Does Centralization Affect the Number and Size of Lobbies?*

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Abstract

Previous research has shown that if countries "merge", (i.e. move to centralized policy choices) the effect is to reduce lobbying. However empirical evidence suggests that this is not always the case. This paper attempts to explain the empirical evidence in a two-jurisdiction political economy model of endogenous lobby formation and policy determination. We measure lobbying in two ways:(i) the number of lobbies formed under the two settings and, (ii) their impact on policy decisions. We show that preference heterogeneity and lobby formation are positively related and that moving from decentralization to centralization can affect both the number and the type of lobbies. Under decentralization, if lobbies form they will always have an effect on policy decision. Under centralization, if lobbies form, lobby competition may completely offset their influence on policy; however it is possible that the threat of lobbying may affect policy even when no lobby forms in equilibrium. Finally, when lobbying affect policy, the equilibrium policy is more moderate that the equilibrium policy without lobbying.

KEYWORDS: Lobby Formation, Common Agency, Pressure Groups, Centralization. *JEL* CLASSIFICATION: H23, H77, D72.

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1. Introduction

This paper studies the effect that policy centralization has on lobbying. Previous research has shown that if countries "merge", (i.e. move to centralized policy choices) the effect is to reduce lobbying. The reason for this result, known in the literature as *preference dilution effect*, is due to the fact that, given that preference heterogeneity increases under centralization, there is a smaller role in determining policy for politically important groups in each of the countries, and this renders decision making less responsive to factional interests, which dilutes the incentive to lobby, (see among others de Melo, Panagariya, Rodrik (1993)).This kind of argument was also used by Madison (1787) in The Federalist Papers in support of a "well constructed Union" of American States¹.

However empirical evidence suggests that this is not always the case: the US has very strong lobby groups at the federal level, and the number of registered lobbies at the European Union level has rapidly increased in the past recent years. Moreover, in a recent empirical study Fishman and Gatti (2002), Root (1999) and Treisman (1999) found evidence of a negative correlation between decentralization and corruption. In Italy, after the "tangentopoli"² political scandal erupted into national politics in 1992, the number of voters in favor of decentralization grew exponentially such that the Northern League, the newly born party whose main political manifesto was the division of Italy into three regions, gained more than 15% of votes in the North of Italy.

This paper attempts to provide an explanation for this evidence by developing a two-jurisdiction political economy model of endogenous lobby formation and public good provision under policy centralization and policy decentralization,³ where the public good provision choices can be affected by the pressure of endogenously formed lobbies.⁴ In particular we address the following questions. Are citizens more likely to organize a lobby if policy decisions are taken at a central or local level? And, once a lobby exists, in which case does it have more influence on policy? Moreover, is the lobby's size affected by the degree of centralization of the government in charge of the policy? As far as we know this is the first attempt to investigate the relationship between centralization and lobby formation despite the fact that this is an important political issue.

¹The smaller the society, the fewer probably will be the distinct parties and interests composing it; the fewer the distinct parties and interests, the more frequently the majority be found of the same party; and the smaller the number of individuals composing a majority, and the smaller the compass within which they are placed, the more easily will they concert and execute their plans of oppression. Extend the sphere, and you take in a greater variety of parties and interests; you make it less probable that a majority of the whole will have a common motive to invade the rights of the other citizens; or if such a common motive exists, it will be more difficult for all who feel it to discover their own strenght and to act in unison with each other (Hamilton and others, (1787), p.22).

²Translation from italian: town of bribes.

 $^{^{3}}$ The role played by the level of government responsible for a policy decision in affecting the policy outcome has been broadly studied since Oates (1972). Recent works on centralization and policy outcomes in a political economy framework are due to Besley and Coate (1998) and Lockwood (2002).

 $^{^{4}}$ The role of lobbies in affecting policy outcome has been recognized both by political scientists and economists and it has lead to a vast literature. Recently economists have started investigating the process of lobby formation (Felli and Merlo (2000), Mitra (1999), Leaver and Makris (2000) and its relationship with the political process (Felli and Merlo, (2000) Besley and Coate (1999)).

We model the political process following a simplified version of the citizen-candidate approach due to Besley and Coate(1997), and Osborne and Slivinski (1996), where policy makers are elected citizens who select the preferred policy choice that maximizes their utility. We describe lobbies' behavior using the menu-auction⁵ model of Bernheim and Whinston (1986) and Dixit, Grossman and Helpman (1997). However this paper differs from these models in some respects. First, the citizen-candidate model assumes that the first stage of the political process is the entry of the candidates before elections; we do not model candidate entry but we assume that there is an exogenous set of candidates available. Second, the citizen candidate model assumes that citizens vote strategically for their preferred candidate; here the assumption is that they vote sincerely. Third, the menu auction approach models the activity of exogenous lobby groups that try to influence the policy choice toward their preferred policy choice by offering contributions to the policy-maker; we do not take the lobbies as given but we model a lobby formation stage⁶.

Our model is very stylized; there are two jurisdictions, two types of citizens with heterogeneous preferences in each jurisdiction, and one type of citizen is common to both jurisdictions. Policy decisions can be centralized or decentralized. In the first case there exists only one government elected by residents in the two jurisdictions; in the latter case each jurisdiction selects a government which decides the policy independently from the other government. After elections determine the identity of the policy-maker (the government), citizens may form a lobby with citizens of their group and "bribe" the policy maker.

We measure the extent of lobbying in two ways: i) the number of lobbies formed under centralization and decentralization; ii) their impact on the policy decision; and we compare the outcomes under policy centralization and policy decentralization.

We show that lobbies are more likely to form when preference heterogeneity among groups is high and when minorities are large. Both results seem to support the idea that lobbying is higher under centralization where we expect both higher preference heterogeneity and groups of bigger size. However, it is also true that lobbying is less likely to form when citizens are divided into many different groups. We think that these aspects should make lobbying less likely to occur under centralization, where it is more likely that population is more fragmented. Moreover, we show that it is more likely that lobbying will be able to influence policy decisions more under decentralization, where it is easier for an extreme group to be more influent.

Three main results emerge from our analysis. The first result is that lobbying matters. Compared to the benchmark case, the version of the model without lobbying, the identity of the policy maker and the policy outcome can be different from what we observe in a model where lobbying is not taken into account, *even* when we have equilibrium with no lobbies.

The second result is that centralization matters. Lobbying is affected by the level of government who decides the policy. However the effect is *ambiguous*, contrary to the idea of a preference dilution effect;

⁵Felli and Merlo (1999) use the citizen- candidate model to explore lobby formation. However the focus of their paper is on the bargaining process between lobbyists and policy maker and its consequences on policy outcomes.

 $^{^{6}}$ Besley and Coate (1999) study the impact of lobbies on political competition and policy outcome combining the citizencandidate model with the menu-auction model, but in a model with endogenous entry of candidates and with respect to the central level of government only.

we present examples where lobbying is higher under centralization and examples where the opposite is true.

The third result is that the extent os lobbying depends on the type of measure used to compare centralization and decentralization. For example we present scenarios where centralization determines higher lobbying from the point of view of resources spent by the lobbyists, but the effects on policy are smaller, and where the opposite is also true.

The paper is organized as follows. The first section briefly describes the economic environment and the model. The second section presents a simplified version of the model where lobbying is not taken into account, we will refer at this case as the benchmark. In the third section we present and discuss the three definitions of lobbying we use, followed by the analysis. Conclusions and possible extensions are in the last part of the paper.

2. The Model

There are two jurisdictions A and B with the same population size n. Residents are identical in their income (normalized to unity) and consume a private good and a public good or service but they differ with respect to their preferences over the public good level provided by the government, as described below.

Output, Y, is identical in each region and is produced from labor, which is inelastically supplied by each individual in an amount equal to unity. The production technology is assumed to be linear in total labor inputs, and without loss of generality, units are normalized so that the wage rate is unity. It follows that Y = n.

Output is used for private consumption and for the provision of the public good. The marginal rate of transformation between private consumption and the public good in production is assumed to be, without loss of generality, equal to unity.

With decentralization, provision of the public good in each region, g_k , k = A, B, is funded by a proportional income tax levied at rate t_k , which is assumed to be the only fiscal instrument available.⁷ The level of private consumption for an individual residing in jurisdiction k is then $p_k^i = 1 - t_k$, k = A, B, and public good provision $g_k = t_k n$. With centralization, $t = t_A = t_B$, so $g = g_A = g_B = 2tn$.

Each citizen i has quasilinear preferences over private consumption x_k^i and public good g_k of the form

$$u(p_k^i, g_k) = x_k^i + h(\theta^i, g_k), \quad k = A, B.$$
 (2.1)

⁷Although our model accounts for preference heterogeneity, preferences are unobservable and thus taxes cannot be conditioned on them, even though policymakers may have full information about the distribution of preferences.

where h(.) is strictly concave and single peaked. The θ term is a public good preference parameter defined in [0, 1], with citizens with higher $\theta's$ having higher valuations of the public good. For convenience to the model, in what follows we will assume $h(\theta^i, g_k) = \theta^i \ln g_k$, k = A, B.

There are three types of citizens $\omega = L, M, H$. If a citizen *i* has preference type ω , his valuation of the public good is $\theta^i = \theta_\omega$ with $\theta_L < \theta_M < \theta_H$. Note that we use superscripts to refer to the preference parameters of individuals, and subscripts to refer to the preference parameters of types. Let the set of citizens of preference type ω be N_ω , and let the number be $\#N_\omega = n_\omega$.

In each jurisdiction two types of citizens reside, L and M in A, and, M and H in B. The reason why we introduce this assumption is that we want to pick up the fact that if the two jurisdictions decide to centralize policy making decision by forming a union, the outcome will be a jurisdiction of bigger size and higher preference heterogeneity. In jurisdiction A there are therefore n_L citizens of type L and $n_M(A)$ citizens of type M and, in jurisdiction B, n_H citizens of type H and $n_M(B)$ citizens of type M. Therefore in the case of centralization, the union will be formed by n_L citizens of type L, $n_M = n_M(A) + n_M(B)$ citizens of type M and n_H citizens of type H.

The level of the public good is chosen by a policy maker elected by plurality voting over a set of candidates in which each type of citizen is represented. All citizens have a vote they may use for one of the candidates, the candidate with most votes is the winner. Moreover in the event of ties, the winner is randomly chosen from among the candidates with most votes. In the case where no candidate gets at least one vote the default policy zero public good is selected. Elections are local or central depending on the level of government responsible for the policy. If the policy choice is decentralized each jurisdiction elects a policy maker; if it is centralized the two jurisdictions elect a common representative.

Unlike Besley and Coate (1997) and Osborne and Slivinski (1996), we do not model candidate entry but we assume that there is a single candidate for each type of citizen.⁸ Simultaneously and independently citizens vote sincerely for the candidate who maximizes their utility, anticipating her policy behavior, and anticipating lobbying activities. We denote by P the preference type of the winner, where $P_A \in \{L, M\}$ and $P_B \in \{M, H\}$, under decentralization and $P \in \{L, M, H\}$ under centralization. In making their voting decisions, citizen *i* chooses P so as to maximize his expected utility, anticipating lobbying and policy choice.

Once elected, the winning candidate can be lobbied by *endogenously* determined groups formed by citizens with the same preferences. The lobbying activity in this model is the exercise of political influence over government's economic policy decision through contributions after elections. We now describe the lobbying stage in more detail.

After elections, all non-elected citizens simultaneously decide whether or not to become a member of a lobby with citizens having the same preferences, i.e. of the same preference type. We can think of this stage as a subscription stage, where each citizen simultaneously offers his contribution schedule and agrees on equally sharing part of the fixed cost K with the other citizens of the same type. At the end of

⁸This is possible, for example, if we assume the the cost of becoming a condidate is very low.

this stage each citizen can observe the subscriptions by the other members and decide accordingly. We assume that a lobby is formed if all non-elected citizens of the same group have decided to contribute and not otherwise. This rule is assumed to overcome the well-known free rider problem with lobby formation⁹.

