weekend)



(c) Active parenting (Weekday)



(d) Passive parenting (Weekday)



Note: This figure shows the relationship between family income and both active and passive parenting hours on weekdays and weekends.

shocks to instrument for endogenous maternal and paternal time inputs, respectively. The identification assumption is that, after controlling for family income, local gender-specific labor demand shocks are uncorrelated with unobservables in the child development production function. This approach is similar to Agostinelli [2021], following the methodology of Attanasio et al. [2020], which used local prices for food, clothing, notebooks, and medication for worms, and Attanasio [2015], which used the price of toys and food in the municipality of residence, to account for the endogeneity of parental monetary investment.

Given the estimated coefficients of the production functions, we then perform a counterfactual analysis by equalizing paternal time investment among all children across different periods of their development. The results indicate that applying this treatment during both early childhood and middle childhood would lead to a 22 percent reduction in the variation of cognitive outcomes and a 49 percent reduction in the variation of health outcomes by adolescence. These significant findings highlight the crucial role that the heterogeneity of paternal time investment plays in the inequality of children's human capital development.

1.4 Literature Review

This paper makes several contributions to the literature. First, it adds to the intergenerational mobility literature [Chetty et al., 2014, Blanden, 2019] by identifying a new mechanism—heterogeneous paternal time input—as a driver of intergenerational persistence. Second, it contributes to the growing body of research applying dynamic latent factor models to the empirical study of child human capital development [Attanasio, 2015, Attanasio et al., 2020, Agostinelli, 2021]. These studies follow the dynamic latent factor framework initially proposed by Cunha and Heckman [2007, 2008], Cunha et al. [2010]. Our work emphasizes the critical role of fathers in explaining the inequality in children's cognitive and health outcomes.

Besides the empirical work, the econometrics literature has also focused on studying the properties of dynamic latent factor models, proposing new approaches for identification and estimation [Agostinelli and Wiswall, 2016a,b, Freyberger, 2021]. Additionally, another branch of literature on child human capital development has employed structural models to study the endogenous parental decision-making process [Del Boca et al., 2014, Yum, 2023].

The rest of the paper is structured as follows. Section 2 documents the data used in this study. Section 3 details the dynamic latent factor model that we estimate. Section 5 presents the estimation results and conducts counterfactual analyses. Finally, Section 6 offers concluding remarks.

2 Data

In this study, we used the PSID Child Development Supplement (CDS), a longitudinal data set that provides comprehensive information on child development, health, and parenting within families from the Panel Study of Income Dynamics (PSID). The CDS was initiated in 1997 and has collected data in multiple waves, including 2001, 2007, 2013, and 2019. Our dataset consists of two distinct groups of children.

The first group includes children who were between the ages of 0 and 17 during the initial waves in 1997, 2001, and 2007. By the 2013 wave, all children in this first group had grown up to adulthood. Consequently, the 2013 and 2019 waves focus on a second group of children, who were aged 0 to 17 years and lived in PSID families during these later years.

In total, our unbalanced panel comprises 5,463 children. Among them, 2,194 children were measured in both the 1997 and 2001 waves, and 1,086 children were observed in all three waves of 1997, 2001, and 2007. This data set offers a rich resource for analyzing developmental trajectories and influences on child outcomes over time, across different cohorts.

Table 1: Child, households and parenting hours

	Mean	SD
Maternal year of schooling	7.73	5.29
Paternal year of schooling	12.94	2.30
Annual total family income (2015 price)	8260.03	7792.47
Annual child care expenditure (2015 price)	163.31	383.45
Girls	0.49	0.50
White	0.50	0.50
Age	6.01	3.66
mother active hours on a weekend	4.61	3.34
father active hours on a weekend	2.73	3.19
mother passive hours on a weekend	3.54	2.88
father passive hours on a weekend	2.04	2.58
mother active hours on a weekday	2.90	2.55
father active hours on a weekday	1.11	1.61
mother passive hours on a weekday	2.43	2.26
father passive hours on a weekday	0.95	1.47

Note: Based on PSID-CDS data in 1997. Monetary measured deflated by CPI index. Parenting is active when a parent participated and passive when the parent was only present in the house. Family total income consists of father and mothers' labor income and household non-labor income or debt.

As shown in Table 1, mothers spend more time in parenting compared to fathers in any kind of measure by passive or active involvement or by weekend or weekday measures. Family income and childcare expenditure are deflated by the CPI index.

