Prerational Social Preferences with Risk, Uncertainty, and Enlivenment

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Outline

Consequentialist Decisions and Expected Utility

- 1988 "Consequentialist Foundations for Expected Utility"
  Theory and Decision 25 (1): 25–78.
- Two chapters in:
  S. Barberà, P.J. Hammond, and C. Seidl (eds.) (1998)
  Handbook of Utility Theory, Vol. 1: Principles
  (Dordrecht: Kluwer Academic Publishers)
  1. "Objective Expected Utility: A Consequentialist Perspective"
     ch. 5, pp. 143–211.
  2. "Subjective Expected Utility", ch. 6, pp. 213–271.
- 1999 "Subjectively Expected State-Independent Utility
  on State-Dependent Consequence Domains"
  In M.J. Machina and B. Munier (eds.)
  Beliefs, Interactions, and Preferences in Decision Making
  (Dordrecht: Kluwer Academic Publishers), pp. 7–21.

Prerational Base Relations

"Prerationality as Avoiding Predictably Regrettable Consequences"
CRETA working paper 72, April 2022, being revised

Prerationality builds on, and perhaps improves,
earlier work on consequentialist decision theory.

This talk will be about applying prerationality to social decisions.

Outline

Consequence Lotteries

- Let Y denote a fixed non-empty consequence domain.
- A simple consequence lottery
  is a probability mapping $Y \ni y \mapsto \lambda(y) \in [0, 1]$ for which:
  1. there exists a finite support $Z \subseteq Y$
     such that $\lambda(y) > 0 \iff y \in Z$;
  2. $\sum_{y \in Y} \lambda(y) = \sum_{y \in Z} \lambda(y) = 1$.
  3. Let $\Delta(Y)$, or sometimes simply L,
     denote the set of all simple consequence lotteries.

Choice Functions

- Let $\mathcal{F}(L) := \{F \in 2^L \mid 0 < \#F < \infty\}$
  denote the family of non-empty finite subsets of L.
- The mapping $\mathcal{F}(L) \ni F \mapsto C(L) \in \mathcal{F}(L)$ is a choice function
  just in case, for all $F \in \mathcal{F}(L)$,
  the choice set $C(F)$ is a non-empty subset of F.

- Let $\mathcal{F}^2(L) := \{F \in 2^L \mid \#F = 2\}$
  denote the family of pair subsets of L.
- The mapping $\mathcal{F}^2(L) \ni F \mapsto C^2(L) \in \mathcal{F}^2(L)$
  is a dichotomous choice function just in case,
  for all $F \in \mathcal{F}^2(L)$,
  the choice set $C^2(F)$ is a non-empty subset of F.

Base Relations

Proposition

The mapping $\mathcal{F}^2(L) \ni F \mapsto C^2(F) \in \mathcal{F}^2(L)$
is a dichotomous choice function if and only if
there is a complete binary base relation $\succsim$ *on* $L = \Delta(Y)$ *such that*
for each $\{\lambda, \mu\} \in \mathcal{F}^2(L)$ *one has* $\lambda \succsim \mu \iff \lambda \in C^2(\{\lambda, \mu\})$.

Strict Preference and Indifference Relations

As usual, we define the strict preference relation $\succ$ and indifference relation $\sim$ on $L = \Delta(Y)$ so that for each $\{\lambda, \mu\} \in \mathcal{F}^2(L)$ one has

$$\lambda \succ \mu \iff \mu \notin C^2(\{\lambda, \mu\})$$
$$\lambda \sim \mu \iff C^2(\{\lambda, \mu\}) = \{\lambda, \mu\}$$

Outline

Finite Decision Trees with Chance Nodes

Consider an unrestricted domain $\mathcal{T}$ of finite decision trees T
with a finite set N of nodes n
which is partitioned into three pairwise disjoint sets:

1. N^d of decision nodes n at which the decision maker determines a move $d(n) \in N_{+1}(n)$ to an immediately succeeding node;
2. N^c of chance nodes at which a roulette lottery is resolved and determines a specified positive probability $\pi(n^+|n)$ of moving to each $n^+ \in N_{+1}(n)$;
3. N^t of terminal nodes n which each lead directly to a specified consequence lottery $\gamma^t(n) \in \Delta(Y)$.

