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Logical Content and Its Discontents

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Abstract

The doctrine that the content of the conclusion of a deductively valid argument is included in the content of its premises, taken jointly, is a familiar one. It has important consequences for the question of what value valid arguments possess, since it indicates the poverty of three traditional answers: that arguments may and should be used as instruments of persuasion, that they may and should be used as instruments of justification; and that they may and should be used to advance knowledge. The truth is, however, that in each of these cases the argument has only a managerial role and, if there is any work done, it is the premises that do it. It will be maintained that this point has little force against the critical rationalist answer, which I shall defend, that the principal purpose of deductive reasoning from an assemblage of premises is the exploration of their content, facilitating their criticism and rejection.

That said, the main aim of the present paper is not to promote critical rationalism but to consider some published objections to the doctrine that a statement asserts every statement that is validly deducible from it. The alleged counterexamples to be considered fall roughly into two groups: statements that emerge with time from a rich mathematical or empirical theory, but were originally unformulated and are deducible from the theory only in a non-trivial way (Frederick 2011, 2014; Williamson 2012); and statements, notably disjunctions, that are easily formulated and are deducible from a theory in a trivial way (Schurz & Weingartner 1987; Gemes 1994; Mura 2008; Yablo 2014). Each of these counterexamples will be evaluated and dismissed.

With minor variations, sections 0–4 comprise the original version of a paper presented, with several cuts, at the colloquium *Does Valid Reasoning Matter?* at the Congress Center ACADEMIA, Stará Lesná, The High Tatras, Slovakia, from September 13 to September 16, 2018. Shortly after the colloquium, Danny Frederick, the respondent to the paper, made available on line a revised version (Frederick 2018) of his comments. Section 5, newly added here, identifies and corrects serious failings in these comments.

I grateful to Diego Rosende and to David Marans for helpful observations on an earlier draft of section 5. In quotations from the writings of others I have quietly brought their notation into line with my own.

0 Introduction

In this paper I shall try to defend, though in an indirect way, the critical rationalist doctrine that the most important intellectual use of deductive arguments (which must be distinguished from deductive inferences and reasoning) and deductive inferences and reasoning (which must be distinguished from deductive arguments) is to enable us to investigate what our hypotheses and assumptions assert, that is, to elucidate aspects of their content, and thereby, if we are lucky, to bring to light in what ways those hypotheses and assumptions are defective. As part of this defence I shall reject three common claims about the proper purposes of deductive inferences: that they may and should be used as instruments of persuasion; that they may and should be used as instruments of justification or validation; and that they may and should be used to advance knowledge. These points were made succinctly in Miller (2005) with regard to the seemingly unstoppable educational fashion known as *critical thinking*, and they were made at greater length, and more generally, in Miller (2006), Chapter 3 (see also Miller 1995). I pointed out that in each case it cannot be the argument (whether valid or invalid) that does the work that is to be done (if indeed any real work is done) — the persuading, the justifying, the contributing to knowledge — but the premises of the argument. If a valid argument plays any part in persuading an agent that its conclusion is true, it is only by revealing that the agent is already persuaded that the premises are true. The argument itself has no persuasive power. If a valid argument plays any part in justifying its conclusion, it is only because the premises are already justified. The argument itself has no authenticating power. If a valid argument plays any part in contributing to our knowledge of the world, it is only by revealing that the premises implicitly contain that contribution. Why validity is so valuable is that a valid argument may be used critically, to reveal that a hypothesis or theory that has been endorsed conflicts with some other item of information (or in some cases, with itself). A valid argument that is used critically, and is successful, does no more than any other argument in uncovering something that is already present, but if that something is an inconsistency, we can use it to reject at least one of the premises from which it was drawn. Our knowledge is not advanced when we learn more; it is abridged.

I have nothing more to say about persuasion and justification, except to ask why either is regarded as valuable. Even if arguments could be used to persuade, or to justify, we should ask why they should be. A rational, open-minded person does not need to be persuaded; nor does he need justification for what he thinks. In sum, it is only in connection with the third rejected claim above, the one about the advancement of knowledge, that there is, I think, much more to be said. But something does have to be said. In recent years there have been put forward several related criticisms of the view, which this paper endorses, that the content of a hypothesis, that is, what it asserts or says, should be identified with the class of statements that are deducible from it. It will be necessary to explain why each of these criticisms is mistaken.

There are two short preliminary sections to be endured before I turn to my main topic, the correctness or incorrectness of the identification of what a statement says with the class of statements that can be deduced from it; that is, everything that it logically implies. In § 1, I shall bring to your attention, but not dwell on further, a noteworthy historical prevision of the central critical rationalist teaching about deductive reasoning, that its primary purpose is inquisitive and inquisitorial. In § 2, I shall report and discuss some definitions, familiar to logicians, of logical content. They ought not to be contentious, unless it is supposed that I am planning to settle a philosophical question by mere stipulation. I have no intention of doing this. The issue that is waiting to be discussed is whether the *content* of a statement or hypothesis h (which may, for the sake of clarity, sometimes be called its *assertoric content*) always includes the *logical content* of h, namely $\mathbf{Cn}(\mathsf{h})$.

In § 3, I shall analyse two fairly recent attempts to bolster the not uncommon view that the existence of mathematical discoveries, and of surprising deductive connections between previously unconnected ideas, shows that deductive reasoning, if undertaken with enthusiasm and perseverance, has a miraculous ability to advance our knowledge. In § 4, I shall turn to a rather different type of objection to the thesis that everything that can be deduced from a statement is asserted by it; according to this line of argument, from almost any statement we may deduce, in a transparent way (in particular, by a single use of the rule $\vee$I of disjunction introduction) a statement (indeed, any number of statements) that cannot realistically be regarded as part of what the original statement says, its assertoric content. My main response to this objection is that it has little force when disjunctions are rewritten as material conditionals. My response to most of the ways in which the objection has been developed into theories of partial truth and verisimilitude is not dissimilar, but I shall say little on that subject on this occasion. For the most part these theories ignore, or brush aside, the problem of language dependence that has been around, and not properly answered, for over 40 years. I have discussed this problem at length in Chapter 11 of my (2006), and shall revert to it, but quite briefly, in § 4.2 below.

1 A 19th century anticipation

Only recently have I come to appreciate to what extent earlier writers have advocated the critical rationalist thesis that the primary role of deductive reasoning is not to extend our knowledge but to criticize and control it. This thesis is really not at all original with Karl Popper, though it is a decisive component (see § VII of his 1957, and the fifteenth thesis of his 1962) of the critical rationalist theory of knowledge of which he was the architect. What is original to Popper is not the thesis that *conjectures* and *refutations* must be sharply separated, but the thesis that they comprise the totality of our intellectual activity; that is, that all reasoning (but not all intelligent thinking) is deductive reasoning.

Without in any way installing the author as an authority, I should here like to quote an extended passage from § II of the Introduction to *Logic, Inductive and Deductive* by William Minto (1893). Minto, whose name is not too well known, was the successor in 1880 to Alexander Bain (the founder of *Mind*) in the Regius Chair of Logic and English Literature at the University of Aberdeen.

> Why describe logic as a system of defence against error? Why say that its main

> end and aim is the organization of reason against confusion and falsehood? Why not rather say, as is now usual, that its end is the attainment of truth? Does this not come to the same thing?
>
> Substantially, the meaning is the same, but the latter expression is more misleading. To speak of logic as a body of rules for the investigation of truth has misled people into supposing that logic claims to be an art of Discovery, that it claims to lay down rules by simply observing which investigators may infallibly arrive at new truths. Now, this does not hold even of the Logic of Induction, still less of the older Logic, the precise relation of which to truth will become apparent as we proceed. It is only by keeping men from going astray and by disabusing them when they think they have reached their destination that logic helps men on the road to truth. Truth often lies hid in the centre of a maze, and logical rules only help the searcher onwards by giving him warning when he is on the wrong track and must try another. It is the searcher's own impulse that carries him forward: Logic does not so much beckon him on to the right path as beckon him back from the wrong. In laying down the conditions of correct interpretation, of valid argument, of trustworthy evidence, of satisfactory explanation, Logic shows the inquirer how to test and purge his conclusions, not how to reach them.

This quotation, along with similar but much shorter quotations from the writings of Cicero and Russell, may be found in § 8 of my (2015). The quotations from Cicero and Minto, and a not dissimilar thought of Condillac, may be found on the pages devoted to these writers in David Marans's entertaining *Logic Gallery* (Marans 2018).

