

Chapter 2:

Learning from *Theaetetus*

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11.8.15

Much learning does not teach understanding.
—Heraclitus.

1. In the previous chapter, we considered an example of a piece of mathematical testimony and an array of questions that arise when one tries to establish whether such testimony can be a source of knowledge. In the course of that chapter, we were presented with G. H. Hardy's characterization of the mathematician as an observer of the layout of mathematical reality. As we saw, Hardy worried that if the analogy were pushed to its extreme, then it might seem to lead to a paradoxical conclusion:

...that there is, strictly, no such thing as mathematical proof; that we can, in the last analysis, do nothing but *point*;

Now whatever the fate of proof, we have as yet been offered no grounds for thinking that proof and pointing are the only ways to instil one's mathematical knowledge in others. In particular, it remains open that there is at least one other way to do so: by passing one's knowledge on to them via testimony. According to a number of philosophers, however, that way is closed. Thus, in the *Republic*, Plato has Socrates say:

Education isn't what some people declare it to be, namely, putting knowledge into souls that lack it, like putting sight into blind eyes.... Education takes for granted that sight is there but that it isn't turned the right way or looking where it ought to look, and it tried to redirect it appropriately. (*Republic* 518b–d.)

On this view, the educator's resources are again restricted to a form of pointing. Knowledge requires seeing things for oneself—albeit, perhaps, on the basis of a thorough familiarity with proofs. (*Meno* 85c, 98a.)

More recently, Bernard Williams has suggested that such a restriction applies specifically to mathematical knowledge. According to Williams,

...I can truly believe a mathematical proposition, which I cannot demonstrate, because I have been authoritatively told that it is true. This would be widely agreed not to constitute knowledge—knowledge, that is, that p , where p is the mathematical proposition. (Williams 1972: 54.)

Williams says very little in favour of the restriction, and reconstructing grounds that might be offered in its favour will take some effort, over the course of this chapter and the next. In this chapter, I shall make a start on reconstructing such grounds by considering the standing of a more general restriction on the epistemic power of testimony that Myles Burnyeat claims to find in some of Plato's dialogues. We will thus be considering the pedigree and prospects of a negative answer to the first question that was raised in the previous chapter: is it ever possible to come to know things by accepting another's claims? Although the proposed restriction is too general, and the attribution to Plato is ultimately unpersuasive, the outcome of reflection on both the proposal and the attribution will be a deeper understanding of the way in which testimony can function as a source of knowledge. That deeper understanding will, in turn, reveal ways in which testimony might fail to deliver knowledge and, so, ways in which attempts might be made to argue that it is incapable in principle of delivering mathematical knowledge. The attempts will be the subject of the following chapter.

2. Late in Plato's *Theaetetus*, the eponymous young mathematician proposes for consideration the hypothesis that knowledge is true judgement. (Since judgement, unlike knowledge, is episodic, it would be reasonable to take Theaetetus's proposal to be either that knowledge is true belief or that knowledge is the capacity to judge truly.) Socrates seeks to refute the hypothesis in the following passage:

...there is a whole art indicating to you that knowledge is not what you say.... The art of the greatest representatives of wisdom—the men called orators and lawyers. These men, I take it, use their art to produce conviction not by teaching people, but by making them judge whatever they themselves choose. Or do you think there are any teachers so clever that within the short time allowed by the clock they can teach adequately the truth of what happened to people who have been robbed or assaulted, in a case where there were no eye-witnesses? ...Then suppose a jury has been justly persuaded of some matter which only an eye-witness could know, and which cannot otherwise be known; suppose they come to their decision upon hearsay, forming a true judgement: then they have decided the case without knowledge, but granted they did their job well, being correctly persuaded? But...they couldn't have done that if true judgement is the same thing as knowledge... (Plato *Theaetetus* 201a–c.)

At first glance, Socrates here supplies a simple counterexample to Theaetetus's hypothesis: having been justly persuaded, the jury comes to a true judgement. However, because the jury's judgement is based neither on teaching (since they were only persuaded) nor on perception (since they were not eye-witnesses), and

since those are in this case the only ways they could have come to know, their judgement does not amount to knowledge. Thus, Theaetetus's hypothesis is refuted.

However, a second look at the passage is apt to be less reassuring. As Myles Burnyeat has emphasised, there is an apparent tension between the two ways in which Socrates seeks to preclude the title of knowledge from the jury's true judgement. The first way appeals to the fact that in the circumstances of the trial—given, that is, the functions of the orators and lawyers and the pressures of time—the jury can only be persuaded and not taught. That appeal suggests that it would be possible in principle—that is, in more propitious circumstances—for the jury to have been taught rather than persuaded. And on the further assumption that being taught is a way of coming to know, the suggestion would then be that in those circumstances the jury could have come to know. By contrast, the second way appeals to the idea that the matter in question is such that “only an eye-witness could know, and which cannot otherwise be known”. The second appeal, then, is to the effect that with respect to the matter in question, only an eye-witness could know. And assuming that the jury were not eye-witnesses (“there were no eye-witnesses”), it would seem to follow that no quantity or quality of teaching could bring them to know. So, the second appeal seems to render the first otiose. (Burnyeat 1980: 178–9)

Even more worrisome than the tension between the two appeals that Socrates makes in the passage, and of greater immediate relevance to our present concerns, is the fact that the second appeal appears independently paradoxical. For it seems to rule out what we would ordinarily take to be a live possibility, that someone who knew something on the basis of witnessing it might pass their knowledge on to someone who hadn't witnessed it by exploiting the power of hearsay or testimony. If there are facts that can be known only by an eye-witness, then the only way for an eye-witness to furnish another with knowledge of those facts would be by manoeuvring them into a position from which they too can be an eye-witness—for example, by pointing. (Barnes 1980: 197; Burnyeat 1980: 181–4.)

The precise strength of scepticism about the epistemic power of testimony that Burnyeat takes Plato to express in the target passage would depend on the extent of the class of facts taken to be knowable only by an eye-witness and, so, not knowable, even in principle, also on the basis of testimony. However, we might reasonably hold that there are no such cases. (At least as a matter of principle, for we should allow the only witness might die before telling.) And Burnyeat suggests that Plato seems independently to be committed to the view that the class of facts that are knowable in only one way—for example, only on the basis of perception or only on the basis of proof—, and so not also on the basis of testimony, is quite extensive. (Burnyeat 1980: 183–4. See also Barnes 1980: 195–6. Barnes is agnostic about Burnyeat's attribution of such a view to Plato, but suggests that Aristotle and Locke may have been so committed.)