Continuation Subtrees and Decision Strategies

Given any tree $T \in \mathcal{T}$ along with any node $\bar{n}$ in T,
any decision maker who reaches $\bar{n}$ is confronted
with a continuation decision subtree $T_{\geq \bar{n}}$ whose initial node is $\bar{n}$.

When $\bar{n}$ is the initial node n_0 of tree T, one has $T_{\geq n_0} = T$.

Let $N_{\geq \bar{n}}$ etc. denote the set of nodes in the subtree $T_{\geq \bar{n}}$.

A decision strategy in subtree $T_{\geq \bar{n}}$
is a mapping $N^d_{\geq \bar{n}} \ni n \mapsto d(n) \in N_{+1}(n)$, or equivalently a list

$$\mathbf{d}_{\geq \bar{n}} = \langle d(n) \rangle_{n \in N^d_{\geq \bar{n}}} \in \mathbf{D}(T_{\geq \bar{n}}) = \prod\nolimits_{n \in N^d_{\geq \bar{n}}} N_{+1}(n)$$

A decision maker's actual decision strategy $\mathbf{d}_{\geq \bar{n}} \in \mathbf{D}(T_{\geq \bar{n}})$
in any continuation subtree $T_{\geq \bar{n}}$ must be dynamically consistent
in the sense of being the continuation
of the actual decision strategy $\mathbf{d} \in \mathbf{D}(T)$ in the whole tree T.

Actual Behaviour Rules

Regardless of a decision maker's plans or intentions, an actual behaviour rule is a mapping $(T, \bar{n}) \mapsto \beta(T_{\geq \bar{n}}, \bar{n})$ defined at every decision node $\bar{n} \in N^d$ of every tree $T \in \hat{\mathcal{T}}$ which satisfies $\emptyset \neq \beta(T_{\geq \bar{n}}, \bar{n}) \subseteq N_{+1}(\bar{n})$.

Each behaviour rule $(T, \bar{n}) \mapsto \beta(T_{\geq \bar{n}}, \bar{n})$ induces an actual decision strategy choice rule whose value $\mathbf{D}_\beta(T_{\geq \bar{n}})$ in every continuation subtree $T_{\geq \bar{n}}$ is the Cartesian product $\prod_{n \in N^d_{\geq \bar{n}}} \beta(T_{\geq n}, n)$.

Outline

Consequences of a Decision Strategy

Given any node $\bar{n} \in N$ and any fixed strategy $\mathbf{d}_{\geq \bar{n}} \in \mathbf{D}(T_{\geq \bar{n}})$
in the continuation subtree $T_{\geq \bar{n}}$,
the hierarchy of lotteries $\gamma(\mathbf{d}_{\geq \bar{n}}; n) \in \Delta(Y)$ at each node $n \in N_{\geq \bar{n}}$
that result from using decision strategy $\mathbf{d}_{\geq \bar{n}}$ throughout $T_{\geq \bar{n}}$
is the unique solution to the backward recurrence relation
defined by the following three-part rule:

1. in case $n \in N^t_{\geq \bar{n}}$ is a terminal node,
   the consequence is the lottery $\gamma(\mathbf{d}_{\geq \bar{n}}; n) = \gamma^t(n)$;
2. in case $n \in N^d_{\geq \bar{n}}$ is a decision node,
   the consequence is the lottery $\gamma(\mathbf{d}_{\geq \bar{n}}; n) = \gamma(\mathbf{d}_{\geq \bar{n}}; d(n))$;
3. in case $n \in N^c_{\geq \bar{n}}$ is a chance node at which the probability of moving to each node $n^+ \in N_{+1}(n)$ is $\pi(n^+|n) > 0$,
   the consequence is the compound lottery

$$\gamma(\mathbf{d}_{\geq \bar{n}}; n) = \sum\nolimits_{n^+ \in N_{+1}(n)} \pi(n^+|n) \, \gamma(\mathbf{d}_{\geq \bar{n}}; n^+)$$

Feasible and Chosen Consequence Lotteries

In each fixed tree $T \in \mathcal{T}$ and node $n \in N$, we can use the consequence mapping $\mathbf{D}(T_{\geq n}) \ni \mathbf{d}_{\geq n} \mapsto \gamma(\mathbf{d}_{\geq n}; n) \in \Delta(Y)$ in order to define:

