# Optimal Disclosure of Test Results by a Strategic Intermediary

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# Motivation

- In a signalling game tests can provide verifiable evidence about hidden information
- We consider the case where the receiver requires the sender to seat a stochastic test provided by a third party intermediary
- The intermediary commits to a test design and a disclosure policy ex-ante
  - Profit maximisers or who have different objectives than the sender or receivers
- The receiver takes a binary decision based on test outcome

# Trade off: Participation vs Profit

- The intermediary's objective in designing the test's informativeness and the disclosure policy encompass a trade off:
- Reveal information to induce participation of the good type sender and the receiver
  - Exploit willingness to pay of the good type to reveal information (Retake)
- Not to reveal information to induce participation of the bad type sender
  - Exploit willingness to pay of the bad type to hide information (Pricing /Disclosure)

# Motivation

- An intermediary with misaligned objective with sender and receiver can manipulate information in different ways to extract benefit:
  - Test Design: Bayesian Persuasion
  - Disclosure Design: Certification Equilibrium
  - Both: Our Case

# Characteristics of the environment

- Receiver's decision is binary (A/R): Test needs to be informative
- Sender's signal is stochastic it can produce positive or negative evidence
- For Sender test result is uncertain and continuation is risky
- Intermediary can both design the test's informativeness and the disclosure policy

# A Research Question

What is the optimal information and disclosure design for the intermediary to induce participation and maximise his pay off

# An Example

- ETS, the organization that manages the GRE exam, introduced a new *ScoreSelect* option in 2012 whereby:
  - students could send their best score(s) to their graduate institutions, and
  - retakes would not be seen by admissions panels.
- Before 2012, all the previous test scores were sent to the colleges.

We will call this a change from **full disclosure** to **delegated disclosure** policy.

# Basic model

- Three (strategic) players: Sender, Receiver and Intermediary
- Sender of type $\theta \in \{G, B\}$ with prior probability of $G$ being $\gamma < 1/2$.
- Sender privately informed of her type.
- Sender wants to be "admitted" by the Receiver.
- Receiver wants to admit the Sender only if she is type $G$.
- Intermediary wants to maximize price $\times$ expected no. of tests.

# Basic model

Distribution of results $(x)$:

$$x = \begin{cases} S & \text{w. p. } 1 \text{ if } \theta = G \text{ and w. p. } b \text{ if } \theta = B \ , \\ F & \text{otherwise.} \end{cases}$$

# Basic model

**TIMING:**

1. The platform decides the price of each test $p$, a cap on the number of maximum tests $K$ and the disclosure policy (given $b$ and $\gamma$).

2. Sender decides how many tests to take sequentially after looking at their results, and discloses some or all of them.

3. Receiver decides whether to admit/reject the sender.

For this talk, we focus on **full disclosure**.

# Full Disclosure

- Consider the equilibrium in which both types are expected to reveal $s$ successes to be admitted
- On-path belief after $s$ successes: $\gamma_s = \frac{\gamma}{\gamma + b^s(1-\gamma)}$
- $\gamma_s \geq \frac{1}{2}$ uniquely pins down the minimum no. of successes required to be admitted.
- Price $p_s$ pinned down by the $\theta = B$'s participation constraint:

$$b^s - p(1 + b + b^2 + ... + b^{s-1}) \geq 0$$
$$\implies p_s = \frac{b^s(1-b)}{1-b^s}$$

- Off-path belief: Any $s' > s$ believed to come from $\theta = H$ and $s' < s$ believed from $\theta = L$

# Full Disclosure

Intermediary can choose $p_s$ to sustain any pooling on $s$ equilibrium conditional on $\gamma_s \geq \frac{1}{2}$. Which one is best for it?

- Classic inverse demand function generated – Increasing price reduces equilibrium $s$
- Intermediary's ex-ante expected profits:

$$\pi(s;\gamma) = \gamma s \frac{b^s(1-b)}{1-b^s} + (1-\gamma)b^s$$

- Where are the profits maximized?

