ACCOUNTING FOR EX ANTE FAIRNESS IN EX POST WELFARE ASSESSMENT

P. Bardier & M. Fleurbaey & S. Zuber

Centre d'Économie de la Sorbonne,
CNRS & Paris School of Economics

Workshop "Social choice under risk and uncertainty", University of Warwick, May 9. 2022

CES
Centre d'Économie de la Sorbonne
UMR 8174

cnrs
dépasser les frontières

pse
PARIS SCHOOL OF ECONOMICS
ECOLE D'ECONOMIE DE PARIS

Introduction

Social choice under risk and uncertainty

- Public policies involve dealing with risk/uncertainty: unemployment, health, environment.
- Key result in social choice theory: Harsanyi's (1955) theorem.
- ***Theorem:*** *Expected utility + Pareto ex ante imply that the social criterion must be a weighted sum of individuals' expected utilities*.

Introduction

Many issues with Harsanyi's result

Harsanyi's result has serious drawbacks:

- Ex ante vs ex post equity: Diamond (1967); Broome (1991).
- Conflict between fairness and Pareto in a multidimensional framework.
  Applied to risk: Gajdos and Tallon (2002); Fleurbaey and Maniquet (2011); Fleurbaey and Zuber (2017).
- Spurious unanimity and conflicting beliefs: Mongin (1995, 1998); Gilboa, Samet, Schmeidler (2004); ; Crès, Gilboa and Vieille (2011); Nascimento (2012); Danan, Gajdos, Hill and Tallon (2014); Alon and Gayer (2016); Qu (2017)...

Introduction

Ex ante and ex post equity

Consider the following lotteries (with $p(s_1) = p(s_2) = 1/2$):

	s_1	s_2
u_1	1	0
u_2	1	0

Lottery 1

	s_1	s_2
u_1	1	0
u_2	0	1

Lottery 2

	s_1	s_2
u_1	1	1
u_2	0	0

Lottery 3

Diamond's (1967) criticism: Lottery 2 is better than Lottery 3 because equal ex ante.
Broome's (1991) criticism: Lottery 1 is better than Lottery 2 because equal ex post.

Introduction

Dealing with the ex ante and/or ex post equity issue

- Some attempts to account for ex ante equity: Epstein and Segal (1992); Grant et al. (2010); Hayashi and Lombardi (2019); Qu (2022)... $\Longrightarrow$ Drop Expected utility.
- Some attempts to account for ex post equity: Fleurbaey (2010); Grant et al. (2012); Fleurbaey and Zuber (2017), Miyagishima (2019)... $\Longrightarrow$ Drop Pareto
- Some attempts to account for both: Gajdos and Maurin (2004); Chew and Sagi (2012); Gajdos, Fleurbaey and Zuber (2015)...
  $\Longrightarrow$ We continue on this path, but working with resources rather than utility numbers (cf. Fleurbaey and Zuber, 2017)

Introduction

Contribution of the paper

- We present a framework for conditional preferences where we can account for ex ante fairness in ex post welfare evaluation.
- We derive conditions under which the distribution of expectations modify the ex post welfare judgment.
- We provide examples to deal with Diamond's equity concern in an ex post framework.
  We also discuss other fairness conditions.

Framework

Alternatives and states of the world

- Population denoted $N = \{1, \cdots, n\}$.
- $\mathbf{X} = \mathbb{R}_+^n$ denotes the set of possible sure allocations. For $\mathbf{x} \in \mathbf{X}$, $x_i \in \mathbb{R}_+$ denotes the consumption of individual i. For any $x \in \mathbb{R}_+$, $x \cdot \mathbf{1_n} \in \mathbf{X}$ denotes the egalitarian allocation (each individual receives x). $\mathbf{X^e}$ the set egalitarian allocations. We use $\mathring{\mathbf{x}} = 0 \cdot \mathbf{1_n}$.
- Set of states of the world S, with typical element $s \in S$. Σ a σ-algebra over S. P a probability measure. We assume that for any $A \in \Sigma$ and $\kappa \in [0, 1]$, there exists $A' \in \Sigma$ such that $A' \subset A$ and $P(A') = \kappa P(A)$.

