Logic (PH133)
Revision Lecture 1

Stephen Butterfill, Philosophy/Warwick

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

A	B	$A \to B$	$B \to A$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

A	B	A→B	B→A
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

(A→B) ∧(B→A)
T
F
F
T

A	B	$A \rightarrow B$	$B \rightarrow A$	$A \leftrightarrow B$	$(A \rightarrow B) \wedge (B \rightarrow A)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

A	B	$A \to B$	$B \to A$	$A \leftrightarrow B$	$(A \to B) \wedge (B \to A)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

A	B	$A \rightarrow B$	$B \rightarrow A$	$A \leftrightarrow B$	$(A \rightarrow B) \wedge (B \rightarrow A)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

1. | $A \leftrightarrow B$

2. | A

3. | B $\leftrightarrow$Elim:1,2

A	B	A→B	B→A	A↔B	(A→B) ∧ (B→A)
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

1.	A↔B	
2.	A	
3.	B	↔Elim:1,2

1.	A↔B	
2.	B	
3.	A	↔Elim:1,2

→A	A↔B	(A→B) ∧ (B→A)
T	T	T
T	F	F
F	F	F
T	T	T

...

11. | | A
12. | | ...
13. | | B ...
14. | A→B →Intro: 11-13

→A	A↔B	(A→B) ∧(B→A)
T	T	T
F	F	F
F	F	F
T	T	T

...

11. | A

12. | ...

13. | B ...

14. A→B →Intro: 11-13

15.

16.

17.

18.

19.

…A	A↔B	(A→B) ∧(B→A)
T	T	T
F	F	F
F	F	F
T	T	T

...

11. | A

12. | ...

13. | B ...

14. A→B →Intro: 11-13

15. | B

16. | ...

17. | A ...

18. B→A →Intro: 15-17

19.

→A	A↔B	(A→B) ∧ (B→A)
T	T	T
F	F	F
F	F	F
T	T	T

...

11. | A
12. | ...
13. | B ...
14. A→B →Intro: 11-13
15. | B
16. | ...
17. | A ...
18. B→A →Intro: 15-17
19. B↔A ↔Intro: 11-13, 15-17

→A	A↔B	(A→B) ∧(B→A)
T	T	T
F	F	F
F	F	F
T	T	T

...

11. | A

12. | ...

13. | B ...

14. A→B →Intro: 11-13

15. | B

16. | ...

17. | A ...

18. B→A →Intro: 15-17

19. B↔A ↔Intro: 11-13, 15-17

A↔B	(A→B) ∧ (B→A)
T	T
F	F
F	F
T	T

…

11. | A
12. | …
13. | B …
14. A→B →Intro: 11-13
15. | B
16. | …
17. | A …
18. B→A →Intro: 15-17
19. B↔A ↔Intro: 11-13, 15-17

A↔B	(A→B) ∧(B→A)
T	T
F	F
F	F
T	T

...

11. A
12. ...
13. B ...
14. $A \rightarrow B$ $\rightarrow$Intro: 11-13
15. B
16. ...
17. A ...
18. $B \rightarrow A$ $\rightarrow$Intro: 15-17
19. $B \leftrightarrow A$ $\leftrightarrow$Intro: 11-13, 15-17

$\rightarrow A$	$A \leftrightarrow B$	$(A \rightarrow B) \wedge (B \rightarrow A)$
T	T	T
F	F	F
F	F	F
T	T	T

Outline

How to pass the exam

Outline

How to ~~pass~~ ace the exam


БРОНЕНОСЕЦ ПОТЕМКИН
1905
ПРОИЗВОДСТВО
ГОСКИНО
ПЕРВОЙ ФАБРИКИ
ПОСТАНОВКА
С. М. ЭЙЗЕНШТЕЙНА

Outline

How to ~~pass~~ ace the exam

Q.1 proofs

a) State the rules of proof for conjunction ($\wedge$) and material implication ($\rightarrow$).

a) State the rules of proof for conjunction (∧) and material implication (→).

∧Elim:

| P1 ∧ P2
| …
| Pi

→Elim:

| *→#
| …
| *
| …
| #

a) State the rules of proof for conjunction (∧) and material implication (→).

∧Elim:

| P1 ∧ P2
| …
| Pi

∧Intro:

| P1
| P2
| …
| P1 ∧ P2

→Elim:

| *→#
| …
| *
| …
| #

→Intro:

| | *
| |---
| | …
| | #
| …
| *→#

a) alternatives …

What is a logically valid argument?

a) alternatives …

What is a logically valid argument?

An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false

a) alternatives …

What is a logically valid argument? (logical consequence, logical truth, tautology, contradiction, counterexample, …)

An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false

a) alternatives …

What is a logically valid argument? (logical consequence, logical truth, tautology, contradiction, counterexample, …)

What is a proof?

An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false

a) alternatives …

What is a logically valid argument? (logical consequence, logical truth, tautology, contradiction, counterexample, …)

What is a proof?

Explain the significance in the Fitch system of multiple vertical lines to the left of proof sentences.

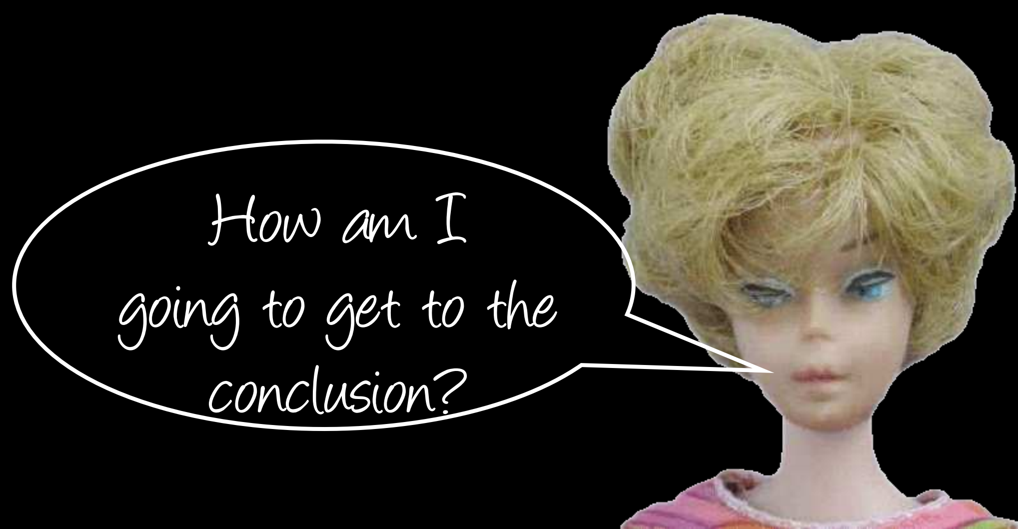
An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false

b) proofs

1. $\neg P \vee R$
2.
3.
4.
5. $P \rightarrow R$

b) proofs

1. $\neg P \vee R$
2.
3.
4.
5. $P \rightarrow R$



b) proofs

1.	$\neg P \vee R$
2.	
3.	
4.	
5.	$P \rightarrow R$

b) proofs

1. $\neg P \lor R$
2.
3.
4.
5. $P \to R$

$\to$Intro: ???

b) proofs

1. $\neg P \lor R$

2. P

3.

4. R

5. $P \to R$ $\to$Intro: 2-4

b) proofs

1. $\neg P \vee R$

2. P

3.

4. R

5. $P \rightarrow R$ — $\rightarrow$Intro: 2-4

b) proofs

1.	$\neg P \lor R$	
2.	P	
3.		
4.	R	
5.	$P \to R$	$\to$Intro: 2-4

b) proofs

1.	$\neg P \vee R$	
2.	P	
3.	$\bot$	$\bot$Intro: 1, 2
4.	R	$\bot$Elim: 3
5.	$P \to R$	$\to$Intro: 2-4

b) proofs

1.	$\neg P \vee R$	
2.	P	
3.	$\bot$	$\bot$Intro: 1, 2
4.	R	$\bot$Elim: 3
5.	$P \rightarrow R$	$\rightarrow$Intro: 2-4

b) proofs

1.	$\neg P \vee R$	
2.	P	
3.	$\perp$	$\perp$Intro: 1, 2
4.	R	$\perp$Elim: 3
5.	$P \rightarrow R$	$\rightarrow$Intro: 2-4

$\perp$Intro

S

…

$\neg S$

…

$\perp$

b) proofs

1.	¬P∨R	
2.	P	
3.	⊥	⊥Intro: 1, 2
4.	R	⊥Elim: 3
5.	P→R	→Intro: 2-4

⊥Intro

S

. . .

¬S

. . .

⊥

b) proofs

1.	$\neg P \lor R$	
2.	P	
3.	$\bot$	$\bot$Intro: 1, 2
4.	R	$\bot$Elim: 3
5.	$P \to R$	$\to$Intro: 2-4

$\bot$Intro

S

...

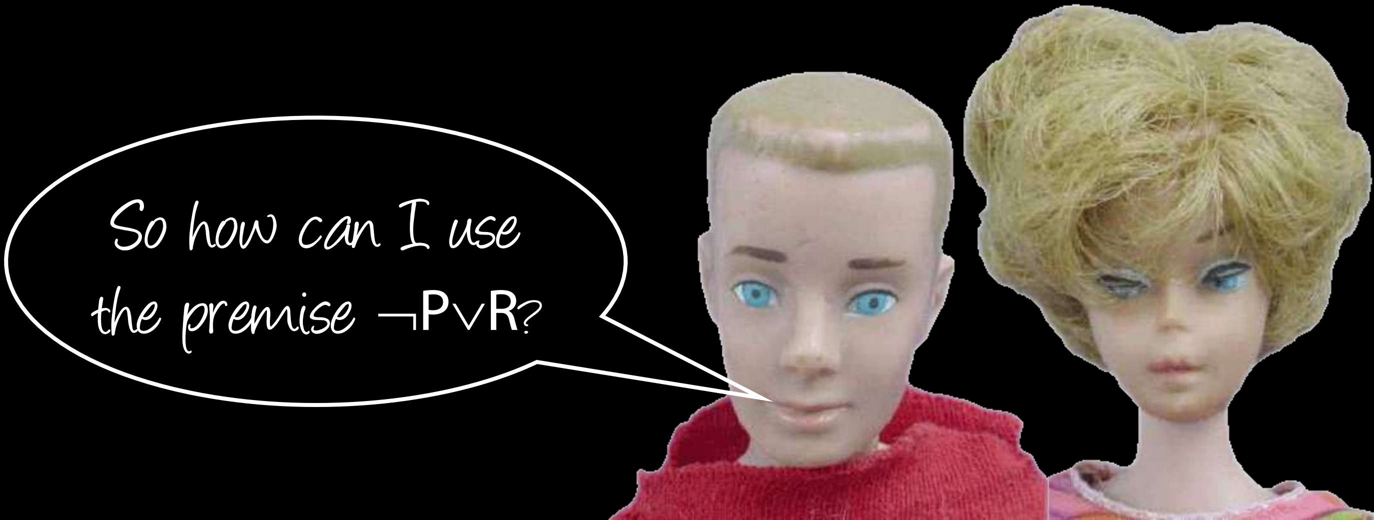
$\neg S$

...

