Logic (PH133)
Lecture 6

Stephen Butterfill,
Philosophy/Warwick

What not to confuse

1

$\forall x$ (Square(x) $\rightarrow$ Blue(x))

"All squares are blue"

2

$\exists x$ (Square(x) $\wedge$ Blue(x))

"Some square is blue"

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"Some square is blue"

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$\exists x \, \mathrm{Square}(x) \wedge \exists x \, \mathrm{Blue}(x)$

"Some object is square and some object is blue"

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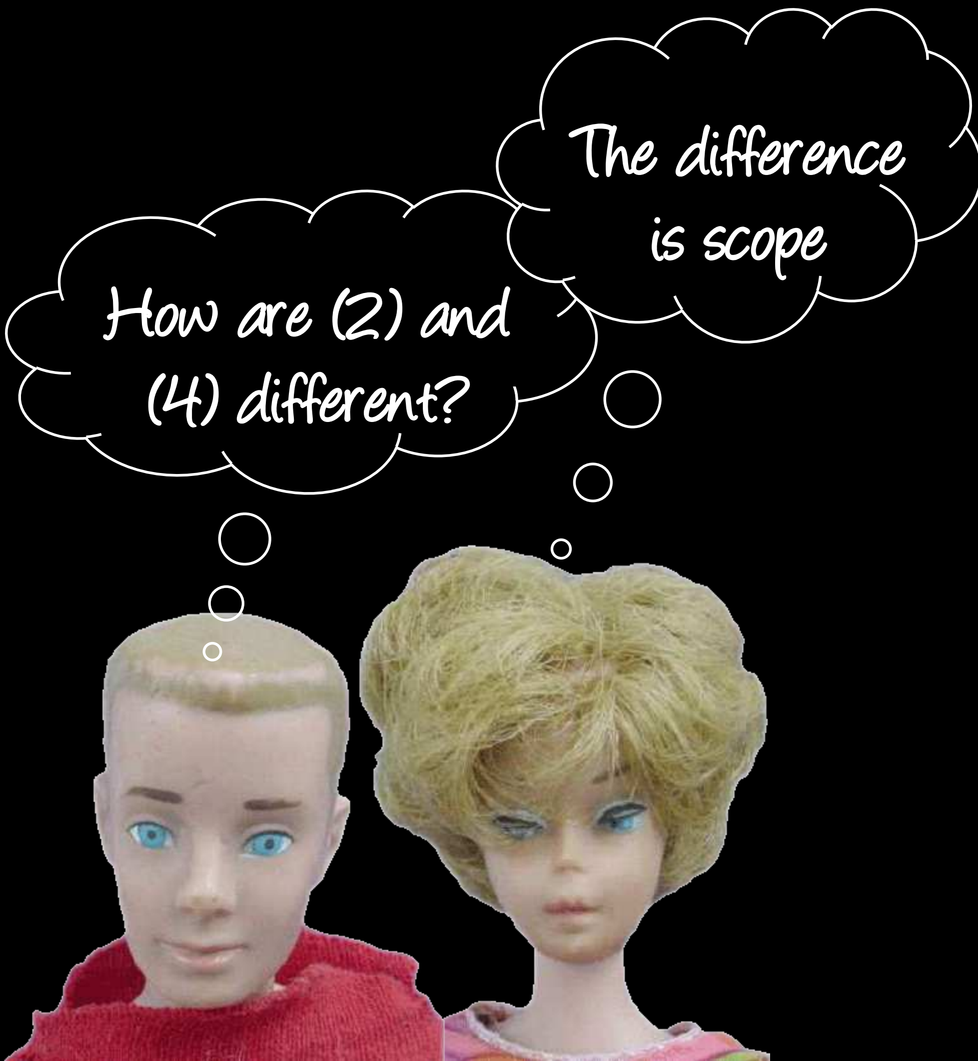

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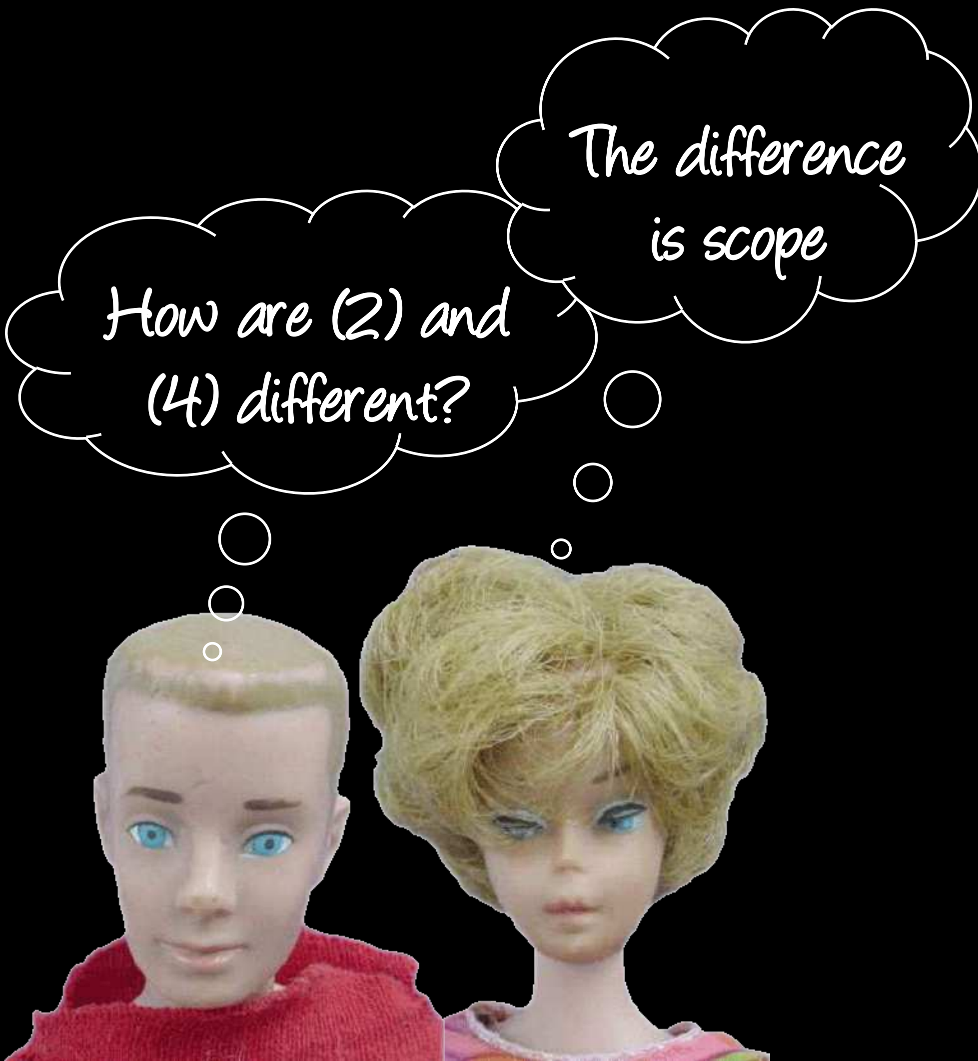

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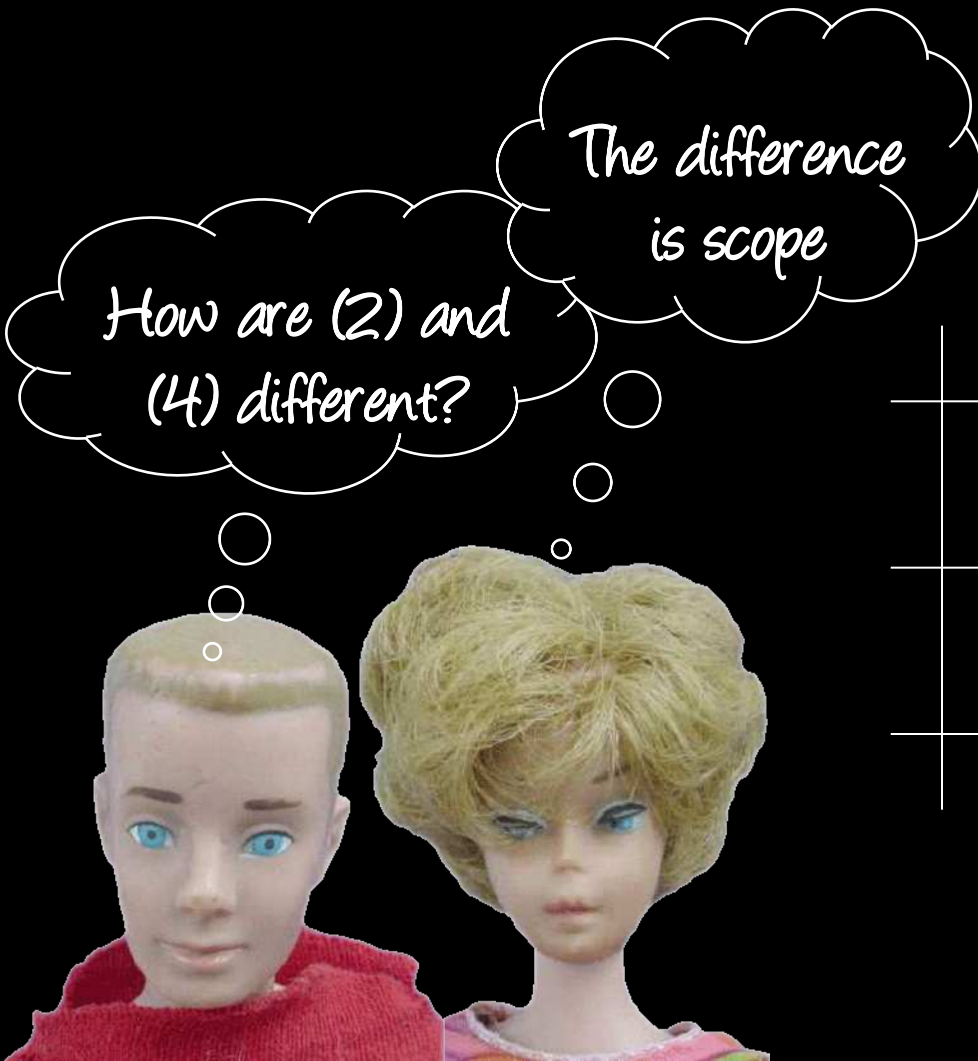

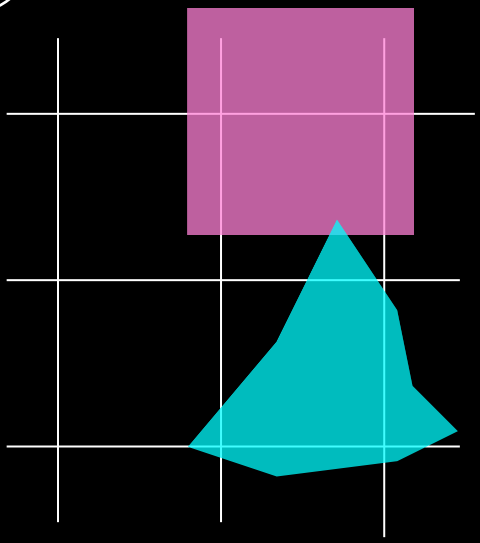

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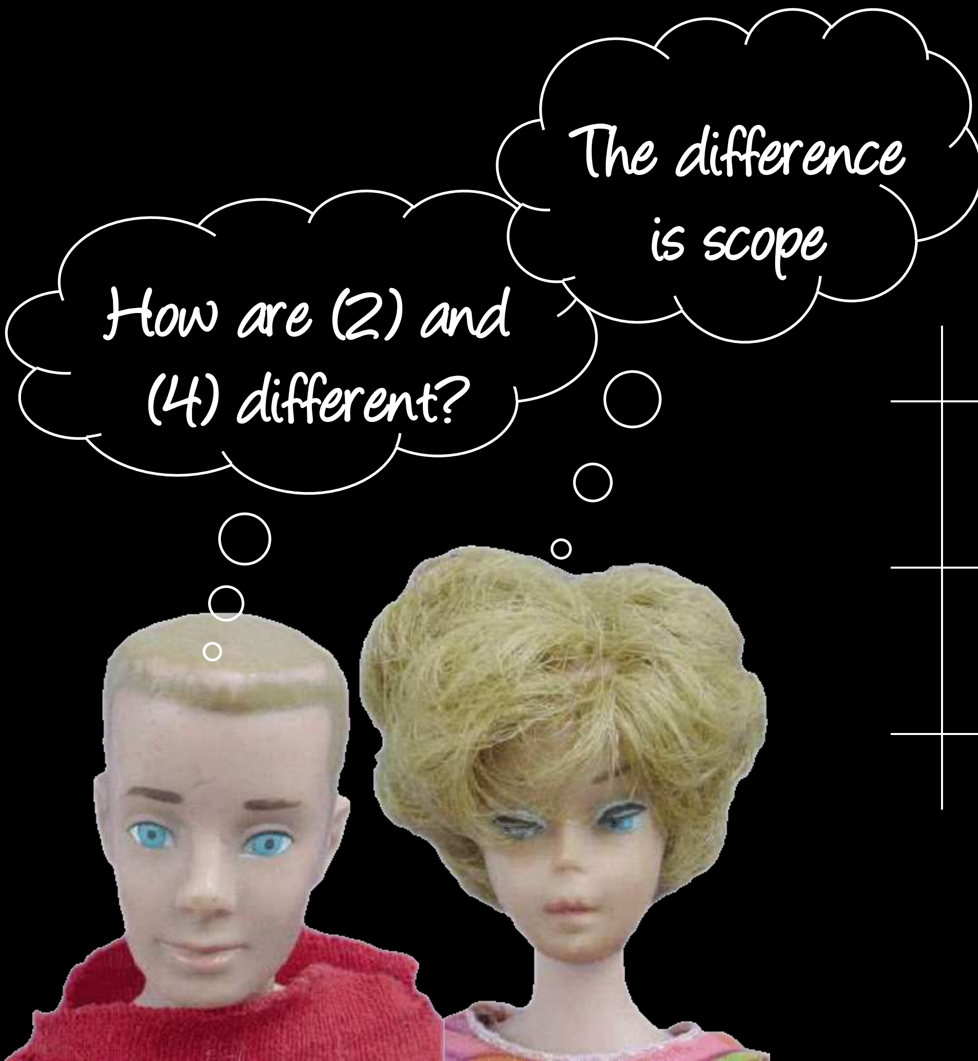

