Bayesian social aggregation with almost-objective uncertainty

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Labex MME-DII (ANR11-LBX-0023-01).

Workshop on social choice under risk and uncertainty
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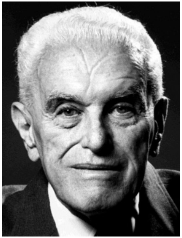

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Also, suppose that society satisfies the ex ante *Pareto axiom.*

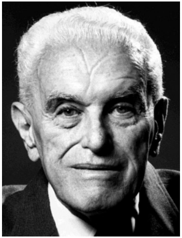
Then the social vNM utility function (i.e. the SWF) *must be a weighted sum of individual vNM utility functions.*

Upshot: (vNM rationality) + (Pareto) $\Rightarrow$ utilitarianism.

Problem: vNM assumes that risks have known, *objective* probabilities.

But in many situations, there is no "objective" way to assign probabilities.

Question: Is there an analogy to Harsanyi's social aggregation theorem in the "Savage" framework, with purely *subjective* probabilities?



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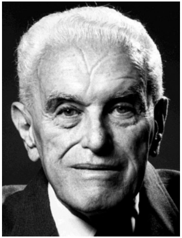

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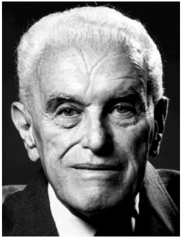

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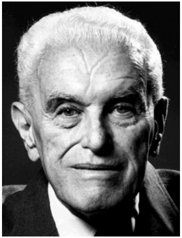

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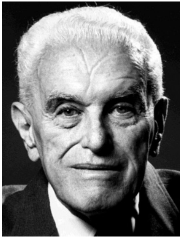

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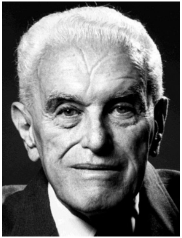

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Mongin (1995):
In the Savage framework, Harsanyi's theorem is false, unless all agents have the same subjective beliefs.

Indeed, if agents have different beliefs, then it is impossible to satisfy the *ex ante* Pareto axiom.

(Related work: Hylland & Zeckhauser 1979 and Hammond 1981.)

Key problem. "*Spurious unanimity*"

Different people might have different utility functions *and* different beliefs.

But these differences might "cancel out", so everyone ends up with the *same* preferences between two acts α and β.

This unanimous preference is "spurious", since it conceals disagreement in the underlying beliefs and utilities.

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GSS Possibility Theorem

Idea. Find a way to exclude "spurious unanimity" ....

Gilboa, Samet & Schmeidler (2004): Restrict *ex ante* Pareto to acts where all agents have the *same* beliefs about the underlying events.

Theorem. *The social planner satisfies this restricted* ex ante *Pareto iff:*

- *The SWF is weighted sum of individual utility functions.*
- *The social beliefs are a weighted average of individual beliefs.*

Upshot: (Gilboa-Samet-Schmeidler "restricted Pareto" axiom) $\Longrightarrow$ (SWF is utilitarian, *and* social beliefs are linear pooling of individual beliefs).

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 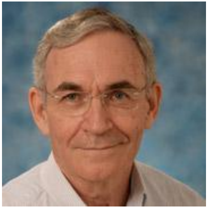 

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Many other important papers have been written on this topic, including:

- Chambers, C., Hayashi, T., 2006. Preference aggregation under uncertainty: Savage vs. Pareto. *Games Econom. Behav.* **54**, 430–440.
- Chambers, C., Hayashi, T., 2014. Preference aggregation with incomplete information. *Econometrica* **82** (2), 589–599.
- Gilboa, I., Samuelson, L., Schmeidler, D., 2014. No-betting Pareto dominance. *Econometrica* **82**, 1405–1442.
- Alon, S., Gayer, G., 2016. Utilitarian preferences with multiple priors. *Econometrica* **84** (3), 1181–1201.
- Danan, E., Gajdos, T., Hill, B., Tallon, J.-M., 2016. Robust social decisions. *Am. Econ. Rev.* **106** (9), 2407–2425.
- Billot, A., Vergopoulos, V. 2016, Aggregation of Paretian preferences for independent individual uncertainties. *Soc. Choice Welf.* **47**(4), 973-984.
- Zuber, S., 2016. Harsanyi's theorem without the sure-thing principle. *Journal of Mathematical Economics* **63**, pp.78-83.
- Qu, X., 2017. Separate aggregation of beliefs and values under ambiguity. *Economic Theory* **63** (2), 503–519.
- Sprumont, Y., 2018. Belief-weighted Nash aggregation of Savage preferences. *Journal of Economic Theory* **178**, 222–245.
- Sprumont, Y., 2019. Relative utilitarianism under uncertainty. *Social Choice and Welfare* **53** (4), 621–639.
- Hayashi, T., Lombardi, M., 2019. Fair social decision under uncertainty and responsibility for beliefs. *Economic Theory* **67** (4), 775–816.
- Ceron, F., Vergopoulos, V., 2019. Aggregation of Bayesian preferences: unanimity vs monotonicity. *Social Choice and Welfare* **52** (3), 419–451.
- Dietrich, F., 2021. Fully Bayesian aggregation. *Journal of Economic Theory* **194**, 105255.
- Brandl, Florian, 2021. Belief-averaged relative utilitarianism. *Journal of Economic Theory* **198**, 105368.

