Speculative Attacks on Debts and Optimum Currency Area: A Welfare Analysis*

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Abstract

Traditionally the issue of an optimum currency area is based on the theoretical underpinnings developed in the 1960s by McKinnon [12], Kenen [11] and mainly Mundell [14], who are concerned with the benefits of lowering transaction costs *vis-à-vis* adjustments to asymetrical shocks. Recently, this theme has been reappraised with new aspects included in the analysis, such as: incomplete markets, credibility of monetary policy and seigniorage, among others. For instance, Neumeyer [15] develops a general equilibrium model with incomplete asset markets and shows that a monetary union is desirable when the welfare gains of eliminating the exchange rate volatility are greater than the cost of reducing the number of currencies to hedge against risks.

In this paper, we also resort to a general equilibrium model to evaluate financial aspects of an optimum currency area. Our focus is to assess the welfare of a country heavily dependent on foreign capital that may suffer a speculative attack on its public debt. The welfare analysis uses as reference the self-fulfilling debt crisis model of Cole and Kehoe ([4], [5]

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and [6]), which is employed here to represent dollarization. Under this regime, the national government has no control over its monetary policy, the total public debt is denominated in dollars and it is in the hands of international bankers. To describe a country that is a member of a currency union, we modify the original Cole-Kehoe model by including public debt denominated in common currency, only purchased by national consumers. According to this rule, the member countries regain some influence over the monetary policy decision, which is, however, dependent on majority voting. We show that for specific levels of dollar debt, to create inflation tax on common-currency debt in order to avoid an external default is more desirable than to suspend its payment, which is the only choice available for a dollarized economy when the foreign creditors decide not to renew their loans.

Keywords: dollarization, optimum currency areas, speculative attacks, debt crisis, sunspots

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Emerging market economies of Latin America and Southeast Asia accumulated high levels of external debt in the 1990's. The sharp demand for foreign credits helped sustain stabilization programs and strengthen the value of national currencies. Reversal of market expectations and contagion effects changed this environment, causing financial crisis for some of these economies. Argentina and Russia actually defaulted, while Mexico, Korea, Thailand, Hong Kong and Brazil experienced severe speculative attacks. With this background in mind, we make an extension to the Cole and Kehoe ([4], [5] and [6]) model on self-fulfilling debt crisis to describe an economy for which there is positive probability of defaulting on its external debt, but that also belongs to a currency union. Under this monetary regime, a default could be avoided by inflation of the common currency, which, however, incurs costs in terms of a fall in productivity. Besides, the decision to inflate depends on majority voting and the welfare of a country with low weight in the voting system is adversely affected by antagonistic choices. We also try to evaluate the contagion among members that results from a loss in confidence of international bankers towards one country being passed on to another. These aspects are considered in a welfare analysis and we do simulations for Brazil as a a minority and majority voter.

In a currency union a speculative attack on one member country can be transmitted to other partners in two ways: one is through coordinated monetary policy and the other by contagion. In the first case, since the central government decision to inflate the common currency depends on majority voting, there may be antagonisms between the optimal choices of the majority and one minority member. In the other case, since the currency union assumes that the member countries have strong common trade and financial ties, the loss in confidence that international bankers feel about one particular country is passed on to another. We take into account both of these cases in the welfare analysis of a country in a currency union when its weight in the voting system is low (below 50 percent) and high (above 50 percent).

On a more methodological ground, this paper can be viewed as part of the literature on general equilibrium with bankruptcy, which asserts that in an incomplete market situation the introduction of the possibility of bankruptcy can be welfare enhancing (see Dubey, Geanakpoplos and Zame [9], for static economies, and Araujo, Páscoa and Torres-Martínez [3], on infinite horizon economies). The introduction of common curency can give rise to the possibility of a better bankruptcy technology through inflation than just the repudiation of the external debt, which can be quite costly.

1 The Cole-Kehoe Model

Cole and Kehoe developed a dynamic, stochastic general equilibrium model in which they consider the probability π of a self-fulfilling crisis of public debt occurring.

1.1 Basic Assumptions

The basic assumptions of the original Cole-Kehoe model are: one good produced with capital, k, and inelastic labor supply, and price normalized at one dollar; three participants — national consumers, international bankers and the government; one exogenous sunspot variable, ζ, which describes the bankers confidence that the government will not default. The sunspot is assumed to be independent and identically distributed with uniform [0,1] distribution and the probability that the bankers confidence is below the critical value π is equal to the probability of a self-fulfilling debt crisis, i.e. $P(\zeta \leq \pi) = \pi$. The model also assumes a stock of dollar debt, B, supposed to be completely in the hands of the bankers and probability π of no rollover if its level lies in the crisis zone. If the government defaults, it is always total. The decision to default is characterized by the government decision variable, z, either equal to zero or one.

In the original model, the representative consumer maximizes the expected utility

$$\max_{c_t, k_{t+1}} E \sum_{t=0}^{\infty} \beta^t \left[\varrho c_t + v\left(g_t\right) \right] \tag{1}$$

subject to the budget constraint

$$c_t + k_{t+1} - k_t \leq (1-\theta)\left[a_t f(k_t) - \delta k_t\right]$$

and given initial capital

$$k_0 > 0$$

At instant t, the consumer chooses how many goods to save for next period, k_{t+1}, and to consume presently, c_t. The utility has two parts: a linear function of private consumption, ϱc_t, and a logarithmic function v of government spending, g_t. The term ϱ is the weight of the utility of private consumption relative to the utility of public consumption. The right hand side of the budget constraint corresponds to the consumer's income, after taxes (θ is the tax rate) and capital depreciation, given by δ. The term a_t is essential to the Cole-Kehoe model. It is the productivity factor. If the government defaults on its debt, then it suffers a permanent drop in national productivity, $a_t = \alpha$, $0 < \alpha < 1$. Otherwise, a_t is equal to one.

The problem of the representative international banker is analogous to the consumer problem, except that the instantaneous utility excludes the term related to government spending, and consists of

$$\max_{x_t, b_{t+1}} E \sum_{t=0}^{\infty} \beta^t x_t \tag{2}$$

s.t.

$$x_t + q_t^* b_{t+1} \leq \overline{x} + z_t b_t$$

$$b_0 > 0$$

At time t, the bankers choose how many goods to consume, x_t, and the amount of government bonds to buy, b_{t+1}. The left hand side of the budget constraint shows the expenditure on new government debt, where q_t^* is the price of one-period bonds that pay one unit of good at maturity if the government does not default. The right hand side includes the revenue received from the bonds purchased in the previous period. The decision variable z indicates government default ($z = 0$) or not ($z = 1$). If it defaults, then the bankers receive nothing.

The government is assumed benevolent, in the sense that it maximizes the welfare of national consumers, and with no commitment to honor its obligations. Its decision variables are: new debt, B_{t+1}, whether or not to default, z_t, and government consumption, g_t. It has a budget constraint given by

$$g_t + z_t B_t \leq \theta \left[a_t f\left(K_t\right) - \delta K_t \right] + q_t^* B_{t+1} \tag{3}$$

where the expenditure, on the left hand side of expression (3), refers to current consumption and the payment of its debt; while the revenue, on the right hand side, includes taxes and the selling of new debt. The government is also assumed to have a strategic behavior since it foresees the optimal decision of the participants, including its own, c_t, k_{t+1}, q_t^*, z_t and g_t, given the initial aggregate state of the economy, s_t, and its choice of B_{t+1}.

The timing of actions within a period t is (the subscript t is omitted):

- the sunspot ζ is realized and the aggregate state is $s = (K, B, a_{-1}, \zeta)$;
- the government, given the price function $q^* = q^*(s, B')$, chooses B';
- the bankers decide whether or not to purchase B';
- the government chooses whether or not to default, z, and how much to consume, g;
- lastly, consumers, given $a(s, z)$, decide about c and k'.

1.2 A Recursive Equilibrium

In the construction of a recursive equilibrium, the first step is to characterize the behavior of the consumers and bankers. The optimal accumulation of capital, k', may take three values

$k^n > k^\pi > k^d$, depending on the consumers expectation about the productivity factor in the next period, $E[a']$. When the expectation is equal to one, k' equals k^n. If consumers expect a debt crisis next period with probability π, then $E[a']$ equals $1-\pi+\pi\alpha$ and they choose k' as k^π. Finally, when consumers know that the government has defaulted or will default for sure, they expect a drop in the productivity factor to α and k' is k^d. Analogously, the price that bankers pay for the new debt may take three values, β, $\beta(1-\pi)$ and 0, depending on their expectation of whether or not the government will default next period, since $q^* = \beta E[z']$. For example, if bankers expect the government will not default, $E[z'] = 1$, then q^* is β.

The second step in the construction of a recursive equilibrium is to define the crisis zone with probability π. For a given maturity of government bonds, the crisis zone is the debt interval for which a crisis can occur with probability π. For one-period government bonds and aggregate state $s = (k^\pi, B, 1, \zeta)$, such that there is probability π of a default, the crisis zone is given by

$$\left(\overline{b}\left(k^n\right), \overline{B}\left(k^\pi, \pi\right)\right]$$

The lower limit, $\overline{b}\left(k^n\right)$, is the highest debt level such that the government payoff of not defaulting, V_g^n, is greater than the payoff of defaulting, V_g^d, when it does not obtain new external loans (the second argument, B', and the third, q^*, are zero). This restriction is called the *no-lending condition* and is given by the expression

$$V_g^n(s,0,0) \geq V_g^d(s,0,0)$$

On the other hand, the upper limit, $\overline{B}\left(k^\pi, \pi\right)$, is the highest debt level such that the government prefers not to default rather than default, as long as it is able to sell new debt at positive price, $\beta(1-\pi)$. This condition may be written as

$$V_g^\pi(s, B', \beta(1-\pi)) \geq V_g^d(s, B', \beta(1-\pi))$$

Given these limits of the crisis zone, if the government chooses new dollar debt below the crisis zone, $B' \leq \overline{b}(k^n)$, then bankers will always renew their loans, as if π were equal to zero. If the new dollar debt is inside the crisis zone, then bankers renew their loans depending on their confidence in the government. There is probability π of no rollover and consequently of default next period. Finally, if $B' > \overline{B}(k^\pi, \pi)$, then there will be default for sure, as if π were equal to one, and the bankers purchaase no new debt at that time.

1.3 Numerical Exercise

Using their model, Cole and Kehoe [4] did a numerical exercise for Mexico for the eight months before the 1994-95 crisis. The parameters they used are: an average maturity of eight months for the public debt; capital share, ν, equal to 0.4 applied to the production function $f(k) = Ak^\nu$ and total productivity factor, $A = 2.0$; tax rate, $\theta = 0.2$; drop in productivity after default of 0.05, meaning $\alpha = 0.95$; discount factor, $\beta = 0.97$; depreciation factor, $\delta = 0.05$; and probability of default, $\pi = 0.02$, corresponding to one minus the fraction of the interest rate on the U.S. Treasury Bills and on the Mexican external dollar indexed bonds — *Tesobonos*, given by

$$\pi = 1 - \frac{(1 + r^*)}{(1 + r)} \tag{4}$$

One result of their simulation is the government debt policy function. For a given current debt level, the curve shown in Figure 1, determines the new debt the government chooses. The dashed line represents the 45-degree line.

Another result of their simulation is the crisis zone for different maturities of the public debt. In Figure 2, they show that the Mexican public debt of 20% relative to GDP (marked by a star) and average maturity of eight months was inside the crisis zone before the crisis.

We have done a similar exercise for Brazil for the 24-month period from June 1999 to May 2001. The parameters we use are the following: average maturity of public debt of 24 months;

capital share, $\nu = 0.5$; tax rate, $\theta = 0.3$; probability of default, $\pi = 0.06$; drop in national productivity equal to the Mexican one ($\alpha = 0.95$); and depreciation factor, $\delta = 0.24$. We make a strong assumption that the total Brazilian public debt is indexed or denominated in dollars and is in the hands of international bankers. For an average maturity of two years, the total Brazilian public debt of 50% relative to GDP is also in the crisis zone, as shown in Figure 3.

2 A Currency Union Model

We modify the original Cole-Kehoe model to assess the welfare of an economy that belongs to a currency union. The currency union model is mainly characterized as one with two currencies (the common one, such as the Euro, and the dollar), I member countries and a central government, equivalent to the Council of the European Union, constituted as the decision-making body for all members. Each country i, $i = 1, \ldots, I$, issues debt in the two currencies: dollar, B^i, and common currency, D^i. Since there is debt denominated in common currency, it is possible for the central government to collect inflation tax, but this decision depends on majority voting.

2.1 Basic Assumptions

The currency union model is very similar to the original Cole-Kehoe model. The basic assumptions are: for each country i, there are four participants in the market for the reference good — national consumers, international bankers, the national government and the central government; the price of the good equals one dollar, or p_t units of the common currency, in all member countries; the debt denominated in dollars, B^i, is only acquired by international bankers, there is probability π^i of no rollover if its level is in the crisis zone and any suspension in payment is always total; the debt denominated in common currency, D^i, is issued by the national government of country i, it is only taken up by consumers from this country, there is always credit rollover and repayment may be suspended partially.

Analogous to the original model, the decision to default on dollar debt is characterized by the national government's decision variable, z^i, being equal to zero and a permanent fall in national productivity, a^i, to α^i, $0 < \alpha^i < 1$. Meanwhile, the decision whether or not to create inflation tax on common-currency debt is described by the central government decision variable, ϑ_t^u, which takes two values: 1 or $\phi = \dfrac{1}{1+\chi}$, $0 < \phi < 1$ and χ, the inflation rate. If the central government decides for no inflation tax, then the common currency bond pays one good, $\upsilon^u = 1$, as the dollar bond does. Otherwise, it pays less than one unit, $\upsilon^u = \phi$. The national government obtains additional revenue by the lower real return of the common-currency debt held by consumers after the central government decision to inflate. If there is inflation, consumers receive ϕ goods per common-currency bond and believe that the government will henceforth start paying this quantity of goods per bond, while country i is affected by a permanent fall in productivity, a^i, to α^ϕ, which is related to the rate of inflation tax chosen. Therefore, the decision to inflate brings a cost to the member countries in terms of lower productivity, despite the benefit of the extra revenue to avoid an external default. An alternative approach to include inflation cost is to suppose a reduction in consumer's utility, but this is still a theoretical proposal.

