Disagreement Aversion

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Slides: bit.ly/disag_av

Motivation

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Disagreement aversion? Sure Project.

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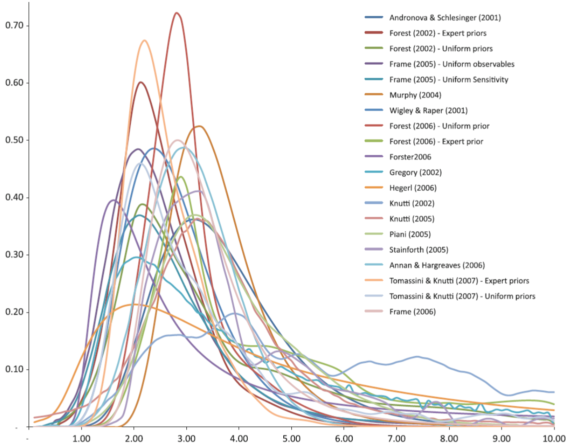
- **Implications of disagreement aversion in concrete applications.**
  - EU-aggregating models: ore ambiguity aversion $\not\Rightarrow$ more cautious choices.
  - Distribution-aggregating models: more disagreement aversion $\Rightarrow$ more cautious choices.

Why do we care?

In various applications, the decision-maker may want to aggregate experts' distributions in a cautious way, e.g.:

- Warming potential of CO_2.
- Asset returns.
- Effectiveness and side effects of a vaccine.

Figure: Estimated probability density functions for climate sensitivity from a variety of published studies, collated by Meinshausen and al. (2009), taken from Millner, Dietz & Heal (2013).



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- Is relevant to applied studies of decision-making under uncertainty.
  - Gollier (ReStud, 11); Millner et al. (13); Berger (14); Berger et al. (21).

Setting

- Set of states of the world: Ω.
- Space of outcomes: $X = [X^-; X^+] \in \mathbb{R}$.
- Choice $\alpha : \Omega \to X$. Its image: $\alpha_1 < \ldots < \alpha_{K_\alpha}$. Sure choice of outcome x: x.
- Expert i. P_i denotes i's belief, i.e. their subjective probability measure on Ω.
- Expertise. $\mathcal{P} = (P_1, \ldots, P_N)$ is the expertise, given N *experts* $\{1; \ldots; N\}$.
- A decision-rule $\succcurlyeq : \mathcal{P} \mapsto \succcurlyeq^{\mathcal{P}}$ maps expertises to preferences over choices.
- Introductory example:

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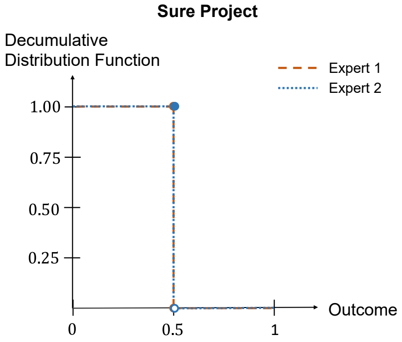

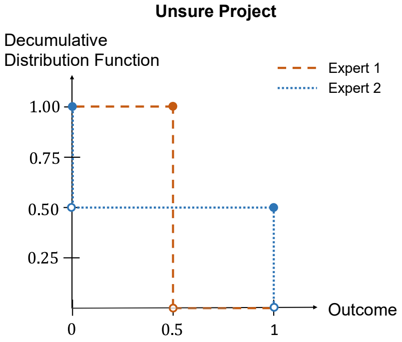

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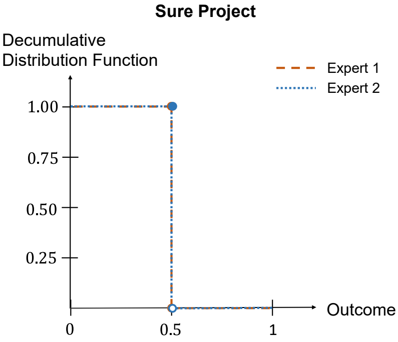

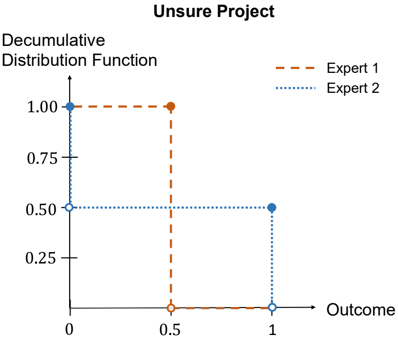

Risk preferences

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$$U(\alpha) = \mathbb{E}_{p^1}[u(\alpha)] \text{ in which } u \text{ is an increasing bijection of } [0,1].$$

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$$\mathbb{E}_{p^1}[u(\alpha)] = \sum_{k=1}^{K_\alpha - 1} u(\alpha_k)\left(p_k^1 - p_{k+1}^1\right) + u(\alpha_{K_\alpha})\, p_{K_\alpha}^1$$

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Notions of consensus

- Utility consensus:
  α is utility-consensual if experts agree on its expected utility level

$$\forall i,\ \mathbb{E}_{p^i}\left[u\left(\alpha\right)\right] = \mathbb{E}_{p^1}\left[u\left(\alpha\right)\right]$$

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EU-aggregating decision-rules

- *Adapted from Cerreia-Vioglio and al. (2011).* $\succcurlyeq$ is said *EU-aggregating* (or MBA) if its risks preferences are EU and if it admits a representation (u, I) of the form:

$$U_{UA}(\alpha) = I\left(\mathbb{E}_{p^1}[u(\alpha)], \ldots, \mathbb{E}_{p^N}[u(\alpha)]\right)$$

Aggregators

Recall the EU-aggregating representation:

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An aggregator I is a continuous function from $[0,1]^N$ to $[0,1]$ which is component-wise increasing and fulfills $I(q, \ldots, q) = q$.

Examples of aggregators:

- Arithmetic mean (linear pooling): $I_{linear}(u_1, \ldots, u_N) = \frac{1}{N}\sum_{i=1}^N u_i$.
- Min: $I_{\min}(u_1, \ldots, u_N) = \min_{1 \leq i \leq N}\{u_i\}$.
- Second-order EU (KMM): $I_{KMM}(u_1, \ldots, u_N) = \psi^{-1}\left(\frac{1}{N}\sum_{i=1}^N \psi(u_i)\right)$ where ψ is smooth and increasing, e.g. $\psi : x \mapsto -e^{-\lambda x}$.