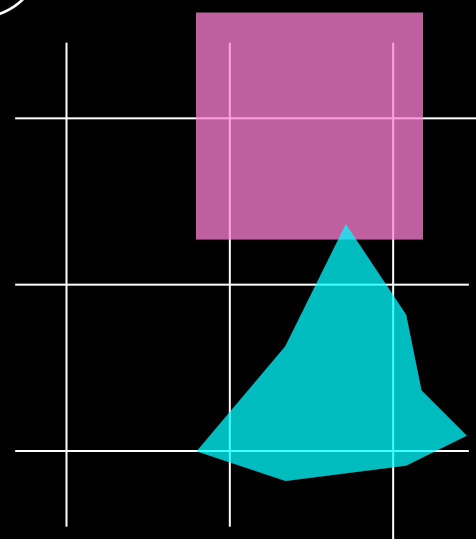

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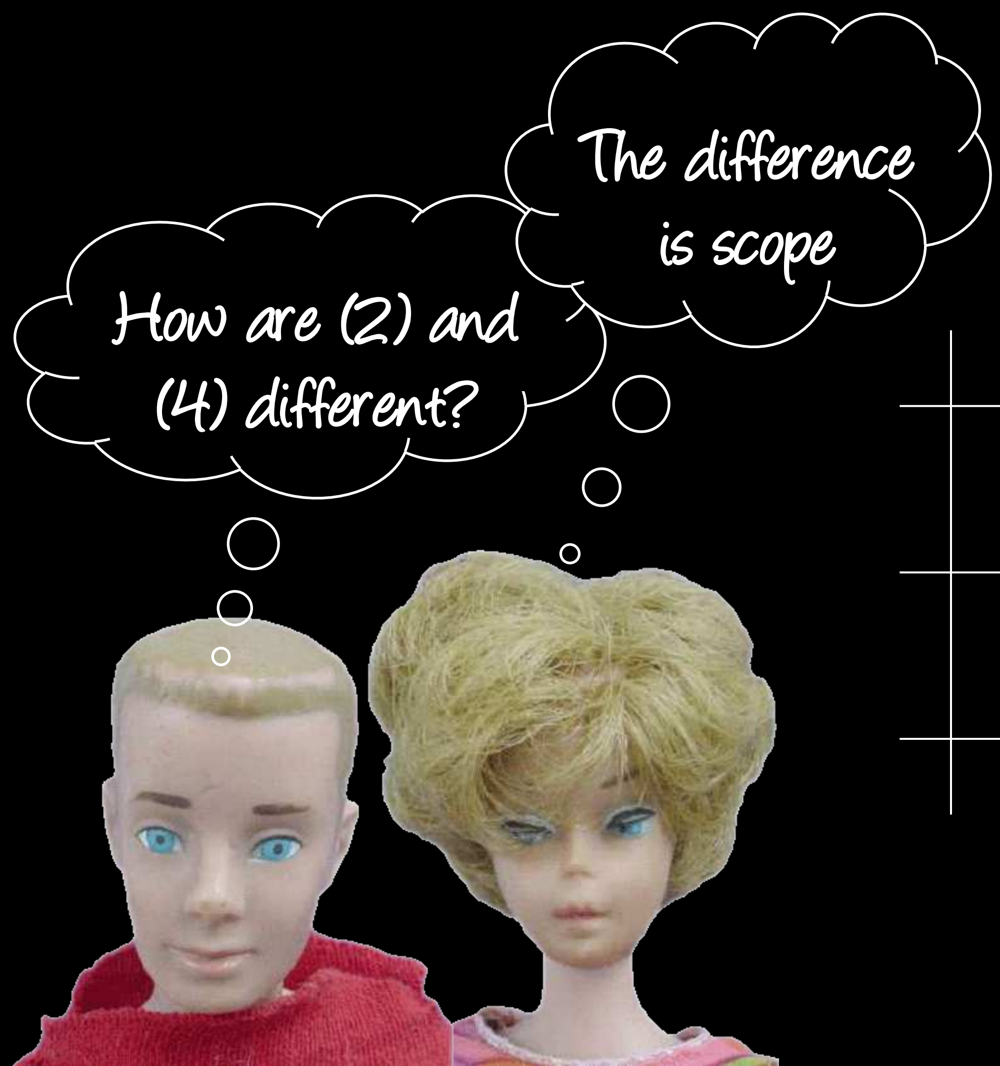

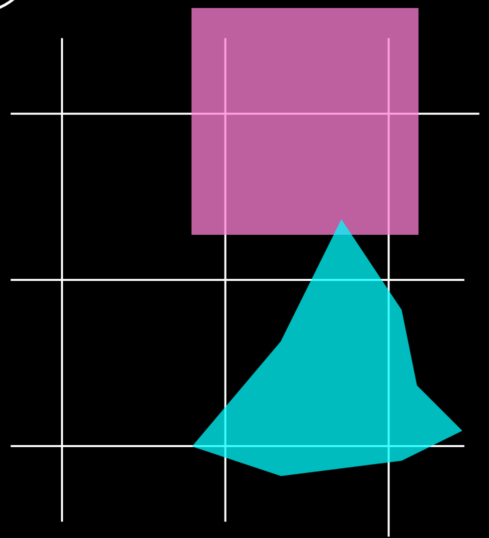

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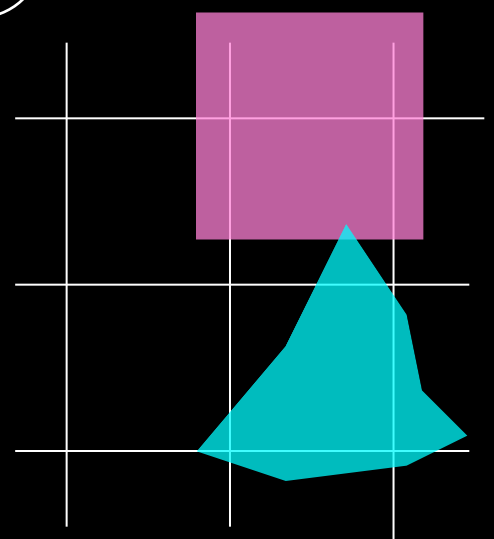
How are (2) and (4) different?

The difference is scope



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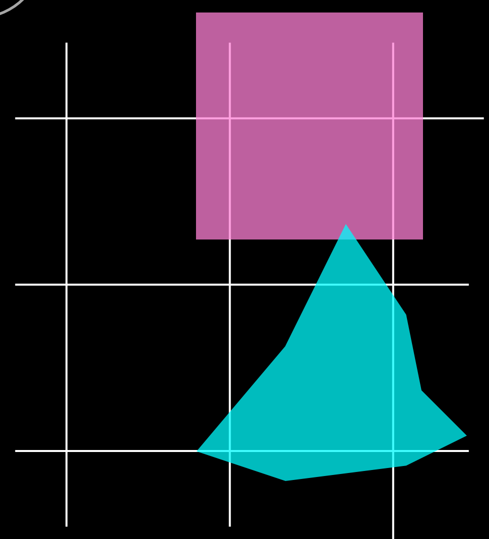

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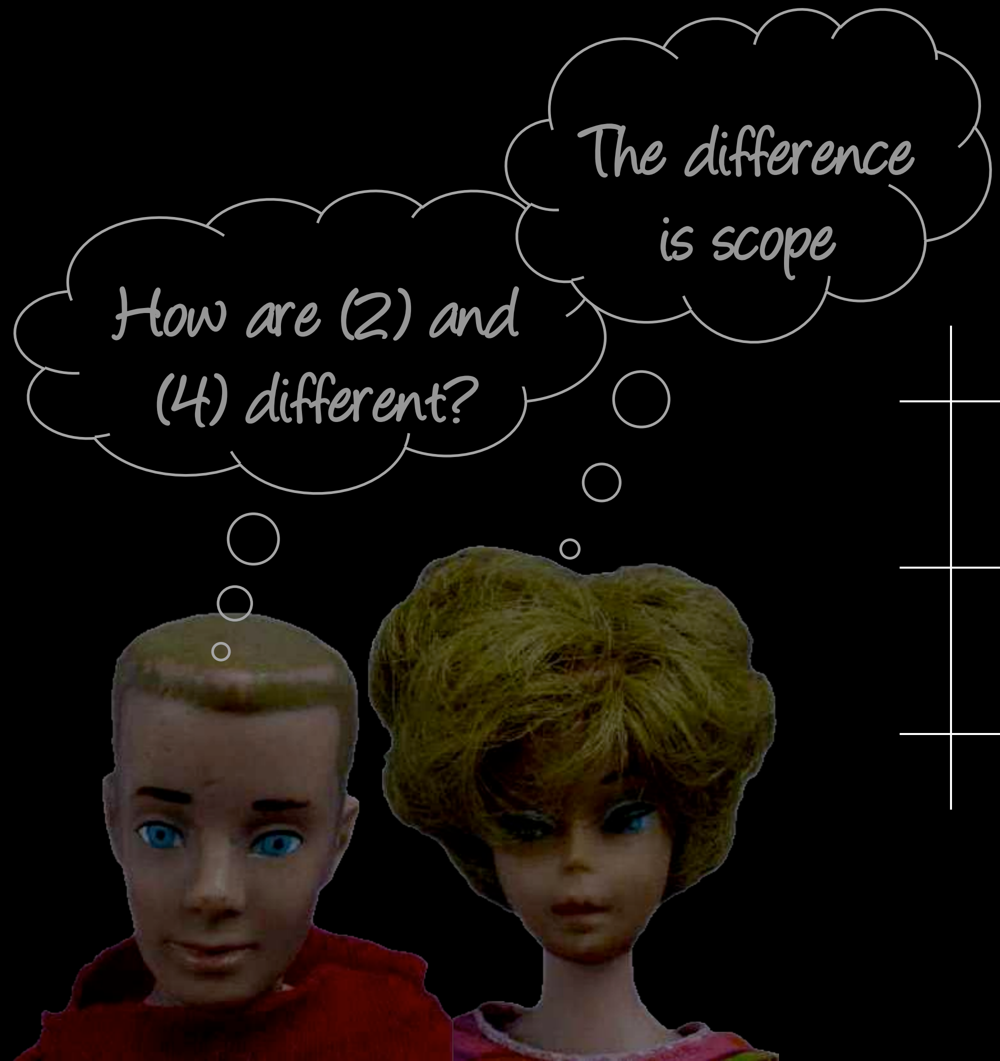

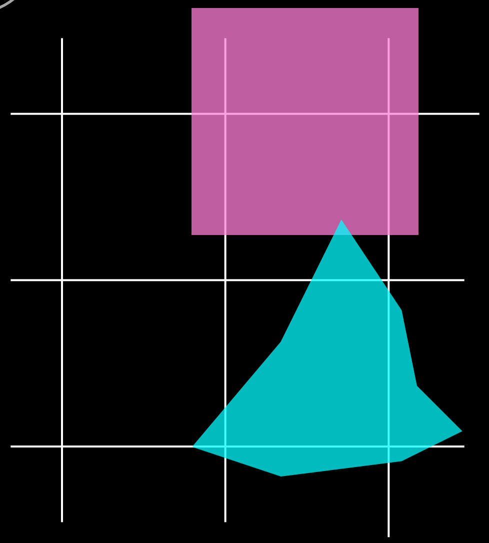

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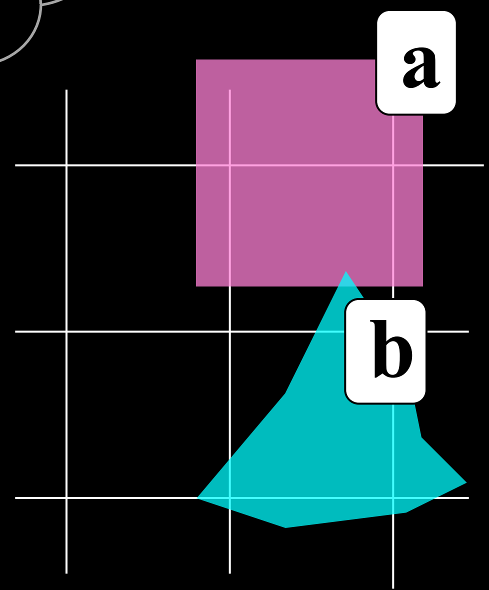

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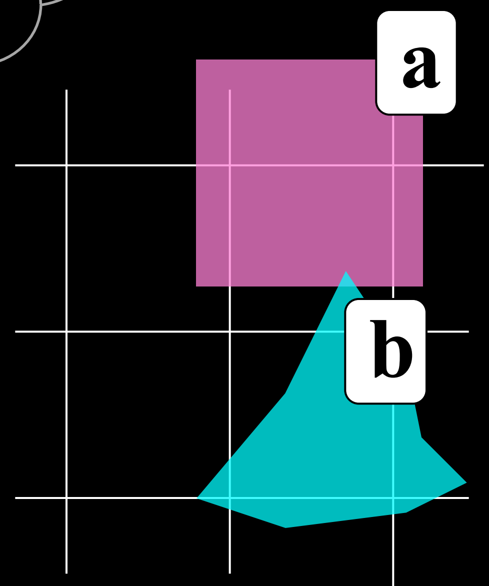