The problem of new information

Recall: (Gilboa-Samet-Schmeidler "restricted Pareto" axiom) $\Longrightarrow$ (SWF is utilitarian, *and* social beliefs are linear pooling of individual beliefs).

Problem: Linear pooling does not respond correctly to new information.

$$\begin{matrix}\text{Bayesian}\\\text{update}\end{matrix}\left[\begin{matrix}\text{Weighted}\\\text{average}\end{matrix}\left(\begin{matrix}\text{individual}\\\text{beliefs}\end{matrix}\right)\right] \neq \begin{matrix}\text{Weighted}\\\text{average}\end{matrix}\left[\begin{matrix}\text{Bayesian}\\\text{update}\end{matrix}\left(\begin{matrix}\text{individual}\\\text{beliefs}\end{matrix}\right)\right].$$

In fact, GSS Pareto axiom does not respond well to new information, either.

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Complementary ignorance

Consider a social decision with two agents, Ann and Bob, and $\mathcal{S} = \{r, s, t\}$.

Consider two acts α and β, which yield the *same* payoff for both agents in each state of nature:

	r	s	t
α	100	0	100
β	0	100	0

Ann and Bob begin with the *same* prior probability p:

$$p(r) = 0.49, \quad p(s) = 0.02, \quad \text{and} \quad p(t) = 0.49.$$

Ann privately observes the event $\{r, s\}$, while Bob privately observes $\{s, t\}$.

After Bayesian updating, they have the following posterior probabilities:

	Info	r	s	t
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Ann	{r,s}	0.96	0.04	0
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Ann & Bob agree: Expected Utility$(\alpha) = 96$, while Expected Utility$(\beta) = 4$.

Thus, $\alpha \succ_{\text{Ann}} \beta$ and $\alpha \succ_{\text{Bob}} \beta$.

Also α and β are measurable relative to the algebra $\mathfrak{B} = \{\mathcal{S}, \{r, t\}, \{s\}, \emptyset\}$.

Ann and Bob have the same beliefs about $\mathfrak{B}$.

Thus, even GSS's restricted *ex ante* Pareto dictates that $\alpha \succ \beta$.

Indeed, if P is the average of Ann's and Bob's beliefs (as GSS recommend), then P also says Expected SWF$(\alpha) = 96$, while Expected SWF$(\beta) = 4$.

However, clearly, the *true* state is s. So β *is actually the better choice*.

Upshot: In some cases, GSS Pareto and linear pooling are not appropriate.

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	r	s	t
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Ann & Bob agree: Expected Utility$(\alpha) = 96$, while Expected Utility$(\beta) = 4$.

Thus, $\alpha \succ_{\text{Ann}} \beta$ and $\alpha \succ_{\text{Bob}} \beta$.

Also α and β are measurable relative to the algebra $\mathfrak{B} = \{\mathcal{S}, \{r, t\}, \{s\}, \emptyset\}$.

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Social welfare vs. collective beliefs

This malfunction of the GSS theorem has a broader message.

Different belief-aggregation rules are suitable in different contexts.

The criteria that determine the best belief-aggregation rule might *not* be the criteria that determine the correct SWF.

The construction of a social welfare function is an ethical problem.

The construction of a collective belief is an epistemic problem.

There is no reason that these two problems should be solved by the same theorem, or even with the same data.

We will focus on the *ethical* problem, leaving the epistemic problem to be solved later by other methods.