1. the feasible set $F(T_{\geq n})$ of consequence lotteries as the entire range $\gamma(\mathbf{D}(T_{\geq n}))$ of the mapping

$$\mathbf{D}(T_{\geq n}) \ni \mathbf{d}_{\geq n} \mapsto \gamma(\mathbf{d}_{\geq n}; n)$$

2. given the behaviour rule $(T, \bar{n}) \mapsto \beta(T_{\geq \bar{n}}, \bar{n})$, the chosen set $\Phi_\beta(T_{\geq n})$ of consequence lotteries as the range $\gamma(\mathbf{D}_\beta(T_{\geq n}))$ of the restricted mapping

$$\mathbf{D}_\beta(T_{\geq n}) \ni \mathbf{d}_{\geq n} \mapsto \gamma(\mathbf{d}_{\geq n}; n)$$

Outline

Definition of Prerationality

The base relation $\succsim$ on $\Delta(Y)$ is prerational
just in case there exists:

1. a choice function $\mathcal{F}(\Delta(Y)) \ni F \mapsto C(F) \in \mathcal{F}(\Delta(Y))$
   whose restriction to the domain $\mathcal{F}^2(\Delta(Y))$
   of dichotomous feasible sets corresponds to $\succsim$;
2. a behaviour rule $(T, n) \mapsto \beta(T_{\geq n}, n)$
   defined on the unrestricted domain $\mathcal{T}$ of finite decision trees
   with the property that, for any decision tree T in $\mathcal{T}$
   with continuation subtrees $T_{\geq n}$ for all $n \in N$,
   the successive sets $\Phi_\beta(T_{\geq n})$ of chosen lotteries
   which result from $(T, n) \mapsto \beta(T_{\geq n}, n)$
   all satisfy $\Phi_\beta(T_{\geq n}) = C(F(T_{\geq n}))$.

Part 2 says that "behaviour can be explained by its consequences".

First Characterization of Prerationality

Theorem

The base relation $\succsim$ on $\Delta(Y)$ is prerational if and only if it is transitive as well as complete, and also satisfies the independence axiom requiring that, whenever $\lambda, \mu, \nu \in \Delta(Y)$ and $0 < \alpha < 1$, then the two compound lotteries $\alpha\lambda + (1-\alpha)\nu$ and $\alpha\mu + (1-\alpha)\nu$ in $\Delta(Y)$ satisfy

$$\alpha\lambda + (1-\alpha)\nu \succsim \alpha\mu + (1-\alpha)\nu \iff \lambda \succsim \mu$$

Continuity on Triangles

Given the base relation $\succsim$ on $\Delta(Y)$,
a Marschak triangle is the convex hull $\Delta(\{\lambda, \mu, \nu\})$
of any three lotteries that satisfy $\lambda \succ \mu$ and $\mu \succ \nu$.

The base relation $\succsim$ on $\Delta(Y)$ is continuous on triangles
just in case, whenever $\Delta(\{\lambda, \mu, \nu\})$ is a Marschak triangle,
the two sets

$$\{\alpha \in [0,1] \mid \alpha\lambda + (1-\alpha)\nu \succsim \mu\}$$
$$\{\alpha \in [0,1] \mid \mu \succsim \alpha\lambda + (1-\alpha)\nu\}$$

are both closed subsets of the unit interval $[0,1]$.

Von Neumann–Morgenstern Utility

The mapping $Y \ni y \mapsto v(y) \in \mathbb{R}$
is a von Neumann–Morgenstern utility function (or NMUF)
that represents the base relation $\succsim$ on $\Delta(Y)$
just in case, for all $\lambda, \mu \in \Delta(Y)$, the difference

$$\mathbb{E}_\lambda v - \mathbb{E}_\mu v = \sum\nolimits_{y \in Y} [\lambda(y) - \mu(y)]\, v(y)$$

in the expected values of v satisfies $\lambda \succsim \mu \iff \mathbb{E}_\lambda v - \mathbb{E}_\mu v \geq 0$.

The two NMUFs $Y \ni y \mapsto v(y) \in \mathbb{R}$ and $Y \ni y \mapsto \tilde{v}(y) \in \mathbb{R}$
are cardinally equivalent just in case
there exist an additive constant $\alpha \in \mathbb{R}$
and a positive mulitplicative constant $\rho \in \mathbb{R}$ such that,
for all $y \in Y$, one has $\tilde{v}(y) = \alpha + \rho\, v(y)$.

