

# PH126 Starting Logic Lecture 6

Lecturer: s.butterfill@warwick.ac.uk

## Truth table example

P	Q	R	$P \vee \neg(Q \wedge \neg(R \vee \neg P))$
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

## Substitution of logical equivalents

Two sentences are *logically equivalent* when each is a logical consequence of the other.

If \* and # are logically equivalent, then you take a complex formula and replace \* with # without changing the truth table of the complex formula.

Example. Because  $\neg(R \vee \neg P)$  is logically equivalent to  $\neg R \wedge P$ , it follows that:  $P \vee \neg(Q \wedge \neg(R \vee \neg P))$  is logically equivalent to  $P \vee \neg(Q \wedge (\neg R \wedge P))$

And because  $\neg(Q \wedge (\neg R \wedge P))$  is logically equivalent to  $\neg Q \vee (R \vee \neg P)$ , it follows that  $P \vee \neg(Q \wedge (\neg R \wedge P))$  is logically equivalent to  $P \vee (\neg Q \vee (R \vee \neg P))$ .

$\vee$ Elim:

$P1 \vee P2$
P1
...
Q
P2
...
Q
...
Q

To prove a conclusion from a disjunction, prove the conclusion from each of the disjuncts.

$\perp, \neg$

P	$\neg P$	$\perp$	$P \wedge \neg P$
T	F	F	F
F	T	F	F

## How to order reference columns

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Annotations:

- Always start with T
- Sentence letters are ordered alphabetically
- Right-most column alternates every row
- Always end with F
- Next right-most column alternates half as often as previous column
- Next right-most column alternates half as often

## Subproofs

The following proof contains a mistake. What is it?

	R	S	$R \vee S$	$R \wedge S$
T				
	1. $R \vee S$			
	2.   R			
	3.   $S \vee R$	$\vee$ Intro: 2		
	4.   S			
	5.   $S \vee R$	$\vee$ Intro: 4		
	6. $S \vee R$	$\vee$ Elim: 1,2-3,4-5		
F	7. $R \wedge S$	$\wedge$ Intro: 2,4		

## Proof example

	1. $P \wedge Q$	
	2. P	$\wedge$ Elim: 1
	3. Q	$\wedge$ Elim: 1
	4. $\neg P \vee \neg Q$	
	5. $\neg P$	
	6. $\perp$	$\perp$ Intro: 2,5
	7. $\neg Q$	
	8. $\perp$	$\perp$ Intro: 3,7
	9. $\perp$	$\vee$ Elim: 4, 5-6, 7-8
	10. $\neg(\neg P \vee \neg Q)$	$\neg$ Intro: 4