Strategic Liquidity and Default Risk with Implications for International Capital Controls

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Abstract: How does the uncertain provision of external finance affect investment projects' default probability and liquidity risk? This paper studies the strategic interaction between many small foreign investors and a single borrower in the context of a two-period investment project requiring external finance. The key working assumptions are that there is maturity mismatch but no currency mismatch, foreign investors are risk-averse, and the information structure is imperfect but symmetric. The equilibrium probabilities of default and illiquidity are obtained under fixed and variable loan rates. It is found that aggregate default risk is decreasing in the project's internal endowment. A range of fundamentals is derived outside of which default and liquidity risk are either zero or one. If aggregate lending is pro-cyclical, it is shown that default risk is also pro-cyclical provided the project disruption caused by early liquidation is small. In that case, financial regulators targeting a stable investment environment should impose fewer restrictions on short-term capital outflows in times of expansion, and more during recessions. In contrast, if project disruption is severe, aggregate default risk becomes counter-cyclical. Prudent regulatory policy should then be relaxing controls on capital outflows during slowdowns.

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1 Introduction

This paper studies the impact of liquidity risk on investment project outcomes and default risk. Liquidity risk is thought of as the risk that a borrower will lose the project's rents due to excessive liquidation incentives of lenders (Diamond (1991)). The provision of liquidity to an investment project requiring foreign funding is modeled as a game between many small risk-averse foreign investors and a single domestic borrower. Unlike Bulow and Rogoff (1989) and Mella-Barral and Perraudin (1997), the foreign debt is privately held and there is no possibility for strategic recontracting and rescheduling. Moreover, the currency risk element in bringing about financial crises is not discussed—thus there is mismatch in maturities but not in currencies.[2]

The project has constant returns to scale with respect to both its internal and external finance components. However, final investment performance is a linear function of a shock to macroeconomic fundamentals which is only realized after lending has been committed. At the outset, foreign investors decide how much to lend based on their individual risk aversion. It is assumed that the maturity of the loan is one period, but the investment project requires two periods to complete. Moreover, all internal finance is domestic and all external finance is foreign; domestic capital markets are relatively under-developed and/or too small to handle the scale of the project.

The recent financial crises in many developing economies and the resulting sharp and protracted output declines have focussed research attention on the role of the available amount of liquidity—and the lack of it—in bringing about crises which become self-fulfilling. As argued by Eichengreen and Hausmann (1999) and Tirole (2002), the mismatched maturity aspect of emerging markets' "original sin" can be traced to agency-based incentive problems. In turn, foreign investors' uncertain prospects of recouping their investment can raise the probability of default and create systemic risk; for a historical perspective see Bordo et al (2001).

Liquidity is fundamentally determined at the micro level by individual

[2]Balance-sheet considerations can create a powerful link between banking crises and the exchange rate regime. See Chang and Velasco (2000a,b, 2001), Calvo (1998) Calvo and Reinhart (2000,2002) and Kaminsky and Reinhart (1999).

decision-makers (foreign creditors/investors) and translated into aggregate liquidity. Holmstrom and Tirole (1996, 1998) show that if markets are incomplete then entrepreneurs may be unable to insure themselves against exogenous liquidity shocks to projects' net worth. Faced with a balance sheet structured with a long-term asset and short-term external liabilities, investment projects that are socially valuable may thus be prematurely terminated because enough foreign investors do not extend their credit.

The probabilities of early liquidation of foreign investment and of project default constitute equilibrium liquidity and default risk, respectively. Unlike Diamond (1991), the borrower cannot choose the maturity structure: at the end of the first period, investors may decide not to roll over their loan and instead withdraw. Early withdrawal attracts a penalty (c) increasing in the amount of individual investment, but bounded by limited liability. The disruption to the project (k) brought about by early liquidation is also linearly increasing in the amount of loans not rolled over. These two parameters proxy for the level of capital controls and the project's relative dependence on foreign lending. The other model parameters are the riskless foreign interest rate, the domestic foreign-denominated loan rate (in Sections 2 and 3), the project's internal endowment and the probability distributions of the domestic fundamentals and foreign investors' risk aversion, the penalty for early liquidation and the disruption it causes to the project.

The empirical link between the level of capital controls and the incidence of currency and banking crises is ambiguous; see Bartolini and Drazen (1997a,b), Eichengreen et al. (1995) and Eichengreen (2002). In that respect, the present paper offers a framework for analyzing the desirability of imposing restrictions on short-term capital flows via the impact on the project's default probability of varying the penalty charged to creditors for liquidating early. In turn, the impact of this penalty on liquidity risk is a function of the disruption that early withdrawal causes the investment project. The disruption is assumed to be increasing in the amount of outside credit, reflecting the project's sensitivity to short-term reversals of investor and market sentiment.

There are three stages in the analysis. In Section 2, the loan rate is fixed exogenously and the game is sequential. Information about the fundamentals is complete and perfect and default risk is exogenous. I solve for the pure dominant strategy equilibria and obtain a range of fundamentals below which all lenders withdraw early, and above which they all roll over. In Section 3, the loan rate is fixed and the game is simultaneous. In Section 4, the loan rate is made a function of current macroeconomic fundamentals. Information about the latter is symmetric but imperfect: by the Harsanyi transformation, this is then equivalent to a simultaneous coordination game of complete but imperfect information. This contrasts with the seminal work of Diamond and Dybvig (1983), where lenders refuse to finance an illiquid borrower because of strategic uncertainty about other lenders' actions.

The main findings when loan rates depend on the underlying fundamentals are as follows. Default risk declines in the initial level of fundamentals and rises in the degree of short-term capital controls. Aggregate liquidity risk declines in the project's internal endowment; the latter strengthens investors' confidence in the project's chances of sucess. Moreover, liquidity risk is increasing in the disruption to the project caused by early liquidation. Assuming fundamentals follow a random walk, aggregate liquidity risk is *pro-cyclical*. Therefore, there is less likelihood of capital flow reversals in slowdown periods than in boom phases.