By definition, with centralization, there are three possible lobbies, l = L, M, H, and with decentralization, there are four possible lobbies overall, two in each jurisdiction, l = L, M in A, and l = M, H in B. Given the lobby formation rule above, the members of the lobby l are the set $S_l = N_l/\{P\}$, where Pis the policy maker. So, if a lobby forms, its size $s_l = \#S_l$ is equal to n_l if the policy maker is not a type θ_l and $n_l - 1$ otherwise.

At the lobbying stage, every non-elected citizen i can choose to make a contribution to the lobby l that represents his preference. This contribution is the sum of a contribution schedule which depends on the tax, $c^i(t) \ge 0$, and a share of the fixed cost, K/s_l . The citizen can also choose not to lobby by making an overall contribution of zero. Define the indicator $\lambda^i = 1$ if i chooses to lobby, and $\lambda^i = 0$ otherwise.

We can now state the lobby formation rule more formally. If for all $i \in N_L/\{P\}$, $\lambda^i = 1$, then lobby L forms, and similarly for lobby H. For lobby M, we need to distinguish centralization and decentralization. In the first case, citizens of type M can form only a single lobby overall; in the second case, citizens of type M can form a lobby in each jurisdiction, depending on their residence, therefore there will be two possible lobbies of type M. So, if lobby l forms, we set an indicator variable $\lambda_l = 1$: otherwise, $\lambda_l = 0$. Also, for future reference, define $\Lambda = \{l \in \{L, M, H\} | \lambda_l = 1\}$, to be the set of lobbies that form in the case of centralization and similarly, let $\Lambda_A = \{l \in \{L, M\} | \lambda_l = 1\}$, $\Lambda_B = \{l \in \{M, H\} | \lambda_l = 1\}$ be the set of lobbies that form in jurisdictions A and B in the case of decentralization.

If an agent $i \in N_l$ makes contribution $c^i(t) + \frac{K}{s_l}$ to lobby l, his utility is given by

$$H^{i}(t) = 1 - t + \theta^{i} \ln 2tn - \lambda_{l} [c^{i}(t) + \frac{K}{s_{l}}] \qquad \text{(centralization)} \\ H^{i}(t_{k}) = 1 - t_{k} + \theta^{i} \ln t_{k}n - \lambda_{l} [c^{i}(t_{k}) + \frac{K}{s_{l}}] \qquad \text{(decentralization)}$$
(2.2)

Once formed, the lobby's objective function is the maximization of the sum of utilities of its members, $\sum_{i \in S_l} H^i$, which, conditional on a given tax, t or t_k , is, using (2.2):

$$H_l(t) = n_l(1-t) + \sum_{i \in S_l} \theta^i \ln 2tn - C_l(t) - K \quad \text{(centralization)} \\ H_l(t_k) = n_l(1-t_k) + \sum_{i \in S_l} \theta^i \ln t_k n - C_l(t_k) - K \quad \text{(decentralization)}$$
(2.3)

where $C_l(t) = \sum_{i \in S_l} c^i(t)$ and $C_l(t_k) = \sum_{i \in S_l} c^i(t_k)$, are the total payment functions offered by lobby *l*.

Finally, in the third stage of the game, the policy makers, P or P_k , implement their favorite policy choices, $t(P, \Lambda)$, in the case of centralization, or $t_k(P_k, \Lambda_k)$, in the case of decentralization, by maximizing their utility functions, taking $C_l(t)$ or $C_l(t_k)$ as given. The policy-maker's utility is

$$H_P(t) = 1 - t + \theta_P \ln 2tn + \sum_{l \in \Lambda} C_l(t) \qquad \text{(centralization)} \\ H_{P_k}(t_k) = 1 - t_k + \theta_P \ln t_k n + \sum_{l \in \Lambda} C_l(t_k) \qquad \text{(decentralization)}$$
(2.4)

⁹Other lobby formation rules, we believe, would lead to broadly the same results as ours. In the absence of some rule, and when the fixed cost of lobby formation is large enough, the free-rider problem is so severe that lobbies never form in equilibrium, an uninteresting possibility (See Leaver and Makris(2000) for more discussion on this point).

In solving the model with centralization and decentralization we proceed backwards. We first characterize the last stage of the game, policy selection, we then characterize lobbying and election.

3. The Benchmarks

In this section we present two versions of the model to use as a benchmark. We first consider the case where lobbying is not taken into account, and we extend the analysis to the case where lobbies are exogenous.

3.1. Political Equilibrium without Lobbying

With no lobbying, our model simply becomes a two stage model of election and policy determination. In the first stage, elections determine the identity of the policy maker, who decides the policy in the following stage. We solve this model under decentralization and under centralization and we will use these results to compare the outcome when we introduce lobbying. In what follows, when the same result applies for decentralization, we report the result for centralization only.

Since there is no *ex-ante* policy commitment, the preferences of the elected policy makers will determine the policy. A type θ_{ω} candidate chooses the public good level that maximizes her utility function, which, using (2.4), and setting $C_l(t) = 0$, and K = 0 is easily calculated to be $t = \theta_{\omega}$.

Moving to the voting stage, we assume that citizens vote sincerely. If each group size differs of more than 1 citizen, i.e. $|n_{\omega} - n_{v}| > 1$, all $\omega, v \in \{L, M, H\}$, in the case of centralization, and $\omega, v \in \{L, M\}$ in jurisdiction A and $\omega, v \in \{M, H\}$ in jurisdiction B in the case of decentralization; it can be shown that sincere voting is an equilibrium strategy for every citizen in the plurality voting game.¹⁰ We will make this assumption in what follows. So, the outcome must be that in any jurisdiction, the elected policy-maker has the preferences of the largest group in that jurisdiction.

Let the type of the majority group be m in the case of centralization (for example, if $n_L > n_M, n_H$ then m = L), and let the type of the majority group in jurisdictions A, B be m(A), m(B) respectively. The above results are summarized in the following lemma:

Lemma 1. When lobbying is not possible, each jurisdiction elects the representative having the same preferences as the majority group (m in the case of centralization, m(A), m(B) in the case of decentralization) who implements her favorite policy choice $(t^* = \theta_m \text{ in the case of centralization}, t_k^* = \theta_{m(k)}, k = A, B$ in the case of decentralization).

3.2. Political Equilibrium with Exogenous Lobbying

We now consider the case where lobbies exist but are exogenous. Of course there are several combinations we could examine, like one, two, three or four lobbies, and also which groups lobby etc. However, we will

¹⁰This guarantees that no citizen's vote can be pivotal, and therefore there is no incentive to deviate from sincere voting.

focus on the case where every citizen group lobbies, i.e. *full-lobbying* scenario. Therefore there will be four lobbies overall under decentralization and three under centralization. The reason is that in this way we represent the opposite case to the no-lobbying benchmark analyzed above, and so it will make sense to compare endogenous lobbying with these two extreme cases.¹¹

The political process has now three stages. First, citizens elect policy makers. Second, lobbies offer their contributions to the policy makers. Third, the policy makers implement their policy.

In order to characterize the equilibrium of the policy selection game, we follow Bernheim and Whinston (1986) Dixit, Grossman and Helpman (1997) and Besley and Coate (2001) in assuming that each lobby chooses a payment schedule that maximizes the utility of their members, taking as given the payment schedules offered by other lobbies and anticipating policy maker's policy choice. Since we can have multiple equilibria, we focus on truthful equilibria.¹²

In truthful equilibrium, the aggregate contribution by a lobby l, conditional on a choice of tax t by the policy-maker P, and the identity of P, exactly compensates the agent (the policy maker P) for every change in the tax by exactly the amount of change in the principal's welfare, provided that the payment both before and after the change is strictly positive. We write this aggregate contribution in the case of centralization $C_l(t; P)$, and the formula is

$$C_{l}(t,P) = [t - t_{-l} + \theta_{P} \ln(t_{-l}/t)] + \sum_{\omega \in \{L,M,H\}/\{l\}} [s_{\omega}(t - t_{-l}) + \theta_{\omega} \ln(t_{-l}/t)], \qquad (3.1)$$

and is derived in the Appendix. Note that t_{-l} is the *tax that would be chosen by the policy-maker if all lobbies but l form.* These contributions compensate the policy maker for the loss of utility due to (i) moving away from her ideal point, which corresponds to the first expression in brackets of (3.1), and/or (ii) the loss of the contributions paid by the other lobbyists, the second expression in brackets of (3.1). A similar formula applies in the case of decentralization.

Given these contributions (3.1) the elected policy maker selects her preferred policy among the set of feasible policies by maximizing her utility function, defined in (2.4). As we have quasi-linear preferences, the policy outcome is a weighted average between policy makers and lobbyists preferences (Dixit, Grossman and Helpman (1997)).

With full lobbying, since here every citizen is either a policy maker or a lobbyist, the policy outcome will correspond to the social optimum; which is

$$t^{**} = \frac{\sum_{\omega \in N} n_{\omega} \theta_{\omega}}{2n}, \ t_k^{**} = \frac{\sum_{\omega \in N(k)} n_{\omega} \theta_{\omega}}{n}$$
(3.2)

in the case of centralization and decentralization respectively. So, no matter who is elected, the policy outcome, will be always the same, and equal to the social optimum.

¹¹ Moreover, it turns out that when the fixed cost K of lobbying is zero, all lobbies will form in equilibrium with endogenous lobbying. So, one possible interpretation of full lobbying is that it is a special case of endogenous lobbying with zero lobbying costs.

¹²These equilibria always exist and have the characteristics of being both efficient and coalition proof Dixit, Grossman and Helpman (1997), p.759.

Turning to the election stage, citizens vote for the candidate who, after election, maximizes their utility. In the case of centralization, given equilibrium taxes, and the fixed set of lobbies, the payoff¹³ of a citizen i of type ω is

$$H^{i}(P) = 1 - t^{**} + \theta_{\omega} \ln 2t^{**}n - \frac{1}{s_{l}}[C_{l}(t^{**}, P) + K]$$
(3.3)

where we have used (2.4) and (3.2).

Since the policy choice is independent on the identity of the policy maker (from (3.2)), it is clear from (3.3) that a citizen of type ω votes for the P who is the "cheapest" to buy, i.e. the candidate that minimizes the amount of contributions to pay, $C_{\omega}(t^{**}, P)$. Define $C_{\omega}(t^{**}, P(\omega)) \leq C_{\omega}(t^{**}, P)$, all P = L, M, H: this $P(\omega)$ is unique, given any fixed ω . In general, $P(\omega)$ need not be the same as ω . Now recall that $m \in \{L, M, H\}$ is the majority preference group in the population i.e. $n_m > n_{\omega}, \omega \neq m$. Then it is clear that a citizen of preference type P(m) is elected. This means that the majority group is still able to elect its preferred candidate, who is not necessarily the one sharing their *ex ante* policy preferences. [This is because if citizens of type m vote for a candidate of type m, the lobby of type m will lose a member and therefore the other members will have to bear higher costs.] However in the next Lemma we show that under full lobbying the majority group will always prefer a candidate with its *ex ante* policy preferences.

Lemma 2. Under full lobbying, the policy outcome is independent on the policy maker type and correspond to the social optimum, t^{**} and t_k^{**} , and policy makers of type P(m)=m and P(m(k))=m(k) are selected.

3.3. Preference Dilution Effect with Exogenous Lobbying

According to the preference *dilution effect* the extent of lobbying increases with decentralization because there is a larger role for politically motivated groups to influence policy; while under centralization groups with opposite interests tend to counteract each other influence. We can now try to establish this result using our lobbying set up, by comparing the policy outcome in the exogenous lobbying model with the benchmark without lobbying.

We have shown, in Lemma 1, that the policy choice in the benchmark without lobbying depends only on preference type of the majority group (m) and (m(k)), and, in Lemma 2, that under *full lobbying* the policy choice is independent on the identity of the policy maker and corresponds to the social optimum. It easy to calculate the policy deviation between the benchmark without lobbying and the benchmark with *full lobbying*, which under centralization is

$$\Gamma^{c} = |t^{*} - t^{**}| = \frac{\left|\sum_{\omega \neq m} n_{\omega} \left(\theta_{m} - \theta_{\omega}\right)\right|}{2n}$$
(3.4)

 $^{^{13}}$ In this setting, since the lobbies exist already the fix cost of lobby formation is set K=0.

and under decentralization is

$$\Gamma^{d} = \sum_{k=A,B, v \neq m(k)} |t_{k}^{*} - t_{k}^{**}| = \sum_{k=A,B, v \neq m(k)} \left| \frac{n_{v}\left(k\right)\left(\theta_{m} - \theta_{v}\right)}{n} \right|, \ k = A, B$$
(3.5)

Lemma 3. With exogenous full lobbying, if the majority groups are the group with moderate preferences for the public good (i.e. m(A)=m(B)=m=M), the policy outcome is affected more under decentralization than under centralization; the direction of the effect is ambiguous otherwise.