$\bot$

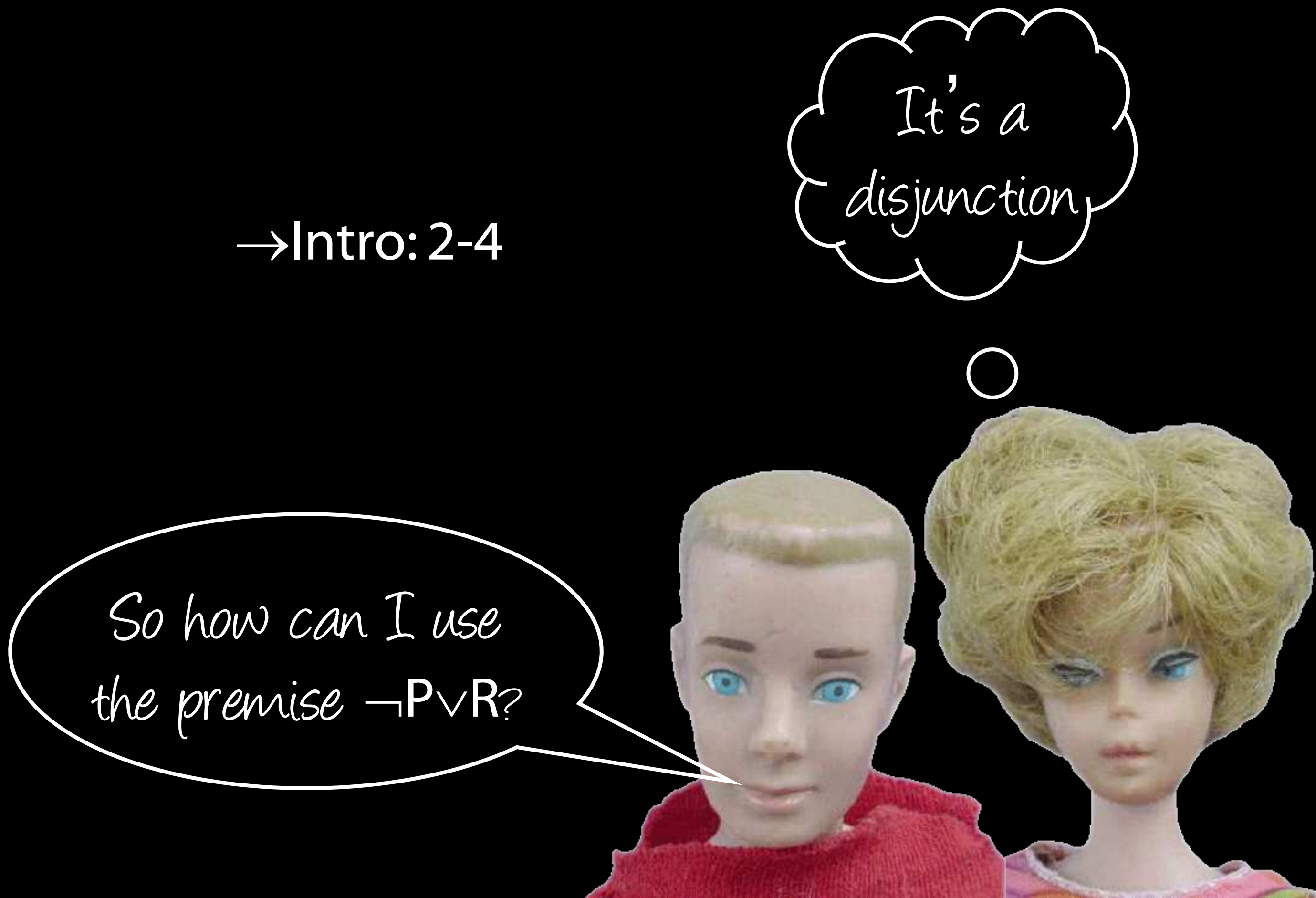
b) proofs

1. $\neg P \vee R$
2. P
3.
4. R
5. $P \rightarrow R$ $\rightarrow$Intro: 2-4



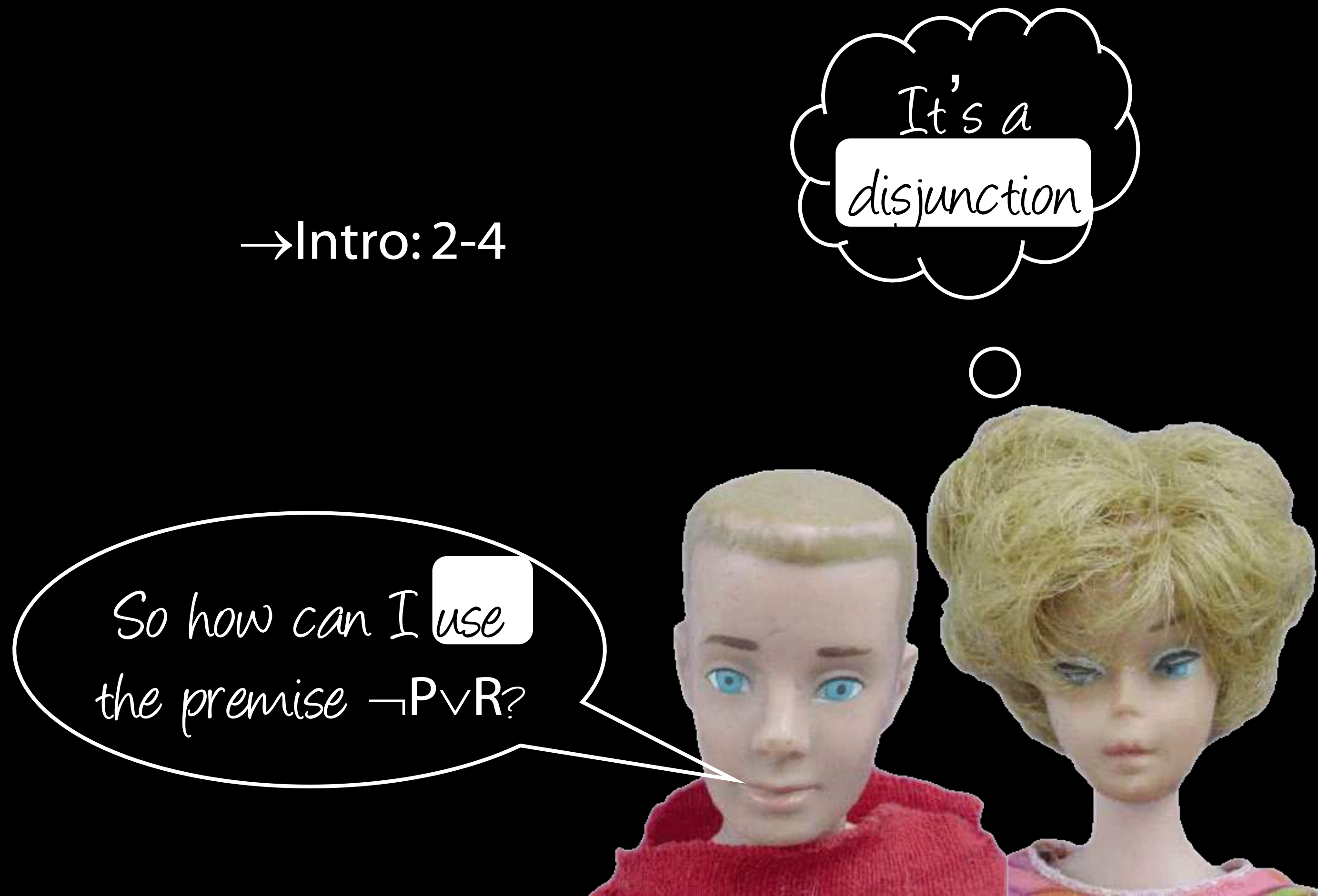
b) proofs

1.	$\neg P \vee R$	
2.	P	
3.		
4.	R	
5.	$P \rightarrow R$	$\rightarrow$Intro: 2-4



b) proofs

1.	$\neg P \vee R$	
2.	P	
3.		
4.	R	
5.	$P \rightarrow R$	$\rightarrow$Intro: 2-4