3. More generally, Burnyeat suggests that Plato seems to hold that facts fall into what we might call (following Barnes 1980: 195–6) *epistemic categories*: categories marked out by limits on the ways in which facts that fall within them can be

known. On this view, there is a non-empty category of facts, EC_1 meeting the following condition:

- (EC₁) For all p such that the fact that p is in EC_1 , and all subjects x , if x is in a position to know that p , then x has seen that p (or has seen some truth-maker for the proposition that p).

Similarly, there might be other such categories specified by appeal to other sensory modalities, and a more general category specified by appeal to perception:

- (EC₂) For all p such that the fact that p is in EC_2 , and all subjects x , if x is in a position to know that p , then x has perceived that p (or has perceived some truth-maker for the proposition that p).

There might also be other such categories specified by appeal to other ways of knowing, for example by appeal to familiarity with a proof, as in (EC₃):

- (EC₃) For all p such that the fact that p is in EC_3 , and all subjects x , if x is in a position to know that p , then x has gained familiarity with a proof that p .

(A more restrictive principle of this sort might make appeal to a particular proof, or specific range or type of proofs, for each fact.) Grounds were offered in the previous chapter for doubting that (EC₃) has application to mathematical facts, for we could find no principled reasons for requiring that there were mathematical facts that can be known only by way of proof as opposed, for example, to intellectual intuition. Thus, as with the case of seeing and perception, we might consider a more general category specified by appeal to intellection:

- (EC₄) For all p such that the fact that p is in EC_4 , and all subjects x , if x is in a position to know that p , then x is aware that p by way of intellection (or is aware by intellection of some truth-maker for the proposition that p). (See e.g. Williams 1972: 54.)

On this type of view, facts that fall within an epistemic category cannot be known in ways disjoint from the way that specifies the category. (Two ways are disjoint if exploiting one of those ways fails to entail exploiting the other and *vice versa*.) Thus, for example, if we assume that perception and intellection are disjoint ways of knowing, then (EC₂) specifies a range of facts that cannot be known by intellection, and (EC₄) specified a range of facts that cannot be known by perception. By contrast, since perception and seeing are not disjoint ways of knowing—since, that is, seeing entails perceiving—, (EC₁) specifies a range of facts that can be known by perception, and (EC₂) specifies a range of facts in a way that leaves open whether or not they can be known by seeing.

Suppose that all facts fall within one or another such category. It doesn't yet follow that no such facts can be known on the basis of testimony. To obtain that result, further premises are needed. The first such premise, (P1), is required to rule

out that the way of testimony marks out its own proprietary epistemic category, disjoint from all other categories:

- (P1) There is no non-empty, *basic* epistemic category—that is, no category disjoint from all other categories—such that, for all p such that the fact that p is in the category, and all subjects x , if x is in a position to know that p , then x is aware that p by way of by receiving testimony to the effect that p .

We might hold that it is reasonable to endorse that premise on the grounds that there are no facts that can be known only on the basis of testimony. And we might accept that on the further grounds that testimony is not itself a basic way of knowing, but rather provides at best a dependent way of knowing—that is, as a means for transmitting between persons knowledge that was acquired in some other way. On the basis of the first additional premise, it follows that testimony can be a way of knowing only if there are ways of knowing from which it is not disjoint. That is to be ruled out by the second and third premises. Thus, as a second premise, we require something to this effect:

- (P2) There is no epistemic category specified by appeal to a way of knowing W such that for all subjects x , if x is in a position to know that p by W , then x is in a position to know that p by receiving testimony to the effect that p .

The second required premise, like the first, is plausible. For it is plausible that neither having perceived that p , nor having gained familiarity with a proof that p , entails having received testimony to the effect that p . The third required premise is the converse of the second:

- (P3) There is no epistemic category specified by appeal to a way of knowing W such that for all subjects x , if x is in a position to know that p by receiving testimony to the effect that p , then x is in a position to know that p by way W .

The third premise, like the second, can be rendered plausible by appeal to examples. It is plausible that one can be told that p without perceiving that p (or perceiving a truth-maker for the proposition that p); and it is plausible that being told that p does not put one in a position to gain familiarity with a proof that p . (One can, of course, be told a proof that p . But being told such a proof is not a general requirement on being told that p .) Furthermore, if testimony were to provide a way of knowing only in concert with the provision of a testimony independent way of knowing, then that would render the testimony redundant: on receipt of testimony with the potential to afford one with knowledge, one would anyway be in a position to gain knowledge independently of any epistemic power possessed by the testimony. (That is a further reason why the possibility of being told a proof that p fails adequately to gainsay the restriction imposed by (P3)).

Putting the three premises together, we have that testimony does not in general provide a way of knowing. The reason for this is that testimony lacks the power to bring about knowledge independently of other ways of knowing ((P1) and (P2)) and, more importantly, lacks the intrinsic power to bring about those other ways of knowing. Thus, the combination of the view that facts fall into epistemic categories with some independently plausible claims about the power of testimony leads to the paradoxical conclusion that it is not possible to come to know by receiving testimony. As a special case, the combination leads, via (EC₄) to a bar on the possibility of acquiring mathematical—or, more generally, intellectual—knowledge by way of testimony.

4. The appeal to epistemic categories institutes a global form of scepticism about the epistemic power of testimony. That might seem bad enough, but as Burnyeat indicates, the appeal to epistemic categories has further unacceptable consequences. In setting out those further consequences, Burnyeat suggests that the appeal to epistemic categories conflicts with features of our ordinary conception of knowledge. Although that claim is stronger than is strictly warranted by the considerations he presents, those considerations provide the basis for an objection with similar force.

As Burnyeat observes, our ordinary conception of propositional knowledge treats knowledge as factive. Thus, if it is true of a subject that they know that p , then p . Furthermore, ordinary competence with the concept of knowledge can put one in a position to exploit its factiveness. Thus, if one were to know that a subject knows that p , ordinarily one would thereby be in a position to deduce that p . Assuming that one's deduction were competent, it is plausible that one would thereby in a position to know that p . (Burnyeat 1980: 181–183, drawing on Hintikka 1962.) However, Burnyeat claims that the possibility of coming to know that p in that way, as apparently written into our ordinary conception of knowledge, conflicts with the appeal to epistemic categories. For this form of competent deduction seems capable, in principle, of putting one in a position to know any known fact, in a way that is inconsistent with the restrictions imposed by the appeal to epistemic categories. (Burnyeat 1980: 184–5.)