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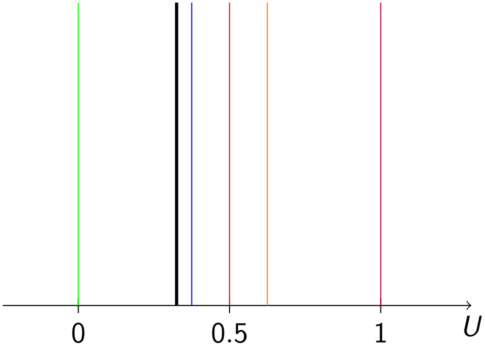
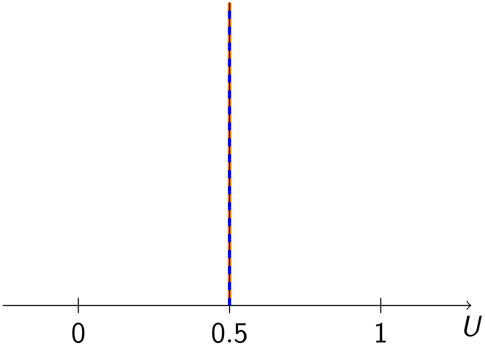
Aggregation of utilities: an illustration



Aggregation of utilities in our example



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- **Definition. Pareto condition.**
A decision-rule $\succcurlyeq$ is *Paretian* if for all expertise $\mathcal{P} = (P_1, \cdots, P_N)$ and choices α, β:

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Disagreement aversion

- **Definition.** $\succcurlyeq$ is said *disagreement averse* if for all expertise $\mathcal{P}$, for all outcome x, for all choice α that is not distribution-consensual:

$$\left(x \succcurlyeq^{P_i} \alpha,\ \forall i\right) \Rightarrow x \succ^{\mathcal{P}} \alpha$$

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Lemma. Violation of the Pareto Condition.

There is no decision-rule with EU risk preferences which is Paretian and exhibits disagreement aversion.

Comparative aversion

- **Definition. Comparative disagreement aversion.**
  Take two decision-rules $\succsim_A$ and $\succsim_B$ that share the same EU preferences (u) on distribution-consensual choices. $\succsim_A$ is *more disagreement averse than* $\succsim_B$ if for all non-**distribution**-consensual choices α, all expertise $\mathcal{P}$, all sure choices x,

$$x \sim_B^{\mathcal{P}} \alpha \Rightarrow x \succ_A^{\mathcal{P}} \alpha$$

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Distribution-aggregating decision-rules

- A decision-rule $\succcurlyeq$ is said *distribution-aggregating*, with representation (u, I), if there exist a utility-index u and an aggregator I s.t.:

$$U_{DA}(\alpha) = \mathbb{E}_{I(p^1,\ldots,p^N)}[u(\alpha)] = \sum_{k=1}^{K_\alpha} \Delta u_k I\left(p_k^1, \ldots, p_k^N\right)$$

- Recall the representation of EU-aggregating rules:

$$U_{UA}(\alpha) = I\left(\mathbb{E}_{p^1}[u(\alpha)], \ldots, \mathbb{E}_{p^N}[u(\alpha)]\right) = I\left(\sum_{k=1}^{K_\alpha} \Delta u_k p_k^1, \ldots, \sum_{k=1}^{K_\alpha} \Delta u_k p_k^N\right)$$

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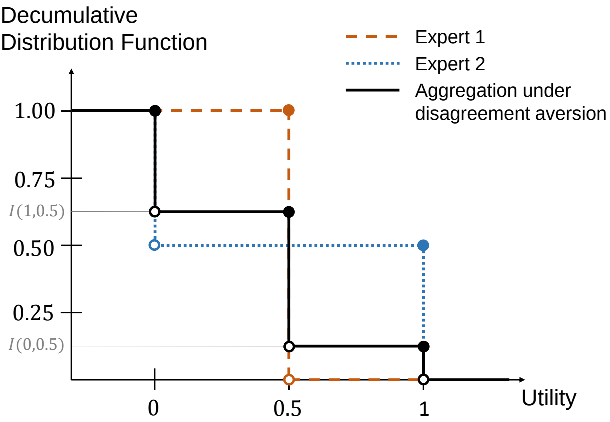
Distribution-aggregating in the example



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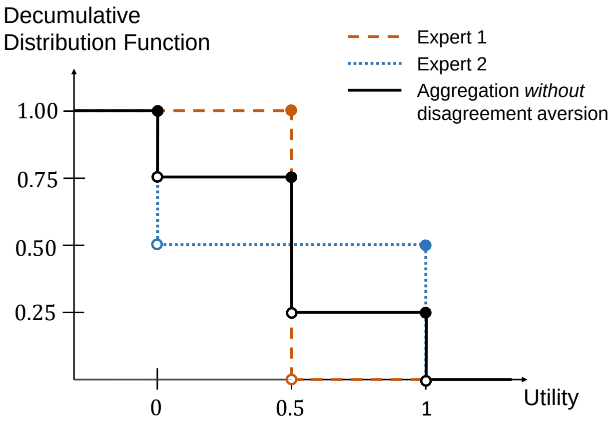

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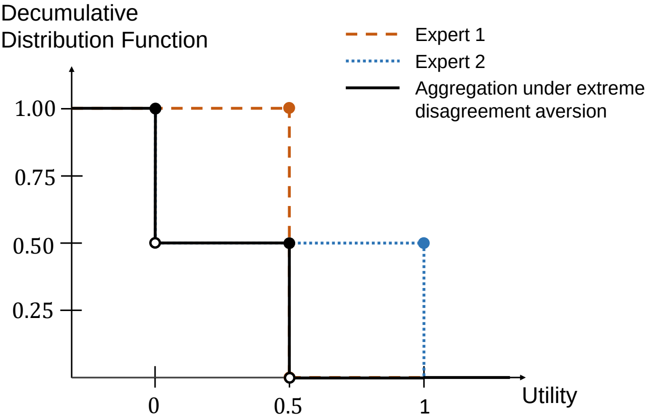

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Characterization of cautiousness

Sufficient conditions for disagreement aversion (each one suffices) ▸ See proof ▸ See NSC

- The aggregator $I : [0,1]^N \to [0,1]^N$ is strictly concave except on constant vectors.
- There exist weights $\lambda_i \geq 0$ summing to 1 s.t. $I(p_1, \ldots, p_N) < \sum_{i=1}^{N} \lambda_i p_i$ for all non constant vector $(p_1, \ldots, p_N) \in [0,1]^N$.

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Consider two distribution-aggregating decision-rules $\succcurlyeq_A$ and $\succcurlyeq_B$ with aggregator I_A and I_B. Then $\succcurlyeq_A$ is more disagreement averse than $\succcurlyeq_B$ if and only if $I_A(\vec{p}) < I_B(\vec{p})$ for all non-constant vector $\vec{p} = (p_1, \ldots, p_N) \in [0,1]^N$ and both share EU preferences u.

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Characterization of comparative ambiguity aversion ▸ See proof

Take $\succcurlyeq_A$ and $\succcurlyeq_B$, either both distribution-aggregating or both EU-aggregating. Then $\succcurlyeq_A$ is more ambiguity averse than $\succcurlyeq_B$ if and only if $I_A(\vec{p}) < I_B(\vec{p})$ for all non-constant vector $\vec{p}$ and both share EU preferences u.