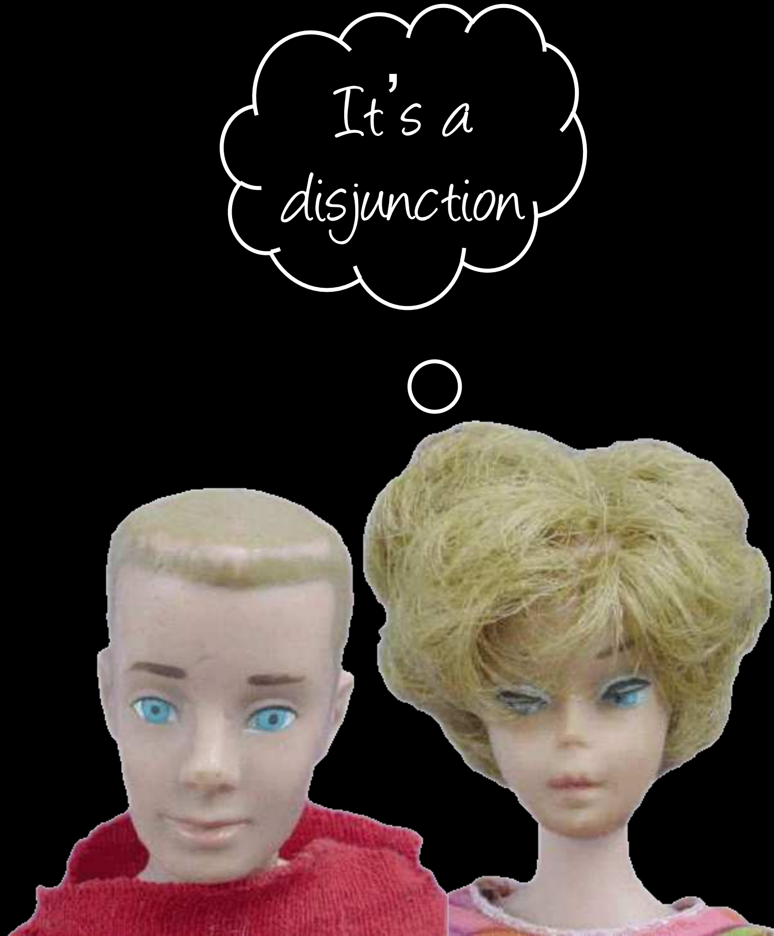


b) proofs

1. $\neg P \vee R$
2.
3.
4.
5.
6.
7.
8.
9.
10. $P \rightarrow R$

b) proofs

1.	$\neg P \vee R$	
2.	P	
3.		
4.		
5.		
6.		
7.		
8.		
9.	R	
10.	$P \rightarrow R$	$\rightarrow$Intro: 2-9



b) proofs

1.	¬P∨R	
2.	P	
3.		
4.		
5.		
6.		
7.		
8.		
9.	R	
10.	P→R	→Intro: 2-9

b) proofs

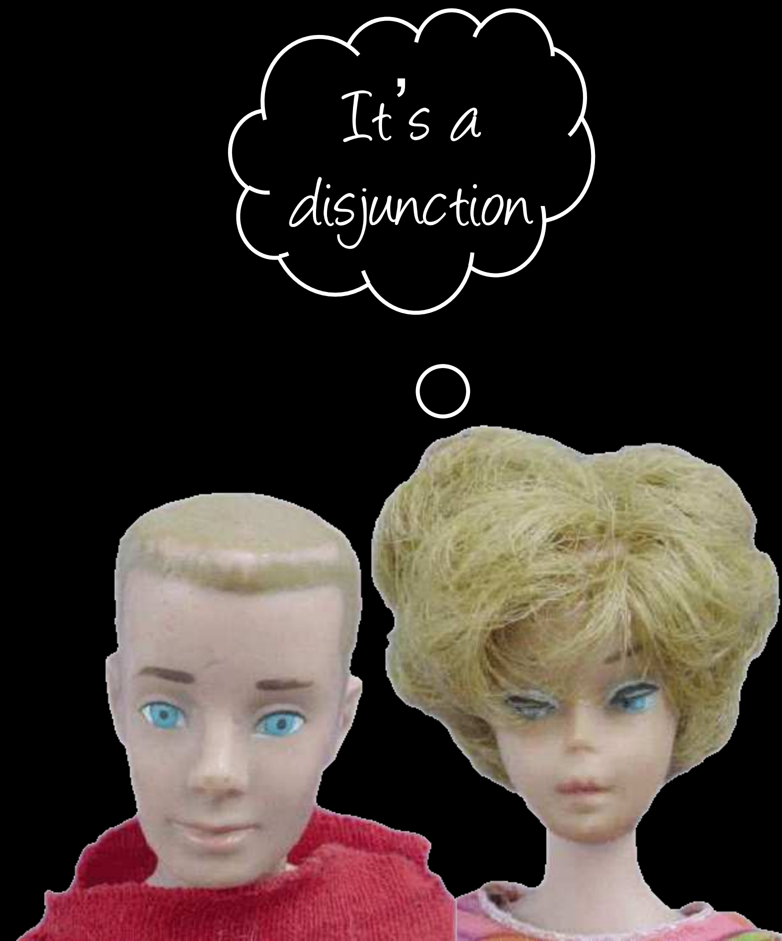
1. ¬P∨R
2. P
3.
4.
5.
6.
7.
8.
9. R
10. P→R →Intro: 2-9

b) proofs

1.	$\neg P \vee R$	
2.	P	
3.	$\neg P$	
4.		
5.		
6.	R	
7.		
8.		
9.	R	
10.	$P \rightarrow R$	$\rightarrow$Intro: 2-9

b) proofs

1. $\neg P \lor R$
2. P
3. $\neg P$
4.
5.
6. R
7.
8.
9. R
10. $P \to R$ — $\to$Intro: 2-9



b) proofs

1. ¬P∨R
2. P
3. ¬P
4.
5.
6. R
7.
8.
9. R
10. P→R →Intro: 2-9

b) proofs

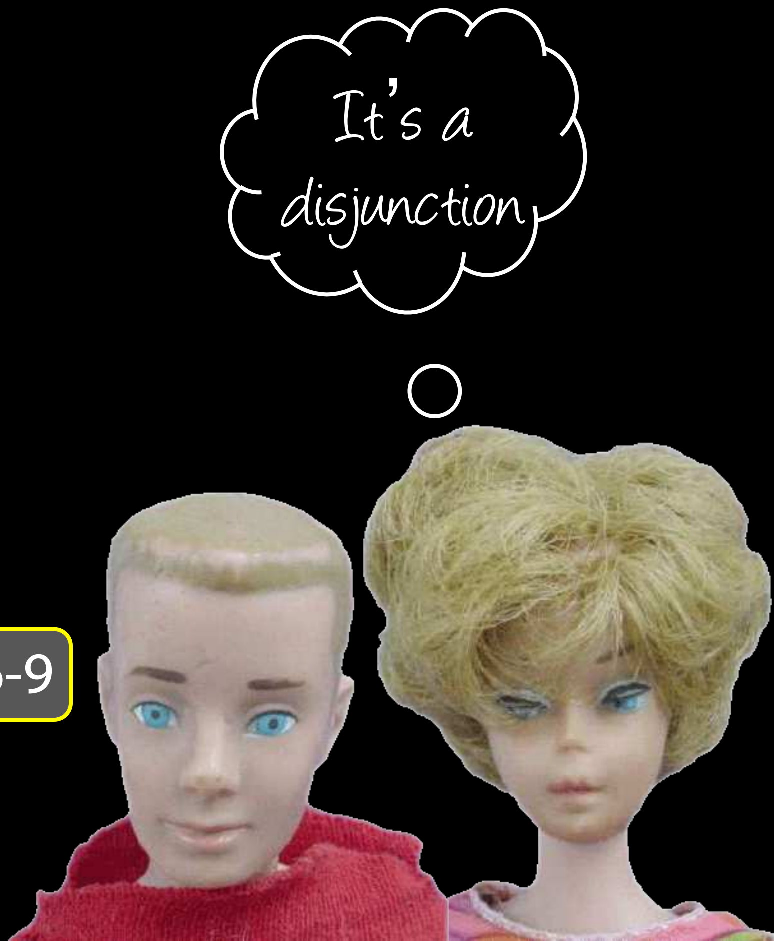
1.	$\neg P \vee R$	
2.	P	
3.	$\neg P$	
4.		
5.		
6.	R	
7.		
8.		
9.	R	
10.	$P \rightarrow R$	$\rightarrow$Intro: 2-9

b) proofs

1. $\neg P \lor R$
2. P
3. $\neg P$
4. ...
5. R
6. R
7. ...
8. R
9. R
10. $P \rightarrow R$ $\rightarrow$Intro: 2-9

b) proofs

1.	$\neg P \vee R$	
2.	P	
3.	$\neg P$	
4.	…	
5.	R	
6.	R	
7.	…	
8.	R	
9.	R	$\vee$Elim: 1, 3-5, 6-9
10.	$P \rightarrow R$	$\rightarrow$Intro: 2-9



b) proofs

1.	$\neg P \vee R$	
2.	P	
3.	$\neg P$	
4.	…	
5.	R	
6.	R	
7.	…	
8.	R	
9.	R	$\vee$Elim: 1, 3-5, 6-9
10.	$P \rightarrow R$	$\rightarrow$Intro: 2-9

b) proofs

1.	¬P∨R	
2.	P	
3.	¬P	
4.	…	
5.	R	
6.	R	
7.	…	
8.	R	
9.	R	∨Elim: 1, 3-5, 6-9
10.	P→R	→Intro: 2-9

Only have to complete these two subproofs

b) proofs

$\lor$Elim

1. $\neg P \lor R$
2. P
3. $\neg P$
4. ...
5. R
6. R
7. ...
8. R
9. R — $\lor$Elim: 1, 3-5, 6-9
10. $P \to R$ — $\to$Intro: 2-9

Which rule to complete this subproof?

these two subproofs

b) proofs

∨Elim

→Intro

1.	¬P∨R	
2.	P	
3.	¬P	
4.	…	
5.	R	
6.	R	
7.	…	
8.	R	
9.	R	∨Elim: 1, 3-5, 6-9
10.	P→R	→Intro: 2-9

Which rule to complete this subproof?

these two subproofs

b) proofs

$\lor$Elim

$\to$Intro

$\neg$Elim

1.	$\neg P \lor R$	
2.	P	
3.	$\neg P$	
4.	…	
5.	R	
6.	R	
7.	…	
8.	R	
9.	R	$\lor$Elim: 1, 3-5, 6-9
10.	$P \to R$	$\to$Intro: 2-9

Which rule to complete this subproof?

these two subproofs

b) proofs

1. ¬P∨R
2. P
3. ¬P
4. …
5. R
6. R
7. …
8. R
9. R
10. P→R

∨Elim

→Intro

¬Elim

⊥Elim

Which rule to complete this subproof?

these two subproofs

∨Elim: 1, 3-5, 6-9

→Intro: 2-9

Q1 (b) iii

1. $\forall x\ S(x)$

2. $\forall x\ \neg S(x)$

3.

4.

5.

6.

7.

8.

9.

10. $\bot$

1. $\forall x\, S(x)$

2. $\forall x\, \neg S(x)$

3.

4.

5.

6.

7.

8.

9.