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The problem of heterogeneous ambiguity attitudes

Another concern. The aforementioned results all assume that all agents are *expected utility maximizers.*

Question. Can *non*-SEU ambiguity attitudes enter into group decisions?

Problem. Different agents might have *different* ambiguity attitudes.

Such heterogeneity yields impossibility theorems (Chambers & Hayashi 2006; Gajdos Tallon & Vergnaud 2008; Mongin & P. 2015; Zuber 2016).

Upshot. To satisfy ex ante Pareto, agents must be SEU maximizers.

Partial solution. Weaken the ex ante Pareto axiom (Alon & Gayer 2016; Danan, Gajdos, Hill & Tallon 2016; Qu 2015; Hayashi & Lombardi 2019).

These papers characterize a SWF *and* a "linear" belief-aggregation rule.

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Goal. An approach to group decisions under uncertainty that is compatible with heterogeneity of beliefs *and* heterogeneity of ambiguity attitudes.

Idea. Use *almost-objective uncertainty* to formulate a weak Pareto axiom.

Main results. This axiom is both necessary and sufficient for the ex post social welfare function to be *utilitarian* —i.e. a weighted sum of the individual utility functions.

This holds for a variety of ambiguity attitudes.

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Overview.

I. Almost objective uncertainty.
II. Axioms and main result for SEU preferences.
III. Main result for non-SEU preferences.

I. Almost objective uncertainty

Notation. For any $K \in \mathbb{N}$, let $\Delta^K := \{\mathbf{q} = (q_1, ..., q_K) \in \mathbb{R}_+^K;\ \sum_{k=1}^{K} q_k = 1\}$, the set of K-dimensional probability vectors.

Let $\mathcal{S}$ be a measurable space.

Let $\mathcal{R}$ be a collection of probability measures on $\mathcal{S}$.

Let $K \in \mathbb{N}$ and let $\mathbf{q} \in \Delta^K$.

For all $n \in \mathbb{N}$, let $\mathfrak{G}^n := \{\mathcal{G}_1^n, \mathcal{G}_2^n, \ldots, \mathcal{G}_K^n\}$ be a K-element partition of $\mathcal{S}$.

Definition. The sequence of partitions $(\mathfrak{G}^1, \mathfrak{G}^2, \mathfrak{G}^3, \ldots\ldots)$ is $\mathcal{R}$*-almost-objectively uncertain* and *subordinate to* $\mathbf{q}$ if, for all $\rho \in \mathcal{R}$, we have

$$\lim_{n\to\infty} \rho(\mathcal{G}_1^n) = q_1, \quad \lim_{n\to\infty} \rho(\mathcal{G}_2^n) = q_2, \ \ldots\ldots \ \lim_{n\to\infty} \rho(\mathcal{G}_K^n) = q_K.$$

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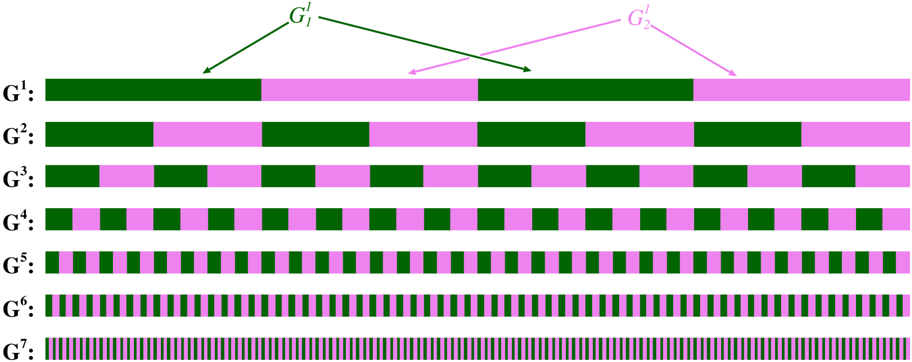
Example. (Poincaré, 1912; Machina, 2004, 2005) Let $\mathcal{S} := [0,1]$.

Let $\mathcal{R} := \{$probability measures on $[0,1]$ with *continuous* density functions$\}$.

Let $K := 2$ and let $\mathbf{q} := (\frac{1}{2}, \frac{1}{2})$.

Consider the partitions $\mathfrak{G}^1, \mathfrak{G}^2, \mathfrak{G}^3, \ldots$, where $\mathfrak{G}^n := \{\mathcal{G}_1^n, \mathcal{G}_2^n\}$ for all $n \in \mathbb{N}$.



