Deliberative Democracy and Utilitarianism

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Homogeneous Beliefs and Utilitarian Aggregation

- Bayesian paradigm
  - Both individuals and society are represented by expected utility functions
- Pareto condition
  - If all the individuals prefer one social alternative to another, then so does society
- Utilitarian aggregation of individual values
  - vNM setting: objective probability [Harsanyi, 1957]
  - State-contingent settings: identical subjective probabilities [Blackorby-Donaldon-Weymark, 1999]
- Pareto condition is equivalent to *utilitarian* aggregation

Heterogeneous Beliefs and Aggregation

Savage: individuals have double disagreement

- Heterogeneity of beliefs:
  - Individuals disagree on likelihood assessments
- Heterogeneity of tastes:
  - Individuals have different tastes over outcomes
- Under Bayesian paradigm, Pareto condition is inconsistent with non-dictatorship aggregation rule [Hylland-Zeckhauser, 1979, Mongin, 1995]
- Preferences unanimity might be spurious if such a unanimity is induced by double disagreement on belief and tastes

Motivating Example

Spurious unanimity

$p_{1,2} \diagdown \Omega$	E	E^c
p_1	0.85	0.15
p_2	0.15	0.85

and

$u_{1,2} \diagdown X$	x	y	z
u_1	1	-5	0
u_2	-5	1	0

- Both individuals prefer act $f = xEy$ to constant act $g = z$
- Pareto condition implies that society prefers f to g
- Yet, they radically diverge in beliefs and tastes
  - Can their judgment of probabilities be simultaneously correct? No...
  - Could information exchanges and probability updating provide an appropriate solution for alleviating spurious unanimity? Maybe...
- In this paper, we argue that, instead of restricting the Pareto Condition, deliberation may facilitate for possible result

Deliberative Democracy and Social Choice

Deliberative democracy

- Social decisions lie in
  - free deliberation of individuals
  - stability of choice setting from individuals to society

Social choice

- Social decisions lie in
  - rationality of individual preferences
  - unanimity principle: the Pareto Condition

— Can deliberative democracy and social choice be convergent?

— Can a deliberation produce a consensus?

— Can a consensus approach solve the issue of preference aggregation?

Deliberation Design: Suggestions

A social matrix D

- Confidence: Individuals put confidence on other individuals according to social experiments, meetings, reputation, etc.
  - Confidence impact on values: The confidence individual i puts on individual j is reflected through a weight $d_{ij}, 0 \leqslant d_{ij} \leqslant 1$, impacting i's prior values, beliefs and tastes, after meeting j

An updating rule φ

- Sequential updating: Each deliberation step, *i.e.*, meeting, debate, information exchange, etc., modifies individual priors through **D**
  - Cumulative updating: As soon as one prior is updated after a meeting, all priors are updated through 'contagion'

The Framework

- Ω is a finite set of m *states*: $\Omega = \{1, \dots, m\}$
- X is a topological space of *outcomes*
- $\mathcal{F} = \{f : \Omega \to X\}$ is the set of *social alternatives* (or *acts*)
- Society $\mathcal{I}$ is defined as a finite set of n individuals: $\mathcal{I} = \{1, \dots, n\}$
- Any individual $i \in \mathcal{I}$ has a *preference relation* $\succsim_i \subset \mathcal{F} \times \mathcal{F}$
- A *probability profile* is a vector $\overrightarrow{p} = (p_1, \dots, p_n) \in \Delta^n$, where p_i is i's subjective probability
- A *utility profile* is a vector $\overrightarrow{u} = (u_1, \dots, u_n) \in \mathbf{U}^n$, where u_i is i's utility function
- A *social profile* s consists of a pair $(\overrightarrow{p}, \overrightarrow{u})$ of a probability profile and a utility one

Bayesian Behavior

Both individual and social preferences are represented by expected utility functions

- Δ: collection of all probability measures on Ω
- $\mathbf{U}$: collection of all continuous functions on X
- for each $p \in \Delta$, $u \in \mathbf{U}$ and $f \in \mathcal{F}$, we denote

$$\mathbf{E}(f|p,u) = \int_{\Omega} u(f)\mathrm{d}p$$

  the expected utility of act f based on probability distribution p and utility function u.