10. $\bot$

1.	$\forall x\ S(x)$	//everything is S
2.	$\forall x\ \neg S(x)$	//everything is $\neg$S
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.	$\bot$	

1. $\forall x\, S(x)$ //everything is S

2. $\forall x\, \neg S(x)$ //everything is ¬S

3.

4.

5.

6.

7.

8.

9.

10. $\bot$

1. $\forall x\, S(x)$ //everything is S

2. $\forall x\, \neg S(x)$ //everything is ¬S

3.

4.

5.

6.

7.

8.

9.

10. $\bot$

1. $\forall x\, S(x)$ //everything is S

2. $\forall x\, \neg S(x)$ //everything is $\neg$S

3. $S(a)$ $\forall$Elim: 1

4.

5.

6.

7.

8.

9.

10. $\bot$

1.	$\forall x\, S(x)$		//everything is S
2.	$\forall x\, \neg S(x)$		//everything is ¬S
3.	$S(a)$	∀Elim: 1	
4.			
5.			
6.			
7.			
8.			
9.			
10.	$\bot$		

1.	∀x S(x)		//everything is S
2.	∀x ¬S(x)		//everything is ¬S
3.	S(a)	∀Elim: 1	
4.			
5.			
6.			
7.			
8.			
9.			
10.	⊥		

1. | $\forall x\ S(x)$ //everything is S

2. | $\forall x\ \neg S(x)$ //everything is $\neg$S

3. | $S(a)$ $\forall$Elim: 1

4. | $\neg S(a)$ $\forall$Elim: 2

5. |

6. |

7. |

8. |

9. |

10. | $\bot$

1. $\forall x\, S(x)$

2. $\forall x\, \neg S(x)$

3. $S(a)$ $\forall$Elim: 1

4. $\neg S(a)$ $\forall$Elim: 2

5.

6.

7.

8.

9.

10. $\bot$

1.	∀x S(x)	
2.	∀x ¬S(x)	
3.	S(a)	∀Elim: 1
4.	¬S(a)	∀Elim: 2
5.		
6.		
7.		
8.		
9.		
10.	⊥	⊥Intro: 3,4

Q3 (a) harder version

1. $\forall x\, S(x)$
2. $\exists x\, \neg S(x)$
3.
4.
5.
6.
7.
8.
9.
10. $\bot$

1. $\forall x\, S(x)$
2. $\exists x \neg S(x)$
3.
4.
5.
6.
7.
8.
9.
10. $\bot$

1.	$\forall x\, S(x)$	
2.	$\exists x\, \neg S(x)$	
3.	$S(a)$	$\forall$Elim: 1
4.		
5.		
6.		
7.		
8.		
9.		
10.	$\bot$	

1.	$\forall x\ S(x)$	
2.	$\exists x\ \neg S(x)$	
3.	$S(a)$	$\forall$Elim: 1
4.	$\neg S(a)$	$\exists$Elim: 2
5.		
6.		
7.		
8.		
9.		
10.	$\bot$	

1.	∀x S(x)	
2.	∃x ¬S(x)	
3.	S(a)	∀Elim: 1
4.	¬S(a)	∃Elim: 2
5.		
6.		
7.		
8.		
9.		
10.	⊥	

1.	$\forall x\ S(x)$		
2.	$\exists x\ \neg S(x)$		// something is not S
3.	$S(a)$	$\forall$Elim: 1	
4.	$\neg S(a)$	$\exists$Elim: 2	// a, the very thing which is S, is not S
5.			
6.			
7.			
8.			
9.			
10.	$\bot$		

1.	$\forall x\, S(x)$		
2.	$\exists x\, \neg S(x)$		// something is not S
3.	$S(a)$	$\forall$Elim: 1	
4.	$\neg S(a)$	$\exists$Elim: 2	// a, the very thing which is S, is not S
5.			
6.			
7.			
8.			
9.			
10.	$\bot$		

0%

Q1 (b) iv - easier version

b) proofs

1. $(P \vee R) \rightarrow (P \rightarrow R)$
2.
3.
4.
5.
6.
7.
8.
9.
10. $P \rightarrow R$

b) proofs

1. | $(P \vee R) \rightarrow (P \rightarrow R)$
2. |
3. |
4. |
5. |
6. |
7. |
8. |
9. |
10. | $P \rightarrow R$

b) proofs

1. $(P \lor R) \to (P \to R)$
2.
3.
4.
5.
6.
7.
8.
9.
10. $P \to R$

b) proofs

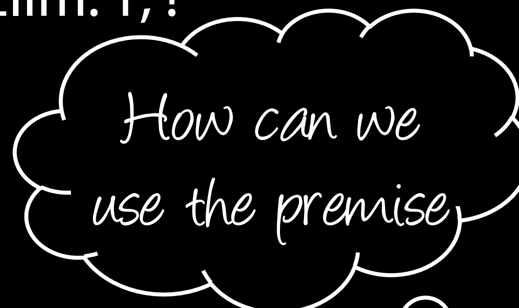
1.	$(P \vee R) \rightarrow (P \rightarrow R)$	
2.	P	
3.		
4.		
5.		
6.		
7.		
8.		
9.	R	
10.	$P \rightarrow R$	→Intro: 2-9

b) proofs

1.	$(P \lor R) \to (P \to R)$	
2.	$\quad P$	
3.		
4.		
5.		
6.		
7.		
8.		
9.	$\quad R$	
10.	$P \to R$	$\to$Intro: 2-9

b) proofs

1.	$(P \vee R) \to (P \to R)$	
2.	P	
3.		
4.	$P \to R$	$\to$Elim: 1, ?
5.		
6.		
7.		
8.		
9.	R	
10.	$P \to R$	$\to$Intro: 2-9



b) proofs

1.	(P∨R)→(P→R)	
2.	P	
3.		
4.	P→R	→Elim: 1, ?
5.		
6.		
7.		
8.		
9.	R	
10.	P→R	→Intro: 2-9

b) proofs

1.	$(P \vee R) \to (P \to R)$	
2.	P	
3.		
4.	$P \to R$	$\to$Elim: 1, ?
5.		
6.		
7.		
8.		
9.	R	
10.	$P \to R$	$\to$Intro: 2-9

b) proofs

1.	(P∨R)→(P→R)	
2.	P	
3.	P∨R	∨Intro: 2
4.	P→R	→Elim: 1, ?
5.		
6.		
7.		
8.		
9.	R	
10.	P→R	→Intro: 2-9

b) proofs

1.	$(P \vee R) \rightarrow (P \rightarrow R)$	
2.	P	
3.	$P \vee R$	$\vee$Intro: 2
4.	$P \rightarrow R$	$\rightarrow$Elim: 1, 3
5.		
6.		
7.		
8.		
9.	R	
10.	$P \rightarrow R$	$\rightarrow$Intro: 2-9

Q1 (b) iv

Last proof

b) proofs

1. $\neg(P\wedge R)\rightarrow(P\rightarrow R)$
2.
3.
4.
5.
6.
7.
8.
9.
10. $P\rightarrow R$

b) proofs

1. $\neg(P \wedge R) \rightarrow (P \rightarrow R)$
2.
3.
4.
5.
6.
7.
8.
9.
10. $P \rightarrow R$

b) proofs

1. $\neg(P \wedge R) \rightarrow (P \rightarrow R)$

2. P

3.

4.

5.

6.

7.

8.

9. R

10. $P \rightarrow R$ $\rightarrow$Intro: 2-9

b) proofs

1. $\neg(P \wedge R) \rightarrow (P \rightarrow R)$ $\quad \Dashv \models \neg\neg(P \wedge R) \vee (P \rightarrow R)$

2. P $\quad \dashv \vdash (P \wedge R) \vee (P \rightarrow R)$

3. $\quad \dashv \vdash P \rightarrow R$

4.

5.

6.

7.

8.

9. R

10. $P \rightarrow R$ $\quad \rightarrow$Intro: 2-9

b) proofs

1. $\neg(P \wedge R) \to (P \to R)$
2. P
3.
4.
5.
6.
7.
8.
9. R
10. $P \to R$ — $\to$Intro: 2-9

b) proofs

1.	$\neg(P \wedge R) \to (P \to R)$	
2.	P	
3.		
4.		
5.		
6.		
7.		
8.		
9.	R	
10.	$P \to R$	$\to$Intro: 2-9

To use the premise I need $\neg(P \wedge R)$?

b) proofs

1.	$\neg(P \wedge R) \rightarrow (P \rightarrow R)$	
2.	P	
3.		
4.		
5.		
6.		
7.		
8.		
9.	R	
10.	$P \rightarrow R$	$\rightarrow$Intro: 2-9

b) proofs

1.	$\neg(P \wedge R) \to (P \to R)$
2.	P
3.	
4.	
5.	
6.	
7.	
8.	
9.	R
10.	$P \to R$ — $\to$Intro: 2-9

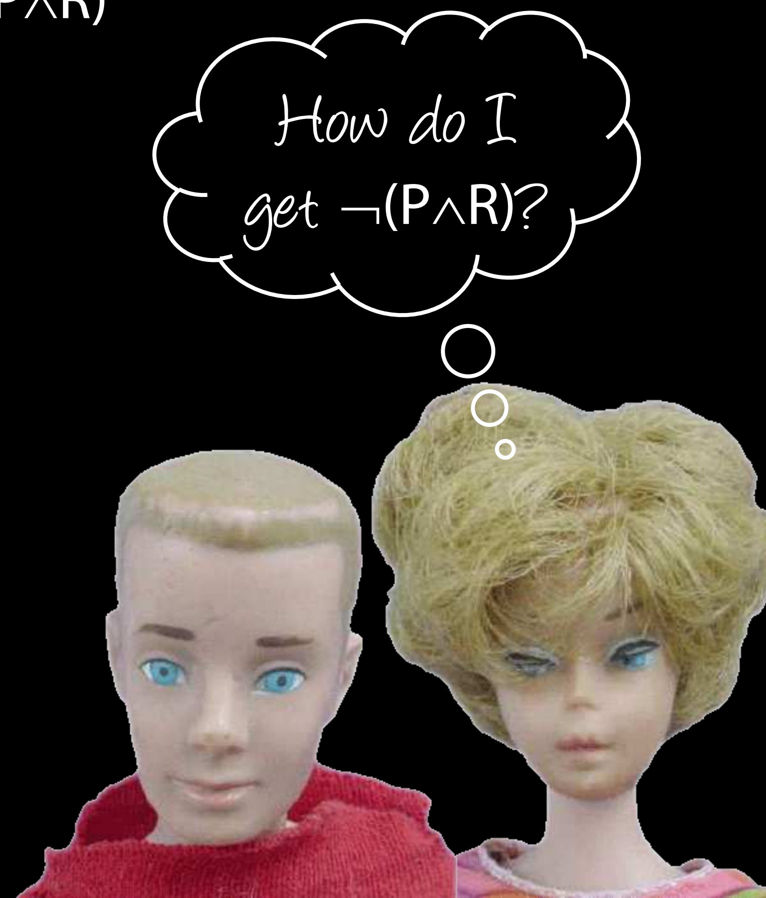
$\neg R$

$P \wedge R$

R

$\bot$

$\neg(P \wedge R)$



b) proofs

$\neg R$

$P \wedge R$

R

$\bot$

$\neg(P \wedge R)$

1. $\neg(P \wedge R) \rightarrow (P \rightarrow R)$
2. P
3.
4.
5.
6.
7.
8.
9. R
10. $P \rightarrow R$ $\rightarrow$Intro: 2-9

