PH126 Starting Logic Lecture 6
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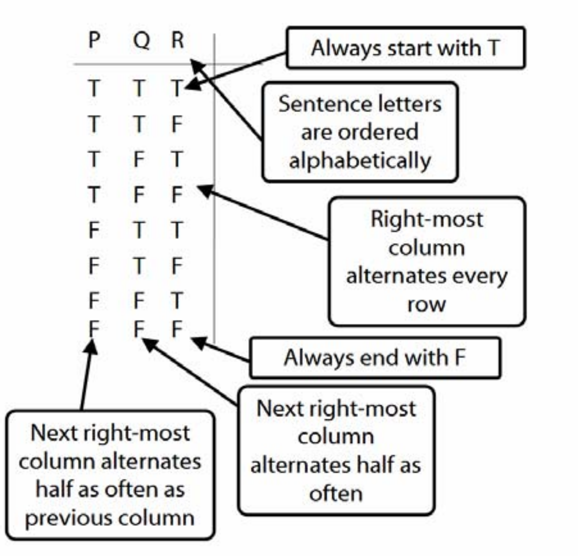
Truth table example

P	Q	R	P ∨ ¬(Q ∧ ¬(R ∨ ¬P))
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

How to order reference columns



Substitution of logical equivalents

Two sentences are *logically equivalent* when each is a logical consequence of the other.

If * and # are logically equivalent, then you take a complex formula and replace * with # without changing the truth table of the complex formula.

Example. Because ¬(R ∨ ¬P) is logically equivalent to ¬R ∧ P, it follows that: P ∨ ¬(Q ∧ ¬(R ∨ ¬P)) is logically equivalent to P ∨ ¬(Q ∧ (¬R ∧ P))

And because ¬(Q ∧ (¬R ∧ P)) is logically equivalent to ¬Q ∨ (R ∨ ¬ P), it follows that P ∨ ¬(Q ∧ (¬R ∧ P)) is logically equivalent to P ∨ (¬Q ∨ (R ∨ ¬ P)).

Subproofs

The following proof contains a mistake. What is it?

R	S	R∨S	R∧S
T	F	T	F

T 1. R∨S
2. R
3. S∨R — ∨Intro: 2
4. S
5. S∨R — ∨Intro: 4
6. S∨R — ∨Elim: 1,2-3,4-5
F 7. R∧S — ∧Intro: 2,4

∨Elim:

P1 ∨ P2
P1
…
Q
P2
…
Q
…
Q

To prove a conclusion from a disjunction, prove the conclusion from each of the disjuncts.

⊥, ¬

P	¬P	⊥	P∧¬P
T	F	F	F
F	T	F	F

Proof example

1. P∧Q
2. P — ∧Elim: 1
3. Q — ∧Elim: 1
4. ¬P ∨ ¬Q
5. ¬P
6. ⊥ — ⊥Intro: 2,5
7. ¬Q
8. ⊥ — ⊥Intro: 3,7
9. ⊥ — ∨Elim: 4, 5-6, 7-8
10. ¬(¬P ∨ ¬Q) — ¬Intro: 4