# PH210 exam revision guide 2015-2016

The material material roughly corresponds to sections 1.1-3.2 in *Logic and Structure*. Much of the same material is also covered in 17.1-17.2, 18.1-18.3, and 19.1-19.8 of *Language, Proof and Logic*. In instances where these treatments differ, you should assume that the presentation in *Logic and Structure* takes precedence.

You should be familiar with the definitions of the concepts mentioned below. Exam problems are likely to resemble problems which have appeared in the problem sets in scope and difficulty. Going through these exercise is hence a good means of revising as is looking at past exams papers.

I) Basic concepts from set theory and discrete mathematics

1) Definitions of basic set theoretic concepts and operations: empty set, subset and superset, order and unordered pairs, union, intersection, cross product, powerset
2) Countability: definition of a countable and examples of countable and uncountable sets
3) Relations and functions: reflexivity, symmetry, transitivity, functionality, totality, the representation of a functions as a relation, equivalence relations, equivalence classes
4) Proofs and definition by mathematical and structure induction (e.g. of $PROP$)

II) Propositional logic

1) Syntax: definition of $PROP$, parse trees, $rank(\varphi)$, $sub(\varphi)$, etc.
2) Semantics: definitions of a valuation $v$, the truth assignment $[\![\cdot]\!]_v$ by $v$ (and why it is unique), definition and properties of the notions(e.g.) tautology, logical consequence (i.e. $\Gamma \models \varphi$), logical equivalence
3) Truth functional completeness (you should at least know the definition, cf. LS pp. 23-25)
4) Proof theory: natural deduction proofs (format, open hypotheses, discharging hypotheses), natural deduction rules for all connectives (including $\vee$), examples (e.g. $\vdash \varphi \vee \neg\varphi$), definition and properties of the notions of derivability with and without hypotheses (i.e. $\Gamma \vdash \varphi$ and $\vdash \varphi$), definitions and properties of $Hyp(\mathcal{D})$, $Concl(\mathcal{D})$ and $DER$
5) Soundness Theorem for PL: soundness of individual proof rules, structure of the overall soundness proof
6) Completeness Theorem for PL: definition and properties of consistent sets and maximally consistent sets, Lindenbaum's Lemma, if $\Gamma$ is a MCS then there is a valuation $v$ s.t. $[\![\varphi]\!]_v = 1$ iff $\varphi \in \Gamma$, Truth Lemma

III) First-order logic

1) Syntax: similarity type, the definitions of $TERM_{\mathcal{L}}$,$FORM_{\mathcal{L}}$, $FV(t)$, closed term, $FV(\varphi)$, $SENT_{\mathcal{L}}$,$\varphi[t/x]$, "$t$ is free for $x$ in $\varphi$"
2) Semantics: Definition of a model, interpretation of a term in a model, Tarski's definition of truth, first-order validity and logical consequence, examples and properties of the foregoing notions – e.g. $\models (\exists x\varphi(x) \rightarrow \psi) \rightarrow \forall x(\varphi(x) \rightarrow \psi)$ if $x \notin FV(\varphi)$)

3) Proof theory: rules for $\forall$, $\exists$ and $\doteq$, examples (e.g. $\vdash \forall x\varphi(x) \leftrightarrow \neg\exists x\neg\varphi(x)$)
4) Examples of theories and models (e.g. partial order, linear order, group theory, $\mathsf{PA}$), using first-order sentences to distinguishing between structures, definability of a set in a model
5) Soundness theorem: soundness of the first-order rules (e.g. $\exists E$)
6) Completeness theorem: definition of a theory, Henkin theory, "Hekinization", theory $S_1$ is a conservative extension of theory $S_2$, Elimination Lemma, theories $T_0 \subseteq T_1 \subseteq \ldots \subseteq T_\omega$, Lindenbaum's Lemma, Model Existence Lemma and construction of the canonical model $\mathcal{M}$ (what is in its domain? why do we have have to take equivalences of terms in $ClTerm_{T_m}$?), Truth Lemma
7) First-order definability and non-definability (e.g. having an infinite domain, having a domain with exactly $n$ members, etc.)
8) Compactess theorem: statement and proof (via the completeness theorem), application to non-definability of finitude, transitive closure, well-foundedness, etc., non-standard model of $\mathsf{PA}$