Suppose R is false

Then $\neg(P \wedge R)$ is true

So we have $(P \rightarrow R)$ from line 1

But we also have P (line 2)

So we have R

Which contradicts the supposition that R is false

Q2 (b) v

1. $\forall x\ (F(x) \rightarrow x=a)$

2.

3.

4.

5.

6.

7.

8.

9.

10. $\neg\exists x\ (F(x) \wedge \neg x=a)$

1. $\forall x\,(F(x) \rightarrow x=a)$

2.

3.

4.

5.

6.

7.

8.

9.

10. $\neg\exists x\,(F(x) \wedge \neg x=a)$

1. $\forall x\,(F(x) \rightarrow x{=}a)$ // If anything is F it's a

2.

3.

4.

5.

6.

7.

8.

9.

10. $\neg\exists x\,(F(x) \wedge \neg x{=}a)$

1. $\forall x\ (\ F(x) \rightarrow x{=}a\ )$ // If anything is F it's a

2.

3.

4.

5.

6.

7.

8.

9.

10. $\neg\exists x\ (\ F(x) \land \neg x{=}a\ )$

1. $\forall x\, (F(x) \rightarrow x{=}a)$ // If anything is F it's a

2.

3.

4.

5.

6.

7.

8.

9.

10. $\neg\exists x\, (F(x) \wedge \neg x{=}a)$ //nothing is F and not a

1. $\forall x\,(F(x) \rightarrow x=a)$ // If anything is F it's a

2.

3.

4.

5.

6.

7.

8.

9.

10. $\neg\exists x\,(F(x) \wedge \neg x=a)$ //nothing is F and not a

1. $\forall x\ (F(x) \rightarrow x=a)$ // If anything is F it's a

2.

3.

4.

5.

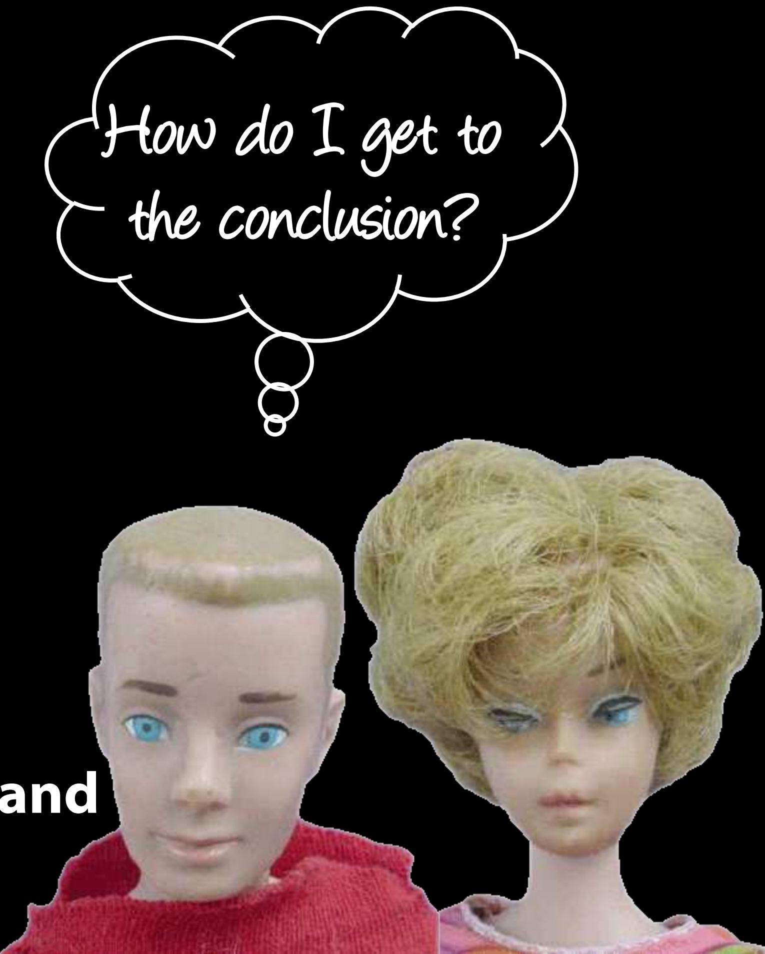
6.

7.

8.

9.

10. $\neg\exists x\ (F(x) \wedge \neg x=a)$ //nothing is F and not a



1. $\forall x\,(F(x) \rightarrow x{=}a)$ // If anything is F it's a

2.

3.

4.

5.

6.

7.

8.

9.

10. $\neg\exists x\,(F(x) \wedge \neg x{=}a)$ ¬Intro

¬Intro

| S
| ...
| ⊥
¬S

1. $\forall x\,(F(x) \rightarrow x{=}a)$ // If anything is F it's a

2. $\exists x\,(F(x) \wedge \neg x{=}a)$

3.

4.

5.

6.

7.

8.

9. $\bot$

10. $\neg\exists x\,(F(x) \wedge \neg x{=}a)$ ¬Intro: 2-9

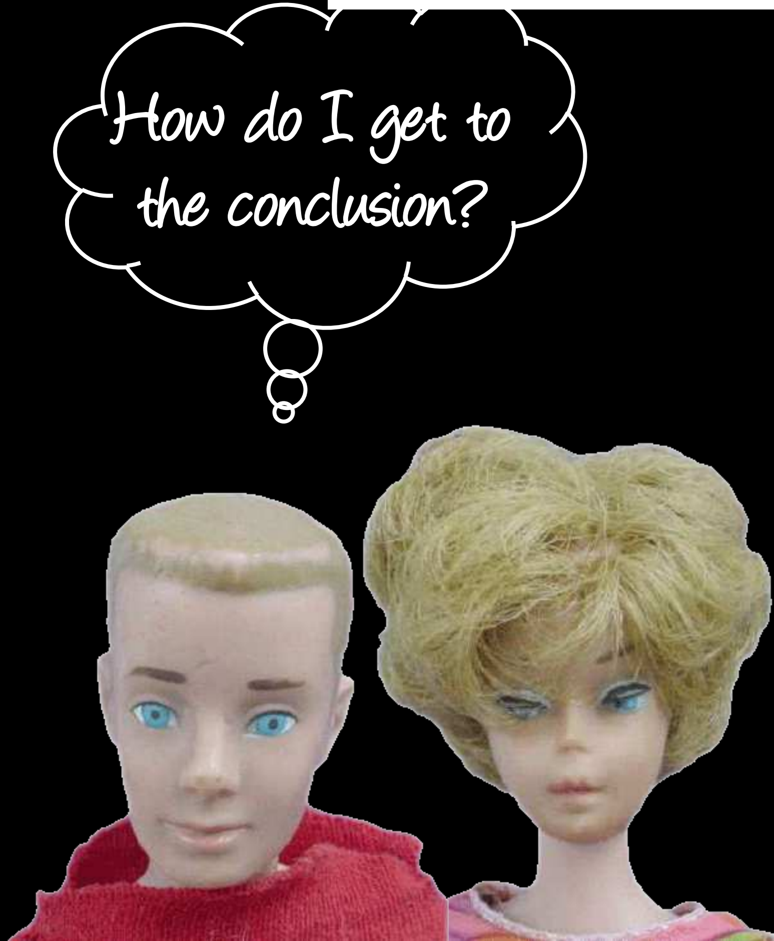
¬Intro

S

…

$\bot$

$\neg S$



$\neg$Intro

| S
| ...
| $\bot$
$\neg S$

1. | $\forall x\,(F(x) \rightarrow x{=}a)$ // If anything is F it's a
2. | | $\exists x\,(F(x) \wedge \neg x{=}a)$
3. | |
4. | |
5. | |
6. | |
7. | |
8. | |
9. | | $\bot$
10. | $\neg\exists x\,(F(x) \wedge \neg x{=}a)$ $\neg$Intro: 2-9

¬Intro

| S
| ...
| ⊥
¬S

1. $\forall x\,(F(x) \rightarrow x=a)$ // If anything is F it's a
2. $\exists x\,(F(x) \land \neg x=a)$
3.
4.
5.
6.
7.
8.
9. $\bot$
10. $\neg\exists x\,(F(x) \land \neg x=a)$ ¬Intro: 2-9

1. $\forall x\,(F(x) \rightarrow x{=}a)$ // If anything is F it's a

2. $\exists x\,(F(x) \land \neg x{=}a)$

3.

4.

5.

6.

7.

8.

9. $\bot$

10. $\neg\exists x\,(F(x) \land \neg x{=}a)$ $\neg$Intro: 2-9

∃Elim

$\exists x\, F(x)$

c $F(c)$

...

S

S

1. $\forall x\,(F(x) \rightarrow x{=}a)$ // If anything is F it's a
2. $\exists x\,(F(x) \land \neg x{=}a)$
3.
4.
5.
6.
7.
8.
9. $\bot$
10. $\neg\exists x\,(F(x) \land \neg x{=}a)$ ¬Intro: 2-9

∃ *before* ∀

∃Elim

$\exists x\, F(x)$

c $F(c)$

…

S

S

1. $\forall x\, (F(x) \rightarrow x=a)$ // If anything is F it's a

2. $\exists x\, (F(x) \wedge \neg x=a)$

3.