Although Burnyeat is right to think that there is a further difficulty here for the appeal to epistemic categories, he is wrong to locate the difficulty in a conflict between the appeal and our ordinary conception of knowledge. In particular, the conflict to which Burnyeat points arises only if we make the further assumption that there are ways of coming to know what someone else knows that are disjoint from ways of coming to know that which they know. (We must also assume that competent deduction based on conceptual knowledge is a way of coming to know, but it would take us too far afield to attend further to that assumption.) In the absence of that assumption, it would be open for us to retain our ordinary conception of knowledge, including its factiveness, in either of two ways. First, we might do so by denying that it is ever possible to come to know that someone else knows that p without independently being in a position to know that p . Second, and connectedly, we might do so by denying that our ways of coming to know that someone else knows that p are disjoint from ways of knowing that specify the

epistemic categories of what they know. It is the implausibility of either of those denials that induces a conflict with the appeal to epistemic categories, rather simply the factiveness of our ordinary conception of knowledge. (See Barnes 1980: 200–201.)

Nonetheless, Burnyeat directs us to what can be seen—in light of the implausibility of the denials required to avoid it—as an additional problematic consequence of the appeal to epistemic categories. In addition to instituting scepticism about testimony, the appeal to epistemic categories has the further sceptical consequence that one can know what others know only when one independently knows that which they know for oneself via exploitation of a category specifying way of knowing. The appeal to epistemic categories would deprive us not only of ways of knowing on the basis of their testimony that which other people know, but also of ways of knowing about the epistemic standing of those other people. In cases in which we do not know whether p , the most we could hope to know about someone else would be that they know *whether* p . Put another way, it would be possible for one to reveal what one knows to someone else only by getting them to acquire that knowledge for themselves via a category acceptable route. That is, one would be confined to analogues of pointing. (Barnes 1980: 200–201; Burnyeat 1980: 181–186.) We will have reason to return to this consequence in the next chapter, when we come to discuss in more detail scepticism about testimony to mathematical fact.

5. The problematic consequence with respect to testimony of the global appeal to epistemic categories has two causes. The first cause is the fact that there are forms of psychological or epistemic attainment that the acceptance of what one is told seems to be incapable of ensuring. In particular, accepting what one is told is incapable of ensuring either that one comes to perceive that something is the case or that one comes to intellect that something is the case. The second cause is the requirement on knowing that is embodied in the appeal to epistemic categories, to the effect that knowing depends on those forms of psychological or epistemic attainment. As we've seen, there are compelling reasons to reject the appeal to epistemic categories, and so to reinstate the natural thought that it is possible to acquire knowledge by accepting testimony. However, it is natural to wonder whether there are other psychological or epistemic attainments that testimony is incapable of ensuring, either because they, unlike knowing, depend on perception or intellection, or for some other reason.

In the previous chapter, we mentioned a number of potential cases of epistemic goods that might not be passed on simply on the basis of testimony to the truth of a particular mathematical theorem but might depend in addition on gaining familiarity with a proof of that theorem: the tracing of connections amongst theorems or domains, the testing and developing of methods of proof (perhaps including the articulation of axioms suitable to be incorporated into those methods), the generalizing of theorems, and, more generally, the deepening of our understanding of mathematical reality. Similarly, Burnyeat suggests that whether or not testimony cannot transmit knowledge, it is arguable that it cannot transmit understanding:

I can of course be given the information that p is connected with q , r , etc., just as I can be given the information that p is true because q is true. What is more, I can accept that this is so with adequate justification and thereby, in the ordinary sense, know it. But every schoolboy is familiar with the fact that it is one thing to know in that external way that the connection holds (e.g., that these propositions constitute a proof of that theorem), and quite another to understand the connection, to see how the elements hang together. That is something one can only do for oneself. (Burnyeat 1987: 41. Burnyeat here develops a suggestion initially made in his 1980: 186–188 in light of an objection made in Barnes 1980: 203.)

It would take us too far afield to pursue the question whether Burnyeat's proposal about understanding is correct. However, as noted in the previous chapter, it is important in seeking an answer to the question whether testimony can be a source of knowledge—in mathematics or more generally—carefully to distinguish that question from closely related questions about understanding and other broadly epistemic goods.

6. As we've seen, the global appeal to epistemic categories has unacceptable consequences and, so, should not be accepted. The implausibility of the appeal to epistemic categories might naturally make us wonder whether Burnyeat's attribution to Plato of such an appeal is fair. Although Burnyeat cites a number of passages in other dialogues in support of the attribution to Plato of an appeal to epistemic categories, none of them speaks decisively in favour of the attribution. If the attribution to Plato of an appeal to epistemic categories is not warranted, that might make us wonder whether his brief discussion of testimony in the *Theaetetus* might have something else to teach us. (Burnyeat 1980: 179. The passages are *Meno* 87b–c, *Symposium* 175d–e, and *Republic* 518b–c, from which I quoted in section 1.)

In assessing both the independent standing of the appeal to epistemic categories, and its attribution to Plato, it is important to observe that the claim that such epistemic categories are non-empty would be even less plausible if the categories were marked out by appeal to what x *can* perceive or *is* aware of by intellection rather than, as above, by what x *has* perceived or intellected. For on plausible assumptions, the former restriction would tend, implausibly, to preclude the preservation of knowledge in memory. (A version of the point is emphasised at *Theaetetus* 163d–164b.) Alternatively, such a claim would be more plausible if the categories were marked out by appeal to what *someone* has perceived or intellected, where it need not be that it is x who has perceived or intellected that p . That weaker restriction would leave open that x might acquire knowledge without perceiving or intellecting that p , for example by receiving testimony from someone else who has perceived or intellected that p . (Barnes 1980: 194–5.) A defence of the claim of non-emptiness with respect to epistemic categories that are subject to the intermediate restriction would therefore need to include a principled explanation for why that specific restriction is to be favoured over either the

stronger or the weaker alternatives. Furthermore, a defence of Burnyeat's attribution to Plato would need to be sensitive to the fact that the types of epistemic categories involved in the attribution are sandwiched between more plausible (weaker) and less plausible (stronger) types. Thus, support would be required not only for the general attribution to Plato of some sort of appeal to epistemic categories, but also for the specific attribution of appeal to epistemic categories of just the required strength.