The equilibrium relation between liquidity risk and default risk is more subtle. The concerns raised by the BIS proposals for risk-sensitive capital adequacy ratios (Basel II) centre on the destabilising potential of pro-cyclical credit quality, that is counter-cyclical aggregate default risk.[3] The level of disruption caused by early liquidation, i.e. lost project output, turns out to be pivotal. In that respect, I find that for default risk to be pro-cyclical—for the default probability to rise during expansions and fall during recessions—the disruption caused by early liquidation has to be small. In that case, financial regulators aiming to maintain a stable investment environment should impose *less* restrictions on short-term international capital flows in times of expansion and *more* in recessions. In contrast, if the disruption caused by early liquidation is severe, aggregate default risk becomes counter-cyclical. Prudent regulatory policy should then be relaxing capital controls over short-term investment flows during times of macroeconomic slowdown.

[3]See the Basel Committee on Banking Supervision (2001) and Borio et al. (2001).

2 The model

2.1 Investment technology

I model a 2-period game between a single risk-neutral domestic entrepreneur and many small risk-averse foreign investors. The entrepreneur's investment project is not self-financing and requires two periods to complete. In order to focus on the implications of aggregate illiquidity for default, I make the simplifying assumptions that borrowing is in the foreign currency only—i.e. there is no liquid debt market in domestic currency—and that there is no devaluation risk, reflecting, for example, the presence of a credible fixed exchange rate regime. Thus, foreign investors' risk aversion is idiosyncratic and unrelated to any currency risk premium.

Gross project income is realized in the second period and accrues in the foreign currency, wlog. I assume a linear investment technology:

$$y_2 \;=\; \theta_2(E + L_0) \;\geq\; 0 \;, \tag{1}$$

where θ_2 is a domestic productivity/technological shock perfectly correlated with macroeconomic fundamentals at $t = 2$. It is assumed to be uniformly distributed over the unit interval, so its unconditional expectation is 0.5. In Section 3, θ_t will be taken to follow a pure random walk. E is the borrower's internal and illiquid endowment, and L_0 is the funding obtained from foreign investors at $t = 0$. The borrower can also access liquid reserve assets A yielding the riskless return r_A. The latter coincides with the world rate of interest, assumed fixed wlog.

2.2 Roll-over uncertainty and liquidity risk

The domestic entrepreneur seeks foreign borrowing at $t = 0$ for up to two periods. However, in the first period ($t = 1$) a proportion $\lambda_1 \in [0, 1]$ of foreign investors may not roll over their loans. There is no further borrowing at $t = 1$. The borrower's stock of liquid assets A can then be used to cover the resulting liquidity shortfall. Thus λ_1 measures roll-over uncertainty, amounting to the project's *liquidity risk*. This setup follows Chui, Gai and Haldane (2000).

In turn, if a foreign investor liquidates early, they incur a marginal cost $c < 1$. This is a constant fraction of their investment and is deducted from their payoff.[4] Early liquidation also results in a marginal cost $k < 1$ to the investment project, borne by the borrower. The parameter k captures the marginal disruption to the project brought about by early liquidation. The total disruption cannot exceed the amount of aggregate foreign borrowing.

Therefore, the investment project will not default iff its terminal net worth, defined as assets minus liabilities, is non-negative. This impies the following solvency constraint:

$$\theta_2(E + L_0) - k\lambda_1 L_0 + (1 + r^A)(A - \lambda_1 L_0) \;\geq\; (1 - \lambda_1)(1 + r_0^L)L_0 \qquad (2)$$

On the LHS is the entrepreneur's gross return, while the RHS is his total payment to the foreign investors who have rolled over their loans at $t = 2$. The project's terminal net worth decreases in the amount of early liquidation at $t = 1$, $\lambda_1 L_0$. The decline is monotonically increasing in the marginal cost of disruption k and in the proportion of foreign investors λ_1 who do not roll over their loans. Thus a proportion $1 - \lambda_1$ of investors stay on to the project's maturity. Some of the borrower's liquid assets A then have to be used to cover the shortfall from liquidating foreign investors. The borrower's liabilities are increasing linearly in the loan rate r_0^L offered to foreign investors at $t = 0$. In Section 4 the loan rate is assumed to be exogenously fixed, while in Section 5 it is a function of the underlying macroeconomic fundamentals.

2.3 Foreign investor risk aversion

The investment project's payoff at $t = 2$ follows a Bernoulli distribution with *Success* corresponding to repayment and *Failure* to default. I initially assume that the borrower's default probability P_2 is positive and exogenously given. From the atomistic viewpoint of each small foreign investor, there is default

[4] Arguably, c is positively correlated with the short-term international capital controls in place. Rapid financial liberalization and lifting of controls can result in abrupt reversals in capital flows—Calvo's (1998, 2000) *sudden stops*—which can lead to sovereign insolvency and default and painful macroeconomic slowdown.

risk but no aggregate liquidity risk. Thus, at $t = 0$ investor i's expected return and return variability upon completion of the project are just:

$$\begin{aligned} E_0 R_2 &= (1 - P_2)(1 + r_0^L) L_0^i \\ \sigma_0^2 &= P_2(1 - P_2)(1 + r_0^L)^2 L_0^{i2} \quad , \end{aligned} \tag{3}$$

where L_0^i is i's total investment amount. At $t = 0$, foreign investor i then maximizes quadratic expected utility:

$$E_0 U_2^i \;=\; ER_0^i \;-\; b_i \sigma_0^2 \tag{4}$$

Foreign investors are differentiated by their individual coefficient of risk aversion b_i. This is assumed to be random and uniformly distributed on $[\underline{b}, 1]$, where $\underline{b} > 0$. The minimum lower bound of the distribution's support is strictly positive to prevent infinite optimal lending arising under risk neutrality. Given the fixed upper bound of one, bigger (smaller) $\underline{b}$ values reflect more (less) aggregate risk aversion.

