Partial Utilitarianism

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Social welfare criteria

Comparing social alternatives requires a social welfare criterion.

- Utilitarianism (Bentham): $\sum_i u_i$.
- Egalitarianism (Rawls): $\min_i u_i$.

Axiomatic foundations can guide the choice of a criterion.

Harsanyi's Aggregation Theorem. If alternatives are risky and both individuals and society conform to the EU model, then the only criteria satisfying the Pareto principle are of the form $\sum_i \theta_i u_i$.

Incomplete preferences

Individual preferences may leave some alternatives unranked.

- Aumann (1962); Bewley (1986); Shapley and Baucells (1998); Ok (2002); Dubra et al. (2004); **?**); Ok et al. (2012); Galaabaatar and Karni (2013); Riella (2015); McCarthy et al. (2021).

Taste uncertainty, multiple selves / rationales, decision frames, ancillary conditions.

- Koopmans (1964); Kreps (1979); Dekel et al. (2001); May (1954); Kalai et al. (2002); Salant and Rubinstein (2008); Bernheim and Rangel (2009); Ambrus and Rozen (2014).

Partial identification of individual preferences by the social planner.

- Manski (2005, 2010, 2013).

Social welfare with incomplete preferences

Incomplete preferences are not representable by a utility function.

Suitable social welfare criteria?

In this paper:

- Risk; assume EU (except completeness) for individuals; more general for society.
- Multi-profile: extension of Sen (1970)'s social welfare functionals.

Multi-profile aggregation

Harsanyi's theorem is a single-profile result:

- Individual preference profile $(\succsim_i)$; social preference $\succsim_0$.
- EU + Pareto $\Rightarrow \sum_i \theta_i u_i$.
- θ generally not unique for fixed (u_i); almost arbitrary without fixing (u_i); varies arbitrarily with $(\succsim_i)$.

Mongin (1994) proved a multi-profile refinement:

- Social welfare functional $(u_i) \mapsto \succsim_0$.
- EU + Pareto + IIA $\Rightarrow \sum_i \theta_i u_i$ with θ independent of (u_i).
- θ unique.
- + Anonymity $\Rightarrow \sum_i u_i$.

Here: extended social welfare functional allowing for incompleteness.

Main results

Impossibility: EU, Pareto, and IIA are incompatible.

- Unlike Harsanyi and Mongin: utilitarianism violates IIA.
- Impossibility persists under weakened IIA (relative utilitarianism).

Characterizations relaxing social EU (Completeness; Independence).

- Partially utilitarian: rely on a set of utilitarian criteria.
- Keep Independence for coherence: unanimity representations.
- Keep Completeness for decisiveness: max-min representations.
- Two-stage representations: first coherence then decisiveness.

Additional results

- Constant-linear representations (generalizez max-min).
- Special cases (Anonymity, Strict Pareto, . . .).
- Interpersonal utility comparisons (informational invariance).

Related literature

Danan et al. (2013): multi-profile aggregation of incomplete EU preferences.

- Characterization relaxing Completeness; impossibility of utilitarian aggregation.
- But more restrictive setting; stronger IIA; no relaxation of Independence.

Danan et al. (2015): single-profile aggregation of incomplete EU preferences.

- Utilitarian aggregation possible.
- But identification issues even more severe.

Relaxing Completeness or Independence in various social choice settings:

- Gajdos et al. (2008); Crès et al. (2011); Pivato (2011, 2013, 2014); Nascimento (2012); Gajdos and Vergnaud (2013); Chambers and Hayashi (2014); Qu (2015); Danan et al. (2016); Alon and Gayer (2016); Zuber (2016); McCarthy et al. (2019, 2020).

Outline

Alternatives, preferences, utility

Social welfare functionals and utilitarianism

Extended social welfare functionals and impossibility of utilitarianism

Partial utilitarianism: unanimity representations

Partial utilitarianism: max-min representations

Partial utilitarianism: two-stage representations

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Alternatives, preference relations

X: set of alternatives.

- Convex subset of some vector space; at least 3 affinely independent alternatives.
- E.g. $X = \Delta(Z)$ with $|Z| \geqslant 3$.

$\succsim \subseteq X \times X$: (weak) preference relation on X.

- $\succ$: strict preference; $\sim$: indifference.