Link between distribution-aggregating and EU-aggregating rules

Proposition: "distribution-aggregating is more cautious than EU-aggregating" ▸ See proof

For any (u, I), take the distribution-aggregating decision-rule $\succcurlyeq_{DA}$ and the EU-aggregating decision rule $\succcurlyeq_{UA}$ with representations (u, I).
If I is strictly concave then $\succcurlyeq_{DA}$ exhibits more ambiguity aversion and more disagreement aversion than $\succcurlyeq_{UA}$.

Proposition: distribution-aggregating $\cap$ Paretian $=$ linear pooling ▸ See proof

A decision-rule is both distribution-aggregating and EU-aggregating if and only if its aggregator I is linear, i.e. there are weights λ_i s.t. $I(p_1, \ldots, p_N) = \sum_i \lambda_i p_i$.

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Distribution-aggregating is more cautious than EU-aggregating

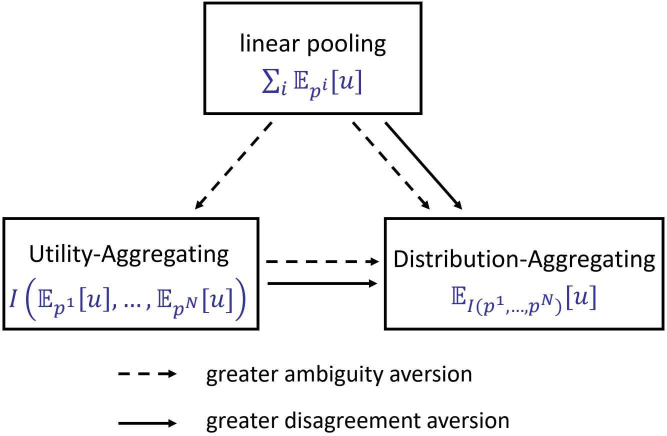

Figure: The relation between decision-rules when I is strictly concave.

Axiomatization

The main axioms are: ▸ See proof

- EU risk preferences.
- monotonicity with respect to first-order stochastic dominance (M-FSD)
- the comonotonic sure-thing principle.
- continuity.

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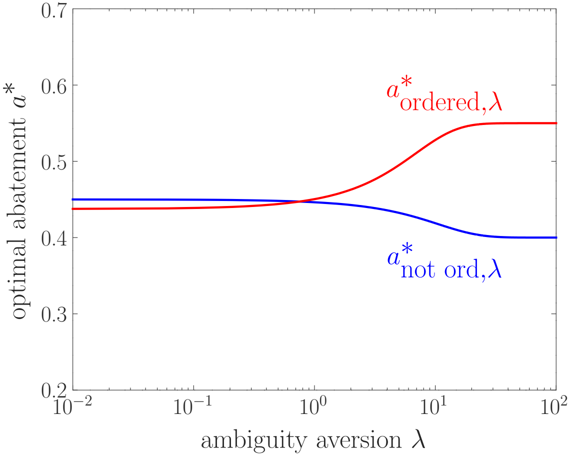
We add a last axiom to separate outcomes and probabilities and simplify the formula.

Impact of greater ambiguity aversion for EU-aggregating models

The ex-post utility has now the form $u(a, \omega)$, with a a choice variable and ω a contingency. With the example of climate, a is abatement and ω is the climate *in*sensitivity:



Impact of greater disagreement/ambiguity aversion

Assume a constant-sign cross-derivative. A's decision is more cautious than B's if: ▸ See why

EU-aggregating case:

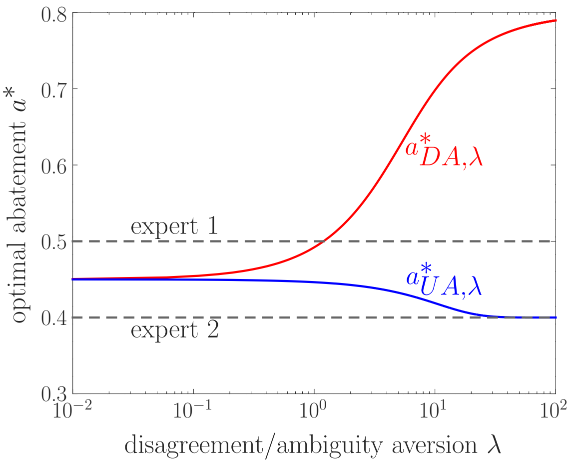
- $I_A < I_B$
- experts' beliefs are ordered in terms of optimism (FOSD), and
- I_A, I_B have KMM forms.

Distribution-aggregating case:

- $I_A < I_B$

Impact of greater ambiguity aversion for utility- vs. distribution-aggregating

Example where experts' beliefs are *not* ordered in terms of optimism (FOSD).



Take home messages

- We provide a cautious, probability-aggregating model where EU uses a certainty-equivalent probability distribution of outcomes.
- It is sensitive to disagreement over underlying beliefs, not only over utilities.
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Thank you!

Working paper: bit.ly/disagreement_aversion

Proof of representation result

Back to properties

1. *Definition of I, $n=1$.* EU on consensual choices gives u. Represent choices by pair: image $\overrightarrow{x}$, probas $\mathrm{p} = (\overrightarrow{p_k})_k$, denote $\overrightarrow{X} = (\underline{X}, \overline{X})$. For $\mathcal{P}$, $\exists! x, p$ such that $\left(\overrightarrow{X}, \overrightarrow{p}\right) \sim \overline{x} \sim \left(\overrightarrow{X}, \overline{p}\right)$. Define $\widetilde{I}\left(\overrightarrow{p}\right) = p$, $I = f \circ \widetilde{I} \circ f^{-1}$.
2. *Case n=2.* Take any $\overrightarrow{y} = (y_0, y_1)$ and let p, y be unique pair s.t. $\left(\overrightarrow{y}, \overrightarrow{p}\right) \sim \overline{y} \sim \left(\overrightarrow{y}, \overline{p}\right)$. By EU on consensual choices $\overline{y} \sim \left(\overrightarrow{y}, \overline{p}\right)$ implies $U\left(\overline{y}\right) = U\left(\overrightarrow{y}, \overline{p}\right)$ and by level-independent disagreement aversion: $\left(\overrightarrow{y}, \overrightarrow{p}\right) \sim \left(\overrightarrow{y}, \overline{p}\right) \Leftrightarrow \left(\overrightarrow{X}, \overrightarrow{p}\right) \sim U\left(\overrightarrow{y}, \overline{p}\right)$ so $U\left(\overline{y}\right) = U\left(\overrightarrow{y}, \overrightarrow{p}\right)$.
3. *General case* Assume result shown when support of size $\leq n$, take $\alpha = \left(\left(x_1, \ldots, x_{n-1}, x_n, x_{n+1}\right), \mathrm{p}\right)$. By continuity, there exist $\widehat{x_n}, \widehat{x}$ s.t. $\alpha \sim \overline{\widehat{x}} \sim \left(\left(x_1, \ldots, x_{n-1}, \widehat{x_n}, \widehat{x_n}\right), \mathrm{p}\right) =: \widehat{\alpha}$. By the comonotonic sure-thing principle: $\left(\left(x_1, \ldots, x_n, x_n, x_{n+1}\right), \mathrm{p}\right) \sim \left(\left(x_1, \ldots, x_n, \widehat{x_n}, \widehat{x_n}\right), \mathrm{p}\right)$. By induction, representation hold for these transformed choices, so we can equate their formulas. Re-arranging terms, we obtain $U\left(\alpha\right) = U\left(\widehat{\alpha}\right)$. As $\widehat{\alpha}$ of size n, $U\left(\alpha\right) = U\left(\overline{\widehat{x}}\right)$. ■