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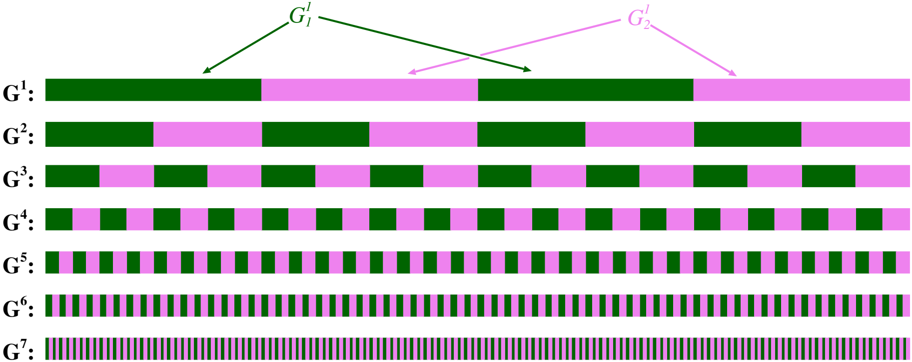
Consider the partitions $\mathfrak{G}^1, \mathfrak{G}^2, \mathfrak{G}^3, \ldots$, where $\mathfrak{G}^n := \{\mathcal{G}_1^n, \mathcal{G}_2^n\}$ for all $n \in \mathbb{N}$.



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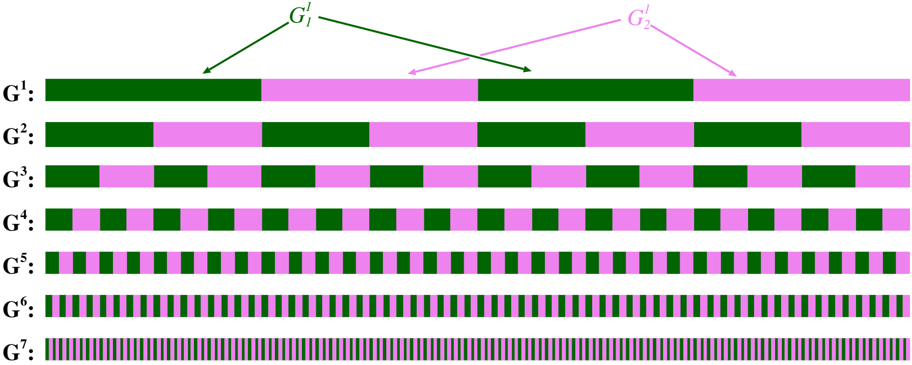

Almost-objective uncertainty on an interval

Claim. *The partition sequence* $(\mathfrak{G}^n)_{n=1}^{\infty}$ *is* $\mathcal{R}$*-almost-objectively uncertain, and subordinate to* $(\frac{1}{2}, \frac{1}{2})$.

To see this, let $\rho \in \mathcal{R}$. Suppose ρ has the density function f shown below.

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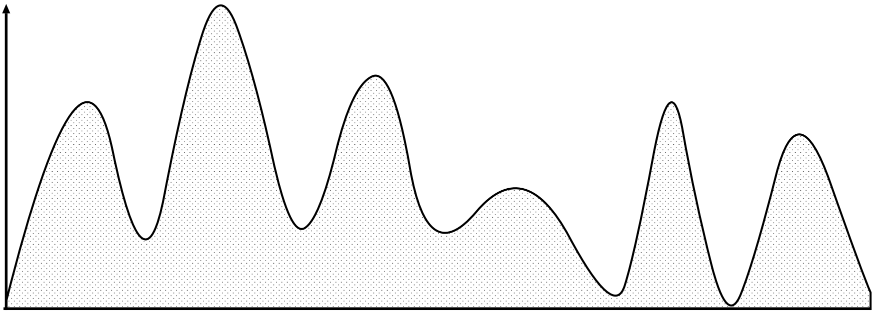

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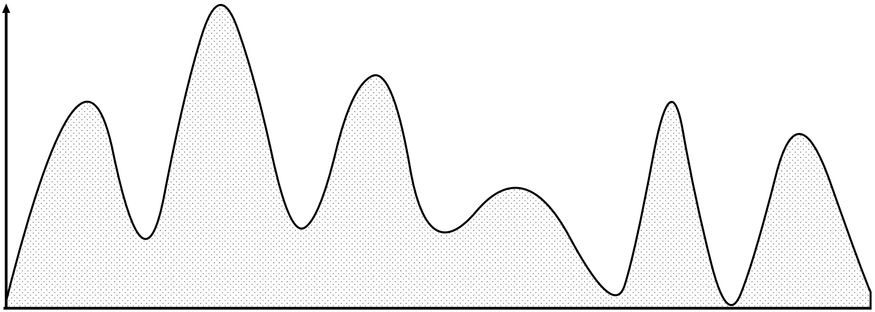