Framework

Prospects

- A prospect f is a measurable function from S to $\mathbf{X}$, with $f(s)$ the allocation in state $s \in S$.
  We say that f is constant on $A \in \Sigma$ if $f(s) = f(s')$ for all $s, s' \in A$.
  In that case, $f(A)$ is the allocation induced on event A
- We assume that for each f, there exists $m \in \mathbb{N}$ and a finite partition $\mathcal{A}(f) = (A_1, \cdots, A_m)$ of S such that for each $k = 1, \cdots, m$ $A_k \in \Sigma$, $P(A_k) > 0$, and f is constant on A_k.
- For any prospects f and $g \in \mathcal{F}$, $f_A g$ denotes prospect $h \in \mathcal{F}$ such that $h(s) = f(s)$ for all $s \in (S \setminus A)$ and $h(s') = g(s')$ for all $s' \in A$.
- For $f \in \mathcal{F}$, $f_i : S \to \mathbb{R}_+$ represents i's individual prospect.
  For any measurable function $v_i : S \to \mathbb{R}$, $i \in N$, we denote $\mathbb{E}[v_i] = \int_S v_i(s) dP(s)$.
- Notation:
  - for any $\mathbf{x} \in \mathbf{X}$, $\mathbf{x}$ also denotes de sure prospect.
  - $\mathcal{F}^e$ the set of egalitarian acts.

Framework

Conditional preferences

- We assume that collective judgments are made using conditional preferences.
- Those conditional preferences may take into account what happens in other states of the world (Skiadas, 1997), hence a departure from the standard Harsanyi's model.

Conditional preferences.

For each $A \in \Sigma$, society has conditional preferences $\succsim^A$ represented by a continuous function $U(., A) : \mathcal{F} \to \mathbb{R}$, and $\succsim^S \equiv \succsim$.
Furthermore, for any finite partition $(A_1, \cdots, A_m)$ of A (with each A_i a non-null event in Σ), and any $f \in \mathcal{F}$:

$$P(A)U(f, A) = \sum_{j=1}^{m} P(A_j)U(f, A_j).$$

In addition, for each $f \in \mathcal{F}$, society has preferences $\succsim^f$ on Σ: for each $A, B \in \Sigma$ with $A \cap B = \emptyset$, $A \succsim^f B$ means that f has better consequences on A than on B. $A \succsim^f B$ if and only if $U(f, A) \geq U(f, B)$.

General results

Basic Principles

- Consequentialism for equal risk: If no issue of ex ante fairness, only consequences ex post matter.
- State independence: We can define conditional preferences that depend only on the final allocation and the whole ex ante prospect.
- Range: Conditional welfare have the same range.
- Pareto for equal risk: Pareto principle applied to egalitarian prospects (Fleurbaey, 2010).

General results

Proposition 1

Proposition 1

If the social ordering $\succsim$ satisfies Conditional preferences, Consequentialism for equal risk, State independence and Range then there exist $v : \mathbb{R}_+ \to \mathbb{R}$ and $\Xi : \mathcal{F} \times \Sigma \to \mathbb{R}_+$ such that for any $f, g \in \mathcal{F}$:

$$f \succsim g \Longleftrightarrow \sum_{A \in \mathcal{A}(f)} P(A) v\Big(\Xi(f, A_i)\Big) \geq \sum_{A' \in \mathcal{A}(g)} P(A') v\Big(\Xi(g, A'_i)\Big).$$

In addition, for any $f \in \mathcal{F}^e$ and $A \in \mathcal{A}(f)$ such that $f(A) = x \cdot \mathbf{1_n}$: $\Xi(f, A) = x$.

If furthermore $\succsim$ satisfies Pareto for equal risk, then there exist weights $(\beta_i)_{i \in N} \in \mathbb{R}^n_{++}$ such that $v(x) = \sum_{i \in N} \beta_i u_i(x)$ (up to a positive affine transformation).

General results

Separability

To obtain more specific formulas, we may want to introduce a separability property.

Separability for given expectations.