So we see that under *full lobbying* the predictions of the preference dilution effect are only confirmed for the case where the majority groups are moderate. In all the other possible combinations it is possible that centralization will increase the effect that lobbies excise on policy.

Moreover, previous studies, like de Melo, Panagariya, Rodrik (1993), in order to illustrate *the preference dilution effect* analyze only the special case of one lobby in each jurisdiction, without any, even informal, discussion about the case of more than one exogenous lobby under decentralization.

In addition, without a model of lobby formation, it is not possible to capture important aspects of lobbying and answer other questions like, how does lobby composition change by moving to centralized policy choice? Does the number of organized groups increases or decreases? In the remaining of the paper we endogenize the lobbying formation and we address these additional issues, by considering different aspects of lobbying.

4. Endogenous Lobbying

4.1. Definition of Equilibrium

We proceed by first describing the lobbying and tax choice. After elections, all non-elected citizens in lobby l decide whether or not to lobby the policy maker by offering contributions $C_l(t, P, \Lambda)$, to move her preferred policy choice toward their preferred policy, where

$$C_l(t, P, \Lambda) = t - t_{-l} + \theta_P \ln(t_{-l}/t) + \sum_{\omega \in \Lambda/\{l\}} s_\omega(t - t_{-l}) + \theta_\omega \ln(t_{-l}/t)$$

As in the benchmark with full lobbying, for any given lobby, the policy maker P maximizes her utility, defined in (2.4), given $C_l(t, P, \Lambda)$, which yields a policy choice under centralization and decentralization respectively of:

$$t(P,\Lambda) = \frac{\theta_P + \sum_{l \in \Lambda} s_l \theta_l}{\sum_{l \in \Lambda} s_l + 1}, \ t_k(P_k,\Lambda_k) = \frac{\theta_P + \sum_{l \in \Lambda_k} s_l \theta_l}{\sum_{l \in \Lambda_k} s_l + 1}, \ k = A, B$$
(4.1)

Compared to (3.3), now t depends on P, Λ . When there are no lobbies ($\Lambda = \emptyset$), $t(P, \Lambda) = \theta_P$; when there are one or more lobbies the policy choice is an average between policy maker and lobbyists' preferences.

Now consider the lobby formation stage. From (2.2), the payoff to a type ω if lobbies Λ form is, under centralization,

$$H_{\omega}(P,\Lambda) = 1 - t(P,\Lambda) + \theta_{\omega} \ln(t(P,\Lambda)2n) - \frac{1}{s_{\omega}} [C_{\omega}(P,\Lambda) + K]$$
(4.2)

If lobby ω does not form, a type ω gets instead an utility equal to

$$H_{\omega}(P,\Lambda/\{\omega\}) = 1 - t(P,\Lambda/\{\omega\}) + \theta_{\omega} \ln\left(t(P,\Lambda/\{\omega\})2n\right).$$

$$(4.3)$$

So, the gain to forming a lobby is $\Delta_{\omega}(P,\Lambda) = H_{\omega}(P,\Lambda) - H_{\omega}(P,\Lambda/\{\omega\})$, which, after simplifications, becomes:

$$\Delta_{\omega}\left(P,\Lambda\right) = \frac{\sum_{l\in\Lambda}s_l+1}{s_{\omega}}\left(t(P,\Lambda/\left\{\omega\right\}) - t(P,\Lambda)\right) + \frac{\sum_{l\in\Lambda}s_l\theta_l+\theta_P}{s_{\omega}}\ln\left(\frac{t(P,\Lambda)}{t(P,\Lambda/\left\{\omega\right\})}\right) - \frac{K}{s_{\omega}} \tag{4.4}$$

Condition (4.4) represents the payoffs difference for a citizen of type ω between forming a lobby for this group and not lobbying, given the identity of the policy maker, and given a set of lobbies $\Lambda / \{\omega\}$ formed by other citizens. So, if $\Delta_{\omega} (P, \Lambda) \geq 0$, lobbying is preferred to non-lobbying by citizens of type ω , given that other lobbies $\Lambda / \{\omega\}$ have formed.¹⁴ This motivates the following definition of an equilibrium set of lobbies, given a policy-maker P.

Definition 1. With centralization, $\Lambda^*(P)$ is an equilibrium set of lobbies, given P, if (i) $\Delta_{\omega}(P, \Lambda^*(P)) \ge 0$, all $\omega \in \Lambda^*(P)$, and (ii) $\Delta_{\omega}(P, \Lambda^*(P) \cup \{\nu\}) < 0$, all $\nu \notin \Lambda^*(P)$. With decentralization, $\Lambda^*_k(P)$ is an equilibrium set of lobbies, given P_k , if (i) $\Delta_{\omega}(P_k, \Lambda^*_k(P_k)) \ge 0$, all $\omega \in \Lambda^*_k(P_k)$, and (ii) $\Delta_{\omega}(P_k, \Lambda^*_k(P_k) \cup \{\nu\}) < 0$, all $\nu \notin \Lambda^*_k(P_k)$.

This is, lobbying choices are Nash equilibrium ones: at the equilibrium, those who lobby are better off lobbying than not, given the decisions of others, and vice versa.

At the first stage of the game, each citizen makes his voting decision voting sincerely among the set of candidates, anticipating lobbying and policy choice. If we substitute the equilibrium set of lobbies $\Lambda^*(P)$ into $H_{\omega}(P,\Lambda)$ in (4.2) we get the following indirect utility function dependent only on P:

$$H_{\omega}(P) \equiv H_{\omega}(P, \Lambda^*(P)) \tag{4.5}$$

So, now sincere voting means that a citizen of type ω will vote for the P that maximizes $H_{\omega}(P)$: let this be $P(\omega)$. In general, $P(\omega)$ need not be the same as ω . Recall that $m \in \{L, M, H\}$ is the majority preference group in the population i.e. $n_m > n_{\omega}, \ \omega \neq m$. Then it is clear that a citizen of preference type P(m) is elected.

¹⁴If we set $\Lambda = \{\omega\}$, (4.4) becomes the condition for a citizen of type ω to form a lobby when no other group lobbies. If Λ includes types other than $\{\omega\}$ (4.4) becomes the lobbying condition for a citizen of type ω , given that other lobbies exist. On the other hand, if $\Delta_{\omega} (P, \Lambda \cup \{\nu\}) < 0$ for some type ν , then citizens of that type do not want to form a lobby given that lobbies Λ already exist.

We are now in a position to define an equilibrium in the game as a whole.

Definition 2. An equilibrium with endogenous lobbying is a triple $\{P^*, \Lambda^*(P^*), t(P^*, \Lambda^*(P^*))\}$ such that (i) $P^* = P(m)$ is the type of the elected policy-maker; (ii) $\Lambda^*(P^*)$ is the set of lobbies that form, given the policy-maker's type, and $t(P^*, \Lambda^*(P^*))$ is the tax chosen by the elected policy-maker.

Note that when the possibility of forming lobbies is introduced, P(m) may not be equal to m. Allowing for lobbying gives citizens, who are not happy with the electoral result, another instrument: to form a lobby and "pay" the policy maker in order to move her policy choice closer to their ideal point. However this action is anticipated by the other citizens who can react in different ways: i) to form a lobby themselves, and/or ii) vote for another candidate in the first stage of the game. This means that the majority group is still able to elect its preferred candidate, who is not necessarily the one sharing their *ex ante* policy preferences.

We have now succeeded in defining an equilibrium with endogenous lobbying for our model. In the next section, we characterize this equilibrium. Before we do so, however, we should note two possible complications. The first is that is, conditional on P^* , it is possible to have several equilibrium sets of lobbies in the resulting subgame.¹⁵ The second is that there may be no equilibrium set of lobbies in pure strategies, $\Lambda^*(P)$, for some P.¹⁶ However, the results stated below apply when there is at least one equilibrium in pure strategies of the lobbying subgame and hold for any selection of an equilibrium lobby sets.

5. Equilibrium Lobbying

5.1. Preliminary Results

We have already obtained a characterization of the equilibrium tax given P, Λ . The next step is to get a characterization of $\Lambda^*(P)$ in the case of both centralization and decentralization. A general characterization is not possible, but we can establish conditions under which citizens of any given type ω want to lobby given a policy-maker P and given a fixed set of other lobbies. We only present this result for the case of centralization: a very similar result holds for decentralization.

First define $\Delta_{\omega}(\theta, \Lambda)$ which is the gain to lobbying by citizens of preference type ω , in the hypothetical situation that the policy-maker has a preference parameter θ (in fact, the preference type is fixed at

¹⁵

The following is a numerical example of multiple equilibrium where there are two Nash equilibria, one involving the formation of two lobbies by the minority groups, the other involving no lobby at all. Consider a jurisdiction in which the two minority groups ω and v, with preferences parameter $\theta_{\omega} = 0.3$ and $\theta_v = 0.9$, have a size of $n_{\omega} = 150$ and $n_v = 100$ respectively. Moreover, the majority group m has preference parameter $\theta_{\omega} = 0.5$. To form a lobby costs an amount K = 15 unit for each lobby. Consider now the lobbying decision by type ω : if no other lobby exists: their gain is $\Delta_{\omega}(M,(\omega)) = -0.0536$, therefore they do not want to lobby alone. However if v types form a lobby, ω types gain becomes $\Delta_{\omega}(M,(\omega,v)) = 0.0383$. Now let us turn to the lobbying decisions by v types; their gain to lobby alone is $\Delta_v(M,(v)) = -0.0220$ and with lobby ω is $\Delta_{\omega}(M,(\omega,v)) = 0.0414$. Therefore it is clear that there exist two Nash equilibria of the lobbying subgame $\Delta(M) = \{\emptyset\}, \{\omega, v\}$.

 $^{^{16}}$ See the proof of proof or Proposition 2 in the Appendix for a characterization of these equilibria.

 θ_{ω}). That is:

$$\Delta_{\omega}(\theta, \Lambda) = \frac{\sum_{l \in \Lambda} s_l + 1}{s_{\omega}} \left(t(\theta, \Lambda / \{\omega\}) - t(\theta, \Lambda) \right) + \frac{\sum_{l \in \Lambda} s_l \theta_l + \theta}{s_{\omega}} \ln \left(\frac{t(\theta, \Lambda)}{t(\theta, \Lambda / \{\omega\})} \right) + \frac{-\frac{K}{s_{\omega}}}{s_{\omega}}.$$

Also, allow θ to take any value. It is shown in the appendix that this function is convex in θ , and has a minimum value of $-K/s_{\omega}$ which is less than zero: this minimum value occurs at $\theta_{\min}(\omega, \Lambda) = \frac{\theta_P + \sum_{l \in \Lambda/\{\omega\}} s_l \theta_l}{\sum_{l \in \Lambda/\{\omega\}} s_l + 1}$. So, there must be two values of θ at which $\Delta_{\omega}(\theta, \Lambda) = 0$. Let these two values be $\underline{\theta}(\omega, \Lambda), \ \overline{\theta}(\omega, \Lambda)$ respectively with $\underline{\theta}(\omega, \Lambda) < \overline{\theta}(\omega, \Lambda)$. Then we have:

Proposition 1. Given any policy maker of type P, and other lobbies $\Lambda/\{\omega\}$, citizens of type ω wish to lobby iff $\theta_{\omega} < \underline{\theta}(\omega, \Lambda)$ or $\theta_{\omega} > \overline{\theta}(\omega, \Lambda)$. Moreover, $\underline{\theta}(\omega, \Lambda) < \theta_{\min}(\omega, \Lambda) < \overline{\theta}(\omega, \Lambda)$, so if there are no other lobbies, $\underline{\theta}(\omega, \{\omega\}) < \theta_P < \overline{\theta}(\omega, \{\omega\})$. Finally, $\underline{\theta}(\omega, \Lambda) \to \overline{\theta}(\omega, \Lambda)$ as $s_{\omega} \to \infty$, so if $\theta_{\min}(\omega, \Lambda) \neq \theta_{\omega}$, there exists \hat{s}_{ω} high enough so that for all $s_{\omega} > \hat{s}_{\omega}$, citizens of type ω wish to lobby.

