Optimal Sovereign Debt Write-downs

Sayantan Ghosal and Kannika Thampanishvong*
University of Warwick and University of St Andrews
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Abstract

At present, the enhanced HIPC initiative and the Gleneagles Proposal for debt write-downs by the G8 are the main mechanisms used to reduce indebtedness of low-income countries. In these countries where poor governance is a key issue, it is naïve to believe that the Millennium Development Goals can be achieved if the current debt relief mechanisms fail to address such problem. In this paper, we develop a model of sovereign debt write-downs, where governance problems reflect domestic distributive conflict between two classes in the society and intertemporal conflict. The main policy issue is how to design the optimal form of debt write-downs and the conditionality requirements attached to it with such governance problems in mind. To deal with the domestic distributive conflict, it is crucial that the conditionality requirements target both provision of public goods and private consumption level of the poor citizens. Addressing the intertemporal conflict problem requires the use of long-run conditionality requirements. Against such a benchmark, we then evaluate the efficacy of the current debt relief initiatives and discuss some policy implications.

Keywords: Debt Relief, Conditionality, HIPC Initiative
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1 Introduction

Despite significant benefits derived from stronger global economic growth and improvement in world-wide living conditions over the past two decades, much of the developing world still remains mired in poverty. As a response, the United Nations have endorsed a set of Millennium Development Goals (MDGs), including health and education, as an international blueprint for promoting development (United Nations, 2006). However, it is difficult for these goals to be attained if debt obligations are excessive as is, by definition, true for the Heavily Indebted Poor Countries (hereafter, HIPCs).

In Sachs (2002), Jeffrey Sachs argues that there is a rationale for granting a "fresh start" to poor countries when they are vulnerable to a poverty trap. He evaluates the efficaciousness of the current mechanisms used to reduce the indebtedness of the HIPCs, which include the HIPC initiative of 1999 and the Gleneagles Proposal for debt write-down by the G8 countries. Under these initiatives, low-income countries that fell into a debt crisis have not received sufficient debt reduction to restore growth and reestablish normal relationship with the creditors. It has been manifest that arbitrary formulas – the criteria such as the ratio of debt to export or ratio of debt to government revenue – have been used in the debt sustainability analysis under the HIPC initiative of 19999 to decide on the level of debt relief. With such criteria, a country cannot truly be judged as sustainable or unsustainable except in the context of its needs, which must be carefully spelled out. To address such problems, Jeffrey Sachs provides a framework for the theory that should underpin debt relief. In particular, he proposes that debt relief should be based on a systematic assessment of each country's needs, measured against the explicit development objectives like the MDGs.

Despite the appealing suggestions put forward by Jeffrey Sachs, his analysis still fails to take into account the "politics" of sovereign debt relief. This issue is extremely important as for these low-income countries, particularly in Africa, there are corruption, conflict between the ruling elites and the poors they govern and excessive issue of sovereign debt by the elite class. When poor governance is a key issue, it is crucial to ensure that the conditions attached to the debt relief help address such problems. However, by examining the current debt relief initiatives – the enhanced HIPC initiative of 1999 and the Gleneagle Proposal – we find that this is not the case as the former includes a good deal of inappropriate requirements and the conditions imposed by the latter are the same as those required to reach the completion point under the former[1];

[1] "Countries must be current on their repayment obligations to the IFIs and must

therefore, it is naïve to believe that the MDGs can be achieved and the well-being of the poor is improved.

With the problem of poor governance in mind, in this paper, we study the optimal form of debt write-down and the design of conditionality requirements attached to it to ensure that the well-being of poor citizens in those low-income countries improves and the MDGs are achieved. We consider a fully liberalised economy, consisting of elites and poor citizens. When the elites have all the bargaining power, they determine relevant policy choices subject to the participation constraint of the poor citizens. The *domestic distributive conflict* exists when the elites curtail public expenditure on state education and public health as well as private consumption of the poor citizens and use the public funds (including the resources freed up by debt relief) to support their private consumption. The elites also make a policy choice on public debts that determines the probability of default: a policy which supports an excessive issuing of public debts raises the default policy. Due to their shortsightedness or myopia, the elites do have an incentive to put in place policies on public debts (which are not optimal from the perspective of poor citizens) that significantly increase the probability of default. This gives rise to the problem of *intertemporal conflict*. Since the elites are effectively insured against the possibility of default, only the poor citizens suffer as they are forced to bear a disproportionately high share of default costs.

In the economy where the problems of *domestic distributive conflict* and *intertemporal conflict* are prevalent, we show how the conditionality requirements attached to the debt write-down could have a crucial impact on the well-being of the poor citizens. We consider two versions of the model, static and dynamic.

In the static model, we use the generalised Nash bargaining framework to study how the distribution of power between the elites and the poor citizens plays a crucial role as it determines the relative utility gains of both classes from debt write-down in the equilibrium. Our results show that for the society with a balanced distribution of power between the two classes, unconditional debt write-down is sufficient to ensure that the well-being of the poor citizens is improved. On the other hand, for the society where all power is in the hands of the elites and where the problem of *domestic distributive conflict* is present, it is necessary that the conditionality requirements attached to the debt write-down target

not have experienced serious lapses, including in governance, such that their IMF programs would be at risk. Finally, the proposal makes several references to transparency and country performance as criteria for the allocation of development assistance," (IMF, 2005).

both provision of public goods[2] and the private consumption level of the poor citizens.

It is important to note that, unlike Acemoglu and Robinson (2000, 2001, 2006), our main emphasis in this paper is not to model power and changes in the institutions. In Acemoglu and Robinson (2000, 2001), they analyse how *de facto* political power drives changes in political institutions and the future distribution of *de jure* political power, while in Acemoglu and Robinson (2006), they study how equilibrium economic institutions emerge from the interaction between political institutions, which allocate *de jure* political power, and the distribution of *de facto* political power. Instead, we study how the outcome of a bargaining situation may be influenced by the exogenously determined bargaining power of the two classes.

We then extend the model to a dynamic framework, which is based on a two-period repeated game approach. In the dynamic setup, the national resource in the second period is stochastic, depending on whether or not the elites choose to default. Our results show that, if the conditions on public goods and private consumption of the poors are imposed in the first period, the elites can get around such conditions by issuing more public debts. While issuing excessive public debts is favouring the elites since they can use the additional resources raised from the issue of new public debts to finance their private consumption and once default occurs they are effectively insured against the default costs, it is not beneficial for the poor citizens. The main reason behind this is that the poor citizens have to suffer from a high share of default costs, particularly from a permanent contraction in national income and everlasting exclusion from the international capital market.

This opposing views of the elites and the poor citizens with regards to issue of public debts lead to an existence of *intertemporal conflict* between the two classes. To keep this problem in check, our results suggest that the long-run conditionality requirements, which are structured in such a way that elites bear the consequences of excessive debts and default. For example, the external funding agency could require the elites to protect public goods and social spending and, at the same time, imposing an explicit limit on the amount of public debts the elites are allowed to issue.

[2]The public goods, such as education and health, are important components of the MDGs. They are not only valuable in themselves but they also increase the income-generating capacity of the low-income countries in question (Stern, Dethier and Rogers, 2005). In particular, for the society where the problem of *domestic distributive conflict* is prevalent, it is very crucial to link debt relief to public investment in public goods as there is a very high tendency that they will be underprovided by the ruling elites.

Against such a benchmark, we then evaluate the efficacy of the current initiatives for debt write-down under the enhanced HIPC initiative and the Gleneagles Proposal for debt write-down by the G8 countries. We discuss about the effectiveness of the conditionality requirements under these two debt relief initiatives as well as the conditionality prescribed by the IMF under its financial support programme, particularly maintenance of primary budget surplus. We then conclude by making some policy recommendations.

The remainder of the paper is structured as follows. In section 2, we develop a static model of debt write-downs, which captures the problem of *domestic distributive conflict* between the two classes in the society. Section 3 is devoted to discuss the design of conditionality requirements attached to the debt write-downs under a static framework. In section 4, we extend the model to a dynamic framework to study the problem of *intertemporal conflict*. In section 5, we evaluate the efficacy of the recent initiatives for debt write-down and discuss some policy recommendations while section 6 concludes. Some of the technical materials are contained in the appendix.

2 The static model

Consider an economy consisting of two classes, namely elites (indexed by E) and poor citizens (indexed by P), who live for one period. In the static model, we assume that the national income, y, is non-stochastic and exogenously determined. In this fully liberalised economy, a new public debt can be issued at a constant rate of interest, $r > 0$. We assume that the external funding agency imposes a limit on the amount of public debts the country is allowed to issue, given by $\tilde{D}$. The debt obligation falling due is $(1+r)D_{-1}$, where D_{-1} is the exogenously given stock of public debt. The national resource, I, characterises a surplus of fixed size and is given by $I = y + \tilde{D} - (1+r)D_{-1}$, where $I > 0$. All variables are measured in "real" terms; therefore, there is no issue of inflation nor foreign exchange rate risk.

The two classes, elites and poor citizens, bargain over the partition of a national resource of size I. The budget constraint is given by $x^E + x^P + cG \leq I$, where G is the provision of public goods, x^E is the private consumption level of the elites, x^P is the private consumption level of the poor citizens and $c > 0$ is a constant unit cost of providing the public goods. The set of possible agreements is Ω, where $\Omega = \left\{ \left(x^E, x^P, G\right) : 0 \leq x^E < I,\ 0 < x^P \leq I \text{ and } G = \frac{I - x^E - x^P}{c} \right\}$.

We impose the non-negativity constraints on G, x^E and x^P thus $G \geq 0$, $x^E \geq 0$ and $x^P \geq 0$. For each class i, $U^i\left(G, x^i\right)$ is the utility

obtained by class i from G and x^i, where $i = E, P$. For all i, $U^i\left(G, x^i\right)$ is strictly increasing and strictly concave in both $\left(G, x^i\right)$ and is twice continuously differentiable. To ensure that the private consumption level of the poor citizens is always strictly positive (i.e. $x^P > 0$), the following boundary condition is being imposed: $\lim_{x^P \to 0} \frac{\partial_2 U^P\left(G, x^P\right)}{\partial x^P} = \infty$.

Before proceeding, we find it very important to address the following points explicitly. First, in this static framework, since the newly issued public debt is included in the national resource, I, which is to be splitted between the elites and the poor citizens in bargaining, granting a debt relief to such country will lead to an expansion in the national resource. Second, we assume that the limit on the amount of new public debt the country is allowed to issue, $\tilde{D}$, is exogenously determined by the external funding agency at the beginning of the period. Third, even though there are several approaches in which we can use to study the problem of *domestic distributive conflict* between the two classes in the society, in this section, we use the generalised Nash bargaining approach because this framework is easy to work with; it possesses sound strategic foundations and it allows us to study how the outcome of a bargaining situation may be influenced by the distribution of power between the two classes and the granting of debt relief.

If the two classes fail to reach agreement in bargaining, then class i obtains a "disagreement income" d^i which we assume to be a fraction of the national resource, i.e. $d^i = \tilde{d}^i I$, where $\tilde{d}^i \in (0, 1)$. We assume that the fractions, $\tilde{d}^E$ and $\tilde{d}^P$, are exogenously determined and $\tilde{d}^E > \tilde{d}^P$. When no agreement can be reached, the level of public good provision is zero and class i spends all the disagreement income on his private consumption. It follows that class i obtains a "disagreement utility" of d_u^i, where $d_u^i = U^i\left(0, \tilde{d}^i I\right)$. Throughout this paper, we refer to the disagreement utility of class i, $U^i\left(0, \tilde{d}^i I\right)$, as d_u^i, $u_i\left(d^i\right)$ or $u_i\left(\tilde{d}^i I\right)$, interchangeably. Since we assume that $\tilde{d}^E > \tilde{d}^P$, it follows that $d_u^E > d_u^P$ thus the elites' disagreement utility is higher than the poor citizens' and our justification is as follows. The elites usually have access to the "outside option", which allows them to opt out when no agreement can be reached in the bargaining. A "mutually beneficial agreement" exists if $\left(x^E, x^P, G\right) \in \Omega$ such that $U^E\left(G, x^E\right) > d_u^E$ and $U^P\left(G, x^P\right) > d_u^P$.