2 Logical content

Throughout this paper, it will be assumed that, unless otherwise stated, the logic of statements includes classical elementary logic in such a way that the familiar soundness and completeness theorems hold. It will therefore be possible systematically to blur the distinction between the relation $\mathsf{k} \vdash \mathsf{h}$ (h *is deducible from* k) and the relation $\mathsf{k} \vDash \mathsf{h}$ (h *is a logical consequence of* k). Nothing of importance will hang on this, but it is useful to be able to supplement the vocabulary of deducibility with the often more flexible vocabulary of logical consequence. In particular, we shall be able to use without discomfort the term *consequence class of* h, and Tarski's notation $\mathbf{Cn}(\mathsf{h})$, to refer to the class of statements that can be deduced from h.[1]

It is true that some writers (including Popper 1976a, § 7) use the word *implication* exactly as I wish to use the word *consequence*, but I prefer to maintain a clean distinction between the relation of implication, for which some writers use the word *entailment* and its second term (its relatum). But using *implication* as a synonym of *logical consequence* is greatly to be preferred to using it as a synonym for *material conditional*.

[1]Logical truths, which are consequences of any statement whatever are sometimes excluded from $\mathbf{Cn}(\mathsf{h})$, but this is a technical artifice that, contrary to what Frederick (2014) seems to suggest, is without deep significance. It ensures that the smallest logical content of all is the empty set, rather than the class of logical truths, and also that no statement h shares any logical consequences with its contradictory $\neg\mathsf{h}$.

It is standard to define the *logical content* $\mathfrak{I}(\mathsf{h})$ of a statement or hypothesis h, as its consequence class $\mathbf{Cn}(\mathsf{h})$, and this is the definition that will be adopted here. $\mathfrak{I}(\mathsf{h})$ is occasionally called the *informative content* of h, but note that this term has a different connotation in Popper *op. cit.*, which will be mentioned in § 3.1 below. It is to be observed that set-theoretical relations among logical contents reflect logical relations among statements only in an upside-down and somewhat distorted (but not inaccurate) manner. The logical content $\mathfrak{I}(\mathsf{h} \vee \mathsf{k})$ of the *disjunction* $\mathsf{h} \vee \mathsf{k}$ of two statements h and k is identical with the *intersection* $\mathfrak{I}(\mathsf{h}) \cap \mathfrak{I}(\mathsf{k})$ of their individual logical contents, but $\mathfrak{I}(\mathsf{h} \wedge \mathsf{k})$, the logical content of the *conjunction* $\mathsf{h} \wedge \mathsf{k}$ of h and k, may not be the same as the *union* $\mathfrak{I}(\mathsf{h}) \cup \mathfrak{I}(\mathsf{k})$ of their logical contents; unless one of h and k is deducible from the other (so that $\mathsf{h} \wedge \mathsf{k}$ is equivalent to either h or k), the logical content $\mathfrak{I}(\mathsf{h} \wedge \mathsf{k})$ includes, but is not included in, $\mathfrak{I}(\mathsf{h}) \cup \mathfrak{I}(\mathsf{k})$. This mismatch is avoided by a popular alternative definition of logical content $\mathfrak{I}$, a definition that is expressed in terms of sets of models (or structures), rather than sets of statements. On this semantic (or model-theoretic) definition, which was proposed by Carnap & Bar-Hillel (1952) and Bar-Hillel & Carnap (1954), the logical content $\mathfrak{I}(\mathsf{h})$ of the statement h is identified with the class $\mathbf{M}(\mathsf{h})$ of models (thinned down, by the relation of elementary equivalence, to set-hood) in which h is false. The two definitions of the function $\mathfrak{I}$ agree at a crucial point: the logical contents of the statements h and k are set-theoretically comparable whenever h and k are logically comparable, that is to say, $\mathfrak{I}(\mathsf{k}) \subseteq \mathfrak{I}(\mathsf{h})$ if and only if k is deducible from h. But they do not agree in every particular. Set-theoretic relations among the various $\mathbf{M}(\mathsf{h})$ reflect much more cleanly the logical relations among statements than do set-theoretic relations among the $\mathbf{Cn}(\mathsf{h})$.[2]

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[2]The lack of precise matching is most obvious when we ask how to define $\mathfrak{I}(\mathsf{h} \mid \mathsf{k})$, the *relative logical content* of h given k, sometimes called the *excess content* or the *surplus content* of h over k (it being supposed for simplicity, but only for simplicity, that k is deducible from h). This problem is sometimes called the problem of defining *logical subtraction*, but the set-theoretical *difference* $\mathfrak{I}(\mathsf{h}) \setminus \mathfrak{I}(\mathsf{k})$ is not a good candidate for the difference between the logical contents of h and k; for only if every logical truth $\top$ is excluded from logical contents (with the result that $\mathbf{Cn}(\mathsf{h}) \setminus \mathbf{Cn}(\mathsf{h}) = \varnothing = \mathbf{Cn}(\top)$) is the difference between two logical contents (in this case, two identical logical contents) itself a logical content. The answer often given (for example, in Carnap & Bar-Hillel *op. cit.* and in Popper & Miller 1983) is that the relative logical content $\mathfrak{I}(\mathsf{h} \mid \mathsf{k})$ is exactly the absolute logical content $\mathfrak{I}(\mathsf{k} \to \mathsf{h})$ of the material conditional $\mathsf{k} \to \mathsf{h}$. Yablo (2014), § 8.3, calls it the *horseshoe theory* in remembrance of a veteran symbol $\supset$ for the material conditional. Although his footnote 10 acknowledges its close resemblance to the standard (and surely uncontroversial) definition of set-theoretical difference, he says that the identification of $\mathfrak{I}(\mathsf{h} \mid \mathsf{k})$ with $\mathfrak{I}(\mathsf{k} \to \mathsf{h})$ is 'pulled out of a hat'. But even if it is not the best answer, it is an answer that deserves to be taken seriously, as may be seen if we define $\mathfrak{I}(\mathsf{h})$ as $\mathbf{M}(\mathsf{h})$. Since a model in which $\mathsf{k} \to \mathsf{h}$ is false is, as a truth table shows, a model in which h is false and k is true, $\mathbf{M}(\mathsf{h}) \setminus \mathbf{M}(\mathsf{k})$, the logical content of h shorn of the logical content of k, is just $\mathbf{M}(\mathsf{k} \to \mathsf{h})$. When Yablo *op. cit.*, § 6.1, says that those who identify $\mathsf{k} \to \mathsf{h}$ as the surplus content of $\mathsf{h} \wedge \mathsf{k}$ over k 'seem to be conceiving logical remainders on the model of numerical remainders' (I have considerably simplified his text) and that, since 'the [logical and numerical] cases are not really analogous', the identification should be rejected, it would be better, as well as simpler, to say that those (including Popper & Miller *op. cit.*) who make this identification are conceiving logical remainders under the syntactic definition on the model of logical remainders under the semantic definition, and that the two senses are quite analogous (though like all analogues, not identical). The semantic definition sets $\mathfrak{I}(\mathsf{h} \mid \mathsf{k})$ equal to $\mathbf{M}(\mathsf{k} \to \mathsf{h})$, and the syntactic definition sets $\mathfrak{I}(\mathsf{h} \mid \mathsf{k})$ equal to

In contrast to those philosophers of science who laud the supposed benefits of the *semantic view of theories* (scientific theories are sets of models) over what is disparagingly referred to as the *syntactic view* (scientific theories are sets of statements), logicians tend not to discriminate fussily between the two definitions of logical content $\mathfrak{I}(h)$, the definition as $Cn(h)$ and the definition as $M(h)$. Even though the senses are distinct, anything said about logical contents in one sense can be easily translated into statements about logical contents in the other sense. Both reformulate, in an alternative vocabulary based on elementary set theory, statements about logical consequence or deducibility. Since the semantic definition of $\mathfrak{I}(h)$ is technically the more docile, it may be thought that it should be preferred. In the context of the present paper, however, it seems to be quite unhelpful. Some imagination is required to entertain the thought that what a statement asserts is the set of models in which it is false. Indeed, a statement h never belongs to $M(h)$, its own content, whereas, of course, h always belongs to $Cn(h)$. We shall accordingly retain the definition of logical content $\mathfrak{I}(h) = Cn(h)$ proposed above.