Atomistic investor i's participation constraint requires her expected return at $t = 0$ to be no less than the opportunity cost of lending to the project, i.e. the riskless (foreign) interest rate r^A, assumed constant:

$$ER_0^i \;\geq\; (1 + r^A) L_0^i \quad \Rightarrow \quad 1 + r_0^L \;\geq\; \frac{1 + r^A}{1 - P_2} \tag{5}$$

This is a necessary condition for the project to attract *any* foreign finance. Provided $P_2 > 0$, the credit spread $r_0^L - r^A$ between the project's loan rate and the riskless rate is always positive, reflecting the underlying default risk. Each foreign investor's lending at $t = 0$ is determined by substituting equations (3) into (4) and maximizing expected utility with respect to L_0^i, to yield

$$L_0^{*i} \;=\; \frac{1}{2 b_i (1 + r_0^L) P_2} \tag{6}$$

Individual optimal lending is finite iff $b_i > 0$ and $P_2 > 0$. It is declining in the loan rate, investor i's risk aversion and the (fixed) default probability.

Integrating (6) over the risk aversion distribution's support $[\underline{b}, 1]$ yields aggregate foreign investment going into the project at $t = 0$:

$$L_0^* = \int_{\underline{b}}^{1} L_0^{i*} db_i = -\frac{\ln \underline{b}}{2(1 + r_0^L)P_2} > 0 \tag{7}$$

Note that L_0^* decreases in foreign investors' aggregate risk aversion, so larger $\underline{b}$ values result in a smaller part of the project being externally financed. The impact of the default probability is discussed in Section 4.

2.4 Strategies and payoffs

In pure strategies, at $t = 1$ all foreign investors *Stay* ($\lambda_1 = 0$) or *Exit* ($\lambda_1 = 1$), while at $t = 2$ the domestic entrepreneur *Defaults* ($P_2 = 1$) or *Repays* ($P_2 = 0$) with certainty. Now assume that at $t = 0$ there is strategic uncertainty on both sides. In mixed strategies, the interpretation is that a proportion $\lambda_1 \in (0, 1)$ of a large population of small foreign investors is opting to liquidate early, incurring the marginal exit cost c, while a proportion $1 - \lambda_1$ stays on to the project's end date. Then, at $t = 2$ the entrepreneur defaults with probability $P_2 \in (0, 1)$ and repays with probability $1 - P_2$. The default probability reflects a strategic decision independently of any private information held by the foreign investors regarding their true type. The two-period game's terminal ($t = 2$) payoffs are in Table 1. For the entrepreneur these follow from the project's viability constraint in (2). Investor i's and the entrepreneur's pure strategies are in rows and columns, and their payoffs are in the top and bottom entries of each cell:

Table 1. The two-period game payoff matrix: $t = 2$

	Repay $[P_2 = 0]$	*Default* $[P_2 = 1]$
Exit $[\lambda_1 = 1]$	$L_0^i(1 - c)$	$L_0^i(1 - c)$
	$\theta_2(E + L_0) - kL_0 + (1 + r^A)(A - L_0)$	$(1 + r^A)(A - L_0)$
Stay $[\lambda_1 = 0]$	$L_0^i(1 + r_0^L)$	0
	$\theta_2(E + L_0) + (1 + r^A)A - (1 + r_0^L)L_0$	$(1 + r^A)A$

Their mixed strategies P_2 and λ_1 correspond to the default and liquidity risk measures of the borrower and foreign investors, respectively. Atomistic investor i's payoff is a function of their individual loan amount, while the borrower's payoffs involve the aggregate amount of lending.

2.5 The sequential investment game

To build intuition, I first analyse the 2-period sequential game with complete and perfect information. The loan rate is fixed at $t = 0$, foreign investors can liquidate early and the borrower observes the proportion which does so. The default probability is taken to be exogenous. The extensive form is in **Figure 1, Panel A**. At $t = 1$, each investor i will roll over their loan—implying $\lambda_1 = 0$ on aggregate—iff her expected return exceeds the certain return from early liquidation:

$$(1 - P_2)(1 + r_0^L)L_0^i \ > \ (1 - c)L_0^i$$

Thus, $\lambda_1 = 0$ requires:

$$P_2 \ < \ P^* = \frac{c + r_0^L}{1 + r_0^L} \ , \tag{8}$$

where $P^* \leq 1$ because $c \leq 1$. Limited liability guarantees foreign investors cannot lose more than their initial investment. The threshold default probability level increases in c and r_0^L, so the range $P_2 \in (P^*, 1]$ yielding early liquidation is smaller for higher c and r_0^L. Thus, imposing a bigger penalty to foreign investors for liquidating early and/or offering them higher loan rates makes liquidation less likely, ceteris paribus. If $P_2 = P^*$ then investor i is indifferent, while if $P_2 > P^*$ then is dominant, implying $\lambda_1 = 1$.

The borrower's best response at $t = 2$ is a function of the exogenous default probability P_2. There are two cases to analyze. First, if $P_2 \ \leq \ P^*$ then $\lambda_1 = 0$, and the best response strategy is to complete the project and repay foreign investors iff:

$$\theta_2(E + L_0) + (1 + r^A)A - (1 + r_0^L)L_0 \ > \ (1 + r^A)A$$

This implies a fundamental insolvency threshold $\underline{\theta}$ as a function of the loan rate r_0^L, the internal endowment E and aggregate foreign lending L_0:

$$\theta_2 > \underline{\theta} = \frac{(1+r_0^L)L_0}{E+L_0} \tag{9}$$

Note that $\underline{\theta}$ increases in r_0^L and L_0 and decreases in E: higher credit spreads and/or smaller internal endowments make default more likely. The positive contribution of aggregate external finance to default risk reflects moral hazard. The fact that there has been no external finance shortfall improves the project's expected return and presents the borrower with a greater incentive to default.

Thus, if the fundamental realization at $t = 2$ is less than $\underline{\theta}$, the borrower will default even if no investor has liquidated early ($\lambda_1 = 0$). However, the strategy combination $\{Stay, Default\}$ is not subgame-perfect because the default risk threshold in (8) is inconsistent with the borrower's best response. Provided $P_2 \leq P^*$, from (9) the borrower will always default if $\underline{\theta} > 1$. Then $(1+r_0^L)L_0 > E+L_0$, equivalently $L_0 > \frac{E}{r_0^L}$, and the unique Nash equilibrium in pure strategies is $\{Stay, Default\}$. In the subgame commencing at $t = 0$, if lenders know that the borrower will certainly default, they will never roll over because $0 < L_0^i(1-c)$ for all $L_0^i > 0$. Indeed, in that case the project will obtain no foreign lending at $t = 0$. Thus, $P^* < 1$ is inconsistent with certain default.