Uncertainty is included in the model by three sunspot variables: two for each country i, ζ^i and η^i, and one for the union, η^u. We assume that the sunspot ζ^i is affected by the sunspot ζ^j from another country in the currency union. Accordingly, π^i and π^{ij} refer respectively to $P\left(\zeta^i \leq \pi^i \mid \zeta^j > \pi^j\right) = \pi^i$ and $P\left(\zeta^i \leq \pi^i \mid \zeta^j \leq \pi^j\right) = \pi^{ij}$, which are the the probability of a self-fulfilling dollar debt crisis occurring in country i, respectively, given that international bankers did not lose confidence in national government j and given that foreign creditors did lose their confidence in country j. Supposing a currency union with only two countries, Table 1 and Table 2 show the conditional probability of ζ^2 given ζ^1 and their joint probability.

Table 2 shows that the marginal probability of no default for country 2 is smaller than under independence between the two sunspots, for $\pi^{21} > \pi^2$.

The sunspot η^i is conditional on ζ^i and describes the confidence that consumers from country i have that the central government will honor its obligations regarding payment of the common currency bonds. The probability that the confidence of the consumers from country i is below the critical value ξ^i is equal to ξ^i, which is the probability that the central government will create an inflation tax, given that international bankers have little confidence in government i, i.e. $P(\eta^i \leq \xi^i \mid \zeta^i \leq \pi^i) = \xi^i$. If the realization of η^i is less than or equal to ξ^i, then the consumers of country i believe that the central government will resort to an inflation tax to avoid a default on the dollar debt and the national government votes for this option, influencing the central government decision. This sunspot variable performs a relevant function when the national governments conjecture on voting to inflate the common money to avoid an external crisis in their economies. Nevertheless, if no member country is under speculative attack on its dollar debt, then the realization of this variable for the group becomes irrelevant, since there is no reason to inflate.

Finally, the sunspot η^u gathers all the η^i and describes the confidence that consumers from the union have about the central government decision not to inflate the common currency. The probability that the confidence of consumers from the union as a whole is below the critical value ξ^u, given that the confidence of the consumers from the majority of countries is below the critical value ξ^i, is ξ^u. We define ξ^u as the average of the ξ^i for all member countries according to their weight in the voting system, and it corresponds to the probability that the central government inflates given that there is a confidence crisis in the majority of countries in the union. Table 3 shows, for a currency union of two countries, the conditional probability of η^u given symmetry and asymmetry between η^1 and η^2, supposing that country 1 has the highest weight in the voting system. Table 4 presents their joint probability. The probability si refers to the joint probability of symmetry (s) between η^1 and η^2 and inflation (i), sni refers to symmetry (s) and no inflation (ni). Analogously, we define the probabilities asi and $asni$, for the case of asymmetry (a).

The different realizations of η^i for each country correspond to the political risk the national government faces in adopting a common currency. The realization of the sunspot variables η^i

and η^u indicates whether the government of country i and the central government are in harmony regarding price versus output stability. Antagonistic types of national governments can result in different preferences regarding the conduct of a common monetary policy. This same question is analyzed by Alesina and Grilli [1], but using a different theoretical approach.

Figure 4 is a tree diagram for two countries in a currency union in one period. The branches of the tree indicate the probabilities that the market participants face before realization of the sunspot variables, when the initial state is such that there has not been inflation tax and both countries may default on their dollar debts.

2.2 Description of Participants

At time t, the representative consumer from country i maximizes the expected utility, given by the expression (1), subject to the new budget constraint

$$c_t^i + k_{t+1}^i - k_t^i + q_t^i d_{t+1}^i \leq \left(1 - \theta^i\right) \left[a_t^i f\left(k_t^i\right) - \delta^i k_t^i\right] + \vartheta_t^u d_t^i$$

Besides k_{t+1}^i and c_t^i, the representative consumer from country i chooses the amount of new common currency debt, d_{t+1}^i. The common-currency debt consists of zero-coupon bonds maturing in one period that pay one unit of the good at the price in common currency effective in the preceding period. The price, in units of common currency, of one bond at time t is $\widetilde{q}_t^i$ and, in units of the good, is $q_t^i = \dfrac{\widetilde{q}_t^i}{p_t}$. The right-hand side of the budget constraint includes the expenditure on new common currency debt, $q_t^i d_{t+1}^i$, and the left-hand side, the payment of the debt purchased in the previous period, $\vartheta_t^u d_t^i$. We also assume that the consumer holds an amount d_0^i of the common-currency debt.

International bankers maximize the expected utility given by the expression (2) and subject to the budget constraint

$$x_t + \sum_{i=1}^{I} q_t^{*i} b_{t+1}^i \leq \overline{x} + \sum_{i=1}^{I} z_t^i b_t^i$$

which includes purchase and redemption of dollar debt of the I member countries. Each banker chooses dollar-denominated bonds of country i at time t, b_{t+1}^{i}, and pays q_t^{*i} goods per bond. Initially, external creditors hold b_0^i dollar debt bonds of each country i. We assume that the supply of credits from international bankers meets the demand for loans from these countries without competition causing a liquidity crisis.

At time t, the national government of country i makes the following choices: new dollar debt, B_{t+1}^{i}, common-currency debt , D_{t+1}^{i}, whether or not to default on its dollar debt, z_t^i, and current government consumption, g_t^i. The budget constraint at time t equals

$$g_t^i + z_t^i B_t^i + \vartheta_t^u D_t^i \leq \theta^i \left[a_t^i f\left(K_t^i\right) - \delta^i K_t^i \right] + q_t^{*i} B_{t+1}^i + q_t^i D_{t+1}^i$$

which is rewritten as,

$$g_t^i \leq \theta^i \left[a_t^i f\left(K_t^i\right) - \delta^i K_t^i \right] - z_t^i B_t^i + q_t^{*i} B_{t+1}^i + \left(1 - \vartheta_t^u\right) D_t^i + q_t^i D_{t+1}^i - D_t^i \tag{5}$$

where $\left(1 - \vartheta_t^u\right) D_t^i$ refers to the additional revenue that the national government obtains by the lower real return of the common-currency debt held by consumers. The central government is also benevolent, since it maximizes the welfare of the consumers from the union. It decides whether or not to inflate the common-currency debt, ϑ_t^u, which depends on the the decisions of the member countries and their relative influence in the voting system, λ^i, $i = 1, \ldots, I$. If the sum of the weights of the countries that do not wish to inflate the common currency is greater than, for example, two-thirds of the total votes, then the central government chooses $\vartheta^u = 1$. Otherwise, it chooses $\vartheta^u = \phi$. We do not model how the inflation ϕ is chosen.

At the initial period for each country i the supply of dollar debt B_0^i is equal to its demand for this debt, b_0^i; the supply of common-currency debt D_0^i is equal to the demand for this type of debt, d_0^i; and the aggregate capital stock per worker, K_0^i, is equal to the individual capital stock, k_0^i.

2.2.1 Timing within a period for country i

- the sunspot variables ζ^i, η^i and η^u are realized and the aggregate state of economy i is s^i $= (K^i, B^i, D^i, a_{-1}^i, \vartheta_{-1}^u, \zeta^i, \zeta^j, \eta^u)$;

- the government of country i, taking the dollar-bond price schedule $q^{*i} = q^{*i}(s^i, B'^i)$ as given, chooses the new dollar debt, B'^i;

- the government of country i, taking the common-currency-bond price schedule, $q^i = q^i(s^i, B'^1, \ldots, B'^I)$ as given, chooses the new common-currency debt, D'^i, and each country knows the supply and demand of common-currency bonds in other member countries;

- international bankers, taking q^{*i} and ϑ^u as given, choose whether to purchase B'^i, $i = 1, \ldots, I$;

- consumers, considering q^{*i}, q^i and ϑ^u as given, decide whether to acquire D'^i;

- the central government decides whether or not to inflate the common currency, ϑ^u;

- the national governments decide whether or not to default on their dollar debts, z^i, and choose their current consumption, g^i;

- consumers, taking $a^i = a^i(s^i, z^i, \vartheta^u)$ as given, choose c_t^i and k'^i.

2.3 A Recursive Equilibrium

Following the Cole-Kehoe model, we define a recursive equilibrium for one country belonging to a currency union supposed to be constituted of two partners. First, we assume that the country under analysis has the highest weight in the voting system, $\varphi > 0.5$, and we call it country 1. The highest weight means that this country actually chooses to inflate, because of its voting power over the central government, however the inflation tax rate depends on the vote of both countries. Secondly, we assume that the country being studied has the lowest weight in the monetary union, $(1-\varphi)$ and it is called country 2. Our purpose is to define a recursive equilibrium for both cases and do welfare analysis under the possibility of a self-fulfilling external debt crisis occuring. First, we describe the behavior of consumers and bankers.

Consumers

At time t, consumers know ϑ_t^u, g_t^i and z_t^i when making their decisions as to c_t^i and k_{t+1}^i and they take these variables as given when deciding on d_{t+1}^i. The optimization problem at time t corresponds to

$$\max_{c_t^i, k_{t+1}^i, d_{t+1}^i} \; c_t^i + \beta E c_{t+1}^i$$

s.t.

$$c_t^i + k_{t+1}^i - k_t^i + q_t^i d_{t+1}^i = \left(1-\theta^i\right)\left[a_t^i \; f\left(k_t^i\right) - \delta^i k_t^i\right] + \vartheta_t^u d_t^i$$

$$c_{t+1}^i + k_{t+2}^i - k_{t+1}^i + q_{t+1}^i d_{t+2}^i = \left(1-\theta^i\right)\left[a_{t+1}^i \; f\left(k_{t+1}^i\right) - \delta^i k_{t+1}^i\right] + \vartheta_{t+1}^u d_{t+1}^i$$

$$c_t^i, c_{t+1}^i, k_{t+1}^i, d_{t+1}^i \geq 0$$

The first order condition for capital accumulation is the same as in the original Cole-Kehoe model, given $f(k) = Ak^{\nu}$. It depends on the consumers' expectation regarding the productivity of the economy in the following period, $E_t\left[a_{t+1}^i\right]$, as can be seen in the expression below

$$k_{t+1}^i = \left\{\left[\left(\frac{1}{\beta}-1\right)\frac{1}{1-\theta^i} + \delta^i\right]\frac{1}{E_t\left[a_{t+1}^i\right] A^i \nu^i}\right\}^{\dfrac{1}{\nu^i - 1}} \tag{6}$$

The price that the consumers pay for the new common-currency debt is given by

$$q_t^i = \beta E_t\left[\vartheta_{t+1}^u\right]$$

For consumers from country 1, the possible choices of capital accumulation are $k^{\pi\xi^u}$, $k^{\pi\phi}$, $k^{n\phi}$, k^d and k^n. Analogously, the optimal amounts of common-currency debt to acquire are $q^{\pi\xi^u}$, q^{ϕ} and q^n. National consumers of country 1 choose $k^{\pi\xi^u}$, because they expect that the productivity factor next period, $E_t\left[a_{t+1}\right]$, equals to $\pi^1\pi^{21}\ \left[(si+asi)\alpha^{\phi}+(sni+asni)\alpha\right]+\pi^1(1-\pi^{21})\ \left[\xi^1\alpha^{\phi}\right.$ $\left.+\ (1-\xi^1)\alpha\right]+(1-\pi^1)$. Also, they choose $q^{\pi\xi^u}$ which is the price they pay when $E_t\left[\vartheta^u_{t+1}\right]$ equals $\pi^1\pi^{21}\ \left[(si+asi)\phi+(sni+asni)\right]+\pi^1(1-\pi^{21})\ \left[\xi^1\phi+(1-\xi^1)\right]+(1-\pi^1)$. In this case the initial state is such that it is possible that one or both countries default on the dollar debt or that a default can be avoided through an inflation tax on debt denominated in common currency (this initial state is the one in Figure 4). In the same fashion, consumers decide on $k^{\pi\phi}$ when there has been inflation tax and it is possible that country 1 defaults on the dollar debt next period. Consequently, $E_t[a_{t+1}]=\left(1-\pi^1\right)\alpha^{\phi}+\pi^1\alpha$, meaning that the productivity factor is maintained at α^{ϕ} if the sunspot ζ^1 is such that the international bankers renew their loans, or falls to α otherwise. If the central government has inflated, then consumers pay $\beta\phi$ for new common-currency debt from then on, regardless of the realization of the sunspot ζ^1. The expectation of productivity associated with the last three optimal capital levels are, respectively: α^{ϕ}, α and 1. The capital level k^d, given by $E_t\left[a_{t+1}\right]=\alpha$, is chosen if the national government defaults on dollar debt. If the private sector is confident the government will not default, then they choose $k^{n\phi}$, with $E\left[a_{t+1}\right]$ equal to α^{ϕ}, in case there has been inflation tax previously or they choose k^n, with $E\left[a_{t+1}\right]$ equal to one, and pay price β for new common-currency bonds otherwise.