The utility pair, $d_u = \left(d_u^E, d_u^P\right)$, is known as the "disagreement point". In order to define the generalised Nash bargaining solution of this bargaining situation, it is useful to first define the set $\Psi = \left\{\left(u_E, u_P\right) : \text{there exists } \left(x^E, x^P, G\right) \in \Omega \text{ such that } U^E\left(G, x^E\right) = u_E \text{ and } U^P\left(G, x^P\right) = u_P\right\}$.

Fix an arbitrary utility, u_E, to the elites, where $u_E \in \left[\underline{U}^E, \bar{U}^E\right]$.

This range of elites' utility (which characterises the domain of the poor citizens' indirect utility function), $\left[\underline{U}^E, \bar{U}^E\right]$, is derived as follows. The lower bound, $\underline{U}^E$, is a solution to the following maximisation problem:

$$\underline{U}^E = \arg\min_{G,x^E} U^E\left(G, x^E\right), \tag{1}$$

subject to $x^E + x^P + cG \leq I$, $G \geq 0$, $x^E \geq 0$ and $x^P > 0$. When $x^P = I$, it follows from the budget constraint that the corner solutions, $G = 0$ and $x^E = 0$, result. It follows that $\underline{U}^E = U^E(0,0)$. The upper bound, $\bar{U}^E$, is a solution to the following problem:

$$\bar{U}^E = \arg\max_{G,x^E} U^E\left(G, x^E\right), \tag{2}$$

subject to $x^E + x^P + cG \leq I$, $G \geq 0$, $x^E \geq 0$ and $x^P > 0$. When $x^P = 0$, the elites make the policy choices for $\left(G, x^E\right)$ in such a way that $U^E\left(G, x^E\right)$ is maximised. Let the optimal choices for $\left(G, x^E\right)$ be represented by $\left(G_{op}, x^E_{op}\right)$. It follows that $\bar{U}^E = U^E\left(G_{op}, x^E_{op}\right)$.

For an arbitrary utility of the elites, $u_E \in \left[\underline{U}^E, \bar{U}^E\right]$, the utility maximisation for the poor citizens is given by

$$\max_{G,x^P} U^P\left(G, x^P\right), \tag{3}$$

subject to $x^E + x^P + cG \leq I$, $G \geq 0$, $x^E \geq 0$, $x^P > 0$ and $U^E\left(G, x^E\right) \geq d^E_u$. The constraints $x^E + x^P + cG \leq I$, $x^E \geq 0$, $x^P > 0$ and $G \geq 0$ can be represented by the "admissible set" Φ_1 while the elites' participation constraint, $U^E\left(G, x^E\right) \geq d^E_u$, can represented by the set Φ_2. Thus, for a given $u_E \in \left[\underline{U}^E, \bar{U}^E\right]$, the poor citizens' indirect utility is given by:

$$u_P = h\left(I, u_E\right) = \max_{G,x^P}\left\{U^P\left(G, x^P\right) \text{ s.t. } \Phi_1 \text{ and } \Phi_2\right\}. \tag{4}$$

It follows from (4) that $h\left(I, u_E\right)$ represents the utility the poor citizens obtain when the elites obtain the utility $u_E \in \left[\underline{U}^E, \bar{U}^E\right]$. It immediately follows that the set of utility possibility frontier is given by

$$\Psi = \left\{\left(u_E, u_P\right) : u_E \in \left[\underline{U}^E, \bar{U}^E\right] \text{ and } u_P = h\left(I, u_E\right)\right\}, \tag{5}$$

that is Ψ is the graph of the function $h : \left[\underline{U}^E, \bar{U}^E\right] \longrightarrow \Re$. Even though, as long as the utility functions for the elites and the poor citizens are well-behaved, the properties of the poor citizens' indirect utility function $h\left(I, u_E\right)$ are standard, for future reference, we summarise such properties in *Lemma 1* below.

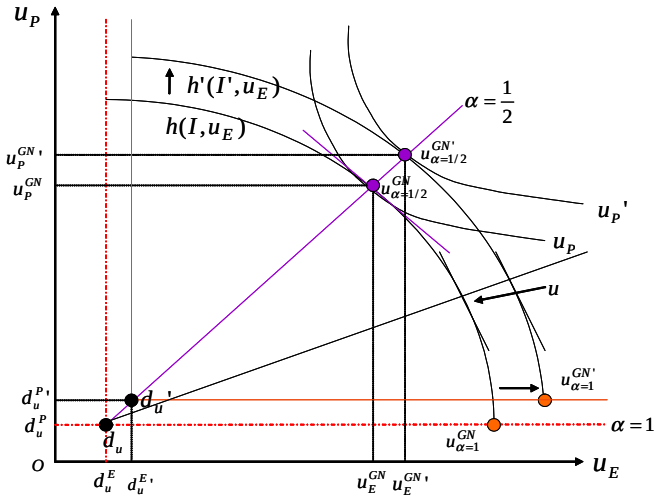

Figure 1: The utility possibility frontier in the static model

Lemma 1 *For a fixed I, the function $h(I, u_E)$ is strictly decreasing in u_E and concave. An increase in I leads to an affine transformation of the graph of function $h(I, u_E)$.*

Proof. See appendix. ■

The utility possibility frontier, $h(I, u_E)$, is depicted in Figure 1.

The utility possibility frontier shown in Figure 1 is strictly decreasing in u_E and concave. Moreover, an increase in I results in an affine transformation or an outward parallel shift in the frontier as discussed in *Lemma 1.*

In what follows, we define the generalised Nash bargaining solution of the bargaining situation described above. For each $\alpha \in [0, 1]$, the generalised (or, asymmetric) Nash bargaining solution[3] is a function

[3]Let us define the bargaining problem in a similar way as in Muthoo(1999, p.22). A bargaining problem is a pair (Ψ, d_u), where $\Psi \subset \Re^2$ and $d_u \subset \Re^2$, where Ψ is the set of possible utility pairs obtainable through agreement and $d_u = \left(d_u^E, d_u^P\right)$ is the disagreement point (or the utility pair obtainable if the players fail to reach agreement. If $(u_E, u_P) \in \Psi$, it follows that there exists an agreement which gives class i, where $i = E, P$, a utility $u_i \in \Re$. The bargaining problem should satisfy the following two assumptions:

Assumption 1 The Pareto frontier Ψ^{eff} of the set Ψ is the graph of a concave function, denoted by h, whose domain is a closed interval $\left[\underline{U}^E, \bar{U}^E\right] \subseteq \Re$. There exists $u_E \in \left[\underline{U}^E, \bar{U}^E\right]$ such that $u_E > d_u^E$ and $u_P > d_u^P$.

Assumption 2 The set Ψ^w of weakly Pareto efficient utility pairs is closed. A utility pair $(u_E, u_P) \in \Psi^w$ if and only if $(u_E, u_P) \in \Psi$ and there does not exist another utility pair $(u'_E, u'_P) \in \Psi$ such that $u'_E > u_E$ and $u'_P > u_P$. Note that $\Psi^w \subseteq \Psi^{eff}$.

$u_\alpha^{GN} : \Sigma \longrightarrow \Re^2$, defined as follows. For each $(\Psi, d_u) \in \Sigma$, $u_\alpha^{GN}(\Psi, d_u)$ is the unique solution to the following maximisation problem[4]:

$$\max_{(u_E, u_P) \in \Theta} \left(u_E - d_u^E\right)^\alpha \left(u_P - d_u^P\right)^{1-\alpha}, \tag{6}$$

where $\Theta \equiv \left\{(u_E, u_P) \in \Psi^{eff} : u_E \geq d_u^E \text{ and } u_P \geq d_u^P\right\}$ and $\Sigma \equiv \{(\Psi, d_u) : \Psi \subset \Re^2, d_u \in \Re^2$ and the pair (Ψ, d_u) satisfies Assumption 1 and 2$\}$.

The maximand of the generalised Nash bargaining problem (sometimes referred to as the *Nash product*)stated in (6) can be rewritten as $\left(u_E - d_u^E\right)^\alpha \left(u_P - d_u^P\right)^{1-\alpha} = k$ or

$$u_P = \left(\frac{k}{(u_E - d_u^E)^\alpha}\right)^{\frac{1}{1-\alpha}} + d_u^P, \tag{7}$$

for an arbitrary constant $k > 0$. The function u_P stated in equation (7) is quasiconcave and continuous. It is shown in Figure 1 as the curve labelled u_P.

The maximisation problem stated in (6) has a unique solution because the maximand is continuous and quasiconcave, $h(I, u_E)$ is strictly decreasing and concave as stated in *Lemma 1* and the set Θ is non-empty[5]. Figure 1 illustrates the generalised Nash bargaining solution. $u_{\alpha=1/2}^{GN}$ is the generalised Nash bargaining solution of the bargaining situation in which the set Ψ of possible utility pairs obtainable through agreement is the graph h and d_u is the disagreement point. Since $u_E^{GN} > d_u^E$ and $u_P^{GN} > d_u^P$, in the generalised Nash bargaining solution the two classes reach agreement on $\left(G^{GN}, x_E^{GN}, x_P^{GN}\right)$.

Claim 1 provides a characterisation of the generalised Nash bargaining solution of the bargaining situation described above (when function h is differentiable):

Claim 1 *For any $\alpha \in (0,1)$ and any bargaining problem $(\Psi, d_u) \in \Sigma$ such that $h(I, u_E)$ is differentiable, the generalised Nash bargaining solution is a unique solution to the following pairs of equations:*

$$-\frac{\partial_2 h(I, u_E)}{\partial u_E} = \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{u_P - d_u^P}{u_E - d_u^E}\right] \text{ and } u_P = h(I, u_E).$$

[4]In this paper, we do not model the bargaining power of the two classes but we instead capture it by the exogenous parameter α. We argue that α can be linked with the degree of impatience of the elites and the poor citizens. The class that is more impatient tends to have a higher bargaining power. We assume that the elites are less impatient than the poor citizens when α is close to 1.

[5]There exists a continuum of utility pairs $(u_E, u_P) \in \Theta$ such that $u_E > d_u^E$ and $u_P > d_u^P$.

Proof. Since the interior generalised Nash bargaining solution exists if $u_E^{GN} > d_u^E$ and $u_P^{GN} > d_u^P$, it may be characterised by finding the value of u_E that maximises $\left(u_E - d_u^E\right)^{\alpha} \left(h\left(I, u_E\right) - d_u^P\right)^{1-\alpha}$. The first-order condition is given by:

$$\left[\begin{array}{c}(1-\alpha)\left(u_E - d_u^E\right)^{\alpha}\left(h\left(I, u_E\right) - d_u^P\right)^{-\alpha}\left(\frac{\partial_2 h(I,u_E)}{\partial u_E}\right) \\ +\alpha\left(h\left(I, u_E\right) - d_u^P\right)^{1-\alpha}\left(u_E - d_u^E\right)^{\alpha-1}\end{array}\right] = 0.$$

Substituting $u_P = h\left(I, u_E\right)$ and re-arranging yield:

$$-\frac{\partial_2 h\left(I, u_E\right)}{\partial u_E} = \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{u_P - d_u^P}{u_E - d_u^E}\right] \text{ and } u_P = h\left(I, u_E\right). \tag{8}$$

Then, *Claim 1* follows immediately from the first-order condition given in expression (8). ■

Note that the above characterisation of the generalised Nash bargaining solution is valid only for $\alpha \in (0, 1)$. When $\alpha = 0$ or 1, we obtain corner solutions for (u_E, u_P). When $\alpha = 0$, all the bargaining power is in the hand of the poor citizens thus the poor citizens tend to choose $\left(G, x^E\right)$ in such a way which ensures that the participation constraint of the elites bind thus $u_E = d_u^E$. On the other hand, when $\alpha = 1$, the elites have all the bargaining power and they will determine the policy choices on $\left(G, x^P\right)$ which ensure that the poor citizens' participation constraint binds.