For any f, f', g, g' such that for any $i \in N$ $\mathbb{E}[f_i] = \mathbb{E}[f'_i] = \mathbb{E}[g_i] = \mathbb{E}[g'_i]$, for all $A \in \Sigma$ such that f, f', g, g' are constant on A, if there exists a subset $M \subset N$ such that (1) $f_k(A) = f'_k(A)$, $g_k(A) = g'_k(A)$ for all $k \in M$; (2) $f_l(A) = g_l(A)$, $f'_l(A) = g'_l(A)$ for all $l \in (N \setminus M)$; then $f \succsim^A g$ if and only if $f' \succsim^A g'$.

We need to complement it with a property of Consequentialism for given expectations.

General results

Proposition 2

Proposition 2

If the social ordering $\succsim$ satisfies Conditional preferences, Pareto for equal risk, Consequentialism for equal risk, State independence, Range, Consequentialism for given expectations and Separability for given expectations, then there exist weights $(\beta_i)_{i\in N} \in \mathbb{R}^n_{++}$, a function $\Phi : \mathbb{R} \times \mathbb{R}^n_+ \to \mathbb{R} \to \mathbb{R}$ and functions $\phi_i : \mathbb{R}_+ \times \mathbb{R}^n_+ \to \mathbb{R}_+$ such that, for any $f, g \in \mathcal{F}$:

$$f \succsim g \iff \sum_{A\in\mathcal{A}(f)} P(A) \left[\sum_{i\in N} \beta_i u_i \left(\Phi\left(\sum_{j\in N} \phi_j\left(f_j(A), (\mathbb{E}[f_k])_{k\in N}\right), (\mathbb{E}[f_j])_{j\in N}\right)\right)\right]$$
$$\geq \sum_{A'\in\mathcal{A}(f)} P(A') \left[\sum_{i\in N} \beta_i u_i \left(\Phi\left(\sum_{j\in N} \phi_j\left(g_j(A), (\mathbb{E}[g_k])_{k\in N}\right), (\mathbb{E}[g_j])_{j\in N}\right)\right)\right].$$

In addition, for all $x, y \in \mathbb{R}_+$:

$$\Phi\left(\sum_{i\in N} \phi_i\left(x, y \cdot \mathbf{1_n}\right), y \cdot \mathbf{1_n}\right) = x.$$

Examples and additional principles

Example 1

A first family of welfare functions is such that (with Ξ the function in Proposition 1 and f constant on A):

$$\Xi(f, A) = \varphi^{-1}\left[\frac{1}{n}\sum_{i\in N}\varphi\left(f_i\left(A\right)\right)\right]\left(\frac{\psi^{-1}\left[\frac{1}{n}\sum_{i\in N}\psi\left(1+\mathbb{E}[f_i]\right)\right]}{1+\frac{1}{n}\sum_{i\in N}\mathbb{E}[f_i]}\right),$$

with φ and ψ increasing concave functions.

The equally-distributed equivalent ex post is adjusted for ex ante inequality (ratio between generalized mean of expectations and average of expectations).

Examples and additional principles

Example 2

A second family of welfare functions is such that (with Ξ the function in Proposition 1 and f constant on A):

$$\Xi(f, A) = \varphi^{-1}\left(\frac{1}{n}\sum_{i\in N}\left[\frac{\psi(\mathbb{E}[f_i])}{\frac{1}{n}\sum_{i\in N}\psi\left(\mathbb{E}[f_i]\right)}\right]\varphi\left(f_i\left(A\right)\right)\right),$$

with φ an increasing concave function and ψ a decreasing function.

Each individual welfare function ex post is weighted by the relative ex ante prospects (people with worse prospects have more weight).

Examples and additional principles

Preference for randomization

Preference for randomization

Let $\mathbf{x} \in \mathbf{X}$ be an unequal allocation and f be a prospect such that there exists a partition $(A_1, \cdots, A_n)$ of S with $P(A_k) = 1/n$ for each k and $f(A_k)$ is a permutation of $\mathbf{x}$ so that $\mathbb{E}[f_i] = \mathbb{E}[f_j]$ for each i, j then $f \succ \mathbf{x}$.