The dataset provides a comprehensive set of attributes that cover various aspects of child development, family dynamics, and parental participation. These attributes are categorized into several key areas, each providing valuable information on the factors that influence child outcomes. Below, we describe the specific measures used in each domain, along with an introduction to the construction of the instrumental variables employed to address the endogeneity of parental time investments.

2.1 Child Mental and Physical Health

Child mental health is assessed using the Child Depression Inventory in the PSID, which evaluates mental health across various dimensions, including appearance, crying, task performance, friendships, irritability, isolation, love, sadness, self-hate, and optimism. Physical health is measured through height, weight status, and a Body Mass Index (BMI) calculation adjusted for the child's age and gender.

2.2 Cognition

Child cognitive abilities are evaluated using the Woodcock-Johnson tests across several domains, including applied problems, broad math, broad reading, calculation, letter-word identification, and passage comprehension. Additionally, for children aged 7 and older, the Memory for Digit Span assessment, a component of the Wechsler Intelligence Scales for Children-Revised (WISC-R), is administered to measure short-term memory [Wechsler, 1974].

2.3 Parenting

Parent-child interactions are measured through indicators of various activities, such as building something together, cleaning the house, discussing books and family matters, engaging in crafts, doing dishes, yard work, shopping, playing games or sports, preparing food, using the computer, and washing clothes together.

Parental warmth is assessed through indicators such as discussing TV programs (for children aged 6 and above), expressing appreciation, engaging in favorite activities, attending parenting classes before birth, playing together, saying "I love you," showing physical affection, discussing current events, interests, relationships, and daily activities.

2.4 Parental Time Investment

Parental time investment is measured using the 24-hour time diary from the PSID, which records detailed information about a child's activities over a randomly selected weekday and weekend day. The diary captures the start and end times, activity types, locations, and social interactions. Parental involvement is categorized into passive and active participation, with each child's activities documented for both a weekday and a weekend day.

2.5 Family Environment

The family environment is measured in two main domains. The first domain assesses family conflict, including the calmness of family discussions, the frequency of family fights, physical altercations, and instances of throwing objects. The second domain evaluates cognitive stimulation in the home, such as the number of books read in the past year, the number of books in the home, and the presence of cellphones. The third domain covers the home and neighborhood environment, including noise levels inside and outside the house, and the cleanliness of the home.

2.6 Instruments for Endogenous Parental Time Investment

To address the potential endogeneity of parental time investment, we propose using local gender-specific labor demand as an instrument for time spent with children. This measure is derived from the IPUMS CPS dataset, which harmonizes microdata from the U.S. labor force survey, the Current Population Survey (CPS), covering the period from 1962 to the present. The instrument is calculated as the residual from a regression of the employment rate on a linear year trend, conducted separately by state, race and gender. We assume that the deviation from this linear prediction reflects local state-race-gender specific labor demand shocks, which do not directly affect child outcomes after controlling for family income, thus serving as an exclusion restriction. Additionally, these demand shocks are standardized to have a mean of 0 and a standard deviation of 1, separately by gender.

3 Dynamic latent factor model

To understand the process of human development and the role of parental investment, we build on the series of seminal works on children's skill formation by Cunha and Heckman [2007] and Cunha et al. [2010], which utilize a dynamic latent factor model. This framework consists of three key components: the technology of skill formation (equivalently, the human capital production function), the instruments used to address the endogeneity of parental investment decisions, and the measurement system that links observable data to unobservable latent factors. In the following, we discuss each component in detail. Three parts: human capital production function, instruments for the endogeneity of time investment, measurement system

3.1 Human capital technology

We begin by specifying that the development of human capital in children encompasses two primary dimensions: cognitive skills (θ_c) and physical health (θ_h), as is commonly addressed in the literature. To account for the various developmental stages of childhood, we categorize children into three distinct age groups: early childhood (ages 0 to 5), middle childhood (ages 6 to 11), and adolescence (ages 12 to 18). From this point forward, we refer to these stages as Stage 1, Stage 2, and Stage 3, respectively. This classification allows us to model the evolution of cognition and health through dynamic production functions that span these stages, resulting in two periods of production: $t = 1, 2$.