In the second case, $P_2 > P^*$. Then early liquidation is dominant, so $\lambda_1 = 1$. Because investors move first, defaulting may be the borrower's dominant strategy. From the Table 1 payoffs, $\{Exit, Default\}$ is the unique Nash equilibrium in pure dominant strategies iff:

$$(1+r^A)(A-L_0) \;>\; \theta_2(E+L_0) - kL_0 + (1+r^A)(A-L_0) \Rightarrow$$

$$\theta_2 \;<\; \overline{\theta} \;=\; \frac{kL_0}{E+L_0} \tag{10}$$

The fundamental upper bound $\overline{\theta}$ is increasing in k and L_0 and decreasing in E. The intuition is that early liquidation lowers the project's *ex post* return, thus encouraging default. Conversely, more internal funding improves the project's chances of completion following early liquidation, all else equal. Moreover, $\{Exit, Default\}$ is a subgame-perfect strategy combination because $P_2 > P^*$ is consistent with certain default. Substituting aggregate foreign lending from equation (7) into (10) it is easy to verify that higher P_2 yields a wider fundamental range for which default is the optimal strategy.

Therefore, equations (9) and (10) imply that foreign investors' perception of P is consistent with the borrower's best response iff $\underline{\theta} < \theta_2$ when $P_2 < P^*$ (good fundamentals), and $\theta_2 < \overline{\theta}$ when $P_2 > P^*$ (bad fundamentals). Thus, the fundamental range $\underline{\theta} < \theta_2 < \overline{\theta}$ supports subgame-perfect pure strategy Nash equilibria.

3 Equilibrium in the simultaneous game with fixed loan rates

3.1 Default risk

This Section analyzes the 2-period game where at $t = 2$ the borrower has not observed investors' action at $t = 1$. Unlike the sequential game of Section 2.5, I now also assume that the fundamental realization at $t = 2$ is not observed by either player, but its expectation at $t = 1$, $\theta_2^e = E_1\theta_2$, is used instead. For a uniform distribution of fundamentals on $[0, 1]$, this expectation is unconditionally 0.5. If a process for θ_t is specified, then expectations are conditional on the last available fundamental realization to enter players' information sets. For example, if fundamentals follow a pure random walk ($\theta_t = \theta_{t-1} + \epsilon_t$ where ϵ is white noise) then $\theta_2^e = \theta_1$.

From the Harsanyi (1967) transformation, this setup is equivalent to a simultaneous game of complete but imperfect information: the entrepreneur's decision node at $t = 1$ is not a singleton. Its extensive form is shown in **Figure 1, Panel B**. Let $E_1\pi_2^i$ and $E_1\pi_2^B$ denote investor i's and the borrower's terminal payoffs expected at $t = 1$. These expectations are linear combinations of their pure strategy payoffs from Table 1, with respective weights given by λ_1 and P_2:

$$
\begin{aligned}
E_1\pi_2^i &= \lambda_1[(1-P_2)L_0^i(1-c)+P_2L_0^i(1-c)]+(1-\lambda_1)[(1-P_2)(1+r_0^L)L_0^i] \\
&= \lambda_1\ L_0^i[1-c-(1+r_0^L)(1-P_2)]+L_0^i(1+r_0^L)(1-P_2) \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
E_1\pi_2^B &= (1-P_2)\lambda_1\left[\theta_2^e(E+L_0)-kL_0+(1+r^A)(A-L_0)\right]\ + \\
&\quad (1-P_2)(1-\lambda_1)\left[\theta_2^e(E+L_0)+(1+r^A)A-(1+r_0^L)L_0\right]\ + \\
&\quad P_2\left[\lambda_1(1+r^A)(A-L_0)+(1-\lambda_1)(1+r^A)A\right] \qquad (12)
\end{aligned}
$$

The relevant solution concept at $t=1$ is mixed strategies Nash equilibrium: the equilibrium probabilities the players assign to each of their pure strategies are obtained from their opponent's optimization. Thus, foreign investor i maximizes her expected payoff function (11) with respect to λ_1. The FOC simultaneously determines equilibrium default risk at $t=2$:

$$
\frac{\partial E_1\pi_2^i}{\partial\lambda_1}=0 \quad \Rightarrow \quad P_2^* \ = \ \frac{c+r_0^L}{1+r_0^L} \qquad (13)
$$

Equation (13) generalizes dominant strategy condition (8). Note that $P_2^*<1$ iff $c<1$; this is guaranteed by limited liability. It also follows that $\frac{\partial P_2^*}{\partial c}=\frac{1}{1+r_0^L}>0$ and $\frac{\partial P_2^*}{\partial r_0^L}=\frac{1-c}{(1+r_0^L)^2}\geq 0$: equilibrium default risk is increasing in r_0^L and non-decreasing in c.

The equilibrium amount of foreign investment in the project at $t=0$ is now simply obtained by substituting P^* into equation (7):

$$
L_0^* \ = \ -\frac{\ln \underline{b}}{2(c+r_0^L)} \ > \ 0 \qquad (14)
$$

Aggregate foreign lending decreases in foreign investors' aggregate risk aversion $\underline{b}<1$ and in the external loan rate r_0^L. Thus, given the internal endowment level E, larger credit spreads and/or higher foreign investor risk aversion imply that relatively more of the project is internally funded. Bigger credit spreads drive risk-averse foreign investors away. A related implication is that a negative shock to aggregate risk aversion would yield more foreign investment. During the Asian financial crises of 1997-98, such a shift in investors' appetite towards more risk-taking has been attributed, inter alia, to misplaced faith in the stability of the countries' currency pegs.[5]

L_0^* also declines in c. If more short-term capital controls are imposed on investors—i.e. if c goes up— then aggregate foreign lending will decrease, and vice versa if capital controls are relaxed, ceteris paribus.