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Square(a)
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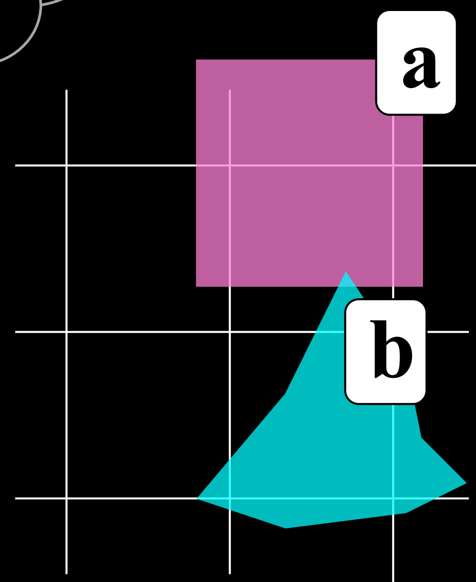

2 is FALSE

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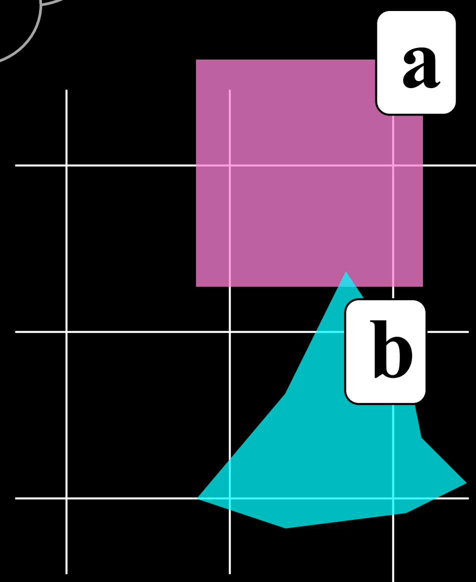

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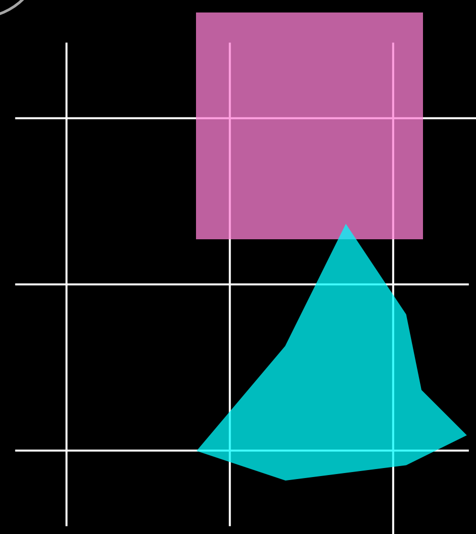

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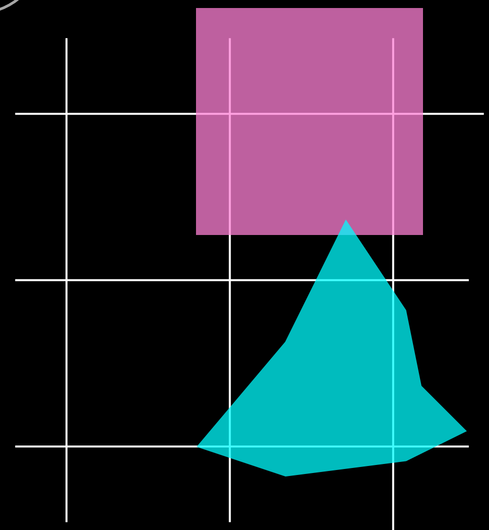

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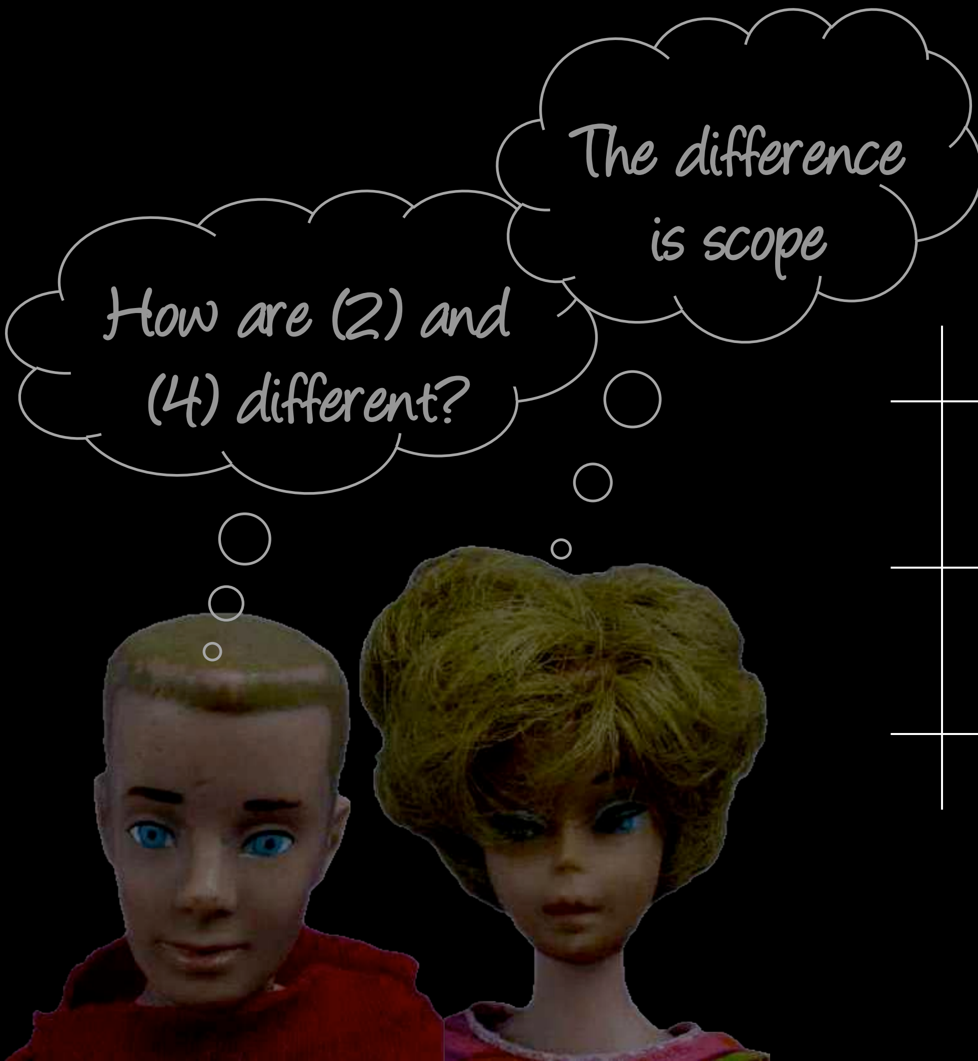

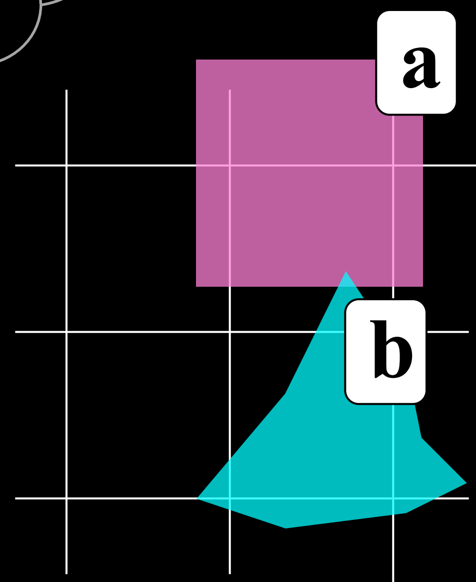

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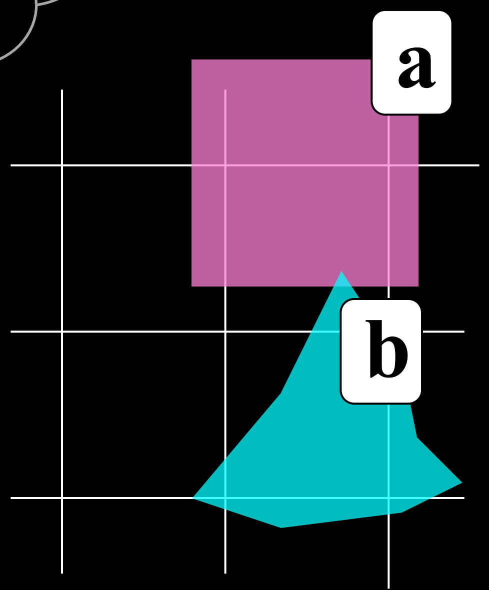

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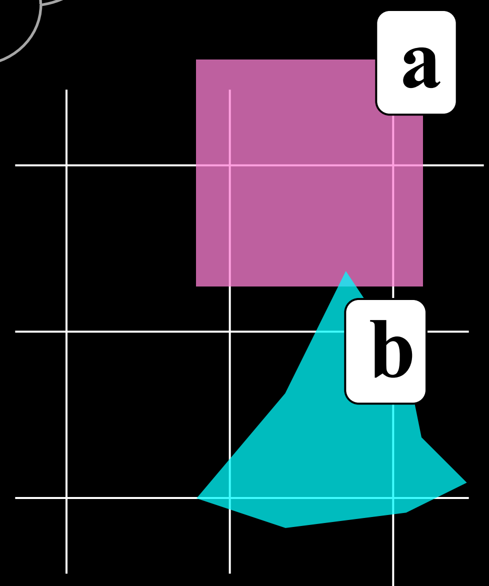

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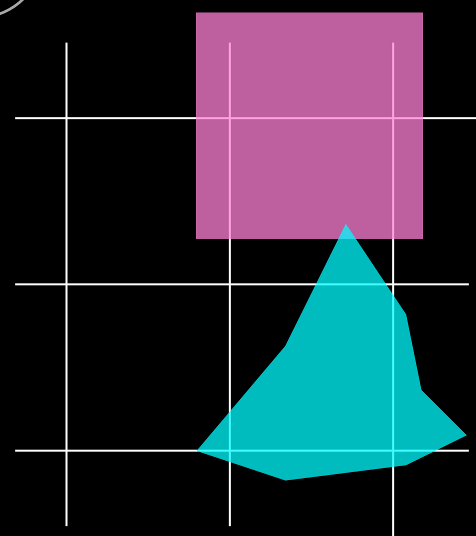

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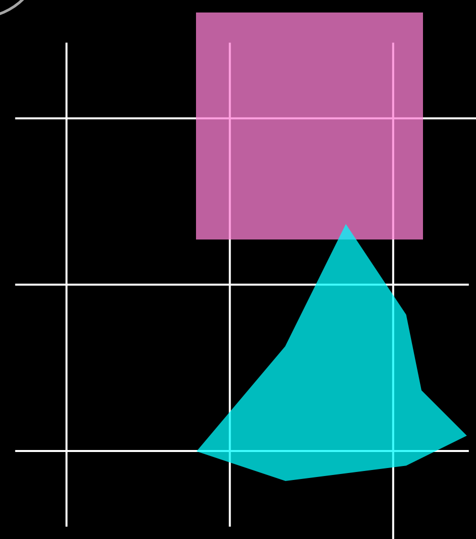

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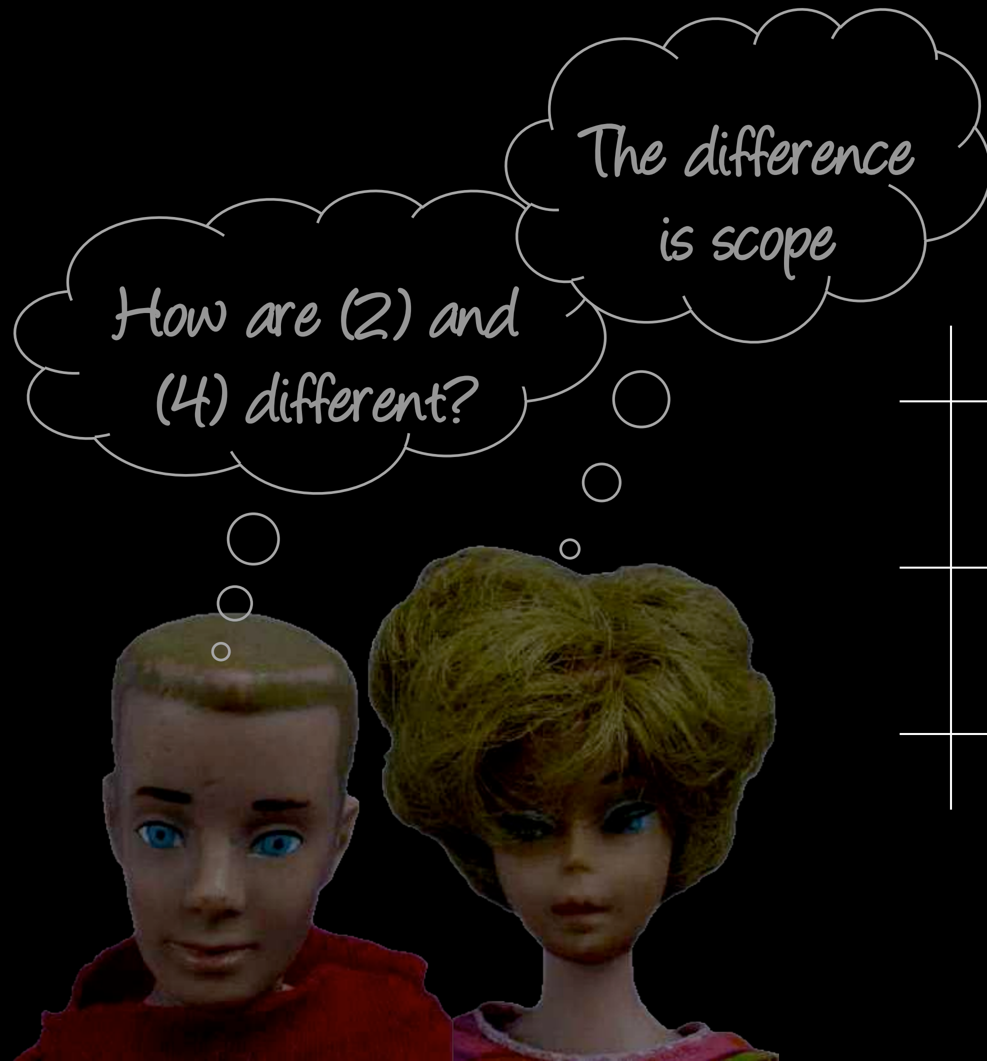