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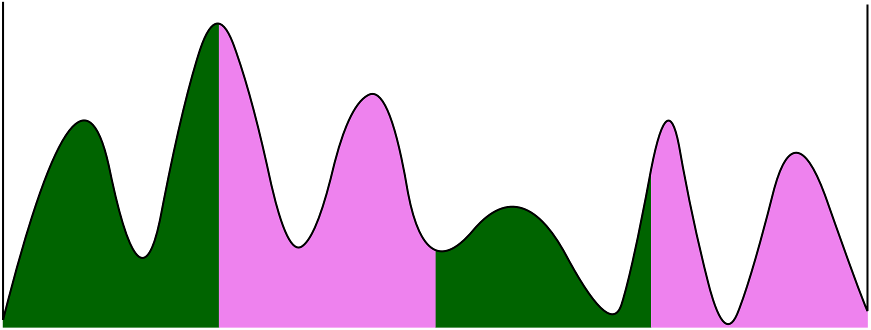

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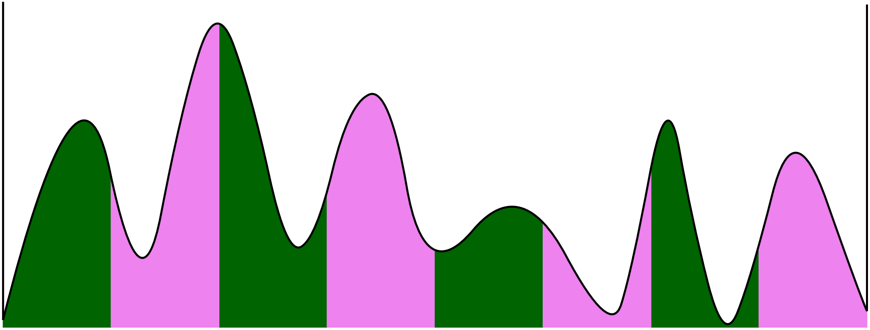

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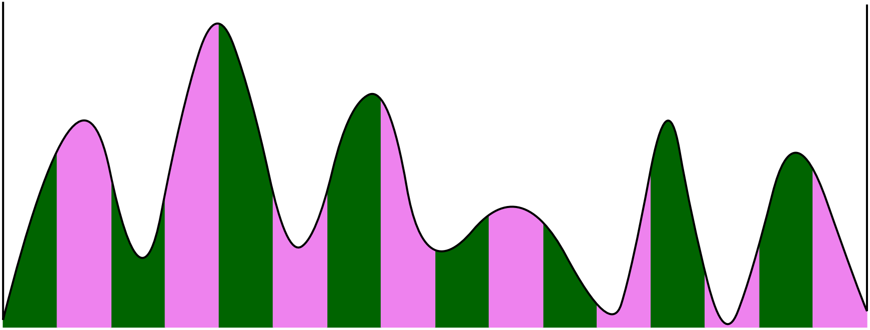

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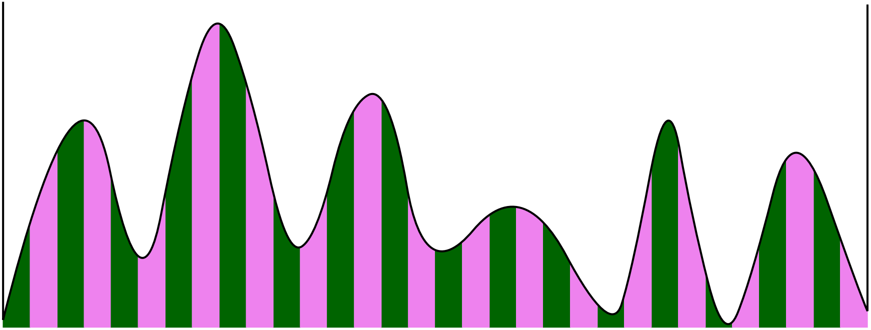

