PH126 Starting Logic Lecture 3
Lecturer: s.butterfill@warwick.ac.uk

Reflexive, symmetric and transitive

A reflexive relation is one that everything bears to itself. (E.g. everything is the SameShape as itself. E.g. of *non*-reflexive: not everything is LeftOf itself).

A symmetric relation is one such that if x bears it to y, then y bears it to x. (E.g. if Adjacent(a,b) then Adjacent(b,a). Not symmetric: LeftOf—LeftOf(a,b) does not imply LeftOf(b,a).)

A transitive relation is one such that if x bears it to y and y bears it to z then x bears it to z. (E.g. LeftOf is transitive; DifferentShape is *not* transitive)

To show that a particular relation lacks one of these properties, we produce and argument and a counterexample.
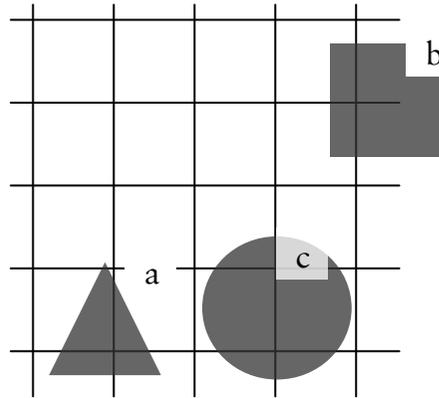
E.g.

Show that NotAdjacent is not transitive.

If NotAdjacent were transitive, the following argument would be logically valid:

1. NotAdjacent(a, b)
2. NotAdjacent(b, c)
3. NotAdjacent(a, c)

Here is a counterexample to this argument:

It's possible to define artificial relations. From the point of view of logic, these are just as good as natural relations.

EqualToOrLeftOf(x, y) iff
      x = y or LeftOf(x, y)

EqualToOrAdjacent(x, y) iff
      x=y or Adjacent(x, y)

JohnOrAyesha(x, y) iff
      x = John and y = Ayesha
      or x = Ayesha and y = John

JohnToAyesha(x, y) iff
      x = John and y = Ayesha

Truth tables and some truth functional connectives

Rough guide:
'∧' means and
'∨' means or
'¬' means not

P	Q	P ∧ Q	P∨Q
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

P	¬P
T	F
F	T

The *truth value* of a sentence is true (T) when the sentence is true and false (F) when the sentence is false.

A *connective* joins one or more sentences to make a new sentence. E.g. 'because', '¬'

The sentences joined by a connective *constituent sentences*.

E.g. in 'P∧Q',
      ∧ is the connective
      P, Q are the constituent sentences

A *truth functional connective* produces a new sentence whose truth value depends

only on the truth values of its constituent sentences.

E.g. '∧' is a *truth functional connective*; 'because' is not.

How do we know 'because' is not truth-functional? Try to construct a truth table for 'P because Q'—what do you put in the first line (where P is T and Q is T)? Consider:

- 'Jack Bauer risked his life because he loves his country' (P is T, Q is T, whole sentence is T)
- 'Jack risked his life because he loves Kim' (P is T, Q is T, whole is F)

Truth tables can be used to show that an argument is valid. For example:

	P ∨ Q	// It went up the left fork or it went up the right fork
	¬P	// It didn't go up the left fork
	R ∨ ¬R	// Therefore John ate figs or John did not eat figs

Truth tables for this argument

P	Q	P ∨ Q	¬P	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F
		∧ premise	∧ premise	∧ conclusion

Exercises 02
for seminars in week 3.

Reading: §§2.4–5, §§3.1–4

Exercises

From Barwise & Etchemendy, *Language, Proof and Logic* (*=optional)

2.21–23 (*2.24–27)
3.5, 3.7
3.8, 3.12–15, (*3.10)

X.2 In the table below, each lines describes a combination of properties. E.g. line 1 specifies a relation that is reflexive, symmetric and transitive, line 2 specifies a relation that is reflexive, symmetric and not transitive. Complete the table by adding an example relation on each line.

(Optional: also find alternatives to the examples already given in the table.)

	Example	Reflexive	Symmetric	Transitive
1	= (identity)	Y	Y	Y
2		Y	Y	N
3		Y	N	Y
4		Y	N	N
5	IdenticalAndNotIdentical [1]	N	Y	Y
6	DifferentShape	N	Y	N
7		N	N	Y
8		N	N	N

[1] definition of IdenticalAndNotIdentical:
IdenticalAndNotIdentical(x, y): x=y and x≠y
(nothing bears this relation to anything)

X.3 Ambiguity Disambiguate the following by giving alternative readings.

Example:
i. Pierre is a foreign coin collector.
– *Pierre is foreign and he collects coins.*
– *Pierre collects foreign coins.*

3.1–2

ii. Tom is kind and brave or foolish
iii. Kids make nutritious snacks.
iv. I want to marry a Norwegian.
v. We are now serving live lobsters.
vi. The door is alarmed.
vii. This is a hospital where doctors are trained.
viii. Ormus said that Vina was happy then.
ix. No one hates a man after he shoots him.
x. A woman and a girl each had a hat and the woman gave hers to the girl.
xi. One can't remove the rods too quickly.
xii. Ayesha clumsily stamped on a snail.

(Some examples are from http://www.vuse.vanderbilt.edu/~jgray/funny.html)