For consumers from country 2, the possible choices of optimal capital are: $k^{\pi\pi^2\xi^u}$, $k^{\pi\xi^1}$, $k^{\pi\pi^{21}\phi}$, $k^{\pi^2\phi}$, $k^{n\phi}$, $k^{n\phi^n}$, k^{π^2}, k^d and k^n. The first one in the list refers to the optimal choice when $E[a_{t+1}]=\pi\pi^{21}\ \left[(si+asi)\alpha^{\phi}+(asni+sni)\alpha\right]+\pi(1-\pi^{21})(\xi^1\alpha^{\phi}+1-\xi^1)+(1-\pi)[1-\pi^2(1-\alpha)]$. Consumers have this belief when it is possible for one or both countries to default on the dollar debt or for a default to be avoided through an inflation tax on common currency debt in the following period (see Figure 4). Also according to these beliefs, consumers from country 2 pay price $q^{\pi\xi^u}$. On the other hand, when it is possible that only country 1 defaults or votes for inflation, then the optimal capital is denoted by $k^{\pi\xi^1}$, meaning that just the sunspots ζ^1 and η^1 matter. For $k^{\pi\xi^1}$, $E\left[a_{t+1}\right]$ corresponds to $1-\pi+\pi si0+\pi asi0\ \alpha^{\phi^n}$ and the price that consumers pay for new common currency debt is $q^{\pi\xi^1}$ equal to $\beta\left(1-\pi+\pi si0+\pi asi0\ \phi^n\right)$, where $asi0$ is the probability of asymmetry and $si0$ of symmetry of the central government decision to inflate the common currency, given that consumers from country 2 would rather not have inflation for sure. Actually, $si0$ equals $\left(1-\xi^1\right)$ and $asi0$ equals ξ^1.

The distinction between α^{ϕ^n} and α^{ϕ} is related to the abatement factor of the common currency debt, ϑ^u, and specifically to the vote of country 2 to inflate. In the common currency model, we assume that in the initial state (which corresponds to the initial one in Figure 4), the sunspot of country 1 always influences country 2. Under this assumption, two situations may happen: sunspot ζ^2 matters or not, depending on the level of its public debt. In the first case, if country 1 prefers not to default and votes for inflation, then the the abatement factor ϑ^u is chosen equal to ϕ and, in the second, ϑ^u equals ϕ^n, indicating that country 2 surely votes for no inflation.

Next, given the initial state of the economy $s^2=(k^{\pi\pi^2\xi^u}, B^2, D^2, a^2_{-1}, \vartheta^u_{-1}, \zeta^1, \zeta^2, \eta^u)$, with $a^2_{-1}=1$, $\vartheta^u_{-1}=1$, $\zeta^1\leq\pi^1$, $\zeta^2\leq\pi^2$ and $\eta^u\leq\xi^u$, the central government decides for inflation and both countries choose not to default on their external debts. The optimal accumulation of capital for the next period may take two values: $k^{\pi\pi^{21}\phi}$, if sunspots ζ^1 and ζ^2 matter in the following period, with the expectation of the productivity factor for the following period, $E_t\left[a_{t+1}\right]$, equal to $\left(1-\pi^1\right)\,\left(1-\pi^2\right)\alpha^{\phi}\,+\,\left(1-\pi^1\right)\pi^2\alpha\,+\,\pi^1\left(1-\pi^{21}\right)\alpha^{\phi}\,+\,\pi^1\pi^{21}\alpha$; or the optimal capital is $k^{\pi^2\phi}$, defined by $E\left[a_{t+1}\right]$ equal to $\pi^2\alpha+\left(1-\pi^2\right)\alpha^{\phi}$, if only the sunspot ζ^2 is taken into account by the private sector. After inflation, consumers pay a price equal to $\beta\phi$ for common currency debt. Since initially both sunspots ζ^1 and ζ^1 matter, the factor ϑ^u is ϕ.

Unlike $k^{\pi\pi^{21}\phi}$, the optimal capital $k^{\pi^2\phi}$ is chosen when sunspot ζ^1 does not matter and only ζ^2 is taken into account by country 2. In this case, the expected productivity factor is $\left(1-\pi^2\right)\alpha^{\phi}$

$+\ \pi^2\alpha$. Lastly, consumers choose k^{π^2}, given by $E\left[a_{t+1}\right] = 1 - \pi^2\left(1-\alpha\right)$, and pay price β for common currency debt. Under this choice, consumers from country 2 know that country 1 has decided to default instead of inflate.

International Bankers

At time t, international bankers solve

$$\max_{x_t, b^1_{t+1}, \ldots, b^I_{t+1}} \quad x_t + \beta E_t\left[x_{t+1}\right]$$

$s.t.$

$$x_t + q^{*1}_t b^1_{t+1} + \ldots + q^{*I}_t b^I_{t+1} = \overline{x} + z^1_t b^1_t + \ldots + z^I_t b^I_t$$

and the first-order condition for b^i_{t+1} is

$$q^{*i}_t = \beta E_t\left[z^i_{t+1}\right]$$

The price that external creditors pay for the new dollar debt of country 1 may take four values: 0, β, $\beta\left(1-\pi\right)$, and $q^{*\pi\xi^u}$. For new dollar debt from country 2, the price that external creditors are willing to pay may take six values: 0, β, $\beta\left(1-\pi^2\right)$, $q^{*\pi\pi^{21}\phi}$ and $q^{*\pi\pi^2\xi^u}$. If the central government has undertaken an inflation tax and bankers believe that the national government will default on the dollar debt with probability π^i in the following period, they pay $\beta\left(1-\pi^i\right)$ for new dollar debt. However, if the national government currently defaults on the dollar debt, then international bankers pay price zero. In case external creditors are sure that the government will not resort to a dollar debt moratorium in the following period, they pay β. For country 1, the price $q^{*\pi\xi^u}$ is defined by $E_t\left[z^1_{t+1}\right]$ equal to $\pi^1\pi^{21}\left(si + asi\right) + \pi^1(1-\pi^{21})\xi^1 + (1-\pi^1)$ and the price $q^{\pi\pi^2\xi^u}$, for country 2, equal to $E_t\left[z^2_{t+1}\right] = \pi\left(1-\pi^{21}\right) + \left(1-\pi\right)\left(1-\pi^2\right) + \pi\pi^{21}\left(si + asi\right)$. In both cases, external creditors expect that one or the other country or both may default on their dollar debt or that a default can be avoided through an inflation tax on debt denominated in common currency. Finally, $q^{*\pi\pi^{21}\phi}$ refers to an expectation of default of $\left[\pi\left(1-\pi^{21}\right)+(1-\pi)\left(1-\pi^{2}\right)\right]$ and is chosen after the central government has decided to inflate and both sunspots ζ^1 and ζ^2 matter.

In the next step to construct an equilibrium, we describe the crisis zone supposing that: (i) more than half of the votes are from member countries with dollar debts in their respective crisis zones; (ii) there has not been any external debt crisis in any of the countries of the monetary union, nor partial moratoria of common-currency debt, up to the initial state (i.e., $a_{-1}^{i}=1$ for all i and $\vartheta_{-1}^{u}=1$); (iii) the common-currency debt is fixed at level D and the inflation the central government can impose is given by the abatement factor for common-currency debt, ϕ.

2.4 The Crisis Zone

The crisis zone that we define corresponds to the dollar debt interval for which the international bankers attribute probability π^i for each member country defaulting, probability ξ^u for the central government to inflate and positive probabilities for symmetrical and asymetrical choices of inflation between the two member countries occurring next period. External creditors evaluate the crisis zones for each country. First we obtain the crisis zone for the country with the highest weight in the voting system and secondly for the country with the lowest weight.

Crisis zone for country 1 (with high weight in voting system)

The crisis zone is given by the interval denoted by $\left(\bar{b}\left(k^{n},D\right),\ \overline{B}\left(k^{\pi\xi^{u}},D,\pi^{1},\xi^{u}\right)\right]$. The lower bound $\bar{b}\left(k^{n},D\right)$ is the highest dollar debt level, B^1, for which the following restriction is satisfied in equilibrium:

$$V^{n}\left(s^{1},0,0,D^{1},\beta\right)\geq V^{d}\left(s^{1},0,0,D,\beta\right) \quad (7)$$

where $s^1=(k^n,\ B^1,\ D,\ 1,\ 1,\ \cdot,\ \cdot,\ \cdot,\ \cdot)$ is an initial state in which country 1 has not defaulted on the dollar debt yet ($a_{-1}^{i}=1$), the central government has not inflated the common-currency debt ($\vartheta^u_{-1} = 1$) and the sunspots do not matter. The welfare levels $V^n\left(s^1,\ 0,\ 0,\ D,\ q^1\right)$ and $V^d\left(s^1,\ 0,\ 0,\ D,\ q^1\right)$ refer to the government decision, respectively, not to default (superscript n) than to default (superscript d) , even if it does not sell new dollar bonds at a positive price in the current period. The second and third positions of the argument of the welfare functions mean that new dollar debt B'^1 and q^{*1} are zero. The fouth and fifth arguments indicate that new debt in common currency, D, is sold for q^1 equal to β. This lower bound is obtained in a similar way as in the original Cole-Kehoe model, except that in the common-currency model the purchase and payment of common-currency debt is included. The welfare levels are:

$$V^n\left(s^1, 0, 0, D^1, q^1\right) = v\left[\theta y^n - B^1 - (1-\beta)D^1\right] + \rho\left[(1-\theta)\, y^n + (1-\beta)\, D\right] + \beta \cdot wnD\,(0)$$

$$wnD\,(0) = \frac{1}{(1-\beta)}\left\{v\left[\theta y^n - (1-\beta)D^1\right] + \rho\left[(1-\theta)\, y^n + (1-\beta)\, D\right]\right\}$$

$$y^n = A\,(k^n)^\gamma - \delta k^n$$

$$V^d\left(s^1, 0, 0, D^1, q^1\right) = v\left[\theta y^{nd} - B^1 - (1-\beta)D^1\right] + \rho\left[(1-\theta)\, y^{nd} + kn - k^d + (1-\beta)\, D\right] + \beta \cdot wdD$$

$$y^{nd} = \alpha A\,(k^n)^\gamma - \delta k^n$$

$$wdD = \frac{1}{(1-\beta)}\left\{v\left[\theta y^d - (1-\beta)D^1\right] + \rho\left[(1-\theta)\, y^d + (1-\beta)\, D^1\right]\right\}$$

The upper bound of the crisis zone, $\overline{B}(k^{\pi\xi^u},\ D^1,\ \pi^1,\ \xi^u)$, is the highest dollar debt for which international bankers extend loans to country 1, given probability π^i for each member country to default and probability ξ^u for the central government to inflate. It is obtained as the highest dollar debt such that the following restrictions are simultaneously satisfied in equilibrium, given initial state $s^1 = (k^{\pi^1\xi^u},\ B^1,\ D^1,\ 1,\ 1,\ \zeta^1,\ \zeta^2,\ \eta^1,\ \eta^2)$, with $a^i_{-1} = 1$, $\vartheta^u_{-1} = 1$ and before the realization of the sunspots

$$V^{\pi^1\xi^u}\left(s^1, B'^1, q^{*1}, D^1, q^1\right) \geq V^d\left(s^1, B'^1, q^{*1}, D^1, q^1\right) \tag{8}$$

and

$$V^{\pi^1}\left(s^1, B'^1, q^{*1}, D^1, \beta\phi\right) \geq V^d\left(s^1, B'^1, q^{*1}, D^1, \beta\right) \tag{9}$$

For the first case, $\zeta^1 > \pi^1$, revealing that bankers are still confident in the government of country 1 and renew their loans. The condition says that the government from country 1 prefers not to default rather than to default and also decides for no inflation tax. For the second case, the realization of the sunspots is $\zeta^1 \leq \pi^1$ and $\eta^1 \leq \xi^1$, meaning that external and internal creditors have lost their confidence in the government. The second condition says that the government prefers not to default rather than to default on the dollar debt as long as they are able to sell new common curency debt at price $\beta\phi$ as a consequence of the central government creating an inflation tax. The simulations show that the decision to default and to create no inflation tax weakly dominates in terms of welfare default and inflation tax.

Cole and Kehoe obtain the upper bound of the crisis zone $\overline{B}\left(k^{\pi}, \pi\right)$, which is the highest debt value for which the government prefers not to default over defaulting as long as it is able to sell new debt at price $\beta\left(1-\pi\right)$. If the sunspot $\zeta^1 \leq \pi^1$, then the government defaults. In the common-currency model, the possibility to create inflation tax allows the country not to default, despite the unfavorable result for the renewal of external loans up to the level of dollar debt $\overline{B}(k^{\pi\xi^u},\ D^1,\ \pi^1,\ \xi^u)$. Therefore, the crisis zone shrinks when compared to the original one.