It is also useful to present some geometric characterisation of the generalised Nash bargaining solution, which is valid when function h is differentiable and follows *Claim 1*. For a given $\alpha \in [0, 1]$, the generalised Nash bargaining solution is the unique point u_α^{GN} on the graph of h with the property that the gradient of the line connecting the points u_α^{GN} and d_u is equal to the absolute value of the gradient of the unique tangent to the graph of h at u_α^{GN}. This is shown in Figure 1.

In this static model, we study two scenarios, namely (i) when there is a balanced distribution of power between the two classes $\left(\alpha = \frac{1}{2}\right)$ and (ii) when all the bargaining power is in the hands of the elites ($\alpha = 1$). We do not consider here another polar case when all the power is in the hands of the poor citizens ($\alpha = 0$). Starting with the case when there is a balanced distribution of power between the two classes $\left(\alpha = \frac{1}{2}\right)$, the generalised Nash bargaining solution is point $u_{\alpha=1/2}^{GN}$. In this case, our result shows that $u_E^{GN} > d_u^E$ and $u_P^{GN} > d_u^P$ thus, in the society where the distribution of power between the two classes is balanced, both the elites and the poor citizens receive sufficient provision of public goods and private goods so their participation constraints do not bind in the equilibrium. Next, let us consider any point u on the graph of h to the

right of $u^{GN}_{\alpha=1/2}$ (when α increases). The gradient of the line joining points d_u and u has decreased relative to the gradient of the line joining points d_u and $u^{GN}_{\alpha=1/2}$, while the absolute value of the gradient of the tangent to the graph of h at u has increased relative to the absolute value of the slope of the tangent at $u^{GN}_{\alpha=1/2}$. Therefore, the gradient of $\overline{d_u u}$ is strictly lower than the absolute value of the gradient of the tangent at u. When $\alpha = 1$, the gradient of the line joining points d_u and $u^{GN}_{\alpha=1}$ (denoted by $\overline{d_u u^{GN}_{\alpha=1}}$) is equal to zero while the absolute value of the gradient of the tangent to the graph of h at point $u^{GN}_{\alpha=1}$ is infinity. When $\alpha = 1$, we find that $u^{GN}_E > d^E_u$ and $u^{GN}_P = d^P_u$. It follows that, when the elites have all the bargaining power, the elites determine the policy choices on $\left(G, x^E, x^P\right)$ subject to a binding participation constraint for the poor citizens.

Therefore, when we α increases from $\frac{1}{2}$ to 1, we observe in Figure 1 that the generalised Nash bargaining solution, u^{GN}_α, moves clockwise along the utility possibility frontier from point $u^{GN}_{\alpha=1/2}$ towards point $u^{GN}_{\alpha=1}$. What are the main implications? It is clear that the utility obtained by the elites at the equilibrium, u^{GN}_E, is increasing in α while the utility obtained by the poor citizens at the equilibrium, u^{GN}_P, is decreasing in α. In sum, when both u^{GN}_E and u^{GN}_P are continuous in α, it follows that

$$\lim_{\alpha\to\frac{1}{2}} u^{GN}_E > d^E_u,\ \lim_{\alpha\to 1} u^{GN}_E > d^E_u \text{ and } u^{GN}_{E,\alpha=1/2} < u^{GN}_{E,\alpha=1},$$

while

$$\lim_{\alpha\to\frac{1}{2}} u^{GN}_P > d^P_u,\ \lim_{\alpha\to 1} u^{GN}_P = d^P_u \text{ and } u^{GN}_{P,\alpha=1/2} > u^{GN}_{P,\alpha=1}.$$

These results show that the relative bargaining power of the elites (captured by parameter α) has a crucial impact on the utility obtained by the two classes at the equilibrium.

We have seen from the above discussion that, when all the bargaining power is in the hands of the elites, the elites make the policy choices on $\left(G, x^E, x^P\right)$ in such a way that most of the national resource is being directed to finance their own private consumption, at the same time, providing minimal amount of G and x^P to ensure that the participation constraint of the poor citizens is satisfied. This problem of inadequate provision of public goods and private goods for the poor citizens typically occurs in the society where most of the bargaining power is in the hands of the elites and it is the key factor behind the existence of "*domestic distributive conflict*" between the two classes in the society.

Next, we study the situation where the external funding agency decides to grant an *unconditional* debt write-down to the country. Our

objective is to show that the impact of an unconditional debt write-down varies between societies, depending on the distribution of power between the two classes. We want to show that, in the society where the elites have all the bargaining power and the problem of *domestic distributive conflict* exists, an unconditional debt write-down benefits the elites and it does not lead to a sufficient improvement in the well-being of the poor citizens. On the other hand, in the society with a balanced distribution of power between the two classes, an unconditional debt write-down leads to a significant improvement in the well-being of the two classes. We now turn to analyse each of these two scenarios in greater details.

Suppose the external funding agency grants an unconditional debt write-down to a country by writing down an amount $\varphi(1+r)D_{-1}$ with no condition attached, where $0 < \varphi \leq 1$. The national resource following an unconditional debt write-down is given by $I' = y + \tilde{D} - (1-\varphi)(1+r)D_{-1}$. For $0 < \varphi \leq 1$, it follows that $I < I'$. As stated in *Lemma 1*, an increase in the national resource, I, results in an affine transformation (an outward parallel shift) in the utility possibility frontier from the curve labelled $h(I, u_E)$ to the one labelled $h'(I', u_E)$. This is illustrated in Figure 1. Moreover, since d^i is assumed to be a fraction of I, an increase in I also causes the disagreement utility of both elites and poor citizens to increase from d_u^E and d_u^P to $d_u^{E\prime}$ and $d_u^{P\prime}$, respectively[6].

We now discuss the impact of an unconditional debt write-down under two scenarios: when $\alpha = \frac{1}{2}$ and when $\alpha = 1$. When $\alpha = \frac{1}{2}$, the generalised Nash bargaining solution is shown in Figure 1 as $u_{\alpha=1/2}^{GN} = \left(u_E^{GN}, u_P^{GN}\right)$. Note that $u_E^{GN} > d_u^E$ and $u_P^{GN} > d_u^P$. After an unconditional debt write-down has been granted, the elites' and poor citizens' utilities are given by $u_E^{GN\prime}$ and $u_P^{GN\prime}$, respectively. It is clear from Figure 1 that $u_E^{GN} < u_E^{GN\prime}$ and $u_P^{GN} < u_P^{GN\prime}$ thus the well-being of both classes improves after an unconditional debt write-down. Therefore, in the society where the distribution of power between the two classes is balanced (when $\alpha = \frac{1}{2}$ or close to $\frac{1}{2}$), the unconditional debt write-down is sufficient to ensure that the well-being of both classes improve.

When $\alpha = 1$, the generalised Nash bargaining solution is shown in Figure 1 as $u_{\alpha=1}^{GN} = \left(\bar{U}^E, d_u^P\right)$. Note that $u_E^{GN} > d_u^E$ but $u_P^{GN} = d_u^P$ thus the participation constraint of the poor citizens binds. After an unconditional debt write-down has been granted, the new generalised Nash bargaining solution is given by $u_{\alpha=1}^{GN\prime} = \left(\bar{U}^{E\prime}, d_u^{P\prime}\right)$. Even though the utility obtained by the poor citizens is higher than before as $d_u^P < d_u^{P\prime}$, the unconditional debt write-down is still not sufficient to ensure that the well-being of the poor citizens improves since the policy choices of the

[6]The rationale for imposing this assumption is discussed in section 4.

elites on $\left(G, x^P\right)$ are still low, implying that their participation constraint still binds in the equilibrium. Therefore, in the society where the elites have all the bargaining power and the problem of *domestic distributive conflict* exists (when $\alpha = 1$ or close to 1), it is crucial to make sure that the appropriately designed conditionality requirements be attached to the debt write-downs. Details of how such conditionality requirements should be designed are discussed in the next section.

We summarise the above discussion with the following proposition:

Proposition 1 *The relative utility gains of the two classes from the debt write-down depends crucially on the distribution of power between the elites and the poor citizens. With a balanced distribution of power, an unconditional debt write-down is sufficient to ensure that the well-being of both classes improve. When the problem of domestic distributive conflict between the two classes is prevalent, it is crucial that the appropriately designed conditionality requirements be attached to the debt write-down.*

3 Conditionality requirements under the static model

In the previous section, we find that when elites have all the bargaining power and the problem of *domestic distributive conflict* is prevalent, an unconditional debt write-down is ineffective as the provision of public goods and private consumption level of the poor citizens are still considered inadequate. Moreover, the poor citizens' participation constraint always binds. In this section, we focus on the scenario where all the bargaining power is in the hands of the elites ($\alpha = 1$). We then study the design of conditionality requirements attached to the debt write-down under this scenario. Our objective is to ensure that the poor citizens' well-being improves after the debt write-down has been granted.

When $\alpha = 1$, the generalised Nash bargaining is given by the following maximisation problem:

$$\max_{x^E, x^P, G} U^E\left(G, x^E\right) - d_u^E,$$

subject to $G \geq 0$, $x^E \geq 0$, $x^P \geq 0$ and $x^E + x^P + cG \leq I$ and $U^P\left(G, x^P\right) \geq d_u^P$.

Since $U^E\left(G, x^E\right)$ is strictly increasing in both G and x^E, the budget constraint binds, that is, $x^E + x^P + cG = I$. The elites' private consumption, x^E, can be rewritten as $x^E = I - cG - x^P$; the non-negativity constraint on the elites' private consumption, $x^E \geq 0$, can be rewritten as $\left(I - cG - x^P\right) \geq 0$; and the elites' utility function, $U^E\left(G, x^E\right)$, can

be rewritten as $U^E\left(G, I-cG-x^P\right)$. The maximisation problem is now given by[7]:

$$\max_{x^E,x^P,G} U^E\left(G, I-cG-x^P\right)-d_u^E,$$

subject to $G \geq 0$, $I-cG-x^P \geq 0$, $x^P > 0$, and $U^P\left(G, x^P\right) = d_u^P$.

To solve the above maximisation problem, we form the Lagrangian function:

$$\begin{aligned}\mathcal{L} = & U^E\left(G, I-cG-x^P\right)-d_u^E+\lambda_1\left[U^P\left(G, x^P\right)-d_u^P\right] \\ & +\lambda_2\left[I-cG-x^P\right],\end{aligned}$$

where λ_1 and λ_2 are the Lagrange multipliers.

The Kuhn-Tucker conditions are given by:

$$\frac{\partial \mathcal{L}}{\partial G} = \left\{\begin{array}{c} \frac{\partial_1}{\partial G}U^E\left(G, I-cG-x^P\right)-c\frac{\partial_2}{\partial G}U^E\left(G, I-cG-x^P\right) \\ +\lambda_1\frac{\partial_1}{\partial G}U^P\left(G, x^P\right)-c\lambda_2 \end{array}\right\} = 0, \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial x^P} = -\frac{\partial_2}{\partial x^P}U^E\left(G, I-cG-x^P\right)+\lambda_1\frac{\partial_2}{\partial x^P}U^P\left(G, x^P\right)-\lambda_2 = 0, \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = U^P\left(G, x^P\right)-d_u^P = 0, \tag{11}$$

$$\lambda_2 \leq 0,\ \left(I-cG-x^P\right) \geq 0 \text{ and } \lambda_2\left(I-cG-x^P\right) = 0. \tag{12}$$

It is important to note that our objective is not to find the exact analytical solutions for the above maximisation problem. We, however, find it instructive and useful to note the following graphic characterisation of the solutions to the above maximisation problem. The graphic characterisations for the Kuhn-Tucker conditions (11) and (12) are discussed below and are given in Figure 2.

We begin with the Kuhn-Tucker condition (11). Since $x^P > 0$, it follows that the participation constraint of the poor citizens always binds, that is $U^P\left(G, x^P\right) = d_u^P$. This binding participation constraint for the poor citizens is shown in Figure 2 as a standard indifference curve, which is strictly convex and has a diminishing marginal rate of substitution.