3.2 Aggregate liquidity risk

The equilibrium liquidity risk measure is λ_1^*. The proportion of foreign investors opting to liquidate at $t = 1$ is obtained from maximizing the borrower's expected payoff function (12) with respect to P_2:

$$\frac{\partial E_1 \pi_2^B}{\partial P_2} = 0 \quad \Rightarrow \quad \lambda_1^* \ = \ \frac{\theta_2^e (E + L_0) - (1 + r_0^L) L_0}{(k - 1 - r_0^L) L_0} \tag{15}$$

Note that $\theta_2^e = E_1 \theta_2$ is affecting liquidity risk at $t = 1$. This is because of the imperfect information assumption and the fact that one player's optimal strategy is obtained from the other player's first-order condition.

The comparative static impact of the model parameters on λ_1^* is as follows:

(a) $\frac{\partial \lambda_1^*}{\partial E} = \frac{\theta_2^e}{(k-1-r_0^L)L_0} < 0$: λ_1^* is falling in the internal endowment. Other things equal, better endowed projects improve foreign investors' confidence in their success, and vice versa for worse endowed projects. Moreover, the absolute magnitude of $\frac{\partial \lambda_1^*}{\partial E}$ is increasing in θ_2^e: expectations of better fundamentals strengthen the marginal impact of the internal endowment on liquidity risk.

(b) $\frac{\partial \lambda_1^*}{\partial \theta_2^e} = \frac{E+L_0}{(k-1-r_0^L)L_0} < 0$. Ceteris paribus, better expected fundamentals at $t = 2$ induce more investors to roll over their loans.

(c) $\frac{\partial \lambda_1^*}{\partial k}$ is negative (positive) for large (small) θ_2^e. The corresponding fundamental insolvency threshold is $\underline{\theta}$, given in equation (9). Thus, when expected fundamentals are good, i.e. above the threshold, λ_1^* is falling in the project disruption caused by early withdrawal. In contrast, if expected fundamentals are bad, then λ_1^* is increasing in k. The intuition is that when θ_2^e is small, investment projects are perceived to be relatively more dependent on foreign lending, ceteris paribus. Whether such projects' eventual failure is

[5] For related theoretical and empirical arguments see the contributions in Krugman (2000) and Eichengreen (2002).

due more to fundamental insolvency than illiquidity depends on the expected value of fundamentals at $t = 2$.

(d) The impact of L_0 on λ_1^* is $\frac{\partial \lambda_1^*}{\partial L_0} = \frac{(E_1\theta_2)E}{(1+r_0^L-k)L_0^2} > 0$ as $k < 1$. At the margin, more aggregate foreign lending induces more foreign investors to withdraw early, other things equal. The marginal impact is decreasing in the level of L_0 and increasing in expected fundamentals for $t = 2$ and in the internal endowment.

The last comparative static property has two implications. First, if aggregate foreign lending is pro-cyclical, the liquidity risk measure is also. Second, it then follows that, on average, the business cycle has a smaller impact on liquidity risk for lower $E_1\theta_2$ values. The intuition is that weaker expected fundamentals make foreign investors less sensitive to abrupt changes in aggregate lending patterns. Conversely, equilibrium liquidity risk is more sensitive to changes in L_0 if fundamentals are expected to improve. Such changes in expectations could also be driven by sudden shifts in sentiment and herding behaviour.[6]

What are sufficient conditions for an interior solution $\lambda_1^* \in (0, 1)$, that is for equilibrium liquidity risk to be a well-defined probability measure?

In order for $\lambda_1^* > 0$, both numerator and denominator in equation (15) have to have the same sign. The denominator $(k-1-r_0^L)L_0$ is always negative because $k < 1$ and $r_0^L > 0$. The numerator is negative (positive) provided $L_0 > (<) \frac{\theta_2^e \, E}{1+r_0^L-\theta_2^e}$. Maintaining the negative case, aggregate foreign lending must exceed a certain level which is decreasing in the loan rate. Rearranging the above expression, this level corresponds to $\theta_2^e < \theta_{MAX} = \frac{(1+r_0^L)L_0}{E+L_0}$. Therefore, positive liquidity risk—a non-zero proportion of foreign investors liquidating at $t = 1$—requires that expected fundamentals are weaker than the θ_{MAX} upper bound. In contrast, if period–2 fundamentals are expected to be stronger than θ_{MAX} then $\lambda_1^* = 0$ and no investor has an incentive to liquidate early.

At the opposite extreme, in order for $\lambda_1^* < 1$ equation (15) implies:

$$(E_1\theta_2)(E + L_0) - (1 + r_0^L)L_0 > L_0(k - 1 - r_0^L) \;\Leftrightarrow\; (E_1\theta_2)E > (k - E_1\theta_2)L_0$$

[6] These comments are also relevant for the IMF's catalytic finance role; see Eichengreen and Mody (2001) and Mody and Saravia (2003).

Rearranging this inequality yields $E_1\theta_2 > \theta_{min} = \frac{kL_0}{E+L_0}$. Thus, fundamental expectations below θ_{min} imply $\lambda_1^* = 1$, so that all foreign investors liquidate early. Combining this with the upper bound for $\lambda_1^* > 0$, an interior solution for liquidity risk exists inside the expected fundamental range $E_1\theta_2 \in (E_1\theta_2^{min}, E_1\theta_2^{MAX})$, where

$$E_1\theta_2^{min} = \frac{kL_0}{E+L_0} \quad , \quad E_1\theta_2^{MAX} = \frac{(1+r_0^L)L_0}{E+L_0} \tag{16}$$

Note that $\theta_{min} < \theta_{MAX}$ requires $k < 1 + r_0^L$, which is always true, so the fundamentals range derived in (16) is non-empty. Substituting optimal aggregate foreign lending L_0^* from equation (14) into (16), the fundamental range becomes

$$E_1\theta_2^{min} = \frac{-k\ln \underline{b}}{2E(c+r_0^L) - \ln \underline{b}} \quad , \quad E_1\theta_2^{MAX} = \frac{-(1+r_0^L)\ln \underline{b}}{2E(c+r_0^L) - \ln \underline{b}} \tag{17}$$

Given the uniform fundamentals distribution, foreign investors' aggregate risk aversion and the fixed loan rate, bigger k values make $E_1\theta_2 < E_1\theta_2^{min}$ more likely, so that no investor will roll over their credit, while bigger c values lower both θ_{min} and θ_{MAX}, making $E_1\theta_2 > E_1\theta_2^{MAX}$ more likely, so that all investors roll over. Prima facie, these equilibrium sensitivities suggest that a smaller disruption to the project and higher controls on capital outflows induce less liquidity risk.