Crisis zone for country 2

As long as we assume that country 1 always chooses dollar debt inside the crisis zone, the crisis zone for country 2 depends on the realization of the sunspots ζ^1 and η^1. Accordingly, the crisis zone is denoted by the interval $\left(\overline{b}\left(k^{\pi\xi^1},\ D,\ \pi^1,\ \xi^1\right), \overline{B}\left(k^{\pi\pi^2\xi^u},\ D,\ \pi^1,\ \pi^2,\ \xi^1,\ \xi^2\right)\right]$. The lower bound $\overline{b}\left(k^{\pi\xi^1},\ D,\ \pi^1,\ \xi^1\right)$ is the highest dollar debt level for which the following restriction is satisfied in equilibrium:

$$V^n\left(s^2, 0, 0, D, q^{\pi\xi^1}\right) \geq V^d\left(s^2, 0, 0, D, q^{\pi\xi^1}\right) \tag{10}$$

where $s^2 = (k^{\pi\xi^1}, B^1, D, 1, 1, \zeta^1, \eta^1, \cdot,)$ is an initial state in which country 2 has not yet defaulted on the dollar debt ($a^i_{-1} = 1$), the central government has not inflated the common-currency debt ($\vartheta^u_{-1} = 1$) and the sunspots ζ^1 and η^1 matter. The welfare levels $V^n\left(s^2, 0, 0, D, q^{\pi\xi^1}\right)$ and $V^d\left(s^2, 0, 0, D, q^{\pi\xi^1}\right)$ refer to the government decision, respectively, not to default rather than to default, even if it does not sell new dollar bonds at a positive price in the current period. New debt in common currency is sold for q^2 equal to $q^{\pi\xi^1}$. The payoffs $V^n\left(s^2, 0, 0, D, q^{\pi\xi^1}\right)$ and $V^d\left(s^2, 0, 0, D, q^{\pi\xi^1}\right)$ are given by:

$$V^n\left(s^2, 0, 0, D, q^{\pi\xi^1}\right) = v\left[\theta y^{\pi\xi^1} - B^2 - (1 - q^{\pi\xi^1})D\right] + \rho\left[(1-\theta)\, y^{\pi\xi^1} + \left(1 - q^{\pi\xi^1}\right) D\right] +$$
$$\frac{\beta}{1 - \beta\,(1 - \pi^1)} \cdot \left[\left(1 - \pi^1\right) u^{\pi\xi^1}\left(k^{\pi\xi^1}, 0, D, 1, 1, \zeta^1, \eta^1, \cdot, 0, 0, D, q^{\pi\xi^1}\right) + \right.$$
$$\left. \pi^1 asi0\ u^{n\phi}\left(k^{\pi\xi^1}, 0, D, 1, 1, \cdot, \cdot, \cdot, 0, 0, D, \beta\phi\right) + \pi^1\,(1 - si0)\ \ u^n\left(k^{\pi\xi^1}, 0, D, 1, 1, \cdot, \cdot, \cdot, 0, 0, D, \beta\right)\right]$$

and

$$u^{\pi\xi^1}\left(k^{\pi\xi^1}, 0, D, 1, 1, \zeta^1, \eta^1, \cdot, 0, 0, D, q^{\pi\xi^1}\right) =$$
$$v\left[\theta y^{\pi\xi^1} - (1 - q^{\pi\xi^1})D\right] + \rho\left[(1-\theta)\, y^{\pi\xi^1} + \left(1 - q^{\pi\xi^1}\right) D\right]$$

where $u^{\pi\xi^1}\left(k^{\pi\xi^1}, 0, D, 1, 1, \zeta^1, \eta^1, \cdot, 0, 0, D, q^{\pi\xi^1}\right)$ is the instantaneous utility of the government from country 2 for initial state $(k^{\pi\xi^1}, 0, D, 1, 1, \zeta^1, \eta^1, \cdot)$, with $\zeta^1 > \pi^1$, $\eta^1 > \xi^1$ and given that consumers pay price $q^{\pi\xi^1}$ for common-currency debt. The next two instantaneous utility functions refer to the realization of sunspots, respectively, $\zeta^1 \leq \pi^1$ and $\eta^1 \leq \xi^1$ and $\zeta^1 \leq \pi^1$ and $\eta^1 > \xi^1$, equivalent to:

$$u^{n\phi}\left(k^{\pi\xi^1}, 0, D, 1, 1, \cdot, \cdot, \cdot, 0, 0, D, \beta\phi^n\right) = v\left[\theta y^{\pi\xi^1\phi^n} - \phi^n(1-\beta)D\right] +$$
$$\rho\left[(1-\theta)\, y^{\pi\xi^1\phi^n} + \phi^n(1-\beta)D - k^{\pi\xi^1} + k^{n\phi^n}\right] +$$
$$\frac{\beta}{(1-\beta)}\left\{v\left[\theta y^{n\phi^n} - \phi^n(1-\beta)D\right] + \rho\left[(1-\theta)\, y^{n\phi^n} + \phi^n\,(1-\beta)\, D\right]\right\}$$

$$y^{\pi\xi^1\phi^n} = \alpha^{\phi^n} A \left(k^{\pi\xi^1}\right)^{\gamma} - \delta k^{\pi\xi^1}$$

$$u^n\left(k^{\pi\xi^1}, 0, D, 1, 1, \cdot, \cdot, \cdot, 0, 0, D, \beta\right) = v\left[\theta y^{\pi\xi^1} - (1-\beta)D\right] +$$
$$\rho\left[(1-\theta)\, y^{\pi\xi^1} + (1-\beta)D + k^{\pi\xi^1} - k^n\right] +$$
$$\frac{\beta}{(1-\beta)}\left\{v\left[\theta y^n - (1-\beta)D\right] + \rho\left[(1-\theta)\, y^n + (1-\beta)\, D\right]\right\}$$

The upper limit of the crisis zone for country 2, $\overline{B}\left(k^{\pi\pi^2\xi^u},\ D,\ \pi^1,\ \pi^2,\ \xi^1,\ \xi^2\right)$, is obtained in an analogous way as for the crisis zone of country 1, given initial state $s^2 = (k^{\pi\pi^2\xi^u}$, B^2, D, 1, 1, ζ^1, ζ^2, η^1, $\eta^2)$. It corresponds to the highest dollar debt level for which the following three restrictions are satisfied simultaneously:

$$V^{\pi\pi^2\xi^u}\left(s^2, B', q^{*\pi\pi^2\xi^u}, D, q^{\pi\xi^u}\right) \geq V^d\left(s^2, B', q^{*\pi\pi^{21}\phi}, D, q^{\pi\xi^u}\right)$$

$$V^{\pi\pi^{21}\phi}\left(s^2, B', q^{*\pi\pi^{21}\phi}, D, \beta\phi\right) \geq V^d\left(s^2, B', q^{*\pi\pi^{21}\phi}, D, \beta\phi\right)$$

$$V^{\pi^2}\left(s^2, B', \beta\left(1-\pi^2\right), D, \beta\right) \geq V^d\left(s^2, B', \beta\left(1-\pi^2\right), D, \beta\right)$$

For the first case, the sunspot realizations are $\zeta^1 > \pi^1$ and $\zeta^2 > \pi^2$, which indicate that international bankers are confident that both countries will not default on their dollar debts. They renew their loans to country 2 up to the level B' such that the national government prefers not to default rather than to default, as long as it is able to sell new dollar-debt at price $q^{*\pi\pi^2\xi^u}$ and new common-currency debt at price $q^{\pi\xi^u}$. The second condition says that the national government prefers not to default rather than to default for sunspot realizations $\zeta^1 \leq \pi^1$, $\eta^1 \leq \xi^1$ and $\zeta^2 \leq \pi^2$. Accordingly, the central government creates infaltion tax on common-currency debt and both countries would rather not default on their external debts, since they sell dollar debt for $q^{*\pi\pi^{21}\phi}$ and common-currency debt for $\beta\phi$. The last condition indicates that it would be better not default rather than to default, after the sunspot results

$\zeta^1 \leq \pi^1$, $\eta^1 > \xi^1$ and $\zeta^2 > \pi^2$. In this case, country 1 defaults and international bankers roll over the dollar debts of country 2. Given initial state $s^2 = (k^{\pi\pi^2\xi^u},\ B^2,\ D,\ 1,\ 1,\ \zeta^1,\ \zeta^2,\ \eta^1,\ \eta^2)$ and before the realization of the sunspots, these three conditions are the ones under which external creditors are sure that government 2 will not default. As long as all the three are satisfied, then they renew the loans to this country. These payoffs are easily characterized applying the Cole-Kehoe methodology and the optimal choices for the market participants described in this section.

2.5 Optimal decisions of national government i

Following the same procedure as Cole and Kehoe [6], we obtain the national government optimal behavior when its dollar debt is in the no-crisis zone and in the crisis zone. We do this exercise just for country 1. For the other country, the procedure is analogous.

Dollar debt in the no-crisis zone

The no-crisis zone is the dollar debt region below the lower limit of the crisis zone. Given $B_t^1 \leq \overline{b}(k^n,\ D,\ \phi)$, K_{t+1}^1 and B_{t+2}^1 the national government of country 1 chooses g_t, g_{t+1} and B_{t+1}^1, to be sure not to have a dollar debt crisis next period and solves the following problem:

$$\max_{B_{t+1}^i} \ \beta^t v\left(g_t\right) + \beta^{t+1} E v\left(g_{t+1}\right)$$

s.t.

$$g_t = \theta y^n + \beta B_{t+1}^1 - B_t^1 - \phi\left(1-\beta\right) D$$

$$g_{t+1} = \theta y^n + \beta B_{t+2}^1 - B_{t+1}^1 - \phi\left(1-\beta\right) D$$

$$g_t, g_{t+1} > 0$$

The expectation refers to the possibility of an inflation tax on common-currency debt in the following period. Since we suppose that it has already occurred, then ϑ^u equals ϕ and q is

$\beta\phi$ forever. Moreover, if the government wishes to have no default always for sure, it can not default and chooses new dollar debt level such that the bankers pay price $q^* = \beta$ and consumers save $k' = k^n$ every time. Accordingly, the first-order condition regarding B^1_{t+1} results in $v'(g_t)$ $= v'(g_{t+1})$ and the optimal behavior of national government 1 consists of holding its current consumption steady, $g_t = g_{t+1}$. Hence, if at the start the dollar debt is B^1_0 falling in the no-crisis zone, then the optimal new dollar debt is to maintain this same level.

Dollar debt in the crisis zone

If the dollar debt is in the crisis zone, the decision regarding B^1_{t+1} is that which provides the highest welfare from the perspective of the national government among the following options: resorting to a moratorium; creating an inflation tax to avoid an external default; lowering the dollar debt to $\overline{b}\,(k^n, D)$ in T periods if no crisis or inflation occur; or never lowering it. To characterize this optimal policy, suppose that (8) and (9) are not active and that there is neither a default on the dollar debt nor an inflation tax on common-currency debt ($z^1_t = 1$ and $\vartheta^u_t =$ 1). Under these hypothesis, the first-order condition for the national government's problem is

$$\begin{aligned} v'\left(g^i_t\right) q^{*\pi\xi^u} &= \beta\left[\left(1-\pi^i\right) v'\left(g^{\pi\xi^u}_{t+1}\right)\right. \\ &\quad + \left[\pi^1\pi^{21}(si+asi) + \pi^1(1-\pi^{21})\xi^1\right] v'\left(g^{\pi\phi}_{t+1}\right) \end{aligned}$$

where,

$$g_t = \theta y^{\pi\xi^u} - B^i_t + q^{*\pi\xi^u} B^i_{t+1} - D^i + q^{\pi\xi^u} D$$

$$g^{\pi\xi^u}_{t+1} = \theta y^{\pi\xi^u} - B^i_{t+1} + q^{*\pi\xi^u} B_{t+2} - D^i + q^{\pi\xi^u} D$$

$$g^{\pi\phi}_{t+1} = \theta y^{\pi\xi^u} - B^i_{t+1} + \beta\,(1-\pi)\,B_{t+2} - \phi\,(1-\beta)\,D$$

$$y^{\pi^i\xi^u} = A\left(k^{\pi\xi^u}\right)^{\gamma} - \delta k^{\pi\xi^u}$$

where $g^{\pi\xi^u}_{t+1}$ and $g^{\pi\phi}_{t+1}$ are the consumption levels of national government 1, when the central government decides, respectively, not to create an inflation tax and to do it, given that it did not default on its dollar debt.

This condition does not result in constant government consumption. The analytic expression for the expected welfare is more complex than that obtained in the original Cole-Kehoe model, since we are considering the possibility of an inflation tax on common-currency debt. The optimal solution for new dollar debt, given its current level, is obtained in numerical form in the simulations.

2.6 Welfare for the national government of country i

One of the advantages of the Cole-Kehoe methodology is to do welfare analysis. We use their approach to evaluate the expected welfare of the national government of country i with high and low weight in the voting system of a monetary union and under the possibility of a self-fulfilling dollar debt crisis. We compare this result with the expected welfare given by the original Cole-Kehoe model, which we characterize as being a dollarization regime, and also to a model with local currency with central bank under political influence.

In assessing the welfare of a country with low weight in the voting system of a monetary union, we consider non-perfect correlation between the decisions of the member countries to create inflation tax. Since the low-weight country has to follow the decision of the majority, which is represented by the high-weight country, its welfare is affected by antagonistic choices.

2.6.1 Welfare for member country with high weight in voting system

- *Dollar debt in the no-crisis zone for initial state* $s^1 = (k^n, B^1, D, 1, 1, \cdot, \cdot, \cdot, \cdot)$

For dollar debt levels in the no-crisis zone, $B^1 \leq \bar{b}(k^n, D)$, external creditors know that the national government always prefers to pay back its debts, no matter what the realization of the sunspot variables is. The expected welfare for country 1 before realization of the sunspots is

$$V^{n}\left(s^{1}\right)= \tag{11}$$

$$\frac{1}{1-\beta}\left\{v\left[\theta y^{n}-(1-\beta)B-(1-\beta)D\right]+\varrho\left[(1-\theta)y^{n}+(1-\beta)D\right]\right\}$$

- *Dollar debt in the crisis zone for initial state* $s^1 = (k^{\pi\xi^u},\ B^1,\ D,\ a^1_{-1},\ \vartheta^u_{-1},\ \zeta^1,\ \eta^1,\ \zeta^2,\ \eta^2)$

When dollar debt is in the crisis zone, the realization of the sunspot variables has bearing. Given initial state $s^1 = (k^{\pi\xi^u},\ B^1,\ D,\ a^1_{-1},\ \vartheta^u_{-1},\ \zeta^1,\ \eta^1,\ \zeta^2,\ \eta^2)$, with $\bar{b}(k^n,\ D) < B^1 \leq \overline{B}\left(k^{\pi\xi^u}, D, \pi^i, \xi^u\right)$, D fixed, $a^1_{-1} = 1$ and $\vartheta^u_{-1} = 1$, the expected welfare for country 1 corresponds to

$$V^{\pi\xi^{u}}\left(s^{1}\right)= \tag{12}$$

$$\left(1-\pi^{1}\right)V^{\pi\xi^{u}}\left(s^{1},B',q^{*\pi\xi^{u}},D,q^{\pi\xi^{u}}\right)+$$

$$\left[\pi^{1}\pi^{21}(si+asi)+\pi^{1}(1-\pi^{21})\xi^{1}\right]V^{\pi\phi}\left(s^{1},B',\beta\left(1-\pi\right),D,\beta\phi\right)+$$

$$\left[\pi^{1}\pi^{21}(sni+asni)+\pi^{1}(1-\pi^{21})\left(1-\xi^{1}\right)\right]V^{d}\left(s^{1},0,0,D,\beta\right)$$

where, $V^{\pi\xi^u}(s^1,\ B',\ q^{*\pi\xi^u},\ D,\ q^{\pi\xi^u})$ is the expected welfare with positive probability that country 1 defaults on the external debt or that an inflation tax is created in the following period, $V^{\pi\phi}(s^1,\ B',\ \beta\,(1-\pi),\ D,\ \beta\phi)$ is the expected welfare after the inflation tax on common-currency debt and the possibility of a moratorium on the dollar debt next period. Finally, $V^d\left(s^1, 0, 0, D, \beta\right)$ refers to the expected welfare level for country 1 after country 1 defaults.