4.

5.

6.

7.

8.

9. $\bot$

10. $\neg\exists x\, (F(x) \wedge \neg x=a)$ ¬Intro: 2-9

1. $\forall x\,(F(x) \rightarrow x{=}a)$ // If anything is F it's a

2. $\exists x\,(F(x) \wedge \neg x{=}a)$

3. b

4.

5.

6.

7.

8.

9. $\bot$

10. $\neg\exists x\,(F(x) \wedge \neg x{=}a)$ ¬Intro: 2-9

∃ before ∀

∃Elim

$\exists x\, F(x)$

c $F(c)$

…

S

S

1. $\forall x\,(F(x) \rightarrow x=a)$ // If anything is F it's a

2. $\exists x\,(F(x) \wedge \neg x=a)$

3. b $F(b) \wedge \neg b=a$

4.

5.

6.

7.

8. $\bot$

9. $\bot$

10. $\neg\exists x\,(F(x) \wedge \neg x=a)$ ¬Intro: 2-9

∃Elim

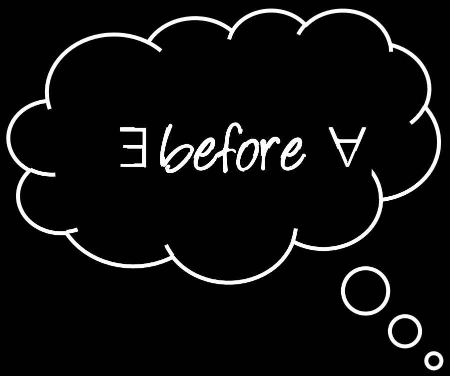
$\exists x\,F(x)$

c $F(c)$

…

S

S



∃Elim

$\exists x\, F(x)$

[c] $F(c)$

…

S

S

1. $\forall x\,(F(x) \rightarrow x{=}a)$ // If anything is F it's a
2. $\exists x\,(F(x) \land \neg x{=}a)$
3. [b] $F(b) \land \neg b{=}a$
4.
5.
6.
7.
8. $\bot$
9. $\bot$ ∃Elim: 2,3-8
10. $\neg\exists x\,(F(x) \land \neg x{=}a)$ ¬Intro: 2-9

Q.2 (b) vi

1. $\exists x \forall y\, [\, F(y) \rightarrow \neg G(x,y)\,]$

2.

3.

4.

5.

6.

7.

8.

9.

10. $\forall y \exists x\, [\, F(y) \rightarrow \neg G(x,y)\,]$

1. $\exists x \forall y$ [F(y)

2.

3.

4.

5.

6.

1. $\exists x \forall y \, [\, F(y) \rightarrow \neg G(x,y) \,]$

2.

3.

4.

5.

6.

7.

8.

9.

10. $\forall y \exists x \, [\, F(y) \rightarrow \neg G(x,y) \,]$

1. $\exists x \forall y\, [\, F(y) \rightarrow \neg G(x,y) \,]$

2.

3.

4.

5.

6.

7.

8.

9.

10. $\forall y \exists x\, [\, F(y) \rightarrow \neg G(x,y) \,]$

1. $\exists x \forall y [F(y) \rightarrow \neg G(x,y)]$

2.

3.

4.

5.

6.

7.

8.

9.

10. $\forall y \exists x [F(y) \rightarrow \neg G(x,y)]$

1. $\exists x \forall y$ blah(x,y)

2.

3.

4.

5.

6.

7.

8.

9.

10. $\forall y \exists x$ blah(x,y)

1. $\exists x \forall y$ **blah(x,y)**

2.

3.

4.

5.

6.

7.

8.

9.

10. $\forall y \exists x$ **blah(x,y)**

1. $\exists x \forall y$ **blah(x,y)**

2.

3.

4.

5.

6.

7.

8.

9.

10. $\forall y \exists x$ **blah(x,y)**

1. | ∃x∀y blah(x,y)
2. |
3. |
4. |
5. |
6. |
7. |
8. |
9. |
10. | ∀y∃x blah(x,y)

∀Intro

| c
| ...
| F(c)

∀x F(x)

∀Intro

c

...

F(c)

∀x F(x)

1. ∃x∀y blah(x,y)

2.

3. b

4.

5.

6.

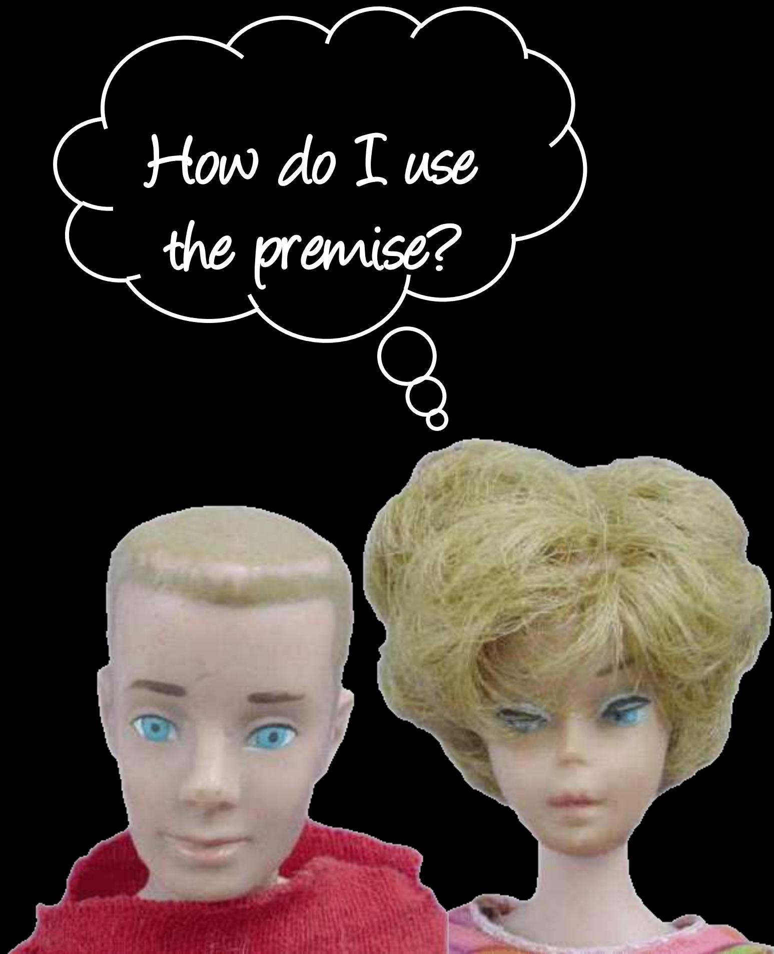
7.

8.

9. ∃x blah(x,b)

10. ∀y∃x blah(x,y) ∀Intro: 3-9

1.	$\exists x \forall y$ blah(x,y)	
2.		
3.	b	
4.		
5.		
6.		
7.		
8.		
9.	$\exists x$ blah(x,b)	
10.	$\forall y \exists x$ blah(x,y)	$\forall$Intro: 3-9



1. ∃x∀y blah(x,y)

2.

3. b

4.

5.

6.

7.

8.

9. ∃x blah(x,b)

10. ∀y∃x blah(x,y) ∀Intro: 3-9

1. ∃x∀y blah(x,y)

2.

3. b

4.

5.

6.

7.

8.

9. ∃x blah(x,b)

10. ∀y∃x blah(x,y) ∀Intro: 3-9

∃Elim

∃x F(x)

c F(c)

…

S

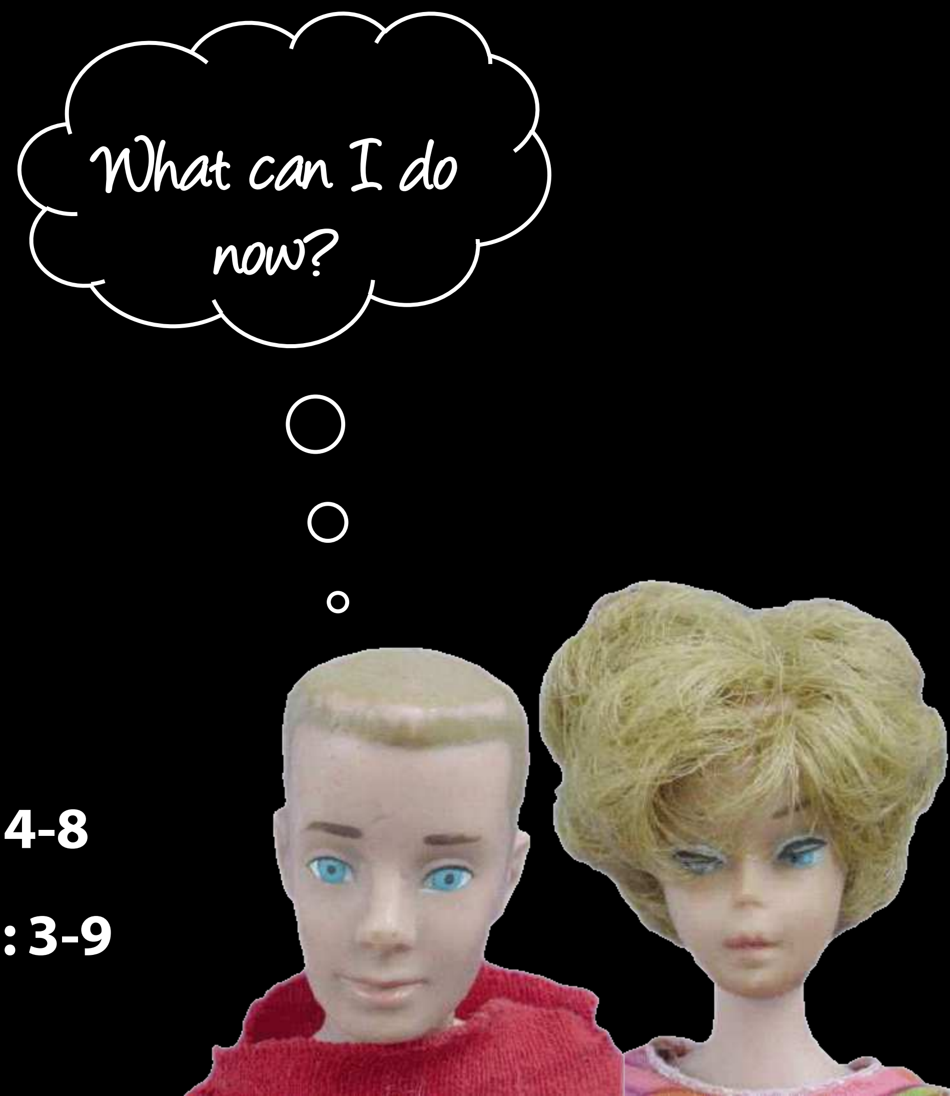
S

∃Elim

	∃x F(x)
c	F(c)
	…
	S
	S

1.	∃x∀y blah(x,y)	
2.		
3.	b	
4.	c ∀y blah(c,y)	
5.		
6.		
7.		
8.	∃x blah(x,b)	
9.	∃x blah(x,b)	∃Elim: 4-8
10.	∀y∃x blah(x,y)	∀Intro: 3-9