3 Novelty

3.0 Summary

Let us now look at two fairly recent attempts to controvert the thesis that is being defended here, the thesis that a hypothesis or theory asserts every one of its consequences, that its assertoric content extends as far as its logical content does. Frederick (2011) offers a counterexample, distilled from § 7 of Popper's *Unended Quest* (1976a), of a theory in the empirical sciences that has remote logical consequences that were not known, and perhaps could not have been known, to the author of the theory. In a slightly different, more conventional, line of attack, Williamson (2012) reminds us that mathematics is full of surprising and unexpected theorems that were proved only many years after the formulation of the axioms and assumptions from which they were eventually deduced. It is maintained by each of these authors that the examples displayed show that the assertoric content of a theory, what it says, may well be less comprehensive than its logical content.

To the interventions of both Frederick and Williamson the critical rationalist (or anyway, this critical rationalist) replies that, although the newly discovered consequences may not have been known to earlier scientists and mathematicians in any subjective or even intersubjective sense — they were neither personal knowledge nor common knowledge —, they were part of the *objective scientific knowledge* of those earlier times; that is, part of knowledge in the only sense that is of much significance to critical rationalism. The existence of the psychological states and processes that feature so largely in traditional, and contemporary, subjectivist epistemology is not denied by critical rationalism, and it

$Cn(k \to h)$. In each case, $\mathfrak{I}(h \mid k)$ is the smallest logical content $\mathfrak{I}(j)$ such that $\mathfrak{I}(h) \subseteq \mathfrak{I}(j) \cup \mathfrak{I}(k) \subseteq \mathfrak{I}(j \wedge k)$. It should be noted, however, that if k, which is here a statement, is replaced by an unaxiomatizable theory K, then there may exist no theory that behaves in the way we expect the material conditional $K \to h$ to behave, so that it is not always possible to identify $\mathfrak{I}(h \mid k)$ with $Cn(K \to h)$. On this difficulty, which is not easily resolved, the reader is referred to the discussion in Miller (2014), § 2.5.

is appreciated that without subjective knowledge there would be no objective knowledge. (Without shoemaking subjects there would be no shoes, but shoes are not subjective entities.) But the world of objective knowledge consists of linguistically formulated hypotheses, theories, problems (and much else besides, but that is enough for today); that is, of items that have discarded as much as possible of their psychological patrimony. This answer, that the deduction of new consequences does not extend our objective knowledge, is pretty much the answer that has been given by many authors, at least since Mill (1843). Does either Frederick or Williamson say anything that overturns it?

In both authors' presentations, the disputed thesis that statements assert all their logical consequences, a thesis that I judge to be true, is committed to trial in the company of another thesis, a different one for each author, that is judged by its sponsor to be false; and although neither associate thesis seems to be a logical consequence of the thesis in dispute, each is adduced as providing an argument against that thesis. In Frederick's case, I agree with him that the associate thesis is false, but I deny that this has any repercussions for the thesis that assertoric content is the same as logical content. Contrary to Williamson, on the other hand, I think that the associate thesis that he interposes is true and that, if it has any repercussions for the disputed thesis that assertoric content is the same as logical content, it is to indicate that it is true. In short, once we disentangle the thesis in dispute from the rogue theses with which it is associated, we shall see that neither author really makes a compelling objection to the objectivist view that a theory asserts everything that in principle may be deduced from it. Both presentations, it seems to me, have an unwelcome justificationist flavour that needs to be exposed and expunged. Let us consider each in turn, first Frederick, and second Williamson.

3.1 Deductive validity and deductive reasoning

Frederick (2011) associates what he calls the 'hoary claim about deductive *validity*' (in his words: 'every deductively valid argument is a petitio principii') with

> an equally hoary theory of deductive *reasoning*, which is found more or less explicitly in both empiricist and rationalist philosophers of the modern period and which is still popular today. The hoary theory says that a deductive reasoner arrives at a conclusion of an argument by analysing the content of its premises.

Frederick tells us that 'the hoary theory of deductive reasoning may seem to entail the hoary claim about deductive validity', but he soon questions this imagined entailment and writes that, if we were to be granted a distinction between

> the objective content of a theory and the part of that content that is available to a particular reasoner in a particular situation, we could consistently retain the hoary theory of deductive reasoning while rejecting the hoary claim about deductive validity. For we could maintain that . . . any deductively valid conclusion from a set of premises to which we can reason deductively must be contained in that part of the content of the premises that is already available to us . . . [but there are other parts of the objective content from which] we are currently (and, in some cases, perhaps

> forever) unable to reason deductively and which would therefore tell us something new even though they are part of the objective content of the premises.

But, going into reverse once more, he then concludes that 'the falsity of the hoary claim about deductive validity casts doubt on the hoary theory of deductive reasoning'. This hardly constitutes a forceful objection to 'the hoary claim about deductive validity'.

Few people who have tried to prove a serious mathematical theorem, and have reflected on what they were doing, can be expected to accredit the theory of deductive reasoning that Frederick says is so hoary. Mathematical work of any depth is far from a smooth process of starting with axioms, or other assumptions, and gaily deducing consequence after consequence. It is much more a business of incessant trial and error in which both the final theorem, if there is one, and its negation, have appeared as trials, perhaps many times (Miller 2006, Chapter 3, § 3). This is not to say that there are not algorismic moments in the midst of the fits and starts. It would be perverse, for example, to resort to trial and error in order to solve a quadratic equation that turns up during an attempt to solve a deeper problem, or to evaluate a familiar integral. But unexpected theorems of interest are indeed uncommon. What is much more common, at the end of a long chain of calculation, is the disappointing conclusion $0 = 0$. Those who forcefully reject the hoary claim about deductive reasoning, as Frederick does, but retain the hoary claim about validity, may wonder why Frederick gives the former claim such prominence. Its falsity tells us nothing about the truth of the latter. Only for someone who thinks that it furnishes a needed argument in favour of that claim, may its falsity appear to be damaging.

The above distinction between the *objective content* of a theory and its *available* (or *accessible*) *content* is attributed by Frederick to Popper's remark that, since the logical content of a theory is usually infinite, '*we never know what we are talking about*' (*op. cit.*, § 7). Popper continued:

> For when we propose a theory, or try to understand a theory, we also propose, or try to understand its logical implications [consequences]; that is, all those statements which follow from it. But this, as we have just seen, is a hopeless task: there is *an infinity of unforeseeable nontrivial statements belonging to the informative content of any theory.* ... We can therefore never know or understand all the implications [consequences] of any theory, or its full significance

and he suggested that if N is Newton's theory of gravitation and E is Einstein's, then '*non-E* belongs to the *logical content* of N, ... a fact which, obviously, could ... not have been known to Newton, or anybody else, before E was discovered'. But this is not quite true. If the theories N and E are spelt out in some common mathematical language (say the language of ZF set theory) in sufficient detail for it to be possible to assess the validity of the argument $N \therefore \neg E$, then its validity could have been discovered, though perhaps not appreciated, before Einstein.

Popper *loc. cit.* provided a simple proof of his assertion that 'there is *an infinity of unforeseeable nontrivial statements belonging to the informative content of any theory*', but he did not specify the conditions under which the proof is valid. One corollary that he did not consider explicitly is that since what holds for a complex and wide-ranging theory must hold also for every one of its consequences (except logical truths), we may not be able to understand these consequences any better than we understand the theory itself (Miller 1998, § 2.6, ¶19). It is not just Newton's theory of gravitation that we can never fully understand, but singular statements such as 'The Moon is smaller than the Earth' (though this particular statement is not a consequence of Newton's theory, which is here assumed to be consistent). Like the peace of God, almost any theory you care to mention 'passes all understanding' (*Philippians* 4:7).

In conclusion, it may be suggested that what Frederick's thesis boils down to is that the *available content* of a statement (which is in a constant state of flux) is seldom identical with its logical content (which is more or less fixed, at least if the object language is fixed). But this distinction is as trite as the distinction between a country and the part of that country that is accessible to a particular explorer at a particular time. I hope that there is more to it than that.