Properties of preference relations

Weak order:

- Completeness: $x \gtrsim y$ or $y \gtrsim x$.
- Transitivity: $x \gtrsim y \gtrsim z \Rightarrow x \gtrsim z$.

Preorder:

- Reflexivity: $x \gtrsim x$.
- Transitivity.

EU weak order / EU preorder:

- Weak order / preorder.
- Independence: $x \gtrsim y \Leftrightarrow \lambda x + (1 - \lambda) z \gtrsim \lambda y + (1 - \lambda) z$.
- Continuity: $\{\lambda : x \gtrsim \lambda y + (1 - \lambda) z\}$ and $\{\lambda : \lambda y + (1 - \lambda) z \gtrsim x\}$ are closed.

Utility

$u \in \mathbb{R}^X$: utility function on X.

$U \subseteq \mathbb{R}^X$: utility set on X.

u is a vNM utility function if $u(\lambda x + (1-\lambda)y) = \lambda u(x) + (1-\lambda)u(y)$.

U is a vNM utility set if it is non-empty, compact, and convex and each $u \in U$ is vNM.

- $\mathbb{R}^X$ with product topology; {vNM utility functions} with subspace topology.

Utility representation

U represents $\succsim$ if $x \succsim y \Leftrightarrow [u(x) \geqslant u(y)$ for all $u \in U]$.

- If $U = \{u\}$ then say that u represents $\succsim$.

$\succsim$ can be represented by some vNM utility function iff $\succsim$ is an EU weak order (von Neumann and Morgenstern, 1944; Herstein and Milnor, 1953).

If X is finite-dimensional, then $\succsim$ can be represented by some vNM utility set iff $\succsim$ is an EU preorder (Shapley and Baucells, 1998; Dubra et al., 2004).

If X is infinite-dimensional, $\succsim$ being an EU preorder is necessary but generally not sufficient for such a representation to exist.

Outline

Social welfare functionals

I: set of individuals.

- Non-empty; finite.

$f : \{\text{vNM utility functions}\}^I \to \{\text{EU weak orders}\}$: social welfare functional.

- $\succsim_{(u_i)} := f((u_i))$.

Axioms

Pareto: if $u_i(x) \geqslant u_i(y)$ for all i then $x \succsim_{(u_i)} y$.

Pareto Indifference: if $u_i(x) = u_i(y)$ for all i then $x \sim_{(U_i)} y$.

IIA: if $u_i(x) = v_i(x)$ and $u_i(y) = v_i(y)$ for all i then $x \succsim_{(u_i)} y \Leftrightarrow x \succsim_{(v_i)} y$.

Mongin's theorem

Theorem (Mongin, 1994). f satisfies Pareto Indifference, IIA, and Non-Triviality iff there exists a $\theta \in \mathbb{R}^I$ such that the vNM utility function

$$\sum_i \theta_i u_i$$

represents $\succsim_{(u_i)}$. Moreover:

- θ is unique up to a positive scale factor.
- f satisfies Pareto iff $\theta \geqslant 0$.

Mongin's theorem – Proof sketch

Pareto Indifference + IIA $\Rightarrow$ Neutrality: $x \lesssim\gtrsim_{(u_i)} y$ fully determined by $(u_i(x), u_i(y))$.

Neutrality $\Rightarrow$ welfarism: F boils down to a weak order $\gtrsim$ on $\mathbb{R}^I$.

All $\gtrsim_{(u_i)}$'s are EU weak orders $\Rightarrow$ $\gtrsim$ is also an EU weak order.

θ is then obtained from any vNM utility representation of $\gtrsim$.

Outline

Extended social welfare functionals

f : {vNM utility functions}$^I \to$ {EU weak orders} : social welfare functional.

F : {vNM utility sets}$^I \to$ {preorders} : extended social welfare functional.

- $\succsim_{(U_i)} = F((U_i))$.

Axioms

Completeness: $\succsim_{(U_i)}$ satisfies Completeness.

Independence: $\succsim_{(U_i)}$ satisfies Independence.

Continuity: $\succsim_{(U_i)}$ satisfies Continuity.

Axioms

Pareto: if $u_i(x) \geqslant u_i(y)$ for all i and all $u_i \in U_i$ then $x \gtrsim_{(U_i)} y$.