The first part of the Proposition says lobby formation is determined both by lobbyists and policy maker preferences; the more ω types are far from either of them the more lobbying will occur. The second part of Proposition 1 says that as preference heterogeneity between a group and the policy maker increases the likelihood for citizens of type ω to form a lobby increases, if no other group lobbies. Finally, the third part of the Proposition says that bigger groups are more likely to organize a lobby, because the costs of forming a lobby are shared between more members.

5.2. Strongly Lobby-Free and Non-Lobby-Free Equilibria

We begin by establishing conditions for an endogenous lobbying equilibrium not to be affected by the introduction of a lobby formation stage. Say that the equilibrium is *strongly lobby-free* if the equilibrium is *exactly the same* as in the model without lobbies. That is, in equilibrium, (i) no lobby forms, and (ii) the elected policy-maker is of type m under centralization and of type m(k) under decentralization, i.e. the same type as in the model without lobbying.

Lemma 4. Under decentralization, the equilibrium is strongly lobby-free iff (i) $\Delta_{\omega}(m, \{\omega\}) < 0$ for $\omega \neq m$. Under centralization, the equilibrium is lobby-free iff (i) $\Delta_{\omega}(m, \{\omega\}), \Delta_{\omega}(m, \{\omega, v\}) < 0$, (ii) $\Delta_{v}(m, \{v\}) < 0$ for $\omega, \nu \neq m$, (iii) $\Delta_{n}(m, \{\omega, v, m\}) < 0$, for some $n \in \{\omega, v, m\}$.

So, we see that under decentralization, where only two types of citizens reside in each jurisdiction, the only condition for an equilibrium not affected by endogenous lobbying is that the minority group will not lobby alone, if the candidate of type m(k) is elected. The intuition for that is very simple, if citizens of the minority group do not want to pay the policy maker when there are no other lobbies, it would be inconsistent if they paid a bigger amount for a smaller policy deviation, if the majority group were induced to lobby. This rules out the possibility of a multiple equilibrium where there are either zero or two lobbies. Note that voting for the candidate with their *ex ante* policy preferences it is also the best voting strategy for the majority group if condition (i) holds, because the policy maker will just implement their ideal policy. So, if the above condition is satisfied, introducing the lobbying stage to the model will not affect the political equilibrium, i.e. the elected policy makers will be the same as in the benchmark, no contributions will be paid, and no lobbies will form.

Note that, under decentralization the above condition is necessary and sufficient for an equilibrium without lobbying. The reason for this is that the majority group will never prefer an equilibrium where the candidate of the minority group is elected, if no lobby exists. This equilibrium is always dominated by the one determined by the election of a candidate of the majority group, whatever the lobbying outcome associated with this voting equilibrium is.

Under centralization, instead, the conditions of Lemma 4 are only sufficient but not necessary for an equilibrium without lobbies. This is because we can not rule out the possibility of an equilibrium where a candidate of type $\omega \neq m$ is elected and no lobbies have formed. This is however possible only when the equilibrium of the lobbying subgame for a policy maker of type m involves an equilibrium with one or more lobbies. However, even if no lobby exist in equilibrium, the political equilibrium is affected because a different policy maker is elected compared to the benchmark without lobbying. In this case the threat of lobbying has affected the political equilibrium, and the policy choice is now the one preferred by the median group and not by the majority group.

In summary, adding a lobby formation stage to the model will not affect the political equilibrium only if groups have very close preferences over the public good and/ or minority groups are very small. So, when lobbying does affect political equilibrium, it is does so in one or both of two ways; either by affecting the policy choice of a given elected policy-maker, or affecting the identity of the chosen policy-maker.

Since we have already discussed the case where every group lobbies in the *full lobbying* section, from now on, we assume that, for at least one group of citizens, lobbying with the other two groups is never preferred to not lobbying. This rules out the possibility of an equilibrium with three lobbies under centralization. In formal way, this requires that $\Delta_n(m, \{\omega, v, z\}) < 0$, for some $n \in \{\omega, v, z\}$. Where $\Delta_n(.)$ is the lobbying- not-lobbying payoffs difference for a citizen of type $n \in \{\omega, v, z\}$ when the set of lobbies is $\{\omega, v, z\}$. This implies that the following condition must hold for at least one type of citizen.¹⁷

$$\frac{s_{\omega} + s_v + s_z + 1}{s_{\omega}} \left(\frac{\theta_v s_v + \theta_z s_z + \theta_P}{s_v + s_z + 1} - \frac{\theta_\omega s_\omega + \theta_v s_v + \theta_z s_z + \theta_P}{n} \right) +$$

$$+ \frac{\theta_\omega s_\omega + \theta_v s_v + \theta_z s_z + \theta_P}{s_{\omega}} \ln \frac{(s_v + s_z + 1) \left(\theta_\omega s_\omega + \theta_v s_v + \theta_z s_z + \theta_P\right)}{n \left(\theta_v s_v + \theta_z s_z + \theta_P\right)} - \frac{K}{s_{\omega}} < 0$$
(5.1)

Note that condition (5.1) must be positive or equal to zero in the exogenous lobbying case, which is always true when K = 0,¹⁸ for a proof. see the proof of Proposition 1.

Similarly we can derive the conditions for a political equilibrium affected by endogenous lobbying. We say that the political equilibrium is *not strongly lobby-free* if any Nash equilibrium in pure strategies

¹⁷For the calculation of the condition refer to the Appendix where we substitute the relevant $t(P, \Lambda)s$.

 $^{^{18}}$ K is zero in the exogenous lobby case, since it is assumed to reflect the cost of forming a lobby, such as gather information, held meetings with citizens etc.

of the lobbying subgame is with lobbies and or a candidate of type $P(m) \neq m$ is elected. The following result gives a necessary and sufficient condition for the equilibrium to be *not strongly lobby free*, and also gives some characterization of how the potential existence of lobbies impacts on the equilibrium.

Proposition 2. Any equilibrium is not strongly-lobby free iff $\Delta_{\omega}(m, \{\omega\}) \ge 0$, for at least an $\omega \ne m$. Assume that this condition holds, then, under decentralization, at least one lobby will form in equilibrium.

Under centralization:

a) If the majority group has moderate preferences (m = M), at least one lobby will form in equilibrium; b) if the majority group has extreme preferences (m = L, H), then it is possible that either no lobbies or at least one lobby will form in equilibrium.

It is interesting to note that, both under decentralization and under centralization (only when the majority group has moderate preferences), the conditions of Proposition 2 implicitly define the conditions for an equilibrium with lobbies, because voting for a candidate with ex ante policy preferences different form the majority group, when the associated lobbying outcome is $\Lambda = (\emptyset)$, is always strictly dominated by any other voting equilibria where the elected policy maker is of is of type m. This is because citizens of type m can always achieve a better outcome when a type m is elected. Therefore, whoever they vote for, there will always be at least one lobby.

Under centralization, instead, when the majority group has extreme policy preferences (either L or H), it is possible that a moderate candidate (of type M) is preferred to the candidate with their *ex ante* policy preference, if this voting equilibrium is associated with an equilibrium without lobbies. This is because citizens of type m vote for a candidate of type M, anticipating the formation of a hostile lobby if they vote sincerely. In this case the political equilibrium is affected by the threat of lobbying, not by the formation of lobbies but by affecting the voting stage. So, we have shown that, in a *not-lobby-free* equilibrium, even when no lobby forms, the majority extreme group either votes for a more centrist candidate, or vote for the candidate with their a priori policy preferences who is lobbied by the other extreme group.

In the next section we propose two ways of measuring the extent of lobbying and we define the neutrality conditions with respect to them.

6. Measuring the Extent of Lobbying

So far, we have just established when an equilibrium is *strongly lobby-free* or *not strongly lobby-free*. Now, we wish to investigate more how lobbying can be measured. Lobbying can be characterized in several ways; from the number of existing lobbies, to how much these lobbies have to pay the policy maker, from how they can affect policy choices, to how they can influence elections.

In particular in this paper we consider lobbying from two points of view described below. In this section we assume that the equilibrium is not strongly lobby-free in the sense of Lemma 3, and we define

the *neutrality conditions* with respect to the two ways we characterize lobbying.

6.1. The number of lobbies

The first way considers lobbying as the participation of citizens to the lobbying process, measured by the number of lobbies/lobbyists. From this point of view lobbying will increase under centralization if the overall number of lobbies and the number of their members rise. Using the analysis developed in the previous section we can derive the conditions under which no lobby will form in equilibrium, and we say that the equilibrium is *lobby-free* if, under centralization, $\Lambda^* (P^*) = \{\emptyset\}$, and, under decentralization, $\Lambda^*_k (P^*_k) = \{\emptyset\}$. So in a *lobby-free equilibrium* the only condition is that the political equilibrium is without lobbies. In addition, if a policy maker of type m is elected $(P^*(m) = m)$, the equilibrium is also strongly *lobby-free*, but not otherwise.

Proposition 3. Assume that the equilibrium is lobby-free (i.e. $\Lambda^*(P^*) = \{\emptyset\}$ and, $\Lambda^*_k(P^*_k) = \{\emptyset\}$). Then, under decentralization: the equilibrium is also strongly lobby-free, so a policy maker of type $\theta_{m(k)}$ is elected and $t^*_k(P^*,\phi) = t^*_k = \theta_m$. Under centralization: if the majority group has moderate preferences (m = M), the equilibrium is also strongly lobby free, i.e. $P^*(m) = \theta_m$; if the majority group has extreme policy preferences (m=L or H), the equilibrium can be not strongly lobby-free, and. $P^*(m) = m, M$. Assume that the equilibrium is not lobby-free and one lobby will form (i.e. $\Lambda^*(P^*) = \{\omega\}$ and $\Lambda^*_k(P^*_k) = \{\omega\}$). Then, both under centralization and decentralization, the lobby to form will never be the friendly one, i.e. the one formed by citizens sharing the same preferences as the policy maker, so $P^*(m) = P$ and $\Lambda^*(P^*) = \{\omega\}, \omega \neq P$.

So we see that , under centralization, when the majority group has extreme policy preference, it is possible that no lobby forms in equilibrium but a policy maker with moderate policy preferences is selected by the majority, so the equilibrium is *lobby free* but not *strongly lobby-free*. This result is driven by the same logic as in the Besley and Coate (2001) paper, where citizens vote strategically in order to ensure that hostile lobbies will have no effect on policy. However, if we add a lobby formation stage and we reduce the possible number of candidates, it is not always possible that the majority group is able to completely offset lobbying influence by strategic voting. So the majority group may have to compromise between minimizing its cost of lobbying and achieving a policy choice as close as possible to its ideal one.

Moreover, when only one lobby forms in equilibrium, it will always be the one opposing the policy maker. Note that the policy maker does not have necessarily to be of type m. This results is consistent with Austen-Smith and Wright (1992), where if a lobby forms, it will be the one which disagrees with the policy maker's *ex-ante* policy preferences, but is in contrast with Leaver and Makris (2000), where, because of free-riding within groups, the only possible lobby is the one sharing the same preferences as the policy maker.

6.1.1. Centralization and the Number of Lobbies

In this section we compare centralization and decentralization outcomes using our first definition of lobbying: the number of lobbies formed. We will develop some scenarios and we will discuss the relationship between the number of lobbies and the level of government in charge of the policy.

Proposition 4. Assume that the majority group under decentralization and under centralization is moderate (i.e. m=m(A)=m(b)=M)). Then, if centralization is strongly lobby-free so is decentralization; but not vice versa.