Second Characterization of Prerationality

Theorem

Suppose that the base relation $\succsim$ on $\Delta(Y)$:

1. *is non-trivial in the sense that there exists a triple $\{\lambda, \mu, \nu\} \subset \Delta(Y)$ that satisfies $\lambda \succ \mu$ and $\mu \succ \nu$;*
2. *is continuous on triangles.*

Then $\succsim$ is prerational if and only if it is represented by the expected value of each NMUF $Y \ni y \mapsto v(y) \in \mathbb{R}$ in a unique cardinal equivalence class.

Outline

Interpersonal Consequences

- Assume that the consequence domain Y is the interpersonal consequence domain consisting of the Cartesian product set $X \times N$ where:
  1. X is the domain of what Arrow would call "social states";
  2. N is the finite set of individuals.
- For each $i \in N$, the interpersonal consequence domain $X \times N$ includes i's personal consequence domain $X \times \{i\}$.
- The simple interpretation is that N is a fixed set of individuals.
- An extension that allows a variable population has $X = \prod_{i \in N} X_i$ where each X_i is a set of i's personal states, which include an individual state $x_0 \in X_i$ signifying that person i never comes into existence.

(with Graciela Chichilnisky and Nicholas Stern) (2020) "Fundamental Utilitarianism and Intergenerational Equity with Extinction Discounting" *Social Choice and Welfare* 54: 397–427.

Interpersonal Prerationality

Theorem

Suppose that:

1. *the base relation* $\succsim$ *on* $\Delta(X \times N)$ *is continuous on triangles;*
2. *there exists a triple* $\lambda, \mu, \nu \in \Delta(X \times N)$ *such that* $\lambda \succ \mu$ *and* $\mu \succ \nu$*.*

Then $\succsim$ *is prerational if and only if there exists a unique cardinal equivalence class of interpersonal NMUFs* $X \times N \ni (x, i) \mapsto u(x, i)$ *such that, for each pair* $\lambda, \mu \in \Delta(X \times N)$*, one has*

$$\lambda \succsim \mu \Longleftrightarrow \sum\nolimits_{(x,i) \in X \times N} [\lambda(x, i) - \mu(x, i)]\, u(x, i) \geq 0$$

Outline

Personal Decision Trees in 18th Century Scotland

Adam Smith (1759) *The Theory of Moral Sentiments*

> *O wad some Pow'r the giftie gie us*
> *To see oursels as ithers see us!*

Robert Burns (1786)
"To A Louse, On Seeing One on a Lady's Bonnet at Church"

Consider a version of Adam Smith's "impartial spectator" when assessing the well-being of one individual $i \in N$.

That impartial spectator would presumably regard the lady as being better off if she had been seen at church without a small parasitic insect on her bonnet.

An Impartial Spectator's Base Relation

Given any individual $i \in N$,
consider the domain $\mathcal{T}_i$ of personal decision trees $T \in \mathcal{T}$
in which the non-empty finite set of feasible consequences
satisfies $F(T) \subset \Delta(X \times \{i\})$.

Consider the interpersonal base relation $\succsim$ on $\Delta(X \times N)$
which is represented by $\mathbb{E}_\lambda v(x, i)$
for a unique cardinal equivalence class
of interpersonal NMUFs $X \times N \ni (x, i) \mapsto v(x, i) \in \mathbb{R}$.

When λ is restricted to the personal domain $\Delta(X \times \{i\})$,
the expected interpersonal utility $\mathbb{E}_\lambda v(x, i)$
reduces to the expectation of a unique equivalence class
of personal NMUFs $X \times \{i\} \ni (x, i) \mapsto v(x, i) \in \mathbb{R}$
that all represent the same personal base relation for individual i
over $\Delta(X \times \{i\})$.

Social Welfare Functionals (SWFLs)

Amartya K. Sen (1970) *Collective Choice and Social Welfare*
(San Francisco: Holden-Day);
later expanded edition (Penguin Books, 2017).

Claude d'Aspremont and Louis Gevers (1977)
"Equity and the Informational Basis of Collective Choice"
Review of Economic Studies 44 (2): 199–209.