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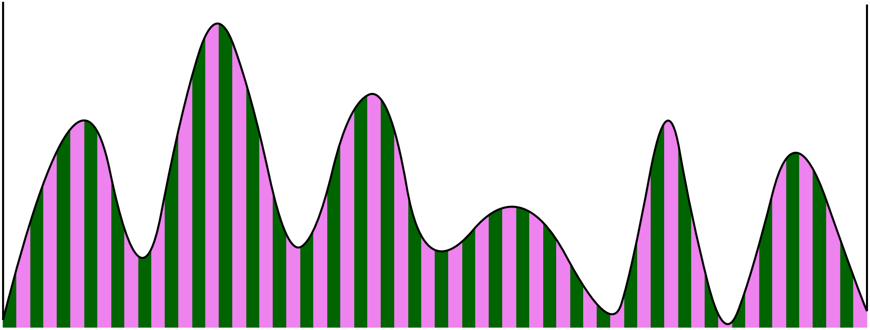

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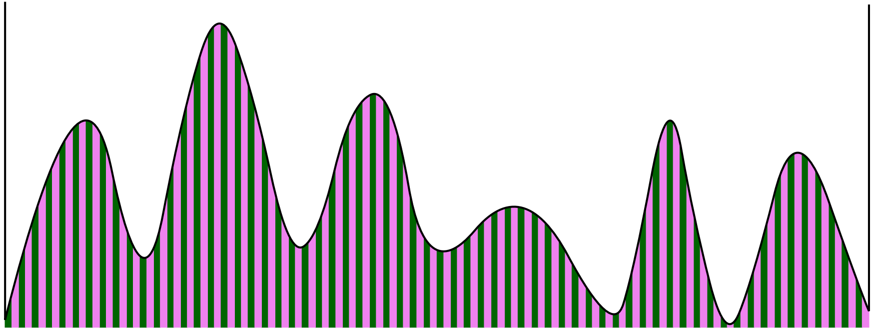

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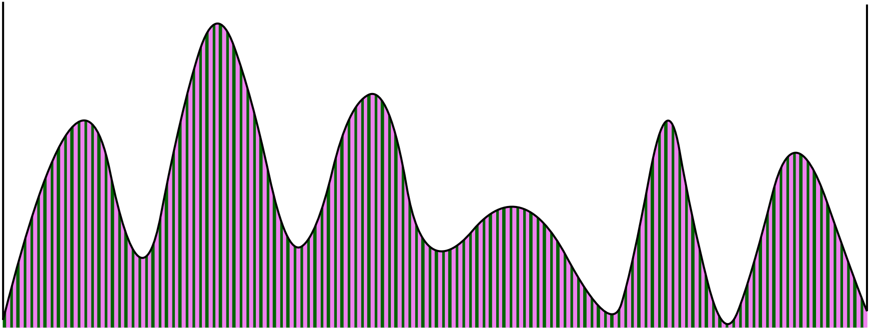

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We will obtain almost-objectively uncertain partitions in any Polish space....

Let $\mathcal{M}(\mathcal{S})$ be the Banach space of signed measures on $\mathcal{S}$.

Terminology. A *closed subspace* of $\mathcal{M}(\mathcal{S})$ is a linear subspace $\mathcal{N} \subseteq \mathcal{M}(\mathcal{S})$ that is closed in the total variation norm topology.

$\mathcal{N}$ is *separable* if it is spanned by a countable subset.

(A subset $\mathcal{H} \subseteq \mathcal{N}$ *spans* $\mathcal{N}$ if $\mathcal{N}$ is the norm-closure of the vector space of all finite linear combinations of elements of $\mathcal{H}$.)

$\mathcal{N}$ is *nonatomic* if all elements of $\mathcal{N}$ are nonatomic.

Notation. Let $\langle\mathcal{N}\rangle := \{\mu \in \mathcal{M}(\mathcal{S});\ \mu$ is absolutely continuous relative to some $\nu \in \mathcal{N}$, and the Radon-Nikodym derivative $\frac{\mathrm{d}\mu}{\mathrm{d}\nu}$ is bounded$\}$.

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Example. Let $\mathcal{S} = [0,1]$. Let $\mathcal{R}$ be the set of all probability measures on $\mathcal{S}$ that are absolutely continuous with respect to Lebesgue, with density functions in $\mathcal{L}^\infty[0,1]$. Then $\mathcal{R}$ is tame.

Recall. A *Polish space* is a topological space homeomorphic to a complete, separable metric space. We equip it with the Borel sigma-algebra.

Proposition. *Let $\mathcal{S}$ be any Polish space.*

Let $\mathcal{R}$ be any tame set of probability measures on $\mathcal{S}$.