First consider the case where the equilibrium is strongly lobby-free, in the sense of Lemma 3, under decentralization. Therefore the political equilibrium is the same as the one depicted in the benchmark without lobbying; i.e. policy makers with preferences m(A) and m(B) are elected in equilibrium and will implement policy choices $t_A = \theta_{m(A)}$ and $t_B = \theta_{m(B)}$ respectively. Suppose now that the two jurisdictions decide to centralize and delegate to a common policy maker the policy decision. In order to make the analysis simpler we focus on the symmetric case where the majority groups in A and B are both either the extreme groups (L in A and H in B), or the group in the middle (M), and, under centralization, the majority group is always of type M. If m(A) = L and m(B) = H, it easy to show that if $\Delta_{\omega}(M, \{\omega\}) > 0^{19}$, for $\omega=L$ or H, i.e. the equilibrium is not lobby-free for Proposition 2, and centralization can lead to the formation of 1 or 2 lobbies. If m(A) = m(B) = M, there will be two lobbies in equilibrium iff $\Delta_L(M, \{H, L\}) > 0$, and $\Delta_H(M, \{H, L\}) > 0$, and zero otherwise. So, it possible that moving from decentralization to centralization determines a formation of lobbies from a strongly lobbyfree equilibrium. When the decision wether or not to centralize policy is taken, this must be taken into account, because not necessarily a strongly lobby free equilibrium can be supported under centralization.

Consider the opposite case of a *strongly lobby-free* equilibrium with centralization, where the majority groups are of type m = m(k), for k=A, B, and consider the decision to decentralize. It is easy to show that, the political equilibrium remains necessarily *strongly lobby-free*, this follows from Lemma 4. If instead $m \neq m(k)$, it is possible to have equilibrium with lobbies under decentralization, since the conditions for a *strongly lobby-free* equilibrium under centralization do not rule out the possibility of an equilibrium with lobbies under decentralization, from Lemma 4.

It is also interesting to consider the situation analyzed in the exogenous *full lobbying* scenario with respect to endogenous lobbying. Consider the case where preference heterogeneity among groups is very high such that under decentralization, every citizen group will form a lobby in A and in B, like under the full lobbying benchmark. If we move to centralization, and M = m, and the two extreme groups offset each others influence by lobbying, the majority group does not need to form a lobby to preserve its interest. In this situation, the majority group will be strictly better off under centralization, since its able to achieve its favorite policy choice without lobbying. A sufficient condition for this is that $\Delta_M(M, \{L, H, M\}) < 0$. So in this situation the number of lobbies has decreased from four overall to only two, and the majority group will benefit from centralization since, if the extreme group have the

¹⁹It easy to check that any Nash equilibrium in pure strategies implies at least one lobby will form.

same power, they can achieve their ideal policy choice without lobbying.²⁰ The overall welfare effect for the society is however ambiguous: on one hand, under decentralization the costs associated with lobby formation are higher because more lobbies will form, on the other hand, compare to the benchmark, the policy choice is the efficient one.²¹ However, when more than one lobby exists the policy maker is able to capture most of the surplus,²² and lobbyists may be worse off, instead.

So we have shown that the number of lobbies can either increase or decrease under centralization. Since the model is very general and allows both for different preference distribution between and within jurisdictions and preference heterogeneity across groups, we cannot establish a unique relationship between centralization and lobbies. However we can make some predictions: i) in general if centralization increases preference heterogeneity, (for sure preference heterogeneity will not decrease!), *ceteris paribus*, lobbying is more likely to occur; ii) if centralization increases the number of groups in the economy, lobbying is less likely to occur because citizens have more options at the election stage and prevent lobbying by voting for alternative candidates; finally, iii) if centralization affects groups size lobbying increases, given the assumption that the cost of lobbying constitution is fixed and equally shared among members.

6.2. The effect of lobbying on policy determination

The second way in which lobbying can be measured considers the difference between the policy choice determined under the benchmark without lobbying ($t^* = \theta_m$ and $t^*_k = \theta_{m(k)}$) and the policy choice determined under the lobbying model ($t^*(P^*, \Lambda^*(P^*))$) and $t^*_k(P^*_k(, \Lambda^*_k(P^*_k)))$). The next Proposition discusses the conditions under which the equilibrium is *policy-neutral*, i.e. the same policy as under the benchmark is selected ($t^*(P^*, \Lambda^*(P^*)) = \theta_m$ and $t^*_k(P^*_k(, \Lambda^*_k(P^*_k)) = \theta_{m(k)})$.

Proposition 5. Suppose that the equilibrium is policy-neutral $(t^*(P^*, \Lambda^*(P^*)) = \theta_m \text{ and } t^*_k(P^*_k(, \Lambda^*_k(P^*_k)) = \theta_{m(k)})$. Under decentralization the equilibrium is also lobby-free. Under centralization; a) if the majority group has moderate policy preferences (m = M) the equilibrium can be not lobby-free, and the political equilibrium can be either $\{P^*, \Lambda^*(P^*), t^*\Lambda^*(P^*)\} = \{m, \emptyset, \theta_m\}$, or $\{P^*, \Lambda^*(P^*), t^*\Lambda^*(P^*)\} = \{m, \{L, H\}, \theta_m\}$, if $n_L |\theta_L - \theta_M| = n_H |\theta_H - \theta_M|$; b) If the majority group has extreme policy preferences (m=L, H), the equilibrium is also strongly lobby free.

Suppose that the equilibrium is not lobby-neutral $(t^*(P^*, \Lambda^*(P^*)) \neq \theta_m \text{ and } t_k^*(P_k^*(, \Lambda_k^*(P_k^*)) \neq \theta_{m(k)})$. Under decentralization, at least one lobby has formed, and the equilibrium tax, $t_k^*(P_k^*(, \Lambda_k^*(P_k^*)))$, will differ from the one without lobbying and, there is always policy moderation i.e. in region A, $\theta_M > t_A(P_A^*, \Lambda_A^*(P_A^*)) > \theta_L$, and in region B, $\theta_M < t_B(P_B^*, \Lambda_B^*(P_B^*)) < \theta_H$.

Under centralization,a) If the majority group has moderate policy preferences (m = M), and i) one lobby forms in equilibrium, there is policy moderation and, in particular, $\theta_m > t^*(P^*, \{l^*\}) > \theta_l$, if

 $^{^{20}}$ Note that if we look at the result of the benchmark, the majority group is indifferent between centralization and decentralization, since they can always achieve their ideal policy.

 $^{^{21}}$ Given the notion of truthful equilibrium develop by Bernheim and Whinston (1986) the policy maker maximizes jointly her and lobbyists' utilities. It follows immediately that in a world where every individual lobbies the optimal provision of public is achieved and the society on the whole achieves the maximum level of utility feasible.

 $^{^{22}}$ See DGH for a discussion about that.

 $\theta_m < \theta_l$, or $\theta_l > t^*(P^*, \{l^*\}) > \theta_m$ if $\theta_m > \theta_l$, and ii) two lobbies form, there is policy moderation, unless $\Lambda^* = \{L, H\}$, and $n_L |\theta_L - \theta_M| = n_H |\theta_H - \theta_M|$.

b) If the majority group has extreme policy preferences (m = L, H), and no lobby have formed, the equilibrium tax, $t^*(P^*, \emptyset) = \theta_M$, so there is policy moderation and the equilibrium is lobby-free. If at least one lobby forms there is always policy moderation so $\theta_H > t^*(P^*, \Lambda^*(P^*)) > \theta_L$.

So we see that it is possible that, even if no lobby will form in equilibrium, the equilibrium policy is *not-policy neutral*, because a policy maker with policy preferences different from the majority group will be elected. This is the case where, under centralization, the majority group has extreme policy preferences and, in order to stop minority groups from lobbying, votes for the candidate with moderate policy preferences.

If, instead, lobbies form, the equilibrium is in general not-policy neutral. This is because, in a onelobby equilibrium, the lobby will not have an effect on policy only if it is formed by citizens sharing the same preference as the policy maker and the latter is of type m. However, we have ruled out that this equilibrium exists in pure strategies, both under centralization and decentralization. Under centralization and in a two-lobby equilibrium, the equilibrium may be policy neutral only in very special case. That is when the majority group has preference type M and that a policy maker of type M is lobbied by both extreme groups, which also must have the same power (i.e. $n_L |\theta_L - \theta_M| = n_H |\theta_H - \theta_M|$). In summary, the policy choice can be unaffected by lobbies only if preference heterogeneity between groups is very high, and the extreme groups are balanced. This is in contrast with the main result of Leaver and Makris (2000), where the only possible equilibrium with lobbies is the one where only the "friendly" lobby will form. In our model, this can only be an equilibrium in mixed strategies and can occur when the majority group is very large relatively to the minority and preference heterogeneity is not very high, such that it is too "expensive" for the minority to lobby when the majority does. Note that in this equilibrium the lobby will not pay any contribution because the policy maker implements her *a priori* policy choice, and the lobby does not affect this choice.

The question whether or not lobbying affects policy in equilibrium has been debated by several papers. On one hand, Besley and Coate (1999) and Leaver and Makris (2000) found that lobbying never affects policy. In the first case, Besley and Coate in their citizen-candidate model with exogenous lobbies, suggest that the reason is that citizens can predict lobbying activity and offset their influence by strategically voting for a candidate of a different type; of course this is possible when the set of possible candidates is wide enough for this choice. In the second case, the authors suggest that free-riding prevents lobbies from forming, so that no contributions are due in equilibrium, and this situation corresponds to the one where only the friendly lobby will form. On the other hand, Felli and Merlo (1999) argue with this result and demonstrate that lobbying always matters. They show that an "extremist" candidate is elected and implements a "centrist" policy, which differs from the median voter preferred outcome. In general, our results extend Felli and Merlo finding, by showing that in general the possibility of lobbying move the policy choice toward moderation, even when no lobby forms in equilibrium.

Another related question is whether or not lobbies pay any contribution in equilibrium. From our previous analysis, as a corollary of the results, it follows that

Corollary 1. If lobbies form they will always pay contributions in equilibrium.

This result is in contrast with Leaver and Makris' (2000) findings, where lobbies never pay contributions in equilibrium. This occurs because if lobbies will form, it can only be the friendly one, which, in absence of lobbies' competition, does not need to pay. In our model, however, the reverse is true: if a single lobby will form in equilibrium it will never be the one sharing the policy maker's same *ex-ante* policy preferences.

6.2.1. Centralization and Lobbying Influence on Policy

We now present two scenarios where moving from decentralization to centralization affect policy in different directions. In the first one, we analyze the case where the equilibrium is *strongly lobby-free* under decentralization, i.e. no lobbies form in equilibrium and the policy maker is the same as in the benchmark, and *lobby-neutral* but not *lobby-free*, i.e. no lobbies will form, under centralization. The second case analyzed corresponds to the situation, where, both under centralization and decentralization, policy makers of type M are elected and lobbied by the two extreme groups.

More in details, consider two jurisdictions where the majority group in A are L type citizens, and, in B, H types, and $n_L > n_H, n_M$. Preference heterogeneity between the extreme groups and the median group is relatively low, such that under decentralization no lobby forms in equilibrium and the candidate of the majority is elected. Under centralization, however, the majority group L, is not able to achieve its optimal policy choice by voting for the candidate with its ex -ante policy preferences, since she will be lobbied by type H. In this situation the extreme group L, may prefer the equilibrium generated by the election of the candidate with of type M, if it is associate with an equilibrium with no lobbies. This scenario is described in more formal way in our next Proposition.

Proposition 6 Suppose that L = m(A) = m, H = m(B). If under decentralization conditions of Lemma 4 hold, the equilibrium is strongly lobby free. If under centralization conditions of Proposition 5 b) hold, the equilibrium is lobby-free but not policy-neutral.