The above analysis has been assuming the loan rate is exogenously fixed. The next Section studies the effects of relaxing this assumption.

4 Equilibrium in the simultaneous game with variable loan rates

The loan rate r^L is now made a function of the contemporaneous fundamental realization. The loan terms offered to foreign investors will depend on the functional relationship between r_L and θ. In particular, I assume the entrepreneur sets the loan rate at time t according to a monotonically decreasing function of θ_t:

$$r_t^L(\theta_t) \;=\; r^A + \frac{1}{\theta_t} - 1 \tag{18}$$

Thus, r_t^L varies counter-cyclically. Its maximum value is $r_{MAX}^L(0) \to \infty$, corresponding to the worst fundamental realisation $\theta_t = 0$. In contrast, the minimum loan rate is $r_{min}^L(1) = r^A$, corresponding to the best possible fundamental $\theta_t = 1$. Underlying this inverse relationship is the standard emerging market scenario where deteriorating fundamentals indicate imply higher country risk, hence bond interest rate spreads widen.

Importantly, because this is a game of complete and imperfect information, the expected fundamental realization for the borrower's optimization in equation (12) is still θ_2^e. The entrepreneur and foreign investors plug the relevant fundamental realization θ_0 and the resulting loan rate r_0^L from equation (18) into default risk in (13) and aggregate liquidity risk in (15) to determine their optimal mixed strategies:

$$P_2^* \quad = \quad \frac{\theta_0(c + r_A - 1) + 1}{1 + r_A\theta_0} \tag{19}$$

$$\lambda_1^* \quad = \quad \frac{\theta_0 \;\theta_2^e \;(E + L_0) - (1 + r_A\theta_0)L_0}{[(k - r_A)\theta_0 - 1]L_0} \tag{20}$$

Note that while default risk depends only on θ_0, liquidity risk is also affected by period-1 expectations of period-2 fundamentals, $\theta_2^e = E_1\theta_2$. The latter is unconditionally 0.5. However, because θ_1 is in both investors' and the borrower's information sets, assuming fundamentals follow a pure random walk, as described in Section 3.1, implies $\theta_2^e = E_1\theta_2 = \theta_1$, hence:

$$\lambda_1^* = \frac{\theta_0 \;\theta_1 \;(E + L_0) - (1 + r_A\theta_0)L_0}{[(k - r_A)\theta_0 - 1]L_0} \tag{21}$$

4.1 Comparative statics

Equation (19) and (21) yields the comparative statics of the equilibrium default and liquidity risk measures. First:

$$\frac{\partial P_2^*}{\partial \theta_0} = \frac{c-1}{(1+r_A\theta_0)^2} \; < \; 0 \tag{22}$$

for all $c < 1$. Thus a better fundamental value at the start of the project unconditionally lowers default risk. This follows directly from the fact that θ_0 is common knowledge.

Second, $\frac{\partial P_2^*}{\partial c} = \frac{\theta_0}{1+r_A\theta_0} > 0$. Thus, lowering the marginal cost of liquidation at $t = 1$—that is, relaxing short-term capital controls—unambiguously lowers default risk. This is as expected and was also obtained in Section 3.1 under fixed loan rates—intuitively, making r_t^L a function of θ_t does not involve c.

Third, $\frac{\partial P_2^*}{\partial r_A} = \frac{\theta_0^2(1-c)}{(1+r_A\theta_0)^2} > 0$ for all $c < 1$. A positive shock to the foreign interest rate raises default risk, all else equal. This is a novel property due to the dependence of r_t^L on the initial fundamental state and on r^A.

I now turn to the comparative statics of equilibrium liquidity risk: First, differentiating equation (21) with respect to θ_1 yields

$$\frac{\partial \lambda_1^*}{\partial \theta_1} \; = \; \frac{\theta_0(E+L_0)}{[(k-r_A)\theta_0 - 1]L_0} \; \leq \; 0 \tag{23}$$

This is non-positive as the denominator is equal to that in equation (15), which is always negative. Under a random walk process for θ_t, better fundamentals at $t = 1$ induce investors to expect the trend to continue in $t = 2$. Hence, a smaller proportion liquidates early, reflecting improved confidence in the project's chances of success.

Second, the impact of the initial fundamentals value θ_0 on liquidity risk is given by

$$\frac{\partial \lambda_1^*}{\partial \theta_0} \; = \; \frac{\theta_1(E+L_0) - r_A L_0 - (k-r_A)(\theta_0\theta_1(E+L_0) - (1+r_A\theta_0)L_0)}{[(k-r_A)\theta_0 - 1]L_0} \tag{24}$$

It can be shown that this expression is *negative* (positive) for *small* (large) values of k, the marginal disruption to the project from early liquidation.[7] The reason is as follows: for small values of k the project is relatively immune

[7]The proof is available upon request.

to external liquidity shocks. Consequently, better initial fundamentals lower liquidity risk.

The third comparative static property relates liquidity risk to the project's internal endowment:

$$\frac{\partial \lambda_1^*}{\partial E} = \frac{\theta_1^2}{[(k - r_A)\theta_0 - 1]L_0} \ \leq \ 0 \tag{25}$$

This is non-positive for all θ_0 and $\theta_1 \in [0, 1]$, suggesting that bigger E mitigates liquidity risk. Intuitively, foreign investors perceive that the project is less dependent on external finance, all else equal.