Next, we consider the Kuhn-Tucker condition (12). The non-negativity constraint on x^E is subject to a complementary slackness in relation to the partial derivative of the Lagrangian function, that is $\left(\frac{\partial \mathcal{L}}{\partial\left(I-cG-x^P\right)}\right)\left(I-cG-x^P\right) = 0$. It follows that (i) when $I-cG-x^P > 0$, $\frac{\partial \mathcal{L}}{\partial\left(I-cG-x^P\right)} = 0$, while (ii)

[7]By imposing the boundary condition, $\lim_{x^P \longrightarrow 0}\frac{\partial_2 U^P\left(G, x^P\right)}{\partial x^P} = \infty$, this ensures that $x^P > 0$.

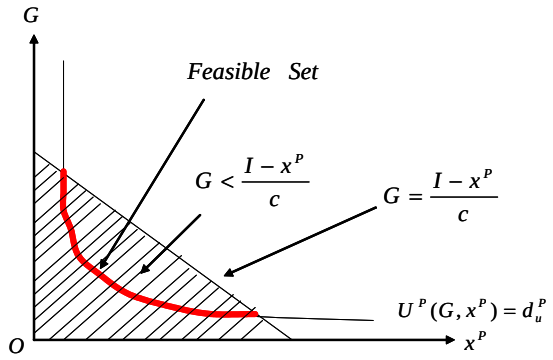

Figure 2: The feasible set

when $I - cG - x^P = 0$, $\frac{\partial \mathcal{L}}{\partial (I - cG - x^P)} < 0$. When the non-negativity constraint on x^E does not bind (i.e. $I - cG - x^P > 0$) this constraint can be represented by the area below the line labelled $G = \frac{I - x^P}{c}$ in Figure 2. However, when the non-negativity constraint binds (i.e. $I - cG - x^P = 0$), it can be represented by the line, $G = \frac{I - x^P}{c}$, itself. Combining the non-negativity constraint on the elites' private consumption with the poor citizens' participation constraint yields the "feasible set" as shown in Figure 2.

The analytical solutions for $\left(G, x^E, x^P\right)$ must satisfy the Kuhn-Tucker conditions given in (9) to (12). The graphic solutions for $\left(G, x^E, x^P\right)$ depend crucially on where the elites' indifference curve intersects the feasible set and the shape of the elites' indifference curve plays a key role. Unfortunately, since the utility function of the elites takes the form of $U^E\left(G, I - cG - x^P\right)$, there is no unique shape for the elites' indifference curve. Therefore, no elites' indifference curve is included in Figure 3 and in other figures throughout this section.

The corner solution exists when the elites' indifference curve intersects the feasible set at either end of the feasible set, i.e. NG_1 or NG_2, in Figure 3. The corner solutions – both NG_1 and NG_2 – are known as the "*non-generic*" cases. On the contrary, the interior solution exists when the elites' indifference curve intersects the feasible set on the interior part of the feasible set. The interior solutions – such as GE – are known as the "*generic*" cases[8]. We begin our discussion with the two *non-generic* cases, namely NG_1 and NG_2, followed by the *generic* case, GE.

When the first *non-generic* case, NG_1, exists, the policy choices for

[8]In fact, there are a continuum of $\left(G, x^E, x^P\right)$ such that the interior solutions exist.

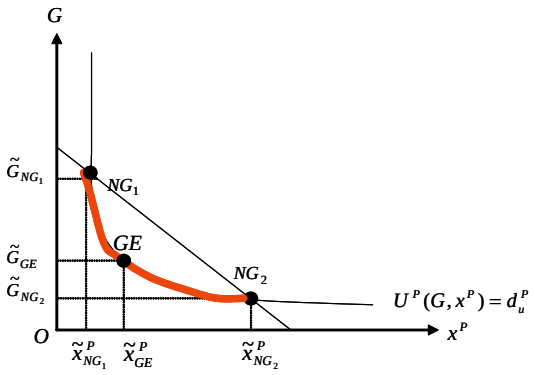

Figure 3: The *non-generic* case vs the *generic* case

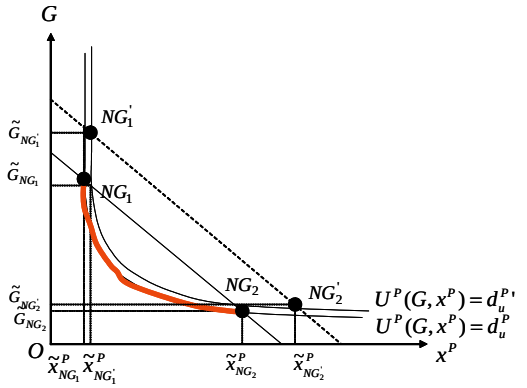

Figure 4: The impact of an unconditional debt write-down under the *non-generic* cases

$\left(G, x^P\right)$ determined by the elites are shown in Figure 3 and Figure 4 as $\tilde{G}_{NG_1}$ and $\tilde{x}^P_{NG_1}$, respectively. What is the impact of an unconditional debt write-down when NG_1 exists? First, an unconditional debt write-down causes the national resource to increase from I to I', which in turn leads to an expansion in elites' non-negativity constraint set as the line labelled $G = \frac{I - x^P}{c}$ shifts to the dashed line labelled $G = \frac{I' - x^P}{c}$ in Figure 4. Moreover, since the disagreement income of the poor citizens, d^P, is proportional to I, an unconditional debt write-down also causes the poor citizens' disagreement income and disagreement utility to increase to $d^{P\prime}$ and $d_u^{P\prime}$, respectively. This leads to a rightward shift in the poor citizens' indifference curve from the curve labelled $U^P\left(G, x^P\right) = d_u^P$ to the one labelled $U^P\left(G, x^P\right) = d_u^{P\prime}$ in Figure 4.

In Figure 4, it is obvious that, under NG_1, the elites use the money freed up by the debt write-down to increase the provision of public goods

from $\tilde{G}_{NG_1}$ to $\tilde{G}_{NG_1'}$, and raise the private consumption level of the poor citizens from $\tilde{x}^P_{NG_1}$ to $\tilde{x}^P_{NG_1'}$. Even though both public good provision and private consumption level of poor citizens increase after the external funding agency granted an unconditional debt write-down, both new policy choices for $\left(G, x^P\right)$ made by the elites still subject the poor citizens to a binding participation constraint. In other words, the poor citizens' utility is still being reduced to the disagreement utility shown in Figure 4 as $d_u^{P\prime}$. It is important to note that $d_u^{P\prime}$ is still considered to be low relative target utility (which allows the MDGs to be achieved) and inadequate level of poor citizens' private consumption is perhaps the key explanation. This suggests that, under the first *non-generic* case NG_1, the conditionality requirement that needs to be attached to the debt write-down should target the private consumption level of the poor citizens.

To ensure that the well-being of the poor citizens improves after the granting of debt write-down, it is crucial that we determine the target level of poor citizens' utility, which is consistent with the MDGs[9]. Suppose that the target level of poor citizens' utility is denoted by $\hat{d}_u^P$. Fixing the level of public goods at $\tilde{G}_{NG_1'}$, if $\underline{x}^P_{NG_1}$ is the solution to the equation $U^P\left(\tilde{G}_{NG_1'}, x^P\right) = \hat{d}_u^P$, $\underline{x}^P_{NG_1}$ will be the level of private consumption for the poor citizens which allows the target level of poor citizens' utility, $\hat{d}_u^P$, to be achieved. The conditionality requirement attached to the debt write-down under the first *non-generic* case, therefore, should take the form of $x^P \geq \underline{x}^P_{NG_1}$. We assume that the target level of poor citizens' utility, $\hat{d}_u^P$, is directly indexed to the conditionality $x^P \geq \underline{x}^P_{NG_1}$ and $\tilde{G}_{NG_1'}$. When G increases from $\tilde{G}_{NG_1}$ to $\tilde{G}_{NG_1'}$ and if the conditionality, $x^P \geq \underline{x}^P_{NG_1}$, is satisfied, $\hat{d}_u^P$ will be attained. When this is the case, the external funding agency is assured that granting debt write-down to the country will result in an improvement in the well-being of the poor citizens in such country.

Next, we study the design of conditionality requirement under the second *non-generic* case, which is characterised by point NG_2 in Figure 4. The policy choices for $\left(G, x^P\right)$ determined by the elites are shown in Figure 4 as $\tilde{G}_{NG_2}$ and $\tilde{x}^P_{NG_2}$, respectively. Similar to the first *non-generic* case, an unconditional debt write-down causes an expansion in the non-negativity constraint set for the elites' private consumption, resulting in an outward parallel shift in the line $G = \frac{I - x^P}{c}$ to $G = \frac{I' - x^P}{c}$, and a

[9] The idea we try to put forward here is similar to the operation of the MDGs. We begin by setting goals, which aim at eradicating poverty and improving the well-being of the poor, and follow through with measures that can be expected to achieve those goals (including actions by both developing and industrialised countries)

rightward shift in the poor citizens' indifference curve[10], from the curve labelled $U^P\left(G, x^P\right) = d_u^P$ to the one labelled $U^P\left(G, x^P\right) = d_u^{P\prime}$. The policy choices for $\left(G, x^P\right)$ determined by the elites following an unconditional debt write-down are given by $\tilde{G}_{NG_2'}$ and $\tilde{x}^P_{NG_2'}$, respectively. The elites still make choices on $\tilde{G}_{NG_2'}$ and $\tilde{x}^P_{NG_2'}$ in such a way which ensures that the poor citizens' participation constraint binds. Considering these new policy choices for $\left(G, x^P\right)$, it is clear that there is a large increase in x^P but a very small increase in G. Although compared to the situation without an unconditional debt write-down, the poor citizens' utility is now higher than before, the poor citizens' utility is still being reduced to the disagreement level, $d_u^{P\prime}$, which might be resulted from an insufficient provision of public goods.

It follows that, under the second *non-generic* case NG_2, the debt write-down granted to such country should target the provision of public goods. Along the same lines as the first *non-generic* case, the external funding agency needs to set the target level of poor citizens' utility, which is linked to the MDGs. Given the target level of poor citizens' utility $\hat{d}_u^P$ and their private consumption $\tilde{x}^P_{NG_2'}$, let $\underline{G}_{NG_2}$ be the solution to the equation $U^P\left(G, \tilde{x}^P_{NG_2'}\right) = \hat{d}_u^P$ thus $\underline{G}_{NG_2}$ is the level of public goods which will ensure that $\hat{d}_u^P$ being achieved. Under NG_2, the conditionality requirement attached to the debt write-down should take the form of $G \geq \underline{G}_{NG_2}$. We assume here that the the target level of the poor citizens' utility, $\hat{d}_u^P$, is directly linked to $G \geq \underline{G}_{NG_2}$ and $\tilde{x}^P_{NG_2'}$ thus when x^P increases to $\tilde{x}^P_{NG_2'}$ and when the conditionality, $G \geq \underline{G}_{NG_2}$, is satisfied, $\hat{d}_u^P$ will be achieved. As a result, there will be an improvement in the well-being of the poor citizens.

In what follows, we discuss the *generic* case, which is shown in Figure 3 as point GE. Even though in fact there exist a continuum of pairs $\left(G, x^P\right)$ such that the point of intersection between the elites' indifference curve and the feasible set lies within the interior part of the feasible set, for the purpose of illustration, we include in Figure 3 one pair of $\left(G, x^P\right)$ labelled $\left(\tilde{G}_{GE}, \tilde{x}^P_{GE}\right)$. Corresponding to the policy choices $\left(\tilde{G}_{GE}, \tilde{x}^P_{GE}\right)$, the poor citizens' participation constraint binds as the utility obtained by the poor is at the disagreement utility, d_u^P.