This scenario shows that the threat of lobbying can change the majority group voting strategy such that the political equilibrium is *lobby-free* but *not policy-neutral*. In other words, lobbies do not have to exist to affect policy, because voters anticipate the political equilibrium associated with the election of every candidate and vote for the one who, in equilibrium, selects the policy choice closest to their ideal one and is the cheapest to buy. Note that, when the majority group has extreme policy preferences, lobbying ensures that a more centrist policy is implemented either via the election of a "moderate" candidate or by the formation of opposing lobbies. However, note that the possibility that an equilibrium that is *lobby-free* but *not policy-neutral* arises only under centralization, where three groups of citizens exist. Under centralization every equilibrium without lobby is always strongly *lobby-free*, since the majority group will never vote for the candidate of the minority if the equilibrium is lobby-neutral. This equilibrium is dominated by any others determined by voting for the candidate of the majority. So, in this situation, the preference dilution effect works in the opposite direction. The welfare consequences of lobbying are therefore an increase in the overall welfare for the society, if the fix costs of lobby constitution are less than the welfare gain, since the policy choice is closer to the average of residents' ideal points, which is the efficient outcome under the Samuelson rule.²³

Let us now turn to the case where the two extreme groups lobby both under centralization and decentralization, with a policy maker of type M being elected.

Proposition 7 Suppose that the majority groups are always moderate, and the extreme groups lobby both under decentralization and under centralization, the effect of lobbying is higher under decentralization.

(For a characterization of these equilibria see the proof of Proposition 2.)

The overall policy deviation under decentralization;

$$\Gamma^{d} = \sum_{k} |t^{*}(k) - t^{*}(k) (M, \{\omega\})| =$$

$$\left| \frac{n_{L} (\theta_{L} - \theta_{M})}{n_{L} + 1} \right| + \left| \frac{n_{H} (\theta_{H} - \theta_{M})}{n_{H} + 1} \right|,$$
with $\omega = L$ if k=A, and $\omega = H$ if $k = B$.
$$(6.1)$$

The policy deviation under centralization is, instead;

$$\Gamma^{c} = |t^{*} - t^{*} (M, \{L, H\})| =$$

$$|\frac{n_{L} (\theta_{L} - \theta_{M})}{n_{L} + n_{H} + 1} + \frac{n_{H} (\theta_{M} - \theta_{H})}{n_{L} + n_{H} + 1}|.$$
(6.2)

So it is clear that $\Gamma^d > \Gamma^c$, since the denominators in Γ^d are always smaller that the denominators in Γ^c while the numerators are the same and also in Γ^c the two fractions have opposite sign. Note also that if lobbies have the same power there is not policy deviation in equilibrium under centralization, since the two groups offset each other influence.

Under decentralization the policy makers will effectively move their policy choice toward lobbies preferences, while under centralization, since the two lobbies have opposite interests, they will just offset each other and they will not be able to move the policy from the *ex ante* policy maker's ideal policy if lobbies have the same power.

²³The notion of efficiency used here is the one determined by the Samuelson rule, where the sum of marginal rate of substitution is equal to the marginal rate of transformation, which in our case is: $G^* = \sum \theta^i$, therefore $t^* = \frac{G^*}{N} = \frac{\sum \theta^i}{N}$

⁽i.e. the optimal policy choice that the economy can achieve is determined by an average of its members preferences.) However under plurality voting, if citizens vote sincerely over the set of candidates, this outcome is not achieved, because the majority group is able to elect its preferred candidate.

So we see that, under decentralization, the policy choice will be biased toward the minority, and therefore the majority of residents will be worse off from lobbying. Under centralization, if the lobbies have the same power, the welfare consequences for the society are negative, there will be no influence on policy in either direction but there will be a waste of resources associated with the cost of lobby formation (which in reality can be thought as the costs of renting offices, organizing people, collecting information etc.).²⁴ Under centralization, if lobbies do not have the same power, the policy choice will move toward the ideal point of the "strongest" lobby, but the welfare consequences are ambiguous: first, we have to take into account the waste of resources associated with lobby constitution, second, we have to consider whether or not the policy choice gets close to the optimal policy choice under the Samuelson rule.

Remark 1. If lobbying does not have any effect on policy it is always harmful, and limiting the possibilities of lobbying by increasing the cost of entrance will improve welfare.

In these situations the society will not have any benefits from lobbying and any measure preventing groups to organize will increase the welfare of the society by K amount for each lobby. For instance it is possible to make lobbying more difficult by restricting the regulations on lobby constitutions.

Moreover, the consequences on policy and the amount of resources spent by the lobbyists are different. In this situation the lobbyists will have to pay a smaller amount to the policy maker for a bigger policy deviation under decentralization.

Remark 2. In an equilibrium where lobbyists and policy maker are the same under decentralization and centralization, the minority groups will prefer decentralization and the majority centralization.

This can be relevant if the choice to centralize or decentralize is endogenous. Consider the proposal of moving from centralization to decentralization by mean of a *referendum*, if votes are aggregated at central level and the number of citizens belonging to the extreme groups is bigger than the majority group, decentralization will pass; it will not otherwise. Suppose that, instead, citizens are asked their opinion about moving from decentralization to centralization, if votes are aggregated at the decentralized level, centralization will passe, because it is preferred by the majority in the two jurisdictions. Note that under the benchmark, where the possibility of lobbying is not taken into account, citizens are indifferent between centralization and decentralization, if you abstract from economies of scale arguments.

We have presented two scenarios where the effect of centralization on lobbying is analyzed. Even with respect to this way of measuring lobbying we have shown that the effect of centralization on lobbying and its direction are not unique. We have shown that, under centralization, it is possible to have an equilibrium no lobbies but an effect on policy. The reason for that is that citizens take into account

 $^{^{24}}$ Note that the potential waste of resources generated by lobbying and stressed by the rent-seeking literature arises from the cost of forming a lobby rather than by the contributions offered to the policy maker (among others Tullock (1967)) or by the additional incentive to run for an office that citizens have in the citizen-candidate model with lobbying (Besley and Coate (2001)).

lobbying generated by the election of each candidate, when they make their voting decision. In this respect the "threat" of lobbying can itself affect the political equilibrium, without observing any lobby formation.

But it is also possible that, under centralization, lobbies may be counteractive, in the sense that they form only to contrast the effect of an opposing lobby. In this situation, in the (very unusual!) event that they have the same power, their overall effect on policy will be zero. Of course, if coordination between groups were possible, both groups will be better off if they would not form a lobby.

In general, in our setting, we have shown that, when lobbies form, what determines their overall effect on policy are the different alternatives available to citizens. More in details, in the benchmark we have shown that the optimal outcome for each group of citizens is the one achieved by the policy maker belonging to their group. However when lobbying is introduced, the policy maker does not necessarily have the same objective function as the citizens of her group anymore, because she can increase her utility by "selling" her policy choices to the highest bidders. Knowing this, the citizens belonging to the majority group can try to offset lobbying by voting for another candidate or by lobbying. Their success depends on several factors:(i) the existence of many voting alternatives which generate better outcomes, (ii) their relative size compared to the other groups, (iii) the cost of forming a lobby.

7. Discussion and Conclusion

This paper has analyzed the relationship between the level of centralization of policy decision and lobby formation. We have developed a formal framework where we combine the citizen-candidate model with the menu-auction model of lobbying, extended to endogenous lobbying. We have discussed the relationship between our results and existing results in the lobbying literature, and their implications for the analysis of endogenous lobby and policy centralization.

First, in our model there is no equilibrium in pure strategies where lobbying can be counteractive in Leaver and Makris (2000) way, in the sense that only the "friendly" lobby²⁵ will form in order to counteract the possibility that other groups will organize; in this case no contributions will be offered to the policy maker. However, in our setting, lobbying can be counteractive in the sense of Austen-Smith and Wright (1992), where two lobbies form in order to offset each others influence, in this case positive contributions are offered to the policy maker. This form of lobby competition can occur when preference heterogeneity between groups is high and groups are large, and, in the special case where lobbies have the same "power", there will not be an effect of lobbying on policy.

Second, apart from the special case described above, lobbying generally induces the policy maker to change her policy; this can happen both when only one lobby exists or when lobbies compete. In the first case the lobby will be the one formed by citizens with preference opposing the policy maker's, and it requires that preferences heterogeneity between groups is not very high. When more than one lobby

²⁵ The one by the group sharing the same preferences of the policy maker.

exists in equilibrium, since the policy choice is an average between lobbies and policy maker preferences, the overall effect will increase the less lobbies are able to counteract each others. Consistently with Felli and Merlo (2002), lobbying may influence policy and move the policy choice toward the center, when the majority group has extreme preferences. This can happen either by the election of a more centrist candidate compare to the candidate with the preference of the majority group, or by the formation of antagonist lobbies.

Finally, we have presented the case where an equilibrium may be *lobby-free* but not *policy-neutral*. This can happen, only under centralization, when the majority group has extreme policy preferences and, in order to prevent the formation of a hostile lobby, the majority group votes for a more moderate candidate. So we see that lobbying can induce policy moderation even when no lobby forms.

In order to compare the political equilibrium under policy decentralization and policy centralization, we have proposed two ways in which the extent of lobbying can be measured: i) the number of lobbies, and ii) their impact on policy. We have shown that lobbying is affected by the level of government who decides the policy. However, contrary to the *preference dilution* effect, which defines an inverse relationship between the level of centralization and lobbying; we have shown that the direction of this relationship is actually *ambiguous*, with respect to the two ways of measuring lobbying. We have presented scenarios where lobbying is higher under centralization and scenarios where the opposite is true.

In general groups are more likely to form a lobby when the minorities have preference quite dissimilar to majority and when they are large (i.e. uniform distribution of preferences). Both results seem to support the idea that lobbying is higher under centralization where we expect both higher preference heterogeneity and groups of bigger size. However we have also shown that lobbying is less likely to form when citizens are divided into many different groups. We have found that lobbies are less likely to form in small jurisdiction, or in jurisdictions where the majority group is very large compare to minority groups, or when where residents are fragmented in many groups.

There is still a lot to be done to develop a more complicated picture of the effect of policy centralization on lobbying. First, in this model we only have accounted for preference heterogeneity among consumers, a more sophisticated representation of the reality could improve the analysis: for example, we could introduce different factor owners or different income distributions. Second, we assume that citizens vote sincerely over the set of candidates. It would be interesting to explore strategic voting and endogenous lobbying.

Finally, centralization and decentralization are here depicted in a very stylized way, it would probably be worth to take into account different concepts of centralization like the existence of different levels of government at the same time.

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8. Appendix

Derivation of the truthful contributions

Following Dixit, Grossman and Helpman (1997), truthful contributions, $C_{\omega}(t, P)$, offered by lobby $\omega \in \Lambda$, are calculated such as: the policy maker P, i) maximizes her utility, taking into account lobbies' contribution payments, $t^* = \arg \max_t H_P(t, \sum_{l \in \Lambda} C_l(t))$; and, ii) receives the same utility he would have got without lobby ω offer; i.e. $H_P(t, \sum_{l \in \Lambda} C_l(t)) = \max_t H_P(t, \sum_{l \in \Lambda/\omega} C_l(t), 0)$, provided that the contributions are positive. Therefore, contributions, $C_{\omega}(t(P, \Lambda))$, offered by lobby ω , for centralization,

are calculated such that $1-t(P,\Lambda)$) + $\theta_P \ln 2t(P,\Lambda)$) $n + C_{\omega}(t(P,\Lambda)) + \sum_{l \in \Lambda/\omega} C_l(t(P,\Lambda)) = 1-t(P,\Lambda/\omega) + \theta_P \ln 2t(P,\Lambda/\omega))n + \sum_{l \in \Lambda/\omega} C_l(t(P,\Lambda/\omega))$, so

$$C_{\omega}(P,\Lambda)) = \max \left[\begin{array}{c} 0, (t(P,\Lambda) - t(P,\Lambda/\omega) + \theta_P \ln \frac{t(P,\Lambda/\omega)}{t(P,\Lambda)} + \\ \sum_{l \in \Lambda/\{\omega\}} s_l(t(P,\Lambda) - t(P,\Lambda/\omega) + \theta_l \ln \frac{t(P,\Lambda/\omega)}{t(P,\Lambda)}) \end{array} \right]$$
(8.1)

Proof of Proposition 1. Calculation of $\Delta_{\omega}(P, \Lambda) = H_{\omega}(P, \Lambda) - H_{\omega}(P, \Lambda/\omega)$.