The payoffs $V^{\pi\xi^u}\left(s^1, B', q^{*\pi\xi^u}, D, q^{\pi\xi^u}\right)$ and $V^{\pi\phi}\left(s^1, B', \beta\left(1-\pi\right), D, \beta\phi\right)$ are obtained numerically in the simulations. Upon request, we can describe these payoffs for a stationary dollar debt.

2.6.2 Welfare for member country with low weight in voting system

- *Dollar debt in the no-crisis zone for initial state* $s^2 = (k^{\pi\xi^1},\ B^2,\ D,\ 1,\ 1,\ \zeta^1,\ \eta^1,\ \cdot,\ \cdot)$

The expected welfare depends on sunspots from country 1 and is equal to

$$V^{\pi\xi^1}\left(s^2\right) = \tag{13}$$

$$(1-\pi)\ V^{\pi\xi^1}\left(s^2, B', \beta, D, q^{\pi\xi^1}\right) +$$

$$\pi\ asi0\ V^{\pi\phi}\left(s^2, B', \beta, D, \beta\phi\right) + \pi\ si0\ V^{d}\left(s^2, B', \beta, D, \beta\right)$$

The payoff $V^{\pi\xi^1}\left(s^2, B', \beta, D, q^{\pi\xi^1}\right)$ represents the expected welfare when the sunspots for country 1 indicates that it is possible to default on the dollar debt or to inflate in the folowing period. If the central government creates inflation tax currently, then the expected welfare for country 2 is given by $V^{\pi\phi}(s^2, B', \beta, D, \beta\phi)$. Finally, $V^{d}(s^2, B', \beta, D, \beta)$ corresponds to the payoff for country 2, when country 1 defaults. As long as dollar debt for country 2 belongs to the no-crisis zone and country 1 has defaulted, then the private sector pays β for debts in both currencies, because consumers and bankers are sure both that the national government from country 2 will not default and the central government will not inflate the common-currency debt.

The expected welfare, when we suppose non-perfect correlation between η^1 and η^2 in the initial period, is given by the expression (13), excluding the term $V^{\pi\phi}(s^2, B', \beta, D, \beta\phi)$, because it is related to asymmetry.

- *Dollar debt in the crisis zone for initial state* $s^2 = (k^{\pi\xi^1},\ B^2,\ D,\ 1,\ 1,\ \zeta^1,\ \eta^1,\ \zeta^2,\ \eta^2)$

$$V^{\pi\pi^2\xi^u}\left(s^2\right) = \tag{14}$$

$$\left(1-\pi^1\right)\left(1-\pi^2\right)\ V^{\pi\pi^2\xi^u}\left(s^2, B', q^{*\pi\pi^2\xi^u}, D, q^{\pi\xi^u}\right) +$$

$$\left[\pi^1\left(1-\pi^{21}\right)(si+asi)+\pi^1\left(1-\pi^{21}\right)\xi^1\right]\ V^{\pi\pi^{21}\phi}\left(s^2,B',q^{*\pi\pi^{21}\phi},D,\beta\phi\right)$$

$$\pi^1\left(1-\pi^{21}\right)\left(1-\xi^1\right)\ V^{\pi^2}\left(s^2,B',\beta\left(1-\pi^2\right),D,\beta\right)+$$

$$\left[\pi^1\pi^{21}(sni+asni)+\left(1-\pi^1\right)\pi^2\right]V^d\left(s^2,0,0,D,\beta\right)$$

The four payoffs that constitute $V^{\pi\pi^2\xi^u}\left(s^2\right)$ are described in a similar way as the ones above. The first one, $V^{\pi\pi^2\xi^u}(s^2,\ B',\ q^{*\pi\pi^2\xi^u},\ D,\ q^{\pi\xi^u})$, assumes that there is positive probability of default in both countries or inflation tax on common-currency debt next period. The second payoff, $V^{\pi\pi^{21}\phi}(s^2,\ B',\ q^{*\pi\pi^{21}\phi},\ D,\ \beta\phi)$, supposes that the central government has created inflation tax currently, but both countries may still default on their dollar debts in the following period. The third, $V^{\pi^2}(s^2,\ B',\ \beta\left(1-\pi^2\right),\ D,\ \beta)$, presumes that country 1 has defaulted and there is only positive probability that country 2 will default in the future. The last payoff characterizes the case of an external default by country 2.

In the simulations, we analyse the expected welfare in case of symmetry between the decision of the two countries to create inflation tax in the initial period. In this case, the expected welfare is similar to the expression (14), except for the joint probabilities related to the payoffs $V^{\pi\pi^{21}\phi}(s^2,\ B',\ q^{*\pi\pi^{21}\phi},\ D,\ \beta\phi)$ and $V^d(s^2,\ 0,\ 0,\ D,\ \beta)$. They are equal to, respectively, $\pi^1\pi^{21}si$ and $[\pi^1\pi^{21}sni+\ (1-\pi^1)\,\pi^2\left(1-\xi^2\right)]$.

2.6.3 Welfare when central bank is under political pressure

Using the Cole-Kehoe methodology, Araujo and Leon (2002) develop a model for an economy under local currency regime, with the possibility that the national central bank be influenced by its government to generate additional budgetary resources through inflation. However, this event is not taken into account by the private sector in their optimization problems and hence national consumers and international bankers are surprised by a decision of the central bank to reduce the real value of its local-currency debt without a crisis in the external sector. For a given initial aggregate state and before realization of the sunspot variables, the private sector attributes a parameter ψ to describe its beliefs regarding the independence of the central bank. When $\psi = 0$, the central bank is free of political pressures (strong), and at the other extreme, if $\psi > 0$, the central bank is subject to political interference (weak).

In the currency union model we could also have allowed a situation in which the central government can inflate the common currency even if the majority of votes are not from countries facing external debt crises. Instead, this decision could have resulted from pressures by some national governments for the purpose of raising extra revenues through an inflation tax.

3 A Numerical Exercise

We carry out simulations for the Brazilian economy, as if Brazil were a member of a monetary union with two member countries. We consider two situations: one in which it has high weight in the voting system ($\varphi > 0.5$) and another with a low weight ($\varphi \leq 0.5$).

3.1 Parameters for the Brazilian Economy

The parameters chosen are mainly the same used in Araujo and Leon (2002). There, we represent the Brazilian economy between June 1999 and May 2001. The 24-month interval equals the average maturity period of Brazilian government debt. The average maturity of debt indexed by the SELIC rate (the basic rate set by the Central Bank of Brazil), which describes common-currency debt, and the average maturity of dollar-indexed bonds, which refers to the dollar debt, are both approximately 24 months for the period under analysis.

The other parameters used in the simulations are: production function specified as $f(k) = Ak^{\nu}$ with capital share, $\nu = 0.5$ and total factor productivity, $A = 0.8$; tax rate, $\theta = 0.3$; utility function of public goods, $v(g) = (1/10)\log(g) + 1$; weight of utility of private relative to public consumption, $\varrho = 0.7$; drop in productivity after default, $\alpha = 0.95$; discount factor, $\beta = 0.93$; depreciation factor, $\delta = 0.20$; total common-currency debt relative to gross domestic product,

$D/GDP = 0.3$.

In the original model, v is a logarithmic function of the type $a \ln(g) + b$, with a and b equal to 1 and zero respectively. We changed these to $a = 1/10$ and $b = 1$ to avoid that the welfare of a central bank influenced by its government to create inflation in the abscence of external debt crisis would be higher than if the central bank suffered no political influence. Besides, to lessen the weight of private consumption, c, in consumer utility, we reduced the parameter ϱ from one to 0.7.

The simulation requires some new parameters. The first one is the real return of common-currency debt effectively paid after inflation, ϕ, which is defined as the inverse of the inflation factor. We suppose that it is a function of ξ^i and, by analogy with the probability of default π, the probability that the central government will create an inflation tax, such that international bankers have little confidence in government i, ξ^i, is defined as

$$\xi^i = 1 - \left(\frac{1 + i^i}{1 + r^i} \right)$$

where r^i and i^i are, respectively, the interest rates on country i bonds denominated in dollars and in common currency. Accordingly, $1/(1 - \xi^i)$ is equal to one plus the rate of devaluation of the common currency relative to the dollar that consumers from country i expect. Assuming ξ^i is small, then we can approximate ξ^i as the expected rate of devaluation of the common currency in country i. The expected rate of devaluation of the common currency by all consumers from the union is defined by

$$\xi^u = \varphi^1 \xi^1 + \left(1 - \varphi^1\right) \xi^2$$

where φ^1 is the weight of country 1 in the voting system. We make the hypothesis that the expected rate of devaluation and the expected rate of inflation are equal and, by rational expectations, the expected rate of inflation actually occurs. Therefore, we have

$$1/(1-\xi^u) = 1+\chi^e = 1+\chi = \frac{1}{\phi}$$

or

$$1-\phi = \xi^u$$

As we can see, the inflation tax on common curency debt $(1-\phi)$ equals the rate of devaluation of the common currency, which in turn depends only on the expected rate of devaluation in each country and the weight of country 1 in the voting system. To simplify the numerical exercises, we assume two cases ($\xi^2 = 0.75\ \xi^1$ and $\xi^2 = 1.25\ \xi^1$) and make a grid of values for $\xi^1 \leq 0.5$.

Inflation Cost

Another parameter to be considered is α^{ϕ}, the productivity of the economy after inflation. Simonsen and Cysne (1994, p.14) have an expression to calculate the cost of inflation in Brazil as a fraction (not percentage) of GDP

$$F(\chi_l) = 1.105 \log(1 + 0.0368\chi_l^{0.475}) \tag{15}$$

where χ_l is the annual periodic inflation rate in logarithmic form and is related to the parameter ϕ by the expression

$$\chi_l = \log(1+\chi) = \log(1/\phi)$$

To obtain α^{ϕ} for different values of ϕ, we compare the cost of inflation given by the expression (15) to the welfare loss after inflation in the Cole-Kehoe model. To simplify the calculations, we suppose the dollar debt is stationary at $\overline{b}(k^n, D)$. For a given ϕ, we have the following equation with α^{ϕ} unknown

$$F(\phi) = \frac{u^{\phi}(s^1) - u^n(s^1)}{(1/2)\, A\, (k^n)^{\gamma}}$$

where, $s^1 = (k^n, \overline{b}(k^n, D), D, 1, 1, \cdot, \cdot, \cdot, \cdot)$, $u^n(s^1)$ and $u^\phi(s^1)$ are the one-period utility without and with inflation, specified as

$$u^\phi(s^1) = v\left[\theta y^{n\phi} - (1-\beta)\,\overline{b}(k^n, D) - \phi\,(1-\beta)\,D\right] + \varrho\left[(1-\theta)\,y^{n\phi} + \phi\,(1-\beta)\,D\right]$$

$$u^n(s^1) = v\left[\theta y^n - (1-\beta)\,\overline{b}(k^n, D) - (1-\beta)\,D\right] + \varrho\left[(1-\theta)\,y^n + (1-\beta)\,D\right]$$

with $y^{n\phi} = \alpha^\phi A\,(k^n)^\gamma - \delta k^n$. We assume unappropriately that the consumer's investiment after inflation is k^n instead of $k^{n\phi}$. Nevertheless, the difference between them is small numerically. A similar procedure is applied to obtain α^{ϕ^n}, in which ϕ^n substitutes for ϕ.

Correlation between ζ^1 and ζ^2

The parameter π^i is the probability that country i defaults, given that other countries with strong comercial and financial ties with it are not under a speculative attack. When we assume that Brazil has more than 50% weight in the voting system, the probability of default for country 1, π^1, may be estimated as the average EMBI$^+$ sovereign spread, calculated by J. P. Morgan. A crisis that affects international bankers' confidence in Brazil also influences another country that is integrated to it, like Argentina. Therefore, when Brazil is country 1, we consider Argentina as country 2. Accordingly, π^2 is the sovereign spread for Argentina when the Brazilian economy is not under a speculative attack and π^{21} when it is. We define π^{21} as

$$\pi^{21} = \pi^2 + \Delta\pi^2$$

and a linear regression model is used to represent the relation between the variation in sovereign spreads of the two countries, $\Delta\pi^1$ and $\Delta\pi^2$, given by

$$\Delta\pi^2 = b\;\Delta\pi^1 \tag{16}$$

in a simplified manner following the procedure of Hernández and Valdés [10]. The regression coefficient is equivalent to the following expression:

$$b = \frac{\left|COR\left(\zeta^1, \zeta^2\right)\right| \sqrt{VAR\left(\zeta^2\right)}}{\sqrt{VAR\left(\zeta^1\right)}}$$

where $COR\left(\zeta^1, \zeta^2\right)$, $VAR\left(\zeta^1\right)$ and $VAR\left(\zeta^2\right)$ are the correlation and variances of the sunspots ζ^1 and ζ^2. If there is no correlation, then $\Delta\pi^2$ is zero and the sovereign spread of country 2 is equal to π^2.

We consider a tranquil period for Brazil: the seven months before the Russian crisis (December 31, 1997 to July 31, 1998). During this time the average Argentine sovereign spread corresponds to 446.8 and the Brazilian one to 542.1. On the other hand, we regard the second half of 1998 (from August 3 until December 31) a period of speculative attack in Brazil. During this period, the average Argentine sovereign spread rose to 816.6 and the Brazilian one to 1163 as a result of the reduced confidence in Brazil after the Russian default.