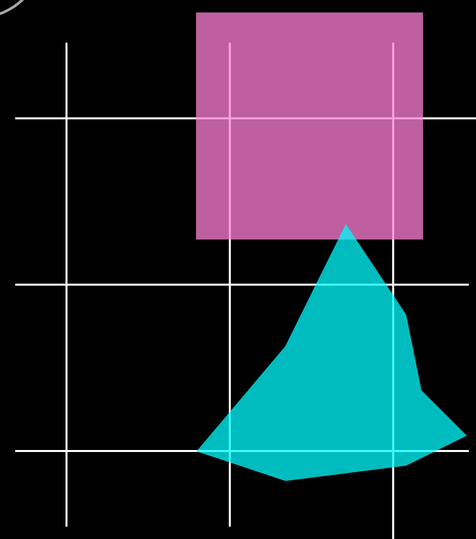

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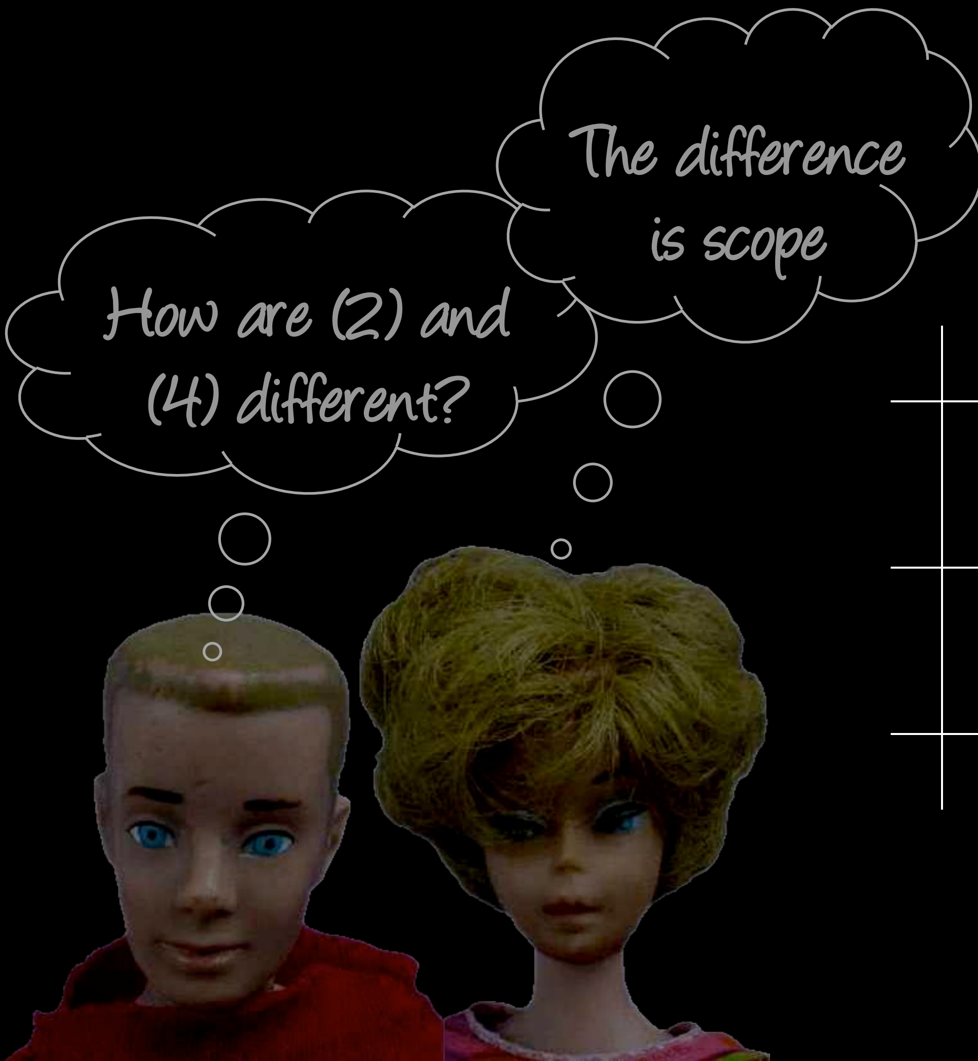

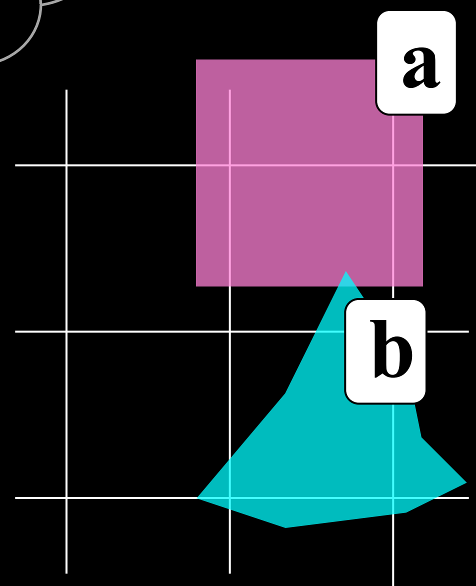

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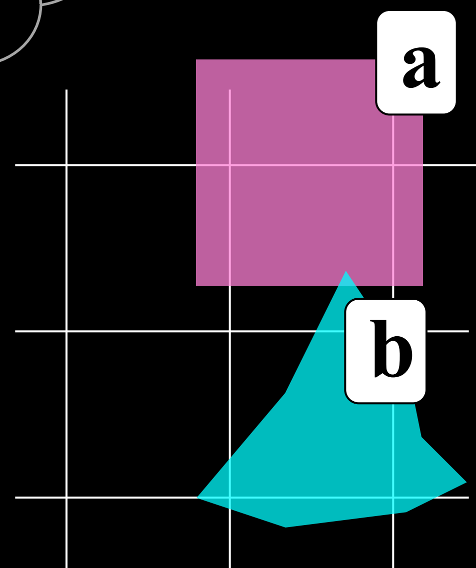

(2) is FALSE

(4) is TRUE

$Square(a) \wedge Blue(a)$

$Square(b) \wedge Blue(b)$

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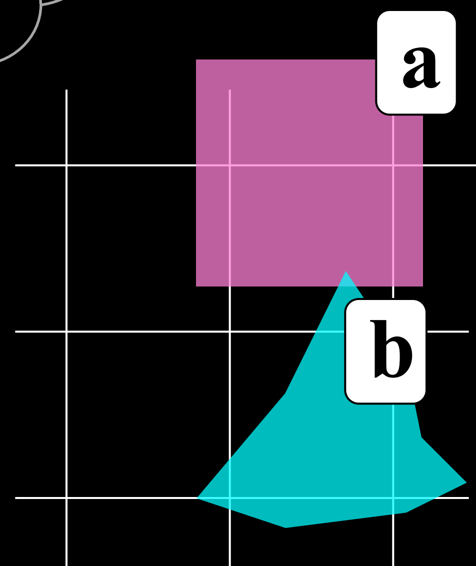

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$Square(b) \wedge Blue(b)$ F

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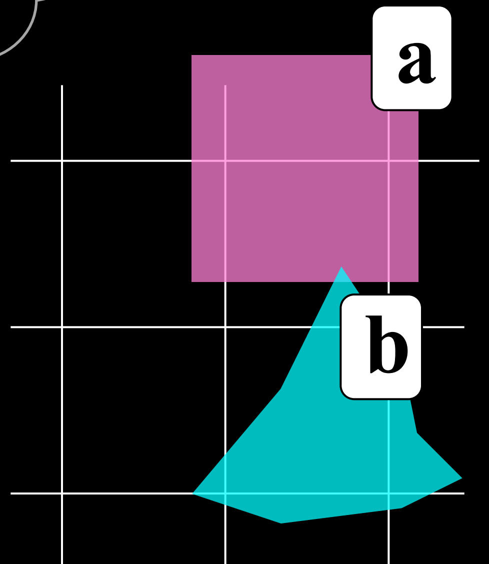

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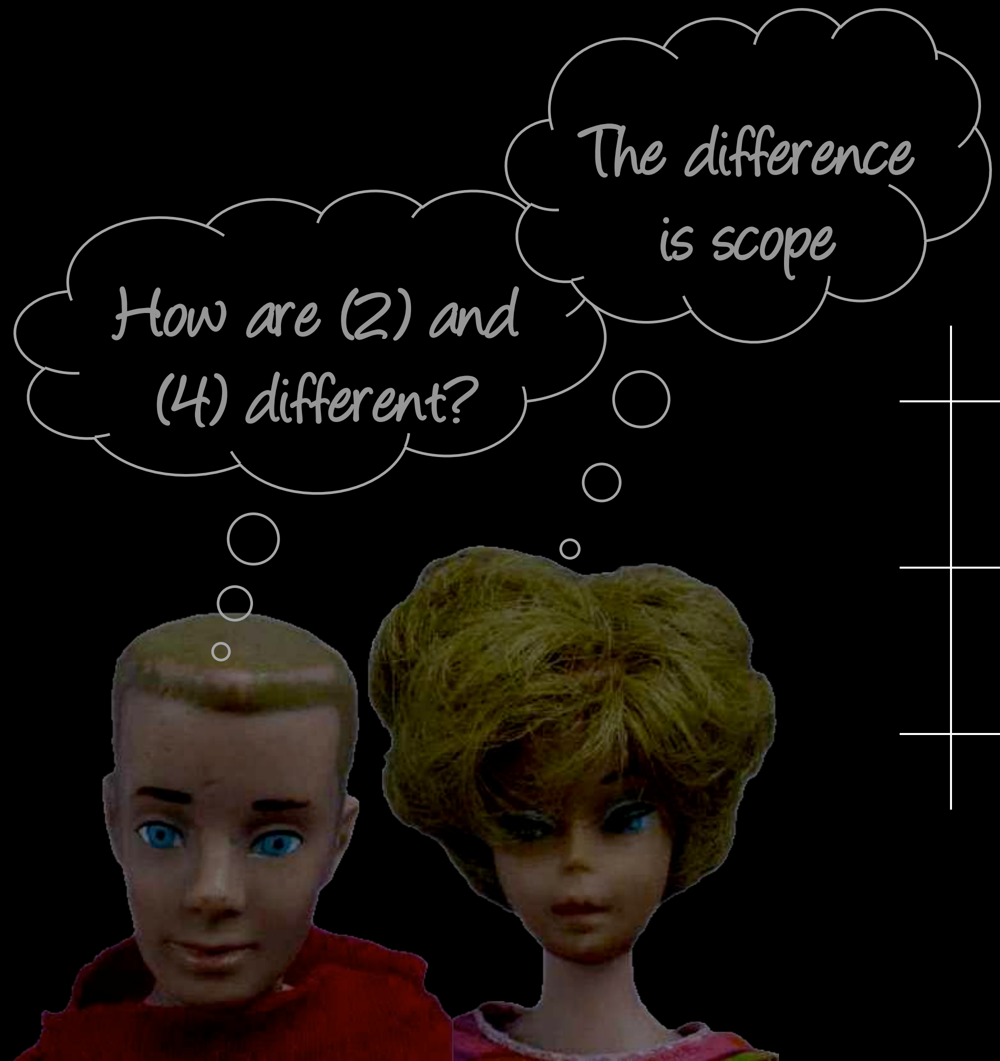

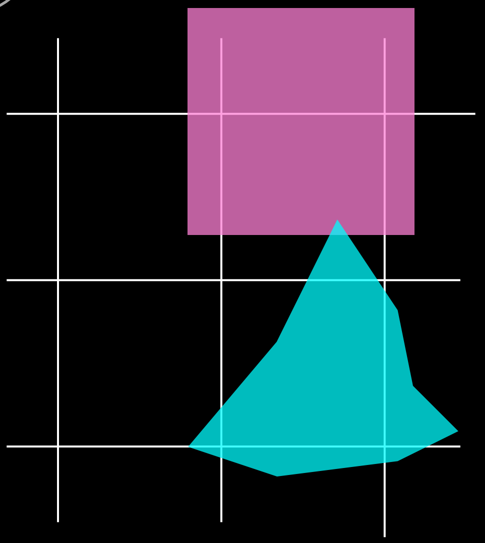

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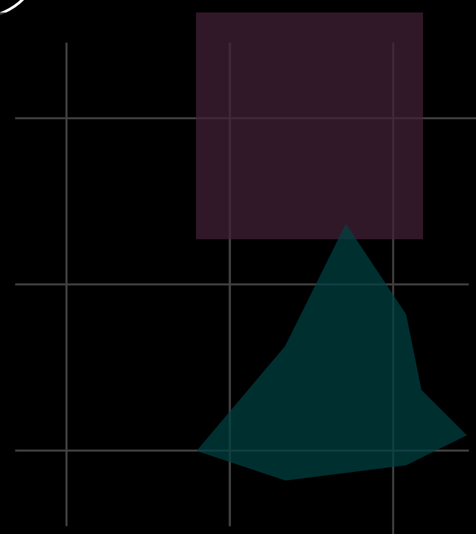

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These 'x's are in the scope of the same quantifier

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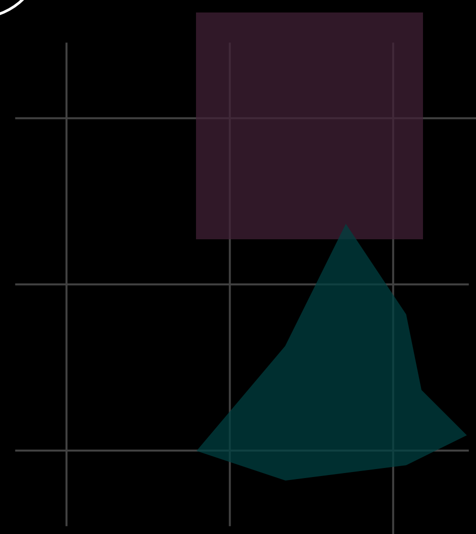
"Some object is square and some object is blue"

is scope

How are (2) and (4) different?