3.2 The debatability of logic

'Logic is just not a controversy-free zone', observes Williamson (2012). 'As in the rest of science, no principle is above challenge. . . . Principles of logic can themselves be debated, and often are, just like principles of any other science.' He cites intuitionistic logic, quantum logic, and paraconsistent logic as domains in which principles dear to classical logic have been contested and rejected, and of course there are numerous other theatres of protest and dissent, notably in modal logic. But if we do not resort to logical reasoning or logical inference — in short, if we do not use principles of logic — we cannot argue rationally about principles of logic, but only squabble about them. From this truism Williamson infers that '[t]he conception of logic as a neutral umpire of debate . . . fails to withstand scrutiny. . . . Whichever side is right, logical theories are players in these debates, not neutral umpires.' We must, he says, reject the idea that

> logic has no substantive content, for otherwise the correctness of that content could itself be debated, which would impugn the neutrality of logic. One way to develop this idea is by saying that logic supplies no information of its own, because the point of information is to rule out possibilities, whereas logic only rules out inconsistencies, which are not genuine possibilities. On this view, logic in itself is totally uninformative, although it may help us extract and handle non-logical information from other sources.

Williamson thinks that this doctrine of the uninformativeness of logic is mistaken, and goes on:

> The power of logic becomes increasingly clear when we chain together such elementary steps into longer and longer chains of reasoning, and the idea of logic as uninformative becomes correspondingly less and less plausible. Mathematics provides the most striking examples, since all its theorems are ultimately derived from a few simple axioms by chains of logical reasoning, some of them hundreds of pages

> long, even though mathematicians usually don't bother to analyze their proofs into the most elementary steps.

He cites as an example of an informative mathematical discovery Wiles's proof of Fermat's Last Theorem, the conjecture that there exists no integer $m > 2$ for which the equation $x^m + y^m = z^m$ has a solution in the positive integers. He says that 'what matters is that together the accepted axioms suffice . . . [to] imply Fermat's Last Theorem' and asks rhetorically 'If logic is uninformative, shouldn't it be uninformative to be told that the accepted axioms of mathematics imply Fermat's Last Theorem?'. But that last statement is not, as its stands, 'a truth of pure logic', since the term 'the accepted axioms' is logically indeterminate. When a sufficient set of axioms is fully spelt out, we do obtain 'a truth of pure logic', but not a convincing example of a genuine discovery. Fermat himself knew that the conjecture is true (though he did not manage to exhibit a justification or proof), and before Wiles got to work on it, it had been extensively tested and never refuted. (That no counterexample has been found does not of course imply that it is true, and it may even be undecidable from any consistent set of axioms so far proposed.) But this is a digression, and it cannot be pursued further.

Williamson takes the thesis that he initially considers, that logic is 'an umpire, a neutral arbitrator between opposing theories, imposing some basic rules on all sides in a dispute', to be false, and the thesis with which he ends, that 'logic is informative', to be true. To the extent that he has something new to say about the informativeness of logic (that is, something beyond the appeal to mathematical surprises, which is decidedly old hat), it is that it is a consequence of the true thesis, with which I fully agree, that *we can debate rationally the correctness of logical principles, and in order to do so we must make use of logical principles*. Where I disagree is with Williamson's assertion that *rational debate about logic is impossible if logic is neutral*. This seems to me to be a characteristic justificationist misapprehension of rationality.

According to critical rationalism, as I said above (and in many other places), the principal purpose of logic is to criticize and to falsify. A rational debate about a logical system typically consists of an attempt to show that the system contains a putative law or rule that is, in some sense, unsatisfactory, not that the system is valid. There are no logical means by which logical axioms and logical rules can be justified, and the history of philosophy is littered with failed attempts to construct such justifications. But errors can be detected. Russell's paradox is a famous example.[3] More generally, the rules embodied in a logical system can be directed against the system itself, and provided that it restricts its ambition remorselessly to self-denial and self-sacrifice, the system preserves its neutrality. 'No one should be a judge in his own case' (*Nemo iudex in causa sua*)

[3] Two less famous examples that are not trivial are the proof by Kleene & Rosser (1935) of the inconsistency of the system of postulates of Church (1932), and the criticism by Montague & Henkin (1956), notes 5f. and text, of the definition of the term *formal deduction* in Church (1942). There are of course many discussions of logical systems that are not themselves logical but combinatorial, for example proofs that classical sentential calculus can be axiomatized using only five distinct variables. Logic's neutrality is not compromised in such proofs.

is a legal principle that is adamantly enforced in many jurisdictions, but it does not prevent a judge with an interest in a case from voluntarily disqualifying himself, that is, from handing down a judgement that may be seriously detrimental to his interest. It is conceivable that, using classical logic, someone might demonstrate that a well thought-of classical principle, for example the rule of *reductio ad absurdum* or the rule of *modus tollens*, has a counterexample. If such a demonstration were to exist, it would not be circular (Miller 1994, Chapter 4, § 3*e*; 2008, § 4). It would use the rule of *reductio ad absurdum* to reduce the rule of *reductio ad absurdum* to absurdity, nothing more. I do not expect any such demonstration ever to emerge, but I do not pretend to prove that it is impossible. Pierre Boutroux may have been right to say that 'in order to combat logic it is necessary to use logic' (Marans *op. cit.*), but his conclusion that '[l]ogic is invincible' does not follow (which is not to say that it is not true).

In this discussion, it is taken for granted that the system or systems being referred to by the word 'logic' contain a rule that is something like the rule of *reductio ad absurdum*; that is, a rule that allows the rejection of one or more assumptions. Despite the protestations of Bunge (1987), § 6, most, perhaps all, systems of paraconsistent logic fulfil this requirement. They may deny the universal validity of the law $h \wedge \neg h \vdash k$ of explosion, but they do admit some cases of it. A logical system that cannot, by some means or other, simulate *reductio ad absurdum* would be of no service to critics, and would be discarded for its methodological, rather than its logical, failings.

3.3 Science and technology

As a parting shot, it may be remarked that many who discuss technological, rather than scientific, innovation, take a view directly contrary to that defended here. That the world is full of wonderful, and sometimes less wonderful, technological surprises is rightly recognized, but technology is often belittled as mere applied science, as no more than the working out of the deductive consequences of our best tested scientific theories. It is nothing of the kind. What successful technological advances depend on is the imaginative realization of previously uncontemplated initial conditions, and these genuinely do go beyond the logical content of theoretical science (Miller 1998, § 3; 2006, Chapter 5, § 3; Petroski 2010). Could it be that it was the mistaken idea that in technology deduction can generate novelty that emboldened the mistaken, but differently mistaken, idea that in science novelty can be generated by deduction?

4 Disjunctions

4.0 Another line of criticism

In § 3 of Chapter 10 of (1963) Popper proposed to partition the logical content $\mathbf{Cn}(h)$ of a statement or theory h into the set of its true consequences, which he called its *truth content* and the set of its false consequences, which he called its *falsity content*. It is easily seen that truth contents are always logical contents, but only empty falsity contents are logical contents (and then only if we adopt the convention that $\mathbf{Cn}(\top) = \varnothing$). He went on to define the comparative *verisimilitude* of two theories h and k in terms of these subsets of $\mathbf{Cn}(\mathsf{h})$ and $\mathbf{Cn}(\mathsf{k})$. It is well known that this theory of verisimilitude is unsuccessful, though perhaps not as disastrous as it is usually represented to be (see Popper 1976b and also Fine 2018), and in the last 40 years or so there have been many attempts to do better. In one of the early ones, Schurz & Weingartner (1987) proposed to modify Popper's definition by replacing the truth and falsity contents of each statement by proper subsets of these sets, composed of what they called *relevant consequences*. This manoeuvre is noteworthy since it appears to be the first explicit challenge to the appropriateness in general, or the relevance (though not the validity), of the argument from h to $\mathsf{h} \vee \mathsf{k}$. 'For example', they wrote (§ 3.2.1), 'if a physicist derives from his theory a sentence h which claims the future existence of a solar eclipse, he will certainly not count sentences like "h or the sun will explode tomorrow" etc. as further consequences from his theory — since they are irrelevant'. Schurz & Weingartner do not deny that the argument from h to $\mathsf{h} \vee \mathsf{k}$ is valid, or that in most systems of logic, including most systems of relevance logic, it is authorized by the rule $\vee$I of disjunction introduction.