**Result:** Intermediary chooses minimum $s$ such that belief just crosses $1/2$: $s^*$, i.e. maximum price

- Enough to show that $s\frac{b^s(1-b)}{1-b^s} + b^s$ decreasing in $s$ – can be shown by induction.
  - Intermediary can benefit by increasing $s$ only from what it gets from the $G$ type sender. This benefit must go down when $\gamma \leq 1/2$. So, more expected loss from increasing $s$.
- $K$ not required – $p_{s^*}$ induces $K = s^*$
- Separating by charging $p = 1$ not optimal

# Full Disclosure – Test design

Say that $s^* > 1$. Consider the following choice for the intermediary:

1. offer a single test with $1 - b'$ difficulty, where $b' = b^{s^*}$, OR
2. offer $s^*$ tests (in equilibrium) each with $1 - b$ difficulty for $B$ type sender

- Note that $b > b^{s^*}$ so that the difficulty of the test has gone up.
- The two tests are outcome equivalent for the receiver. The expected payoff of the receiver in both cases is $\gamma - (1 - \gamma)b^{s^*}$ since in either case the probability of making the wrong decision is the same.
- **BUT one is better for the platform than the other.**

# Full Disclosure – Test design

Under Option 1, max price that the intermediary can charge is $b^{s^*}$, which is $> p_{s^*} = \frac{b^{s^*}(1-b)}{1-b^{s^*}}$.

Intermediary's ex-ante expected profits under Option 1:

$$\pi_1(1;\gamma) = \gamma b^{s^*} + (1-\gamma)b^{s^*} = b^{s^*},$$

which can be verified to $\leq \pi(s^*;\gamma)$.

$\Longrightarrow$ Option 2 does better for the testing intermediary, i.e. incentive to make the test easier for $B$ type.

# Full Disclosure – Test design

What does the intermediary gain by "breaking one test into many"?

Write down the profits of the firm under the two options:

$$\pi_1(1;\gamma) = \gamma b^{s^*} + (1-\gamma)b^{s^*}$$
$$\pi(s^*;\gamma) = \gamma s^* \frac{b^{s^*}(1-b)}{1-b^{s^*}} + (1-\gamma)b^{s^*}$$

- Same amount of money to be made from $B$ types, but more money made from $G$ types by splitting the test.
- What is the optimal splitting of tests? Is there a limit to this behaviour?

# Full Disclosure – Test design: Optimal $b$

We need to write $\pi(s^*(b);\gamma)$ to optimize over $b$.

Note that for the choice of any $b$, the best equilibrium for the intermediary is the one that flips the receiver's just above a half. It must be

$$\begin{aligned}
\gamma_{s^*} &= \frac{\gamma}{\gamma + b^{s^*}(1-\gamma)} = \frac{1}{2} \\
\implies b^{s^*} &= \frac{\gamma}{1-\gamma} \\
\implies s^*(b) &= \frac{\ln(\frac{\gamma}{1-\gamma})}{\ln b}
\end{aligned}$$

So, when the platform increases $b$, it is forced to choose a price in a way that $b^s$ remains the same. So $s^*(b)$ increases accordingly.

# Full Disclosure – Test design: Optimal $b$

$$\pi(b;\gamma) = \gamma s^*(b)\frac{b^{s^*(b)}(1-b)}{1-b^{s^*(b)}} + (1-\gamma)b^{s^*(b)}$$

For the purposes of maximization, note that $\gamma\frac{b^{s^*(b)}}{1-b^{s^*(b)}}$ and $(1-\gamma)b^{s^*(b)}$ are constants. So, we essentially maximize

$$\tilde{\pi}(b;\gamma) = (1-b)\frac{\ln(\frac{\gamma}{1-\gamma})}{\ln b},$$

which is maximized at $b = 1$.

- **BUT the sender-receiver equilibrium breaks down at $b = 1$.**

# Full Disclosure – Test design: Optimal $b$

To get back the equilibrium (and the optimal $b$), assume an $\epsilon$ cost to the intermediary for conducting the test each time.

**STEP 1:** What is the best price to choose for a given $b$?

- The sender-receiver equilibrium is the same and the WTP of the sender is the same.
- Profits of the intermediary:

$$\pi(s;\gamma) = \left(\gamma s \frac{b^s(1-b)}{1-b^s} + (1-\gamma)b^s\right) - \epsilon\left(\gamma s + (1-\gamma)\frac{1-b^s}{1-b}\right)$$

- Still decreasing in $s \in \{1, 2, ...\}$ because the expected costs are increasing.
- Intermediary chooses min $s$ for any given $b$.

# Full Disclosure – Test design: Optimal $b$

**STEP 2:** What is the optimal $b$?

- Profits of the intermediary can be written as a function of $b$ as before by substituting $s^*$ with $s^*(b)$ where $b^{s^*(b)}$ is constant.