In each period, the cognition ($\theta_{c,t+1}$) and health ($\theta_{h,t+1}$) of a child in the subsequent stage depend on several factors: their cognition ($\theta_{c,t}$) and health ($\theta_{h,t}$) in the current period, the mother's time investment ($I_{m,t}$), the father's time investment ($I_{f,t}$) during that stage, and the positive total factor productivity (TFP): ($A_{c,t}, A_{h,t}$), which captures the efficiency of inputs in producing cognitive skills and health outcomes, respectively.

We assume a CES (Constant Elasticity of Substitution) form of the technology:

$$\begin{aligned}\theta_{c,t+1} &= [\alpha_{c,t}(\theta_{c,t})^{\rho_t} + \alpha_{h,t}(\theta_{h,t})^{\rho_t} + \alpha_{m,t}(I_{m,t})^{\rho_t} + \alpha_{f,t}(I_{f,t})^{\rho_t}]^{\frac{1}{\rho_t}} A_{c,t} \\ \theta_{h,t+1} &= [\beta_{c,t}(\theta_{c,t})^{\zeta_t} + \beta_{ht}(\theta_{h,t})^{\zeta_t} + \beta_{m,t}(I_{m,t})^{\zeta_t} + \beta_{f,t}(I_{f,t})^{\zeta_t}]^{\frac{1}{\zeta_t}} A_{h,t}\end{aligned} \tag{1}$$

With input shares α_ts and β_ts summing to 1 respectively, we are particularly interested in the values of (ρ_t, ζ_t), which determine the elasticities between the various inputs in the production process. When they are equal to 1, the production function described in Equation 1 is simplified to a linear function, indicating that all inputs are perfectly substitutable. In contrast, when the they are less than 1, the inputs become complementary. Specifically, when they are equal 0, the system reduces to the Cobb-Douglas form where the elasticity of substitution equals to 1.

An alternative functional form to consider is the trans-log form, as discussed in Freyberger [2021], which offers additional flexibility to capture interactions between inputs.

The next equations determine the lop-TFP:

$$\begin{aligned}\log A_{c,t} &= d_{0t} + d'_{Xt} X_t + u_{c,t} \\ \log A_{h,t} &= g_{0t} + g'_{Xt} X_t + u_{f,t}\end{aligned} \tag{2}$$

We assume that the log-TFP is a linear function of a set of family characteristics from the data, incorporated in the covariate X_t. These characteristics include race, gender of the children, and time-varying family income. Family income, which captures material parental investment, is incorporated as part of the TFP rather than as a separate input to human capital. This is done for simplicity, for two main reasons: first, our focus in this paper is specifically on the effects of parental time investment, and second, the findings of Del Boca et al. [2014] using the same PSID dataset suggest that monetary inputs are less productive compared to parental time investment.

3.2 Control for the edogeneity of parental time investments

Next, we introduce additional equations to account for parental time investments. This step would be unnecessary if parental time investment were exogenous, conditional on the current stage of the child's cognition and health status, as well as household characteristics. However, endogeneity arises because parents may base their decisions on the evolving human capital of their child. For example, a father might spend more time reading with his child upon realizing the child's aptitude for learning. In such cases, identification requires instruments that influence parental investment but are excluded from the child's human capital production function.

As explained in Section 2 , we use residuals from the deviation from gender-, race-, and state-specific linear trends in employment as proxies for local labor demand shocks. The identification assumption is that race- and gender-specific variation in local labor demand is uncorrelated with omitted inputs in the child's human capital production function after controlling for family income, which is incorporated into the household characteristics. The empirical specification of parental time investments $(I_{m,t}, I_{f,t})$ is:

$$\begin{aligned} \ln I_{m,t} =& \gamma_0 + \gamma_{c,t} \ln \theta_{c,t} + \gamma_{h,t} \ln \theta_{h,t} + \gamma'_{X,t} X_t + \gamma'_{Z,t} \ln Z_{m,t} + v_{m,t} \\ \ln I_{f,t} =& \eta_0 + \eta_{c,t} \ln \theta_{c,t} + \eta_{h,t} \ln \theta_{h,t} + \eta'_{X,t} X_t + \eta'_{Z,t} \ln Z_{f,t} + v_{f,t} \end{aligned} \tag{3}$$