Deliberation and Consensus I

When individual $i \in \mathcal{I}$ meets individuals j, he updates his prior beliefs $p_i \in \Delta$ (or u_i) accordingly:

First step: i weights j's opinion by a coefficient d_{ij}, $d_{ij} \geq 0$, translating the confidence i puts on j's information about Ω (or X)
Second step: i's posterior opinion is defined as a weighted average of all j's opinions he heard during deliberation

- A deliberation is then described by:
  - A $(n \times n)$ nonnegative, stochastic and aperiodic matrix $\mathbf{D} = (d_{ij})_{i,j \in \mathcal{I} \times \mathcal{I}}$
  - A mapping φ call *updating rule* such that, for any $i \in \mathcal{I}$, all $k \in \mathbb{N}$:

$$p_i(k+1) = \varphi\big(\mathbf{D}, p_i(k)\big)$$

  - φ satisfies: $\varphi^k(\mathbf{D}, p_i) = \varphi(\mathbf{D}^k, p_i)$, for $p_i = p_i(0)$

Deliberation and Consensus II

- A *belief k-deliberation* is as a mapping ρ_{Δ} such that, for all $k \in \mathbb{N}$, $\rho_{\Delta}(s) = \left(\varphi^k(\mathbf{D}, \overrightarrow{p}), \overrightarrow{u}\right)$
- A *taste k-deliberation* is as a mapping $\rho_{\mathbf{U}}$ such that, for all $k \in \mathbb{N}$, $\rho_{\mathbf{U}}(s) = \left(\overrightarrow{p}, \varphi^k(\mathbf{D}, \overrightarrow{u})\right)$
- A *belief consensus* is a probability profile $\overrightarrow{p}^* \in \Delta^n$ such that:

$$\lim_{k \to \infty} \varphi^k(\mathbf{D}, \overrightarrow{p}) = p^*$$

  where $p_i^* = p^*$, for all $i \in \mathcal{I}$
- A *taste consensus* is a utility profile $\overrightarrow{u}^* \in \mathbf{U}^n$ such that $\lim_{k \to \infty} \varphi^k(\mathbf{D}, \overrightarrow{u}) = \overrightarrow{u}^*$ where $u_i^* = u^*$, for all $i \in \mathcal{I}$
- The pair (p^*, u^*) designates a general consensus

Self-confidence and Consensus

Self-Confidence $(\textsc{sc})$ For any $i \in \mathcal{I}$, $0 < d_{ii} < 1$.

- For social beliefs p_0 and social utility u_0:

Theorem

Under $\textsc{sc}$*, after a deliberation* $(\rho_{\mathbf{\Delta}}, \rho_{\mathbf{U}})$*, there exists a consensus* (p^*, u^*) *such that* $p_0 = p^*$ *and* $u_0 = u^*$*.*

(The proof is based on DeGroot, 1974)

- The possibility of preference aggregation is not conceptually depending on the Pareto condition as suggested by Harsanyi
- A deliberation can play the same role than the Pareto condition as long as individuals respect self-confidence
- However, the consensus (p^*, u^*) cannot be proved to be unique since depending on the updating rule φ

Updating Rule: the Linear Case I

- A social updating rule φ is *linear average* (φ_Σ) if, for $i \in \mathcal{I}$, for all $k \in \mathbb{N}$:

$$p_i(k+1) = \varphi_\Sigma\big(\mathbf{D}, p_i(k)\big) = \sum_{j=1}^{n} d_{ij} p_j(k)$$

Proposition Assume all individuals update their beliefs according to the linear average rule. Under $\textsc{sc}$, the deliberation ρ_Δ reaches a belief consensus $\overrightarrow{p}^*_\Sigma \in \Delta^n$ such that:

$$\rho_\Delta(s) = \big(\varphi_\Sigma(\mathbf{D}^*, \overrightarrow{p}), \overrightarrow{u}\big) = \overrightarrow{p}^*_\Sigma$$

where $p^*_\Sigma = \sum_{j \in \mathcal{I}} d^*_{ij} p_j$, for all $i \in \mathcal{I}$.

Updating Rule: the Linear Case II

Suppose $\mathcal{I}$ is made of two individuals $\{1, 2\}$ and $\mathbf{D}$ defined as follows:

$$\begin{pmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{pmatrix}$$

$\mathbf{D}$ admits a limit matrix $\mathbf{D}^*$:

$$\mathbf{D}^* = \lim_{k \to \infty} \mathbf{D}^k = \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & 2/3 \end{pmatrix}$$

Suppose Ω contains two different states.
1's beliefs are given by $p_1 = (1/4, 3/4)$ and 2's beliefs by $p_2 = (3/4, 1/4)$.
The belief consensus p^*_Σ based on φ_Σ is then given by:

$$\begin{pmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{pmatrix} \cdot \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & 2/3 \end{pmatrix} = \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & 2/3 \end{pmatrix}$$

Hence, the belief consensus is $p^*_\Sigma = (1/3, 2/3)$.