A crucial weakness of the extensive literature on SWFLs
was its failure to explain the source
of the embodied interpersonal comparisons of utility.

Interpersonally Comparable Individual Utilities

(with Marc Fleurbaey) "Interpersonally Comparable Utility"
In S. Barberà, P.J. Hammond, and C. Seidl (eds.) (2004)
Handbook of Utility Theory, Vol. 2: Extensions
(Dordrecht: Kluwer Academic Publishers) ch. 21, pp. 1179–1285.

The issue is how to construct a profile of individual utility functions that explicitly embody interpersonal comparisons.

This is what we have done.

For each fixed $i \in N$,
any personal NMUF $X \times \{i\} \ni (x, i) \mapsto v(x, i) \in \mathbb{R}$
is trivially equivalent to the function $X \ni x \mapsto v_i(x) = v(x, i) \in \mathbb{R}$.

Each interpersonal NMUF $X \times N \ni (x, i) \mapsto v(x, i) \in \mathbb{R}$
in the unique cardinal equivalence class of NMUFs
that represent $\succsim$ on $\Delta(X \times N)$ does embody
relevant interpersonal comparisons of the profile $\langle v_i \rangle_{i \in N}$
of individual NMUFs that represent $\succsim$ on the domains $\Delta(X \times \{i\})$.

Interpreting Individual Utility

There is no presumption
that individual i's personal NMUF $X \times \{i\} \ni (x, i) \mapsto v(x, i) \in \mathbb{R}$
represents individual i's own preferences.

Rather, following Harsanyi and Griffin,
individual i's personal NMUF represents
the "laundered preferences" which an impartial spectator thinks
that individual i should want to maximize.

James Griffin (1986)
Well-Being: Its Meaning, Measurement And Moral Importance
(Oxford University Press).

If an individual's expected utility
really should be ethically relevant to decisions,
its normalized value can be included as part of each consequence.

Outline

Impersonal Consequence Lotteries

Following Vickrey and Harsanyi,
consider the restricted domain $\Delta^I(X \times N)$
of impersonal consequence lotteries $\lambda \in \Delta(X \times N)$
for which the probability measure $Z \mapsto \lambda(Z)$
that λ induces on finite subsets $Z \subseteq X \times N$
satisfies the impartiality condition that, for each $i \in N$,
one has $\lambda(X \times \{i\}) = 1/\#N$.

For impersonal consequence lotteries $\lambda \in \Delta^I(X \times N)$,
the expected utility expression $\sum_{(x,i) \in X \times N} \lambda(x, i)\, u(x, i)$
that represents the base relation $\succsim$ over the whole of $\Delta(X \times N)$
reduces to $\sum_{x \in X} \xi(x) \frac{1}{\#N} \sum_{i \in N} u(x, i)$ where,
for each social state $x \in X$, one has $\xi(x) := \sum_{i \in N} \lambda(x, i)$,
which is the marginal probability of x.

An Impartial Benefactor's Base Relation

The expected utility expression $\sum_{x \in X} \xi(x) \frac{1}{\#N} \sum_{i \in N} u(x,i)$
represents a prerational base relation
over the set of social consequence lotteries $\xi \in \Delta(X)$.

It represents a possible interpretation of the values
that Vickrey and Harsanyi might ascribe to an impartial observer.

But interpreting this impartial observer's values
has been far from straightforward. (Pattanaik, Weymark, etc.)

Prasanta K. Pattanaik (1968)
"Risk, Impersonality, and the Social Welfare Function"
Journal of Political Economy 76 (6) : 1152–1169.

And anyway we want an agent
who acts rather than merely observes,
so I prefer the term "impartial benefactor".

Outline

Conditional Anscombe–Aumann Lotteries

Savage introduced a state space S
whose elements s are uncertain states of the world.

For simplicity, we assume initially that S is finite.

An event E is a non-empty subset of S.

Conditional on any event E, we introduce the space

$$L^E := \prod_{s \in E} \Delta(X_s \times N)$$

of interpersonal Anscombe–Aumann lotteries $\lambda^E = \langle \lambda_s \rangle_{s \in E}$
in the form of a horse lottery $E \ni s \mapsto \lambda_s \in \Delta(X_s \times N)$
which, for each $s \in E$,
specify an interpersonal roulette lottery $\lambda_s \in \Delta(X_s \times N)$.