Fourth, it is easy to check that:

$$\frac{\partial \lambda_1^*}{\partial k} = \frac{\theta_0[(1 + r_A\theta_0)L_0 - \theta_1(E + L_0)]}{[(k - r_A)\theta_0 - 1]^2 L_0} \tag{26}$$

This is positive unless E is very large. Thus, liquidity risk is increasing in the disruption caused to the investment project by early withdrawals. However, if the project is financed more by internal endowment (large E) then λ_1^* can be decreasing in k. Internal funding then exerts a mitigating influence on liquidity risk.

Finally, the sensitivity of liquidity risk to aggregate foreign lending is just:

$$\frac{\partial \lambda_1^*}{\partial L_0} \ = \ -\frac{\theta_0 \ \theta_1 E}{[(k - r_A)\theta_0 - 1]L_0^2} \ > \ 0 \tag{27}$$

The denominator is always negative, therefore $\frac{\partial \lambda_1^*}{\partial L_0} \geq 0$. The comparative static result from Section 3 is thus maintained. A positive shock to external finance induces a larger foreign investor proportion to liquidate early, and vice versa for a negative shock. Therefore, if aggregate foreign lending is taken to be pro-cyclical—positive correlation between L_0 and the international business cycle—liquidity risk will be also.[8] This result suggests that reversals in short-term capital flows (liquidity crises) are less likely in slowdown periods than they are in booms.

[8] Pro-cyclicality of liquidity risk also follows from $\frac{\partial \lambda^*}{\partial E} < 0$, to the extent that the internal endowment's relative share of the project's finance is likely to increase during slowdown periods.

4.2 Discussion: the cyclical properties of λ and P

The equilibrium relationship between default and liquidity risk can now be determined. From the gradient theorem, dividing equations (22) by (24) yields $\frac{\partial P_2^*}{\partial \lambda_1^*} = \frac{\partial P_2^*}{\partial \theta_0} / \frac{\partial \lambda_1^*}{\partial \theta_0}$. Combining the above two results for the sensitivity of default and liquidity risk to θ_0 implies $\frac{\partial P_2^*}{\partial \lambda_1^*} > (<) \; 0$ for small (large) values of k. Therefore, given $\frac{\partial \lambda_1^*}{\partial L_0} > 0$, the default probability is pro-cyclical if the marginal disruption to the investment project caused by early liquidation is small, and counter-cyclical if k is large.

Pro-cyclicality of aggregate default risk exerts a stabilizing influence on the business cycle. In contrast, negative correlation between macroeconomic growth and default risk—or average credit quality—amplifies business cycle fluctuations; see the BIS Committee on Banking Supervision (2001).

Therefore, if aggregate lending is *pro-cyclical*, default risk is also pro-cyclical provided the project disruption caused by early liquidation is small. Given $\frac{\partial P_2^*}{\partial c} > 0$, the tentative policy implication for improving international financial stability and limiting systemic risk is that regulators should impose fewer restrictions on capital outflows in times of expansion than during recessions. But if project disruption is severe, aggregate default risk becomes *counter-cyclical*. Prudent regulatory policy should then be relaxing controls on capital outflows in slowdown periods.

This framework can also be used to address the question of whether more foreign lending reduces aggregate liquidity risk in projects with maturity mismatch.[9] In (17), an open range of expected period-2 fundamentals was obtained such that liquidity risk is non-zero: $\lambda_1^* \in (0,1)$. Using (18) to substitute the loan rate as a function of fundamentals, the range of fundamental expectations outside $(\theta_{min}, \theta^{MAX})$ can be established. At one extreme, the *unstable* range $E_1\theta_2 \leq \theta_{min}$ captures liquidity crises amounting to total liquidation of outside funding ($\lambda_1^* = 1$). At the other extreme lies the *stable* range of fundamentals $E_1\theta_2 \geq \theta^{MAX}$ such that there is no early liquidation ($\lambda_1^* = 0$).

The stable fundamentals range is a non-linear function of θ, suggesting

[9] For example, see Cooper (1999) and Obstfeld (1998).

that liquidity risk may change discontinuously in response to small shocks. In the absence of a specific stochastic process for fundamentals, the stable range corresponds to fundamental expectations in the unit interval. A numerical investigation could shed more light into the dynamics of the stable and the unstable range, and prudent policy would sensibly aim to design a regulatory framework (k, c) so as to minimize the latter.

5 Concluding remarks

This paper studied the strategic interaction between many risk-averse lenders and a single borrower in a two-period investment project with long-term assets and short-term liabilities. The proportion of investors liquidating early was characterised as a function of the credit spread, the lenders' penalty for early withdrawal and the disruption it causes the project, the contribution of the borrower's internal endowment, and the probability distributions of fundamentals and lenders' risk aversion. The cyclical properties of the risk measures and their implications for international financial regulation were obtained and stable and unstable fundamental ranges derived such that the risk of illiquidity is either zero or one. It was found that default risk is procyclical if the disruption caused by early liquidation is small. In contrast, if the disruption caused by early liquidation is severe, aggregate default risk becomes counter-cyclical. Prudent regulatory policy should then be relaxing short-term controls on capital ouflows in times of macroeconomic slowdown and tightening them during expansions.

There are several directions in which the model could be extended. First, the stable and unstable ranges of fundamentals are sensitive to the underlying probability distribution functions; these were assumed to be uniform to simplify the exposition. Second, foreign investors' risk aversion could be made endogenous to the fundamental realization, thus introducing currency risk into the framework. These extensions are the subject of current research.

References

[1] Bartolini, L. and A. Drazen 1997a. "Capital Account Liberalization as a Signal". *American Economic Review* 87: 138-154.

[2] Bartolini, L. and A. Drazen 1997b. "When Liberal Policies Reflect Shocks, What Do We Learn?" *Journal of International Economics* 42: 249-273.

[3] Bordo, M., B. Eichengreen, D. Klingebiel and M. Martinez-Peria 2001. "Financial Crises: Lessons from the Last 120 Years". *Economic Policy* 32: 53-82.

[4] Borio, C., C. Furfine and P. Lowe 2001. Procyclicality of the Financial System and Financial Stability: Issues and Policy Options. *BIS Papers* No. 1, Part 1.