Next, we analyse the impact of an unconditional debt write-down on the well-being of the poor citizens. An unconditional debt write-down results in an increase in the national resource from I to I'. This

[10] This is due to the assumption that $d^P = \tilde{d}^P I$ and $d_u^P = u_P\left(\tilde{d}^P I\right)$; therefore, when the national resource increases from I to I', d_u^P also increases.

causes the non-negativity constraint on the elites' private consumption to expand, leading to an outward parallel shift in the line $G = \frac{I - x^P}{c}$ to $G = \frac{I' - x^P}{c}$. Moreover, it also causes a rightward shift in the poor citizens' indifference curve to a new curve labelled $U^P\left(G, x^P\right) = d_u^{P\prime}$. Even though the poor citizens receive a higher utility than before as $d_u^{P\prime} > d_u^P$, such new level of poor citizens' utility, $d_u^{P\prime}$, is still considered to be low relative to one which is compatible with the MDGs. This supports our result from the previous section that, when $\alpha = 1$ and under the *generic* cases, the *domestic distributive conflict* exists as the elites always have incentive to provide minimal amount of public goods and private consumption to the poor citizens – the levels which are just sufficient to ensure that poor citizens' participation constraint binds. Therefore, for the society where the problem of *domestic distributive conflict* is prevalent, under the *generic* case, the unconditional debt write-down is ineffective.

When improvement in the well-being of the poor citizens is the main objective of the external funding agency for granting debt write-down to the low-income country and since the external funding agency cannot make a direct transfer of utility to the poor citizens, the type of conditional debt write-down and the details of conditionality requirements attached to it matter. We now discuss different types of conditional debt write-down under the *generic* case.

We begin with the first type of conditional debt write-down, where the conditionality requirement targets the provision of public goods. First, the external funding agency determines the target level of poor citizens' utility, $\hat{d}_u^P$. Given $\hat{d}_u^P$ and fixing the level of x^P at $\tilde{x}_{GE}^P$, let $\underline{G}$ be the solution to the equation $U^P\left(G, \tilde{x}_{GE}^P\right) = \hat{d}_u^P$. If $\hat{d}_u^P$ is directly indexed by $G \geq \underline{G}$ and $\tilde{x}_{GE}^P$, when the conditionality $G \geq \underline{G}$ is satisfied and when x^P remains at $\tilde{x}_{GE}^P$, the target level of poor citizens' utility, $\hat{d}_u^P$, will be achieved. However, this is not consistent with the elites' incentive. Consider the point GE'. Without imposing any condition on x^P, the elites have a strategic incentive to reduce expenditure on poor citizens' private consumption from $\tilde{x}_{GE}^P$ to $\tilde{x}_{GE'}^P$. The elites then use the additional national resource freed up from a reduction in x^P to support the additional provision of public goods (equals to $\Delta G = \underline{G} - \tilde{G}_{GE}$) to the condition imposed by the external funding agency. The elites then divert all the national resources freed up by debt write-down to increase in their private consumption, x^E. When this is the case, the target level of poor citizens' utility, $\hat{d}_u^P$, cannot be achieved. As a result, this first type of conditional debt write-down is ineffective in improving the well-being of the poor citizens. This is illustrated in Figure 5.

Next, we consider the second type of conditional debt write-down,

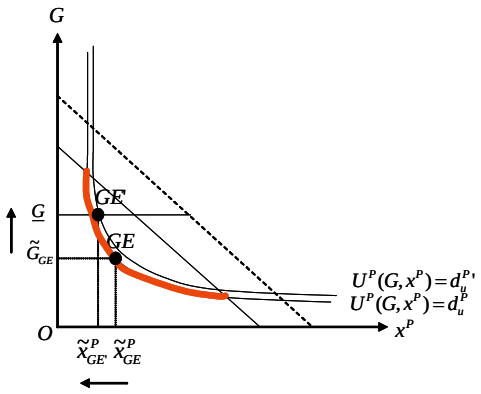

Figure 5: The first type of conditional debt write-down

where the conditionality requirement attached to debt write-down targets the private consumption level of the poor citizens. Given the target level of poor citizens' utility, $\hat{d}_u^P$ and assuming that G is constant at $\tilde{G}_{GE}$, let $\underline{x}^P$ denote the solution to the equation $U^P\left(\tilde{G}_{GE}, x^P\right) = \hat{d}_u^P$. When $\hat{d}_u^P$ is directly indexed by the conditionality $x^P \geq \underline{x}^P$ and $\tilde{G}_{GE}$; when the conditionality is satisfied; and if G remains at $\tilde{G}_{GE}$, the target level of poor citizens' utility, $\hat{d}_u^P$, will be achieved. However, this is not likely to be the case as the elites do not have an incentive to make policy choices consistent with what we predict. Without imposing any condition on the provision of public goods, the elites have a strategic incentive to reduce G. By doing so, the elites can then use the national resource freed up by a reduction in G (equal to $\Delta G = \tilde{G}_{GE} - \tilde{G}_{GE'}$) to finance the additional private consumption of the poor citizens required by the external funding agency, given by $\Delta x^P = \underline{x}^P - \tilde{x}^P_{GE}$, and divert all the additional national resources freed up by the debt write-down, $\Delta I = I' - I$, to support their private consumption. Consequently, there is only a slight increase in the poor citizens' utility from d_u^P to $d_u^{P\prime}$ thus the target level of poor' utility, $\hat{d}_u^P$, cannot be achieved. Hence, this second type of conditional debt write-down is also ineffective in improving the well-being of the poor citizens. This is shown in Figure 6.

Finally, we consider the third type of conditional debt write-down, where the conditionality requirements target both provision of public goods and private consumption level of the poor citizens. First, the target level of poor citizens' utility is set by the external funding agency to be at $\hat{d}_u^P$. Given $\hat{d}_u^P$, the external funding agency determine the policy choices for $\left(G, x^P\right)$ by finding the values of $\left(G, x^P\right)$ that solve the equation $U^P\left(G, x^P\right) = \hat{d}_u^P$. Let $\underline{G}$ and $\underline{x}^P$ denote the solutions to the

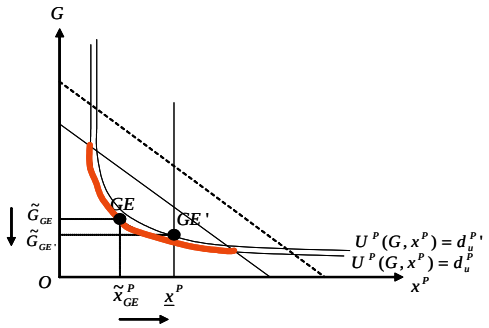

Figure 6: The second type of conditional debt write-down

above equation. When $\hat{d}_u^P$ is directly indexed by $G \geq \underline{G}$ and $x^P \geq \underline{x}^P$ and when both conditionality requirements are satisfied, it is clear that $\hat{d}_u^P$ will be achieved. This is illustrated in Figure 7. From the figure, the poor citizens' indifference curve shift rightwards from the curve labelled $U^P\left(G, x^P\right) = d_u^P$ to the one labelled $U^P\left(\underline{G}, \underline{x}^P\right) = \hat{d}_u^P$ (showing that there is an improvement in the well-being of the poor citizens to the target level) as there is an increase in both G and x^P to the levels required by the external funding agency. Thus, for the society where the elites have all the bargaining power and the problem of *domestic distributive conflict* between the elites and the poor citizens is prevalent, it is crucial that the conditionality requirements attached to the debt write-down under the *generic* case target both provision of public goods and private consumption level of the poor citizens[11].

Before proceeding to the next section, we want to clarify the following points. It is important to note that the elites actually have two options opened to them. The first option is not to accept the debt write-down contract offered by the external funding agencies. If the elites choose this option, they can ensure that the poor citizens' participation constraint binds as before and, at the same time, ensure that their own private consumption is high. Let $U^E\left(G, I - cG^* - x^{P*}\right)$ denote the elites' indirect utility if they decide not to accept the debt write-down, where

[11] In this model, the relative bargaining power of the two classes, captured by the parameter α, has been taken as given; therefore, to ensure that the utility of the poor citizens increases to the target level, it is crucial that the bargaining power is tilted towards the poor citizens. This is achieved by making the conditionality requirements attached to debt write-down target both provision of public goods and private consumption level of the poor. It is interesting to note that the conditionality requirement, which can lead to a decrease in α, can also lead us to the same end. An example of such conditionality requirement includes the policy which supports democracy. Such policy can tilt the bargaining power towards the poor.

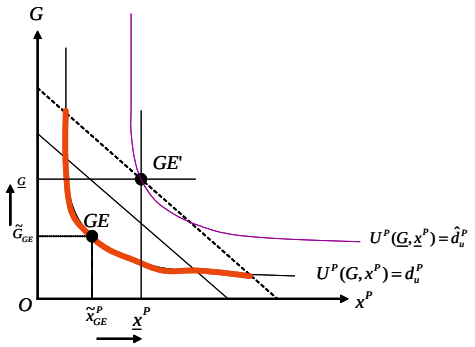

Figure 7: The third type of conditional debt write-down

G^*, x^{E*} and x^{P*} denote the solutions to the maximisation problem presented at the beginning of this section and they satisfy the Kuhn-Tucker conditions (9) to (12). The second option that is opened to the elites is to accept the debt write-down contract and the two conditionality requirements attached to it, that is $G \geq \underline{G}$ and $x^P \geq \underline{x}^P$. By accepting the debt write-down contract, on one hands, the elites become better off as the national resource (I) becomes larger. On the other hand, if the external funding agency sets $\underline{G}$ and $\underline{x}^P$ to be too high, the private consumption of the elites might turn out to be lower than the amount they would otherwise receive if they choose not to accept the debt write-down contract, i.e. x^{E*}. Therefore, in order to ensure the elites have an incentive to accept such debt write-down contract, it is very crucial that the following incentive compatibility constraint for the elites be satisfied: $U^E\left(G, I - c\underline{G} - \underline{x}^P\right) \geq U^E\left(G, I - cG^* - x^{P*}\right)$, where $U^E\left(G, I - c\underline{G} - \underline{x}^P\right)$ denotes the utility the elites obtain if they accept the debt write-down contract. To ensure that this constraint is satisfied, the external funding agency should make sure that the minimum requirements on the public good provision and private consumption level of the poor citizens should not be too high; otherwise, the elites will choose not to participate in the debt write-down after all.

There are other three important issues that must be explicitly addressed: "*enforceability*", "*verifiability*" and "monitoring". The idea of linking debt write-down to public good provision and private consumption level of the poor citizens is attractive; however, such conditional debt write-down contract will not be implementable if the conditions on provision of public goods and private consumption level of the poor citizens cannot be enforced and the implementation of such conditions by recipient government is difficult to be verified by the external funding agency. The question that usually arises is what are the public goods

and private goods of the poor citizens meant by the external funding agency in the debt write-down contract? Moreover, once the elites accepted the debt write-down contract and the conditionality requirements attached to it, how can the external funding agency monitor and verify that the elites actually comply with those conditionality requirements? These issues of verifiability, enforceability and monitoring remain to be investigated further in the future research.

We summarise the above discussion with the following proposition:

Proposition 2 *For the society where the elites have all the bargaining power and the problem of domestic distributive conflict is prevalent, it is crucial that the conditionality requirements attached to the debt write-down target both provision of public goods and private consumption level of the poor citizens.*

4 The dynamic model

In this section, we extend the model to a dynamic framework, which allows us to capture another important aspect of poor governance, namely the problem of "*intertemporal conflict*". With this framework, we show that the elites have a dynamic incentive to issue excessive public debts to get around the conditionality requirements recommended by the static model, which consist of $G \geq \underline{G}$ and $x^P \geq \underline{x}^P$ (hereafter, known as the "*static-* or *short-run conditionality requirements*").