Case i) $\Lambda = \{\omega\}$. So $t(P, \{\omega\}) = \frac{\theta_{\omega} s_{\omega} + \theta_P}{s_{\omega} + 1}$ and $t(P, \{\emptyset\}) = \theta_P$.

Then substituting $C_{\omega}(P,\Lambda)$ into $H_{\omega}(P,\Lambda)$, we get: the payoff to a type ω if lobbies Λ form:

$$\Delta_{\omega}(P, \{\omega\}) = \begin{cases} -\frac{K}{s_{\omega}-1} = \Delta_1 & \text{if } \omega = P\\ \theta_P - \theta_{\omega} + \frac{\theta_P + \theta_{\omega} s_{\omega}}{s_{\omega}} \ln \frac{\theta_P + \theta_{\omega} s_{\omega}}{(s_{\omega}+1)\theta_P} - \frac{K}{s_{\omega}} = \Delta_2 & \text{if } \omega \neq P \end{cases}$$
(8.2)

Case ii) $\Lambda = \{\omega, v\}$. So $t(P, \{\omega, v\}) = \frac{\theta_v s_v + \theta_\omega s_\omega + \theta_P}{s_\omega + s_v + 1}$ and $t(P, \{v\}) = \frac{\theta_v s_v + \theta_P}{s_v + 1}$. Then:

$$\Delta_{\omega}(P, \{\omega, v\}) = \begin{cases} \frac{n_v(\theta_v - \theta_\omega) + (\theta_P - \theta_\omega)}{(n_v + 1)} + \frac{n_v \theta_v + n_\omega \theta_\omega + \theta_P}{n_\omega} \ln \frac{(n_v + 1)(n_v \theta_v + n_\omega \theta_\omega + \theta_P)}{(n_v + n_w + 1)(\theta_P + n_v \theta_v)} - \frac{K}{n_\omega} = \Delta_3 & \text{if } \omega \neq P \text{and } v \neq P \\ \frac{s_v \theta_v + (n_\omega - 1)\theta_\omega + \theta_P}{n_\omega - 1} \ln \frac{(s_v + 1)(s_v \theta_v + (n_\omega - 1)\theta_\omega + \theta_P)}{(n_v + n_\omega)(\theta_P + n_v \theta_v)} + \frac{n_v(\theta_v - \theta_\omega)}{(n_v + 1)} - \frac{K}{n_\omega - 1} = \Delta_4 & \text{if } \omega = P \text{and } v \neq P \\ (\theta_v - \theta_\omega) + \frac{s_v \theta_v + n_\omega \theta_\omega}{n_\omega} \ln \frac{s_v \theta_v + n_\omega \theta_\omega}{(n_v + n_\omega)\theta_v} - \frac{K}{n_\omega} = \Delta_5 & \text{if } \omega \neq P \text{and } v = P \end{cases}$$

Case iii) $\Lambda = \{\omega, v, z\}$. So $t(P, \{\omega, v, z\}) = \frac{\theta_{\omega} s_{\omega} + \theta_v s_v + \theta_z s_z + \theta_P}{n}$ and $t(P, \{v, z\}) = \frac{\theta_v s_v + \theta_z s_z + \theta_P}{s_v + s_z + 1}$. Then, as before

$$\Delta_{\omega}(P, \{\omega, v, z\}) = \frac{s_{\omega} + s_v + s_z + 1}{s_{\omega}} \left(\frac{\theta_v s_v + \theta_z s_z + \theta_P}{s_v + s_z + 1} - \frac{\theta_\omega s_\omega + \theta_v s_v + \theta_z s_z + \theta_P}{2n} \right) + \frac{\theta_\omega s_\omega + \theta_v s_v + \theta_z s_z + \theta_P}{s_{\omega}} \ln \frac{\frac{\theta_\omega s_\omega + \theta_v s_v + \theta_z s_z + \theta_P}{2n}}{\frac{\theta_v s_v + \theta_z s_z + \theta_P}{s_v + s_z + 1}} - \frac{K}{s_{\omega}}.$$

The first part of the Proposition analyses the relationship between preference heterogeneity, calculated as the distance between different groups preference parameter θ . In what follow we study analytically the different Δ functions which represents, the lobbying non-lobbying decisions faced by each group, once the identity of the policy maker is known, and anticipating other groups' behavior. i) and ii) preference heterogeneity:

$$\frac{\partial \Delta_1}{\partial \theta_\omega} = 0 \qquad \text{for any } \theta
\frac{\partial \Delta_2}{\partial \theta_\omega} = \ln \frac{\theta_P + n_\omega \theta_\omega}{(n_\omega + 1)\theta_P} \qquad \text{for } \theta_\omega = \theta_P \qquad A = \frac{\partial^2 \Delta_2}{\partial \theta_\omega} = \frac{n_\omega}{\theta_P + n_\omega \theta_\omega} > 0
\frac{\partial \Delta_3}{\partial \theta_\omega} = \ln \frac{(n_v + 1)}{(n_v + n_\omega + 1)} \frac{n_v \theta_v + n_\omega \theta_\omega + \theta^p}{(\theta_v + n_v \theta_v)} \qquad \text{for } \theta_\omega = \frac{\theta_P + n_v \theta_v}{n_\omega + 1} \qquad B = \frac{\partial^2 \Delta_3}{\partial \theta_\omega} = \frac{n_\omega}{n_v \theta_v + \theta^p + n_\omega \theta_\omega} > 0
\frac{\partial \Delta_4}{\partial \theta_\omega} = \ln \frac{(n_v + 1)}{(n_v + n_\omega)} \frac{(n_v + n_\omega \theta_\omega)}{(\theta_\omega + n_v \theta_v)} \qquad \theta_\omega = \theta_v \qquad C = \frac{\partial^2 \Delta_5}{\partial \theta_\omega} = \frac{n_\omega - 1}{n_v \theta_v + n_\omega \theta_\omega} > 0
\frac{\partial \Delta_5}{\partial \theta_\omega} = \ln \frac{n_v \theta_v + n_\omega \theta_\omega}{(n_\omega + n_v) \theta_v} \qquad \theta_\omega = \theta_P \qquad D = \frac{\partial^2 \Delta_5}{\partial \theta_\omega} = \frac{n_\omega}{n_v \theta_v + n_\omega \theta_\omega} > 0$$

Conditions in (8.3) are monotonically decreasing until they reach their minimum at the value defined in (8.3), and then they become monotonically increasing. Let us implicitly define $\underline{\theta}(\omega, \Lambda)$ and $\overline{\theta}(\omega, \Lambda)$ the lower and upper values that make (8.3) equal to 0. At these values citizens of type ω are indifferent between lobbying and not lobbying. Note that as θ_{ω} move away from θ_P the functions increases, and therefore the likelihood for lobbying increases.

iii) groups size. Moreover if we take the derivative of A, B, C and D with respect to group size n_{ω} and n_{v} , we get:

$$\begin{array}{l} \frac{\partial A}{\partial n_{\omega}} = \frac{\theta_P}{\left(\theta_P + n_{\omega}\theta_{\omega}\right)^2} > 0\\ \frac{\partial B}{\partial n_{\omega}} = \frac{n_v \theta_v + \theta_P}{\left(n_v \theta_v + \theta_P + n_{\omega}\theta_{\omega}\right)^2} > 0; \quad \frac{\partial B}{\partial n_v} = -\frac{n_{\omega}}{\left(n_v \theta_v + \theta_P + n_{\omega}\theta_{\omega}\right)^2} < 0\\ \frac{\partial C}{\partial n_{\omega}} = \frac{1}{\theta_v} \frac{n_v + 1}{\left(n_v + n_{\omega}\right)^2} > 0 \qquad \qquad \frac{\partial C}{\partial n_v} = -\frac{1}{\theta_v} \frac{n_v - 1}{\left(n_v + n_{\omega}\right)^2} < 0\\ \frac{\partial D}{\partial n_{\omega}} = \frac{n_v \theta_v}{\left(n_v \theta_v + n_{\omega}\theta_{\omega}\right)^2} > 0 \qquad \qquad \frac{\partial D}{\partial n_v} = -\frac{n_{\omega} \theta_{\omega}}{\left(n_v \theta_v + n_{\omega}\theta_{\omega}\right)^2} < 0 \end{array}$$

This implies that the slope of A, B, C and D increases in n_{ω} and decreases in n_v . Note that as n_{ω} increases the minimum value of the above functions increases.

Proof of Lemma 2. We show the case under centralization and we omit decentralization, since similar results apply. Define the function $\Psi_m(P) = C(m, \overline{\Delta}) - C(P, \overline{\Delta})$:

$$\Psi_m(P) = \frac{\sum_{l \in \bar{\Delta}/\{m\}} n_l + 1}{n_m - 1} \left[t(P, \overline{\Lambda}) - t(m, \overline{\Lambda}/\{m\}) \right] + \frac{\sum_{l \in \bar{\Delta}/\{m\}} \theta_l n_l + \theta_m}{n_m - 1} \ln\left(\frac{t(m, \overline{\Lambda}/\{m\})}{t(P, \overline{\Lambda})}\right) + \frac{\sum_{l \in \bar{\Delta}/\{m\}} n_l}{n_m} \left[t(P, \overline{\Lambda}) - t(\omega, \overline{\Lambda}/\{m\}) + \theta_l \ln\left(\frac{t(\omega, \overline{\Lambda}/\{m\})}{t(P, \overline{\Lambda})}\right) \right]$$

Where $t(P,\overline{\Lambda}) = \frac{\sum_{l \in \overline{\Lambda}/\{m\}} \theta_l n_l}{2n} = t^{**}, t(m,\overline{\Lambda}/\{m\}) = \frac{\sum_{l \in \overline{\Lambda}/\{m\}} \theta_l n_l + 1}{\sum_{l \in \overline{\Lambda}/\{m\}} n_l + 1} \text{ and } t(\omega,\overline{\Lambda}/\{m\}) = \frac{\sum_{l \in \overline{\Lambda}/\{m\}} \theta_l n_l}{\sum_{l \in \overline{\Lambda}/\{m\}} n_l}.$ The candidate of type m is elected if $\Psi_m(P) \leq 0.$

In order to check the sign of $\Psi_m(P)$, consider first the case where $t(m,\overline{\Lambda}/\{m\}) = t(\omega,\overline{\Lambda}/\{m\}) = x$, the function then becomes $\frac{\sum_{l\in\overline{\Lambda}/\{m\}}n_l+n_m}{n_m(n_m-1)} [t^{**}-x] + \frac{\sum_{l\in\overline{\Lambda}/\{m\}}\theta_l n_l+n_m\theta_m}{n_m(n_m-1)} \ln \frac{x}{t^{**}}$. This value of the function is 0 when it is evaluated at $x = t^{**}$, and this point is also the maximum value of the function, since $\frac{\partial\Psi_m_{t(m,\overline{\Lambda}/\{m\})=t(\omega,\overline{\Lambda}/\{m\})}{\partial t(m,\overline{\Lambda}/\{m\})} \ge 0$ for $x \leqslant t^{**}$ and negative otherwise. So $\Psi_{m_{t(m,\overline{\Lambda}/\{m\})=t(\omega,\overline{\Lambda}/\{m\})}$ is always negative if $x \neq t^{**}$; now consider the case where $t(m,\overline{\Lambda}/\{m\}) = x + d$, and substitute it into $C(m,\overline{\Lambda})$, the derivative of $\frac{\partial C(m,\overline{\Lambda})}{\partial d}$ is positive when $d < x - t(\omega,\overline{\Lambda}/\{m\})$, which is never true by construction. So $\Psi_m \leqslant 0$ is always negative and therefore $P^*(m) = m.\Box$

Proof of Lemma 3. First, for convenience, define $|\theta_L - \theta_M| = d + x$ and $|\theta_H - \theta_M| = d$. Case i) if m=m(A)=m(B)=M, and substituting the correspondent values into (3.4) and (3.5), we get that $\Gamma^c < \Gamma^d$, since $\frac{n_L(d+x)+n_Hd}{n} > \left|\frac{n_L(d+x)-n_Hd}{2n}\right|$. Case ii) if m=M, m(A)=L and m(B)=H, and substituting the correspondent values into (3.4) and (3.5), we get that $\Gamma^c < \Gamma^d$ if $\left|\frac{n_Md+n_M(A)x}{n}\right| > \left|\frac{(n_L-n_H)d+n_Lx}{2n}\right|$, which is true only for some values of our parameters. It easy to check that all the other possible combinations are also ambiguous, so we omit the proof. \Box

