

PH133 Sample Exam

Version 0.4*

* Please email s.butterfill@warwick.ac.uk with errors. (NB the marks are not intended to add up to 100; if they do not, that is no error.)

References to proofs and rules in first order logic are to the system presented in Barwise & Etchemendy, Language, Proof & Logic.

Q1

a) State the rules of proof for the following two connectives: \wedge
 \rightarrow

[10 marks]

b) Give formal proofs of the following arguments.

i)

$$\left| \begin{array}{l} \neg P \wedge R \\ \neg P \end{array} \right.$$

[20 marks]

ii)

$$\left| \begin{array}{l} \neg P \vee R \\ P \rightarrow R \end{array} \right.$$

[20 marks]

iii)

$$\left| \begin{array}{l} \forall x S(x) \\ \forall x \neg S(x) \\ \perp \end{array} \right.$$

[20 marks]

iv)

$$\left| \begin{array}{l} \neg(P \wedge R) \rightarrow (P \rightarrow R) \\ P \rightarrow R \end{array} \right.$$

[10 marks]

v)

$$\left| \begin{array}{l} \forall x [F(x) \rightarrow x=a] \\ \neg \exists x [F(x) \wedge \neg x=a] \end{array} \right.$$

[20 marks]

vi)

$$\left| \begin{array}{l} \forall x F(x) \rightarrow \exists x G(x) \\ \exists x [F(x) \rightarrow G(x)] \end{array} \right.$$

[10 marks]

Q2

a) What is an argument? Define the term 'logically valid argument'.

[10 marks]

b) Use truth tables to establish whether the following arguments are valid. If any arguments are invalid, state counterexamples to them. If any arguments are valid, explain carefully using the truth tables why they are valid.

i.

$$\left\{ \begin{array}{l} P \rightarrow Q \\ \neg P \vee Q \end{array} \right.$$

[15 marks]

ii.

$$\left\{ \begin{array}{l} P \leftrightarrow (Q \rightarrow Q) \\ P \vee Q \end{array} \right.$$

[15 marks]

iii.

$$\left\{ \begin{array}{l} P \vee \neg(Q \wedge R) \\ P \vee (\neg Q \wedge R) \end{array} \right.$$

[15 marks]

c) For each of the following sentences of FOL, give a logically equivalent sentence of idiomatic English using the specified interpretation. Your English sentences should be as concise as possible.

Domain: { people and actions }

$D(x)$: x is desirable

$V(x)$: x is virtuous

$A(x)$: x is an action

$P(x,y)$: x performed y

i. $\forall x [D(x) \rightarrow V(x)]$

ii. $\forall x [[A(x) \wedge D(x)] \rightarrow V(x)]$

iii. $\exists x [A(x) \wedge \neg D(x) \wedge V(x)]$

iv. $\neg \exists x [\exists y [P(x,y) \wedge A(y) \wedge \neg V(y)] \wedge \neg \exists z [P(x,z) \wedge A(z) \wedge V(z)]]$

[10 marks each]

d) Translate the following sentences of English into FOL using the interpretation below:

$L(x,y)$: x is a logical consequence of y

$N(x,y)$: x is the negation of y

$S(x)$: x is a sentence

a : 'Fire melts ice'

i. 'Fire melts ice' is a sentence

ii. There is a sentence

iii. There is a sentence which is the negation of 'Fire melts ice'

iv. Some sentences are contradictions and all contradictions are logically equivalent.

[10 marks each]