Similar opinions, often more strongly worded and seemingly of greater compass, have been articulated by others. Gemes (1994), § 2, has noted that if we accept the thesis that the assertoric content of a statement includes the whole of its logical content, 'not only do Relativity theory and Newtonian mechanics share common content but also so do Relativity theory and your favorite crackpot theory, say, Dianetics'. A little later he said that 'the crucial point is that many of the needs of philosophers, especially philosophers of science, are better served by using a non-classical notion of consequence. In particular, we need a notion of consequence that does not automatically count $\mathsf{h} \vee \mathsf{k}$ as a consequence of h.' Likewise, Mura (2008), § 2, has said that although[4]

> the basic dogma that the content of a proposition coincides with the set of its logical consequences . . . sounds prima facie very natural from a logical point of view[, it has] some consequences that are, in my view, completely unacceptable (see also Gemes 1994). . . . For example, this view entails that two propositions always share a common content, except when they are logically disjunct, i.e. when their disjunction is a logical truth. In fact, only in that case should two propositions have no factual logical consequence in common. So 'Napoleon was defeated at Waterloo' and 'The population of Chicago amounted in 1990 to 2,783,726 inhabitants' would have a common content. This appears to be completely counterintuitive.

According to Yablo *loc. cit.*, who does not at this point offer any new argument, '*Snow is white* does not in any sense whatsoever share content with *Charlemagne was Holy Roman Emperor*'.

[4]Mura has another argument against the identification of assertoric content and logical content, provoked by the variety of functions that may be used to measure partial deducibility (which he calls partial entailment), and their connection with measures of relative content (which are discussed in footnote 2 above). The argument is too technical to be examined here, but I do not doubt that the difficulty that Mura mentions can be satisfactorily resolved.

Since one of the main purposes and main achievements, of scientific and philosophical investigation is to identify, criticize, and correct faulty intuitive judgements, which are usually just dressed-up prejudices, I am little moved by Mura's diagnosis, or by similar complaints of unintuitiveness that appear throughout Gemes *op. cit.* What matters is whether the judgements stand up to criticism. I shall argue that the judgement that $h \vee k$ is not always a part of the content of both h and k is one intuitive judgement that does not stand up very successfully to criticism.

4.1 Material conditionals

The disjunction $h \vee k$ is logically equivalent, in classical logic (but not in some other systems, such as intuitionistic logic), to the material conditional $\neg h \to k$, and also of course to $\neg k \to h$. Does $\neg k \to h$ belong to the assertoric content of h; that is, does h assert $\neg k \to h$? If we understand the material conditional (as I for many years taught students to understand it) as asserting h *under the condition* $\neg k$ (or h *in the circumstance* $\neg k$), and h as asserting h *under all conditions* (or h *in all circumstances*), the question becomes a special case of the question of whether the rule UI of universal instantiation can add anything to assertoric content; that is, whether there can be an instance of a universal generalization that it does not assert. In the case of the proposal of Schurz & Weingartner *op. cit.*, the answer to this question is negative: in § 3.4 they list $\forall x \mathfrak{F}x \to \mathfrak{F}a$ among what they call the *relevant valid implications*. I do not know if the other authors mentioned are as explicit, but I should be surprised if anyone wishes to exclude the instance $\mathfrak{F}a$ from the assertoric content of the universal generalization $\forall x \mathfrak{F}x$.

It should be noted, however, that in each of the examples quoted from the writings of Mura and of Yablo, at least one of the disjuncts in the offending disjunction ('Napoleon was defeated at Waterloo', 'Charlemagne was Holy Roman Emperor') is in the past tense. Now it could be questioned whether, if k is a statement in the past tense, the material conditional $\neg k \to h$ can fairly be read as h *under the condition* $\neg k$. Does *If Charlemagne was not Holy Roman Emperor then snow is white* say the same as *Snow is white under the condition that Charlemagne was not Holy Roman Emperor*? Really, I do not know. Those who like to analyse conditionals in terms of possible worlds may have an easy answer to this question, but for the present discussion it seems to me not to be too important. A restriction to statements formulated in the present and future tenses is enough to show the frailty of the intuitions to which the authors quoted defer.

What I am trying to emphasize here is that no decent argument has been given against the inclusion of the disjunction $h \vee k$ in the assertoric content of both h and k. Intuitions (as Yablo *op. cit.*, § 1.3, quoted below, seems to be prepared to admit) are not enough.

4.2 Partial truth

It seems appropriate, almost in conclusion, to call attention to a discussion that unwittingly shows what a linguistic and logical quagmire is entered when it is maintained that the assertoric content of a statement may exclude some consequences that can be obtained by the rule ∨I. Early in his book, Yablo introduces the problem of partial truth in the following words (*loc. cit.*).

> What is it for a hypothesis to be partly true? [footnote] Here is the naivest possible idea about this:
>
> 1 A hypothesis is partly true iff it has parts that are wholly true.
>
> Now we must ask what is meant by *part* of a hypothesis. The naivest possible idea about part/whole as a relation on hypotheses is
>
> 2 One hypothesis is part of another iff it is implied by the other. [footnote]
>
> The naivest possible idea about about partial truth is on the right track, I think ... [b]ut the naivest possible idea about what it takes for h to include k is questionable.
>
> A paradigm of inclusion ... is the relation that simple conjunctions bear to their conjuncts — the relation *Snow is white and expensive* bears, for example, to *Snow is white*. A paradigm of noninclusion is the relation disjuncts bear to disjunctions. *Snow is white* does not have *Snow is white or expensive* as a part. This is not predicted by (2).
>
> You might say that paradigm case intuitions are a poor base for theory. But the intuitions here are systematic.

In support of this judgement, Yablo lists a number of other contexts in which, it is claimed, our intuitions dictate that a disjunction may not be a part of either of its disjuncts. It seems clear that this latter claim closely resembles the thesis whose truth this lecture has been contesting.[5]

[5] One of the contexts mentioned by Yablo is, or is very similar to, a celebrated conundrum in the logic of imperatives, the question whether the imperative *slip the letter into the letter-box!* implies the disjunction *slip the letter into the letter-box or burn it!* (Ross 1941, pp. 61f.; the more common formulation is *post the letter!*). According to Yablo (*loc. cit.*), who does not mention Ross's paradox by name,

> ... [o]rdering Smith to eat pork chops is ordering her to eat pork. Ordering her to eat pork is not ordering her to eat pork or human flesh, though eating pork or human flesh is no less implied by eating pork than is eating pork implied by eating pork chops. One commands (normally) the parts of what one commands, but not its implications more generally. ...

What, as far as I know, has not been adverted to in the discussions of Ross's paradox is the well known and pervasive ambiguity in everyday English of the particle *or*, which is obliged to perform as the translation of each of the Latin particles *vel* and *aut*. The former is represented in most systems of formal logic by the connective ∨, the *inclusive disjunction* of the items connected, while *aut*, which expresses the *exclusive disjunction* of the items connected, has no established symbolization; △, called *symmetric difference*, $\nleftrightarrow$, the negation of the biconditional, and $\overset{e}{\vee}$ (called *eeyore* by Mary Cresswell), have all been used. If the connective *or* is read as the exclusive disjunction *aut*, then it is quite in order that *post the letter or burn it!* is not a consequence of *post the letter!*. If *or* is read as the inclusive disjunction *vel*, then *post the letter or burn it!* says no more than does *If you do not burn the letter, then post it!* (or perhaps more accurately, *Bring it about that, in the event that you do not burn the letter, you post it!*). To the extent that this latter command is in idiomatic English, it does appear to be a consequence of *post the letter!*.

The thesis that disjunctions are not normally parts of their disjuncts may be supposed — but this may be just my imagination — to apply to material conditionals too, since $h \to k$ is logically equivalent to $\neg h \vee k$; hence $h \to k$ is not a part of k, and therefore — but again this may be my imagination — not a part of $h \wedge k$. In any case, it evidently serves to prevent every conjunction of false statements from having a true part, that is, from being partly true. For $h \wedge k$ is logically equivalent to $h \wedge (h \to k)$, whose second conjunct is true if h and k are both false. (A similar point is made in the discussion in *op. cit.*, § 8.3, of what Yablo calls the *horseshoe theory* of relative or surplus content, adverted to in footnote 2 above.) The claim of the biconditional $h \leftrightarrow k$ to be a part of the conjunction $h \wedge k$ cannot be dismissed in quite the same way. The logical equivalence of $h \wedge k$ and $h \wedge (h \leftrightarrow k)$ is uncontroversial, and once again the second conjunct of the latter conjunction is true if h and k are both false. I imagine that Yablo's response here would be to plead that when he says that '[a] paradigm of inclusion . . . is the relation that simple conjuncts bear to their conjuncts', he intends the word 'simple' seriously, to mean something like *primitive*. The conditional *If snow is white, then it is expensive* and the biconditional *Snow is white if and only if it is expensive* are compound statements and, under the paradigm, do not qualify as parts of *Snow is white and expensive*. It is well known, however, that a language in which h and k are primitive is equivalent, for every logical purpose, to a language that differs from it only in having h and $h \leftrightarrow k$ instead as primitive. This is the simplest instance of the phenomenon of language dependence that was mentioned at the end of § 0 above.