We use a two-period repeated game framework where, in each period, the elites and the poor citizens bargain over the partition of the national resource. In any period t, if the elites and the poor citizens fail to reach agreement, class i obtains a disagreement income $d_t^i = \tilde{d}^i I_t$, where $\tilde{d}^i \in (0,1)$ and $t = 1, 2$. We still assume that the fractions, $\tilde{d}^E$ and $\tilde{d}^P$, are exogenously determined and $\tilde{d}^E > \tilde{d}^P$. When no agreement can be reached, the level of public good provision is zero and class i spends all the disagreement income on his private consumption. It follows that class i obtains a "disagreement utility" of $d_{u,t}^i$, where $d_{u,t}^i = U^i\left(0, \tilde{d}^i I_t\right)$, which can also be denoted by $u_i\left(d_t^i\right)$ or $u_i\left(\tilde{d}^i I_t\right)$, interchangeably. Since we assume that $\tilde{d}_t^E > \tilde{d}_t^P$, it follows that $d_{u,t}^E > d_{u,t}^P$ thus the elites' disagreement utility is higher than the poor citizens'. We focus on the scenario where the elites have all the bargaining power ($\alpha = 1$); therefore, they are entitled to make decisions on the level of public goods provision, private consumption for the poor citizens , their own private consumption level, and the amount of public debts to be issued in each period. A "mutually beneficial agreement" exists if $\left(x^E, x^P, G\right) \in \Omega$

such that $U^E\left(G_t, x_t^E\right) > d_{u,t}^E$ and $U^P\left(G_t, x_t^P\right) > d_{u,t}^P$. The utility pair, $d_{u,t} = \left(d_{u,t}^E, d_{u,t}^P\right)$, is known as the "disagreement point".

Let $D_0 > 0$ denote the exogenously given stock of public debt at the beginning of $t = 1$. The debt obligation falling due at in the first period is given by $(1 + r)D_0$. Without allowing the elites to issue new public debts at $t = 1$, the national resource is given by $\tilde{I}_1 = y_1 - (1 + r)D_0$. After allowing the elites to issue public debts with an amount D_1, the first-period national resource is given by $I_1 = \tilde{I}_1 + D_1$. Note that the external funding agency does not place any restriction on the amount of new public debts the elites are allowed to issue in the first period (that is, there is no limit on D_1).

Because of the uncertainty of their tenure and their myopic behaviour, we aim to show that the elites have a strong incentive to issue an excessive amount of public debts in the first period. It is very important to note that the resources generated from the issue of new public debts by the elites are not very likely being directed by the elites to support provision of public goods nor the poor citizens' private consumption but instead being used by them to finance their own private consumption. The policy choice on public debt made by the elites is very crucial as it determines the probability of default: the policy which leads to an excessive issue of public debts in the first period increases the probability of default. When default occurs, while the elites are insured against the adverse impact of default, the poor citizens bear a disproportionately high share of default costs as the country is permanently denied access to the international capital market from that point onwards and there is a permanent contraction in the national output. Such strategic incentive of the elites to issue excessive amount of public debts is the key explanation for the existence of *intertemporal conflict* between the elites and the poor citizens. We show in this section that, when the *intertemporal conflict* exists, it is necessary that the external funding agency puts an explicit limit on the amount of public debts the elites are allowed to issue in the first period together the imposition of the short-run conditionality requirements in both periods.

The national income at $t = 2$ is y_2. The national resource in the second period, I_2, is stochastic, depending on whether or not the elites choose to default in the second period. We assume that, if default occurs, there will be a permanent contraction in the national output by a fraction $(1 - \beta)$, where $\beta \in (0, 1)$, and the country will be permanently excluded from the international capital market from that period onwards.

For a given r, the probability of default is denoted by $p\left(D_1, r\right)$.[12] If

[12]It is important to note that the rate of interest, r, also has a big impact on the probability of default. Specifically, a higher interest rate makes it more difficult

the elites chooses to default at $t = 2$, an event which occurs with probability $p(D_1, r)$, the national resource in the second period is $I_2^D = \beta y_2$. On the contrary, if the elites choose to repay the debts, an event which occurs with probability $1-p(D_1, r)$, the second-period national resource is $I_2^{ND} = y_2 - (1+r)D_1 + \bar{D}_2$, where $\bar{D}_2$ is the limit the external funding agency placed on the amount of public debts the elites are allowed to issue in the second period. This limit on the second-period public debts helps prevent the elites from issuing an excessive amount of public debts in the second period and leaving the poor citizens saddled with a large amount of debts to be serviced.

Consider an arbitrary u_t^P. The possible values of u_t^P should lie in following closed interval[13] $\left[\underline{U}_t^P, \bar{U}_t^P\right]$, which is derived as follows. The lower bound, $\underline{U}_t^P$, is a solution to problem:

$$\underline{U}_t^P = \arg\min_{G_t, x_t^P} U^P\left(G_t, x_t^P\right) \text{ s.t. } \Phi_1',$$

where Φ_1' represents the admissible set, which contains the following constraints: $x_t^E + x_t^P + cG_t \leq \tilde{I}_t$, $G_t \geq 0$, $x_t^E \geq 0$ and $x_t^P \geq 0$, and the upper bound, $\bar{U}_t^P$, is a solution to the problem:

$$\bar{U}_t^P = \arg\max_{G_t, x_t^P} U^P\left(G_t, x_t^P\right) \text{ s.t. } \Phi_1'.$$

For an arbitrary $u_t^P \in \left[\underline{U}_t^P, \bar{U}_t^P\right]$, the elites' indirect utility is a solution to the following maximisation problem of the elites in period t:

$$u_t^E = \tilde{h}\left(\tilde{I}_t, u_t^P\right) = \max_{G, x_t^E} \left\{ \begin{array}{c} U^E\left(G_t, x_t^E\right) \\ \text{s.t. } \Phi_1' \text{ and } \Phi_2' \end{array} \right\},$$

where Φ_2' denotes the poor citizens' participation constraint, $U^P\left(G_t, x_t^P\right) \geq d_{u,t}^P$.

The set of utility possibility frontier is given by

$$\Psi' = \left\{ \left(u_t^P, u_t^E\right) : u_t^P \in \left[\underline{U}_t^P, \bar{U}_t^P\right] \text{ and } u_t^E = \tilde{h}\left(\tilde{I}_t, u_t^P\right) \right\}.$$

Consider the following generalised Nash bargaining problem. For each $\gamma \in [0, 1]$, the generalised Nash bargaining solution is a unique pair of utilities denoted by $\left(u_t^P, u_t^E\right)$ which solves the following problem:

$$\max_{\left(u_t^P, u_t^E\right) \in \Theta'} \left(u_t^P - u_P\left(\tilde{d}^P \tilde{I}_t\right)\right)^{\gamma} \left(u_t^E - u_E\left(\tilde{d}^E \tilde{I}_t\right)\right)^{1-\gamma},$$

for the elites to repay the public debts in the second period thus raising the default probability, $p(D_1, r)$.

[13] Again, this constitutes the domain of the function $\tilde{h}\left(\tilde{I}_t, u_t^P\right)$ which we discuss below.

where $\Theta' \equiv \left\{ \left(u_t^P, u_t^E\right) \in \Psi' : u_t^P \geq u_P\left(\tilde{d}^P \tilde{I}_t\right) \text{ and } u_t^E \geq u_E\left(\tilde{d}^E \tilde{I}_t\right) \right\}$ and γ denotes the exogenously given bargaining power of the poor citizens.

From *Lemma 1*, for a fixed I, the function $\tilde{h}\left(\tilde{I}_t, u_t^P\right)$ is strictly decreasing in u_t^P and concave. An increase in I results in an affine transformation (an outward parallel shift) in the graph of function $\tilde{h}\left(\tilde{I}_t, u_t^P\right)$. For a given γ, since the maximand of this generalised Nash bargaining problem is continuous and quasiconcave; the function $\tilde{h}\left(\tilde{I}_t, u_t^P\right)$ is strictly decreasing in u_t^P and concave and the set Θ' is nonempty, the above generalised Nash bargaining problem has a unique solution[14] and should satisfy the first-order condition (similar to that stated in *Claim 1*), which states that for $\gamma \in (0,1)$, such that the function $\tilde{h}\left(\tilde{I}_t, u_t^P\right)$ is differentiable, the *interior* generalised Nash bargaining solution is a unique solution to the following pairs of equations:

$$-\frac{\partial_2 \tilde{h}\left(\tilde{I}_t, u_t^P\right)}{\partial u_t^P} = \left(\frac{\gamma}{1-\gamma}\right)\left[\frac{u_t^E - u_E\left(\tilde{d}^E \tilde{I}_t\right)}{u_t^P - u_P\left(\tilde{d}^P \tilde{I}_t\right)}\right] \text{ and } u_t^E = \tilde{h}\left(\tilde{I}_t, u_t^P\right).$$

However, for the corner solutions – when $\alpha = \{0,1\}$ – the above first-order condition does not apply. When $\alpha = 0$, $u_t^P = u_P\left(\tilde{d}^P \tilde{I}_t\right)$ and $u_t^E = \tilde{h}\left(\tilde{I}_t, u_P\left(\tilde{d}^P \tilde{I}_t\right)\right)$, while, when $\alpha = 1$, $u_t^P = \bar{U}_t^P$ and $u_t^E = \tilde{h}\left(\tilde{I}_t, \bar{U}_t^P\right)$.

In this section, since we focus on the society where the elites have all the bargaining power, we study the above generalised Nash bargaining under the assumption that $\alpha = 0$. Figure 8 provides a graphic characterisation of the generalised Nash bargaining problem under the case where $\alpha = 0$. The generalised Nash bargaining solution is illustrated in figure as point GN.

The utility possibility frontier, $\tilde{h}\left(\tilde{I}_t, u_t^P\right)$, shown in Figure 8 has the properties which are consistent with those presented in *Lemma 1*: for a given I, $\tilde{h}\left(\tilde{I}_t, u_t^P\right)$ is strictly decreasing and concave. Moreover, since an increase in I leads to an affine transformation in the graph of function $\tilde{h}\left(\tilde{I}_t, u_t^P\right)$, it follows that an increase in national resource by allowing the elites to issue public debts of an amount D_t leads to an outward parallel shift in the utility possibility frontier from the curve labelled $\tilde{h}\left(\tilde{I}_t, u_t^P\right)$ to the one labelled $\tilde{h}\left(\tilde{I}_t + D_t, u_t^P\right)$. Since the disagreement income of

[14] ★Similar to the discussion we made in section 2, in fact there exists a continuum of $\left(u_t^P, u_t^E\right)$, which satisfy $u_t^P > u_P\left(\tilde{d}^P \tilde{I}_t\right)$ and $u_t^E > u_E\left(\tilde{d}^E \tilde{I}_t\right)$.★ CHECK!!

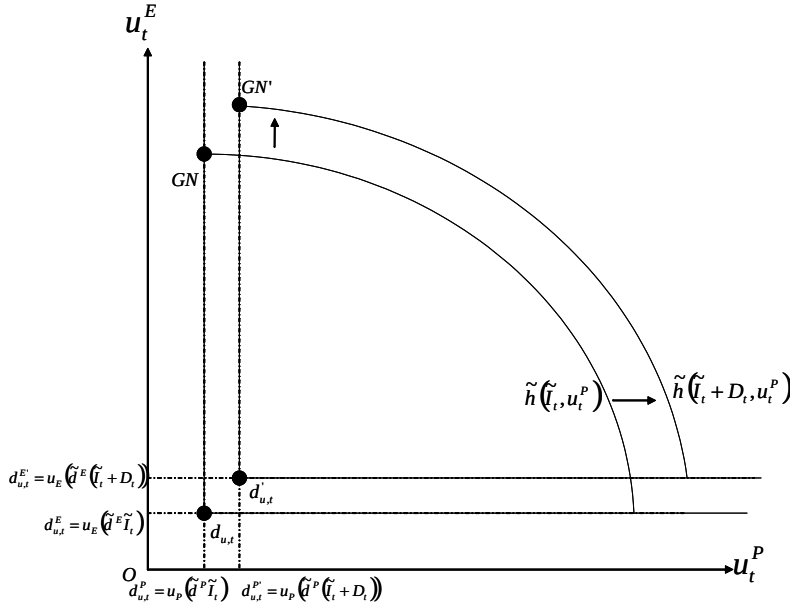

Figure 8: Utility possibility frontier in the dynamic model

both classes are proportional to the national resource, it follows that the disagreement utility of both classes increases after an amount D_t of public debts has been issued. Specifically, the disagreement utilities of the elites and the poor citizens increase from $u_E\left(\tilde{d}^E\tilde{I}_t\right)$ and $u_P\left(\tilde{d}^P\tilde{I}_t\right)$ to $u_E\left(\tilde{d}^E\left(\tilde{I}_t + D_t\right)\right)$ and $u_P\left(\tilde{d}^P\left(\tilde{I}_t + D_t\right)\right)$, respectively. The new generalised Nash bargaining solution is shown in Figure 8 as GN'.