Often, but not always, we assume that X_s as independent of s.

Conditional Base Relations

Following Savage, given each event E
we consider a separate conditional base relation $\succsim^E$
on the domain L^E of interpersonal Anscombe–Aumann lotteries.

We will consider complete families $\langle \succsim^E \rangle_{E \in 2^S \setminus \{\emptyset\}}$
of conditional base relations $\succsim^E$, one for each event E in S.

Finite Interpersonal Decision Trees, I

Consider an unrestricted domain $\mathcal{T}$ of finite decision trees T with a finite set N of nodes n.

For each node $n \in N$, there must exist an event $E(n)$ consisting of all states that are possible after reaching node n.

Finite Interpersonal Decision Trees, II

Moreover, the set N is partitioned into four pairwise disjoint sets:

1. N^d of decision nodes n at which the decision maker determines a move $d(n) \in N_{+1}(n)$
   to an immediately succeeding node, where $E(d(n)) = E(n)$;
2. N^c of chance nodes at which a roulette lottery is resolved and determines a specified positive probability $\pi(n^+|n)$
   of moving to each $n^+ \in N_{+1}(n)$, where $E(n^+) = E(n)$;
3. N^e of event nodes at which a horse lottery is resolved which partitions the event $E(n)$ into the collection
   $\langle E(n^+) \rangle_{n^+ \in N_{+1}(n)}$ of pairwise disjoint non-empty subsets;
4. N^t of terminal nodes n which each lead directly
   to a specified consequence lottery $\gamma^t(n) \in L^{E(n)}$.

Outline

Complete Conditional Probabilities

Given the fixed finite set S of possible states of the world, define the domain $\mathcal{E} := \{(E, E_0) \in 2^S \times 2^S \mid E \subseteq E_0 \neq \emptyset\}$ of conditioned events.

Then a complete conditional probability system (or CCPS) is a mapping $\mathcal{E} \ni (E, E_0) \mapsto \mathbb{P}(E|E_0) \in [0, 1]$ such that:

1. for all $E \in 2^S \setminus \{\emptyset\}$, one has $\mathbb{P}(E|E) = 1$;
2. for all $E, E' \in 2^S$ satisfying $E, E' \subseteq S \neq \emptyset$ and $E \cap E' = \emptyset$, one has $\mathbb{P}(E \cup E'|S) = \mathbb{P}(E|S) + \mathbb{P}(E'|S)$;
3. for all $E, E', S \in 2^S$ with $E \subseteq E' \subseteq S$ and $E' \neq \emptyset$, one has $\mathbb{P}(E|S) = \mathbb{P}(E|E')\,\mathbb{P}(E'|S)$.

The CCPS $\mathcal{E} \ni (E, E_0) \mapsto \mathbb{P}(E|E_0) \in [0, 1]$ is refined just in case $\mathbb{P}(E|E_0) > 0$ for all $(E, E_0) \in \mathcal{E}$ with $E \neq \emptyset$.

Bayesian Rationality and a Refinement

The complete family $\langle \succsim^E \rangle_{E \in 2^S \setminus \{\emptyset\}}$ of conditional base relations over the respective lottery domains L^E is Bayesian rational just in case there exist a CCPS $\mathcal{E} \ni (E, E_0) \mapsto \mathbb{P}(E|E_0) \in [0,1]$ and an NMUF $\cup_{s \in S}(X_s \times N) \ni (x,i) \mapsto v(x,i) \in \mathbb{R}$ such that, for each event E in S, the conditional base relation $\succsim^E$ is represented by the conditional subjective expected utility (or SEU) function $L^E \ni \lambda^E = \langle \lambda_s \rangle_{s \in E} \mapsto U^E(\lambda^E) \in \mathbb{R}$ which is defined, for all $\lambda^E \in L^E$, by

$$U^E(\lambda^E) := \sum\nolimits_{s \in E} \mathbb{P}(\{s\}|E) \sum\nolimits_{(x,i) \in X_s \times N} \lambda_s(x,i)\, v(x,i)$$

That is, for all pairs $\lambda^E, \mu^E \in L^E$, one has

$$\lambda^E \succsim^E \mu^E \iff U^E(\lambda^E) \geq U^E(\mu^E)$$

The family $\langle \succsim^E \rangle_{E \in 2^S \setminus \{\emptyset\}}$ is refined Bayesian rational just in case it is Bayesian rational with a CCPS that is refined.