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2. $\exists x\,(\,\mathrm{Square}(x) \wedge \mathrm{Blue}(x)\,)$

"Some square is blue"

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"Some object is square and some object is blue"

These 'x's are in the scope of the same quantifier

These 'x's are in the scope of different quantifiers

How are (2) and (4) different?

the difference is scope

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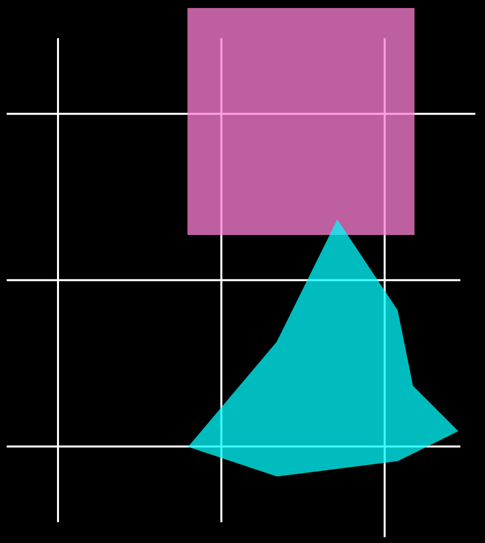

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"All squares are blue"

2 $\exists x\,(\,\mathrm{Square}(x) \wedge \mathrm{Blue}(x)\,)$

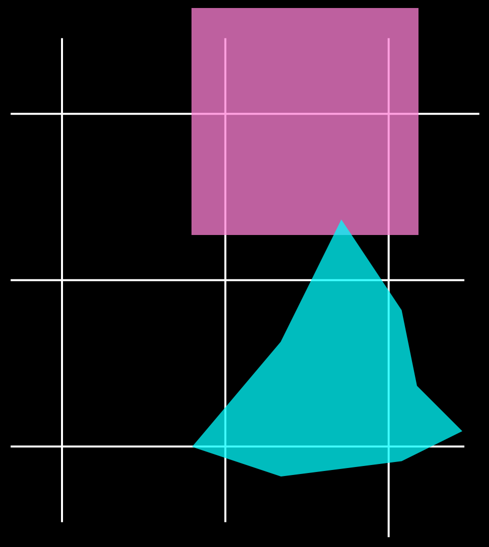
"Some square is blue"

3 $\forall x\,\mathrm{Square}(x) \rightarrow \forall x\,\mathrm{Blue}(x)$

"If everything is square, everything is blue"

4 $\exists x\,\mathrm{Square}(x) \wedge \exists x\,\mathrm{Blue}(x)$

"Some object is square and some object is blue"



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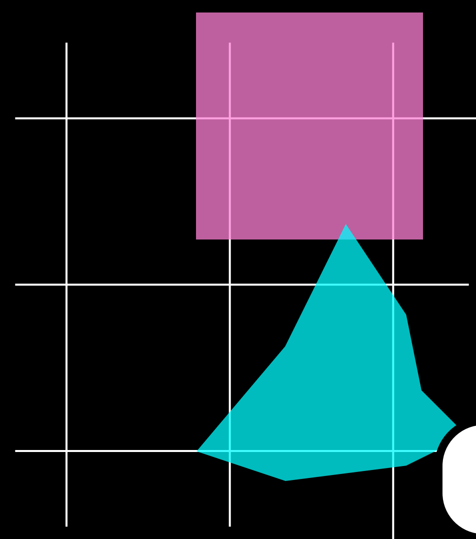
"If everything is square, everything is blue"

4 $\exists x\,\mathrm{Square}(x) \wedge \exists x\,\mathrm{Blue}(x)$

"Some object is square and some object is blue"



1 is FALSE

1 $\forall x\,(\text{Square}(x) \rightarrow \text{Blue}(x))$

"All squares are blue"

2 $\exists x\,(\text{Square}(x) \wedge \text{Blue}(x))$

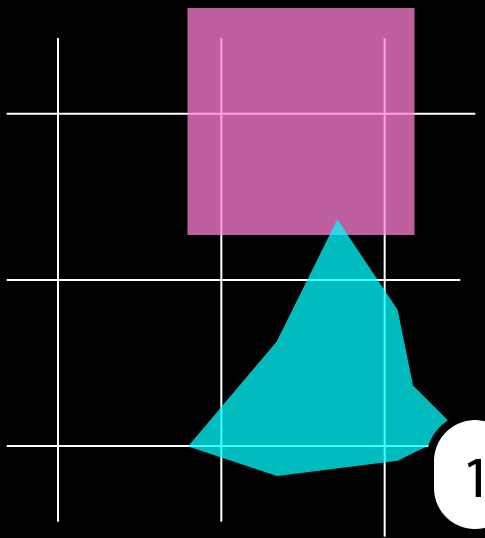
"Some square is blue"

3 $\forall x\,\text{Square}(x) \rightarrow \forall x\,\text{Blue}(x)$

"If everything is square, everything is blue"

4 $\exists x\,\text{Square}(x) \wedge \exists x\,\text{Blue}(x)$

"Some object is square and some object is blue"



1 is FALSE

3 is TRUE

1 $\forall x\,(\,Square(x) \rightarrow Blue(x)\,)$

"All squares are blue"

2 $\exists x\,(\,Square(x) \wedge Blue(x)\,)$

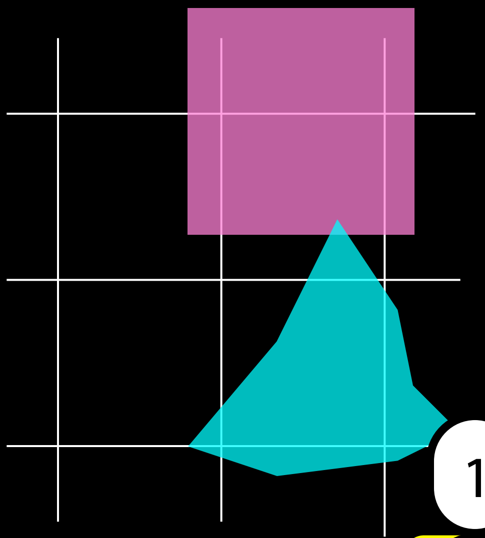
"Some square is blue"

3 $\forall x\, Square(x) \rightarrow \forall x\, Blue(x)$

"If everything is square, everything is blue"

4 $\exists x\, Square(x) \wedge \exists x\, Blue(x)$

"Some object is square and some object is blue"



1 is FALSE

3 is TRUE

1. $\forall x\,(\,Square(x) \rightarrow Blue(x)\,)$

   "All squares are blue"

2. $\exists x\,(\,Square(x) \wedge Blue(x)\,)$

   "Some square is blue"

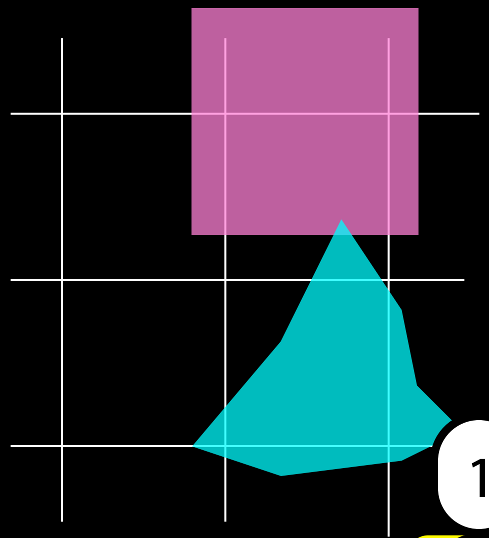
3. $\forall x\, Square(x) \rightarrow \forall x\, Blue(x)$

   "If everything is square, everything is blue"

4. $\exists x\, Square(x) \wedge \exists x\, Blue(x)$

   "Some object is square and some object is blue"

Ex. Explain why 1 is true and 3 is false in this world by appeal to the meaning of '∀'



1 is FALSE

3 is TRUE

What not to confuse

The negation of a disjunction	A disjunction of two negations
$\neg(P \vee Q)$	$\neg P \vee \neg Q$

The negation of a disjunction

A disjunction of two negations

What is the scope of $\vee$ in this formula?

$\neg(P \vee Q)$

$\neg P \vee \neg Q$

The negation of a disjunction

A disjunction of two negations

What is the scope of $\vee$ in this formula?

$\neg(P \vee Q)$

$\overbrace{\neg P \vee \neg Q}$

The negation of a disjunction

A disjunction of two negations

What is the scope of $\vee$ in this formula?

$\neg(P \vee Q)$

What is the scope of $\vee$ in this formula?

$\neg P \vee \neg Q$

The negation of a disjunction

What is the scope of ∨ in this formula?

$$\neg(\overbrace{P \vee Q})$$

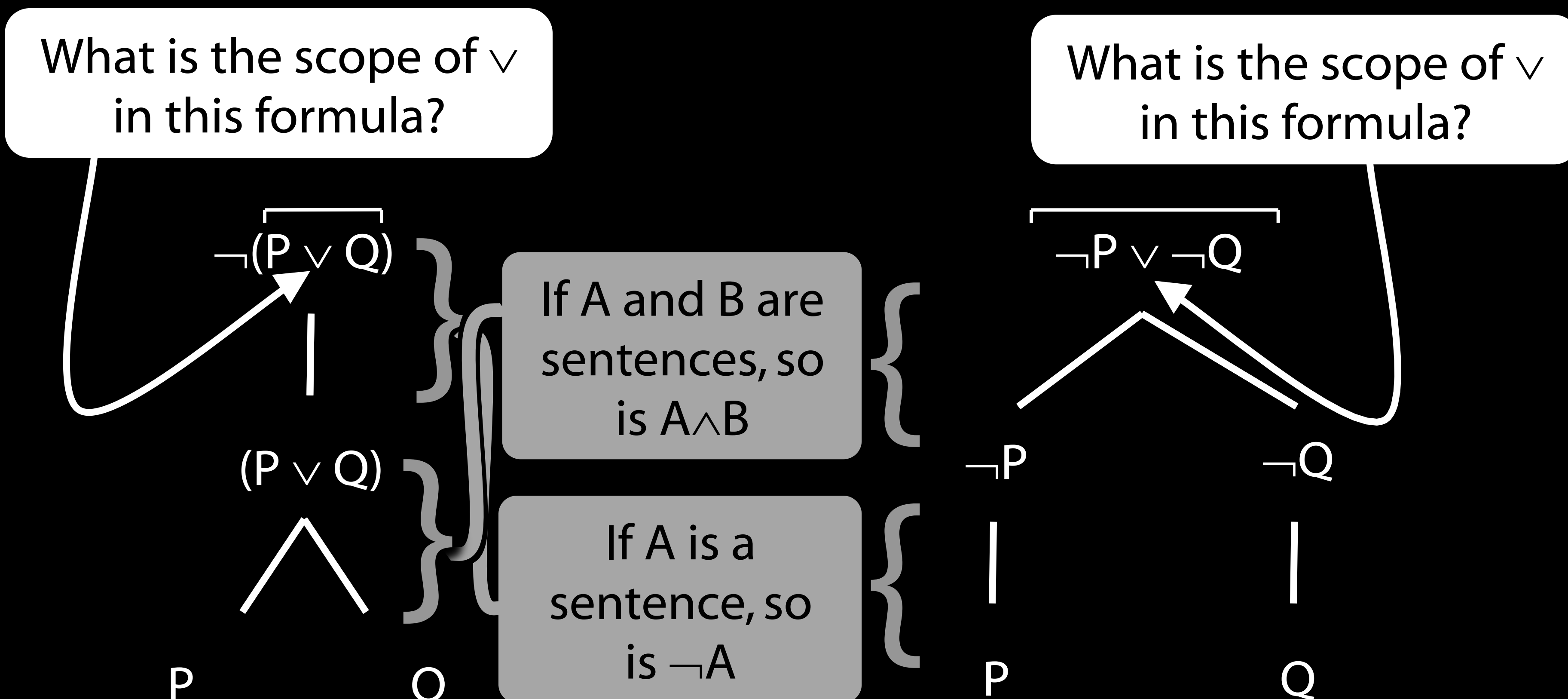
A disjunction of two negations

What is the scope of ∨ in this formula?

$$\overbrace{\neg P \vee \neg Q}$$

The negation of a disjunction

A disjunction of two negations



The negation of a disjunction

A disjunction of two negations

$\neg(P \vee Q)$

$\neg P \vee \neg Q$

The negation of a disjunction

$\neg(P \vee Q)$

$\neg P$

A disjunction of two negations

$\neg P \vee \neg Q$

$\neg P$

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T					
T	F	T	F	F	T	T
F	T					
F	F					

$$\begin{array}{|l} \neg(P \vee Q) \\ \hline \neg P \end{array}$$

$$\begin{array}{|l} \neg P \vee \neg Q \\ \hline \neg P \end{array}$$

P	Q	P∨Q	¬(P ∨ Q)	¬P	¬Q	¬P ∨ ¬Q
T	T					
T	F	T	F	F	T	T
F	T					
F	F					

| ¬(P ∨ Q)
|—
| ¬P

T | ¬P ∨ ¬Q
|—
| ¬P

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T					
T	F	T	F	F	T	T
F	T					
F	F					

$\neg(P \vee Q)$

$\neg P$

T $\neg P \vee \neg Q$

F $\neg P$

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T					
T	F	T	F	F	T	T
F	T					
F	F					

$\neg(P \vee Q)$

$\vdash$

$\neg P$

T $\neg P \vee \neg Q$

$\vdash$

F $\neg P$

P	Q
T	F

is a counterexample to this argument

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T					
T	F	T	F	F	T	T
F	T					
F	F					

