PH126 Starting Logic
Handout 9
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modus ponens

modus tollens

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

→Intro:

```
| | *
| |----
| | ...
| | #
| ...
| *→#
```

→Elim:

```
| *→#
| ...
| *
| ...
| #
```

```
| 1. P→Q
| 2. ¬Q
|----
| | 3. P
| |----
| | 4. Q        →Elim: 1, 3
| | 5. ⊥        ⊥Intro: 2,4
| 6. ¬P         ¬Intro: 3-5
```

¬Intro:

```
| | *
| |----
| | ...
| | ⊥
| ...
| ¬*
```

Interesting feature of →Intro:
Suppose a proof is a logically valid argument. Then if we add a step using →Intro, it will still be a logically valid argument.

All rules of Fitch have this feature.

Paradoxes of material implication
Adams (Inquiry 1965): "The following … 'inferences' are all ones which, when symbolized in the ordinary way in the propositional calculus and analysed truth-functionally, are valid":

1. John will arrive on the 10 o'clock plane. Therefore, if John does not arrive on the 10 o'clock plane, he will arrive on the 11 o'clock plane. [because Q entails $(P \rightarrow Q)$]
2. John will arrive on the 10 o'clock plane. Therefore, if John misses his plane in New York, he will arrive on the 10 o'clock plane. [because Q entails $(P \rightarrow Q)$]
3. If Brown wins the election, Smith will retire to private life. Therefore, if Smith dies before the election and Brown wins it, Smith will retire to private life. [because $(P \rightarrow R)$ entails $((P \wedge Q) \rightarrow R)$]
4. If Brown wins the election, Smith will retire to private life. If Smith dies before the election, Brown will win it. Therefore, if Smith dies before the election, then he will retire into private life. [because $(P \rightarrow Q)$ and $(Q \rightarrow R)$ entails $(P \rightarrow R)$]
5. It is not the case that if John passes history, he will graduate. Therefore, John will pass history. [because $\neg(P \rightarrow Q)$ entails P]
6. If you throw both switch S and switch T, the motor will start. Therefore, either if you throw switch S the motor will start, or if you throw switch T the motor will start. [because $((P \wedge Q) \rightarrow R)$ entails both $(P \rightarrow R)$ and $(P \rightarrow R)$]

From rules to truth tables
On the assumption that ∧Intro and ∧Elim are sound, there is only one possible truth table for ∧.

Material implication (→) again

1.		
2.	$\bot$	
3.	$\bot$	$\bot$Elim: 2
4.	$\bot \to \bot$	$\to$ Intro: 2-3

1.		
2.	$\bot$	
3.	$P \lor \neg P$	$\bot$Elim: 2
4.	$\bot \to (P \lor \neg P)$	$\to$ Intro: 2-3

1.		
2.	$P \lor \neg P$	
3.	$P \lor \neg P$	$\bot$Elim: 2
4.	$P \lor \neg P \to (P \lor \neg P)$	$\to$ Intro: 2-3

Truth functional completeness
Example: for any truth function, we can find a logically equivalent one using only ¬, ∨, ∧.

DeMorgan's Laws

$\neg P \wedge \neg Q$ ⫤⊨ $\neg(P \vee Q)$

$\neg P \vee \neg Q$ ⫤⊨ $\neg(P \wedge Q)$

Substution Theorem

If A ⫤⊨ B then S(A) ⫤⊨ S(B)

Exercises
Little bit more than usual because we have no lectures or seminars in reading week.

7.1–7.8

7.12.1–7.12,19 odd numbers only
FOR 7.12.* ALSO SUBMIT YOUR ANSWERS ON PAPER/BY EMAIL.

8.1
8.3, 8.5, 8.7, 8.9 odd nos only
8.12
8.16
8.17
8.18–8.25

8.26–30
FOR 8.26–30 YOU DO NOT HAVE PERMISSION TO USE TAUT CON TO PROVE $P \vee \neg P$