

Reit

$$\begin{array}{c} | \varphi \\ \vdots \\ \triangleright | \varphi \end{array}$$

$\wedge$ Elim

$$\begin{array}{c} | \varphi_1 \wedge \varphi_2 \\ \vdots \\ \triangleright | \varphi_i \end{array}$$

$\wedge$ Intro

$$\begin{array}{c} | \varphi_1 \\ \vdots \\ | \varphi_2 \\ \vdots \\ \triangleright | \varphi_1 \wedge \varphi_2 \end{array}$$

$\vee$ Intro

$$\begin{array}{c} | \varphi_i \\ \vdots \\ \triangleright | \varphi_1 \vee \varphi_2 \end{array}$$

$\vee$ Elim

$$\begin{array}{c} | \varphi_1 \vee \varphi_2 \\ \vdots \\ | \varphi_1 \\ \vdots \\ | \psi \\ \vdots \\ | \varphi_2 \\ \vdots \\ | \psi \\ \vdots \\ \triangleright | \psi \end{array}$$

$\perp$ Intro

$$\begin{array}{c} | \varphi \\ \vdots \\ | \neg \varphi \\ \vdots \\ \triangleright | \perp \end{array}$$

$\perp$ Elim

$$\begin{array}{c} | \perp \\ \vdots \\ \triangleright | \psi \end{array}$$

$\neg$ Elim

$$\begin{array}{c} | \neg \neg \varphi \\ \vdots \\ \triangleright | \varphi \end{array}$$

$\neg$ Intro

$$\begin{array}{c} | \varphi \\ \vdots \\ | \perp \\ \vdots \\ \triangleright | \neg \varphi \end{array}$$

$\rightarrow$ Elim

$$\begin{array}{c} | \varphi \rightarrow \psi \\ \vdots \\ | \varphi \\ \vdots \\ \triangleright | \psi \end{array}$$

$\rightarrow$ Intro

$$\begin{array}{c} | \varphi \\ \vdots \\ | \psi \\ \vdots \\ \triangleright | \varphi \rightarrow \psi \end{array}$$

$\leftrightarrow$ Elim

$$\begin{array}{c} | \varphi \leftrightarrow \psi \text{ [or } \psi \leftrightarrow \varphi] \\ \vdots \\ | \varphi \\ \vdots \\ \triangleright | \psi \end{array}$$

$\leftrightarrow$ Intro

$$\begin{array}{c} | \varphi \\ \vdots \\ | \psi \\ \vdots \\ | \psi \\ \vdots \\ | \varphi \\ \vdots \\ \triangleright | \varphi \leftrightarrow \psi \end{array}$$

$\forall$ Elim

$$\begin{array}{|l} \forall v\varphi(v) \\ \vdots \\ \varphi(c/v) \end{array} \triangleright$$

$\forall$ Intro

$$\begin{array}{|l} \boxed{c} \\ \vdots \\ \varphi(c/v) \\ \forall v\varphi(v) \end{array} \triangleright$$

Where  $c$  does not occur outside the subproof in which it is introduced.

$\exists$ Elim

$$\begin{array}{|l} \exists v\varphi(v) \\ \vdots \\ \boxed{c} \varphi(c/v) \\ \vdots \\ \psi \end{array} \triangleright \psi$$

Where  $c$  does not occur outside the subproof in which it is introduced.

$\exists$ Intro

$$\begin{array}{|l} \varphi(c) \\ \vdots \\ \exists v\varphi(v) \end{array} \triangleright$$

Identity rules

=Elim

$$\begin{array}{|l} \varphi(n) \\ \vdots \\ n=m \text{ (or } m=n) \\ \vdots \\ \varphi(m) \end{array} \triangleright$$

=Intro

$$\triangleright \mid n=n$$