To estimate the effect of an increase of $\Delta\pi^1$ on $\Delta\pi^2$ during periods of speculative attacks in Brazil, we run a linear regression from August 3, 1998 until December 31, 1998. The regression coefficient is equal to 0.88, the correlation is 0.9357 and the variances of Argentina and Brazil, respectively, equal 5069 and 5741. According to the results, we assume that the ratio of standard errors and the correlation are equal to one. The average EMBI$^+$ sovereign spread for Brazil during the period under study (June 1, 1999 until May 31, 2001) is 801.1 and, for Argentina, 723.3, which are rather similar. Consequently, in the simulations we assume that the sovereign spreads of both countries, π^1 and π^2, are the same.

The other part of the numerical exercise assumes that Brazil is a small country, denominated country 2. In this case, the linear regression model (16) shows that the confidence loss in Brazil is caused by a drop in confidence in all emerging market countries, excluding Brazil, and is estimated as:

$$\Delta\pi^2 = 1.5\ \Delta\pi^1$$

The correlation is 0.75 and the ratio of standard errors between Brazil and the emerging markets indicates that the Brazilian sovereign spread is two times bigger. For the period under analysis, we assume that $\Delta\pi^1$ is zero, since the higher sovereign spread relative to the calm period is supposed to be caused by the country itself. Moreover, π^1 is approximately 0.05 or 462 bps, which is the average EMBI$^+$ sovereign spread, except for Brazil and Argentina, during the period December 31, 2001 and April 30, 2002. This short period of time starting in December 31, 2001 is the only one available from J. P. Morgan that excludes Brazil. Also, the sample ends in April 30 to leave out the uncertainties from the 2002 Brazilian presidential elections.

3.2 Preliminary Results

The first result obtained in the simulations is that, for a given positive risk of devaluation of the common currency, ξ^u, the expected welfare of a small country in a monetary union with two member countries is an increasing function of its weight in the voting system. Figure 5 shows the expected welfare levels for Brazil, supposing that its dollar debt relative to GDP is 0.50 and the probability of default equal to 0.04. (For the period under analysis, the probability of default for Brazil is estimated at between 0.04 and 0.08 (see Araujo and Leon (2002)). In Figure 5, we also assume for all curves that consumers from the small country attribute probability ξ^2 equal to 0.75 times ξ^1 that the central government will create inflation tax. For Brazil as a small country, it is analytically the same to assume that the correlation between the sunspots ζ^1 and ζ^2 is null, while for Brazil as a big country, the correlation is one. However, the results are not affected significantly by the change in the correlation. By changing the correlation from one to zero, the welfare of the small country increases by 0.004 to 0.007 percent and is inversely proportional ξ^1. The welfare levels for Brazil, as a small country, is represented by each of the curves in the interval for the weight in the voting system from zero to 0.50. Taking, for example, the light-crossed-solid curve, it refers to ξ^1 equal to 0.45 and it is an increasing function of the weight in the voting system.

Given a weight in the voting system, Figure 5 also shows that the expected welfare for a small country is a decreasing function of the probability of creating inflation tax ξ^u, which is the inflation tax itself and equivalent to $(0.75+0.25\varphi)\xi^1$. As the decision to inflate is the decision of the majority, it may be in opposition to the small country's wish. Therefore, the welfare loss is higher for the small country as the beliefs of the consumers from the big country, ξ^1, are in favor of an inflation tax. On the contrary, Figure 5 also shows that for the big country, the expected welfare is an increasing function of ξ^1 and varies very little with the weight in the voting system.

Figure 6 is very similar to Figure 5, except for the assumption that the probability ξ^2 equals 1.25 times ξ^1. For ξ^1 in the interval 0 to 0.4, the expected welfare levels for the small and big countries are very close to the ones presented in Figure 5. For ξ^1 equal to 0.45 and the weight in the voting system of 0.5 and 0.55, the expected welfare collapses, meaning that the big country would rather default on the dollar debt than vote for inflation.

A second result obtained in the simulations is that for the country with the majority vote in the union, the expected welfare is equal to the welfare of a country with independent local currency and strong national central bank. However, for the country with the minority vote in the union, the expected welfare is greater than the welfare of a country with an independent local currency as long as the central bank is politically influenced by the national government. This result is presented in Figure 7. The horizontal axis refers to the degree of central bank dependence or quality for the local currency model. It also concerns the weight in the voting system in the common currency model. When the central bank of a country with local currency is considered strong, its dependence is zero and we are considering the position zero on the horizontal axis. In this case, the expected welfare is at the same level as for the majority country in a monetary union. As the degree of dependence increases (higher postive values on the horizontal axis), the welfare of the local currency regime decreases. The group of convex curves refers to the expected welfare for different probabilities of inflating the local currency. For a non-zero dependence of the national central bank, the highest welfare is with the lowest inflation tax to be collected for political purposes. Figure 7 shows that, for this particular parametrization, the country with minority voting in the union is better off belonging to the monetary union for higher degrees of central bank dependence (above 0.1).

In Figure 7, the horizontal diamond-marked solid line refers to the expected welfare for a country under dollarization, with dollar debt relative to GDP of 80 percent. This regime is characterized by debt denominated only in dollars and no possibility to inflate it.

In Figures 5 to 7, we are not taking into account whether or not the decision of the two countries to create inflation tax in the initial period are symmetrical. Both cases of assymetry and symmetry are considered simultaneously. In Figure 8, only the case of symmetry is evaluated and the consequence is a sharp fall in the expected welfare for the small country, compared with the previous figures. This fall is a result of the elimination of some states of nature that otherwise provides positive payoffs initially. In the tree diagram, the states of nature removed initially are the ones in accordance with the following probabilities: $\pi^1\pi^{21}asi$, $\pi^1\pi^{21}asni$, $\pi^1(1-\pi^{21})\xi^1$ and $(1-\pi^1)\pi^2\xi^2$. Now the dollarization regime is Pareto superior to the common-currency regime for the country with low weight in the voting system.

A third result obtained in the numerical exercises is the following: suppose that Brazil has a 60 percent weight in the voting system, dollar debt of 50 percent relative to GDP and probability of default equal to 0.07, then its welfare is highest when the inflation of the common currency is expected to be above 20 percent. This result does not change significantly according to the relation between ξ^1 and ξ^2 and is shown in Figure 9. The curves in Figure 9 are isolines for the expected welfare levels of a country with high weight in the monetary union. The welfare levels vary according to the inflation of the common currency and the probability of default. The figure also shows that, for a probability of default of 0.05, the expected welfare is highest for inflation rates above 40 percent (see the isoline correponding to welfare of 16.845).

Figure 10 is analogous to Figure 9, except that we are considering the country as if it has low weight (40 percent). Supposing dollar debt of 50 percent relative to GDP and probability of default of 0.07, the expected welfare levels are highest for expected inflation rates below 20 percent. For an inflation rate of 60 percent, the welfare level is 16.80 and for an inflation rate of 20 percent, 16.82. This result also does not change significantly with either assumption made about the relation between ξ^1 and ξ^2.

A fourth result is that as the weight in the voting system increases, there are expected welfare gains for the small country, which moves upward the higher the probability of default is and the greater the possible inflation of the common currency is. Figure 11 shows isolines of the percent increase in expected welfare level for the small country when its weight changes from 10 to 40 percent in the voting system and when we assume there is no perfect symmetry between the decisions to inflate of the central government and the small country. For a probability of default of 0.08, the welfare gains from increasing the weight in the voting system rise as the inflation rate becomes bigger. There is a 0.05 percent increase in expected welfare when the possible inflation is in the range of 70 to 80 percent in the period.

The last result compares the percent variation in the expected welfare levels between the common currency and local currency regimes with weak and strong national central bank. Figure 12 shows, for dollar debt in the crisis zone, the isolines that describe the difference between expected welfare of the common and local currency regimes relative to the common currency one. The solid line refers to the difference in welfare levels between the two regimes considering that the national central bank, under the local currency regime, is politically influenced by its government, while the dashed line assumes that it is not. We observe that the local currency regime with strong central bank provides higher expected welfare than the common currency regime for the country with low weight in the voting system. The dashed isolines have negative levels, indicating that welfare is lower for the common currency than for the local one. On the contrary, the solid isolines have positive values and they show that when the central bank is under strong political influence, it is better for the country with low weight in a monetary to keep this regime.

4 Conclusions and Extensions

The paper brings into discussion one aspect of the debate about monetary regimes for countries heavily dependent on international lending, like the Latin-American and emerging market economies of Southeast Asia. This task is accomplished by means of a macroeconomic model that incorporates microfundamentals, rational expectations and dynamic optimization.

The Cole-Kehoe model for obtaining the welfare of an economy subject to a speculative attack on its external debt is the starting point to describe alternative monetary regimes. In the present paper, we develop an extension of the original model in which we decribe a monetary union with common currency. In a forthcoming paper, we will introduce a model for a local currency regime with a central bank subject to influence by its national government.

The currency union model includes public debt denominated in common currency, thus allowing the central government to resort to lowering the real return on these debts owned by domestic consumers. The inflation tax so extracted is used to avoid a default on the external debt, whose consequences could be even worse in terms of welfare. Besides, we have also taken into account the inflation costs associated with raising this revenue and the symmetry between the national and central governments' decisions about whether or not to inflate. We went a bit further to describe the contagion that results from a loss in confidence regarding one government's commitment to repay its external debt being passed on to another country with which the first country has strong commercial and financial ties.

The model is used to run simulations of the Brazilian economy for the 24-month period from June 1999 until May 2001. In the numerical exercises, Brazil is assumed to belong to a monetary union as a country with high weight in the voting system (greater than 50 percent) and also as a country with low weight (less than 50 percent). When we assume that it is a high-weight-country, Argentina is the other country with strong commercial and financial ties with it and the one that suffers contagion from a bad realization of the sunspot variable corresponding to international bankers' confidence in the Brazilian government.

The preliminary results indicate that, for a given risk of devaluation of the common currency, the expected welfare for Brazil as a country with low weight in a monetary union of two member countries increases the higher is its weight in the voting system and, for a given weight (less than 50 percent), it is a decreasing function of the probability of creating inflation tax in the union. The second result indicates that Brazil, as a majority voter in a monetary union, has an expected welfare equal to the the welfare of a country with independent local currency and central bank under no political pressure (i.e., that only resorts to an inflation tax in case of an external crisis). Nevertheless, if Brazil is a minority voter, the expected welfare is greater than the welfare of a country with independent local currency, as long as the central bank is under political influence of its national government and as long as we are not taking into account whether or not the decision of the two countries to inflate in the initial period are symmetrical. Another result obtained with the numerical exercises is to quantify the gain in welfare for a small country when its weight in the voting system rises from 10 to 40 percent. The increase in expected welfare varies positively with its probability of defaulting on the dollar debt and also on the possible inflation rate of common currency. Finally, the simulations evaluate the variation in the expected welfare levels between the common currency and the local currency regime with a national central bank under political pressures or not.

Our extension of the original Cole-Kehoe model is also suitable to compare the expected welfare levels between currency union and dollarization regimes. This exercise would particularly apply to Mexico. A numerical exercise of the currency union model could provide the welfare gain that this country could achieve if its weight in the voting system of a monetary union with the United States were increased.

Future extensions should be aimed at carrying out simulations in which debt in common currency is not fixed, but instead results from an optimization exercise as is the case for dollar debt. In addition, the currency union model could be applied to describe the government dollar debt policy function and to depict the crisis zone for different dollar-debt maturities, as in the original Cole-Kehoe model.

Other recent works that deal quantitatively with the issue of whether or not a country should keep its own currency are Cooley and Quadrini [7], Schmitt-Grohé and Uribe [16] and Mendoza [13].

A Tables and Figures

Table 1: Conditional Probability of ζ^2 given ζ^1

	$\zeta^1 > \pi^1$	$\zeta^1 \leq \pi^1$
$\zeta^2 > \pi^2$	$(1 - \pi^2)$	$(1 - \pi^{21})$
$\zeta^2 \leq \pi^2$	π^2	π^{21}

Table 2: Joint Probability of ζ^1 and ζ^2

	$\zeta^1 > \pi^1$	$\zeta^1 \leq \pi^1$
$\zeta^2 > \pi^2$	$(1 - \pi^2)\,(1 - \pi^1)$	$(1 - \pi^{21})\,\pi^1$
$\zeta^2 \leq \pi^2$	$\pi^2(1 - \pi^1)$	$\pi^{21}\pi^1$

Table 3: Conditional Probability of η^u given η^1 and η^2

	Symmetrical η^1 and η^2	Asymmetrical η^1 and η^2
$\eta^u \leq \xi^u$	$\varphi^1\xi^1 + (1 - \varphi^1)\,\xi^2 \equiv \xi^u$	$\varphi^1\xi^1 + (1 - \varphi^1)\,(1 - \xi^2) \equiv \xi^{uu}$
$\eta^u > \xi^u$	$\varphi^1(1 - \xi^1) + (1 - \varphi^1)\,(1 - \xi^2) \equiv (1 - \xi^u)$	$\varphi^1(1 - \xi^1) + (1 - \varphi^1)\,\xi^2 \equiv (1 - \xi^{uu})$

Table 4: Joint Probability of η^1, η^2 and η^u

	Symmetrical η^1 and η^2	Asymmetrical η^1 and η^2
$\eta^u \leq \xi^u$	$[\xi^1\xi^2 + (1 - \xi^1)\,(1 - \xi^2)]\,\xi^u \equiv si$	$[\xi^1(1 - \xi^2) + (1 - \xi^1)\,\xi^2]\,\xi^{uu} \equiv asi$
$\eta^u > \xi^u$	$[\xi^1\xi^2 + (1 - \xi^1)\,(1 - \xi^2)]\,(1 - \xi^u) \equiv sni$	$[\xi^1(1 - \xi^2) + (1 - \xi^1)\,\xi^2]\,(1 - \xi^{uu}) \equiv asni$