For any $K \in \mathbb{N}$ and $\mathbf{q} \in \Delta^K$, there is an $\mathcal{R}$-almost-objectively uncertain sequence of partitions of $\mathcal{S}$ that is subordinate to $\mathbf{q}$.

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II. Axioms and main result

(for SEU preferences)

Let $\mathcal{S}$ and $\mathcal{X}$ be measurable spaces.

$\mathcal{S}$ is the *state space*. $\mathcal{X}$ is the *outcome space*.

An *act* is a measurable function $\alpha : \mathcal{S} \longrightarrow \mathcal{X}$ taking finitely many values.

Let $\mathcal{A}$ be the set of all acts.

Let $\succeq$ be a preference order on $\mathcal{A}$ (e.g. some agent's ex ante preferences).

A *representation* of $\succeq$ is a function $V : \mathcal{A} \longrightarrow \mathbb{R}$ such that

$$\text{for all } \alpha, \beta \in \mathcal{A}, \quad \Big(\alpha \succeq \beta\Big) \quad \Longleftrightarrow \quad \Big(V(\alpha) \geq V(\beta)\Big).$$

Example. A representation V is *subjective expected utility* (SEU) if there is some $\rho \in \Delta(\mathcal{S})$ and a bounded measurable function $u : \mathcal{X} \longrightarrow \mathbb{R}$ such that

$$V(\alpha) \quad = \quad \int_{\mathcal{S}} u \circ \alpha \; \mathrm{d}\rho, \qquad \text{for all } \alpha \in \mathcal{A}.$$

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Let $\mathcal{S}$ and $\mathcal{X}$ be measurable spaces.

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Almost-objective acts

Let $\mathcal{R}$ be a collection of probability measures on the statespace $\mathcal{S}$.

Let $\boldsymbol{\alpha} = (\alpha^1, \alpha^2, \alpha^3, \ldots\ldots)$ be a sequence of acts.

Definition. $\boldsymbol{\alpha}$ is an *$\mathcal{R}$-almost-objective act* if there exists some $\mathbf{x} = (x_1, \ldots, x_K) \in \mathcal{X}^K$, and an $\mathcal{R}$-almost-objectively uncertain sequence of K-cell partitions $\boldsymbol{\mathcal{G}} = (\mathfrak{G}^n)_{n=1}^{\infty}$, with $\mathfrak{G}^n := \{\mathcal{G}_1^n, \ldots, \mathcal{G}_K^n\}$ for all $n \in \mathbb{N}$, such that for all $n \in \mathbb{N}$ and $k \in [1 \ldots K]$ we have $\alpha^n(s) = x_k$ for all $s \in \mathcal{G}_k^n$.

Suppose $\boldsymbol{\mathcal{G}}$ is subordinate to the probability vector $\mathbf{q} = (q_1, \ldots, q_K) \in \Delta^K$.

Then we say that $\boldsymbol{\alpha}$ is *subordinate to* $(\mathbf{q}, \mathbf{x})$.

Idea: $(\alpha^1, \alpha^2, \ldots)$ "converges" to the objective lottery $\begin{pmatrix} q_1 & q_2 & \ldots & q_K \\ x_1 & x_2 & \ldots & x_K \end{pmatrix}$.

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Let $\succeq$ be a preference order on $\mathcal{A}$. Let V be a representation of $\succeq$.

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If V_1 and V_2 are contiguous representations for $\succeq$, and there exist $N \in \mathbb{N}$ and $\epsilon_1 > 0$ such that $V_1(\alpha^n) > V_1(\beta^n) + \epsilon_1$ for all $n \geq N$, then there exists $\epsilon_2 > 0$ such that $V_2(\alpha^n) > V_2(\beta^n) + \epsilon_2$ for all $n \geq N$.

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Almost-objective Pareto

Let $\mathcal{I}$ be a set of individuals.

Let o be the social observer. Let $\mathcal{J} := \mathcal{I} \sqcup \{o\}$.

For all $j \in \mathcal{J}$, let $\succeq_j$ be a preference order on $\mathcal{A}$.

We require $\succeq_o$ to satisfy the following axiom, relative to $\{\succeq_i\}_{i \in \mathcal{I}}$ and $\mathcal{R}$:

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Remark. We do *not* require $\boldsymbol{\alpha} \succ_o^\infty \boldsymbol{\beta}$; we simply require the social planner not to form the *opposite* asymptotic preference to that of the individuals.