Preference for randomization represents ex ante fairness in the sense of Diamonds (1967).

It is satisfied by Family 1 and Family 2.

Examples and additional principles

Fairness

Preference for redistribution to those with worse prospects

If $f, g \in \mathcal{F}$ and $A \in \Sigma$ are such that there exists i, j and ε with $g_i(A) + \varepsilon = f_i(A) \leq f_j(A) = g_j(A) - \varepsilon$, $\mathbb{E}[g_j] = \mathbb{E}[f_j] \geq \mathbb{E}[f_i] = \mathbb{E}[g_i]$, $\mathbb{E}[f_k] = \mathbb{E}[g_k]$ and $f_k(A) = g_k(A)$ for all $k \neq i, j$, then $f \succ_A g$.

The principle states that we want to make a transfer from the rich j to the poor i when j also has better prospects than i.

Again, this is satisfied by Family 1 and Family 2.

A stronger fairness principle is that we actually may accept to compensate ex post for worse prospects ex ante.

Compensation for worse prospects

If $g \in \mathcal{F}$ and $A \in \Sigma$ are such that there exists i, j with $\mathbb{E}[g_j] \geq \mathbb{E}[g_i]$ and $g_i = g_j$ then there exists ε such that if $f \in \mathcal{F}$ satisfies $g_i(A) + \varepsilon = f_i(A)$, $f_j(A) = g_j(A) - \varepsilon$, $\mathbb{E}[f_i] = \mathbb{E}[g_i]$, $\mathbb{E}[f_j] = \mathbb{E}[g_j]$, $\mathbb{E}[f_k] = \mathbb{E}[g_k]$ and $f_k(A) = g_k(A)$ for all $k \neq i, j$, then $f \succ_A g$.

Only Family 2 satisfies this principle. Indeed, Family 1 is symmetric given that expectations are kept constant.

Conclusion

Summary

- Exploration of conditional social preferences to account for ex ante fairness.
- We have adopted the Skiadas model but we try to keep the departure from consequentialism to a minimum: only the distribution of ex ante prospects .
- We characterize very large families and provide simple natural examples.
- We formulate principles of fairness ex ante and ex post.

Conclusion

Future work

- Work in progress!!!
- We would like to have more specific characterizations but for the moment our fairness principles are not enough to restrict attention to simple families.
- Note that we could also have more general families: not evaluating prospects through the expectation only!

Consequentialism for equal risk

For all $f, g \in \mathcal{F}^e$, for all A such that f and g are constant on A, if $f(A) \geq g(A)$ then $f \succsim^A g$.

↩

State independence

For all $f \in \mathcal{F}$, A, $B \in \Sigma$ with $A \cap B = \emptyset$, if $f(s) = f(s')$ for all $s \in A$ and $s' \in B$ then $A \sim^f B$.

↩

Range

For any $f, g \in \mathcal{F}$ and any $A \in \mathcal{A}(f)$, there exist $x \in \mathbb{R}_{++}$ such that $\mathring{\mathbf{x}}_A(x \cdot \mathbf{1_n}) \succsim^A f \succsim^A \mathring{\mathbf{x}}$.

$\hookleftarrow$

Appendix

Pareto for equal risk

Pareto for equal risk

There exists functions $u_i : \mathbb{R}_+ \to \mathbb{R}$ such that for all $f, g \in \mathcal{F}^e$, if $\mathbb{E}[u_i \circ f_i] \geq \mathbb{E}[u_i \circ g_i]$ for all $i \in N$, then $f \succsim g$. If, furthermore, $\mathbb{E}[u_j \circ f_j] > \mathbb{E}[u_j \circ g_j]$ for some $j \in N$, then $f \succ g$.

↩

Consequentialism for given expectations

For any $f, g \in \mathcal{F}$ and $A \in \Sigma$ such that f and g are constant on A, if $\mathbb{E}[f_i] = \mathbb{E}[g_i]$ for all $i \in N$, then $f(A) \geq g(A)$ implies $f \succsim^A g$. If furthermore $f(A) \neq g(A)$ then $f \succ^A g$.

↩