More generally, the appeal to epistemic categories in order to rule out the possibility of knowledge being acquired by way of testimony seems insufficiently explanatory of the restriction. More would anyway be required before it would be possible to see clearly why the plausible idea that individual perception and intellection are required to afford us with initial access to proprietary, and disjoint, domains of fact should lead to the implausible idea that they are required to afford us with any form of access to those domains.

In addition to interpretative charity, a further reason for doubting the attribution to Plato is that it struggles to deal adequately with the interpretative difficulty with which we began. That difficulty, recall, was that one of the reasons that Socrates offers for denying that the jury's judgement manifests knowledge—that in this case, only an eye-witness could know—is in tension with the other—that the jury's judgement is, at best, a response to persuasion rather than teaching. The appeal to epistemic categories is a generalisation of the first reason and, to that extent, aggravates, rather than easing, the difficulty. Burnyeat suggests two potential salves: first, we might blame the apparent tension on hasty composition, and excise or revise the aspects of the passage that induce it; second, we might sever the apparent link between teaching and knowledge that underpins the tension. He suggests that the latter solution is the most economical. In taking it, we are able to retain the idea that the jury cannot come to know—that is, we retain the idea ensconced in the more general appeal to epistemic categories—whilst allowing that in propitious circumstances, they might nonetheless be taught, just as long as the teaching didn't result in their knowing. (Burnyeat 1980: 179–180.) However, the idea that teaching results in knowledge seems solid and seems, moreover, to be affirmed by Socrates just prior to the target passage. (Plato *Theaetetus* 198b, as noted by Burnyeat, 1980: 179.) It would be reasonable, then, to consider alternative potential routes to salvation.

7. Let's consider, then, the option of excising or revising aspects of the passage that induce the tension. Given Plato's overarching emphasis on the negative impact of persuasion by contrast with teaching, together with the problematic consequences that arise from the imposition of epistemic categories, the most plausible point of excision would be the suggestion that there are facts that can be known only by an eye-witness.

Jonathan Barnes makes one such proposal for excision, by suggesting that we “strike out the offending words” (Barnes 1980: 193.) His suggestion is that it would leave Socrates' argument unimpaired to delete the following:

...which only an eye-witness could know, and which cannot otherwise be known... (Plato *Theaetetus* 201b.)

In doing so, Barnes suggests, we would remove the suggestion that Socrates sought to impose an epistemic category specified by appeal to eye-witnessing. We would thus remove the source of tension with the opposing suggestion that, in more propitious circumstances, the jury could have been taught and, so, could have come to know. Furthermore, Barnes claims, there would be no cost to Socrates' argument against the hypothesis that knowledge is true judgement.

Barnes' proposal is tempting. However, it faces at least three objections. The first objection is that the proposed excision is the second time that Socrates mentions eye-witnesses in the target passage. (They are mentioned earlier in Plato *Theaetetus* 201b: "...in a case in which there were no eye-witnesses..."). Minimally, that suggests that the second occurrence wasn't alone in being the upshot of hasty composition. Less minimally, it suggests that the appeal to eye-witnesses plays an operative role in Socrates' argument. The second objection is that a plausible role for the appeal is precisely to foreclose on one way in which the jury might otherwise have come to judge, not only truly, but knowledgably: one or more members of the jury might have been eye-witnesses. (The first two objections are due to Stramel 1989.) The third objection is that, shorn of Socrates' appeal to the absence of eye-witnesses, or to the machinery of epistemic categories, it is not clear why his supposition that the jury has "come to their decision upon hearsay" would have as a consequence that "they have decided the case without knowledge". (Plato *Theaetetus* 201b–c.) The objections suggest that we might reasonably seek a less costly revision.

James Stramel proposes a less extravagant revision. According to Stramel, Socrates' overall argument makes appeal to two possible ways in which the jury might come to a knowledgeable decision: they might come to such a decision either, first, on the basis of being taught or, second, on the basis of being eye-witnesses. Since Socrates treats those as the only two possible ways in which the jury can come to a knowledgeable decision, Stramel views him as imposing a disjunctive necessary condition on knowledge (as opposed to the simple necessary condition apparently imposed in the passage). Socrates' argument then proceeds by eliminating each disjunct: there were no eye-witnesses, so that way is foreclosed; and the circumstances of the trial preclude teaching, so that way is also foreclosed. (Stramel 1989: 8–10.)

Stramel's proposal is plausible and a version of it will be derivable from my own solution to the difficulty. However, it cannot be accepted without supplementation. To see why, we must consider an objection and an alternative proposal, both due to Tamer Nawar. (Nawar 2013.)

Nawar's main objection to Stramel's disjunctive proposal is based on the observation that, as it stands, such a proposal would not be acceptable to Plato, for it fails to explain any commonality between the two disjuncts. It is thus of a piece with the sorts of case-based accounts of knowledge that Socrates rejected at length earlier in the dialogue. (Plato *Theaetetus* 148d. See also *Meno* 72a–d.) And we might well agree with Plato that, all else being equal, a non-disjunctive account would be preferable. What is required, then, are non-disjunctive grounds, deriving

from the nature of knowledge in general, for holding, first, that eye-witnessing and teaching are both potential ways of knowing and, second, that other resources that might be available to the jury—in particular, their openness to being persuaded—are not potential ways of knowing. Since Stramel fails to provide such grounds, his proposal is not acceptable without supplementation. (Nawar 2013: 14–15.)

In place of Stramel’s merely disjunctive proposal, Nawar presents a unified alternative. According to his alternative proposal, Socrates’s discussion is organised by the following “virtue intuition”:

...if *S* knows something, *S* gets things right *as a result of* a cognitive capacity or virtue in *S*. (Nawar 2013: 7.)

Nawar’s central claim is that the necessary condition specified in the virtue intuition—or an adequate precisification thereof—can be used to explain the three target judgements: (1) that it is possible for perceiving to result in knowing; (2) that it is possible for being taught to result in knowing; and (3) that it is not possible for being persuaded to result in knowing. Nawar’s fundamental idea here is that, at least according to Plato, in cases in which one gets things right on the basis of being taught or on the basis of perceiving, one does so as a result of the virtuous exercise of one’s own cognitive capacities; by contrast, in cases in which one gets things right on the basis of being persuaded, one does so as a result of the exercise of the persuader’s capacities, rather than one’s own.

Like Stramel’s proposal, Nawar’s sheds light on Plato’s thinking in this area, and on the issues with which Plato is wrestling. However, it too is subject to difficulties and cannot be accepted without further development.