1.	$\exists x \forall y$ blah(x,y)	
2.		
3.	**b**	
4.	**c** $\forall y$ blah(c,y)	
5.		
6.		
7.		
8.	$\exists x$ blah(x,b)	
9.	$\exists x$ blah(x,b)	$\exists$Elim: 4-8
10.	$\forall y \exists x$ blah(x,y)	$\forall$Intro: 3-9



1. ∃x∀y blah(x,y)

2.

3. b

4. c ∀y blah(c,y)

5.

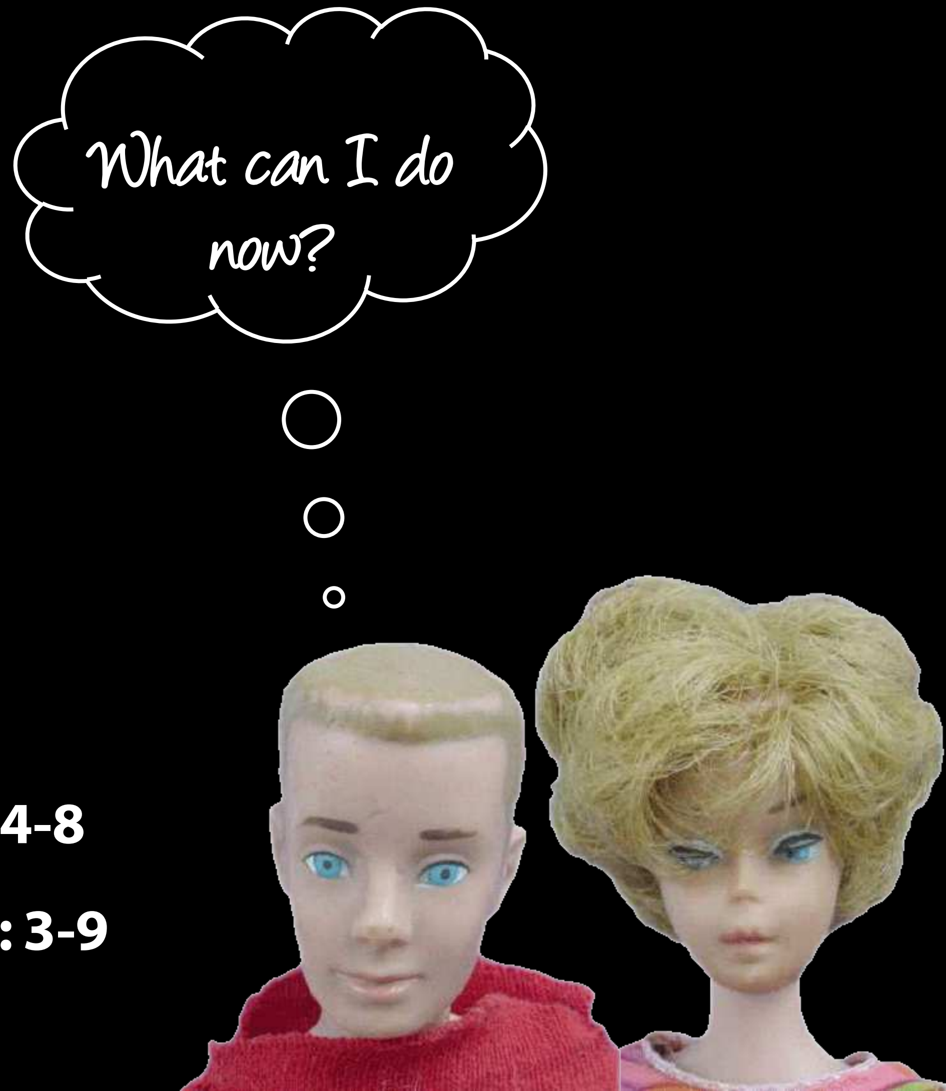
6.

7.

8. ∃x blah(x,b)

9. ∃x blah(x,b) ∃Elim: 4-8

10. ∀y∃x blah(x,y) ∀Intro: 3-9



1. ∃x∀y blah(x,y)

2.

3. b

4. c ∀y blah(c,y)

5. blah(c,b) ∀Elim: 4

6.

7.

8. ∃x blah(x,b)

9. ∃x blah(x,b) ∃Elim: 4-8

10. ∀y∃x blah(x,y) ∀Intro: 3-9

1. $\exists x \forall y$ blah(x,y)

2.

3. b

4. c $\forall y$ blah(c,y)

5. blah(c,b) — $\forall$Elim: 4

6.

7.

8. $\exists x$ blah(x,b)

9. $\exists x$ blah(x,b) — $\exists$Elim: 4-8

10. $\forall y \exists x$ blah(x,y) — $\forall$Intro: 3-9

1. ∃x∀y blah(x,y)

2.

3. b

4. c ∀y blah(c,y)

5. blah(c,b) ∀Elim: 4

6.

7.

8. ∃x blah(x,b)

9. ∃x blah(x,b) ∃Elim: 4-8

10. ∀y∃x blah(x,y) ∀Intro: 3-9

1. ∃x∀y blah(x,y)

2.

3. b

4. c ∀y blah(c,y)

5. blah(c,b) ∀Elim: 4

6.

7.

8. ∃x blah(x,b) ∃Intro: 5

9. ∃x blah(x,b) ∃Elim: 4-8

10. ∀y∃x blah(x,y) ∀Intro: 3-9

1. ∃x∀y blah(x,y)

2.

3. b

4. c ∀y blah(c,y)

5. blah(c,b) ∀Elim: 4

6.

7.

8. ∃x blah(x,b) ∃Intro: 5

9. ∃x blah(x,b) ∃Elim: 4-8

10. ∀y∃x blah(x,y) ∀Intro: 3-9

Q.2 truth & meaning

a)

$P \to Q$

$\neg P \lor Q$

a)

$$P \rightarrow Q \vdash \neg P \vee Q$$

P	Q
T	T
T	F
F	T
F	F

a)

$P \to Q$

$\neg P \lor Q$

P	Q	$P \to Q$	$\neg P \lor Q$
T	T		
T	F		
F	T		
F	F		

^ premise ^ conclusion

a)

$$\begin{array}{|l} P \rightarrow Q \\ \hline \neg P \vee Q \end{array}$$

P	Q	$P \rightarrow Q$	$\neg P \vee Q$
T	T	T	
T	F	F	
F	T	T	
F	F	T	

^ premise ^ conclusion

a)

$P \to Q$

$\neg P \lor Q$

P	Q	$P \to Q$	$\neg P \lor Q$	$\neg P$
T	T	T		F
T	F	F		F
F	T	T		T
F	F	T		T

		^ premise	^ conclusion	

a)

$$\begin{array}{|l} P \rightarrow Q \\ \hline \neg P \vee Q \end{array}$$

P	Q	$P \rightarrow Q$	$\neg P \vee Q$	$\neg P$
T	T	T	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T
		^ premise	^ conclusion	

a) $P \to Q$ ⊢ $\neg P \lor Q$

P	Q	$P \to Q$	$\neg P \lor Q$	$\neg P$
T	T	T	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

		^ premise	^ conclusion	

An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false

a) T | P→Q
T | ¬P∨Q

P	Q	P → Q	¬P ∨ Q	¬P
T	T	T	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

	^ premise	^ conclusion	

An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false

a) F | $P \to Q$

F | $\neg P \lor Q$

P	Q	$P \to Q$	$\neg P \lor Q$	$\neg P$
T	T	T	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

^ premise ^ conclusion

An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false

a) T T | P→Q ⊢ ¬P∨Q

P	Q	P → Q	¬P ∨ Q	¬P
T	T	T	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

^ premise ^ conclusion

An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false

a)

T	$P \to Q$
T	$\neg P \lor Q$

P	Q	$P \to Q$	$\neg P \lor Q$	$\neg P$
T	T	T	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T
		^ premise	^ conclusion	

An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false

F	premise
T	conclusion

P	Q	$P \rightarrow Q$	$\neg P \vee Q$	$\neg P$
T	T	T	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T
		^ premise	^ conclusion	

An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false

Another one: Q2 (a) (iii)

a) iii. $\models P \lor \neg(Q \land R)$

$P \lor (\neg Q \land R)$

P	Q	R	
T	T	T	
T	T	F	
T	F	T	
T	F	F	

…

a) iii. $\vdash \begin{array}{l} P \vee \neg(Q \wedge R) \\ P \vee (\neg Q \wedge R) \end{array}$

P	Q	R	$Q \wedge R$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F

…

a) iii. $P \lor \neg(Q \land R) \vdash P \lor (\neg Q \land R)$

P	Q	R	$Q \land R$	$\neg(Q \land R)$
T	T	T	T	
T	T	F	F	
T	F	T	F	
T	F	F	F	

…

a) iii. $\vdash$ $P \lor \neg(Q \land R)$

$P \lor (\neg Q \land R)$

P	Q	R	$Q \land R$	$\neg(Q \land R)$	$P \lor \neg(Q \land R)$
T	T	T	T		
T	T	F	F		
T	F	T	F		
T	F	F	F		