Pareto Indifference: if $u_i(x) = u_i(y)$ for all i and all $u_i \in U_i$ then $x \sim_{(U_i)} y$.

Given $Y \subset X$, $U|_Y := \{(u(x))_{x \in Y} : u \in U\}$.

IIA: if $U_i|_{\{x,y\}} = V_i|_{\{x,y\}}$ for all i then $x \gtrsim_{(U_i)} y \Leftrightarrow x \gtrsim_{(V_i)} y$.

Non-Triviality: $x \not\sim_{(U_i)} y$ for some (U_i) and some x, y.

Impossibility result

Theorem 7. F cannot satisfy Pareto Indifference, IIA, Completeness, Independence, and Non-Triviality.

Impossibility does not rely on Continuity.

Unlike Harsanyi and Mongin's theorems.

Utilitarianism, in the sense that $\succsim_{(U_i)}$ can be represented by $\sum_i \theta_i u_i$ for some selection $(u_i) \in \prod_i U_i$, violates IIA.

Impossibility persists under weakening of IIA (relative utilitarianism).

Impossibility result – Proof sketch

Pareto Indifference + IIA $\Rightarrow$ Neutrality: $x \lesssim\gtrsim_{(U_i)} y$ fully determined by $(U_i|_{\{x,y\}})$.

Neutrality $\not\Rightarrow$ welfarism: $U_i|_{\{x,y\}}$ not fully determined by $U_i|_{\{x\}}$ and $U_i|_{\{y\}}$.

But two key properties follow from Independence:

1. $x \lesssim\gtrsim_{(U_i)} y$ fully determined by $(U_i|_y^x := \{(u_i(x) - u_i(y) : u_i \in U_i\})$.
2. If $U_i|_y^x \subseteq V_i|_y^x$ for all i then $x \gtrsim_{(V_i)} y \Rightarrow x \gtrsim_{(U_i)} y$.

Now for any $(U_i), x, y$:

- There exists (V_i) such that $U_i|_y^x \subseteq V_i|_y^x = V_i|_x^y$ for all i,
- Hence $x \sim_{(V_i)} y$ by key property 1 and Completeness,
- Hence $x \sim_{(U_i)} y$ by key property 2.

This contradicts Non-Triviality.

Impossibility result – Weakening IIA

Restricted IIA: if $U_i|_{\{x,y\}} = V_i|_{\{x,y\}}$ then $x \succsim_{(U_i,(\{0\})_{j\neq i})} y \Leftrightarrow x \succsim_{(V_i,(\{0\})_{j\neq i})} y$.

Satisfied by relative utilitarianism (Dhillon, 1998; Dhillon and Mertens, 1999).

Extended PI: if $x \sim_{((U_i)_{i\in J},(\{0\})_{i\notin J})} y$ and $x \sim_{((\{0\})_{i\in J},(U_i)_{i\notin J})} y$ then $x \sim_{(U_i)} y$.

Under Restricted IIA, Extended PI $\Rightarrow$ PI.
Under IIA and Independence, Extended PI $\Leftrightarrow$ PI.

Theorem 7 (ct'd). F cannot satisfy Extended PI, Restricted IIA, Completeness, Independence, and Non-Triviality.

Outline

Notation

$\Delta_I := \{\theta \in \mathbb{R}_+^I : \sum_i \theta_i = 1\}.$

$\Delta_{2I} := \{(\beta, \gamma) \in \mathbb{R}_+^{2I} : \sum_i \beta_i + \gamma_i = 1\}.$

Unanimity representation – Pareto Indifference

Theorem 8. F satisfies Pareto Indifference, IIA, Independence, Continuity, and Non-Triviality iff there exists a non-empty, compact, and convex $\Phi \subseteq \Delta_{2I}$ such that the vNM utility set

$$\left\{ \sum_i \beta_i u_i - \gamma_i v_i : (\beta, \gamma) \in \Phi, (u_i, v_i) \in \prod_i U_i^2 \right\}$$

represents $\succsim_{(U_i)}$. Moreover, Φ is unique up to "redundant" weights.

Partially utilitarian: unanimity across a set of utilitarian criteria.

The larger this set, the more incomplete social preferences.

$\Phi = \Delta_{2I}$: Pareto-indifference relation.