Figure 1: Mexico 1994: Government Public Debt Policy Function

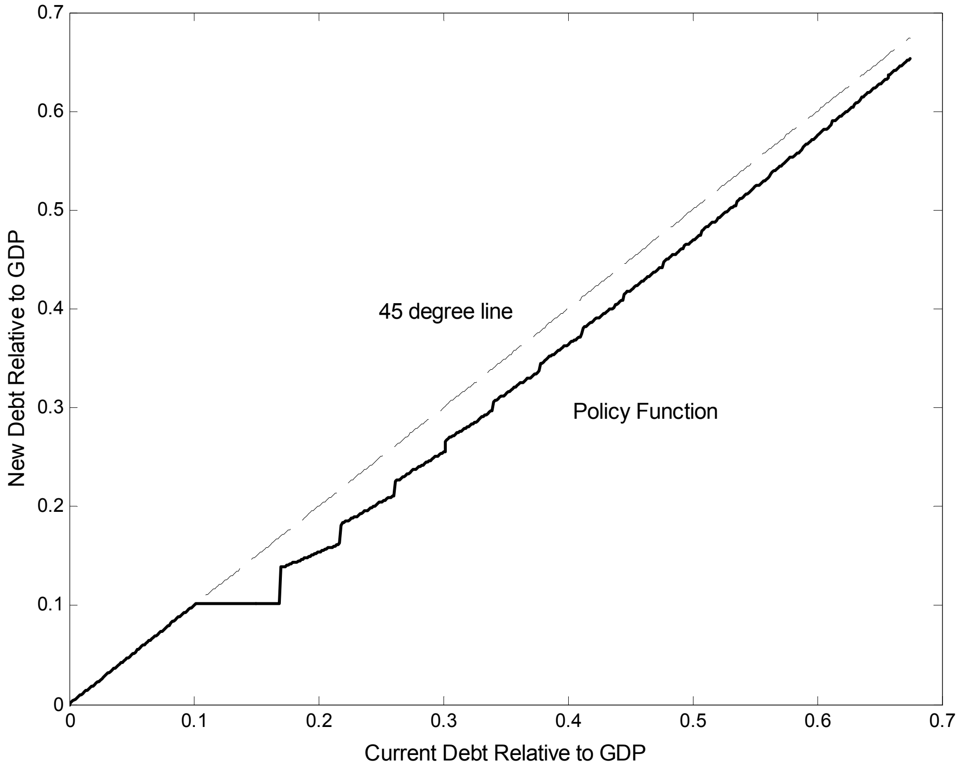


Figure 2: Mexico 1994: The Crisis Zone for Different Maturities of Public Debt

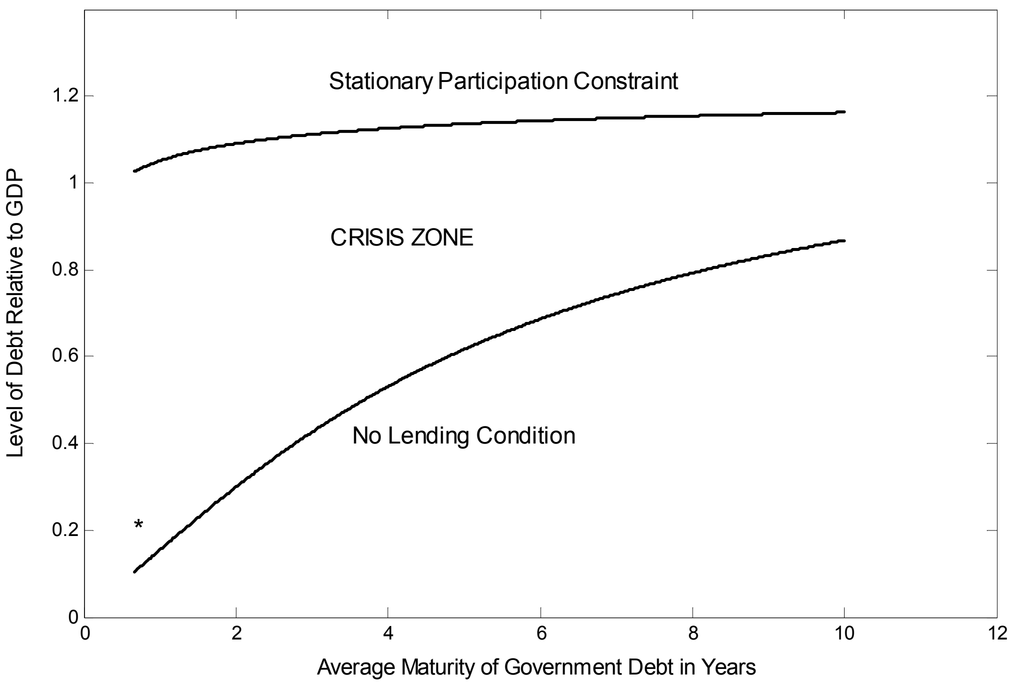


Figure 3: Brazil 1999-2002: The Crisis Zone for Different Maturities

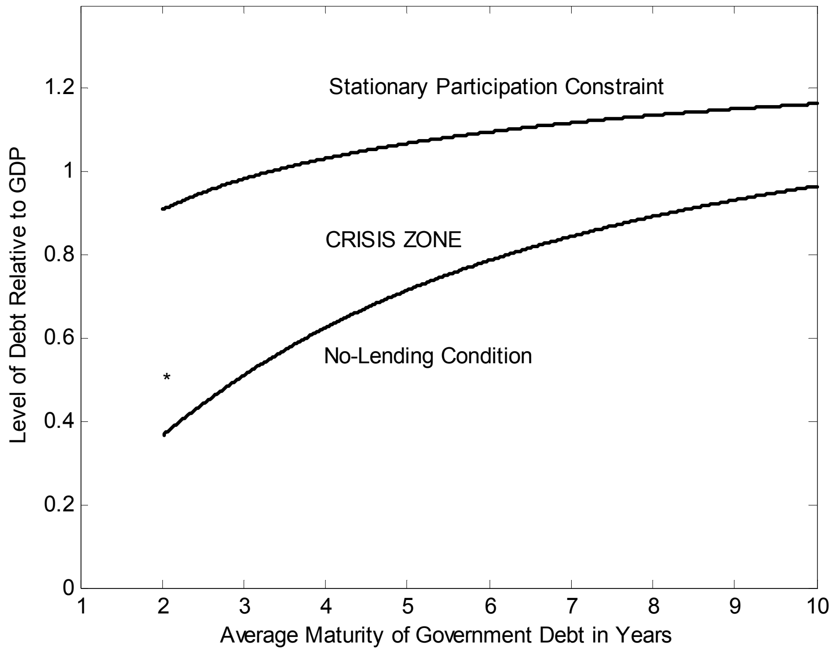


Figure 4: Tree Diagram

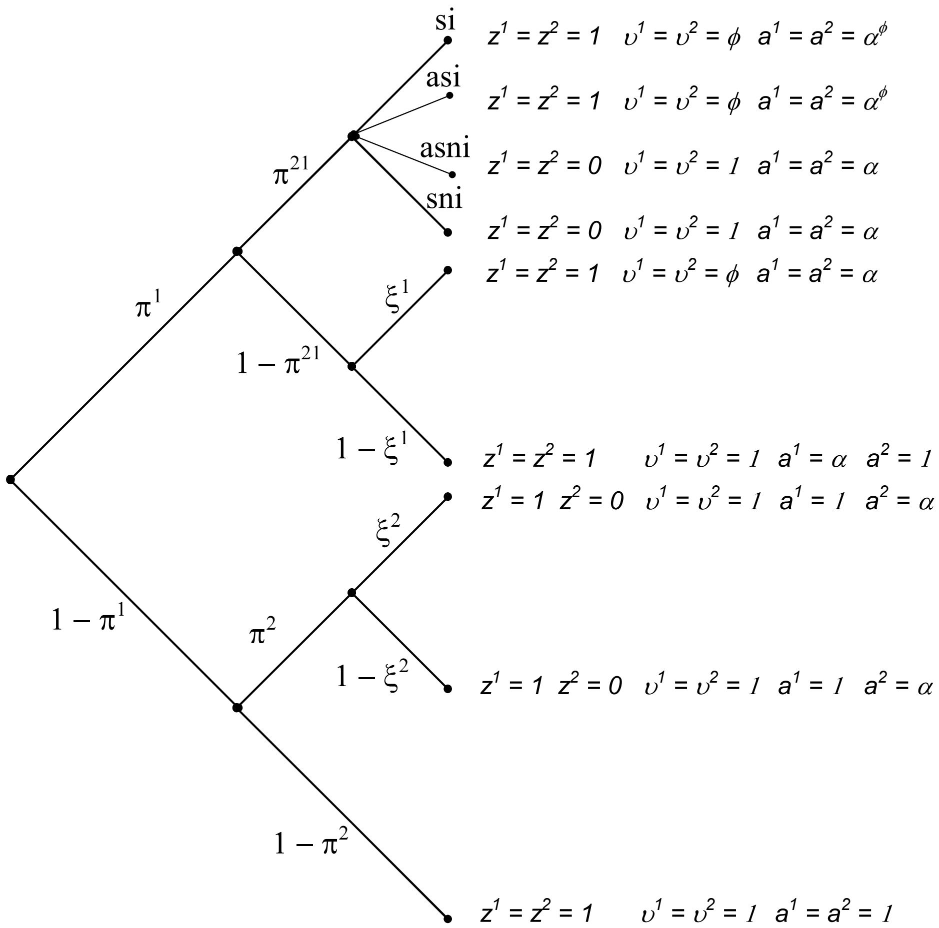


Figure 5: Expected Welfare for a Country in a Monetary Union ($\xi^2 = 0.75\xi^1$)

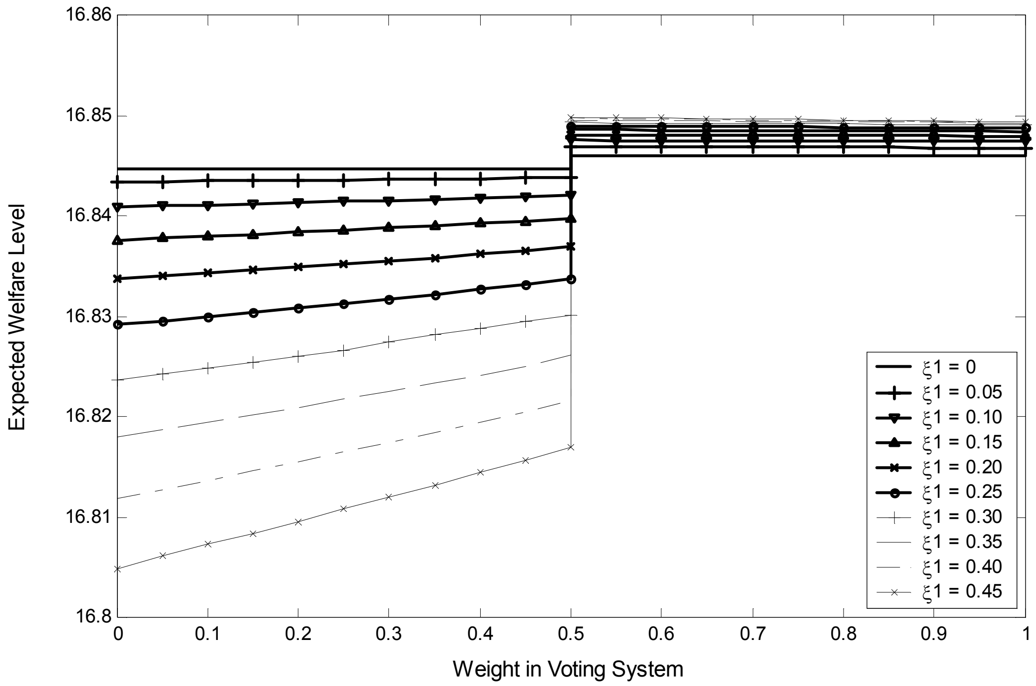


Figure 6: Expected Welfare for a Country in a Monetary Union ($\xi^2 = 1.25\xi^1$)