To deny the importance of language independence in logical investigations seems to me to be nothing short of a belief in word magic (to use a term of Isaac Levi's). It is discouraging that Chapter 11 of my (2006), which is devoted to rebutting a host of attempts to defend language-dependent theories of verisimilitude, has been so pointedly ignored by the main protagonists in this debate. Only Oddie, in his (2014) article in *The Stanford Encyclopedia of Philosophy*, even mentions it, but he does not respond to it. It needs to be understood that even in colloquial language there is no sharp distinction between primitive and defined terms, and in science and in mathematics the situation is even starker (for one or two examples, see Miller & Taliga 2008). In other words, we cannot do without the possibility of reformulating in new vocabularies, in other words, and rarely entirely accurately, what it is that we are trying to say. That is how things are, and no amount of semantical sophistication should be allowed to conceal it.

5 A response to Frederick's comments

5.0 Justification, persuasion, discovery

The old-fashioned, and stodgily conventional, thesis defended in § 3 and § 4 above is that a statement asserts, explicitly or implicitly, all its logical consequences; that is to say, its

If the argument in the text is correct then, contrary to what Yablo asserts, an order to eat pork chops is, at the same time and in the same action, an order to eat pork or to eat human flesh.

assertoric content (which may be called also its *informative content*) includes its logical content. I neither affirmed nor denied the converse inclusion (that a statement asserts nothing but its logical consequences), though I think that it is true. In his comments (2018) Frederick suggests that the assertoric content of a statement should be identified instead with its *semantic content*, which, unlike its logical content, is normally finite and can be known in full. He provides no interesting new argument against my thesis. He simply contradicts it, and says that three associated theses put forward in Chapter 3 of my (2006), and more tersely in § 0 above, are startling. That may be so, but two of these theses are far from being avant-garde. A version of the thesis that every valid argument is a *petitio principii*, and unable to furnish justification for its conclusion, was endorsed by Sextus Empiricus and by Mill, and also by Gassendi and by Unamuno (both quoted in Marans *op. cit.*), and the thesis that valid arguments are not used to generate new knowledge was endorsed by Cicero, and emphasized by Hamilton,[6] by Minto, and by many of the leading logical empiricists.

Against the third thesis, that '[o]nly those who are impressed by style rather than by substance could be convinced by an argument, rather than by what it is that the premises assert' (Miller 2005, p. 66), Frederick offers some speculations about the thought processes of Hobbes and of Frege, but he does not offer much more. I submit that this response misconstrues Frederick's own examples, and misses the point of what I wrote in § 0 (and especially in 2006). It would be more appreciative of Hobbes's searching spirit to say that he examined Euclid's proof of Pythagoras' theorem in order to test the theorem, rather than to say that his purpose was to persuade himself of its truth (or, for that matter, to discover it). His investigation may have had the effect of persuading him that the theorem is true, but had he been more critical of Euclid's fifth postulate than he seems to have been, the repercussion might have been different. As for Frege, Russell reports that he was led to what we call Russell's paradox through a critical consideration of 'Cantor's proof that there is no greatest cardinal number' (1959, pp. 75f.). His letter to Frege was devoted to the mathematical task of criticizing Frege's axioms, not the psychological task of changing Frege's mind. Indeed, to a considerable extent Frege did not change his mind. Although he acknowledged that there was a problem, he soon recovered enough to propose a modification that, if effective, would have rescued his system (see Quine 1955).

The thesis, plainly stated by Minto, that a logical argument is not typically used in

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[6]On p. 44 of Volume III of his (1859–1860), Hamilton writes:

> But an extension of any science through Logic is absolutely impossible; for by conforming to logical canons we acquire no knowledge, receive nothing new, but are only enabled to render what is already obtained more intelligible, by analysis and arrangement. Logic is only the negative condition of truth. To attempt by a mere logical knowledge to amplify a science, is an absurdity as great as if we should attempt by a knowledge of the grammatical laws of a language to discover what was written in this language, without a perusal of the several writings themselves.

This passage, as well as one on p. 45, in which Hamilton recognizes that 'Logic alone affords us the means requisite to accomplish a rational criticism' (Marans *op. cit.*), could well have been quoted in § 1 above.

order to bring to light its conclusion is not threatened by the undoubted possibility that an adventitious discovery (of an error, for example) may be made in the process; and equally, the thesis that a logical argument is not typically used in order to persuade someone of its conclusion is not threatened by the possibility that the interlocutor may unwittingly be persuaded of something else (an error, for example). Frederick's fables are as misguided as the traditional views concerning the purpose of argument that he seeks to defend.

5.1 Semantic content

Frederick says that 'the semantic content of a sentence ... is its meaning as determined by its words and its grammatical structure'. It is hard to make much of this, since the same may truly be said about logical content (with which semantic content is contrasted). He goes on to say that statements that 'have different connectives and quantifiers and ... are constructed differently ... have different semantic contents'; and in § 3 of his comments some simple truth-table equivalents, such as the disjunction $k \vee h$ and the material conditional $\neg k \to h$, are deemed not to be comparable in semantic content (see § 5.4 below). He does not elucidate any conditions under which two different statements have the same semantic content, or conditions under which the semantic content of one statement is part of (that is, is contained or included within) the semantic content of another statement (or set of statements). That is, he does not specify any conditions under which one statement, or set of statements, asserts what another statement, or set of statements, asserts. It is unclear, for example, if h and k are distinct statements, whether $h \wedge k$ and $k \wedge h$, which have the same connectives (and quantifiers), and are not constructed differently, make the same assertions (but I suppose that they do). It is not even clear whether the statements h and $h \wedge h$ and $h \vee h$ make the same assertions (but it seems that they may not, since they contain different connectives). Frederick may even embrace the minimalist view that distinct statements have distinct, even incomparable, semantic contents, and that h and $h \wedge h$ have incomparable semantic contents. On this matter, and on a great deal else about what semantic content is meant to be, his treatment is unrelentingly unspecific.

The brief explanation of semantic content that Frederick provides, and I have quoted above, suggests that the semantic content of a statement is as objective or intersubjective as is its logical content. But in § 3 of his comments he repudiates this reading when he starkly affirms that my claim that 'the instance $\mathcal{F}a$ ... [is part of] the assertoric content of the universal generalization $\forall x \mathcal{F}x$' (see the beginning of § 4.1 above) 'is false if assertoric content = semantic content'. In explanation of this remarkable judgement, he says

> the sentence 'everything is made of atoms' logically implies the sentence 'David Miller is made of atoms.' But the semantic content of the latter sentence goes beyond the semantic content of the former in mentioning David Miller. There are, presumably, many physicalists who understand and assert the universal sentence who have not the faintest idea who David Miller is.

It seems therefore that Frederick has reverted to the relativistic or subjectivistic understanding of content inherent in the distinction made in his (2011) between the objective content of a theory and its available content (see § 3.1 above), and to be saying that the assertoric content of a statement depends idiosyncratically on the limited knowledge of the person who asserts it. The warning *Trespassers will be prosecuted*, that is, does not assert *if Rumpelstiltskin is a trespasser, he will be prosecuted* if the landowner has never heard of Rumpelstiltskin; until 1846 Kepler's theory did not say that *if Neptune is a planet then it travels in an ellipse round the sun*; Newton's law of universal gravitation says nothing about those physical bodies with which no one is personally acquainted. I do not know what Frederick takes to be the assertoric content of his local telephone directory, but it must be much less extensive than what we normally think of as the information that it contains. No doubt a distinction may be made between what is objectively asserted by a statement (which, I claim, is best identified with its logical content) and whatever it is that is asserted (or simply, what is meant) by speakers or writers who utter the statement (which seems to be what Frederick wants to call its semantic content). The latter sense of assertion belongs to the theory of speech acts, and it may be that, thanks to the ubiquity of conversational implicatures, there are always elements in the semantic content of a statement that are not in its logical content; statements that, according to my position, it hints at but does not properly assert. But this sense is of little significance in the *epistemology without a knowing subject* that is cherished by critical rationalism.