…

a) iii. $\vdash$ $P \lor \neg(Q \land R)$ / $P \lor (\neg Q \land R)$

P	Q	R	$Q \land R$	$\neg(Q \land R)$	$P \lor \neg(Q \land R)$	$\neg Q$	$\neg Q \land R$	$P \lor (\neg Q \land R)$
T	T	T	T					
T	T	F	F					
T	F	T	F					
T	F	F	F					

…

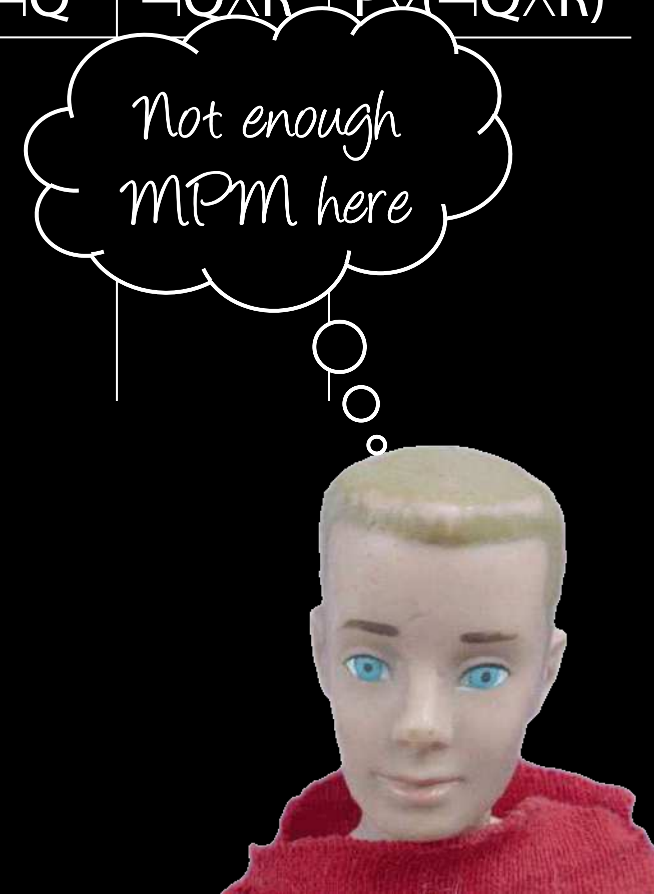
a) iii.

$P \lor \neg(Q \land R)$

$P \lor (\neg Q \land R)$

P	Q	R	$Q \land R$	$\neg(Q \land R)$	$P \lor \neg(Q \land R)$	$\neg Q$	$\neg Q \land R$	$P \lor (\neg Q \land R)$
T	T	T	T					
T	T	F	F					
T	F	T	F					
T	F	F	F					

…



a) iii.

$P \lor \neg(Q \land R)$

$P \lor (\neg Q \land R)$

P	Q	R	$Q \land R$	$\neg(Q \land R)$	$P \lor \neg(Q \land R)$	$\neg Q$	$\neg Q \land R$	$P \lor (\neg Q \land R)$
T	T	T	T					
T	T	F	F					
T	F	T	F					
T	F	F	F					

…

a) iii. $P \vee \neg(Q \wedge R) \vdash P \vee (\neg Q \wedge R)$

P	Q	R	$Q \wedge R$	$\neg(Q \wedge R)$	$P \vee \neg(Q \wedge R)$	$\neg Q$	$\neg Q \wedge R$	$P \vee (\neg Q \wedge R)$
T	T	T	T					
T	T	F	F					
T	F	T	F					
T	F	F	F					

…

a) iii.

$$P \lor \neg(Q \land R)$$

$$P \lor (\neg Q \land R)$$

P	Q	R	$Q \land R$	$\neg(Q \land R)$	$P \lor \neg(Q \land R)$	$\neg Q$	$\neg Q \land R$	$P \lor (\neg Q \land R)$
T	T	T	T					
T	T	F	F					
T	F	T	F					
T	F	F	F					

…

a) iii.

$$\begin{array}{l|l} & P \lor \neg(Q \land R) \\ \hline & P \lor (\neg Q \land R) \end{array}$$

P	Q	R	$Q \land R$	$\neg(Q \land R)$	$P \lor \neg(Q \land R)$	$\neg Q$	$\neg Q \land R$	$P \lor (\neg Q \land R)$
T	T	T	T					
T	T	F	F					
T	F	T	F					
T	F	F	F					

…

a) iii. $P \lor \neg(Q \land R)$

$P \lor (\neg Q \land R)$

$\dashv \vDash P \lor \neg Q \lor \neg R$

P	Q	R	$Q \land R$	$\neg(Q \land R)$	$P \lor \neg(Q \land R)$	$\neg Q$	$\neg Q \land R$	$P \lor (\neg Q \land R)$
T	T	T	T					
T	T	F	F					
T	F	T	F					
T	F	F	F					

…

a) iii.

$$\begin{array}{l} P \lor \neg(Q \land R) \\ \hline P \lor (\neg Q \land R) \end{array}$$

⊣ ⊢ P ∨ ¬Q ∨ ¬R

a) iii. $P \vee \neg(Q \wedge R)$ / $P \vee (\neg Q \wedge R)$

$\dashv\vdash P \vee \neg Q \vee \neg R$

Q	R	$\neg(Q \wedge R)$	$\neg Q \vee \neg R$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

a) iii.

$P \lor \neg(Q \land R)$

$P \lor (\neg Q \land R)$

$\dashv \vdash P \lor \neg Q \lor \neg R$

Q	R	$\neg(Q \land R)$	$\neg Q \lor \neg R$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

a) iii.

$P \lor \neg(Q \land R)$

$P \lor (\neg Q \land R)$

$\dashv\vdash P \lor \neg Q \lor \neg R$

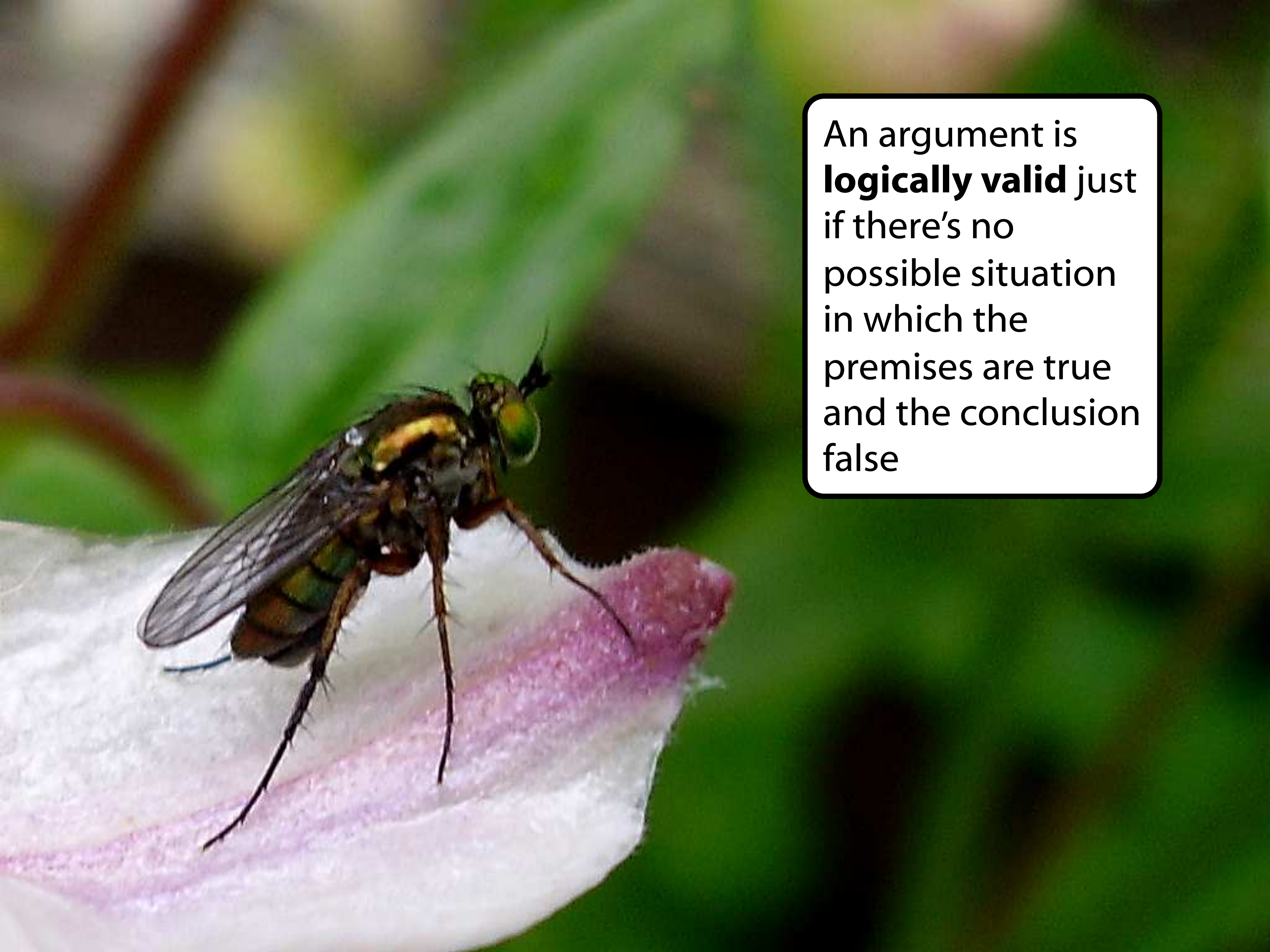
Q	R	$\neg(Q \land R)$	$\neg Q \lor \neg R$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

a) iii. $\begin{array}{|l} P\vee\neg(Q\wedge R) \\ \hline P\vee(\neg Q\wedge R) \end{array}$

$\dashv\vdash P \vee \neg Q \vee \neg R$

P	Q	R	$Q\wedge R$	$\neg(Q\wedge R)$	$P\vee\neg(Q\wedge R)$	$\neg Q$	$\neg Q\wedge R$	$P\vee(\neg Q\wedge R)$
T	T	T	T					
T	T	F	F					
T	F	T	F					
T	F	F	F					

…


An argument is **logically valid** just if there's no possible situation in which the premises are true and the conclusion false