[5] Bulow, J. and K. Rogoff 1989a. A Constant Recontracting Model of Sovereign Debt. *Journal of Political Economy* 97: 155–178.

[6] Calvo, G. 1998. Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops. *Journal of Applied Economics* 1: 35-54.

[7] Calvo, G. and C. Reinhart 2000. When Capital Flows Come to a Sudden Stop: Consequences and Policy, in P. Kenen and A. Swoboda (eds.) *Reforming the International Financial and Monetary System*, Washington, D.C: International Monetary Fund.

[8] Calvo, G. and C. Reinhart 2002. Fear of Floating. *Quarterly Journal of Economics* 107: 379-408.

[9] Chang, R. and A. Velasco 2000a. Financial Fragility and the Exchange Rate Regime. *Journal of Economic Theory* 90: 1-34.

[10] Chang, R. and A. Velasco 2000b. Banks, Debt Maturity and Financial Crises. *Journal of International Economics* 51: 169-94.

[11] Chang, R. and A. Velasco 2001. Monetary Policy in a Dollarized Economy Where Balance Sheets Matter. *Journal of Development Economics* 81: 1-21.

[12] Chui, M., P. Gai and A. Haldane 2000. International Financial Crises and Public Policy: Some Welfare Analysis. *Bank of England Quarterly Bulletin* 368–76.

[13] Cooper, R. 1999. Should Capital Controls Be Abolished? *Brookings Papers on Economic Activity* 89–125.

[14] Diamond, D. 1991. Debt Maturity Structure and Liquidity Risk. *The Quarterly Journal of Economics* 106: 709–37.

[15] Diamond, D. and P. Dybvig. 1983. Bank Runs, Deposit Insurance and Liquidity. *Journal of Political Economy* 91: 401–19.

[16] Eichengreen, B., 2002. *Financial Crises and What to Do About Them.* Oxford: Oxford University Press.

[17] Eichengreen, B. and R. Hausmann. 1999. Exchange Rates and Financial Fragility. *NBER* working paper 7418.

[18] Eichengreen and Mody 2001. Bail-Ins, Bail-Outs and Borrowing Costs, *IMF Staff Papers* 47, 155-187.

[19] Eichengreen, B., J. Tobin and C. Wyplosz. 1995. Two Cases for Sand in the Wheels of International Finance. *The Economic Journal* 105: 162–172.

[20] Harsanyi, J. 1967. Games with Incomplete Information Played by 'Bayesian' Players, I: The Basic Model. *Management Science* 14: 159-82.

[21] Holmstrom, B. and J. Tirole 1996. Modeling Aggregate Liquidity. *American Economic Review* Papers and Proceedings 86: 187–91.

[22] Holmstrom, B. and J. Tirole 1998. Private and Public Supply of Liquidity. *Journal of Political Economy* 106: 1–40.

[23] Kaminsky, G. and C. Reinhart 1999. The Twin Crises: The Causes of Banking and Balance-of-Payments Problems. *American Economic Review* 89: 473–500.

[24] Krugman, P. 2000 (ed.) *Currency Crises*. NBER and University of Chicago Press.

[25] Mella-Barral, P. and W. Perraudin 1997. Strategic Debt Service. *Journal of Finance* 50(2).

[26] Mody, A. and Saravia, D. 2003. Catalyzing Private Capital Flows: Do IMF-Supported Programs Work as Commitment Devices?, *IMF Working Paper* WP/03/100.

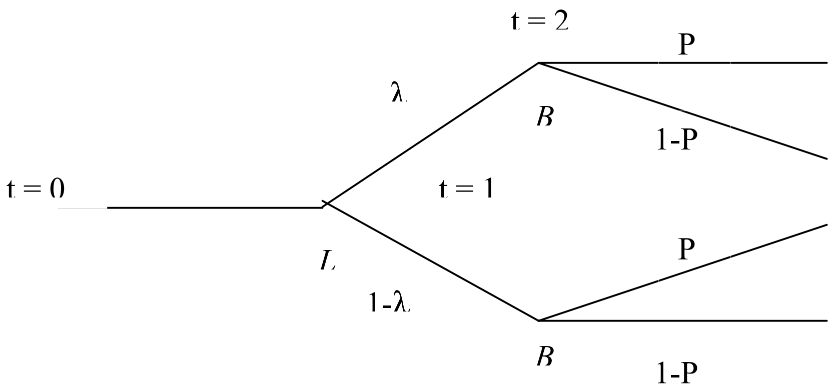
[27] Obstfeld, M. 1998. The Global Capital Market: Benefactor or Menace?, *Journal of Economic Perspectives* 12, 9-30.

[28] The New Basel Capital Accord, *Bank of International Settlements* Consultative Document, Basel Committee on Banking Supervision, 1/2001.

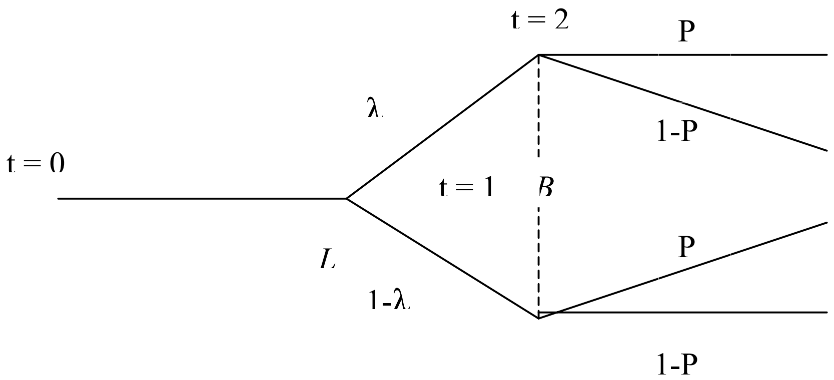
[29] Tirole, J. 2002. Inefficient Foreign Borrowing: A Dual- and Common-Agency Perspective. *IDEI* working paper.

FIGURE 1
The extensive form games*

A. Complete and perfect information



B. Complete and imperfect information



* The dotted lines link decision nodes that are not singletons. The payoffs at *t*=2 are given in Table 1.