$\neg(P \vee Q)$

$\neg P$

T $\neg P \vee \neg Q$

F $\neg P$

P	Q
T	F

is a counterexample to this argument

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$
T	T		F			
T	F	T	F		T	T
F	T		F			
F	F		T	T		

$$
\begin{array}{|l}
\neg(P \vee Q) \\
\hline
\neg P
\end{array}
$$

$$
\begin{array}{r|l}
\text{T} & \neg P \vee \neg Q \\
\hline
\text{F} & \neg P
\end{array}
$$

P	Q
T	F

is a counterexample to this argument

P	Q	P∨Q	¬(P ∨ Q)	¬P	¬Q	¬P ∨ ¬Q
T	T		F			
T	F	T	F		T	T
F	T		F			
F	F		T	T		

$$\begin{array}{l|l} & \neg(P \vee Q) \\ \hline & \neg P \end{array}$$

$$\begin{array}{r|l} \text{T} & \neg P \vee \neg Q \\ \hline \text{F} & \neg P \end{array}$$

P	Q
T	F

is a counterexample to this argument

P	Q	P∨Q	¬(P ∨ Q)	¬P	¬Q	¬P ∨ ¬Q
T	T		F			
T	F	T	F		T	T
F	T		F			
F	F		T	T		

$$\neg(P \vee Q) \vdash \neg P$$

T $\neg P \vee \neg Q$

$\vdash$

F $\neg P$

This is a logically valid argument

P	Q
T	F

is a counterexample to this argument

The negation of a disjunction

A disjunction of two negations

$\neg(P \vee Q)$

$\neg P \vee \neg Q$

Things not to confuse

$\neg(P \vee Q)$ vs. $\neg P \vee \neg Q$

Things not to confuse

$\neg(P \lor Q)$	vs.	$\neg P \lor \neg Q$
$\neg(P \land Q)$	vs.	$\neg P \land \neg Q$

Things not to confuse

$\neg(P \vee Q)$	vs.	$\neg P \vee \neg Q$
$\neg(P \wedge Q)$	vs.	$\neg P \wedge \neg Q$
$\neg(P \rightarrow Q)$	vs.	$P \rightarrow \neg Q$


my happy ending.
Her problems
have become
THE PART OF THE DAY I LOOK FORWARD TO THE MOST, IS WHEN I GET TO PEE IN THE SHOWER
I FEEL SECRETLY

State the following rules:

$\vee$Intro

$\rightarrow$Intro

State the following rules:

∨Intro

→Intro

→Intro:

$*$

$\dots$

$\#$

$\dots$

$* \to \#$

State the following rules:

∨Intro

→Intro

→Intro:

| *

| ...

| #

...

*→#

∨Intro:

Pi

...

P1 ∨ P2

1.	$R \vee S$	
2.	R	
3.	$S \vee R$	$\vee$Intro: 2
4.	S	
5.	$S \vee R$	$\vee$Intro: 4
6.	$S \vee R$	$\vee$Elim: 1,2-3,4-5
7.	$R \wedge S$	$\wedge$Intro: 2,4

$\vee$Intro:

Pi

…

$P1 \vee P2$

$\vee$Elim:

$P1 \vee P2$

P1

…

Q

P2

…

Q

…

Q

$\wedge$Elim:

$P1 \wedge P2$

…

Pi

$\wedge$Intro:

P1

P2

…

$P1 \wedge P2$

1. R ∨ S

2. R

3. S ∨ R ∨Intro: 2

4. S

5. S ∨ R ∨Intro: 4

6. S ∨ R ∨Elim: 1,2-3,4-5

7. R ∧ S ∧Intro: 2,4

∨Intro:

Pi

…

P1 ∨ P2

∨Elim:

P1 ∨ P2

P1

…

Q

P2

…

Q

…

Q

∧Elim:

P1 ∧ P2

…

Pi

∧Intro:

P1

P2

…

P1 ∧ P2

1.	$R \vee S$	
2.	R	
3.	$S \vee R$	$\vee$Intro: 2
4.	S	
5.	$S \vee R$	$\vee$Intro: 4
6.	$S \vee R$	$\vee$Elim: 1,2-3,4-5
7.	$R \wedge S$	$\wedge$Intro: 2,4

$\vee$Intro:

Pi

…

$P1 \vee P2$

$\vee$Elim:

$P1 \vee P2$

P1

…

Q

P2

…

Q

…

Q

$\wedge$Elim:

$P1 \wedge P2$

…

Pi

$\wedge$Intro:

P1

P2

…

$P1 \wedge P2$

1.	$R \vee S$	
2.	$\quad R$	
3.	$\quad S \vee R$	$\vee$Intro: 2
4.	$\quad S$	
5.	$\quad S \vee R$	$\vee$Intro: 4
6.	$S \vee R$	$\vee$Elim: 1,2-3,4-5
7.	$R \wedge S$	$\wedge$Intro: 2,4

$\vee$Intro:

P_i
…
$P1 \vee P2$

$\vee$Elim:

$P1 \vee P2$
$\quad P1$
$\quad$ …
$\quad Q$
$\quad P2$
$\quad$ …
$\quad Q$
…
Q

$\wedge$Elim:

$P1 \wedge P2$
…
P_i

$\wedge$Intro:

$P1$
$P2$
…
$P1 \wedge P2$

1. $R \vee S$

2. R

3. $S \vee R$ $\quad$ $\vee$Intro: 2

4. S

5. $S \vee R$ $\quad$ $\vee$Intro: 4

6. $S \vee R$ $\quad$ $\vee$Elim: 1,2-3,4-5

7. $R \wedge S$ $\quad$ $\wedge$Intro: 2,4

$\wedge$Elim:

$P1 \wedge P2$

…

Pi

$\wedge$Intro:

$P1$

$P2$

…

$P1 \wedge P2$

$\vee$Intro:

Pi

…

$P1 \vee P2$

$\vee$Elim:

$P1 \vee P2$

$P1$

…

Q

$P2$

…

Q

…

Q

R	S	R ∨ S	R ∧ S
T	F	T	F

1. R ∨ S

2. R
3. S ∨ R — ∨Intro: 2

4. S
5. S ∨ R — ∨Intro: 4

6. S ∨ R — ∨Elim: 1,2-3,4-5
7. R ∧ S — ∧Intro: 2,4

∨Intro:

Pi
…
P1 ∨ P2

∨Elim:

P1 ∨ P2
P1
…
Q
P2
…
Q
…
Q

∧Elim:

P1 ∧ P2
…
Pi

∧Intro:

P1
P2
…
P1 ∧ P2

R	S	$R \vee S$	$R \wedge S$
T	F	T	F

T 1. $R \vee S$

2. R

3. $S \vee R$ $\vee$Intro: 2

4. S

5. $S \vee R$ $\vee$Intro: 4

6. $S \vee R$ $\vee$Elim: 1,2-3,4-5

7. $R \wedge S$ $\wedge$Intro: 2,4

$\vee$Intro:

Pi

…

$P1 \vee P2$

$\vee$Elim:

$P1 \vee P2$

P1

…

Q

P2

…

Q

…

Q

$\wedge$Elim:

$P1 \wedge P2$

…

Pi

$\wedge$Intro:

P1

P2

…

$P1 \wedge P2$

R	S	R ∨ S	R ∧ S
T	F	T	F

T 1. R ∨ S

2. R

3. S ∨ R ∨Intro: 2

4. S

5. S ∨ R ∨Intro: 4

6. S ∨ R ∨Elim: 1,2-3,4-5

F 7. R ∧ S ∧Intro: 2,4

∨Intro:

Pi

…

P1 ∨ P2

∨Elim:

P1 ∨ P2

P1

…

Q

P2

…

Q

…

Q

∧Elim:

P1 ∧ P2

…

Pi

∧Intro:

P1

P2

…

P1 ∧ P2

R	S	$R \vee S$	$R \wedge S$
T	F	T	F

T 1. $R \vee S$

2. R

3. $S \vee R$ $\vee$Intro: 2

4. S

5. $S \vee R$ $\vee$Intro: 4

6. $S \vee R$ $\vee$Elim: 1,2-3,4-5

F 7. $R \wedge S$ $\wedge$Intro: 2,4

$\vee$Intro:

Pi

…

$P1 \vee P2$

$\vee$Elim:

$P1 \vee P2$

$P1$

…

Q

$P2$

…

Q

…

Q

$\wedge$Elim:

$P1 \wedge P2$

…

Pi

$\wedge$Intro:

$P1$

$P2$

…

$P1 \wedge P2$

T	1.	$R \vee S$	
	2.	R	
	3.	$S \vee R$	$\vee$Intro: 2
	4.	S	
	5.	$S \vee R$	$\vee$Intro: 4
	6.	$S \vee R$	$\vee$Elim: 1,2-3,4-5
F	7.	$R \wedge S$	$\wedge$Intro: 2,4

R	S	$R \vee S$	$R \wedge S$
T	F	T	F

$\vee$Intro:

Pi

…

$P1 \vee P2$

$\vee$Elim:

$P1 \vee P2$

$P1$

…

Q

$P2$

…

Q

…

Q

$\wedge$Elim:

$P1 \wedge P2$

…

Pi

$\wedge$Intro:

$P1$

$P2$

…

$P1 \wedge P2$

T	1.	$R \lor S$	
	2.	R	
	3.	$S \lor R$	$\lor$Intro: 2
	4.	S	
	5.	$S \lor R$	$\lor$Intro: 4
	6.	$S \lor R$	$\lor$Elim: 1,2-3,4-5
F	7.	$R \land S$	$\land$Intro: 2,4

R	S	$R \lor S$	$R \land S$
T	F	T	F

$\lor$Intro:

P_i

…

$P1 \lor P2$

$\lor$Elim:

$P1 \lor P2$

$P1$

…

Q

$P2$

…

Q

…

Q

$\land$Elim:

$P1 \land P2$

…

P_i

$\land$Intro:

$P1$

$P2$

…

$P1 \land P2$

R	S	$R \vee S$	R
T	F	T	

T

1. $R \vee S$
2. R
3. $S \vee R$ — $\vee$Intro: 2
4. S
5. $S \vee R$ — $\vee$Intro: 4
6. $S \vee R$ — $\vee$Elim: 1,2-3,4-5
7. $R \wedge S$ — $\wedge$Intro: 2,4

F

$\wedge$Elim:

P1 $\wedge$ P2

...

Pi

$\wedge$Intro:

P1

P2

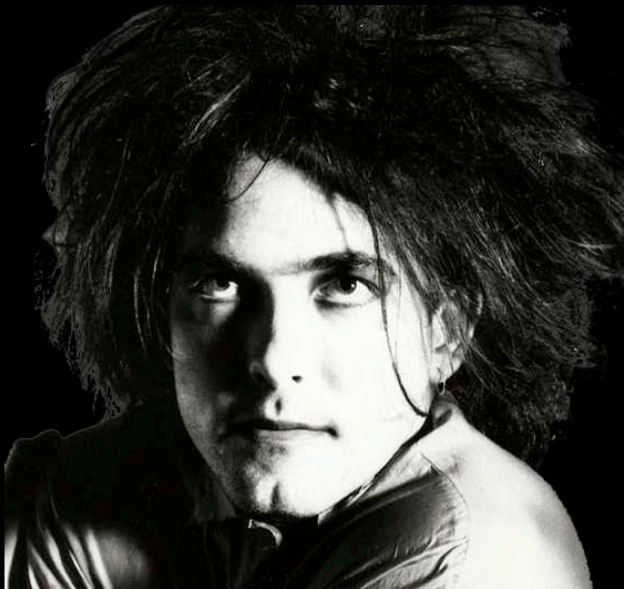
...

P1 $\wedge$ P2

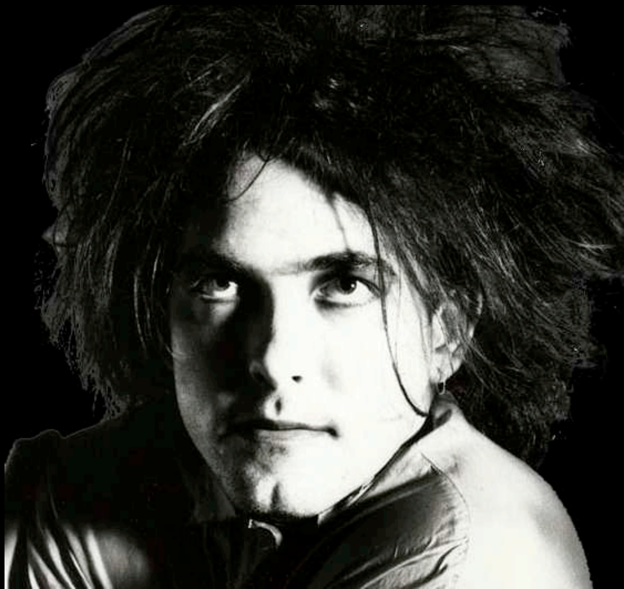
Rules of Proof for Quantifiers

“Everything’s coming to a grinding halt”

"Everything's coming to a grinding halt"

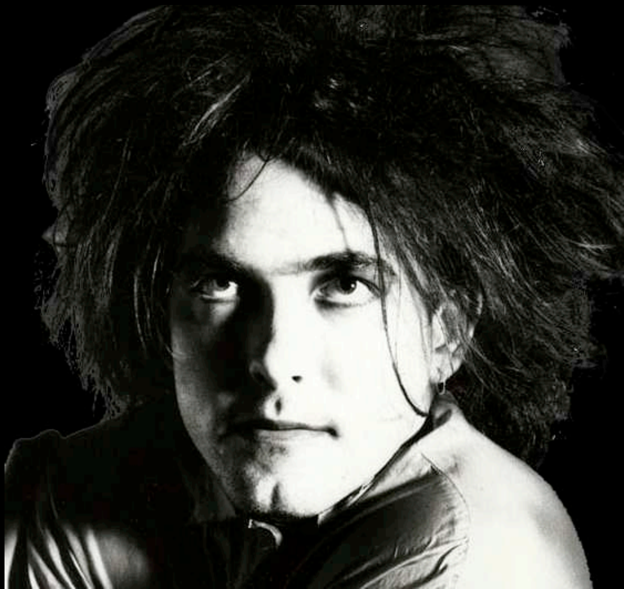


"Everything's coming to a grinding halt"