A transition from point GN to GN' as new public debts have been issued by the elites ensures that the utility obtained by both classes in the equilibrium increases. Although the poor citizens' participation constraint still binds even after D_t has been issued, as $u_P\left(\tilde{d}^P\tilde{I}_t\right) < u_P\left(\tilde{d}^P\left(\tilde{I}_t + D_t\right)\right)$, it follows that the poor citizens are better off with D_t than without it. Therefore, issue new public debts of an amount D_t is beneficial for both elites and poor citizens. It is very crucial to note that the conclusion drawn here is done basing on the assumption that no short-run or static conditionality requirement is being imposed by the external funding agency. In sum, when no short-run conditionality requirements are imposed by the external funding agency, the poor citizens always prefer that new public debts, D_t, being issued.

In what follows, to show that the intertemporal conflict exists, we solve this two-period repeated game by using the backward induction

approach. We begin by solving the second-period generalised Nash bargaining problem for the scenario where $\gamma = 0$ and derive the value function or the second-period indirect utility for the elites. Given the value function, we then move backwards to the first period to solve the first-period generalised Nash bargaining.

The second period generalised Nash bargaining. The national resource in the second period, I_2, is stochastic, depending on whether or not the elites choose to default. If the elites choose to default at $t = 2$, they obtain the utility $V_2^E(\beta y_2)$, while if they choose to honour the debts, their utility is $V_2^E\left(y_2 - (1+r)D_1 + \bar{D}_2\right)$. The elites' strategy in the second period is that they will choose to default if $V_2^E(\beta y_2) \geq V_2^E\left(y_2 - (1+r)D_1 + \bar{D}_2\right)$.

Suppose the elites use the threshold strategy such that, for some $\hat{D}_1$ and for a given r, (i) whenever $D_1 > \hat{D}_1$, the elites choose to default with probability 1 in the second period, (ii) whenever $D_1 < \hat{D}_1$, the elites choose not to default with probability 1 in the second period and (iii) whenever $D_1 = \hat{D}_1$, the elites are indifferent between defaulting and not defaulting. It follows the threshold strategy of the elites that, for a given r, the probability of default is discontinuous in D_1. Given $p(D_1, r)$, the value function of the elites in the second period, $V_2^E(D_1) = \max\left\{V_2^E(\beta y_2), V_2^E\left(y_2 - (1+r)D_1 + \bar{D}_2\right)\right\}$, where, whenever $D_1 < \hat{D}_1$, $V_2^E(D_1) = V_2^E\left(y_2 - (1+r)D_1 + \bar{D}_2\right)$, and whenever $D_1 > \hat{D}_1$, $V_2^E(D_1) = V_2^E(\beta y_2)$.

Consider two levels of first-period public debts, D_1, D_1', where $D_1 < D_1'$. If the elites choose not to default in the second period, a higher D_1 lowers the elites' value function as $V_2^E\left(y_2 - (1+r)D_1' + \bar{D}_2\right) < V_2^E\left(y_2 - (1+r)D_1 + \bar{D}_2\right)$. However, if the elites choose to default, they receive the utility $V_2^E(\beta y_2)$, which is independent of D_1. Therefore, from their perspective, the elites are realise that they are indeed effectively insured against the cost of default since whenever default occurs, they are always guaranteed a minimum utility of $V_2^E(\beta y_2)$ thereafter. However, this is not the case for the poor citizens[15]. The poor citizens are forced to bear a disproportionate cost of default. When no default occurs, an increase in D_1 leads to a reduction in the poor citizens' utility. Consider two level of first-period public debts, D_1, D_1', where $D_1 < D_1'$. Our results show that $u_P\left(\tilde{d}^P\left(y_2 - (1+r)D_1' + \bar{D}_2\right)\right) <$

[15]Without the static or short-run conditionality requirements being imposed by the external funding agency in the second period, when $\gamma = 0$, the elites choose the policy choices on $\left(G_2, x_2^E, x_2^P, \bar{D}_2\right)$ which ensure that the poor citizens' participation constraint binds. Therefore, the poor citizens' utility at $t = 2$ is given by $u_2^P = u_p\left(\tilde{d}^P I_2\right)$.

$u_P\left(\tilde{d}^P\left(y_2-(1+r)D_1+\bar{D}_2\right)\right)$. Moreover, as higher D_1 implies that the probability of default is higher and default implies a permanent contraction in the national output and a permanent exclusion from the international capital market, it follows that the poor citizens are worse off with more public debts being issued in the first period.

The first-period generalised Nash bargaining problem. Consider the national resource $\tilde{I}_1$, which does not include the issue of public debt in the first period by the elites. Suppose that the external funding agency imposes the short-run conditionality requirements in the first period, which consist of $G_1 \geq \underline{G}_1$ and $x_1^P \geq \underline{x}_1^P$, where $\left(\underline{G}_1, \underline{x}_1^P\right)$ are determined by the external funding agency with the target level of poor citizens' utility, $\hat{d}_{u,1}^P$ in mind. The poor citizens' first-period utility is given by $U^P\left(\underline{G}_1, \underline{x}_1^P\right)$ or $\hat{d}_{u,1}^P$, where $U^P\left(\underline{G}_1, \underline{x}_1^P\right)$ is independent of D_1. We have shown in the previous section that $U^P\left(\underline{G}_1, \underline{x}_1^P\right) > u_P\left(\tilde{d}^P\tilde{I}_1\right)$. To determine the optimal first-period public debts from the poor citizens' perspective, $D_{1,opt}^P$, let the poor citizens choose D_1 to maximise their discounted utility:

$$\max_{D_1} f^P = U^P\left(\underline{G}_1, \underline{x}_1^P\right) + \delta_P\left\{u_P\left(\tilde{d}^P\left(y_2-(1+r)D_1+\bar{D}_2\right)\right)\right\},$$

where f^P denotes the discounted utility for the poor citizens and $\delta_P \in (0,1)$ is the discount factor for the poor citizens.

For the poor citizens, since their first period utility, $U^P\left(\underline{G}_1, \underline{x}_1^P\right)$, is independent of D_1 but their second-period utility, $u_P\left(\tilde{d}^P\left(y_2-(1+r)D_1+\bar{D}_2\right)\right)$, is decreasing in D_1, the poor citizens prefer that $D_{1,opt}^P = 0$.

When the short-run conditionality requirements are imposed in the first period and the elites are not allowed to issue new public debts, their indirect utility in the first period is given by $\tilde{h}\left(\tilde{I}_t, U^P\left(\underline{G}_1, \underline{x}_1^P\right)\right)$. Next, we discuss how the elites respond to the presence of these short-run conditionality requirements. Our results show that the elites can get around these conditions by issuing new public debts in the first period. When the new public debts, D_1, has been issued, the national resource is now given by $\tilde{I}_1 + D_1$ and the indirect utility of the elites is given by $\tilde{h}\left(\tilde{I}_t + D_1, U^P\left(\underline{G}_1, \underline{x}_1^P\right)\right)$. Since the elites utility is strictly increasing in G_1 and x_1^E, it follows that $\tilde{h}\left(\tilde{I}_t + D_1, U^P\left(\underline{G}_1, \underline{x}_1^P\right)\right) > \tilde{h}\left(\tilde{I}_t, U^P\left(\underline{G}_1, \underline{x}_1^P\right)\right)$. Thus, the elites are better off with more D_1 so they always have an incentive to issue public debts thus the first-period indirect utility of the elites is represented by $\tilde{h}\left(\tilde{I}_t + D_1, U^P\left(\underline{G}_1, \underline{x}_1^P\right)\right)$, as $D_1 > 0$. This leads us to the first result of this dynamic model: the

elites can always get around the short-run conditionality requirements by issuing new public debts in the first period. Next, we show that the *intertemporal conflict* exists. This requires us to derive the optimal first-period public debts from the elites' perspective, $D^E_{1,opt}$.

We derive $D^E_{1,opt}$ from the elites' maximisation problem. The elites choose D_1 to maximise their discounted utility:

$$\max_{D_1} f^E = \tilde{h}\left(\tilde{I}_t + D_1, U^P\left(\underline{G}_1, \underline{x}^P_1\right)\right) + \delta_E\left\{V^E_2\left(D_1\right)\right\},$$

where f^E denotes the discounted utility for the elites, $\delta_E \in (0,1)$ is the discount factor for the elites and $\delta_E = \delta_P$.

For the elites, as being discussed earlier, they always prefer to issue strictly positive public debts to get around the short-run conditionality requirements and since their first-period indirect utility is increasing in D_1, it follows that $D^E_{1,opt} > 0$. Next, to show that $D^E_{1,opt}$ is high, let us consider two levels of first-period public debts, D_1, D'_1, where $D_1 < D'_1$. Since $\tilde{h}\left(\tilde{I}_t + D_1, U^P\left(\underline{G}_1, \underline{x}^P_1\right)\right) < \tilde{h}\left(\tilde{I}_t + D'_1, U^P\left(\underline{G}_1, \underline{x}^P_1\right)\right)$, issuing more public debts in the first period raises the elites' first-period indirect utility. In the second period, even though a higher D_1 lowers their indirect utility as $V^E_2\left(y_2 - (1+r)D'_1 + \bar{D}_2\right) < V^E_2\left(y_2 - (1+r)D_1 + \bar{D}_2\right)$ when the elites choose not to default, the elites can still prefer that more public debts being issued in the first period as they can always opt out by choosing to default. Even though the poor citizens will be worse off when default occurs as they have to bear a high cost of default (in the form of permanent contraction in national output and permanent exclusion from borrowing in the international capital market), the elites are effectively insured against this cost of default as they are always guaranteed a floor in the utility they obtain given by $V^E_2(\beta y_2)$. It follows that the elites always have an incentive to issue a large amount of D_1 so $D^E_{1,opt}$ tends to be excessively high.

Since it is always true that $D^E_{1,opt} > D^P_{1,opt}$, an intertemporal conflict always exists even though both classes have the same discount rates. The fact that an intertemporal conflict exists even though both classes have the same discount rate distinguishes our result from the literature. In Easterly (1999), the elites' behaviour to run up excessive public debts can also be captured in the model which assumes that the elites have a very high discount rate. In such model, the elites have a lower discount factor than the poor citizens so $\delta_E < \delta_P$. According to Easterly (1999, p.7), "a country that has got an excessive external debt is the one with a high discount rate against the future perhaps because of a profligate government and/or because of political stability of interest group polarization". This is the second main result of dynamic model.

To summarise, in the society where the elites have all the bargaining power, an *intertemporal conflict* exists and the short-run conditionality requirements are insufficient to ensure that the well-being of the poor citizens improves after the debt write-down has been granted as the elites can always get around such conditions by issuing more public debts in the first period. With the presence of these problems, we propose that the external funding agency attaches the "dynamic- or long-run conditionality requirements" to the debt write-down granted to the country in the first period. Details of such conditionality requirements are discussed below.

Suppose the external funding agency wants to grant a conditional debt write-down to such economy in the first period. When designing the conditionality requirements to be attached to the debt write-down, the external funding agency needs to be fully aware of the situation both at $t = 1$ and at $t = 2$. The set of conditionality requirements for the first period should target the provision of public goods, G_1, the private consumption level of the poor citizens, x_1^P, and the amount of public debts the elites allowed to issue in the first period, D_1. Formally, the first-period conditionality requirements should consist of $G_1 \geq \underline{G}_1$, $x_1^P \geq \underline{x}_1^P$ and $D_1 \leq \bar{D}_1$, where $\bar{D}_1$ denotes a limit on the amount of public debt the elites are allowed to issue in the first period thus the last condition, $D_1 \leq \bar{D}_1$, prevents the elites from issuing excessive amount of public debts at $t = 1$. The first two requirements, $G_1 \geq \underline{G}_1$ and $x_1^P \geq \underline{x}_1^P$, help ensure that the poor citizens receive sufficient public goods and private consumption in the first period.