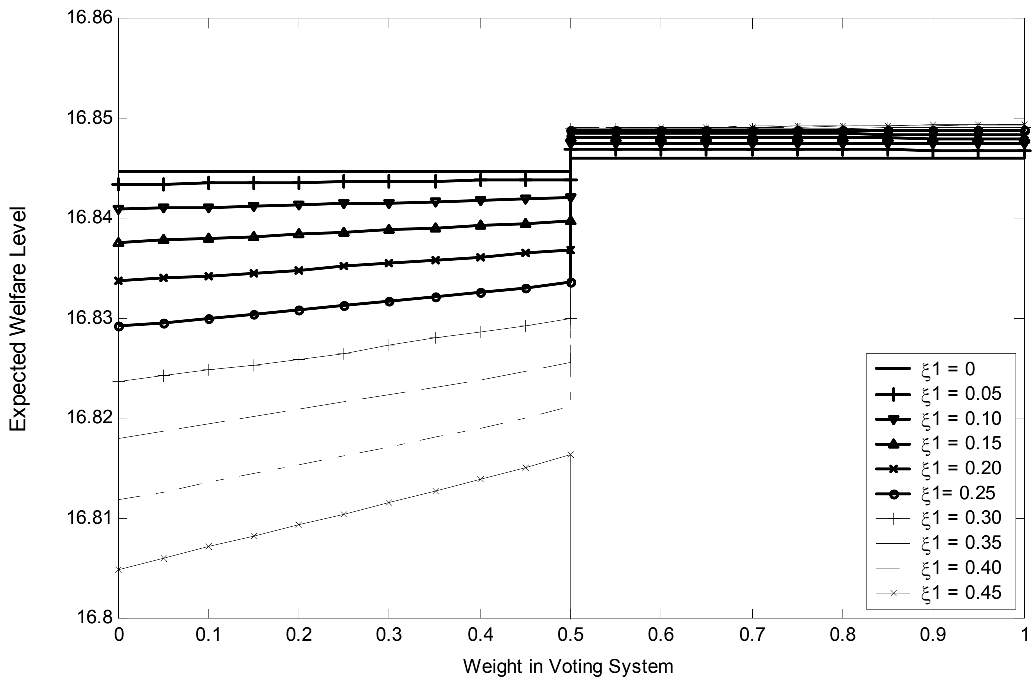


Figure 7: Welfare Levels under Alternative Monetary Regimes

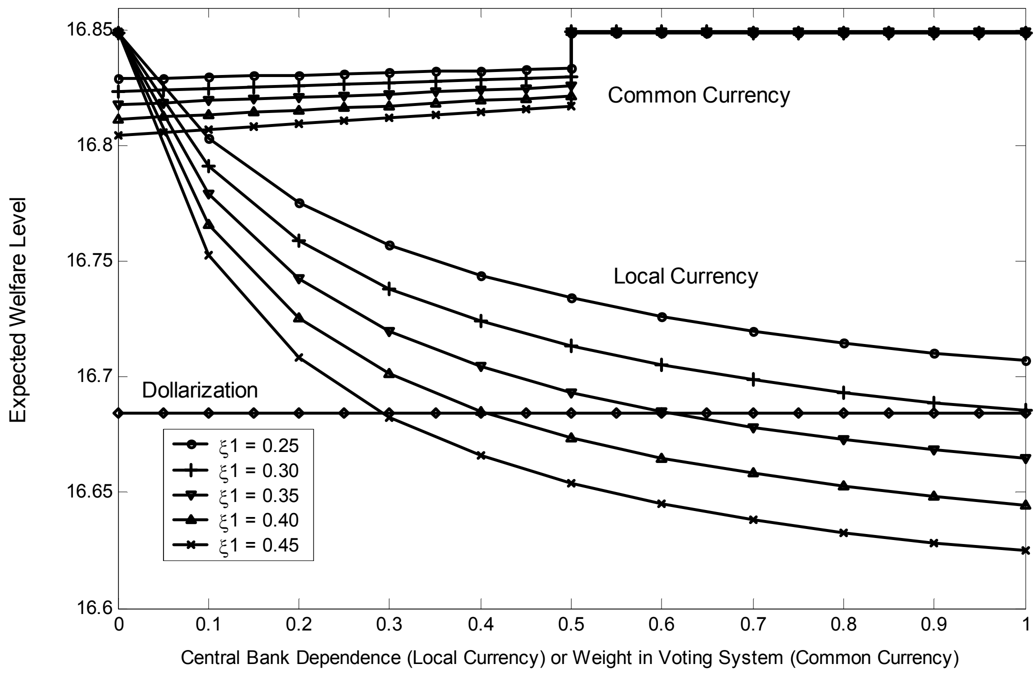


Figure 8: Welfare Levels under Alternative Monetary Regimes (No perfect symmetry)

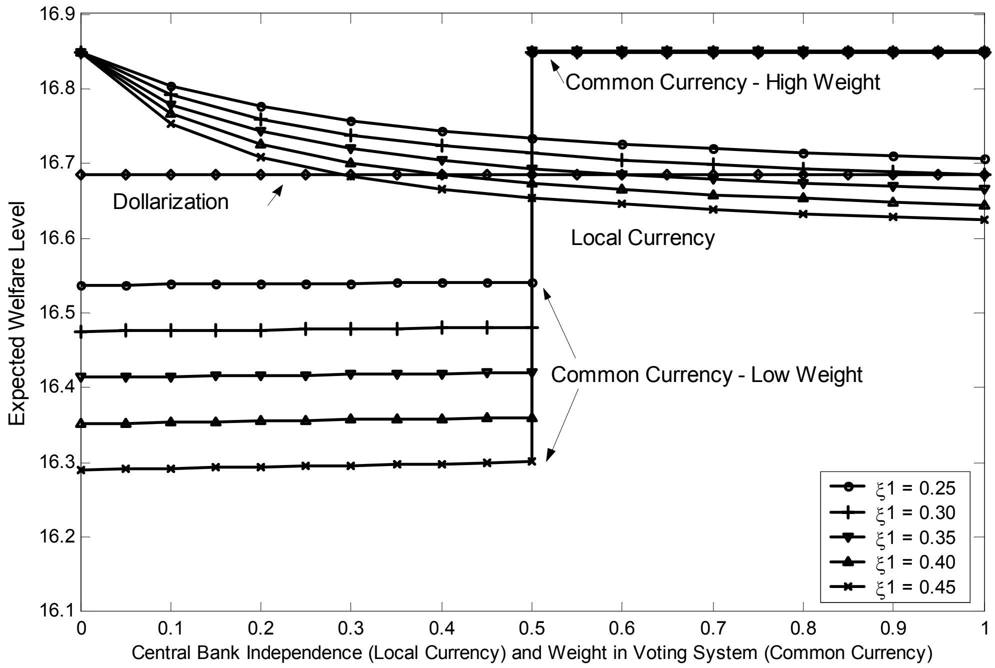


Figure 9: Expected Welfare Curves for Country with High Weight in Voting System

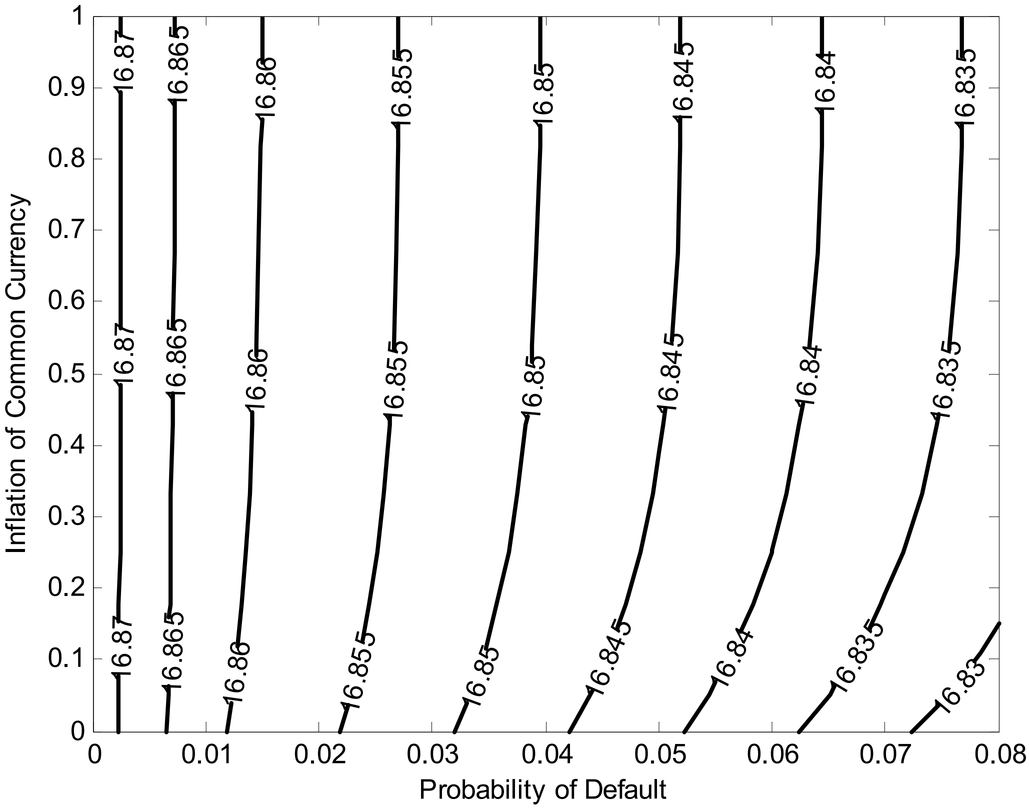


Figure 10: Expected Welfare Curves for Country with Low Weight in Voting System

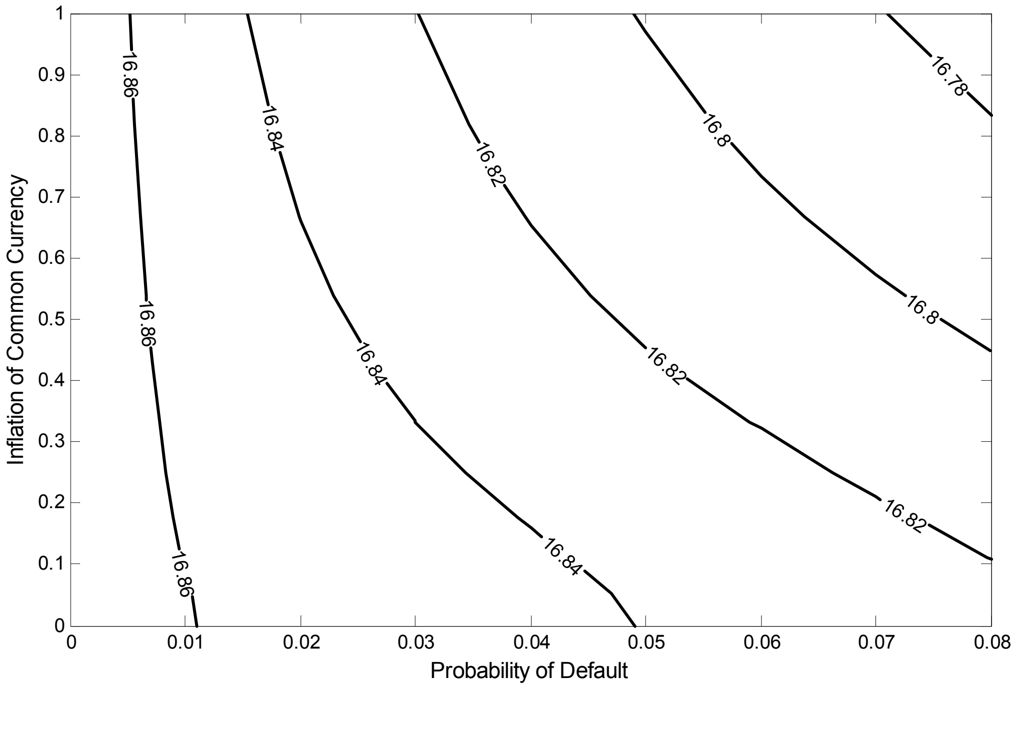


Figure 11: Welfare Gains from a Weight Increase in Voting System for Small Country

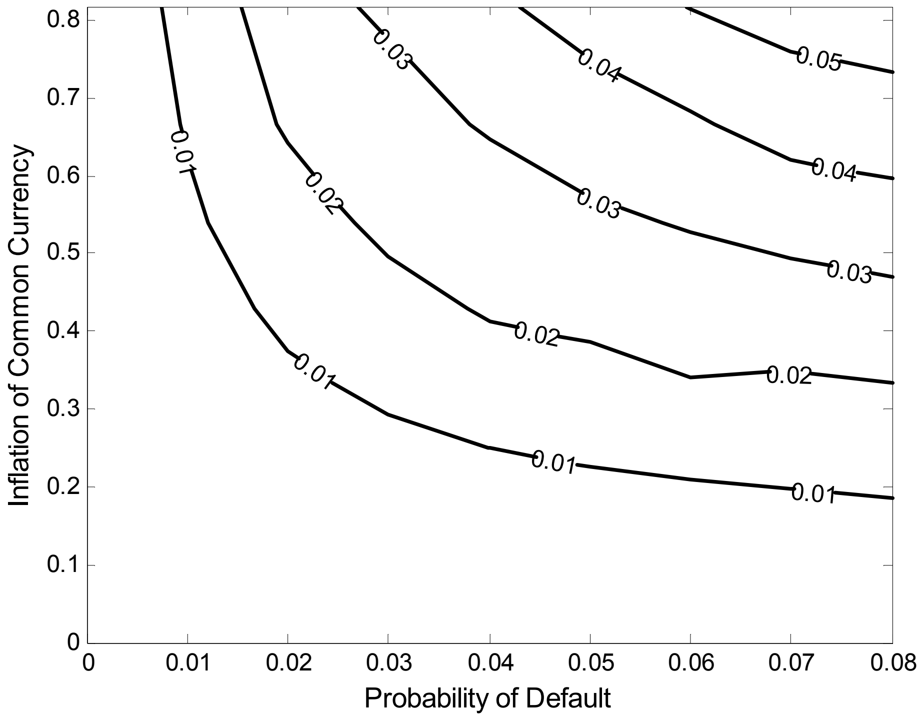
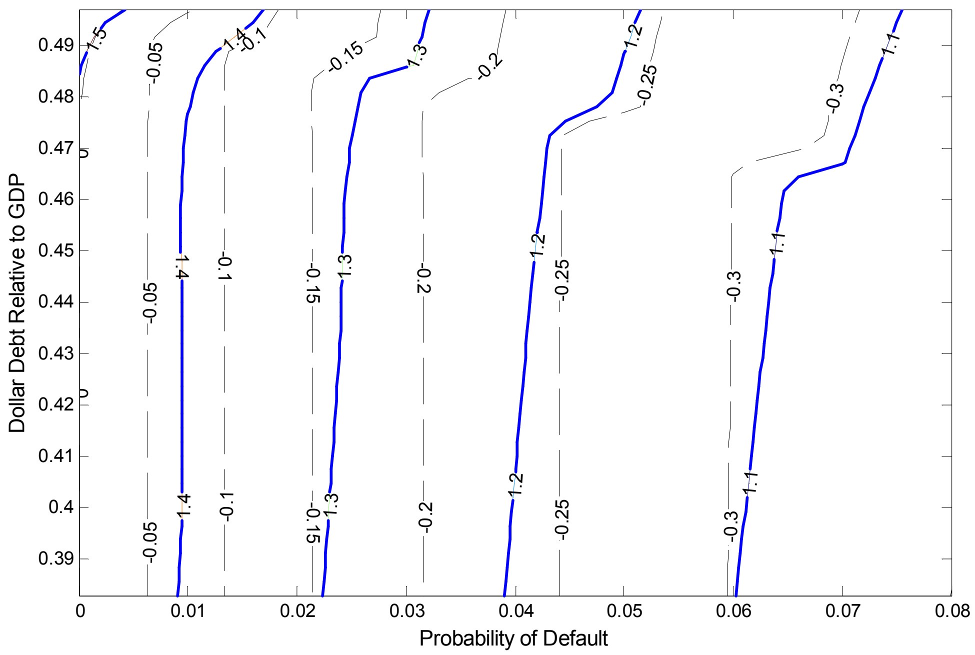


Figure 12: Percent Variation in Expected Welfare Between Common and Local Currency Regimes



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