where $(Z_{m,t}, Z_{f,t})$ are the instruments. With these instruments, we apply a control function approach [Gronau, 1974, Heckman, 1979, Heckman and Navarro-Lozano, 2004]. Specifically, endogeneity arises because the error terms $(v_{m,t}, v_{f,t})$ in the parental time investment equation 3 are correlated with the error terms $(u_{c,t}, u_{h,t})$ in the production function 1. Therefore, we assume a linear specification conditional on the information set:

$$\begin{aligned} E\left(u_{ct} \mid Q_t, Z_t\right) &= \kappa_{c,m} v_{m,t} + \kappa_{c,f} v_{f,t} \\ E\left(u_{ht} \mid Q_t, Z_t\right) &= \kappa_{h,m} v_{m,t} + \kappa_{h,f} v_{f,t} \end{aligned} \tag{4}$$

where Q_t is the full set of variables in the production functions, including parental time investments, and Z_t are the instruments. The control function approach involves including the regression residuals $(\tilde{v}_{m,t}, \tilde{v}_{f,t})$ from Equation 3 as additional regressors in Equation 1.

3.3 The measurement system

Since many of the variables in the production function have multiple measurements in the dataset, this raises the question of how to efficiently use the available data. The simple selection of one of the measures as a proxy could lead to bias with unknown signs due to the non-linearity of the production function [Griliches and Ringstad, 1970].

To address this challenge, we apply the latent factor framework proposed in Cunha et al. [2010]. Instead of the flexible non-parametric identification procedure discussed in that paper, we assume a specific functional form as in Attanasio et al. [2020] for simplicity.

We begin by assuming a semi-log mapping between the measures and latent variables:

$$m_{j,k,t} = a_{j,k,t} + \lambda_{j,k,t} \ln\left(\theta_{k,t}\right) + \epsilon_{j,k,t} \tag{5}$$

Here, $m_{j,k,t}$ denotes the jth measure related to the kth latent variable at time t, and $\lambda_{j,k,t}$ is the corresponding factor loading, for which it is more convenient to handle in log-form as it appears in the production function. Note that the crucial assumption here is that the measurement error $\epsilon_{j,k,t}$ is separable, normally distributed, and independent of both the latent factor $\theta_{k,t}$ and each other.

We can then rewrite Equation 5 by wrapping all variables with subscripts j, k, and t in a matrix form for compactness:

$$M = \mathbf{A} + \Lambda \ln \theta + \Sigma \varepsilon \tag{6}$$

Let I denote the number of measures and J the number of latent factors. Then, M is a vector of length I representing all measures from the dataset, A is a vector of length I representing the constant terms, and ϵ is the vector of error terms of length I. Σ is the $I \times I$ diagonal matrix, and Λ is the factor loading matrix of shape $I \times J$, with $\ln \theta$ being the vector of length J representing all the log-transformed latent factors.

We now consider the joint distribution of the log-transformed latent factors. Due to the assumed semi-log mapping in Equation 5, a natural choice for the functional form would be joint normality, given its closure under linear combinations. However, assuming joint normality would impose additional restrictions on Equation 1, effectively reducing the CES production function to a Cobb-Douglas form (which is linear in the log-form). To avoid this, we instead assume that the joint distribution is a mixture of Gaussians, preserving the desirable property of closure under linear combinations while allowing for greater flexibility.

The next question is determining the appropriate number of clusters for the Gaussian mixture. In our implementation, we experimented with up to 5 clusters and found that when the number of clusters is greater than or equal to 3, the distance between clusters becomes insignificant, indicating that increasing the number of clusters did not efficiently improve the approximation of the true underlying distribution. Therefore, we adopt a parsimonious approach, using only 2 clusters in our model:

$$F_\theta = \tau \Phi \left(\mu_A, \Omega_A\right) + (1-\tau) \Phi \left(\mu_B, \Omega_B\right) \tag{7}$$

where τ represents the mixture weights, or the probability that each realized observation is drawn from the first Gaussian distribution. (μ_A, Ω_A) and (μ_B, Ω_B) are the mean and variance of the first and second groups, respectively. The semi-log mapping between measures and latent factors then gives rise to the mixture Gaussian distribution for the measurement system:

$$F_M = \tau \Phi \left(\Pi_A, \Psi_A\right) + (1-\tau) \Phi \left(\Pi_B, \Psi_B\right) \tag{8}$$

where (Π_A, Ψ_A) and (Π_B, Ψ_B) are the mean and variance of the measures in the first and second groups, respectively. This allows us to construct the mapping between the parameters characterizing the mixture distribution of latent factors and those of the measures.