For the second period, since the debt limit on D_2 is already present, the conditionality requirements attached to the debt write-down should include $G_2 \geq \underline{G}_2$ and $x_2^P \geq \underline{x}_2^P$. These two conditionality requirements help ensure that the poor citizens are not much adversely affected if the elites indeed choose to default in the second period. This recommendation to attach the long-run conditionality requirements to the debt write-down is the third result of this section.

Finally, similar to the static framework, even though it might be easy for the external funding agency and the debtor's government to write a debt write-down contract which incorporate the long-run conditions, the issues of verifiability, enforceability and monitoring still remain to be investigated further.

We summarise the above discussion with the following proposition:

Proposition 3 *In the society where the elites have all the bargaining power, the short-run conditionality requirements are no longer sufficient in the long-run as the elites can always get around these conditions by issuing more public debts. Since an intertemporal conflict exists, the*

long-run conditionality requirements should instead be attached to the debt write-down.

5 The current debt relief initiatives and policy recommendations

At present, the HIPC initiative of 1999 (also known as the enhanced HIPC initiative) and the Gleneagles Proposal for debt write-down by the G8 countries are the main mechanisms used to reduce the indebtedness of the low-income countries. In this section, we aim to evaluate the efficacy of these two debt relief initiatives: discussing their additivity to the current debt relief operation as well as addressing their key criticisms. We begin with the enhanced HIPC initiative, followed by the Gleneagles Proposal.

The enhanced HIPC initiative has been notable not only because it emphasises debt stock reduction but also it keys the extent of debt stock reduction to a sustainable level, it has been subject to several serious criticisms. To name some, first, the foreign creditors have failed to include serious analysis of country's needs in their debt relief operation; instead, they use arbitrary formulas to judge the countries' debt sustainability[16]. Second, debt reduction under the HIPC initiative is usually tied to inappropriate conditionality requirements, particularly those related to the IMF programme.

The myriad of conditions set forth by the IMF, even though they are well-intentioned, sometimes turn out to be counter productive. In particular, they may distract the government from addressing the pressing-concern issues, especially the problems of poor governance and inadequate investment in education and health. They generally do not succeed in engendering development. In the view of one of its critics, "the IMF conditionality sometimes leaves the country just as impoverished but with more debts and even richer ruling elite," (Stiglitz, 2003). The IMF ignores the issue of governance and fails to take into account the impact of distribution of power on the well-being of the poor citizens. Moreover, the IMF typically prescribes the policy to maintain fiscal surplus, which makes it difficult for such countries to focus on their priorities, including enhancing governance and accountability on the part of government and promoting social contract. With such conditionality requirements being

[16] Under the enhanced HIPC initiative, the benchmark for debt sustainability was set at 150 percent debt-to-export ratio (on a net present value basis) or 250 percent NPV of debt to tax revenue for countries with open economies (a minimum of 30 percent export-to-GNP ratio) and substantial tax revenue (a minimum of 15 percent of GNP) (Birdsall and Williamson, 2002).

imposed, the ruling elites can ensure that such conditionality requirements are satisfied even though no money freed up by debt write-down is directed to investment in health and education, the factors which are central to the progress of improving the well-being of the poor citizens and achieving the MDGs[17].

Since it is difficult for the MDGs to be achieved and development to be promoted if debt obligations are excessive, recently the Commission for Africa called for a 100 percent debt write-down and a boost in aid (Commission for Africa, 2005). Responding to the request made by the Commission for Africa, the G8 nations reached a landslide debt relief deal at the G8 Summit in Gleneagles, Scotland, in July 2005, agreeing to write down $40 billion debt owed by 18 countries, mainly in Africa, which reached the completion point under the enhanced HIPC initiative. Moreover, the G8 countries also reached an agreement to boost foreign aid to such countries. This move provided relief for poor indebted nations, freeing up much needed revenue.

It is crucial to note that, since the debt write-down granted by the G8 countries has not been tied to explicit set of conditionality requirements, there is no guarantee that the government of such countries will use money, which would otherwise go to service debt payment, to support provision of necessary public goods and better the lives of its people. It, therefore, becomes a challenge on the part of the government of such countries how they will spend the money they have been paying on debts.

We argue that a decision to cancel debts of the Heavily Indebted Poor Countries (HIPCs) can free such countries from excessive debt burdens; however, in overall, it cannot lead to improvement in the well-being of the poor citizens and achievement of the MDGs without an improvement in governance and if the governments of such countries do not adhere to fiscal discipline. The results from our formal analysis suggest that the government of such countries should undertake the following set of mutually reinforcing reforms.

First, the foreign creditors should support a greater transparency in public financial management by focusing on enhancing transparency in revenues, budgets and expenditure together with taking credible action against corruption. A mechanism akin to the "Poverty Action Fund" used in Uganda could be a useful starting point. The money that would have been spent on servicing debt can be directed to the Poverty Action Fund and from there the money will be channelled into the country's health and education systems (BBC, 2005). This type of mechanism

[17]By no mean restricted to low-income countries, this model may be applied to study similar problems in Latin America, where the elites divert public funds to safe havens overseas.

can help minimise the chance that government diverts the public funds to support corruption and transfers them to safe havens overseas.

Once the country has shown a satisfactory improvement in governance, the next policy recommendation enters into consideration, including some put forward by the G8 nations in the Gleneagles Summit in Scotland (the Gleneagles Communiqué, 2005). It is important that the government supports strategies to improve provision of public goods, especially health, education, food security and basic infrastructure. Whenever possible, these public goods should be provided free of charge.

6 Conclusion

To address the two problems related to poor governance, namely *domestic distributive conflict* and *intertemporal conflict*, we develop a model of debt write-down, where the conditionality requirements attached to debt write-down are designed so as to ensure that the well-being of the poor citizens is improved. We consider two versions of the model, static and dynamic. The results from our static model show that an unconditional debt write-down is only effective in the society with a balanced distribution of power between the elites and the poor citizens as it leads to an improvement in the well-being of both classes. However, for the society where the elites have all the bargaining power and the problem of *domestic distributive conflict* is prevalent, the external funding agency should attach the appropriately designed conditionality requirements, which target both provision of public goods and private consumption of the poor citizens, to the debt write-down.

Our results in the dynamic framework show that, over time, the elites can get around the static conditionality requirements by issuing more new public debts. The problem of *intertemporal conflict* arises because of the conflicting views of the poor citizens and the elites with regards to the issue of new public debts in the first period. With the presence of the static conditionality requirements, the poor citizens do not benefit from the newly issued public debts in the first period as they are guaranteed a target level of utility in any case; moreover, they are forced to bear a disproportionately high share of default costs in the second period. However, for the elites, issuing more public debts in the first period is beneficial for them as that increases their utility; at the same time, they are effectively insured against the cost of default in the second period. With the problem of *intertemporal conflict* in mind, we propose that the long-run conditionality requirements should, instead, be attached to the debt write-down.

Against such a benchmark, we then evaluate the efficacy of the cur-

rent proposals for debt write-down under both the HIPC and the G8 initiatives. The recent agreement reached by the G8 countries to grant a complete debt write-down on poor countries' debts and the promise to boost aid are not adequate to get development in those countries back on track if the money freed up by debt relief and aid is not used to better the lives of the poor people. This can only be viewed as the first step in the reinvention of development assistance process.

We argue that such initiatives to cancel debts of the Heavily Indebted Poor Countries (HIPCs) can free such countries from excessive debt burdens; however, in overall, it cannot lead to improvement in the well-being of the poor citizens and achievement of the MDGs without an improvement in governance. The results from our formal analysis suggest that the external funding agency supports a greater transparency in public financial management by focusing on enhancing transparency in revenues, budgets and expenditure together with taking credible action against corruption. Provided that the country has shown a satisfactory improvement in governance, then each government should support strategies to improve provision of public goods, especially health, education, food security and basic infrastructure.

In the future, we aim to extend the model to study the situation where debt is issued for the consumption smoothing purpose. We aim to investigate how the degree of risk aversion of the elites affect the amount of self-enforcing debt which can be sustained in the equilibrium. Another plausible extension is to study the situation where the ruling elites have a hyperbolic preference. It would be interesting to show that the elites may end up issuing an excessive amount of pubic debts due to myopia.

A Proof of Lemma 1

The proof of *Lemma 1* proceeds in two steps. First, we begin by proving that, for a fixed I, the function $h(I, u_E)$ is strictly decreasing in u_E. The utility possibility frontier, $h(I, u_E)$, is derived by solving the maximisation problem stated in (3) for an arbitrary level of $u_E \in \left[\underline{U}^E, \bar{U}^E\right]$, subject to Φ_1 and Φ_2. There is no interior solution for this problem as the objective function, $U^P\left(G, x^P\right)$, is strictly increasing in G and x^P. It follows that a corner solution, $\left(G, x^P\right) = (\infty, \infty)$, exists . Moreover, the four constraints in the admissible set, Φ_1, are linear thus the admissible set Φ_1 is a convex set. Since $U^E\left(G, x^E\right)$ is concave in both $\left(G, x^E\right)$, it follows that $U^E\left(G, x^E\right) \geq u_E$ is convex; therefore, Φ_2 is also a convex set. When both Φ_1 and Φ_2 are convex sets, it follows that $\Phi_1 \cap \Phi_2$ is a convex set; therefore, the constraint set is quasiconcave.

Next, in order to establish that the function $h(I, u_E)$ is strictly de-

creasing in u_E, we choose $u_E = \underline{U}^E$ thus the set Φ_2 does not have any impact. Increasing u_E causes the set Φ_1 to shrink. With a smaller admissible set, the indirect utility of the poor citizens, $h(I, u_E)$, declines. This implies that, given two arbitrary utilities of the elites, $u_E^A \in \left[\underline{U}^E, \bar{U}^E\right]$ and $u_E^B \in \left[\underline{U}^E, \bar{U}^E\right]$, where $u_E^A > u_E^B$, it follows that $h\left(I, u_E^A\right) < h\left(I, u_E^B\right)$. This proves that the utility possibility frontier, $h(I, u_E)$, is strictly decreasing in u_E.

We then prove that, for a fixed I, the utility possibility frontier, $h(I, u_E)$, is concave in u_E. It follows from the *Envelope Theorem* that, if the objective function is concave in both decision variables and the constraint set is quasiconcave, the value function is concave. In this context, the objective function, $U^P\left(G, x^P\right)$, is concave in both G and x^P, the constraint set is quasiconcave, and the function $h(I, u_E)$ is the value function. Applying the *Envelope Theorem*, we find that, for a fixed I and for two arbitrary utilities of the elites, $u_E^A \in \left[\underline{U}^E, \bar{U}^E\right]$ and $u_E^B \in \left[\underline{U}^E, \bar{U}^E\right]$, $(1-\lambda) h\left(I, u_E^A\right) + \lambda h\left(I, u_E^B\right) \leq h\left[I, (1-\lambda) u_E^A + \lambda u_E^B\right]$. Therefore, the utility possibility frontier, $h(I, u_E)$, is concave.

Next, we study the impact of an increase in I on the utility possibility set and the graph of function $h(I, u_E)$. First, we establish that the utility possibility set is increasing in I. For any two arbitrary national resources, $I, I' > 0$, where $I' > I$, since anything that is feasible with I' is not necessarily feasible with I, it follows that an increase in I leads to an expansion in the utility possibility set. Second, we show that an increase in I leads to an affine transformation in the utility possibility frontier, $h(I, u_E)$. We multiply both I and u_E, by any arbitrary scalar, $\theta > 0$. Since $h(\theta I, \theta u_E) = \theta h(I, u_E)$, increasing I leads to an affine transformation in the utility possibility frontier. Hence, increasing the national resource from I to I' leads to an outward parallel shift in the graph of function $h(I, u_E)$ to $h'(I', u_E)$ as shown in Figure 1.

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