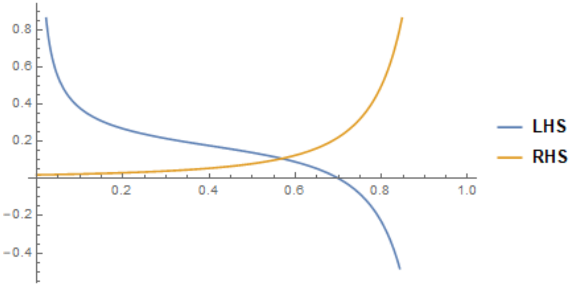
- Simplifying the problem and taking the FOC yields

$$-\gamma \ln \frac{\gamma}{1-\gamma}\left[\frac{A(1-b(1-\ln b))-\epsilon}{b\ln^2 b}\right]=\epsilon\frac{1-2\gamma}{(1-b)^2}$$

where $A=\frac{\gamma}{1-2\gamma}$, which has an interior solution $b^*$.

# Full Disclosure – Test design: Optimal $b$

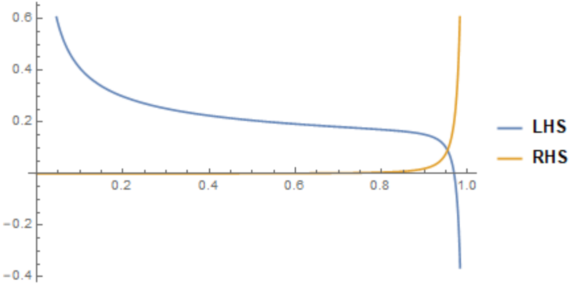
Consider for example, the case of $\gamma = .4$ and $\epsilon = .1$. This yields a fairly difficult test in which the $\Pr(\text{success of } \theta = B) \approx .57$.




**Result:** The intermediary designs the easiest possible test given the cost of conducting the test.

# Full Disclosure – Test design: Optimal $b$

This is different from what happens when $\gamma = .4$ (still) but $\epsilon = .001$. This yields an extremely easy test in which $\Pr(\text{success of } \theta = B) \approx .95$.




**Result:** The difficulty of the test (for the worse type senders) is decreasing in the cost of conducting tests.

# Full Disclosure – Test design: Optimal $b$

The **intuition** is straightforward: The intermediary, by choosing the easiest test for $B$ type sender, forces the $G$ type to take *more* tests to prove herself.

- There is an implicit price discrimination – more money spent by the good types to take more tests.

When the costs of conducting tests are lower, greater incentive to make an easier test to push better senders to take more tests.

# Delegated Disclosure

Equilibrium in which $s$ successes are needed out of $K$ attempts

- Belief after $s$ successes

$$\beta_s = \frac{\beta}{\beta + (1-\beta) b^s \sum_{n=0}^{K-s} \binom{s+n-1}{n} (1-b)^n} \geq \frac{1}{2}$$

- Price that makes this an equilibrium: $p(s, K)$ is still a hyperbolic function.

- (Conjectured)**Delegated disclosure cannot do any better than full disclosure.**

# Issues

- Price too sensitive to changes in expected number of tests (hyperbolic demand function).
- Sender only has signalling value – Receiver understands this and really punishes more tests in case of delegated disclosure.
- If price is fixed, then delegated disclosure becomes the preferred option.

# A more general model

- Three (strategic) players: Sender, Receiver and Intermediary
- Sender of type $\theta \in \{H, M, L\}$ with prior probability of each type being $\lambda_\theta$.
- Sender privately informed of her type.
- Sender wants to be "admitted" by the Receiver.
- Receiver wants to admit the Sender who yields positive pay off.
- Intermediary wants to maximize price $\times$ expected no. of tests, within each disclosure policy

# The Test

Test result: $\{S, F\}$

Probability of Success $(S)$ of each type:

$$Pr = \begin{cases} g & \text{if } \theta = H \ , \\ m & \text{if } \theta = M \ , \\ b & \text{if } \theta = L \ , \ . \end{cases}$$

Test design:

$$0 \leq g, m, b \leq 1$$

# Sender/Receiver pay off

Any Type of Sender:

$$\pi_S = \begin{cases} 1 & \text{if Admitted ,} \\ 0 & \text{if Rejected , .} \end{cases}$$

Receiver:

$$\pi_R = \begin{cases} \pi_\theta - C & \text{if admitts type } \theta \text{ ,} \\ 0 & \text{if rejects , .} \end{cases}$$