$$\begin{aligned} \Psi_A &= \Lambda^T \Omega_A \Lambda + \Sigma, \qquad \Pi_A = \mathbf{A} + \Lambda \mu_A \\ \Psi_B &= \Lambda^T \Omega_B \Lambda + \Sigma, \qquad \Pi_B = \mathbf{A} + \Lambda \mu_B \end{aligned} \tag{9}$$

Finally, in addition to the latent variables in our model, there are also observable variables used in the production functions and as instruments in the time investment functions. These variables are considered error-free, meaning they are present in both the measurement system and the latent variable system. This inclusion results in an augmented system of mixture Gaussian distributions for the latent factors:

$$F_{\theta,X} = \tau \Phi \left(\mu_A^{\theta,X}, \Omega_A^{\theta,X} \right) + (1-\tau) \Phi \left(\mu_B^{\theta,X}, \Omega_B^{\theta,X} \right) \tag{10}$$

This also leads to an augmented system of mixture Gaussian distributions for the measurement system, along with an expanded mapping between the parameters of the latent variable distributions and those of the measurement system. To avoid unnecessary notation complexity, the detailed equations are omitted here.

4 Estimation strategy

In the following, we discuss and explain the estimation strategy we adapt following Attanasio, Meghir, and Nix [2020], which consists of three steps:

Step 1: Use the expectation-maximization (EM) algorithm [Dempster, Laird, and Rubin, 1977, McLachlan and Krishnan, 2007] to estimate all the parameters that characterise the mixture Gaussian measurement system: the weights τ, the mean vectors (Π_A, Π_B) and the covariance matrices (Ψ_A, Ψ_B).

In the context of the Gaussian Mixture Model (GMM), the Expectation-Maximization (EM) algorithm is an iterative update scheme that alternates between Expectation (E-step) and Maximization (M-step), each accepting a closed-form formula for the new estimate. Specifically, let N denote the number of observations, I the number of dimensions (number of measures) of each observation, K the number of clusters, y the observed dataset with shape (N, I), and Z the random variable with shape (N, K) representing the unobserved latent cluster memberships. Using the notation for the mixture distribution of the measurement system from Equation 8, we have:

E-step:

$$\mathbb{E}[Z_k | y; \theta] = \frac{\pi_k \exp\left(-\frac{1}{2} \text{diag}((y - \Pi_k) \Psi_k^{-1} (y - \Pi_k)^T)\right)}{\sqrt{(2\pi)^I \det(\Psi_k)}} \tag{11}$$

where $\mathbb{E}[Z_k | y; \theta]$ is the responsibility that cluster k takes for observation y_i.

M-step:

$$\begin{aligned} \Pi_k &= \frac{Y^T \hat{Z}_k}{1_N^T \hat{Z}_k}, \\ \Psi_k &= \frac{(Y-\Pi_k)^T \text{diag}(\hat{Z}_k)(Y-\Pi_k)}{1_N^T \hat{Z}_k}, \\ \pi_k &= \frac{1_N^T \hat{Z}_k}{N}, \end{aligned} \tag{12}$$

where $\hat{Z}_k = \mathbb{E}[Z_k|y;\theta]$ is a vector of responsibilities for each observation in cluster k, Y is the matrix of observations with shape (N, I), Π_k is the updated mean vector for cluster k, Ψ_k is the updated covariance matrix for cluster k, π_k is the updated mixing coefficient for cluster k, and 1_N is a column vector of ones of length N.

However, while there is a closed-form solution for both the E-step and M-step in each iteration due to the Gaussian assumption, in practice, our case involves a non-negligible amount of missing data. This is largely due to the structure of the PSID dataset, where most children are observed in only one or two waves of data collection. This data structure necessitates that updates in the EM algorithm be conditional on the observed data for each observation, requiring us to rely on looping over each observation rather than using the more efficient vectorized operations as in Equations 11 and 12. This reliance on loops significantly slows the computational time. Additionally, in each iteration, the temporarily obtained parameters must be handled carefully to prevent ill-conditioned matrices in subsequent rounds.