An initial difficulty for Nawar’s proposal can be set aside. The difficulty is that the virtue intuition specifies only a necessary condition and so cannot be used to explain either (1) that it is possible for perceiving to result in knowing or (2) that it is possible for teaching to result in knowing. At most, it can be used to show that (3)—that it is not possible for being persuaded to result in knowing—can be explained in a way that does not itself rule out (1) or (2). For present purposes, however, we can rest content with the weaker conclusion and so set the initial difficulty aside.

A second, more pressing difficulty is that the virtue intuition is too blunt an instrument to cleanly draw all of the distinctions that must be drawn in this area. As we noted, one distinction that must be drawn is that between perceiving (e.g. eye-witnessing) and being persuaded. If we attend just to the virtue intuition, then we can see that the distinction is to be drawn in the following way. In cases in which one knows by perceiving, one must get things right as a result of one’s own cognitive capacity or virtue. By contrast, in cases in which one gets things right on the basis of having been persuaded, one does not get things right as a result of one’s own cognitive capacity or virtue. The difficulty is that it is hard to see why that is the right way to construe the respective cognitive consequences of perceiving and being persuaded. For in both cases, it seems that one’s getting things right is the result of the effects on one of something else: in the case of perceiving, an environmental element that one perceives; in the case of being persuaded, the speech of the person doing the persuading. Nawar works hard to

defend the claim that, for Socrates in the *Theaetetus*, “strictly speaking, the cause of the mental state that results from *S* perceiving *x* is a psychological capacity in *S*.” However, talk of *the* cause here is misleading: we might be willing to allow that a psychological capacity in *S* figures as *a* cause of the mental state that results from their perceiving *x*, but it would be hard to deny that *x* figures as another cause, at least via figuring as an essential constituent of *S*’s perceiving it. Alternatively, insofar as we were willing to allow that the necessary condition imposed in the virtue intuition would be met if a psychological capacity of the subject figured as *a* cause of the resulting mental state, it would be hard to deny that such a capacity figures as such a cause of the mental state that results from being persuaded. For becoming persuaded of something seems to require the cognitive capacities needed to understand and to accept what one is told. The suggestion here is not that there is no way to mark the required distinctions, or even that there is no way that makes appeal to a precisification of the virtue intuition. The present suggestion is only that taken as it stands, the virtue intuition fails cleanly to draw the required distinctions.

Nawar seeks a precisification of the virtue intuition by appeal to a further distinction between processes of mental state formation wherein the subject is active and responsible and those wherein the subject is passive. The attempted precisification figures in the way that Nawar applies the intuition in order to draw the required distinctions. Thus, for example, Nawar appeals to the distinction in characterising cases of teaching and cases of persuasion as, respectively, meeting, and failing to meet, the necessary condition set by the virtue intuition:

...in the *Meno*...Socrates...wishes to show that teaching is not a case of transmitting knowledge and that he is not passing on either geometrical knowledge or even the relevant geometric answers. Rather, the slave is meant to have come to the answers *by himself* and this it is emphasized that the slave answers ‘for himself’ (85b8–9) and as the result of his own ability: ‘and is not this recovery of knowledge, *in himself and by himself*, recollection?’... (85d5–6) (Nawar 2013: 8, with interpolated quotations from Plato *Meno*.)

Here, then, the slave is taken to come to know from his own resources and to that extent actively as opposed to passively. Plato’s description of the slave’s achievement in the *Meno* contrasts starkly with his characterisation of the persuasive power of rhetoric:

In the *Euthedymus*, the art...of speech-writers is compared with that of magicians: ‘the sorcerer’s art is the charming of snakes and tarantulas and scorpions and other beasts and diseases, while the other [rhetoric] is just the charming and soothing of juries, assemblies, mobs, and so forth’ (290a1–4). Similarly, in the *Gorgias*, we are told that ‘with this power you will hold the doctor as your slave, the trainer as your slave’ (...452e4–6). (Nawar 2013: 9–10.)

Although Plato seems here to distinguish sharply between the slave’s active achievement of knowledge and the passivity of the results of persuasion, Nawar’s

exploitation of Plato's distinction is problematic. An initial difficulty arises because Nawar hopes to use Plato's differential treatment of the cases to underwrite the distinction between the respective results of being taught and being persuaded. However, Plato seems not to view the slave's achievement in the *Meno* as the result of teaching:

And he [the slave] will know it *without having been taught* but only questioned, and find the knowledge within himself? (Plato *Meno* 85d3–4, emphasis added. Nawar quotes this passage without noting the obvious problem that it raises for his proposal. (Nawar 2013: 8.))

A second difficulty comes to light when we inquire after Plato's reasons for denying that the slave case involves teaching. For plausibly it can only be because none of the cases in which the slave gets things right about geometry are due to Socrates having told the slave those things. Thus, a natural extrapolation would be that if this had been a case of teaching, then it would have involved Socrates telling the slave things about geometry; and, moreover, that in that case, it would have been far less clear that the slave's getting things right would have been the result of his own activity, as opposed to being the passive upshot of Socrates's teaching. We will return to this difficulty in a moment, after explaining a third difficulty.

The third difficulty would arise even if we were to accept that, in the passages that Nawar cites, Plato aimed to draw a distinction between the results of teaching and persuasion. For the distinction is drawn too crudely to be plausible and it is therefore hard to believe that Plato took it as seriously as Nawar's proposal requires. One issue here concerns the idea that the slave's achievement is entirely his own and so in no respects the passive upshot of Socrates's activity. For the systematic line of questioning that Socrates employs in order to trigger the slave's geometric 'recollections' are notoriously leading. Another issue concerns the idea that being persuaded is, in all cases, entirely passive. The latter issue becomes especially pressing if we assume that the class of cases in which someone comes to a view based on accepting what another person says divide exhaustively into cases of teaching and cases of persuasion. If we assume in addition that if someone is taught something, then they come to know that thing, then cases of persuasion will include any case in which someone forms a view on the basis of accepting what another person says without thereby acquiring knowledge. It is as hard to accept that in all of those cases, such views must be formed purely passively as it is to accept that in all cases of teaching, knowledge is acquired purely actively. To take the most pressing case, someone might be persuaded rather than taught, despite their valiant and virtuous efforts critically to scrutinise the information with which they were presented, simply because their interlocutor failed to have appropriate knowledge to transmit.