5.2 Question-begging arguments

What looks like a similar relapse into subjectivism or relativism occurs in § 3 of Frederick's comments when he condemns as 'question-begging in *the ordinary sense*' two arguments that he attributes to me. Against the thesis that all valid arguments are question-begging, which I share with Mill and many others, he maintains that '[a]n argument is question-begging in the ordinary sense *only if* [and presumably also *if*] the conclusion is evidently presupposed by the premises on account of their semantic content'. I shall not attempt to guess what logical relation is referred to by the word 'presuppose'. But what is evident to one person on one occasion may not be evident to that person, or to another person, on the same or on another occasion. It looks as if Frederick's thesis is that an argument is question-begging or circular '*in the ordinary sense*' if and only if the semantic content of its conclusion is included within the semantic content of its premises. Does this thesis amount to saying just that $\mathsf{h} \therefore \mathsf{h}$ and $\mathsf{h} \wedge \mathsf{k} \therefore \mathsf{h}$ and $\{\mathsf{h}, \mathsf{k}\} \therefore \mathsf{h}$ (or more generally $\Gamma \therefore \mathsf{h}$, where h is evidently an element of the set Γ) are exemplary cases of question-begging arguments? I suppose that it must amount to this, though he does not say so explicitly.[7]

[7] All valid arguments are circular, but some, I agree, are more patently (or evidently) circular than others are. There have been occasions in the history of mathematics when much work was needed to isolate the point or points at which, in a purported demonstration, a suspect axiom (such as the axiom of parallels or the axiom of choice) was silently called on. Kaye (2014) investigates the interesting contrast between the soundness theorem for elementary logic, which is rather obviously assumed in any valid proof of the soundness theorem that uses elementary logic, and the completeness theorem, whose precise role in the proof of the completeness theorem is unearthed only with some care.

But whatever Frederick may mean exactly, or even roughly, by calling an argument 'question-begging in *the ordinary sense*', the penultimate sentence of his comments:

> Miller's arguments for his claim that assertoric content = logical content are question-begging, in the ordinary sense in which begging the question is improper.

is at fault in more than one way. To start with, the purpose of my paper was not to provide 'arguments for ... [my] claim that assertoric content = logical content'. As I announced in § 0 above, what I set out to do was to show the unsoundness, and the unattractiveness, of some recent arguments, including one broached by Frederick (2011), that have been made against this identification. No serious critical rationalist supposes that the absence of compelling arguments against a thesis constitutes an argument in its favour.

The two arguments, cited in the quotation above, that Frederick attributes to me and decries as circular are the subject of § 3 of his comments; one, labelled (A)–(E), he finds in § 4.1 above, the other, labelled (a)–(e), he excavates from Chapter 3 of my (1994) (and from p. 65 of my 1995). I readily admit that these arguments, when tidied up, are question-begging in my sense, for they are deductively valid. But I do not admit that they are of no value, or that they are doomed as 'question-begging in *the ordinary sense*'.

5.3 The argument labelled (a)–(e)

Let us look first at (a)–(e), which is formulated by Frederick in the following way:

- (a) Suppose that 'h' entails 'k.'
- (b) Then, 'h' is logically equivalent to 'k and (if k then h)' [truth table].
- (c) Logically equivalent propositions have the same content.
- (d) Therefore, 'k and (if k then h)' asserts that k [from (b) and (c)]
- (e) Therefore, 'h' asserts that k [from (b), (c) and (d)].

We do well to make a number of modifications to the presentation of this argument (and to its substance too). Its use of the letters 'h' and 'k' in statements of entailment and assertion not as variables for sentences (or statements), but as placeholders that are to be replaced by sentences (so that '"h"' and '"k"' are variable quotation names for sentences), has the drawback that the logical content of a statement cannot be specified in the usual mathematical way; that is, by the formula $\mathbf{Cn}(h) = \{k \mid h \text{ entails } k\}$. (Exactly the same difficulty arises for assertoric content.) We shall instead understand entailment and assertion to be relations between statements (as in 'h entails k' and in 'h asserts k') rather than as sentence-forming operators flanked sometimes by names of sentences (as in '"h" entails "k"') and sometimes by *that*-clauses (as '"h" asserts that k'). This change will not affect the correctness, or the intelligibility, of the discussion that follows. Secondly, in line with Frederick's remark that '[the] argument is valid [if &] only if the word "content" in (c) means *assertoric content*', we shall replace the words 'have the same content' in (c) by 'makes the same assertions'. Thirdly, the conditional 'if k then h', which appears in both (b) and (d) but was absent from the argument in my (2005) that Frederick cites as a source, may be replaced without loss by 'h'. Fourthly, reading is eased if we remove some distracting punctuation, and if we introduce the wedge $\wedge$ to represent conjunction. Fifthly, I insist that (d) be recognized as an additional premise, not as the conclusion of a valid inference from (b) and (c) (which it is not). In the reformulations below, (d) could well be replaced by a general statement independent of h and k. The argument becomes

(a) h entails k
(b) hence, h is logically equivalent to $k \wedge h$
(c) logically equivalent statements make the same assertions
(d) $k \wedge h$ asserts k
(e) h asserts k

Note that the assumption (a) and the intermediate conclusion (b) are logically equivalent for all statements h and k. Discharging (a), we may replace (b) by (b−), which is a logical truth, and move to an entirely equivalent argument (in which the variables in (b−), (d), and (e−) may be understood to be universally quantified) that has the advantage that the truth of its conclusion (e−) is something that is in dispute. I am happy to endorse it.

(b−) if h entails k, then h is logically equivalent to $k \wedge h$
(c) logically equivalent statements make the same assertions
(d) $k \wedge h$ asserts k
(e−) if h entails k, then h asserts k

According to Frederick, the ancestor of this argument is question-begging (in his sense), which presumably means that the purified version too is question-begging; that is, that the conclusion (e−) 'is evidently presupposed by the premises [(b−), (c), and (d) together] on account of their semantic content'. It is not denied that these premises together entail (e−), and in this sense they assert (e−). But this does not imply (evidently or otherwise) that the premises 'evidently presuppose' (e−), or that the argument is 'question-begging in the ordinary sense'. Frederick's explanation of what is going on here is that

> if assertoric content = logical content, then (c) is true; but if assertoric content = semantic content, then (c) is false.... Miller's argument assumes that assertoric content = logical content and thus improperly begs the question.

The second sentence here is demonstrably incorrect. Neither individually nor collectively do the premises (b−), (c), and (d) assume (or presuppose, or state, or assert) that 'assertoric content = logical content' (that is, that for every statement j is the assertoric content of j identical with the logical content of j).[8] Together the three premises (b−),

[8]This may be seen by interpreting *assert* (and, as a consequence, *make the same assertions* too) to be relations that invariably hold between any two statements. All the other terms are to be interpreted standardly. Interpreted in this way, (c) and (d) are true, and (b−), which does not mention assertion, is also true, but the assertoric content of a consistent statement is never the same as its logical content. It is to be observed that, when *assert* is interpreted in the non-standard way here suggested, the converse of (e−), which says that the assertoric content of j is included in the logical content of j, is false for all consistent statements j. Indeed, matters are as stated in the text: the premises (b−), (c) and (d), supplemented with the converse of (e−), do entail the identity of assertoric content and logical content.

(c), and (d) do of course entail that the assertoric content of j includes the logical content of j, which is what the conclusion (e−) says; and were we to add the converse of (e−) as a further premise, then 'assertoric content = logical content' would indeed be validly derivable. But it does not follow from the three premises given, and it is not assumed or asserted by them. If Frederick wishes to maintain that, on his etiolated understanding of what a statement (or set of statements) asserts or assumes, the identity 'assertoric content = logical content' is assumed by (b−), (c), and (d), then he has more explaining to do.