Non-Trivial States

Given any fixed state $s \in S$,
the base relation $\succsim^{\{s\}}$ on $\Delta(X_s \times N)$ is non-trivial just in case there exist three consequences $(a, i), (b, j), (c, k) \in X_s \times N$ such that the three corresponding degenerate lotteries

$$\delta_{(a,i)}, \delta_{(b,j)}, \delta_{(c,k)} \in \Delta(X_s \times N)$$

satisfy $\delta_{(a,i)} \succ^{\{s\}} \delta_{(b,j)}$ and $\delta_{(b,j)} \succ^{\{s\}} \delta_{(c,k)}$.

Generalized State Independence

Given the fixed state space S and fixed family $\langle X_s \rangle_{s \in S}$
of state-dependent domains of social states,
define the union domain $X^* := \cup_{s \in S} X_s$ of all possible social states.

Say that the complete family $\langle \succsim^E \rangle_{E \in 2^S \setminus \{\emptyset\}}$
of conditional base relations on L^E
satisfies generalized state independence just in case
there exists a state-independent base relation $\succsim^*$ on $\Delta(X^* \times N)$
whose restriction to $\Delta(X_s \times N)$, for each state $s \in S$,
equals the relation $\succsim^{\{s\}}$ on $L^{\{s\}}$.

Characterization of Prerationality

Theorem

Suppose that the collection $\langle \succsim^{\{s\}} \rangle_{s \in S}$ *of conditional base relations satisfies non-triviality and generalized state independence.*

Then the complete family $\langle \succsim^E \rangle_{E \in 2^S \setminus \{\emptyset\}}$
of conditional base relations $\succsim^E$
on the respective contingent lottery domains L^E
is continuous on triangles and prerational
if and only if it is refined Bayesian rational.

Outline

The Interpersonal Base Relations

Bayesian rationality requires that, given any event E in S,
the conditional base relation $\succsim^E$ on L^E
is represented by the subjective expected utility function

$$U^E(\lambda^E) := \sum_{s \in E} \mathbb{P}(\{s\}|E) \sum_{(x,i) \in X_s \times N} \lambda_s(x,i)\, v(x,i)$$

defined on the lottery space $L^E = \prod_{s \in E} \Delta(X_s \times N)$.

The Impartial Spectator and Impartial Benefactor

According to the impartial spectator,
the subjective expected utility of any individual $i \in N$ is

$$U_i^E(\lambda^E) := \sum\nolimits_{s \in E} \mathbb{P}(\{s\}|E) \sum\nolimits_{x \in X_s} \lambda_s(x,i)\, v(x,i)$$

defined on the lottery space $L_i^E = \prod_{s \in E} \Delta(X_s \times \{i\})$.

The impartial benefactor's expected utility is

$$\bar{U}^E(\lambda^E) := \frac{1}{\# N} \sum\nolimits_{i \in N} U_i^E(\lambda^E)$$

Three Ingredients

The three ingredients for these measures
for the impartial spectator's and impartial benefactor's
expected utility are:

1. the subjective CCPS $\mathcal{E} \ni (E, E_0) \mapsto \mathbb{P}(E|E_0) \in [0, 1]$
   that determines the outcome s of the horse lottery;
2. the objective probabilities in the interpersonal roulette
   lotteries $\lambda_s \in \Delta(X_s \times N)$;
3. the interpersonal NMUF $\cup_{s \in S}(X_s \times N) \ni (x, i) \mapsto v(x, i) \in \mathbb{R}$.

Only item 1 is new when the risky outcomes of roulette lotteries
are supplemented
by the uncertain outcomes $s \in S$ of horse lotteries.

Whose Beliefs?

The beliefs underlying the subjective CCPS $(E, E_0) \mapsto \mathbb{P}(E|E_0)$ are those of the impartial spectator/benefactor.

Individuals' actual beliefs matter only to the extent that:

1. they may be used to help determine the subjective CCPS;
2. and/or they feature in the description of the consequences that are embodied in each "social state" x.