In light of the three difficulties, it seems clear that the contrast between active and passive processes cannot without supplementation yield a distinction between the respective results of teaching and persuasion. The most that could be hoped is that getting things right as a result of teaching is liable to be more active than is getting things right as a result of persuasion, and even that seems optimistic in

light of the third difficulty. It is anyway hard to see how such comparative judgements might be parlayed into a usable necessary condition on knowing.

8. The most pressing difficulty for Nawar's proposal arises from a pair of assumptions that were made in presenting the third difficulty. The first assumption is that an autonomously virtuous subject might find it impossible to tell that they faced an attempt at persuasion rather than an attempt at teaching. The second assumption is that, nonetheless, such a subject might in either case fully exercise their autonomous virtue in critically scrutinising either attempt before accepting what they were told. For given those two assumptions, a pair of subjects might form their views in ways that are autonomously virtuous and yet, due to the difference between teaching and persuasion, only one of them acquires knowledge. Since the two assumptions are plausible, Nawar's proposal lacks the resources needed to explain the difference between the respective results of teaching and persuasion and, so, cannot be accepted. What the assumptions indicate is that an adequate development of Nawar's proposal must be sensitive not only to the autonomous intellectual virtues and capacities of the subject, but also to the virtues and capacities of the interlocutor, or chain of interlocutors, on which they depend.

That first point connects with a second. Both Stramel and Nawar treat perceiving and being taught as independent ways of coming to know. However, with respect to the case that Plato presents it is implausible to treat being taught in that way.

The reason that it is implausible to treat being taught as independent of perception is that teaching is, in general, a *transmissive*, rather than a *generative*, way of knowing. A *generative* way of knowing is an autonomous way of coming to know something, in principle for the first time—for example, by way of perception or intellection. It is a way of being independently sensitive to the facts about which one can come to know, or to reasons that are determined by those facts. (In what follows, I will assume that knowledge depends ultimately on sensitivity to facts, but I'll sometimes speak of a disjunction between facts and reasons in order to leave it open that there might be facts that are not reasons or reasons that are not facts.) By contrast, a *transmissive* way of knowing is a way of preserving or transmitting a sensitivity to facts or reasons that was acquired originally via a generative way of knowing. Thus, remembering is a transmissive way of knowing, because it preserves within an individual a sensitivity to facts or reasons that was acquired originally in some other way, ultimately by way of perception or intellection. Similarly, being taught is a transmissive way of knowing, because it preserves between individuals a form of sensitivity to facts or reasons that was acquired originally in some other way—again, ultimately by way of perception or intellection. Thus, in cases in which the only available generative way of knowing is perception, it is possible for someone to acquire knowledge by perception without exploiting any other way of knowing. However, although in such cases it may be possible for someone to acquire knowledge by being taught, it is not possible for them to acquire knowledge unless someone has exploited another way of knowing. The availability of such a transmissive way of knowing is

dependent on the prior operation of a generative way of knowing—in the case that Socrates presents, by way of perception. Plausibly, the transmissive way of knowing would be dependent in addition on the retention in memory of the sensitivity to facts or reasons instituted by the operation of the generative source. And it might also be dependent on further intervening transmissions, if the immediate teacher had acquired their knowledge by being taught, and so forth. So, although it is possible for perception to take place without teaching, it is not possible for teaching to take place in this case without perception, and more generally without the operation of one or another generative way of knowing.

(There are at least two ways of knowing that don't cleanly fit this rough distinction between generative and transmissive ways of knowing: proof; and the way, or ways, we have of knowing what others know. The way of proof is problematic due to the fact, noted in the previous chapter, that its capacity to deliver knowledge often depends upon prior knowledge of an array of initial premises. There is a good sense, then, in which a proof transmits sensitivity to facts or reasons from premises to theorems. However, the fruitfulness of proof indicates that it is also a way of acquiring initial awareness of facts, and to that extent a generative source. We might therefore decide to treat proof as a member of a distinct class of combinatorial ways of knowing: ways that enable us to operate on things that we already know in order to acquire new knowledge. Some forms of induction might also be included amongst the combinatorial ways of knowing. Similarly, the way, or ways, we have of knowing what others know is problematic because it depends on the prior possession of knowledge by others. To that extent, it might seem to be a transmissive source. On the other hand, it seems possible for one to come to know that someone knows that p in a case in which neither they, nor anyone else, knows that they know that p . So, there are also grounds for treating this as a generative way of knowing.)

Because transmissive ways of knowing depend ultimately on generative ways of knowing, it is crucial, in assessing whether what an orator offers to the jury is teaching or persuasion, that one consider not only the circumstances in which, or the intentions with which, the offer is made—for example, whether the orator's intention is to teach rather than to persuade—but also the orator's capacities successfully to implement their intention—for example, whether or not the orator can speak knowledgeably on the matter in question. Thus, the question whether an orator can teach a jury, rather than merely persuading them, sometimes depends on whether the orator is also an eye-witness, or whether the orator has been taught in a way that depends ultimately on the resources of an eye-witness. It follows that with respect to cases like the one that Plato presents, in which the ultimate source of knowledge has to be the perception of an eye-witness, something like Stramel's disjunctive necessary condition holds sway. For in that case, someone who knows either must have been taught, and so exposed to the testimony of someone else who was in a position to know, or must have come to know on the basis of their own perceiving. However, we are now in a position to see that the disjunctive condition holds not only with respect to the different sources that are available to the jury, but also with respect to one of those sources considered alone, namely the orator's hearsay: for it will only possible to come to

know by way of accepting that hearsay if the orator was an eye-witness or was taught by someone who was taught by someone...who was an eye-witness.

Furthermore, we are now in a position to discern, and to close, a gap in Stramel's proposal. For Stramel's proposal leaves open that there might be teachers who were not appropriately connected with eye-witnesses. As we have now seen, the capacity to teach depends ultimately on the operation of a generative way of knowing: in the case that Plato presents, it depends on the operation of perception. Thus, we can make better sense of the structure of Socrates' argument in the *Theaetetus* if we allow that there are at least two types of reason why an orator may be able only to persuade and not to teach: first, that the intentions with which the orator speaks—intentions to persuade, rather than to teach—prevent them from making available to others any knowledge that they possess; second, that the orator is not appropriately connected with an eye-witness.

9. Coming this far puts us in the market to reconsider the type of weakening of epistemic category (EC₂) that we briefly considered earlier in the chapter:

- (EC₅) For all p such that the fact that p is in EC_5 , and all subjects x , if x is in a position to know that p , then there is at least one subject y such that y has perceived that p (or has perceived some truth-maker for the proposition that p).