Proof of Lemma 4. (a) Decentralization. Also, $\Delta_{\omega}(m, \{\omega\}) > \Delta_{\omega}(m, \{\omega, m\})$ because both functions reach their minimum at $\theta = \theta_m$, where both assume the value -K/s_{ω} but the latter is smoother $\frac{\partial \Delta^2_{\omega}(m, \{\omega, m\})}{\partial \omega} < \frac{\partial \Delta^2_{\omega}(m, \{\omega\})}{\partial \omega}$. So, $0 > \Delta_{\omega}(m, \{\omega\}) > \Delta_{\omega}(m, \{\omega, m\})$. As agents of type ω never want to lobby, it remains to show that majority group m will never want to lobby a policy-maker of type m on

their own , i.e. $\Delta_m(m, \{m\}) < 0$. But it is shown in the Appendix (eqn 3.31) that $\Delta_m(m, \{m\}) < 0$. (b) Centralization. First, note that the conditions stated rule out an equilibrium where just one minority group lobbies, and we know also that $\Delta_m(m, \{m\}) < 0$. Second, the condition $\Delta_{\omega}(m, \{\omega, v\}) < 0$ rules out an equilibrium where both minority groups lobby. Third, as $0 > \Delta_{\omega}(m, \{\omega\}) > \Delta_{\omega}(m, \{\omega, m\})$, there is no equilibrium where the majority and one minority group lobby. Finally, condition (iii) rules out a lobby of all three types. \Box

Proof. of Proposition 2. We characterize all possible lobbying equilibria when $\Delta_{\omega}(m, \{\omega\}) \geq$ 0.Decentralization. Assume first a candidate of type *m* has been elected. There are two possibilities: (i) $\Delta_m(m, \{\omega, m\}) < 0$ and (ii) $\Delta_m(m, \{\omega, m\}) \ge 0$. In case (i), clearly an equilibrium with no lobbies is impossible, and $\Lambda = \{\omega\}$ is an equilibrium. In case (ii), $\Lambda = \{m, \omega\}$ is an equilibrium if $\Delta_{\omega}(m, \{\omega, m\}) \geq 0$. Also: otherwise, there is a one-equilibrium lobby, $\{\omega\}$ if $\Delta_{\omega}(m, \{\omega, m\}) \geq 0$ and $\Delta_m(m, \{\omega, m\}) < 0$ and there is an equilibrium in mixed strategies otherwise. We can calculate the associate mix strategy equilibrium which is $\left(\frac{\Pi_\omega(m, \{m,\omega\}) - \Pi_\omega(m, \emptyset)}{\Pi_\omega(m, \{m,\omega\}) - \Pi_\omega(m, \{\omega\})}, - \frac{\Delta_\omega(m, \{\omega\})}{\Delta_\omega(m, \{m,\omega\})}\right)$ for players of type m and $\left(\frac{\Delta_m(m,\{m,\omega\})}{\Delta_m(m,\{m,\omega\})+\frac{K}{s_m}}, \frac{\frac{K}{s_m}}{\Delta_m(m,\{m,\omega\})+\frac{K}{s_m}}\right)$ for players type ω . Now suppose a candidate of type ω has been elected and no lobby forms: this is impossible, as all citizens of type m would prefer to vote for m, as $H_m(\omega, \emptyset) < H_m(m, \Lambda)$ for all $\Lambda = \emptyset, \{\omega\}, \{m\}, \{m, \omega\}$. This is because if m is elected, the policy outcome will always be between θ_m and θ_{ω} , whereas if ω is elected and there is no lobbying, the policy outcome is θ_{ω} i.e. worst for them. (b) Centralization: if $\Delta_{\omega}(m, \{\omega, v\}) \ge 0$ there are four possibilities: (i) $\Delta_v(m, \{v\}) \ge 0$ and $\Delta_v(m, \{\omega, v\}) \ge 0$. In case (i) $\Lambda = \{\omega, v\}$ is an equilibrium. Case (ii) $\Delta_v(m, \{v\}) \ge 0$ and $\Delta_v(m, \{\omega, v\}) < 0$. In case (ii) $\Lambda = \{\omega\}$ is an equilibrium. Case (iii) $\Delta_v(m, \{v\}) < 0$ and $\Delta_v(m, \{\omega, v\}) < 0$. In case (iii) $\Lambda = \{\omega\}$ is an equilibrium. Case (iv) $\Delta_v(m, \{v\}) < 0$ and $\Delta_v(m, \{\omega, v\}) \ge 0$. In case (iv) $\Lambda = \{\omega, v\}$ is an equilibrium. if $\Delta_\omega(m, \{\omega, v\}) < 0$ there are four possibilities: (i) $\Delta_v(m, \{v\}) \ge 0$ and $\Delta_v(m, \{\omega, v\}) \ge 0$. In case (i) $\Lambda = \{v\}$ is an equilibrium. Case (ii) $\Delta_v(m, \{v\}) \ge 0$ and $\Delta_v(m, \{\omega, v\}) < 0$. In case (ii), $\Lambda = \{\omega\}, \{v\}$ is an equilibrium. rium. Case (iii) $\Delta_v(m, \{v\}) < 0$ and $\Delta_v(m, \{\omega, v\}) < 0$. In case (iii) $\Lambda = \{\omega\}$ is an equilibrium. Case (iv) $\Delta_v(m, \{v\}) < 0$ and $\Delta_v(m, \{\omega, v\}) \ge 0$, there is no equilibrium in pure strategies, the equilibrium in mixed strategies is for $v\left(\frac{\Delta_v(m,\{v,\omega\})}{\Delta_v(m,\{v,\omega\})-\Delta_v(m,\{v\})}, \frac{\Delta_v(m,\{v\})}{\Delta_v(m,\{v,\omega\})-\Delta_v(m,\{v\})}\right)$ and for $\omega\left(\frac{\Delta_\omega(m,\{v,\omega\})}{\Delta_\omega(m,\{v,\omega\})-\Delta_\omega(m,\{\omega\})}, \frac{\Delta_v(m,\{v,\omega\})}{\Delta_\omega(m,\{v,\omega\})-\Delta_\omega(m,\{\omega\})}\right)$ $\frac{\Delta_{\omega}(m,\{v,\omega\}) - \Delta_{\omega}(m,\{v,\omega\}) - \Delta_{\omega}(m,\{w,\omega\}) - \Delta_$ librium, the political equilibrium will be anyway affected by lobbying through election, even when no lobby forms, so $P^*(m) = \omega, \ \omega \neq m$. However we can show that when the majority group is of type $M, \{\omega, \emptyset, \theta_{\omega}\}$ can not be a political equilibrium. Suppose that if $P(m) = \omega^*$, then $\Lambda(\omega^*) = \emptyset$. So if P(m) = m, it is easy to check that the only possible lobbying equilibria are $\Lambda^*(m) = \{\emptyset\}, \{\omega\}$. This is because if $\Lambda(\omega^*) = \emptyset$, then $\Delta_v(\omega, \{\omega\}), \Delta_v(\omega, \{v, \omega\}) \leq 0$, so also $\Delta_v(m, \{\omega\}), \Delta_v(m, \{v, \omega\}) \leq 0$ for Proposition 1. So in both cases the associate policy outcome $t(M, \Lambda(M)) = \theta_M$, $\frac{\theta_M + n_\omega \theta_\omega}{n_\omega + 1}$ is preferred to $t(\omega, \Lambda(\omega)) = \theta_{\omega}$. When, instead the majority group is extreme (L or H), we can not rule out that P(m) = M is a voting equilibrium when $\Lambda^*(M) = \emptyset$, but for the same reasoning as before we can rule out that $P(m) = \omega$ is a voting equilibrium if the associated lobbying equilibrium is $\Lambda^*(\omega) = \emptyset$, with $\omega \neq M, m.\square$

Proof of Proposition 3. See the proof of Proposition 2. Derivation of an equilibrium lobby free

but not strongly lobby free under centralization. From Proposition 2, if $\Delta_{\omega}(m, \{\omega\}) \ge 0$ the equilibrium is not lobby-free. $\Delta_{\omega}(M, \{\omega\}) < 0$ and $\Delta_m(M, \{m\}) < 0$ and $\Delta_m(M, \{m, \omega\}) < 0$ or $\Delta_{\omega}(M, \{m, \omega\}) < 0$ ensure that $\Lambda^*(M) = \emptyset$ is an equilibrium set of lobbies. Part (ii) says that if $H_m(M, \emptyset) > H_m(m, \Lambda)$ than $M^* = P(m).\square$

Proof of Corollary 1. Given the truthful contribution functions (8.1), $C_{\omega}(t(P, \Lambda)) = 0$ iff $t(P, \Lambda) = t(\omega, \{\omega\})$, for $\omega = L, M, H$. Is it proved in Proposition 2 that this can never be an equilibrium in pure strategies of the lobbying subgame because for any $\omega \Delta_{\omega}(\omega, \{\omega\}) < 0$. A full characterization of this equilibrium is in Proposition 2. \Box

Proof of Proposition 5. Decentralization. In order that $t^*(P^*, \Lambda^*) = t(m, \emptyset), \Lambda^* \neq \emptyset$, the only possibility with two types of citizens is that $P^* = m$ and $\Lambda^* \neq \{m\}$. However it is easy to show that this can never be an equilibrium in pure strategy. In order that at least one lobby will form, we require that the condition for an equilibrium with lobbies of Proposition 2 holds (i.e. $\Delta_{\omega}(m, \{\omega\}) \ge 0$). Moreover, we need that the additional conditions $\Delta_{\omega}(m, \{m, \omega\}) < 0$ and $\Delta_m(m, \{m, \omega\}) > 0$ hold. Combining all these three conditions together and remembering that $\Delta_m(m, \{m\})$ is always negative, it easy to check that an equilibrium in pure strategies does not exist. See the same case in Proposition 3. Centralization. In order $t^*(P^*, \Lambda^*) = t(m, \{\emptyset\}), \Lambda^* \neq \emptyset$, we require either that (i) $P^* = m$ and $\Lambda^* = \{m\}$ or, if m = M and $n_{\omega} |\theta_{\omega} - \theta_P| = n_v |\theta_v - \theta_P|$, that $P^* = m$, and $\Lambda^* = \{v, \omega\}$. Part sub (i): for the same arguments as before the only possible equilibrium is in mixed strategies. Part sub (ii): for m = M and $\omega, \nu \neq m$, if $\Delta_{\omega}(m, \{\omega\}) \ge 0$ and $\Delta_{\omega}(m, \{\omega, \nu\}) \ge 0$ there is a lobbying equilibrium where $\Lambda = (\omega, v)$ for Proposition 3 part (i) and since $n_{\omega} |\theta_{\omega} - \theta_P| = n_v |\theta_v - \theta_P|$ the policy choice will be $t^*(m^*, \{\omega, v\}^*) = \theta_M$. Note that this is also the most preferred political equilibrium for type m since $H_m(m, \{v, \omega\}) = H_m(m, \{\emptyset\})$, and $H_m(m, \{\emptyset\})$ (the payoffs under the benchmark). Finally a three-lobby equilibrium, where lobbies do not affect policy is ruled out, since the necessary condition on preference distribution is the same as the one for two-lobby equilibrium $(n_{\omega} | \theta_{\omega} - \theta_P | = n_v | \theta_v - \theta_P |)$, however if this is the case and $\Delta_{\omega}(m, \{v, \omega, m\}) \ge 0$ for both extreme groups, it follows that $\Delta_m(m, \{v, \omega, m\})$ is necessarily negative since $H_m(m, \{m, v, \omega\}) < H_m(m, \{v, \omega\}) = H_m(m, \{\emptyset\})$, because in all three cases $t^*(P^*, \Lambda^*) = \theta_P$ but in the first one group type m will have to pay contributions to the policy maker. \Box