Outline

More History of Confusion

George L.S. Shackle (1953) “The Logic of Surprise”
Economica 20: 112–117.

> *In contrast with this I define an unexpected event*
> *as one which has never been formulated*
> *in the individual’s imagination,*
> *which has never entered his mind*
> *or been in any way envisaged. (page 113)*

(2007) “Schumpeterian Innovation in Modelling Decisions, Games, and Economic Behaviour”
History of Economic Ideas XV: 179–195.

Schumpeterian innovation involves a transition to new techniques that could not be included in any previous economic model.

“All models are wrong; some are useful.”

More Confusion

- Struggles to deal with complexity
- Herbert Simon on “bounded rationality”.
- Uncountable finite sets,
  like the number of fish in a large pond.
- Nassim Nicholas Taleb (2007) *The Black Swan: The Impact of the Highly Improbable*.

  This is actually a book about “grey swans”.

  Taleb recognizes his inability to model “true black swans” that, by definition, like *cygnus atratus* in Western Australia, cannot be modelled in advance.
- Asymmetric imagination may be more important than asymmetric information.

The Master of Go

[The following passage was omitted from the slides,
but read out during the presentation.]

> *" 'This is what war must be like,' said Iwamoto gravely.*
>
> *He meant of course that in actual battle*
> *the unforeseeable occurs and fates are sealed in an instant.*
> *Such were the implications of White 130.*
> *All the plans and studies of the players,*
> *all the predictions of us amateurs*
> *and of the professionals as well had been sent flying.*
>
> *As an amateur, I did not immediately see that White 130*
> *assured the defeat of the 'invincible Master.' "*

From Kawabata's own shortened version of *The Master of Go*,
end of chapter 37; translated by Edward G. Seidensticker
(New York: Alfred A. Knopf, 1972).

Paul on Transformative Experiences

Paul, Laurie Ann (2014) *Transformative Experience*
(Oxford U. Press)

Decisions where one of the possible options offers
a radically new experience that cannot be assessed in advance,
such as deciding to become a parent,
or choosing to alter one's physical or mental capabilities.

. . . [choices] such as having a child, converting to a religion,
or medically altering one's physical and mental capacities,
are transformative experiences
that are structurally similar to becoming a vampire.
A transformative experience changes the values and preferences
held before making the decision.

Extends argument in earlier working paper, eventually published as:

Paul, L. A. (2015) "What You Can't Expect When You're
Expecting" *Res Philosophica* [online] 92(2): 1–22.

A Philosophers' Debate

Barnes, E. (2015) "What You Can Expect
When You Don't Want to be Expecting"
Philosophy and Phenomenological Research 91(3): 775–786.

Pettigrew, R. (2015)
"Transformative Experience and Decision Theory"
Philosophy and Phenomenological Research 91(3): 766–774.

Paul, L. A. (2015) "Transformative Experience:
Replies to Pettigrew, Barnes and Campbell"
Philosophy and Phenomenological Research 91(3): 794–813.

Bykvist, Krister, and Stefánsson, H Orri (2017)
"Epistemic Transformation and Rational Choice
Economics and Philosophy 33(1): 125–138.

And more . . .

Outline

Enlivened Decision Trees

A decision tree is enlivened
by the inclusion of enlivenment nodes.

These are a special kind of event node following which,
if enlivenment occurs,
a new continuation decision subtree springs up
which was not part of the original model.

By definition, the details of this new subtree cannot be modelled.

Nevertheless, corresponding to each enlivenment node, we can introduce some new states of the world:

1. a $\{0, 1\}$ variable indicating whether enlivenment occurs;
2. contingent on enlivenment,
   possible values of a normalized ex post NMUF.

A Philosophical Implication

Friedrich A. Hayek (1940) "Socialist Calculation:
The Competitive 'Solution'" *Economica* 8: 125–149.

Partha Dasgupta (1980) "Decentralization and Rights"
Economica 47: 107–123.

Claims that Hayek's arguments against socialism
concern the effectiveness of decentralizing decisions,
especially by granting individual rights.

Dasgupta's case is based on a model of asymmetric information;
it presumably works *a fortiori*
when there is asymmetric imagination.

Envoi

Many thanks to Eric Danan for organizing this conference.

Many thanks to you for your patience and attention.