As we noted earlier, the claim that (EC₅) is non-empty is far more plausible than is the analogous claim about (EC₂). Its greater plausibility derives from the fact that it allows that, despite there being facts that can be known only if they are known on the basis of perception, such facts can nonetheless be known by people who haven't themselves done the perceiving. However, although (EC₅) is sufficiently weak to be plausible, its weakness makes it comparatively uninformative. In particular, it fails to specify any connection between those who know by way of perceiving and those who know without perceiving. Thus, it fails to preclude someone from coming to know that p in the absence either of their perceiving that p , or their being appropriately connected by way of testimony with someone who has perceived that p ; it requires only that someone has perceived that p . In so failing, it shows itself unable to account for two sorts of ways in which someone might be prevented from acquiring knowledge by accepting what a speaker tells them. First, it is unable to account for cases in which, despite there being people who know, the speaker is not amongst them. Second, it is unable to account for cases in which, despite the speaker knowing, their circumstances, or the intentions with which they speak, prevent them from transmitting their knowledge. Moreover, (EC₅) fails to suggest any factor that it might have in common with epistemic categories based on other ways of knowing. It is thus susceptible to a version of the complaint that Nawar lodged against Stramel's disjunctive proposal, in that it doesn't clearly issue from a unified condition on knowing.

10. With respect to generative ways of knowing, the appeal to epistemic categories has some plausibility. For example, it is plausible that there are facts initial knowledge of which is possible only by way of perception, and not, say, by intellection, and that there are facts initial knowledge of which is possible only by way of intellection, and not, say, by perception. The implausibility of the global appeal to epistemic categories derived mainly from its preclusion of one or more transmissive ways of knowing. Furthermore, with respect to generative ways of knowing, there is some plausibility to a version of Nawar's proposal that knowing is subject to the requirement outlined in the virtue intuition. For one of the main difficulties facing that proposal arises with respect to transmissive ways of knowing, insofar as the fully virtuous operation of those ways can be dependent on factors without the control of individual subjects' autonomous virtuosity: in particular, it can be dependent on the autonomous virtuosity of putative teachers. However, even with respect to generative ways of knowing like perception, we noted that Nawar's emphasis on the activeness or responsibility of subjects made the proposal difficult to apply. For it is natural to hold that the objects of perception are in some way active and responsible for our coming to know about them by way of perception. Nonetheless, if we focus on virtuosity, rather than individual activity or responsibility, it seems plausible to retain the core of Nawar's proposal with respect to generative capacities. So construed, the central idea would be that virtuosity—roughly, the proper exercise of well-formed sensory and intellectual capacities—is a requirement on acquiring appropriate sensitivity to facts or reasons; and appropriate sensitivity to facts or reasons is a requirement on knowing. Thus, virtuosity, or proper exercise of well-formed sensory or intellectual capacities, is a requirement on the successful exploitation of generative ways of knowing.

Furthermore, we can generalise that insight about virtuosity in a way that enables it to capture transmissive ways of knowing. The two central ideas here are the following: first, that the successful exploitation of transmissive ways of knowing requires that the transmission preserves appropriate sensitivity to facts or reasons; and, second, that it does so via its preserving appropriate sensitivity to the outputs of a generative way of knowing that constitutes an appropriate sensitivity to those facts or reasons. Its doing so requires virtuosity, or the proper exercise of well-formed sensory and intellectual capacities, at a number of points in the chain linking the subject with facts or reasons. First, it requires generative virtuosity on behalf of the ultimate source of the transmitted sensitivity: the person who first acquired knowledge or awareness of the target facts or reasons. Second, it requires the original source to exercise transmissive virtuosity in preserving their initially acquired sensitivity to facts or reasons in memory. Third, it requires what we can think of as their *productive* virtuosity, in making the sensitivity that they have acquired potentially available to others, typically by telling others the things that they know. Fourth, it requires what we can think as the *consumptive* virtuosity of the people to whom the ultimate source tells things. Since there may be a number of intervening producers and consumers between the ultimate source and any given consumer, a fifth requirement is that each such transaction virtuously preserves the required sensitivity. In that case, each consumer is made appropriately sensitive by their participation in the chain of

producers and consumers leading back to the ultimate generative source. And in that way, their participation in the chain can put them in a position to know that is broadly equivalent to that of the ultimate source.

Often, the links in a transmissive chain of this sort will each acquire, and so transmit, knowledge to their consumers. However, we have left open whether their doing so is a requirement on any given consumer being put into a position to acquire knowledge by their participation in such a chain. Although it would be a straightforward exercise to build into the account such a requirement on the proper operation of transmissive chains, there are grounds for hesitancy.

The first ground was mentioned in the previous chapter. There might be cases in which a subject is in fact appropriately sensitive to a specific range of facts or reasons, but in which they are precluded from exploiting that sensitivity in order to acquire knowledge. The case of this sort that we considered in the previous chapter was that of an intuitionist mathematician who would be in a position to know a theorem except for their renunciation of non-constructive methods of proof. Despite the fact that such a subject fails to know, we might want to allow that their underlying sensitivity to facts or reasons makes it possible for them to produce testimony that is capable of putting other subjects in a position to know. (See Graham 2000; Lackey 1999; Thompson 1970.)

The second ground for hesitancy more obviously affects memory rather than testimony. There might be cases in which a subject acquired a perceptual sensitivity to a range of facts without exploiting that sensitivity in order to acquire knowledge. Perhaps, for example, the subject saw a cufflink in a drawer, but didn't notice it, and so didn't come to know that it was there. It might be thought possible in such a case for the subject to retain in memory their earlier perceptual sensitivity. And it might be thought possible for their doing so to put them in a position to come to know later about the location of the cufflink—for example, on the basis of attending to their recollection of the way that the contents of the drawer looked. (Dretske 1969: 18; Martin 1992.)

We might reasonably want to leave it open whether cases of either of those two sorts are possible. In that case, we should leave open whether any link in a transmissive chain is required to know in order to be in position later, or to put others in a position, to know those facts.

It is possible, then, to generalise a development of Nawar's appeal to virtue by requiring that in order for someone to know, they must possess and exercise appropriate virtues, and every link in any transmissive chain on which their sensitivity to the facts depends must also possess and exercise appropriate virtues. Indeed, if we are willing to countenance virtues whose possession and proper exercise is dependent upon external circumstances, including the possession and proper exercise of virtues by others, then we can require that in order for someone to know, they must possess and properly exercise such a dependent virtue. We can thus develop and extend Nawar's virtue intuition in order to allow for the possibility of transmissive ways of knowing.