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Proof of proposition on the impact of disagreement aversion

Back to applications

The first-order condition of the maximization problem is:

$$\underbrace{\sum_{k=2}^{K} \left(\partial_1 u(a, t_k) - \partial_1 u(a, t_{k-1})\right) I_J \left(p_k^1, \ldots, p_k^N\right) + \partial_1 u(a, t_1)}_{U_J'(a)} = 0,$$

given that $I_J\left(p_1^1, \ldots, p_1^N\right) = 1$. Since decision-maker A is more disagreement averse than B, we have $I_A\left(p_k^1, \ldots, p_k^N\right) \leq I_B\left(p_k^1, \ldots, p_k^N\right)$ for all k with strict inequality for some k since the group of experts disagrees. If $\partial_1 u(a, t)$ strictly increases with t, we have $U_A'(a) < U_B'(a)$ and $a_A^* < a_B^*$. If $\partial_1 u(a, t)$ strictly decreases with t, we have $U_A'(a) > U_B'(a)$ and $a_A^* > a_B^*$.

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Back to applications

The first-order condition of the maximization problem is:

$$\sum_{i=1}^{N} \underbrace{\frac{\lambda_i \psi_J' \left(\sum_{k=1}^{K} \Delta u_k p_k^i\right)}{\sum_{l=1}^{N} \lambda_l \psi_J' \left(\sum_{k=1}^{K} \Delta u_k p_k^l\right)}}_{\tilde{\lambda}_i(\psi_J,a)} \cdot \underbrace{\left(\sum_{k=2}^{K} \left(\partial_1 u(a,t_k) - \partial_1 u(a,t_{k-1})\right) p_k^i + \partial_1 u(a,t_1)\right)}_{\rho_i(a)} = 0.$$

We can view $\tilde{\lambda}_i(\psi_J, a)$ as a distribution function where i would be the random variable. With $\psi_C = h \circ \psi_D$, the likelihood ratio of $\tilde{\lambda}_i(\psi_C, a)$ and $\tilde{\lambda}_i(\psi_D, a)$ writes:

$$\frac{\tilde{\lambda}_i(\psi_C, a)}{\tilde{\lambda}_i(\psi_D, a)} = h'\left(\psi_D\left(\sum_{k=1}^{K} \Delta u_k p_k^i\right)\right) \cdot \frac{\sum_{l=1}^{N} \lambda_l \psi_D' \left(\sum_{k=1}^{K} \Delta u_k p_k^l\right)}{\sum_{l=1}^{N} \lambda_l \psi_C' \left(\sum_{k=1}^{K} \Delta u_k p_k^l\right)}.$$

Thus $\tilde{\lambda}_i(\psi_D, a)$ first-order stochastically dominates $\tilde{\lambda}_i(\psi_C, a)$. If $\partial_1 u(a,t)$ increases with t, we get $\sum_{i=1}^{N} \tilde{\lambda}_i(\psi_C, a)\rho_i(a) < \sum_{i=1}^{N} \tilde{\lambda}_i(\psi_D, a)\rho_i(a)$ for a given a, and $a_C^* \leq a_D^*$.

Characterization of disagreement aversion characterization ▸ Back

A distribution-aggregating DM is disagreement averse iff I is such that for any $(q_k^i) \in [0,1]^{N\times K}$ s.t. $q_1^i \geq \ldots \geq q_K^i$, and any $(\Delta u_1, \ldots, \Delta u_K) \in [0,1]^K$ s.t. $\sum_k^K \Delta u_k \leq 1$:

$$\sum_{k=1}^{K} \Delta u_k I\left(q_k^1, \ldots, q_k^N\right) \leq \max_{1\leq j\leq N} \sum_{k=1}^{K} \Delta u_k q_k^i$$

where the inequality is strict as soon as $q_k^i \neq q_k^j$ and $\Delta u_k > 0$ for some i, j, k. **Proof:**

- There is a correspondence between a choice α and its pair $(\Delta u_k)_k$, $\left(q_k^i\right)_{i,k}$.
- By EU, if α is distribution-consensual, we have $U^{P_i}(\alpha) = U^{\mathcal{P}}(\alpha), \forall i$ so that $\sum_{k=1}^{K_\alpha} \Delta u_k I\left(q_k^1, \ldots, q_k^N\right) = U^{\mathcal{P}}(\alpha) = \max_i U^{P_i}(\alpha) = \max_{1\leq i\leq N} \sum_{k=1}^{K_\alpha} \Delta u_k q_k^i$.
- Take α non distribution-consensual $\Leftrightarrow$ there are i, j, k. s.t. $q_k^i \neq q_k^j$ and $\Delta u_k > 0$.
- Take x s.t. $x \succcurlyeq^{P_i} \alpha$, $\forall i \Leftrightarrow u(x) \geq \max_i U^{P_i}(\alpha)$
- Then $x \succ^{\mathcal{P}} \alpha \Leftrightarrow \max_i \sum_{k=1}^{K_\alpha} \Delta u_k q_k^i > \sum_{k=1}^{K_\alpha} \Delta u_k I\left(q_k^1, \ldots, q_k^N\right)$, so both disagreement aversion or the Proposition's property imply the other.