In the absence of (d), which should be uncontroversial, the premise (c) is logically weaker than the conclusion (e−), but in the presence of (d) they are logically equivalent. The intent of the argument from (b−), (c), (d) to (e−), and of similar arguments, was quite modest: it was not to establish (e−), but to show that, provided that (d) holds,

> if (c) h asserts k *whenever* h *and* k *are logically equivalent*, then (e−) h asserts k *whenever* h *entails* k; or, put contrapositively, *if the conclusion* (e−) *is rejected, then the logically weaker premise* (c) *must be rejected too.*

Frederick is happy (even determined) to reject (c) as a general proposition, but not everyone will be. (In § 5.5 below I say a few more words about the inadvisability of dropping (c).) For him, therefore, the argument under discussion, though valid, is unsound (that is, it contains a false premise). But judging a valid argument to be unsound is not the same as judging it to be question-begging in any recognizable sense. If the argument is question-begging in Frederick's sense, then we lack an intelligible explanation why.

5.4 The argument labelled (A)–(E)

Frederick reproaches as question-begging another argument of mine, which he sets out so:

(A) A sentence of the form 'if not-k then h' asserts that h under the condition that not-k, whereas the sentence of the form 'h' asserts that h under all conditions.

(B) An instance, '$\mathfrak{F}$a,' is part of the assertoric content of the universal generalisation '∀x$\mathfrak{F}$x.'

(C) Therefore, a sentence of the form 'h' asserts that if not-k then h.

(D) A sentence of the form 'if not-k then h' is logically equivalent to the corresponding sentence of the form 'k or h'.

(E) A sentence of the form 'h' asserts that k or h.

In § 5.1 I defended the first leg of this argument, from (A) and (B) to (C), especially the premise (B). Here I shall not repeat the defence, but I am moved to point out that even if Frederick were right, and that when the object named by 'a' is unknown to the speaker, the universal statement ∀x$\mathfrak{F}$x does not assert the instance $\mathfrak{F}$a, this would not show that, in many cases, including those held up as counterexamples by Schurz & Weingartner, Gemes, Mura, and Yablo, and quoted in § 4.0 above, the disjunction h ∨ k of two completely independent statements h and k is not asserted by both h and k. The second leg, from (C) and (D) to (E), may be crisply written in this way:

(C) h asserts $\neg\mathsf{k} \rightarrow \mathsf{h}$
(D) $\neg\mathsf{k} \rightarrow \mathsf{h}$ is logically equivalent to $\mathsf{k} \vee \mathsf{h}$
(E) h asserts $\mathsf{k} \vee \mathsf{h}$

It has to be acknowledged that this argument, as it stands, and as it was presented in § 4.1 above, is invalid, and that a third premise is needed to make it valid. Had I stated the argument more carefully, I would, I suppose, have added a premise analogous to (c) above, to the effect that *logically equivalent statements are asserted collectively*. Let us give the makeshift name (F) to this third premise, which may be formulated less awkwardly as

(F) if h and k are logically equivalent, then j asserts h if and only if j asserts k.

The validity of the resultant argument, from (C), (D), and (F) to (E), is assured.

But without considering this possible repair, or any other, Frederick coolly designates the identity 'assertoric content = logical content', which on its own entails the conclusion (E), as 'the suppressed premise'. If included, it would make the argument trivially circular.

> (E) follows from the conjunction of (C) and (D) if assertoric content = logical content, because logically equivalent sentences have the same logical content. . . . Miller's inference of (E) from (C) and (D) depends upon the suppressed premise that assertoric content = logical content, which is the question at issue. Again, Miller improperly begs the question.

This diagnosis cannot be countenanced. To augment the premises of an invalid argument with the conclusion of the argument, or with a statement that implies the conclusion, is a notoriously cheap way of restoring validity, and is rightly discredited, since there are countless other weaker statements, such as (F) here, that do the job as efficiently and do not introduce circularity (Miller 2018). The conclusion (E) of the argument is not assumed (or presupposed, or stated, or asserted) by either (C) or (F) individually though, because (D) is a logical truth, it is logically implied (and, I say, asserted) by (C) and (F) together. It is a question-begging argument only in the sense that it is deductively valid.

It may be remarked that the two premises (c) and (F), which appear separately in the two arguments that I defend, postulate distinguishable properties of the assertion relation. Whereas (c) says that logically equivalent statements are interchangeable in the first place of the relation, (F) says that logically equivalent statements are interchangeable in its second place. The two premises are indeed equivalent if it is assumed that *assertoric content = logical content*, for in that case the relations *asserts* and *implies* are themselves interchangeable. But that is neither here nor there. Neither that identity, nor the equivalence of (c) and (F), is assumed (or denied) at any stage in the two arguments of mine that Frederick imprudently declares to be question-begging 'in the *the ordinary sense*'.[9]

[9] The conclusion (E) of the argument considered in this subsection (and previously in § 4.1) is itself an immediate consequence of the conclusion (e−) of the argument considered in the previous subsection § 5.3. If the argument has any additional interest, it is because of its premises, not because of its conclusion.

5.5 Conclusion

The premises (c) and (F) that appear in the arguments analysed in the previous subsections state succinctly the principle that, in the context of assertion, logically equivalent statements are interchangeable. As I said at the end of § 4.2 above, this principle seems to be unassailable in any context, such as any of the mathematical sciences, not to mention everyday life, in which hypotheses, or proposals, are often reformulated as a matter of course in a strikingly different vocabulary. For two simple examples, let mention first the two standard accounts of logical content, in terms of consequences, and in terms of models, that were compared in § 2 above, and the coexistence of cartesian and polar coordinates in geometry (Miller 2006, Chapter 11, § 5.2). Those who dissent from (c) and (F) really ought to tell the rest of us how it is that such reformulation is so amiably accepted.[10]

The independence of (F) from (c), and of (c) from (F), may be seen informally by noting that two persons of the same sex and the same (or different) parents must have different offspring (unless both are childless): Cain and Abel were brothers, but Enoch, the son of Cain, was not a son of Abel. Let us, rather laxly, use the relation of siblinghood to interpret logical equivalence. To show that (c) is independent of (F), take *begets* to be the interpretation of the relation 'asserts' (so that *is born to* is the interpretation of the converse relation 'is asserted by'). To show that (F) is independent of (c), take *is born to* to be the interpretation of 'asserts'. To ensure that assertion is interpreted as a reflexive relation, that is, that j always asserts j, the interpretations have to be amplified and complicated (in an uncomplicated way).

A more rigorous proof of the independence of (F) from (c) takes the domain of interpretation to be the set S of sentences of an elementary language. Choose two elements j and h of S that are not logically equivalent; for h and for each sentence k that is logically equivalent to h interpret its assertoric content to be the result of discarding from the logical content of k the single sentence j. The assertoric content of every other sentence or set of sentences is identified with its logical content. It is evident that (c) holds. Since h asserts $\neg\neg j$, but does not assert j, (F) fails. The independence of (c) from (F) may be shown as follows: choose a consistent sentence j, and let its assertoric content be the set S of all sentences (which is equal to $\mathtt{Cn}(\bot)$). The assertoric content of every other sentence or set of sentences is identified with its logical content. Then (F) holds, since all assertoric contents are also logical contents. But (c) fails, since the assertoric content of j is S, while the assertoric content of its logical equivalent $\neg\neg j$ is $\mathtt{Cn}(\neg\neg j)$, that is to say $\mathtt{Cn}(j)$, which is not S. Under both these interpretations, assertion is a reflexive relation.

Worth recording in passing is the straightforward proof that the reflexivity of the assertion relation, together with its transitivity (*if* h *asserts* j*, and* j *asserts* k*, then* h *asserts* k) implies that (c) and (F) are equivalent assumptions. For it follows from (c) and reflexivity that if h and k are logically equivalent then each of them asserts the other. By transitivity, if h and k are logically equivalent then if j asserts h, then j asserts also k. In short, (F) is deducible from (c). The converse implication is proved identically.

It is easily proved, however, that (c) and (F) taken together are insufficient to ensure the identity *assertoric content* = *logical content*. Interpret the assertoric content of j as the set of statements logically equivalent to j; that is, h asserts k if and only if h and k are logically equivalent. It follows that if h and k are logically equivalent, and h asserts [or is asserted by] j, then h and j are logically equivalent, whence k and j are logically equivalent, whence k asserts [or is asserted by] j. In brief, (c) and (F) hold, but the assertoric content of a statement j is a proper subset of its logical content (unless j is a logical truth).

[10] Fine (2013), pp. 415f., considers the possibility of rejecting similar assumptions in response to difficulties encountered in characterizing partial content in anything but the natural way (that is: the content of k is part of the content of h if and only if h logically implies k). Fine acknowledges the obvious connection of his work with the proposals of Gemes and Yablo briefly discussed in § 4.0 and § 4.2 above.

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