Step 2: Use minimum distance estimation to recover the parameters needed to characterize the joint distribution of latent variables θ: the shift vector $\mathbf{A}$, factor loading matrix Λ, measurement error matrix Σ, the mean vectors μ_A, μ_B and the covariance matrices Ω_A, Ω_B from the parameters obtained from Step 1.

An important procedure before adopting the minimum distance estimation, which is a nonlinear optimization program, is to impose a set of restrictions to reduce the degrees of freedom in the space of parameters, which we state as follows:

1. **Normalization on the latent factor:** For the augmented latent factor (θ, X), we normalize those time-invariant variables to have the mean of log equal to zero. For the time-varying variables, we normalize only the first period to have the mean of log equal to zero, such that the log value of the subsequent periods can be identified as the growth relative to the first period.

2. **Restrictions on the factor loading:** We assume that each measure links to only one underlying factor, so only the corresponding factor loading is non-zero. Additionally, we set the scale of the latent factor equal to one of the measurements, which is denoted as the primary measurement, with the associated factor loading set to one. For example, for child cognition, we normalize the loading on the Woodcock-Johnson

letter-word test to one, and for child health, we normalize on the z-scores of height. For parental time investment, we normalize on active hours spent with the child on a weekday. Note that this assignment of primary measurements should be consistent across periods.

3. **Restriction on the constant term:** We assume that the growth of the measurement is due only to the growth of the associated latent variables. This, together with the normalization of loading to the first measurement for each latent variable as described in point 2, imposes a restriction on the constant vector **A** such that the elements in **A** corresponding to each first measurement remain unchanged dynamically. Therefore, we only need to determine their values in the first period. Point 1 implies that we can set the term in **A** for the first period of all measurements equal to the mean of the measurements from the data.

Step 3: Draw a synthetic dataset from the parametric joint distribution of the latent factors estimated from Step 1 and Step 2. We then estimate the parameters of interest in the system of production functions 1 using the control function approach, which is essentially a two-stage estimation scheme. First, we perform OLS to regress Equation 3 and obtain the regression residuals. These residuals are then added as additional regressors (to the linear part), and we proceed to estimate the log-form of the production function 1 using a nonlinear least squares approach.

5 Results and counterfactual analysis

We follow the estimation procedure discussed in Section 4, employing a bootstrap approach with 100 iterations to construct the 90% empirical confidence intervals.

5.1 Estimates

We begin by presenting the estimated joint distribution of the latent factors, obtained from Steps 1 and 2 of the estimation procedure. Table 2 displays the results, showing the weights and means of the mixture for each cluster in the Gaussian mixture model that characterizes the joint distribution of these latent factors. For brevity, the covariance matrix is not included.

The estimated mixture weights indicate that approximately one third of the observations belong to the first cluster (Mixture A), while the remaining two-thirds belong to the second cluster (Mixture B). This suggests a meaningful division within the population, with Mixture B representing the majority.

The table reflects differences in parental investment between the two mixtures. For instance, both maternal and paternal investments in their children's development are consistently higher in Mixture A across all ages. This may imply that families in Mixture A prioritize time spent with their children more than those in Mixture B, potentially contributing to the observed differences in cognitive and health outcomes.

These results collectively highlight the necessity and effectiveness of using a Gaussian mixture model to capture the heterogeneity in the population. The significant differences in the means of most latent variables between the two clusters underscore a substantial departure from a simple multivariate normal distribution. This supports our decision to implement the mixture model, which provides a more nuanced and accurate representation of the underlying data structure.

Table 2: Weights and Means of the Gaussian Mixture Model for Latent Factors

	Mixture.A	Mixture.B
Weights	0.382 [0.36,0.39]	0.618 [0.61,0.64]
Mean Cognition Stage 3	2.235 [2.17,2.266]	2.254 [2.236,2.32]
Mean Cognition Stage 2	1.846 [1.816,1.871]	1.635 [1.617,1.652]
Mean Cognition Stage 1	0.408 [0.352,0.436]	−0.251 [−0.269,−0.206]
Mean Health Stage 3	2.153 [1.33,2.219]	2.265 [2.21,2.281]
Mean Health Stage 2	1.697 [1.659,1.718]	1.377 [1.361,1.397]
Mean Health Stage 1	−0.067 [−0.106,−0.042]	0.042 [0.025,0.065]
Mean Mom's Investment Stage 3	1.603 [1.568,1.655]	0.121 [0.073,0.185]
Mean Mom's Investment Stage 2	1.334 [1.306,1.358]	−0.836 [−0.857,−0.819]
Mean Mom's Investment Stage 1	0.544 [0.511,0.599]	−0.336 [−0.361,−0.305]
Mean Dad's Investment Stage 3	2.109 [2.071,2.156]	0.519 [0.471,0.603]
Mean Dad's Investment Stage 2	1.932 [1.908,1.962]	−0.847 [−0.866,−0.829]
Mean Dad's Investment Stage 1	0.928 [0.883,1.01]	−0.573 [−0.613,−0.517]