Proof of sufficiency conditions for disagreement aversion Back

- For the first condition:

$$I\left(q_k^1,\dots,q_k^N\right) \leq \sum_{i=1}^{N} \lambda_i q_k^i \Rightarrow \sum_{k=1}^{K} \Delta u_k I\left(q_k^1,\dots,q_k^N\right) \leq \sum_{i=1}^{N} \lambda_i \sum_{k=1}^{K} \Delta u_k q_k^i \leq \max_{1\leq i\leq N} \sum_{k=1}^{K} \Delta u_k q_k^i \quad (1)$$

- For the second condition: Set $\Delta u_{K+1} = 1 - \sum_{k=1}^{K} \Delta u_k$ and $q_{K+1}^i = 0$ for all $1 \leq i \leq N$. Then, using successively that I is concave and increasing:

$$\sum_{k=1}^{K+1} \Delta u_k I\left(q_k^1,\dots,q_k^N\right) \leq I\left(\sum_{k=1}^{K+1} \Delta u_k q_k^1,\dots,\sum_{k=1}^{K+1} \Delta u_k q_k^N\right) \leq \max_{1\leq i\leq N} \sum_{k=1}^{K} \Delta u_k q_k^i \quad (2)$$

- In both cases the first inequality is strict when one has $q_k^i \neq q_k^j$ and $\Delta u_k > 0$ for some indices i, j, k.

Proof of characterization of comparative aversions

Back to comparative disagreement | Back to comparative ambiguity

$\Rightarrow$ Take any non-constant vector $\vec{p} = (p_1, \ldots, p_N)$ and any expertise $\mathcal{P}$.

- Let $\vec{q} = (p_1, \ldots, p_N)$. Define $x = u^{-1}(I_A(\vec{q}))$. Denote by $\left(\vec{X}, \vec{p}\right)$ the choice α with only extremal outcomes s.t. $D_\alpha^{P_i}(X^+) = p_i, \forall i$.
- By comparative disagreement aversion, $\left(\vec{X}, \vec{p}\right) \sim_A^{\mathcal{P}} x \Rightarrow \left(\vec{X}, \vec{p}\right) \succ_B^{\mathcal{P}} x$. By definition, $\left(\vec{X}, \vec{p}\right) \sim_A^{\mathcal{P}} x$ iff $U_A\left(\left(\vec{X}, \vec{p}\right)\right) = u(x) = I_A(\vec{q})$, which holds by assumption. Thus, $\left(\vec{X}, \vec{p}\right) \succ_B^{\mathcal{P}} x$, i.e. $I_B(\vec{p}) > I_A(\vec{p})$.

$\Leftarrow$ Take $\mathcal{P}$, α non distribution-consensual and β distribution-consensual s.t. $\alpha \sim_A^{\mathcal{P}} \beta$.

- Defining $\left(p_k^i\right)$ the probas of α, $\vec{p_k}$ is non-constant for some k, for which $I_A(\vec{p_k}) < I_B(\vec{p_k})$; and for remaining k $\vec{p_k}$ is constant so $I_A(\vec{p_k}) = I_B(\vec{p_k}) = p_k$.
- As $\succcurlyeq_A$ and $\succcurlyeq_B$ share u and by Definition, this implies $U_B^{\mathcal{P}}(\beta) = U_A^{\mathcal{P}}(\beta) = U_A^{\mathcal{P}}(\alpha) < U_B^{\mathcal{P}}(\alpha)$, i.e. $\alpha \succ_B^{\mathcal{P}} \beta$.

Proof that DA is more averse than UA ▸ Back

- Take any $\mathcal{P}$, α (with at least one non-extremal outcome), and distribution-consensual choice β.
- As $\succcurlyeq_{DA}$ and $\succcurlyeq_{UA}$ share u, they coincide on distribution-consensual choices, and 1. we can denote $U^{\mathcal{P}}(\beta) := U^{\mathcal{P}}_{UA}(\beta) = U^{\mathcal{P}}_{DA}(\beta)$; 2. if α is distribution-consensual, $\alpha \sim^{\mathcal{P}}_{DA} \beta \Rightarrow \alpha \sim^{\mathcal{P}}_{UA} \beta$.
- Take α non distribution-consensual, i.e. there are i, j, k. s.t. $p^i_k \neq p^j_k$ and $\Delta u_k > 0$. Set $\Delta u_{K_\alpha+1} = 1 - \sum_{k=1}^{K_\alpha} \Delta u_k$ and $p^i_{K_\alpha+1} = 0, \forall i$.
- The strict concavity inequality yields: $\sum_{k=1}^{K_\alpha+1} \Delta u_k I\left(p^1_k, \ldots, p^N_k\right) < I\left(\sum_{k=1}^{K_\alpha+1} \Delta u_k p^1_k, \ldots, \sum_{k=1}^{K_\alpha+1} \Delta u_k p^N_k\right)$. i.e. $U^{\mathcal{P}}_{DA}(\alpha) < U^{\mathcal{P}}_{UA}(\alpha)$
- Thus, $\alpha \succcurlyeq^{\mathcal{P}}_{DA} \beta \Rightarrow U^{\mathcal{P}}_{UA}(\alpha) > U^{\mathcal{P}}_{DA}(\alpha) \geq U^{\mathcal{P}}(\beta) \Rightarrow \alpha \succ^{\mathcal{P}}_{UA} \beta$.

Proof that distribution-aggregating $\cap$ Paretian $=$ linear pooling ▸ Back

$\Leftarrow$ By assumption, $U_{DA}^{\mathcal{P}} = \sum_{i=1}^{N} \lambda_i U^{P_i}$. Take any $\alpha, \beta, \mathcal{P}$ s.t. $\beta \succcurlyeq^{P_i} \alpha, \forall i$. Then $U_{DA}^{\mathcal{P}}(\beta) = \sum_{i=1}^{N} \lambda_i U_{DA}^{P_i}(\beta) \geq \sum_{i=1}^{N} \lambda_i U_{DA}^{P_i}(\alpha) = U_{DA}^{\mathcal{P}}(\alpha)$, so that $\beta \succcurlyeq^{\mathcal{P}} \alpha$.

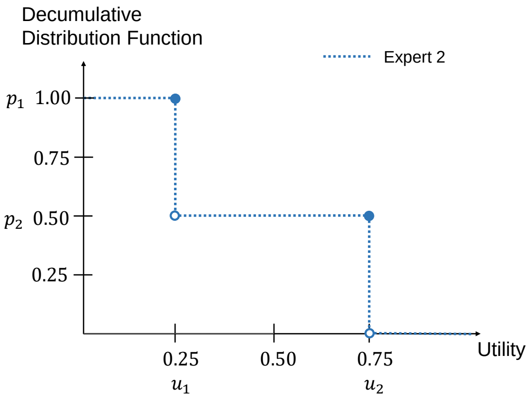
$\Rightarrow$ Sketch of proof of a weaker result: DA $\cap$ UA $\Rightarrow$ linear (see paper for full proof).

