PH126 Sample Exam

Version 0.4
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Marking: 40% = pass, 70% = first class. In the exam you will be asked to answer exactly two questions on logic. There is no advantage to answering more than two questions (only two will be marked). Marks may be awarded for incomplete or incorrect answers, especially where clear explanations are provided.

References to proofs and rules in first order logic are to the system presented in Barwise & Etchemendy, Language, Proof & Logic.

Q1

a) State the rules of proof for conjunction ($\wedge$) and material implication ($\rightarrow$).

[20 marks]

b) Prove the following arguments. Only the standard rules may be used.

i)

$\neg P \vee R$

$P \rightarrow R$

[20 marks]

ii)

$(P \vee R) \rightarrow (P \rightarrow R)$

$P \rightarrow R$

[20 marks]

iii)

$\neg(P \wedge \neg Q)$

$P \rightarrow Q$

[20 marks]

iv)

$\neg(P \wedge R) \rightarrow (P \rightarrow R)$

$P \rightarrow R$

[20 marks]

Q2

a) Use truth tables to establish whether the following arguments are valid. If any arguments are invalid, state counterexamples to them. If any arguments are valid, explain carefully using the truth tables why they are valid.

i.

$P \to Q$

$\vdash \neg P \lor Q$

[10 marks]

ii.

$P \leftrightarrow (Q \to Q)$

$\vdash P \lor Q$

[10 marks]

iii.

$P \lor \neg(Q \land R)$

$\vdash P \lor (\neg Q \land R)$

[10 marks]

b) For each of the following produce logically equivalent formulae using only $\neg$, $\land$ and $\lor$:

i. $P \leftrightarrow Q$ [5 marks]

ii. $\bot$ [5 marks]

iii. $\neg(P \leftrightarrow Q) \land R$ [10 marks]

iv. $(P \land Q) \leftrightarrow (R \land S)$ [10 marks]

c) For each of the following produce logically equivalent formulae using only $\neg$, $\to$:

i. $P \leftrightarrow Q$ [10 marks]

ii. $\bot$ [10 marks]

d) State the substitution principle and describe one application of it.

[20 marks]

Q3

For each of the following arguments, give a formal proof of the argument if it is valid and give a counterexample to the argument if it is not valid.

a)

$\forall x\ S(x)$

$\forall x\ \neg S(x)$

$\vdash$

$\bot$

[20 marks]

b)

$\forall x\ (\ F(x) \rightarrow x{=}a\ )$

$\vdash$

$\neg\exists x\ (\ F(x) \wedge \neg x{=}a\ )$

[20 marks]

c)

$\exists x\ \exists y\ \neg(x{=}y)$

$\vdash$

$\exists x\ \forall y\ (x{=}y)$

[20 marks]

d)

$\exists x \forall y\ [\ F(y) \rightarrow \neg G(x,y)\ ]$

$\vdash$

$\forall y \exists x\ [\ F(y) \rightarrow \neg G(x,y)\ ]$

[20 marks]

e)

$\forall x\ F(x) \rightarrow \exists x\ G(x)$

$\vdash$

$\exists x\ [\ F(x) \rightarrow G(x)\ ]$

[20 marks]

Q4

a) Explain what it means for a relation to be (i) transitive and (ii) symmetric.

[10 marks]

b) Translate the following sentences into FOL using no predicates other than these three:

L(x,y) : x is a logical consequence of y

N(x,y) : x is the negation of y

S(x) : x is a sentence

Explain any difficulties that arise.

i. Logical consequence is a transitive relation
ii. Logical consequence is not a symmetric relation
iii. Some sentences are logically equivalent.
iv. Some sentences are contradictions.
v. All contradictions are logically equivalent.

[45 marks, 9 each]

c) For each of the following sentences of FOL, give a logically equivalent sentence of idiomatic English using the specified interpretation. Your English sentences should be as concise as possible. Describe any difficulties involved in producing English equivalents.

Domain: { people and actions }

D(x) : x is desirable

V(x) : x is virtuous

A(x) : x is an action

P(x,y) : x performed y

i. $\forall x [D(x) \rightarrow V(x)]$
ii. $\forall x [A(x) \rightarrow [D(x) \rightarrow V(x)]]$
iii. $\exists x [A(x) \wedge \neg[D(x) \rightarrow V(x)]]$
iv. $\exists x \forall y [[A(y) \wedge P(x,y)] \rightarrow V(y)]$
v. $\neg\exists x [\exists y[P(x,y) \wedge A(y) \wedge \neg V(y)] \wedge \neg\exists z[P(x,z) \wedge A(z) \wedge V(z)]]$

[45 marks, 9 each]