Notes: 90% confidence intervals based on 100 bootstrap replications in square brackets.

We next present the results from Step 3, showing the estimated coefficients of the production function in Table 3. A crucial finding is that the estimated elasticity parameters (ρ_t, ζ_t) are all between 0 and 1, indicating that the elasticity of the production function is less than 1. This suggests that the various inputs in the production of a child's cognition and health are complementary, which aligns with the findings reported in Attanasio et al. [2020]. Another important observation is that a child's health status in the initial period has a dominant impact not only on its subsequent health (indicating a strong self-productivity effect) but also on cognition during childhood (ages 6–12). This influence remains significant as the child transitions from childhood to adolescence. Conversely, the self-productivity effect of cognition is less pronounced during childhood but becomes more significant during adolescence.

Regarding parental time investments, the estimation results suggest that both maternal and paternal time investments have an insignificant direct impact on health. This is plausible since a child's physical health status might be more directly influenced by material investments. However, parental time investment plays a crucial role in cognitive development. Specifically, paternal time investment is significant during the child's early years, while maternal time investment becomes significant during adolescence.

Finally, we find that the coefficients for the paternal time investment residuals are significantly different from zero for health during childhood and cognition during adolescence, indicating that paternal time investment is endogenous and correlated with unobservable factors in the production function at different stages of child development.

Table 3: Production of Cognition and Health with Endogenous Time Investments

	Cog, Stage 2	Health, Stage 2	Cog, Stage 3	Health, Stage 3
Cognition	−0.019 [−0.082,0.038]	0.013 [−0.077,0.026]	0.399 [0.288,0.435]	0.198 [0.154,0.315]
Health	0.946 [0.874,0.993]	0.961 [0.9,1.019]	0.406 [0.352,0.595]	0.685 [0.551,0.909]
Mom's time	−0.113 [−0.211,0.046]	−0.043 [−0.119,0.168]	0.165 [0,0.264]	0.053 [−0.193,0.159]
Dad's time	0.186 [0.036,0.273]	0.069 [−0.082,0.151]	0.03 [−0.121,0.173]	0.064 [−0.172,0.15]
Mom's time ctrl func	−0.02 [−0.069,0.094]	−0.037 [−0.083,0.092]	0.02 [−0.034,0.066]	−0.023 [−0.186,0.017]
Dad's time ctrl func	0 [−0.044,0.088]	−0.159 [−0.22,−0.031]	0.134 [0.006,0.211]	0.107 [−0.104,0.172]
elasticity parameter	0.221 [−0.118,0.868]	0.416 [−0.438,0.736]	0.336 [0,0.635]	0.711 [−0.098,0.884]
Family income	0.128 [0.098,0.138]	−0.003 [−0.027,0.015]	0.082 [0.064,0.1]	0.024 [−0.007,0.035]
White	0.08 [0.036,0.125]	−0.036 [−0.076,−0.004]	0.079 [0.061,0.129]	−0.004 [−0.045,0.042]
Female	0.084 [0.047,0.114]	0.018 [−0.012,0.051]	0.001 [−0.039,0.023]	−0.228 [−0.259,−0.189]
constant	1.568 [1.501,1.634]	1.465 [1.386,1.557]	0.707 [0.612,0.755]	0.785 [0.213,0.866]
Residual standard error	0.451 [0.426,0.479]	0.248 [0.202,0.294]	0.35 [0.32,0.386]	0.34 [0.304,0.599]

Notes: 90% confidence intervals based on 100 bootstrap replications in square brackets.

5.2 Counterfactual analysis

Using the estimated coefficients of the production function shown in Table 3, we conducted a counterfactual exercise to assess the impact of equalizing paternal time investments on the inequality of child development. Specifically, we aim to evaluate how much inequality in child development could be reduced if all children received an equal amount of paternal time investment.