- Both UA and DA representations are equal up to an increasing bijection. Considering special choices, we see that both representations share u, and I.
- Considering choices s.t. $\Delta u_k = \frac{1}{K_\alpha}, \forall k$, we obtain a functional equation for I:

$$\sum_{k=1}^{K_\alpha} \frac{1}{K_\alpha} I\left(p_k^1, \ldots, p_k^N\right) = I\left(\sum_{k=1}^{K_\alpha} \frac{1}{K_\alpha} p_k^1, \ldots, \sum_{k=1}^{K_\alpha} \frac{1}{K_\alpha} p_k^N\right),$$

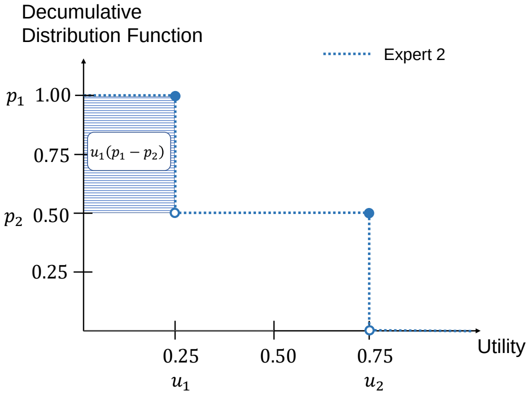
  This is Jensen's functional equation, whose solution is known to be affine (hence linear as $I(0, \ldots, 0) = 0$), modulo a domain restriction: $p_1^i \geq \ldots \geq p_{K_\alpha}^i, \forall i$.
- To handle the domain restriction: as solution applies locally to any neighborhood in the interior of the domain, we use the connectedness of the domain to show that the linear function is the same on all these neighborhoods.

Understanding risk preferences with the example ▸ Back



Expert 2: $U^{P_2}(\alpha) = \sum_k u(\alpha_k)\left(p_k^2 - p_{k+1}^2\right) = \sum_k \Delta u_k p_k^2$

Understanding risk preferences with the example ▸ Back



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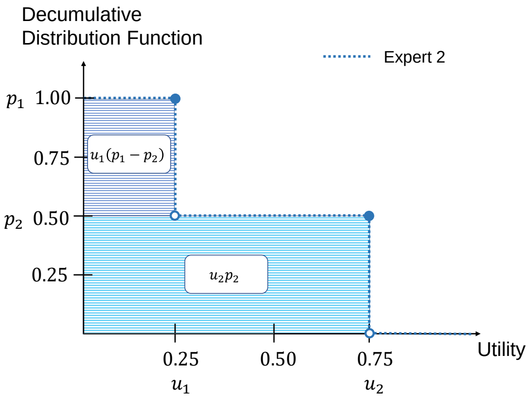
Understanding risk preferences with the example ▸ Back



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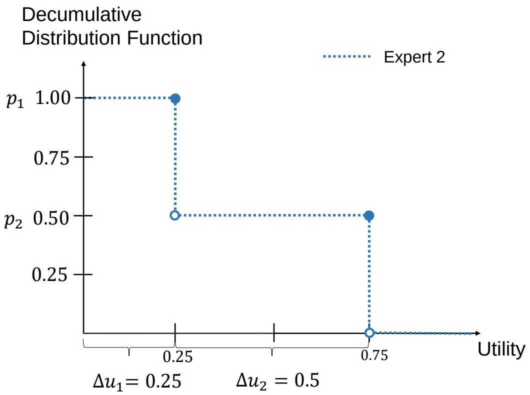
Understanding risk preferences with the example ▸ Back



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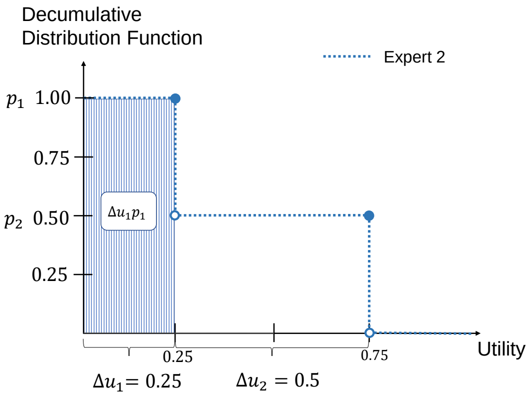
Understanding risk preferences with the example ▸ Back



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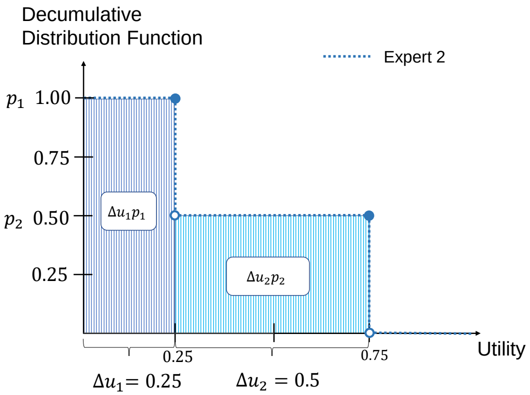
Understanding risk preferences with the example ▸ Back



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Applications ▸ Back

The ex-post utility has now the form $u(a, \omega)$, with a a choice variable and ω a contingency.

Applications ▸ Back

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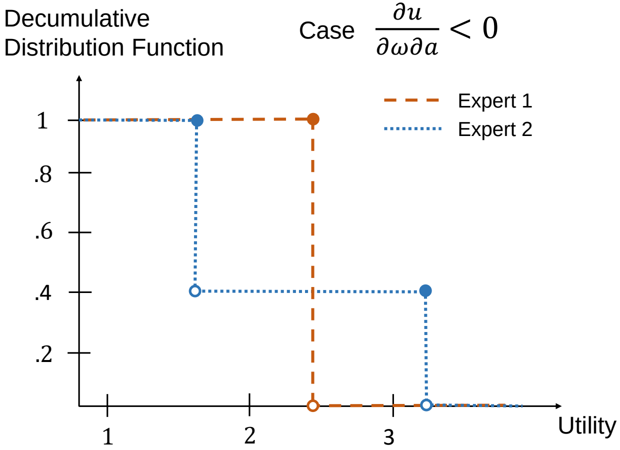

Applications ‣ Back

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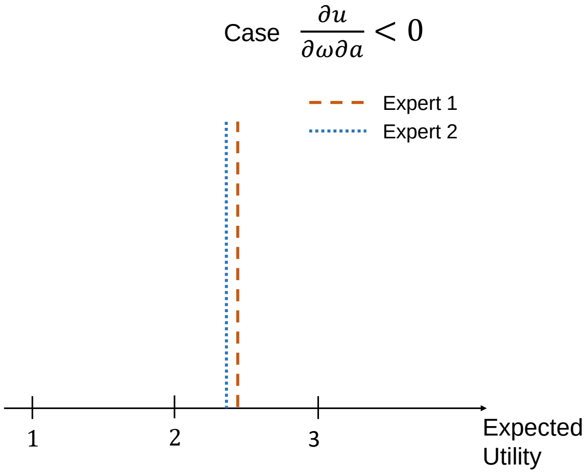