Unanimity representation – Pareto

Theorem 2. F satisfies Pareto, IIA, Independence, Continuity, and Non-Triviality iff there exists a non-empty, compact, and convex $\Theta \subseteq \Delta_I$ such that the vNM utility set

$$\left\{ \sum_i \theta_i u_i : \theta \in \Theta, (u_i)_i \in \prod_i U_i \right\}$$

represents $\succsim_{(U_i)}$. Moreover Θ is unique.

$\Theta = \Delta_I$: Pareto-dominance relation.

Unanimity representations – Proof sketch

Pareto Indifference + IIA + Independence $\Rightarrow$ the two key properties above.

By key property 1, it suffices to characterize the set $K = \{(U_i|_y^x) : x \succsim_{(U_i)} y\}$.

Because each $U_i|_y^x$ is a compact real interval, K is essentially a subset of $\mathbb{R}^{2I}$. By Independence and Continuity, K is a closed convex cone.

$\Phi \subseteq \mathbb{R}^{2I}$ is then obtained from the polar cone of K. Key property 2 and Non-Triviality ensure $\Phi \subseteq \Delta_{2I}$.

Finally, Pareto ensures $\beta = 0$ for all $(\alpha, \beta) \in \Phi$, so we set $\Theta = \{\alpha : (\alpha, 0) \in \Phi\}$.

Outline

Axioms

Two weakenings of Independence and one strengthening of Non-Triviality.

x is egalitarian in (U_i) if $u_i(x) = u_j(x)$ for all i, j and all $u_i \in U_i, u_j \in U_j$.

Egalitarian Independence: if z is egalitarian in (U_i) then
$x \succsim_{(U_i)} y \Leftrightarrow \lambda x + (1-\lambda)z \succsim_{(U_i)} \lambda y + (1-\lambda)z$.

Inequality Aversion $x \sim_{(U_i)} y \Rightarrow 0.5x + 0.5y \succsim_{(U_i)} y$.

Formally similar to Gilboa and Schmeidler (1989).

Egalitarian Non-Triviality: $x \not\sim_{(U_i)_{i\in I}} y$ for some (U_i) and some egalitarian x, y in (U_i).

Notation

$$\hat{\Delta}_{2l} := \{(\beta, \gamma) \in \Delta_{2l} : \sum_i \beta_i - \gamma_i \neq 0\}.$$

Max-min representation – Pareto Indifference

Theorem 9. F satisfies Pareto Indifference, IIA, Completeness, Egalitarian Independence, Inequality Aversion, Continuity, and Egalitarian Non-Triviality iff there exists a non-empty, compact, and convex $\Phi \subseteq \hat{\Delta}_{2I}$ such that the utility function

$$x \mapsto \min_{(\beta,\gamma)\in\Phi} \frac{\sum_i \beta_i \min_{u_i \in U_i} u_i(x) - \gamma_i \max_{v_i \in U_i} v_i(x)}{\left|\sum_i \beta_i - \gamma_i\right|}$$

represents $\succsim_{(U_i)}$. Moreover, Φ is unique up to "redundant" weights.

Partially utilitarian: least favorable of a set of utilitarian criteria.

The larger this set, the more violations of Independence.

Max-min representation – Pareto

Theorem 3. F satisfies Pareto, IIA, Completeness, Egalitarian Independence, Inequality Aversion, Continuity, and Non-Triviality iff there exists a non-empty, compact, and convex $\Theta \subseteq \Delta_I$ such that the utility function

$$x \mapsto \min_{\theta \in \Theta} \sum_i \theta_i \min_{u_i \in U_i} u_i(x)$$

represents $\succsim_{(U_i)}$. Moreover, Θ is unique.

$\Theta = \Delta_I$: (extended) egalitarianism.

Max-min representations – Proof sketch

Pareto Indifference + IIA + Egalitarian Independence + Inequality Aversion $\Rightarrow$ the two key properties above hold provided y is egalitarian.

Completeness and Egalitarian Non-Tvitiality $\Rightarrow$ every alternative has an "egalitarian equivalent."

Hence by key property 1, it suffices to characterize the set $\hat{K} = \{(U_i|_y^x) : x \succsim_{(U_i)} y, y \text{ egalitarian}\}$.

This is done as above, using Inequality Aversion to prove convexity of $\hat{K}$.