Updating Rule: the Geometric Case I

- An individual i's updating rule is *geometric average* (φ_Π) if, for $i \in \mathcal{I}$, for all $k \in \mathbb{N}$,

$$p_i(k+1) = \varphi_\Pi\big(\mathbf{D}, p_i(k)\big) = \frac{\prod_j p_j(k)^{d_{ij}}}{\sum_i \prod_j p_j(k)^{d_{ij}}}$$

Proposition Assume all individuals update their beliefs according to the geometric average rule and all individual beliefs are full-support on Ω. Under $\textsc{sc}$, the deliberation ρ_Δ reaches a belief consensus $\overrightarrow{p}^*_\Pi \in \Delta^n$ where $p^*_\Pi = (\prod_j p_j^{d^*_{ij}})/(\sum_i \prod_j p_j^{d^*_{ij}})$, for all $i \in \mathcal{I}$.

Updating Rule: the Geometric Case II

Assume that both individuals follow now the geometric average rule. According to the above result:

$$p^*_\Pi(s_1) = \frac{\frac{1}{4}^{\frac{1}{3}} \cdot \frac{3}{4}^{\frac{2}{3}}}{\frac{1}{4}^{\frac{1}{3}} \cdot \frac{3}{4}^{\frac{2}{3}} + \frac{3}{4}^{\frac{1}{3}} \cdot \frac{1}{4}^{\frac{2}{3}}} \approx 0.4.$$

Therefore, the consensus belief based on the geometric average rule is $p^*_\Pi = (0.4, 0.6) \neq p^*_\Sigma$.

- This example illustrates that, for the same set of individuals and the same deliberation matrix, the consensus belief differs according to the updating rule individuals choose to use
- The question is then natural of the existence of a criterion whereby social beliefs and utilities are of a utilitarian shape

Utilitarian Rules

Definition

A belief aggregation rule ρ_{Δ} is *utilitarian* if for any social state s, there exist $\{\beta_i(s)\}_{i=1}^n$ with $\sum_{i=1}^n \beta_i(s) = 1$ such that $\rho_{\Delta}(s) = \sum_{i=1}^n \beta_i(s) \cdot p_i$.

Definition

A utility aggregation rule ρ_U is *utilitarian* if, for any social state s, there exist $\{\alpha_i(s)\}_{i=1}^n$ with $\sum_{i=1}^n \alpha_i(s) = 1$ such that $\rho_{\mathbf{U}}(s) = \sum_{i=1}^n \alpha_i(s) \cdot u_i$.

Back to Pareto

Pareto condition (PC) Given a social profile s, two acts $f, g \in \mathcal{F}$,

— if $\mathbf{E}(f|p_i, \rho_{\mathbf{U}}(s)) \geq \mathbf{E}(g|p_i, \rho_{\mathbf{U}}(s))$ for every $i \in \mathcal{I}$, then

$$\mathbf{E}(f|\rho_\Delta(s), \rho_{\mathbf{U}}(s)) \geq \mathbf{E}(g|\rho_\Delta(s), \rho_{\mathbf{U}}(s))$$

— if $\mathbf{E}(f|\rho_\Delta(s), u_i) \geq \mathbf{E}(g|\rho_\Delta(s), u_i)$ for every $i \in \mathcal{I}$, then

$$\mathbf{E}(f|\rho_\Delta(s), \rho_{\mathbf{U}}(s)) \geq \mathbf{E}(g|\rho_\Delta(s), \rho_{\mathbf{U}}(s))$$

- PC means that if all individuals prefer f to g while they are equipped with their own opinions but have already accepted to switch them for deliberated consensual opinions, then society also prefers f to g.

Theorem

PC *is satisfied if and only if the deliberation* $(\rho_{\mathbf{U}}, \rho_{\boldsymbol{\Delta}})$ *is utilitarian.*

Conclusion

- Evidences demonstrate
  - Unanimity induced by conflicting beliefs and tastes is sometimes spurious
  - The possibility of spurious unanimity makes preference aggregation impossible
- This paper argues
  - Heterogeneity of individual beliefs and tastes can be solved through deliberations
  - If a deliberation gets a consensus, this consensus is not unique
  - The Pareto condition allows to select among consensus the one that corresponds to a utilitarian aggregation