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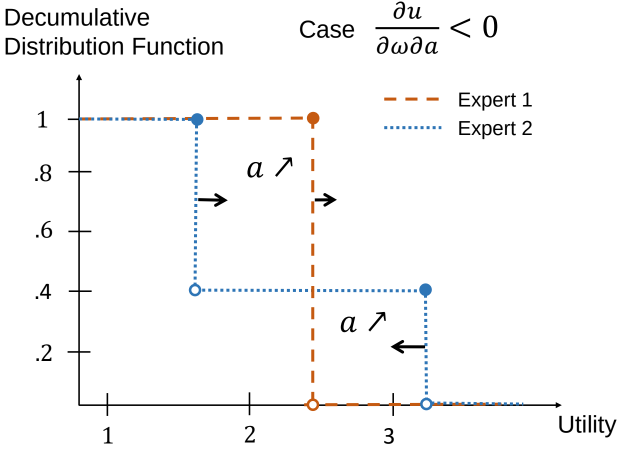

Applications ‣ Back

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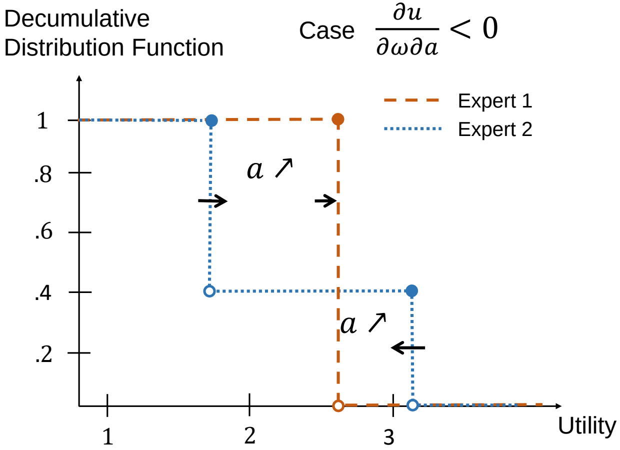

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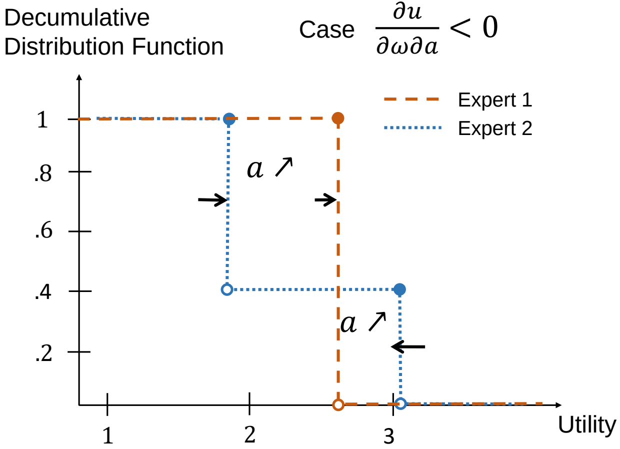

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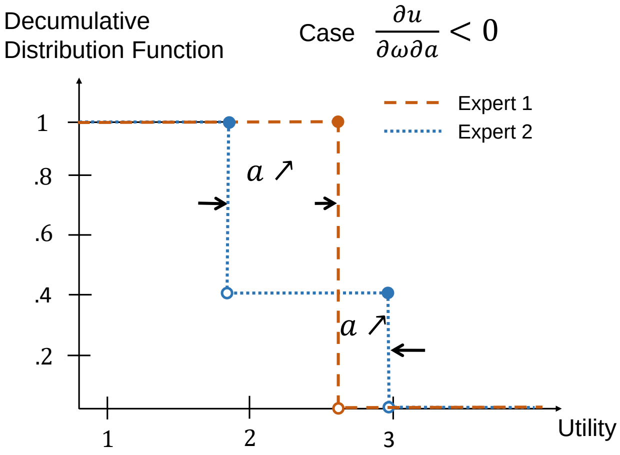

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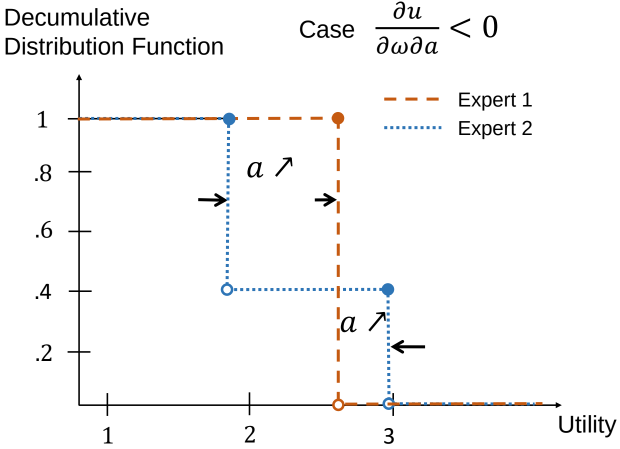

DA model: Disagreement aversion $\nearrow \Rightarrow a^* \nearrow$

Applications ▸ Back

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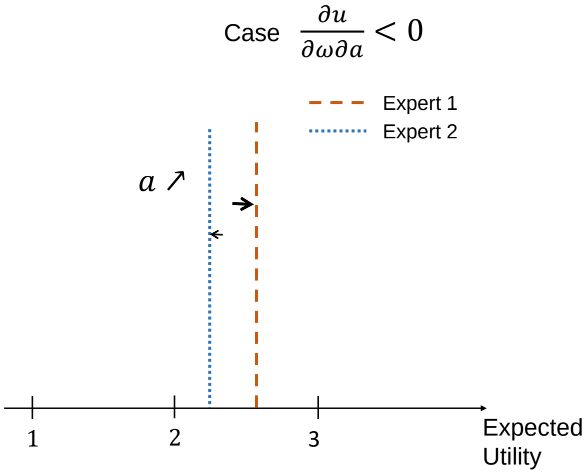

Applications ▸ Back

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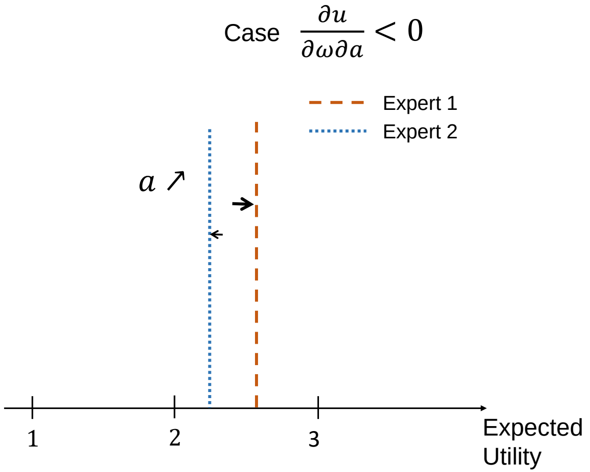