11. Virtue, and the proper exercise of virtue, figures in underwriting the sorts of appropriate sensitivity to facts or reasons on which individual subjects' knowing

depends. Similarly, the absence of virtue, or the improper exercise of virtue, can figure in various ways in helping to explain the absence of appropriate sensitivity and, so, individual subjects' ignorance.

With respect to subjects' views that depend only on the operation of generative ways of knowing, the failure of those views to constitute knowledge will trace to the absence, or improper operation, of virtues associated with those ways of knowing. For example, a subject might fail to know by perception that there is a red-throated loon before them for any of the following reasons: it might be too dark for them to see; they might be looking in the wrong direction or otherwise insufficiently attentive; they might lack a sense-perceptual sensitivity to the colour red, and so be unable to see the colour of the thing before them; they might lack a concept of red-throated loons; they might possess such a concept, but lack the capacity to recognise red-throated loons by sight, or appropriately to tell them apart from similar birds in these circumstances; they might possess misleading information about the looks of local birds; they might fail to take sufficient care in discerning relevant ways that the thing before them looks and thus make too hasty a classification for the outcome to amount to knowledge; and so forth.

It is important to observe that if we focus just on autonomous virtues and their proper exercise and, so, ignore virtues whose possession and proper exercise can be dependent on external circumstances, then we are liable to miss some of the ways in which knowledge can fail to be generated. For example, someone might have a capacity to tell red-throated loons apart from other birds in some circumstances and not in others. In that case, answering the question whether they know might depend not only on whether they possessed and properly exercised an autonomous virtue, but also on whether their virtuosity was suitable to their present circumstances.

With respect to subjects' views that depend on the operation of transmissive ways of knowing, the failure of those views to constitute knowledge will trace to the absence, or improper operation, either of virtues associated with the generative ways of knowing that are ultimately required for the existence of a transmissive chain, or with any of the productive or consumptive links in the transmissive chain. Thus, for example, a subject might fail to know on the basis of testimony that there was a red-throated loon before them for any of the following reasons: the ultimate source of the information might not have appropriately operated their generative way of knowing, as per the example given just above; they might have known, or been in a position to know, but something else might have gone wrong with respect to the transmissive chain. For example, the ultimate source might have failed properly to operate their productive power, either by seeking to persuade rather than teach, or by reporting more than they knew, or in some other way; they might have produced appropriate testimony, based appropriately on what they knew, but the consumer of that testimony might not have been appropriately sensitive to the testimony. For example, the consumer might not have heard, or might have heard but been inattentive; the consumer might have heard and attended, but lacked the capacity to understand what they were told; they might have understood what they were told, but refused to accept it; or, they might have accepted it, but too gullibly, without appropriately exercising their critical faculties; and so forth.

As with the case of generative ways of knowing, it is important to observe that if we focus just on autonomous virtues and their proper exercise, and ignore virtues that are dependent on external vicissitudes—including, in this case, other people—then we are liable to miss some of the ways in which transmission can fail. For example, there might be pairs of transmissive chains with respect to which each link in the chain possesses and properly exercises appropriate autonomous virtues, but such that the last link of only one of the chains is put in a position to know. That might happen, for example, if the penultimate links of both chains spoke only French whilst the ultimate link of only one of the chains could understand French. Similarly, it might happen if the ultimate links of both chains were generally virtuously sensitive to signs of insincerity in general, but the ultimate link of only one of the chains was appropriately sensitive to the distinctive signs of insincerity liable to be manifested by the penultimate links in each of the chains.

12. The purpose of this extended discussion of Plato's brief reflections on testimony in the *Theaetetus*, and of Burnyeat's attempt to make sense of those reflections by appeal to epistemic categories, has been to shed light on the operation of transmissive ways of knowing. More specifically, the purpose has been to consider whether Plato's discussion can illuminate the idea that there might be domains of fact with respect to which transmissive ways of knowing are unavailable. More specifically still, the purpose has been to consider whether mathematical facts might constitute such a domain.

If we had been able to find in Plato a presentation and defence of a basic epistemic category of mathematical ways of knowing—or, more generally, a basic epistemic category specified by appeal to proof or intellection—then that presentation and defence might have helped us to uncover a reasonable source of resistance to the transmissiveness of mathematical knowledge. However, the global appeal to epistemic categories that Burnyeat discerned in Plato's work is implausibly restrictive. Furthermore, that appeal is insufficiently explanatory of the restrictions that it seeks to impose. In particular, while there is some plausibility to the idea that perception and intellection might be disjoint *generative* ways of knowing, the more general restriction imposed by the appeal to epistemic categories is dependent upon the preclusion of *transmissive* ways of knowing the facts to which perception and intellection afford original access. However, although the global appeal to epistemic categories is a dead end, reflection on its failings has put us in a position to consider more articulate routes to the type of restriction on mathematical testimony that the appeal to epistemic categories sought to impose. For we are now in a position to consider whether there might be specific reasons why mathematical knowledge is not transmissible.

Given what we have said about ways in which the transmission of knowledge might be blocked in specific cases, we can foresee two broad ways in which attempts might be made to argue that the transmission of mathematical knowledge is in general impossible. The first way in which such an attempt might be made would be via the fact that the transmission of knowledge depends upon the possession of a productive capacity suitable to make available to others an

appropriate sensitivity to facts or reasons. Taking the first route, an attempt might be made to argue that we could not possess a *productive* capacity of the required sort—perhaps, for example, because mathematical knowledge is taken to require access to a type of reason or guarantee of truth that no contingent manifestation in speech could make available to others. The second way in which an attempt to argue that the transmission of mathematical knowledge is not possible would be via the fact that the transmission of knowledge depends upon the possession of a consumptive capacity suitable to acquire appropriate sensitivity to facts or reasons on the basis of others’ productions. Taking the second route, an attempt might be made to argue that we could not possess or properly exercise a *consumptive* capacity of the required sort. Such an attempt might be motivated by the idea that the possession or proper exercise of a consumptive capacity depends on the obtaining of disparities in intellectual power or expertise that are not to be found in the mathematical domain. That is, it might be held, first, that the apt consumption of mathematical testimony would depend on there being producers who are in a better position to know mathematical facts and, second, that mathematical competence and opportunity are evenly spread. In the next chapter, we will pursue the prospects of such attempts in more detail, with particular attention to those developed by Locke and Kant.

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