PH133 Logic Lectures 5 & 6

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Quantifiers

Everything is broken: ∀x Broken(x)

Something is broken: ∃x Broken(x)

What does ∃ mean? We give the meaning of ∃ by specifying what it takes for a sentence containing ∃ to be true:

1. Give every object a name.
2. For each name in turn, create a new sentence like this: delete the quantifier and replace all instances of the variable it binds with that name
3. If ANY OF the new sentences are true, so is the original.

First quantifier rule of proof: ∀Elim

∀Elim

∀x S(x)

…

S(c)

Proof example with →

1. P→Q
2. ¬Q

6. ¬P

Truth table for →

Assuming that the rules of Fitch are such that it is impossible to prove an argument which is not logically valid, the truth-table for → is fixed if we accept →Elim and →Intro.

How do the rules of proof for → fix its truth table?

A	B	A → B
T	T	
T	F	
F	T	
F	F	

¬Intro proof example

1. A ∧ B

¬(¬A ∨ ¬B)

Scope

In P ∧ (Q∨R), the scope of ∧ is P∧ (Q∨R)

In P ∧ (Q∨R), the scope of ∨ is (Q∨R)

In (P∧Q) ∨R, the scope of ∧ is (P∧Q)

In (P∧Q) ∨R, the scope of ∨ is (P∧Q) ∨R

The scope of a connective is the smallest constituent expression which contains that connective.

Wrong proofs

Step 7 of this proof is wrong. Why?

R	S	R ∨ S	R ∧ S
T	F	T	F

```
T | 1. R ∨ S
  |---
  |  2. | R
  |     |---
  |  3. | S ∨ R        ∨Intro: 2
  |
  |  4. | S
  |     |---
  |  5. | S ∨ R        ∨Intro: 4
  |
  |  6. S ∨ R          ∨Elim: 1,2-3,4-5
F |  7. R ∧ S          ∧Intro: 2,4
```

Which step of this proof is wrong? Why?

```
| 1. ¬¬(¬A ∧ ¬¬ A)
|---
| 2. (¬A ∧ ¬¬A)        ¬Elim: 1
| 3. (¬A ∧ A)          ¬Elim: 2
```

Fubar rules

```
∧Fubar:
  | *
  | …
  | * ∧ #
```

Q1. What would be wrong with adding ∧Fubar to Fitch?
Q2. What would be wrong with having ∧Fubar in any *system of proof?*

Exercises (for Seminar 4)

9.2 (quantifiers)

9.4–5, 9.8–10, 9.12 (quantifiers)

9.15–17, (translation)

EITHER 6.17–20 (proof)

OR 6.33, 6.40 (proofs)

DO NOT USE TAUT CON. EVER.

What not to confuse

∃x (Square(x) ∧ Blue(x)) vs.
∃x Square(x) ∧ ∃x Blue(x)

¬(P∨Q) vs. ¬P∨¬Q

¬(P∧Q) vs. ¬P∧¬Q

¬ (P→Q) vs. P→¬Q