UA model: Ambiguity aversion $\nearrow$ $\not\Rightarrow$ $a^* \nearrow$

Applications ▸ Back

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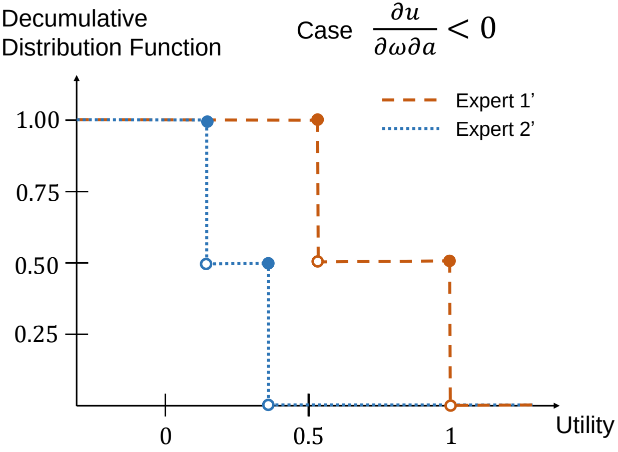

Applications ▸ Back

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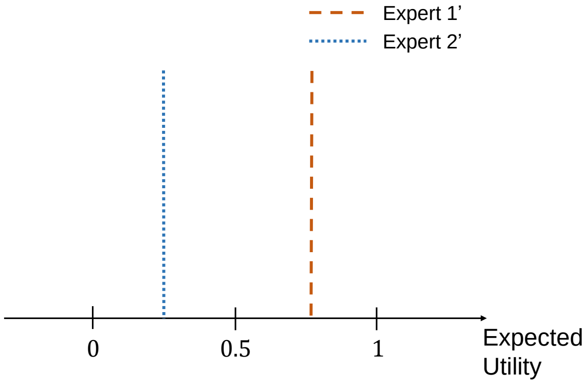

Applications ▸ Back

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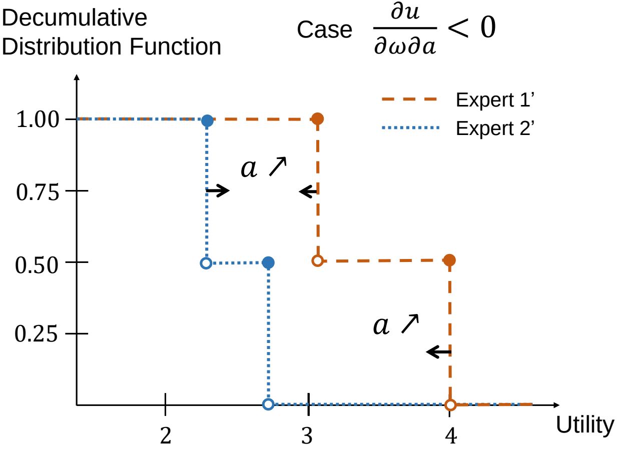

Applications ▸ Back

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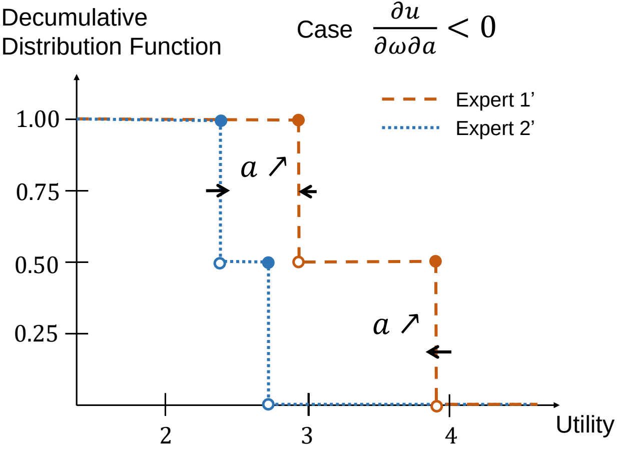

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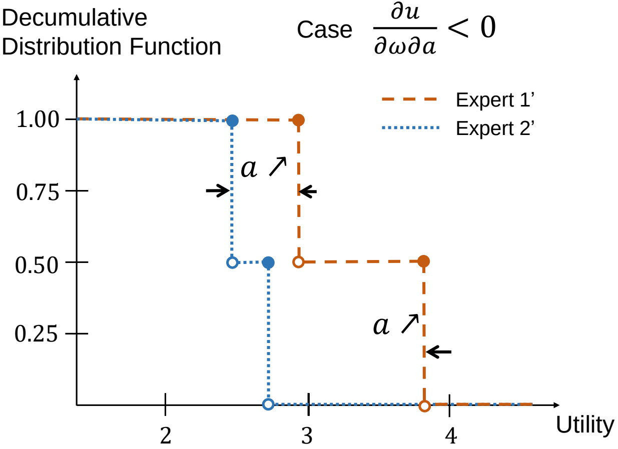

Applications ▸ Back

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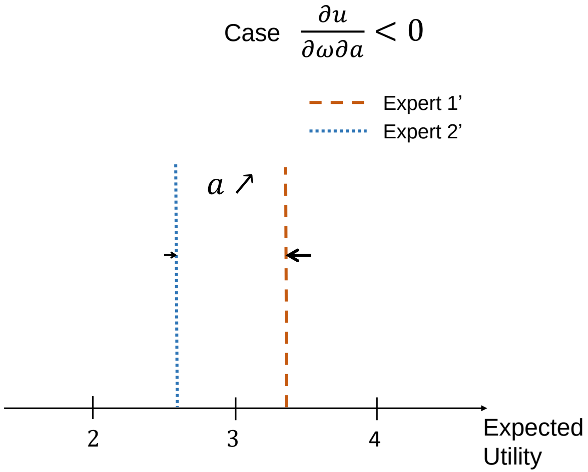

Applications ▸ Back

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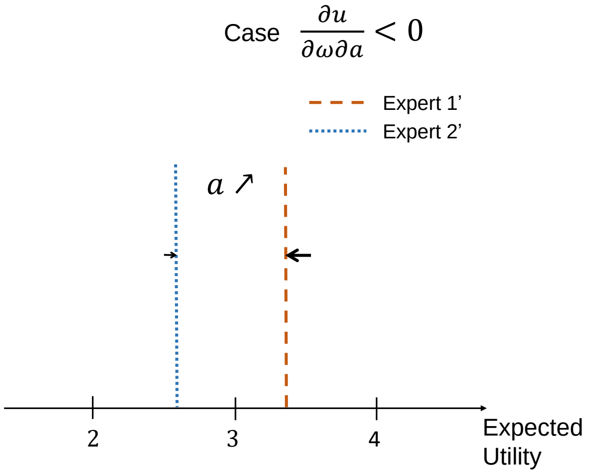

UA model: Ordered experts + Ambiguity aversion $\nearrow \Rightarrow a^* \nearrow$