"Everything's coming to a grinding halt"

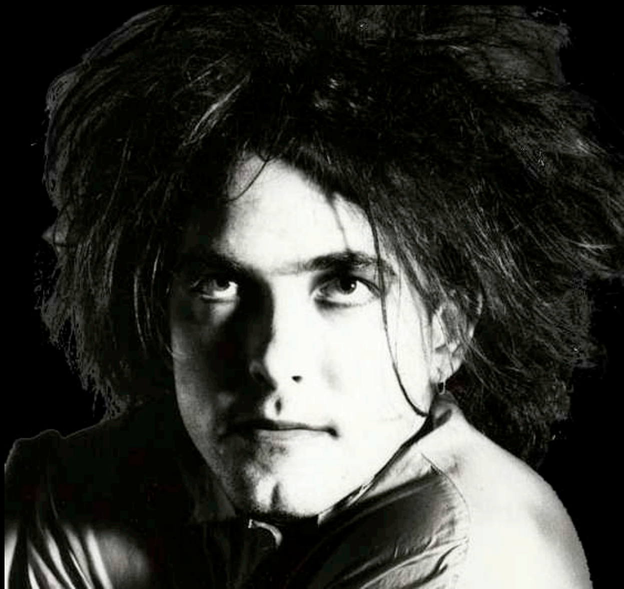
$\forall x$ **ComingToAGrindingHalt(x)**



“Everything’s coming to a grinding halt”

$\forall x$ **ComingToAGrindingHalt(x)**

“This lecture is coming to a grinding halt”

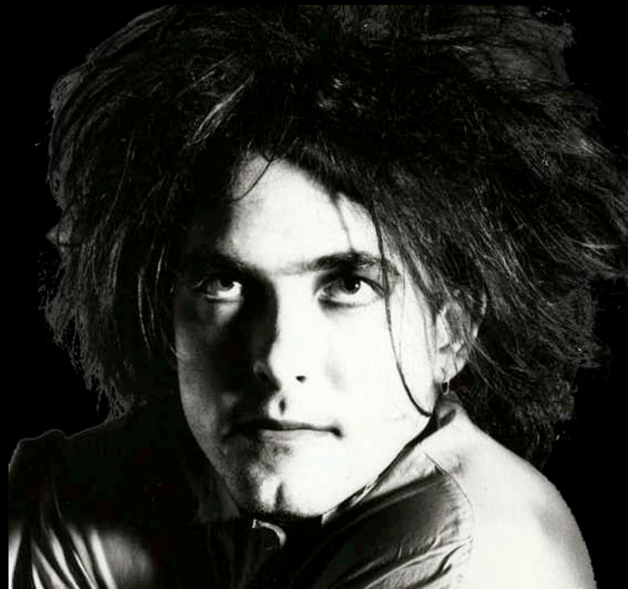


"Everything's coming to a grinding halt"

$\forall$x ComingToAGrindingHalt(x)

"This lecture is coming to a grinding halt"

ComingToAGrindingHalt(a)

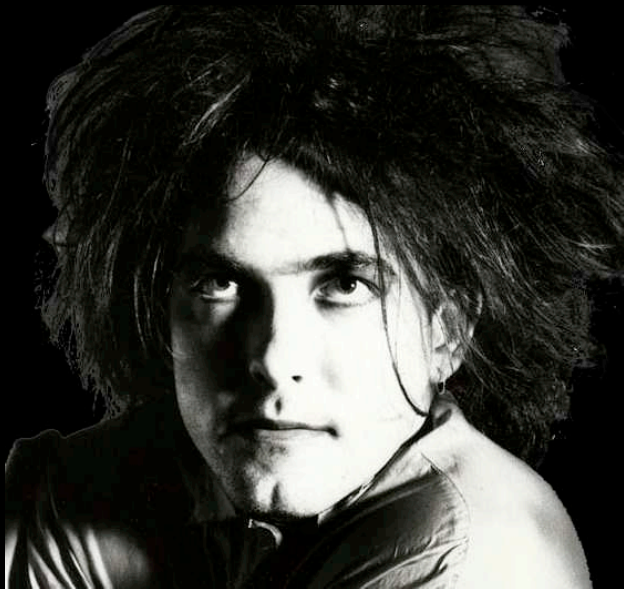


"Everything's coming to a grinding halt"

$\forall x$ **ComingToAGrindingHalt(x)**

"This lecture is coming to a grinding halt"

ComingToAGrindingHalt(a)



"Everything's coming to a grinding halt"

$\forall$x ComingToAGrindingHalt(x)

"This lecture is coming to a grinding halt"

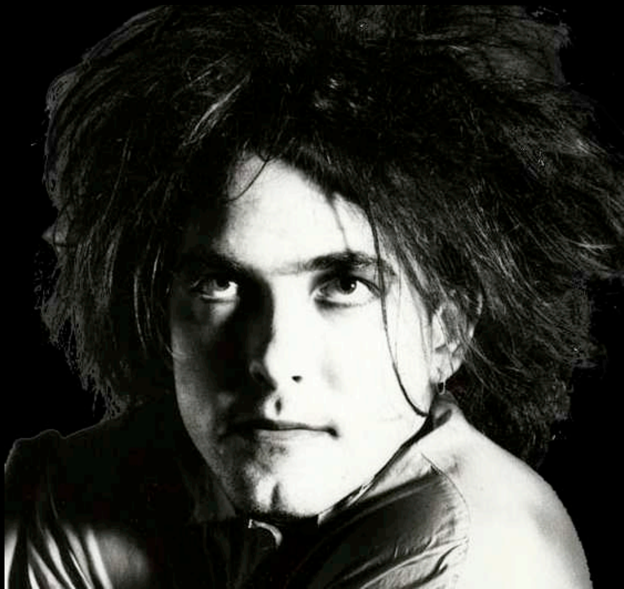
ComingToAGrindingHalt(a)

$\forall$Elim

$\forall$x S(x)

…

S(c)



$\forall x$ ComingToAGrindingHalt(x)

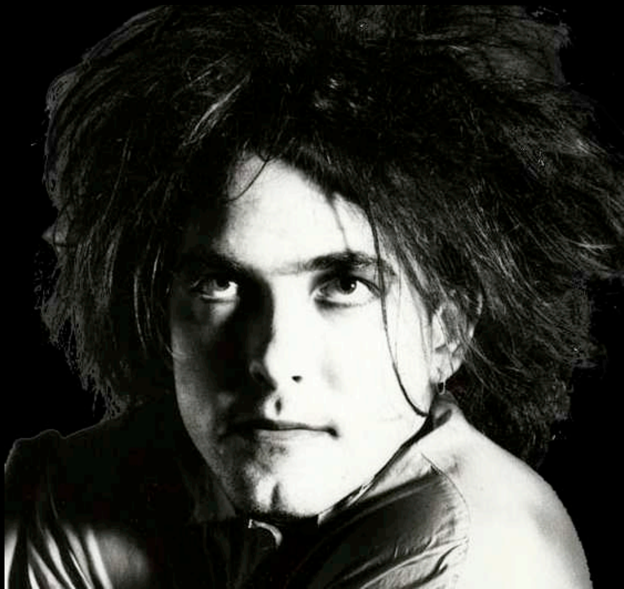
ComingToAGrindingHalt(a)

$\forall$Elim

$\forall x\ S(x)$

…

S(c)



$\forall x$ ComingToAGrindingHalt(x)

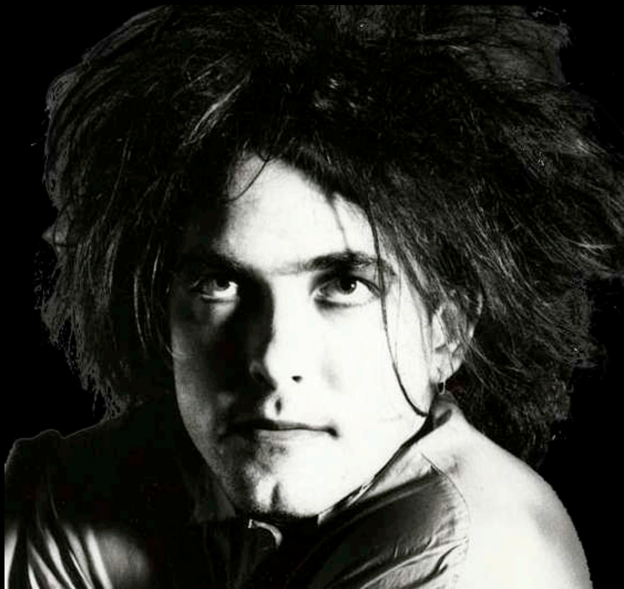
ComingToAGrindingHalt(a)

$\forall$Elim

$\forall x$ S(x)

...

S(c)



$\forall x$ ComingToAGrindingHalt(x)

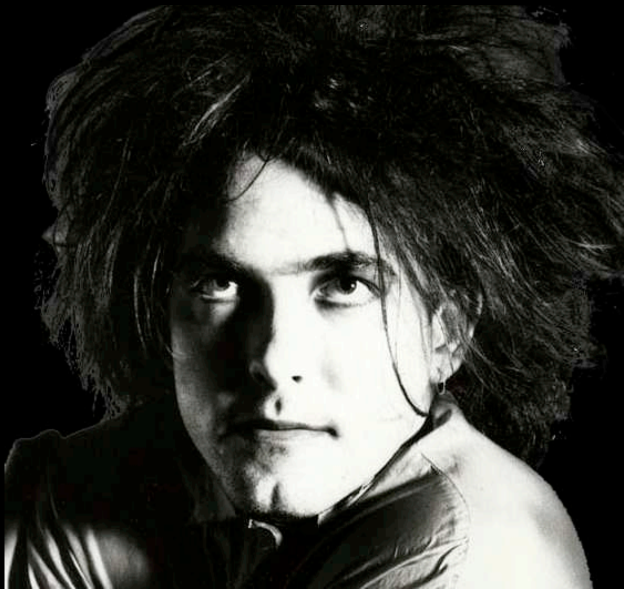
ComingToAGrindingHalt(a)

$\forall$Elim

$\forall x$ S(x)

...

S(c)



$\forall x$ ComingToAGrindingHalt(x)

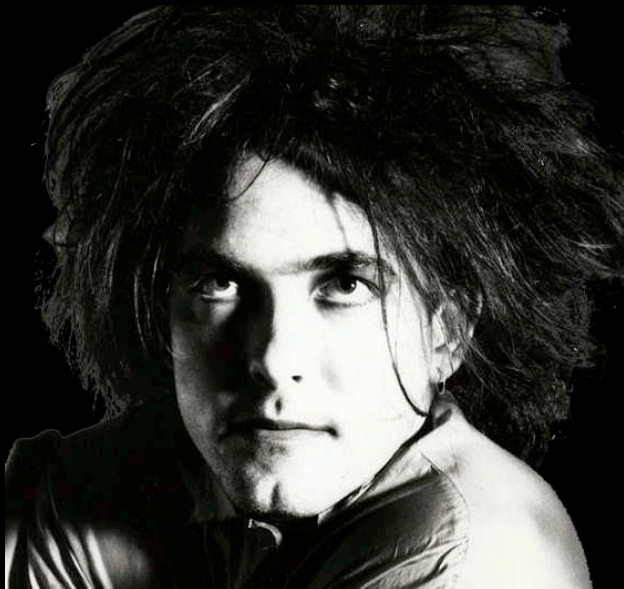
ComingToAGrindingHalt(a)

∀Elim

$\forall x\ S(x)$

...

$S(c)$



1. $\forall x$ ComingToAGrindingHalt(x)

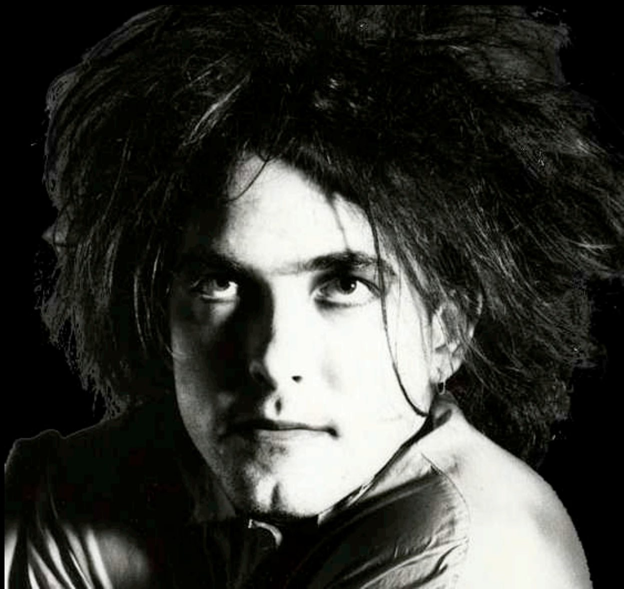
2. ComingToAGrindingHalt(a)

$\forall$Elim

$\forall x$ S(x)

...

S(c)



1. $\forall x$ ComingToAGrindingHalt(x)

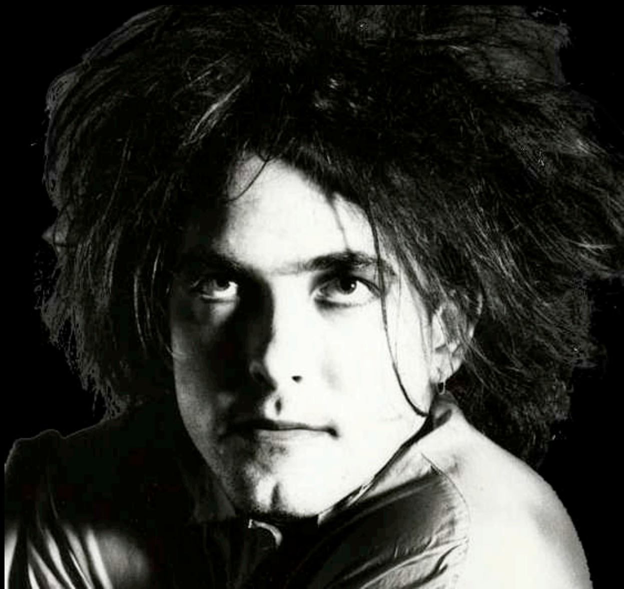
2. ComingToAGrindingHalt(a) $\forall$Elim: 1

$\forall$Elim

$\forall x$ S(x)

...