$\Phi \subseteq \mathbb{R}^{2I}$ is again obtained from the polar cone of K. Key property 2 and Egalitarian Non-Triviality ensure $\Phi \subseteq \hat{\Delta}_{2I}$.

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Partial utilitarianism: unanimity representations

Partial utilitarianism: max-min representations

Partial utilitarianism: two-stage representations

Completeness vs. Independence

Completeness and Independence fulfill different goals:

- Completeness enables social preferences to guide every possible decision to be made.
- Independence ensures social preferences provide a coherent guidance.

Since these two goals are incompatible, society could give up one of them.

Or it could adopt a two-stage decision process:

- First seek to rely on a coherent guidance.
- When it is indecisive, fall back to a less coherent but fully decisive guidance.

Two-stage social decision process

Two extended social welfare functionals:

- A coherent one $F^* = \succsim^*_{(\cdot)}$ satisfying Independence.
- A decisive one $F^\wedge = \succsim^\wedge_{(\cdot)}$ satisfying Completeness.

Formally similar to Gilboa et al. (2010), although no incompatibility in their setting.

Axioms

Consistency: $x \succsim^*_{(U_i)} y \Rightarrow x \succsim^\wedge_{(U_i)} y$.

Egalitarian Default: if y is egalitarian in (U_i) then $x \not\succsim^*_{(U_i)} y \Rightarrow y \succsim^\wedge_{(U_i)} x$.

x egalitarian dominates y in (U_i) if $z \succsim^*_{(V_i)} x \Rightarrow z \succsim^*_{(V_i)} y$ and $y \succsim^*_{(V_i)} z \Rightarrow x \succsim^*_{(V_i)} z$ whenever z is egalitarian in (V_i) and $U_i|_{\{x,y\}} = V_i|_{\{x,y\}}$ for all i.

Egalitarian Dominance: if x egalitarian dominates y in (U_i) then $x \succsim^\wedge_{(U_i)} y$.

Consistency + Egalitarian Default $\Rightarrow$ Egalitarian Dominance $\Rightarrow$ Consistency.

Two-stage representation – Egalitarian Default

Theorem 4. The following are equivalent:

- F^* satisfies Pareto, IIA, Independence, and Continuity.
  $F^\wedge$ satisfies IIA, Completeness, Egal. Independence, Continuity, and Non-Triviality.
  $(F^*, F^\wedge)$ satisfy Consistency and Egalitarian Default.
- There exists a non-empty, compact, and convex $\Theta \subseteq \Delta_I$ representing F^* as per Theorem 2 and $F^\wedge$ as per Theorem 3.

Moreover, Θ is unique.

Alternative foundation for the max-min representation.

Diamond (1967)'s critique of Harsanyi's theorem.

Two-stage representation – Egalitarian Dominance

Theorem 6. The following are equivalent:

- F^* satisfies Pareto, IIA, Independence, and Continuity.
  $F^\wedge$ satisfies IIA, Completeness, Egal. Independence, Continuity, and Non-Triviality.
  $(F^*, F^\wedge)$ satisfy Egalitarian Dominance.
- There exists a non-empty, compact, and convex $\Theta \subseteq \Delta_I$ representing F^* as per Theorem 2 and a constant $\alpha \in [0, 1]$ such that the utility function

$$x \mapsto \alpha \min_{\theta \in \Theta} \sum_i \theta_i \min_{u_i \in U_i} u_i(x) + (1 - \alpha) \max_{\theta \in \Theta} \sum_i \theta_i \max_{u_i \in U_i} u_i(x)$$

  represents $\succsim^\wedge_{(U_i)}$.

Moreover, Θ and α are unique.

Allows more general inequality attitudes.

Conclusion

Extend Mongin (1994)'s multi-profile refinement of Harsanyi's aggregation theorem by allowing individual preferences to be incomplete.

Impossibility result: social preferences cannot be utilitarian.

Characterize two forms of partial utilitarianism by relaxing EU axioms at the social level:

- A coherent one relying on unanimity across a set of utilitarian criteria.
- A decisive one relying on the least favorable of these criteria.

Distinction between coherent and decisive social preferences allows in a sense to retain all the EU axioms, albeit not simultaneously.

Could alternatively look for a single social preference relation reflecting some compromise between coherence and decisiveness.

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