We perform the following steps in our counterfactual analysis: We simulate a synthetic population based on the estimated joint distribution of parameters. We focus on children in Stage 1 and consider three scenarios of interventions: (1) Equalizing paternal time investment solely in Stage 1, (2) Equalizing paternal time investment solely in Stage 2, and (3) Equalizing paternal time investment in both stages. In each scenario, other inputs and household characteristics contributing to the production function of cognition and health are held constant. This setup creates an artificial experiment where paternal time investment is treated as the "treatment." We then compute the standard deviation of cognition and health scores for the population in Stage 3 under the different interventions.

Table 4: Results on Counter-Factual Intervention

	Cog std.	Health std.	Cog std.(%)	Health std.(%)
No Intervention	0.063	0.085	100.0	100.0
Intervention in Stage 1	0.059	0.078	92.7	92.4
Intervention in Stage 2	0.054	0.047	85.0	55.7
Intervention in Stage 1 and 2	0.049	0.043	78.0	51.0

As shown in Table 4, the table compares the standard deviation of cognition and health scores among individuals with different intervention experiences. The first column indicates whether the individual had an intervention experience and at what stage. The second and third columns show the standard deviation of the cognition and health scores, respectively. The fourth and fifth columns display the percentage of the standard deviation of cognition and health scores relative to the group without intervention experience (that is, 100%).

From the table, it is evident that children who had an intervention experience at Stage 1 or Stage 2 exhibit a lower standard deviation in cognition and health scores than those who did not have any intervention experience. Specifically, individuals with intervention experience at Stage 1 show a 7.3% lower standard deviation in cognition and a 7.6% lower standard deviation in health scores compared to the group with no intervention experience. Individuals with intervention experience at Stage 2 show a 15% lower standard deviation in cognition and a 44.3% lower standard deviation in health scores compared to the group with no intervention experience. These findings suggest that equalizing paternal time investment, particularly in Stage 2, could significantly reduce inequality in child development outcomes.

This artificial experiment suggests that to reduce inequality in children's development, both in cognition and health, policymakers could consider implementing policies that influence fathers' decision-making, thereby balancing the inequality of endogenous paternal time investment across different groups. Further investigation into parental decision-making and the design of such policies may be required, though these aspects fall outside the scope of this paper.

6 Conclusion

This paper contributes to the literature on child development and intergenerational mobility by investigating the impact of parental time investment on the development of children's human capital, with a particular focus on distinguishing between maternal and paternal contributions.

Specifically, we begin by presenting our empirical findings, which reveal significant heterogeneity and endogeneity in paternal time investment. To quantify these impacts, we apply a dynamic factor model framework to estimate the production function of children's human capital, measured through cognitive and health outcomes, with both maternal and paternal time investments as inputs. More precisely, we assume a semi-log linear relationship between the observed data and unobserved latent factors. Assuming that the joint distribution of the log of latent factors, and so as the measurement system, follows a mixture Gaussian distribution, we employ the EM algorithm to estimate the joint distribution of the measurement system. We then use minimum distance estimation to map the measurement system to the latent factors. For the estimation of the production function, we utilize a control function approach, with local labor demand shocks as instruments, to account for the endogeneity of parental time investment.

Through a counterfactual analysis, we demonstrate that equalizing paternal time investment across different periods of child development significantly reduces the inequality in cognitive and health outcomes by adolescence. Specifically, our results indicate a 22 percent reduction in the variation of cognitive outcomes and a 49 percent reduction in health disparities when paternal time investment is equalized during both early and middle childhood. These findings underscore the critical role that the heterogeneity of paternal time investment plays in shaping the inequality of children's human capital development.

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A Appendix Data

The Child Development Supplement (CDS) to the PSID collected comprehensive data in the following areas: (i) reliable, age-graded assessments of the cognitive, behavioral, and health status of 3,600 children (including approximately 250 immigrant children), obtained from the mother, a second parent or parent figure, the teacher or childcare provider, and the child; (ii) a detailed accounting of parental and caregiver time inputs to children, as well as other aspects of how children and adolescents spend their time; (iii) teacher-reported time use in elementary and preschool programs; and (iv) non-time use measures of other resources—such as the learning environment at home, teacher and administrator reports of school resources, and parent-reported measures of neighborhood resources.