S(c)



All puffins have yellow beaks

Ayesha is a puffin

Ayesha has a yellow beak

All puffins have yellow beaks

Ayesha is a puffin

Ayesha has a yellow beak

All puffins have yellow beaks	P
Ayesha is a puffin	Q
Ayesha has a yellow beak	R

All puffins have yellow beaks

Ayesha is a puffin

Ayesha has a yellow beak

P

Q

R

There is a counterexample to this argument

All puffins have yellow beaks | P

Ayesha is a puffin | Q

Ayesha has a yellow beak | R

P	Q	R
T	T	F

There is a counterexample to this argument

All puffins have yellow beaks

Ayesha is a puffin

Ayesha has a yellow beak

P

Q

R

P	Q	R
T	T	F

There is a counterexample to this argument

All puffins have yellow beaks

Ayesha is a puffin

Ayesha has a yellow beak

P

Q

R

There is a counterexample to this argument

P	Q	R
T	T	F

a : Ayesha

YelBk(x) : x has a yellow beak

All puffins have yellow beaks

Ayesha is a puffin

Ayesha has a yellow beak

P

Q

R YelBk(a)

P	Q	R
T	T	F

There is a counterexample to this argument

a : Ayesha

YelBk(x) : x has a yellow beak

All puffins have yellow beaks

Ayesha is a puffin

Ayesha has a yellow beak

P

Q

R YelBk(a)

There is a counterexample to this argument

P	Q	R
T	T	F

a : Ayesha

YelBk(x) : x has a yellow beak

All puffins have yellow beaks

Ayesha is a puffin

Ayesha has a yellow beak

P

Q

R   YelBk(a)

There is a counterexample to this argument

P	Q	R
T	T	F

a : Ayesha

YelBk(x) : x has a yellow beak

Puf(x) : x is a puffin

All puffins have yellow beaks

Ayesha is a puffin

Ayesha has a yellow beak

P	
Q	Puf(a)
R	YelBk(a)

P	Q	R
T	T	F

There is a counterexample to this argument

a : Ayesha

YelBk(x) : x has a yellow beak

Puf(x) : x is a puffin

All puffins have yellow beaks	P		
Ayesha is a puffin	Q	Puf(a)	
Ayesha has a yellow beak	R	YelBk(a)	

P	Q	R
T	T	F

There is a counterexample to this argument

a : Ayesha

YelBk(x) : x has a yellow beak

Puf(x) : x is a puffin

All puffins have yellow beaks	P		
Ayesha is a puffin	Q	Puf(a)	
Ayesha has a yellow beak	R	YelBk(a)	

P	Q	R
T	T	F

There is a counterexample to this argument

a : Ayesha

YelBk(x) : x has a yellow beak

Puf(x) : x is a puffin

from last lecture

In most cases we will need to use the form

$$\forall x\ (\ F(x) \rightarrow G(x)\ )$$

"All Fs are Gs"

All puffins have yellow beaks	P	$\forall x\ (\ Puf(x) \rightarrow YelBk(x)\ )$
Ayesha is a puffin	Q	$Puf(a)$
Ayesha has a yellow beak	R	$YelBk(a)$

There is a counterexample to this argument

P	Q	R
T	T	F

a : Ayesha

YelBk(x) : x has a yellow beak

Puf(x) : x is a puffin

from last lecture

In most cases we will need to use the form

$$\forall x\ (\ F(x) \rightarrow G(x)\ )$$

"All Fs are Gs"

All puffins have yellow beaks	P	$\forall x\ (\ Puf(x) \rightarrow YelBk(x)\ )$
Ayesha is a puffin	Q	$Puf(a)$
Ayesha has a yellow beak	R	$YelBk(a)$

P	Q	R
T	T	F

There is a counterexample to this argument

a : Ayesha

YelBk(x) : x has a yellow beak

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from last lecture

In most cases we will need to use the form

$$\forall x\ (\ F(x) \rightarrow G(x)\ )$$

"All Fs are Gs"

All puffins have yellow beaks	1.	$\forall x\ (\ Puf(x) \rightarrow YelBk(x)\ )$
Ayesha is a puffin	2.	$Puf(a)$
Ayesha has a yellow beak	x.	$YelBk(a)$

All puffins have yellow beaks	1.	$\forall x\ (\ Puf(x) \rightarrow YelBk(x)\ )$
Ayesha is a puffin	2.	$Puf(a)$
Ayesha has a yellow beak	x.	$YelBk(a)$

$\forall$Elim

$\forall x\ S(x)$

$\ldots$

$S(c)$

All puffins have yellow beaks	1.	$\forall x\ (\, Puf(x) \rightarrow YelBk(x)\,)$
Ayesha is a puffin	2.	$Puf(a)$
	3.	$Puf(a) \rightarrow YelBk(a)$
Ayesha has a yellow beak	x.	$YelBk(a)$

$\forall$Elim

$\forall x\ S(x)$

…

$S(c)$

All puffins have yellow beaks	1.	$\forall x\,(\,Puf(x) \rightarrow YelBk(x)\,)$
Ayesha is a puffin	2.	$Puf(a)$
If A is a puff’ n, she has a Y.B.	3.	$Puf(a) \rightarrow YelBk(a)$
Ayesha has a yellow beak	x.	$YelBk(a)$

$\forall$Elim

$\forall x\ S(x)$

$\ldots$

$S(c)$

All puffins have yellow beaks	1.	$\forall x\ (\mathrm{Puf}(x) \rightarrow \mathrm{YelBk}(x))$	
Ayesha is a puffin	2.	$\mathrm{Puf}(a)$	
If A is a puff’ n, she has a Y.B.	3.	$\mathrm{Puf}(a) \rightarrow \mathrm{YelBk}(a)$	$\forall$Elim:1
Ayesha has a yellow beak	x.	$\mathrm{YelBk}(a)$	

$\forall$Elim

$\forall x\ S(x)$

…

$S(c)$

All puffins have yellow beaks	1.	$\forall x\,(\,Puf(x) \rightarrow YelBk(x)\,)$	
Ayesha is a puffin	2.	$Puf(a)$	
If A is a puff' n, she has a Y.B.	3.	$Puf(a) \rightarrow YelBk(a)$	$\forall$Elim:1
Ayesha has a yellow beak	x.	$YelBk(a)$	

$\forall$Elim

$\forall x\; S(x)$

$\dots$

$S(c)$

All puffins have yellow beaks	1.	$\forall x\ (\ Puf(x) \rightarrow YelBk(x)\ )$	
Ayesha is a puffin	2.	$Puf(a)$	
If A is a puff' n, she has a Y.B.	3.	$Puf(a) \rightarrow YelBk(a)$	$\forall$Elim:1
Ayesha has a yellow beak	x.	$YelBk(a)$	

$\forall$Elim

$\forall x\ S(x)$

...

$S(c)$

All puffins have yellow beaks	1.	$\forall x\, (\mathrm{Puf}(x) \rightarrow \mathrm{YelBk}(x))$	
Ayesha is a puffin	2.	$\mathrm{Puf}(a)$	
If A is a puff' n, she has a Y.B.	3.	$\mathrm{Puf}(a) \rightarrow \mathrm{YelBk}(a)$	$\forall$Elim:1
Ayesha has a yellow beak	x.	$\mathrm{YelBk}(a)$	

$\forall$Elim

$\forall x\ S(x)$

...

$S(c)$

All puffins have yellow beaks	1.	$\forall x\ (\ Puf(x) \rightarrow YelBk(x)\ )$	
Ayesha is a puffin	2.	$Puf(a)$	
If A is a puff' n, she has a Y.B.	3.	$Puf(a) \rightarrow YelBk(a)$	$\forall$Elim:1
Ayesha has a yellow beak	x.	$YelBk(a)$	

$\forall$Elim

$\forall x\ S(x)$

…

$S(c)$

All puffins have yellow beaks	1.	$\forall x\ (\mathrm{Puf}(x) \rightarrow \mathrm{YelBk}(x))$	
Ayesha is a puffin	2.	$\mathrm{Puf}(a)$	
If A is a puff’ n, she has a Y.B.	3.	$\mathrm{Puf}(a) \rightarrow \mathrm{YelBk}(a)$	$\forall$Elim:1
Ayesha has a yellow beak	x.	$\mathrm{YelBk}(a)$	

$\forall$Elim

$\forall x\ S(x)$

…

$S(c)$

All puffins have yellow beaks	1.	$\forall x\ (\ Puf(x) \rightarrow YelBk(x)\ )$	
Ayesha is a puffin	2.	$Puf(a)$	
If A is a puff’ n, she has a Y.B.	3.	$Puf(a) \rightarrow YelBk(a)$	$\forall$Elim:1
Ayesha has a yellow beak	x.	$YelBk(a)$	$\rightarrow$Elim: 3,2

$\forall$Elim

$\forall x\ S(x)$

…

$S(c)$

All puffins have yellow beaks	1.	$\forall x\ (\ Puf(x) \rightarrow YelBk(x)\ )$	
Ayesha is a puffin	2.	$Puf(a)$	
If A is a puff' n, she has a Y.B.	3.	$Puf(a) \rightarrow YelBk(a)$	$\forall$Elim:1
Ayesha has a yellow beak	x.	$YelBk(a)$	$\rightarrow$Elim: 3,2

$\forall$Elim

$\forall x\ S(x)$

…

$S(c)$

another rule, another argument

Ayesha has a yellow beak

Something has a yellow beak

$\forall$Elim

$\forall x\ S(x)$

...

$S(c)$

Ayesha has a yellow beak	4.		YelBk(a)
Something has a yellow beak	5.		???

∀Elim

∀x S(x)

…

S(c)

Ayesha has a yellow beak	4.	YelBk(a)
Something has a yellow beak	5.	∃x YelBk(x)

∀Elim

∀x S(x)

...

S(c)

Ayesha has a yellow beak	4.	YelBk(a)
Something has a yellow beak	5.	∃x YelBk(x)

∀Elim

∀x S(x)

…

S(c)

∃Intro

S(a)

…

∃x S(x)

Ayesha has a yellow beak	4.	YelBk(a)
Something has a yellow beak	5.	∃x YelBk(x)

∀Elim

∀x S(x)

...

S(c)

∃Intro

S(a)

...

∃x S(x)

Ayesha has a yellow beak	4.	YelBk(a)
Something has a yellow beak	5.	∃x YelBk(x)

∀Elim

∀x S(x)

. . .

S(c)

∃Intro

S(a)

. . .

∃x S(x)

Ayesha has a yellow beak	4.	YelBk(a)	
Something has a yellow beak	5.	∃x YelBk(x)	∃Intro: 4

∀Elim

$$\forall x\, S(x)$$

$$\ldots$$

$$S(c)$$

∃Intro

$$S(a)$$

$$\ldots$$

$$\exists x\, S(x)$$

Ayesha has a yellow beak	4.	$YelBk(a)$	
Something has a yellow beak	5.	$\exists x\ YelBk(x)$	$\exists$Intro: 4

$\forall$Elim

$\forall x\ S(x)$

…

$S(c)$

$\exists$Intro

$S(a)$

…

$\exists x\ S(x)$

All puffins have yellow beaks	1.	$\forall x\,(\,Puf(x) \rightarrow YelBk(x)\,)$	
Ayesha is a puffin	2.	$Puf(a)$	
If A is a puff’ n, she has a Y.B.	3.	$Puf(a) \rightarrow YelBk(a)$	$\forall$Elim:1
Ayesha has a yellow beak	4.	$YelBk(a)$	$\rightarrow$Elim: 3,2

Ayesha has a yellow beak	4.	$YelBk(a)$	
Something has a yellow beak	5.	$\exists x\, YelBk(x)$	$\exists$Intro: 4

$\forall$Elim

$\forall x\, S(x)$

…

$S(c)$

$\exists$Intro

$S(a)$

…

$\exists x\, S(x)$

All puffins have yellow beaks	1.	$\forall x\,(\,Puf(x) \rightarrow YelBk(x)\,)$	
Ayesha is a puffin	2.	$Puf(a)$	
If A is a puff’ n, she has a Y.B.	3.	$Puf(a) \rightarrow YelBk(a)$	$\forall$Elim: 1
Ayesha has a yellow beak	4.	$YelBk(a)$	$\rightarrow$Elim: 3,2
Something has a yellow beak	5.	$\exists x\, YelBk(x)$	$\exists$Intro: 4

$\forall$Elim

$\forall x\, S(x)$

…

$S(c)$

$\exists$Intro

$S(a)$

…

$\exists x\, S(x)$

All puffins have yellow beaks	1.	$\forall x\ (\ Puf(x) \rightarrow YelBk(x)\ )$	
Ayesha is a puffin	2.	$Puf(a)$	
Something has a yellow beak	5.	$\exists x\ YelBk(x)$	$\exists$Intro: 4

$\forall$Elim

$\forall x\ S(x)$

…

$S(c)$

$\exists$Intro

$S(a)$

…

$\exists x\ S(x)$

All puffins have yellow beaks	1.	$\forall x\ (\ Puf(x) \rightarrow YelBk(x)\ )$	
Ayesha is a puffin	2.	$Puf(a)$	
If A is a puff' n, she has a Y.B.	3.	$Puf(a) \rightarrow YelBk(a)$	$\forall$Elim: 1
Ayesha has a yellow beak	4.	$YelBk(a)$	$\rightarrow$Elim: 3,2
Something has a yellow beak	5.	$\exists x\ YelBk(x)$	$\exists$Intro: 4

$\forall$Elim

$\forall x\ S(x)$

…

$S(c)$

$\exists$Intro

$S(a)$

…

$\exists x\ S(x)$