Cost of admittance: C

$$C < \pi_M, C = \pi_M, C > \pi_M$$

# Full Disclosure: Test Design-case 1

Case 1: $C = \pi_M$

$$\pi_R = \begin{cases} 1 & \text{if admits } \theta = H \ , \\ 0 & \text{if admits } \theta = M \ , \text{ or rejects} \\ -1 & \text{if admits } \theta = L \ , \end{cases}$$

R admits only if $\lambda_H \leq \lambda_L$

- A more general case of the Basic model (wherein $\lambda_L \neq 1 - \lambda_H$)
- $m$ plays no role in defining price and the number of successes required $s \rightarrow$ go back to two type case
- Designer will put $m = g$ to extract maximum price discrimination

# Full Disclosure: Test Design- Case 2

Case 2: $C < \pi_M$

$$\pi_R = \begin{cases} 1 & \text{if admits } \theta = H \ , \\ 1/2 & \text{if admits } \theta = M \\ -1 & \text{if admits } \theta = L \ , \\ 0 & \text{if rejects} \end{cases}$$

R admits only if $\lambda_H + \lambda_M/2 \leq \lambda_L$

- $m$ matters in obtaining number of successes required $s$
- Lower threshold belief is required than the previous case to get admitted
- Designer needs to choose optimum $m$ too to maximise profit

# Full Disclosure: Test Design- Case 2

LEMMA 1:

Number of successes needed to jump above the threshold $s$ decreases with increase in $m$ and Number of successes needed to jump above the threshold $s$ increases with increase in $b$

Proof by Induction.

# Full Disclosure: Test Design- Case 2

- Consider the equilibrium in which both types are expected to reveal $s$ success to be admitted
- On path belief after s success: $\lambda_H^s = \frac{g^s \lambda_H}{g^s \lambda_H + m^s \lambda_M + b^s \lambda_L}$ ...
- Number of success required is pinned down by: $g^s \lambda_H + m^s \lambda_M / 2 \leq b^s \lambda_L$
- Price $p_s$ is pinned down by participation constraint of the low type: $p_s = \frac{b^s(1-b)}{1-b^s}$
- Price is decreasing in number of success required $s$

# Full Disclosure: Test Design- Case 2

- A test that requires s number of success pinned down by :

$$g^s \lambda_H + m^s \lambda_M/2 \leq b^s \lambda_L$$

is equivalent to a test:

$$\lambda_H + (m/g)^s \lambda_M/2 \leq (b/g)^s \lambda_L$$

- With higher price $p_s = \frac{(b/g)^s(1-(b/g))}{1-(b/g)^s}$

**Conjecture:** There should exist an optimum design with $g = 1, m = b$ with $b$ high enough to maximise intermediary's pay off

# Full Disclosure: Test Design-case 3

Case 3: $C > \pi_M$

$$\pi_R = \begin{cases} 1 & \text{if admits } \theta = H \ , \\ -1/2 & \text{if admits } \theta = M \\ -1 & \text{if admits } \theta = L \ , \\ 0 & \text{if rejects} \end{cases}$$

R admits only if $\lambda_H \leq \lambda_L + \lambda_M/2$

- $m$ matters in obtaining number of successes required $s$
- Higher threshold belief is required than the previous case to get admitted
- Designer needs to choose optimum $m$ too to maximise profit

# Full Disclosure: Test Design- Case 3

- Consider the equilibrium in which both types are expected to reveal $s$ success to be admitted
- On path belief after s success: $\lambda_H^s = \frac{g^s \lambda_H}{g^s \lambda_H + m^s \lambda_M + b^s \lambda_L}$ ...
- Number of success required is pinned down by: $g^s \lambda_H \leq b^s \lambda_L + m^s \lambda_M / 2$
- Price $p_s$ is pinned down by participation constraint of the low type: $p_s = \frac{b^s(1-b)}{1-b^s}$
- Price is decreasing in number of success required $s$

# Full Disclosure: Test Design- Case 2

- A test that requires s number of success pinned down by :
$$g^s\lambda_H \leq b^s\lambda_L + m^s\lambda_M/2$$
is equivalent to a test:
$$\lambda_H \leq (b/g)^s\lambda_L + (m/g)^s\lambda_M/2$$

- With higher price $p_s = \frac{(b/g)^s(1-(b/g))}{1-(b/g)^s}$