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Remark. We do *not* require $\boldsymbol{\alpha} \succ_o^\infty \boldsymbol{\beta}$; we simply require the social planner not to form the *opposite* asymptotic preference to that of the individuals.

Almost-objective Pareto

Let $\mathcal{I}$ be a set of individuals.

Let o be the social observer. Let $\mathcal{J} := \mathcal{I} \sqcup \{o\}$.

For all $j \in \mathcal{J}$, let $\succeq_j$ be a preference order on $\mathcal{A}$.

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Definition. A set of utility functions $\{u_i\}_{i\in\mathcal{I}}$ satisfies *Minimal Agreement* if there exist probability measures μ_1 and μ_2 on $\mathcal{X}$ such that

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Idea. There exist two "objective lotteries" over outcomes, for which all individuals have the same strict preference.

Suppose u_o is the ex post utility function for the social preference order $\succeq_o$.

Defn. u_o is *weakly utilitarian* if there exist constants $c_i \geq 0$ for all $i \in \mathcal{I}$ and $b \in \mathbb{R}$ such that $u_o = b + \sum_{i\in\mathcal{I}} c_i \, u_i$. u_o is *utilitarian* if $c_i > 0$ for *all* $i \in \mathcal{I}$,

Under mild hypotheses, (weak utilitarianism) $\implies$ (utilitarianism).

Consequence. Our focus is on establishing *weak* utilitarianism.

We now come to our main result....

Utilitarianism and weak utilitarianism

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Main result for SEU preferences

Theorem 1. *Let $\mathcal{S}$ be a Polish space.*

Let $\mathcal{R}$ be a tame set of probability measures on $\mathcal{S}$.

For all $j \in \mathcal{J}$, let $\succeq_j$ be a preference order on $\mathcal{A}$ admitting an SEU representation with $\rho_j \in \mathcal{R}$.

Suppose $\{u_i\}_{i \in \mathcal{I}}$ satisfy Minimal Agreement. *Then:*

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III. Non-SEU decision theories

Notation and terminology

Recall.

$\mathcal{S}$ is the state space.

$\mathcal{X}$ is the outcome space.

$\mathcal{A}$ is the set of all *acts* (finitely valued measurable functions from $\mathcal{S}$ to $\mathcal{X}$)

Let $\succeq$ be a preference order on $\mathcal{A}$.

A *representation* of $\succeq$ is a function $V : \mathcal{A} \longrightarrow \mathbb{R}$ such that

$$\text{for all } \alpha, \beta \in \mathcal{A}, \quad \left(\alpha \succeq \beta\right) \quad \Longleftrightarrow \quad \left(V(\alpha) \geq V(\beta)\right).$$

Generalized Hurwicz representations.

A representation V is *generalized Hurwicz* (GH) if there is a convex set $\mathcal{P} \subset \Delta(\mathcal{S})$ and a bounded function $u : \mathcal{X} \longrightarrow \mathbb{R}$, such that for all $\alpha \in \mathcal{A}$,

$$\inf_{\rho \in \mathcal{P}} \int_{\mathcal{S}} u \circ \alpha \; \mathrm{d}\rho \quad \leq \quad V(\alpha) \quad \leq \quad \sup_{\rho \in \mathcal{P}} \int_{\mathcal{S}} u \circ \alpha \; \mathrm{d}\rho.$$

Idea. $\mathcal{P} = \{$all probability distributions over $\mathcal{S}$ that are *possible*$\}$.

Examples. SEU; Hurwicz; maximin SEU; variational prefs, etc.

- Cerreia-Vioglio, Ghirardato, Maccheroni, Marinacci & Siniscalchi, 2011: *Any monotone, Bernoullian, Archimedean* (MBA) *preference has GH repr.*
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where....

- $\mathcal{P}$ is a set of probability measures on $\mathcal{S}$, with the weak* topology;
- μ is a Borel probability measure on $\mathcal{P}$ ("second order belief");
- $u : \mathcal{X} \longrightarrow \mathbb{R}$ is a bounded measurable ("utility") function; and
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Conclusion

We have introduced a new *Almost-objective Pareto* axiom.

It is based on *asymptotic preferences* along sequences of acts that exhibit "almost objective uncertainty".

For agents with a variety of SEU or non-SEU preferences, with beliefs in a tame collection of probability measures on any Polish space, *Almost-objective Pareto* implies utilitarianism.

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Thank you.